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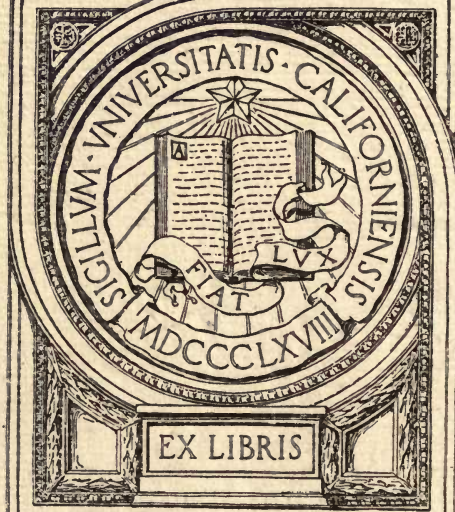

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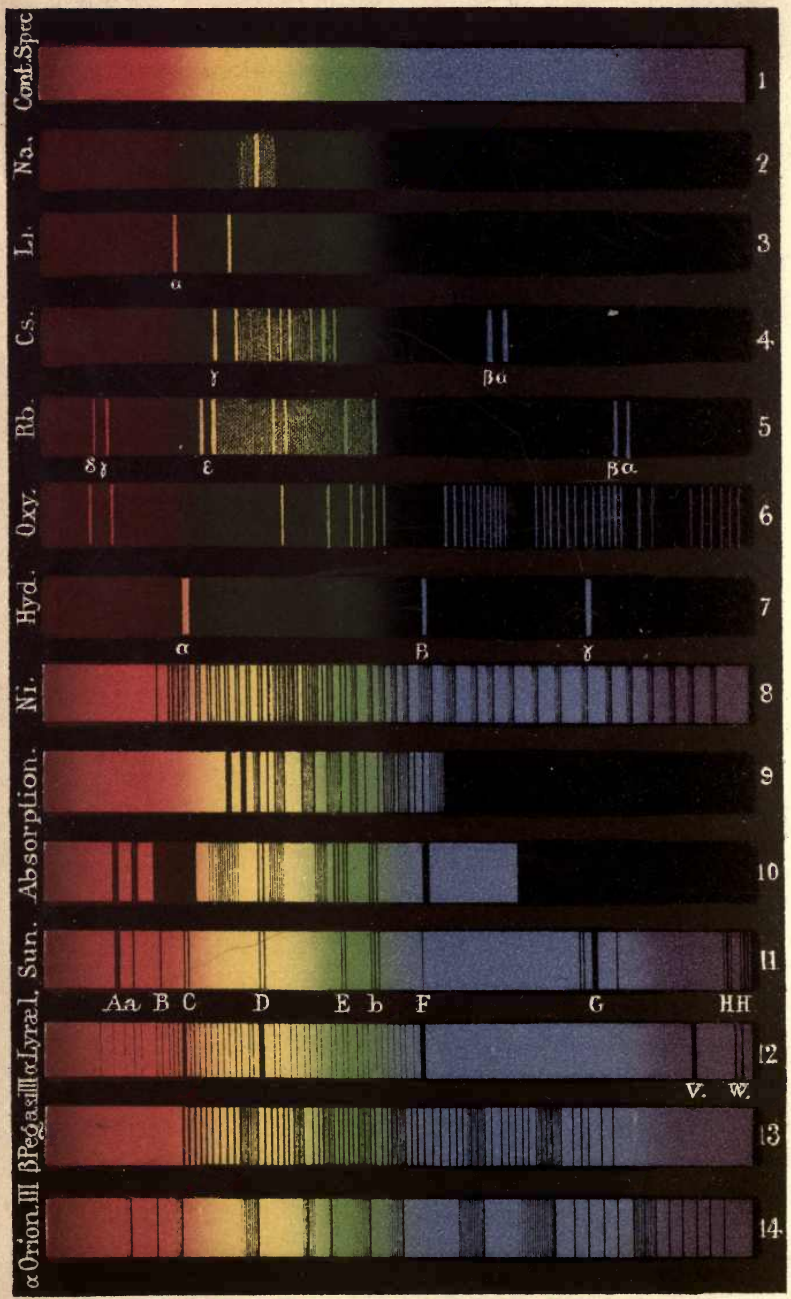
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ELEMENTARY TREATISE

ON

PHYSICS

EXPERIMENTAL AND APPLIED

FOR THE USE OF COLLEGES AND SCHOOLS

TRANSLATED AND EDITED FROM

GANOT'S ÉLÉMENTS DE PHYSIQUE

(*with the Author's sanction*)

BY

E. ATKINSON, PH.D., F.C.S.

LATE PROFESSOR OF EXPERIMENTAL SCIENCE IN THE STAFF COLLEGE

FOURTEENTH EDITION, REVISED AND ENLARGED

ILLUSTRATED by 9 COLOURED PLATES and MAPS and 1028 WOODCUTS

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TO

THE FOURTEENTH EDITION

In the present edition the additions made have increased by thirty-two pages the size of the work as it stood in the last edition. The new matter contains also forty-one additional illustrations.

I have to express my acknowledgments to Dr. G. Lindsay Johnson for his kindness in revising the chapter on the Eye. I am also indebted to Mr. D. K. Morris for having kindly furnished me with a list of errata.

The continued favour with which the work has been received, as a Text-book for Colleges and Schools, and also as a book of reference for the general reader, renders any apology for omissions perhaps unnecessary; it may, however, be as well once more to point out that the book is intended to be a general Elementary Treatise on Physics, and that, while it accordingly aims at giving an account of the most important facts and general laws of all branches of Physics, an attempt to treat completely and exhaustively of any one branch would both be inconsistent with the general plan of the book and impossible within the available space.

E. ATKINSON.

Portesbery Hill, Camberley: *Dec.* 1892.

148386

EXTRACT FROM ADVERTISEMENT TO THE TWELFTH EDITION

SOME alterations have been made in Book I.: in making these I have availed myself of an introductory chapter which Prof. Nipher, of the University of Missouri, prepared for the use of his classes, and which he kindly placed at my disposal.

E. A.

TRANSLATOR'S PREFACE TO FIRST EDITION

THE *Éléments de Physique* of Professor GANOT, of which the present work is a translation, has acquired a high reputation as an Introduction to Physical Science. In France it has passed through Nine large editions in little more than as many years, and it has been translated into German and Spanish.

This reputation it doubtless owes to the clearness and conciseness with which the principal physical laws and phenomena are explained, to its methodical arrangement, and to the excellence of its illustrations. In undertaking a translation, I was influenced by the favourable opinion which a previous use of it in teaching had enabled me to form.

I found that its principal defect consisted in its too close adaptation to the French systems of instruction; and accordingly, my chief labour, beyond that of mere translation, has been expended in making such alterations and additions as might render it more useful to the English student.

I have retained throughout the use of the Centigrade thermometer, and in some cases have expressed the smaller linear measures on the metrical system. These systems are now everywhere gaining ground, and an apology is scarcely needed for an innovation which may help to familiarise the English student with their use in the perusal of the larger and more complete works on Physical Science to which this work may serve as an introduction.

E. A.

ROYAL MILITARY COLLEGE, SANDHURST:
1863.

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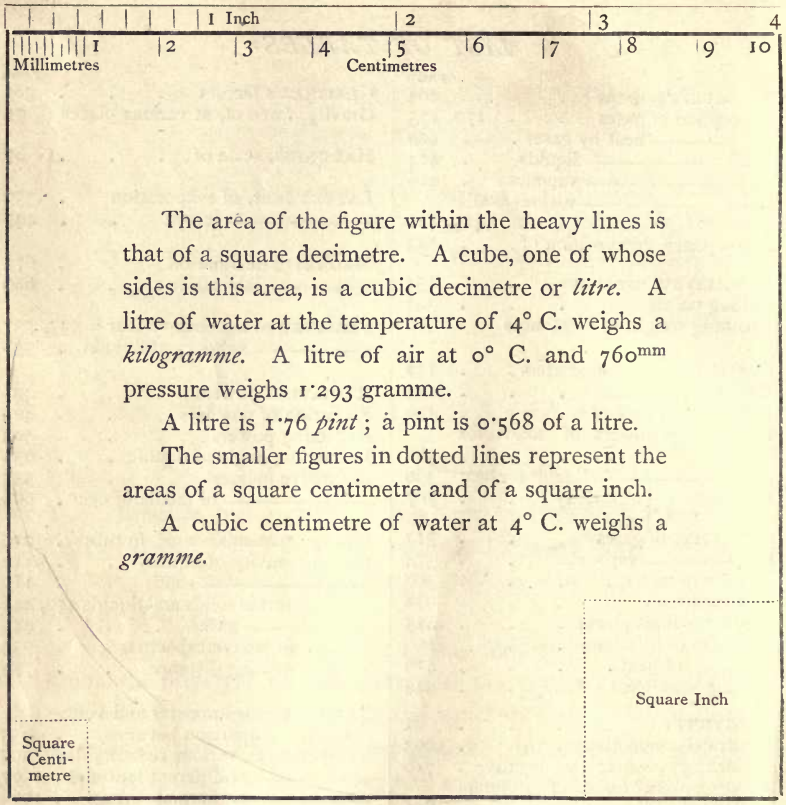

	Inches	Feet
Millimetre	0·03937	0·003281
Centimetre	0·39371	0·032819
Decimetre	3·93708	0·328090
Metre	39·37079	3·280899
Kilometre	39370·70000	3280·899167

A Hectare or 10,000 square metres is equal to 2·47114 acres, each of which is 43,560 square feet. A kilometre is 0·6214 of a statute mile. A statute mile is 1·609 kilometre. A knot (in telegraphy) is 2,029 yards or 1·1528 statute mile.

Measures of Capacity

	Cubic Inches	Cubic Feet 1,728 c. in. = 1 c. ft.
Cubic centimetre or millimetre	0·06103	0·000035
Litre or cubic decimetre	61·02705	0·035317
Kilolitre or cubic metre	61,027·05152	35·316581

Measures of Weight

	English Grains	Avoirdupois Pounds of 7,000 grains
Milligramme	0·01543	0·0000022
Gramme	15·43235	0·0022046
Kilogramme	15,432·34880	2·2046213

1 grain = 0·064799 gramme; 1 pound avoirdupois is 0·453593 kilogramme.

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ELEMENTARY TREATISE

ON

PHYSICS

BOOK I

ON MATTER, FORCE, AND MOTION

CHAPTER I

GENERAL PRINCIPLES

1. **Object of Physics.**—The object of *Physics* is the study of the phenomena presented to us by bodies. It should, however, be added, that changes in the nature of the body itself, such as the decomposition of one body into others, are phenomena whose study forms the more immediate object of *chemistry*.

2. **Matter.**—That which possesses the properties whose existence is revealed to us by our senses, we call *matter* or *substance*.

All substances at present known to us may be considered as chemical combinations of sixty-seven *elementary* or *simple* substances. This number, however, may hereafter be diminished or increased by the discovery of some more powerful means of chemical analysis than we at present possess.

3. **Atoms, molecules.**—From various properties of bodies, we conclude that the matter of which they are formed is not perfectly continuous, but consists of an aggregate of an immense number of exceedingly small portions or *atoms* of matter. These atoms cannot be divided physically; they are retained side by side, without touching each other, being separated by distances which are great in comparison with their supposed dimensions.

A group of two or more atoms forms a *molecule*, so that a body may be considered as an aggregate of very small molecules, and these again as aggregates of still smaller atoms. The smallest masses of matter we ever obtain artificially are *particles*, and not molecules or atoms. Molecules retain their position in virtue of the action of certain forces called *molecular forces*.

From considerations based upon various physical phenomena Sir W. Thomson has calculated that in ordinary solids and liquids the average

distance between contiguous molecules is less than the one hundred-millionth but greater than the one two-thousand-millionth of a centimetre.

To form an idea of the degree of the size of the molecules Sir W. Thomson gives this illustration :—'Imagine a drop of rain, or a glass sphere the size of a pea, magnified to the size of the earth, the molecules in it being increased in the same proportion. The structure of the mass would then be coarser than that of a heap of fine shot, but probably not so coarse as that of a heap of cricket-balls.'

The number of molecules of gas in a cubic centimetre of air is calculated at twenty-one trillions.

By dissolving in alcohol a known weight of fuchsine, and diluting the liquid, it was observed that a solution containing not more than 0·00000002, or as it may conveniently be written $0{\cdot}0_72$, of a gramme in one cubic centimetre had still a distinct colour; that is, that a weight of not more than the $\frac{1}{50}$-millionth of a gramme can be perceived by the naked eye. As the molecular weight of this substance is 337 times that of hydrogen, it follows that the weight of an atom of hydrogen cannot be greater than the one 20,000-millionth of a gramme. Wiener determined the thickness of a layer of silver as not exceeding 0·2 $\mu\mu$, the expression $\mu\mu$ standing for the millionth of a millimetre. As hydrogen, the smallest of all molecules, has a diameter of at most 0·1 to 0·2 $\mu\mu$, this must represent a line of molecules.

Loschmidt gives the diameter of the molecules of hydrogen at $0{\cdot}0_74$ of a centimetre ; and according to Mousson and Quincke the diameter of the sphere within which one molecule can act upon an adjacent one, or what is called the *double radius of molecular action*, is between the $0{\cdot}0_43$ and $0{\cdot}0_44$ of a millimetre, and is therefore from 5 to 10 times less than the wave-length of light (637).

4. **Molecular state of bodies.**—With respect to the molecules of bodies three different stages of aggregation present themselves.

First, the solid state, as observed in wood, stone, metals, &c., at the ordinary temperature. The distinctive character of this state is, that the relative positions of the molecules of the bodies is fixed and cannot be changed without the expenditure of more or less force. Solid bodies tend, therefore, to retain whatever form may have been given to them by nature or by art.

Secondly, the liquid state, as observed in water, alcohol, oil, &c. Here the relative position of the molecules is no longer fixed, the molecules glide past each other with the greatest ease, and the body assumes with readiness the form of any vessel in which it may be placed.

Thirdly, the gaseous state, as in air and in hydrogen. In gases the mobility of the molecules is still greater than in liquids ; but the distinctive character of a gas is its incessant struggle to occupy a greater space, in consequence of which a gas has neither an independent form nor an independent volume, for this depends upon the pressure to which it is subject.

The general term *fluid* is applied to both liquids and gases.

Most simple bodies, and many compound ones, may be made to pass successively through all the three states. Water presents the most familiar example of this. Sulphur, iodine, mercury, phosphorus, and zinc are other instances.

5. **Physical phenomena, laws, and theories.**—Every change which can happen to a body, actual alteration of its chemical constitution being excepted, may be regarded as a *physical phenomenon.* The fall of a stone, the vibration of a string, and the sound which accompanies it, the attraction of light particles by a rod of sealing-wax which has been rubbed by flannel, the rippling of the surface of a lake, and the freezing of water, are examples of such phenomena.

A *physical law* is the constant relation which exists between any phenomenon and its cause. As an example, we have the phenomenon of the diminution of the volume of a gas by the application of pressure; the corresponding law has been discovered, and is expressed by saying that *the volume of a gas is inversely proportional to the pressure.*

In order to explain the cause of whole classes of phenomena, suppositions, or *hypotheses*, are made use of. The utility and probability of a hypothesis or theory are the greater the simpler it is, and the more varied and numerous are the phenomena which are *explained* by it; that is to say, are brought into regular causal connection among themselves and with other natural phenomena. Thus the adoption of the undulatory theory of light is justified by the simple and unconstrained explanation it gives of all luminous phenomena, and by the connection it reveals with the phenomena of heat.

6. **Physical agents.**—In our attempts to ascend from a phenomenon to its cause, we assume the existence of *physical agents*, or *natural forces* acting upon matter; as examples of such we have *gravitation*, *heat*, *light*, *magnetism*, and *electricity*.

Since these physical agents are disclosed to us only by their effects, their intimate nature is completely unknown. In the present state of science, we cannot say whether they are properties inherent in matter, or whether they result from movements impressed on the mass of subtile and imponderable forms of matter diffused through the universe. The latter hypothesis is, however, generally admitted. This being so, it may be further asked, are there several distinct forms of imponderable matter, or are they in reality but one and the same? As the physical sciences extend their limits, the opinion tends to prevail that there is a subtile, imponderable, and eminently elastic fluid called the *ether* distributed through the entire universe; it pervades the mass of all bodies, the densest and most opaque, as well as the lightest or the most transparent. It is also considered that the ultimate particles of which matter is made up are capable of definite motions varying in character and velocity, and which can be communicated to the ether. A motion of a particular kind communicated to the ether can give rise to the phenomenon of heat; a motion of the same kind, but of greater frequency, produces light; and it may be that a motion different in form or in character is the cause of electricity. Not merely do the atoms of bodies communicate motion to the atoms of the ether, but this latter can impart it to the former. Thus the atoms of bodies are at once the sources and the recipients of the motion. All physical phenomena, referred thus to a single cause, are but transformations of motion.

CHAPTER II

GENERAL PROPERTIES OF BODIES

7. **Different kinds of properties.**—By the term *properties*, as applied to bodies, we understand the different ways in which bodies present themselves to our senses. We distinguish *general* from *specific* properties. The former are shared by all bodies, and amongst them the most important are *impenetrability*, *extension*, *divisibility*, *porosity*, *compressibility*, *elasticity*, *mobility*, and *inertia*.

Specific properties are such as are observed in certain bodies only, or in certain states of these bodies; such are *solidity*, *fluidity*, *tenacity*, *ductility*, *malleability*, *hardness*, *transparency*, *colour*, &c.

With respect to the above general properties, *impenetrability* and *extension* might, perhaps, be more aptly termed essential attributes of matter, since they suffice to define it; while *divisibility*, *porosity*, *compressibility*, and *elasticity* do not apply to atoms, but only to bodies or aggregates of atoms (3).

8. **Impenetrability.**—*Impenetrability* is the property in virtue of which two portions of matter cannot at the same time occupy the same portion of space. Thus when a stone is placed in a vessel of water the volume of the water rises by an amount depending on the volume of the stone; this method, indeed, is used to determine the bulk of irregularly shaped bodies by means of graduated measures.

Strictly speaking, this property applies only to the atoms of a body. In many phenomena bodies appear to penetrate each other; thus, the volume of a compound body is always less than the sum of the volumes of its constituents; for instance, the volume of a mixture of water and sulphuric acid, or of water and alcohol, is less than the sum of the volumes before mixture. In all these cases, however, the penetration is merely apparent, and arises from the fact that in every body there are interstices, or spaces unoccupied by matter (13).

9. **Extension.**—*Extension* or *magnitude* is the property in virtue of which every body occupies a limited portion of space.

Many instruments have been invented for measuring linear extension or lengths with great precision. Two of these, the vernier and micrometer screw, on account of their great utility deserve to be here mentioned.

10. **Vernier.**—The *vernier* forms a necessary part of all instruments where lengths or angles have to be estimated with precision; it derives its name from its inventor, a French mathematician, who died in 1637, and consists essentially of a short graduated scale, *a b* (fig. 1), which is made to

slide along a fixed scale, AB, so that the graduations of both may be compared with each other. The fixed scale AB, being divided into equal parts the whole length of the vernier, *ab*, may be taken equal to nine of those parts, and is itself divided into ten equal parts. Each of the parts of the vernier, *ab*, will then be less than a part of the scale by one-tenth of the latter.

This being granted, in order to measure the length of any object, *mn*, let us suppose that the latter, when placed as in the figure, has a length greater than four but less than five parts of the fixed scale. In order to determine by what fraction of a part *mn* exceeds four, one of the ends, *a*, of the vernier, *ab*, is placed in contact with one extremity of the object, *mn*, and the division on the vernier is sought which coincides with a division on the scale, AB. In the figure this coincidence occurs at the eighth division of the vernier, counting from the end, *n*, and indicates that the fraction to be measured is equal to $\frac{8}{10}$ths of a part of the scale, AB. In fact, each of the parts of the vernier being less than a part of the scale by $\frac{1}{10}$th of the latter, it is clear that on proceeding towards the left from the point of coincidence the divisions of the vernier are respectively one, two, three, &c. tenths,

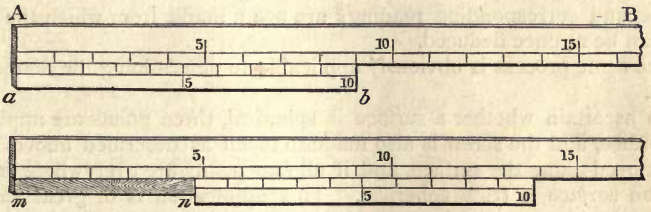

Fig. 1.

behind the divisions of the scale; so that the end, *n*, of the object (that is to say, the eighth division of the vernier) is $\frac{8}{10}$ths behind the division 4 on the scale; in other words, the length of *mn* is equal to $4\frac{8}{10}$ths of the parts into which the scale AB is divided. Consequently if the scale AB were divided into inches the length of *mn* would be $4\frac{8}{10} = 4\frac{4}{5}$ inches. The divisions on the scale remaining the same, it would be necessary to increase the length of the vernier in order to measure the length *mn* more accurately. For instance, if the length of the vernier were equal to nineteen of the parts on the scale, and this length were divided into twenty equal parts, the length *mn* could be determined to the twentieth of a part on the scale, and so on. In instruments like the theodolite, intended for measuring angles, the scale and vernier have a circular form, and the latter usually carries a magnifier in order to determine with greater precision the coincident divisions of vernier and scale.

11. **Micrometer screw.**—Another useful little instrument for measuring small lengths with precision is the *micrometer screw*. It is used under various forms, but the principle is the same in all, and may be conveniently illustrated by reference to the *spherometer*. This consists of an accurately turned screw with a blunt point which works in a companion supported on three steel points (fig. 2). To one of these is fixed a vertical graduated scale, each division of which is equal to the distance between two threads of the screw,

This distance may be accurately determined by measuring a given length of the screw by compasses, and counting the number of the threads in this length. A milled head attached to the screw is graduated at the periphery into any given number of parts, say 500. Suppose now the distance between the threads is 1 millimetre, when the head has made a complete turn it will have risen or sunk through one millimetre, and so on in proportion for any multiple or fraction of a turn.

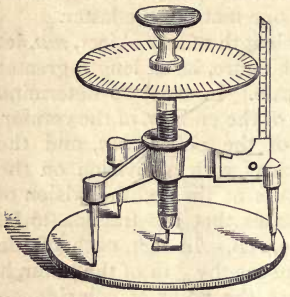

Fig. 2.

In order to determine the thickness of a piece of glass, for instance, the apparatus is placed on a perfectly plane polished surface, and the point of the screw is brought in contact with the glass. The division on the vertical scale immediately above the limb, and that on the limb, are read off. After removing the glass plate the point is brought in contact with the plane surface, and corresponding readings are again made, from which the thickness can be at once deduced.

The same process is obviously applicable to determining the diameter of a wire.

To ascertain whether a surface is spherical, three points are applied to the surface, and the screw is also made to touch as described above. It is then moved along the surface, and if all four points are everywhere in contact the surface is truly spherical. This application is of great value in ascertaining the exact curvature of lenses.

The diameter of a sphere may also be measured by its means; for it can be shown by a simple geometrical construction that the distance of the movable point from the plane of the fixed points, multiplied by the diameter of the sphere, is equal to the square of the distance of the movable point from one of the fixed points. If h is the distance of the movable point from the plane of the fixed points, c the distance of the movable point from the fixed point when in the same plane, and which is known once for all, and d the diameter of the circle, then it can be shown by a simple geometrical construction that $d = \frac{c^2}{h} + h$.

12. **Divisibility.**—*Divisibility* is the property in virtue of which a body may be separated into distinct parts.

Numerous examples may be cited of the extreme divisibility of matter (3). Leslie stated that the tenth part of a grain of musk will continue for years to fill a room with its odoriferous particles, and at the end of that time will scarcely be diminished in weight. Blood is composed of red, flattened globules, floating in a colourless liquid called *serum*. In man the diameter of one of these globules is less than the 3,500th part of an inch, and the drop of blood which might be suspended from the point of a needle would contain about a million of globules.

Again, the microscope has disclosed to us the existence of insects smaller even than these particles of blood; the struggle for existence reaches even

to these little creatures, for they devour still smaller ones. If blood runs in the veins of these devoured ones, how infinitesimal must be the magnitude of its component globules !

Although experiment fails to determine whether there be a limit to the divisibility of matter, many facts in chemistry, such as the invariability in the relative weights of the elements which combine with each other, would lead us to believe that such a limit does exist. It is on this account that bodies are conceived to be composed of extremely minute and indivisible parts called *atoms* (3).

13. **Porosity.**—*Porosity* is the quality in virtue of which interstices or *pores* exist between the molecules of a body.

Two kinds of pores may be distinguished : *physical pores*, where the interstices are so small that the surrounding molecules remain within the sphere of each other's attracting or repelling forces ; and *sensible pores*, or actual cavities across which these molecular forces cannot act. The contractions and expansions resulting from variations of temperature are due to the existence of physical pores, whilst in the organic world the sensible pores are the seat of the phenomena of exhalation and absorption.

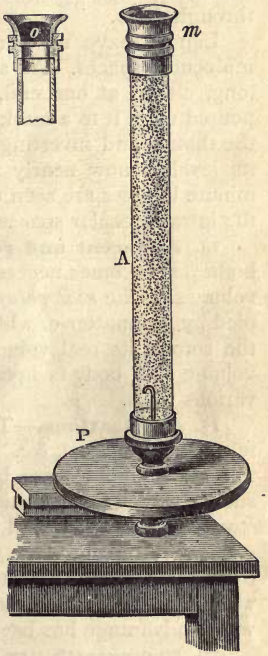

Fig. 3.

In wood, sponge, and a great number of stones—for instance, pumice stone—the sensible pores are apparent ; physical pores, or intramolecular spaces, never are. Yet, since the volume of every body may be diminished, we conclude that all possess physical pores.

The existence of sensible pores in leather or wood may be shown by the following experiment :—A long glass tube, A (fig. 3), is provided with a brass cup at the top, and a brass foot made to screw on to the plate of an air-pump. The bottom of the cup consists of a thick piece of leather. After pouring mercury into the cup so as entirely to cover the leather, the air-pump is put in action, and a partial vacuum produced within the tube. By so doing a shower of mercury is at once produced within the tube, for the atmospheric pressure on the mercury forces that liquid through the pores of the leather. In the same manner water or mercury may be forced through the pores of wood by replacing the leather in the above experiment by a disc of wood cut perpendicular to the fibres.

When a piece of chalk is thrown into water, air-bubbles at once rise to the surface, in consequence of the air in the pores of the chalk being expelled by the water. The chalk will be found to be heavier after immersion than it was before, and knowing its volume, the volume of its pores may be easily determined from the increase of its weight.

The porosity of agate, flint, and marble is evident from the fact that they are penetrated by liquids such as oil; on this, indeed, depends the artificial coloration of these minerals.

The porosity of gold was demonstrated by the celebrated Florentine experiment made in 1661. Some academicians at Florence, wishing to try whether water was compressible, filled a thin globe of gold with that liquid, and, after closing the orifice hermetically, they exposed the globe to pressure with a view of altering its form, knowing that any alteration in form must be accompanied by a diminution in volume. The consequence was, that the water forced its way through the pores of the gold, and stood on the outside of the globe like dew. More than twenty years previously the same fact was demonstrated by Francis Bacon by means of a leaden sphere; the experiment has since been repeated with globes of other metals, and similar results obtained. At a red heat both platinum and iron allow gases to diffuse through them.

Liquids have no pores in the ordinary sense of the word, but intramolecular spaces. As an illustration of this, a glass tube about a metre long, closed at one end, is half filled with water, and then pure alcohol poured upon it to a mark near the top; on then closing the open end with the thumb and inverting the tube several times the mixture shrinks so that its level is now nearly an inch below the mark; at the same time very minute bubbles are seen to rise, owing to the water having penetrated into the intramolecular spaces of the alcohol and expelled the air present.

14. **Apparent and real volumes.**—In consequence of the porosity of bodies, it becomes necessary to distinguish between their real and apparent volumes. The *real volume* of a body is the portion of space actually occupied by the matter of which the body is composed; its *apparent volume* is the sum of its real volume and the total volume of its pores. The real volume of a body is invariable, but its apparent volume can be altered in various ways.

15. **Applications.**—The property of porosity is utilised in filters of paper, felt, stone, charcoal, &c. The pores of these substances are sufficiently large to allow liquids to pass, but small enough to arrest the passage of any substances which these liquids may hold in suspension. Again, large blocks of stone are often detached in quarries by introducing wedges of dry wood into grooves cut in the rock. These wedges being moistened, water penetrates their pores, and causes them to swell with considerable force. Dry cords, when moistened, increase in diameter and diminish in length—a property of which advantage has been taken in order to raise great weights.

16. **Compressibility.**—*Compressibility* is the property in virtue of which the volume of a body may be diminished by pressure. This property is at once a consequence and a proof of porosity.

Bodies differ greatly with respect to compressibility. The most compressible bodies are gases; by sufficient pressure they may be made to occupy ten, twenty, or even some hundred times less space than they do under ordinary circumstances. In most cases, however, there is a limit beyond which, when the pressure is increased, they become liquids.

The compressibility of solids is much less than that of gases, and is found in all degrees. Cloths, paper, cork, woods, are amongst the most com-

pressible. Metals are so also to a great extent, as is proved by the process of coining, in which the metal receives the impression from the die. There is, in most cases, a limit beyond which, when the pressure is increased, bodies are fractured or reduced to powder.

The compressibility of liquids is so small as to have remained for a long time undetected: it may, however, be proved by experiment, as will be seen in the chapter on Hydrostatics.

17. **Elasticity.**—*Elasticity* is the property owing to which bodies resume their original form or volume, when the force which altered that form or volume ceases to act. Elasticity may be developed in bodies by pressure, by traction or *pulling*, flexion or *bending*, and by torsion or *twisting*. In treating of the general properties of bodies, the elasticity developed by pressure alone requires consideration; the other kinds of elasticity, being peculiar to solid bodies, will be considered amongst their specific properties (arts. 89, 90, 91).

Gases and liquids are perfectly elastic; in other words, after undergoing a change in volume they regain exactly their original volume when the pressure becomes what it originally was. Solid bodies present different degrees of elasticity, though none present the property in the same perfection as liquids and gases, and in all it varies according to the time during which the body has been exposed to pressure. Caoutchouc, ivory, glass, and marble possess considerable elasticity; lead, clay, and fats scarcely any.

There is a limit to the elasticity of solids, beyond which they either break or are incapable of regaining their original form and volume. This is called the *limit of elasticity*; within this limit all substances are perfectly elastic. In sprains, for instance, the elasticity of the tendons has been exceeded. In gases and liquids, on the contrary, no such limit can be reached; they always regain their original volume when the original pressure is restored (152).

If a ball of ivory, glass, or marble be allowed to fall upon a slab of polished marble, which has been previously slightly smeared with oil, it will rebound and rise to a height nearly equal to that from which it fell. On afterwards examining the ball a circular blot of oil will be found upon it, more or less extensive according to the height of the fall. From this we conclude that at the moment of the shock the ball was flattened, and that its rebound was caused by the effort to regain its original form.

18. **Mobility, motion, rest.**—*Mobility* is the property in virtue of which the position of a body in space may be changed.

Motion and rest may be either relative or absolute. By the *relative motion* or *rest* of a body we mean its change or permanence of position with respect to surrounding bodies; by its *absolute motion* or *rest* we mean the change of permanence of its position with respect to ideal fixed points in space.

Thus a passenger in a railway carriage may be in a state of relative rest with respect to the train in which he travels, but he is in a state of relative motion with respect to the objects, such as trees, houses, &c., past which the train rushes. These houses again enjoy merely a state of relative rest, for the earth itself which bears them is in a state of incessant relative motion with respect to the celestial bodies of our solar system, inasmuch as it moves at the rate of more than eighteen miles in a second. In short, absolute

motion and rest are unknown to us ; in nature, relative motion and rest are alone presented to our observation.

19. **Inertia.**—*Inertia* is a purely negative though universal property of matter (26) ; it is the property that matter cannot of itself change its own state of motion or of rest. If a body is at rest, it remains so until some force acts upon it ; if it is in motion, this motion can only be changed by the application of some force.

This property of inertia is what is expressed by Newton's first law of motion.

A body, when unsupported in mid-air, does not fall to the earth in virtue of any inherent property, but because it is acted upon by the force of gravity. A billiard ball gently pushed does not move more and more slowly, and finally stop, because it has any preference for a state of rest, but because its motion is impeded by the friction on the cloth on which it rolls, and by the resistance of the air. If all impeding causes were withdrawn, a body once in motion would continue to move for ever in a straight line with unchanging velocity.

20. **Illustrations.**—Numerous phenomena may be explained by the inertia of matter. For instance, before leaping a ditch we run towards it, in order that the motion of our bodies at the moment of leaping may add itself to the muscular effort then made.

On descending carelessly from a carriage in motion, the upper part of the body retains its motion, whilst the feet are prevented from doing so by friction against the ground ; the consequence is we fall towards the moving carriage. A rider falls over the head of a horse if it suddenly stops. In striking the handle of a hammer against the ground the handle suddenly stops, but the head, striving to continue its motion, fixes itself more firmly on the handle.

By the property of inertia may also be explained the following experiments :—Let a card be placed upon a tumbler, and a shilling on the card ; if the edge of the card be smartly flicked with the finger the card is driven away and the coin falls into the tumbler. A gentle push with the finger will move a door on its hinges ; but if a pistol bullet be fired against the door it perforates the door without moving it. So, too, a pistol shot fired through a window-pane produces a sharp round hole, while a less violent shock will smash the pane. A clay tobacco pipe, which is suspended by two vertical hairs, may be cut in two by a powerful stroke with a sharp sword without breaking the hairs.

A string which gently applied will raise a weight, snaps at once when a sudden pull is exerted. Substances which explode with great rapidity, such as fulminating mercury, chloride of nitrogen, cannot be used with fire-arms, because there is not sufficient time to transfer the motion to the projectiles, and hence the weapons are burst.

The terrible accidents on our railways are chiefly due to inertia. When the motion of the engine is suddenly arrested the carriages strive to continue the motion they had acquired, and in doing so are shattered against each other. Hammers, pestles, stampers are applications of inertia. So are also the enormous iron fly-wheels, by which the motion of steam-engines is regulated.

CHAPTER III

ON FORCE, EQUILIBRIUM, AND MOTION

21. **Measure of time.**—To obtain a proper measure of force it is necessary, as a preliminary, to define certain conceptions which are presupposed in that measure; and, in the first place, it is necessary to define the unit of time. Whenever a *second* is spoken of without qualification, it is understood to be a second of *mean solar time*. The exact length of this unit is fixed by the following considerations. The instant when the sun's centre is on an observer's meridian—in other words, the instant of the *transit* of the sun's centre—can be determined with exactitude, and thus the interval which elapses between two successive transits also admits of exact determination, and is called an *apparent* day. The length of this interval differs slightly from day to day, and therefore does not serve as a convenient measure of time. Its *average* length is not open to this objection, and therefore serves as the required measure, and is called a *mean solar day*. The short hand of a common clock would go exactly twice round the face in a mean solar day if it went perfectly. The mean solar day consists of 24 equal parts called *hours*, these of 60 equal parts called *minutes*, and these again of 60 equal parts called *seconds*. Consequently, the second is the 86,400th part of a mean solar day, and is the generally received unit of time.

22. **Measure of space.**—Space may be either *length* or *distance*, which is space of one dimension; *area*, which is space of two dimensions; or *volume*, which is space of three dimensions. In England the standard of length is the British Imperial Yard, which is the distance between two fixed points on a certain metal rod, kept in the Tower of London, when the temperature of the whole rod is 60° F. = 15°·5 C. It is, however, usual to employ as a unit, a *foot*, which is the third part of a yard. In France the standard of length is the *metre*; this is approximately equal to the ten-millionth part of a quadrant of the earth's meridian, that is of the arc from the Equator to the North Pole; it is practically fixed by the distance between two marks on a certain standard rod. The standard metre, adopted by an International Committee for weights and measures, is constructed of an alloy of 90 per cent. platinum and 10 per cent. iridium, which is characterised by great hardness, and unalterability. Its length is somewhat over a metre, and its cross section is represented in its natural size in figure 4. This shape has the advantage of giving the greatest rigidity and of soon acquiring the temperature of the surrounding medium. The exact length of the metre is

Fig 4.

marked by two fine lines on the surface. The relation between these standards is as follows :

1 yard = 0·914401 metre.
1 metre = 1·093612 yard.

The unit of length having been fixed, the units of area and volume are connected with it thus : the *unit of area* is the area of a square, one side of which is the unit of length. The *unit of volume* is the volume of a cube, one edge of which is the unit of length. These units in the case of English measures are the square yard (or foot) and the cubic yard (or foot) respectively; in the case of French measures, the square metre and cubic metre respectively. The length of the seconds pendulum, in lat. 45°, which is about that of Milan, is 0·9935 m., and thus only differs from a metre by 6·5 millimetres.

23. **Measure of mass.**—The mass of a body is that which remains unchanged in all the transformations which it may undergo. Two bodies are said to have equal masses when, if placed in a perfect balance *in vacuo*, they counterpoise each other. Suppose we take lumps of any substance, lead, butter, wood, stone, &c., and suppose that any one of them when placed on the one pan of a balance will exactly counterpoise any other of them when placed on the opposite pan—the balance being perfect and the weighing performed *in vacuo* ; this being the case, these lumps are said to have equal masses.

The British unit of mass is the standard pound (avoirdupois), which is a certain piece of platinum kept in the Exchequer Office in London. This unit having been fixed, the mass of a given substance is expressed as a multiple or submultiple of the unit.

It need scarcely be mentioned that many distances are ascertained and expressed in yards which it would be physically impossible to measure directly by a yard measure. In like manner the masses of bodies are frequently ascertained and expressed numerically which could not be placed in a balance and subjected to direct weighing.

24. **Density and relative density.**—If we consider any body or portion of matter, and if we conceive it to be divided into any number of parts having equal volumes, then, if the masses of these parts are equal, in whatever way the division be conceived as taking place, that body is one of *uniform density*. The *density* of such a body is the mass of the *unit of volume*. Consequently, if M denote the mass, V the volume, and D the density of the body, we have

$$M = VD.$$

If now we have an equal volume V of any second substance whose mass is M′ and density D′, we shall have

$$M' = VD'.$$

Consequently, $D : D' :: M : M'$; that is, the densities of substances are in the same ratio as the masses of equal volumes of those substances. If now we take the density of distilled water at 4° C. to be unity, the relative density of any other substance is the ratio which the mass of any given volume of that substance at that temperature bears to the mass of an equal volume of water. Thus it is found that the mass of any volume of platinum

is 22·069 times that of an equal volume of water, consequently the relative density of platinum is 22·069.

The relative density of a substance is generally called its *specific gravity*. Methods of determining it are given in Book III.

In the table below the densities D of various substances, expressed in pounds to the cubic foot, are given, and column G gives the relative densities of the same substances.

It is evident that column G is obtained by dividing the values in column D by 62·42.

	D.	G.
Water	62·42	1·000
Anthracite	112·36	1·800
Cast iron	449·86	7·207
Cast copper	548·55	8·788
„ lead	708·59	11·352
„ platinum	1,269·43	20·337
Melting ice	58·05	0·930

In the metric system, since the mass of the cubic centimetre of water is one gramme, it is evident that the density D, in grammes to the cubic centimetre, has the same numerical value as the relative density referred to water.

25. **Velocity and its measure.**—When a material point moves, it describes a continuous line which may be either straight or curved, and is called its *path* and sometimes its *trajectory*. Motion which takes place along a straight line is called *rectilinear* motion; that which takes place along a curved line is called *curvilinear* motion. The rate of the motion of a point is called its *velocity*. Velocity may be either uniform or variable; it is *uniform* when the point describes equal spaces or portions of its path in all equal times; it is *variable* when the point describes unequal portions of its path in any equal times.

Uniform velocity is measured by the number of units of space described in a given unit of time. The units commonly employed in this country are feet and seconds. If, for example, a velocity 5 is spoken of without qualification, this means a velocity of 5 feet per second. Consequently, if a body moves for t seconds with a uniform velocity v, it will describe vt feet.

The following are a few examples of different degrees of velocity expressed in this manner. A snail 0·005 foot in a second; the Rhine between Worms and Mainz 3·3 feet; military quick step 4·6; moderate wind 10; fast sailing vessel 18·0; Channel steamer 22·0; railway train 36 to 75 feet; racehorse and storm 50 feet; wave in a tempest 72 feet; eagle 110 feet; carrier pigeon 120 feet; a hurricane 160 feet; sound at 0° 1,090; a shot from an Armstrong gun 1,180; a Martini-Henry rifle bullet 1,330; a point on the Equator in its rotation about the earth's axis 1,520; velocity of the vibratory motion of particles of air, 1,590; maximum tide rate 3,005; velocity of the centre of the earth 101,000 feet; light, and also electricity in a medium destitute of resistance, 192,000 miles.

Variable velocity is measured at any instant by the number of units of space a body would describe if it continued to move uniformly from that instant for a unit of time. Thus, suppose a body to run down an inclined

plane, it is a matter of ordinary observation that it moves more and more quickly during its descent; suppose that at any point it has a velocity 15, this means that at that point it is moving at the rate of 15 ft. per second, or, in other words, if from that point all increase of velocity ceased, it would describe 15 ft. in the next second.

26. **Force.**—Forces manifest themselves to us by the changes which they produce, or tend to produce, in the motion of matter. The action of forces in causing motion is best expressed in Newton's laws: The first law is, *Every body continues in its state of rest or of uniform motion in a straight line, except as it is compelled by forces to change that state.*

A body may be at rest, or may be moving uniformly in a straight line, while acted upon by a system of forces. In this case the forces are said to balance each other. If a constant unbalanced force act upon a body, it will no longer move uniformly. The velocity will increase continually, at a uniform rate. A familiar case of this kind is found in the attraction of the earth for other bodies. According to Newton's law of gravitation, the attraction between two masses, one of which contains m and the other m' units of mass, is $\frac{mm'}{r^2}$, where r is the distance between the centres of the masses (62). If one of the masses be the unit mass, or one pound, the other being the earth, the above expression represents the pull which the earth exerts upon a pound of matter: this pull is the weight of a pound.

It is important to distinguish very carefully between a pound—the unit of mass—and the weight of a pound, which is a *force*. Weight is not a necessary property of matter. If physical conditions were such that we could visit the centre of the earth, we should find matter without weight, although its other properties would remain unchanged. A bullet fired from a gun, although weightless, would have the same effect as at the surface of the earth, this effect being dependent, as will be shown, upon the amount of matter (mass) in the bullet and the velocity imparted, and having no relation whatever to the weight of the bullet. A pound of sugar at the centre of the earth would have precisely the same sweetening properties as at the surface. The commercial value of provisions, drugs, &c., is therefore strictly proportional to the number of units of mass purchased, and has no necessary relation to the weights of those masses.

It is also to be observed that, if masses are counterpoised on a lever balance at any one locality, they would remain balanced at any other point, since the weights of the masses would change in the same ratio. Hence the lever balance with standard 'weights' really measures the mass of a body, and not its weight, and the standard 'weights' should really be called masses. A spring balance determines weight and not mass, since its indications change as the weight of the mass changes.

At the centre of the earth, masses could not be determined by means of abalance, since they weigh nothing, and any mass would counterpoise any other mass.

27. **Measure of force.**—In devising a unit in which to measure force, it is most convenient to make use of the attractive force of the earth. Suppose that two equal masses, P, are balanced on a pulley with fixed axle, that the string and pulley are without mass, and that there is no friction or air-resistance.

The masses P are then perfectly inert. The tension on the string is the pull of the earth on *one* of the masses P, or, in other words, the weight of P. If the pulley is started by a force which then ceases to act, the masses will thereafter move uniformly according to the first law of motion, the tension on the string being, as before, the weight of P. This will be found true, whatever may be the amount of matter in the masses P. If, now, the masses P being at rest, an additional mass m be placed on one side, the system will begin to move. The tension on the string is now greater than the weight of P, and less than the weight of $P + m$. The force which causes the motion is the pull of the earth on m, or the weight of the added mass. The motion is now uniformly accelerated. At the instant of starting, the velocity is zero. At the end of the first second, the velocity will be—say a; at the end of the second second, $2a$; and at the end of t seconds, the velocity will be at. The increase in the velocity per second is a, which is called the *acceleration*.

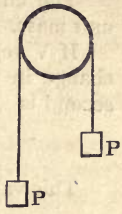

Fig. 5.

If the mass m be entirely disconnected from the masses P and allowed to fall freely, it also falls with a uniformly accelerated motion; but experiment shows that the acceleration is greater than in the former case. This acceleration of a freely falling body is usually denoted by g. The force which causes the motion is, however, the same as before, being the weight of m. The difference in the two cases is, that, in the latter case, the pull of the earth on m is employed in setting in motion the mass m only; while, in the former case, the two inert masses P are attached to m, and are constrained to move with it, the mass to be moved being thus increased without a corresponding increase of the force employed in moving.

It is evident that if the masses P should diminish to zero, or the mass m should increase until it became very large, or infinite, the weight of m would impart a greater and greater acceleration, until finally the acceleration would become g. On the other hand, if the masses P should become very large, or infinite, or the mass m very small, or zero, the acceleration would become zero. It is shown by experiment that if the mass m is made n times as great (so that the moving force is n times as great), and the masses P are equally diminished—so that $2P + m$ is unchanged—the acceleration becomes n times as great, so that, the mass to be moved being unchanged, the acceleration is directly proportional to the force applied. If, however, the mass m be made n times as great, and it is desired to have the acceleration remain unchanged, it is found that the masses P must be equally increased in such a way that $2P + m$ has also become n times as great. This shows that, the acceleration remaining constant, the force applied must change in the same ratio as the mass.

From these experiments it follows that if any force F is applied in giving uniformly accelerated motion to a mass M, the acceleration being a, then

$$F = KMa.$$

Here M is measured in pounds, and the acceleration a measures the change in velocity of M in feet per second. K is a constant, the numerical value of which will depend upon the unit which we now adopt in which to measure F. If, as is customary, we adopt as the unit force that force which

will make $a = 1$ when $M = 1$, then we at the same time necessarily make the remaining quantity K in the last equation equal to 1; and, measured in these units,

$$F = Ma.$$

The unit force is then that force which can impart unit acceleration to unit mass.

If V represent the initial velocity of a body, and v its final velocity, the change in velocity having taken place in t seconds, then the change per second is

$$a = \frac{v - V}{t}.$$

This value of a introduced in the previous equation gives

$$F = \frac{Mv - MV}{t}.$$

28. **Momentum.**—It thus appears that the number of units of force in any force which, acting for t seconds on a mass M, is capable of changing its velocity from V to v, is measured by the change per second in the product Mv. This quantity Mv, being thus an important one, has received a special name—*momentum*. We may now say that the number of units in a force is measured by the change in momentum which it can produce per second, which is the substance of Newton's second law of motion.

29. **Acceleration of gravity.**—At London, the force with which the earth attracts a pound of matter is capable of imparting to the pound an acceleration of 32·1912. At other places, the acceleration is different, and may be denoted by g. Hence, at London, the weight of a pound, expressed in the units which we have chosen for measuring forces, will be 32·1912. At any other point on the earth, or in the interior of the earth, or at any point outside, where the acceleration of a falling body is g, the number of units of force in the weight of a pound is g. The number w of units of force in the weight of m pounds is given by the equation

$$w = mg.$$

If at some point where the acceleration is 32 it is found that the weight of 10 lb., or 320 units of force, is sufficient to serve as the driving-weight to a certain clock, then at some other point, where the acceleration is 16, it would be necessary to use the weight of 20 lb. in order to secure the same effect.

The weight of $\frac{1}{32{\cdot}1912}$ lb., or 0·49 oz., at London, is a unit of force. At any other point, where the acceleration is g, the weight of $\frac{1}{g}$ lb. is the unit of force. Where great accuracy is not required, it is customary to take the weight of the pound as the unit of force, and then the intensity of the force is given in pounds weight, a unit which varies slightly for different places on the earth, as g varies. In like manner, for ordinary purposes, a land surveyor does not find it necessary to make corrections for the varying length of his chain due to changes in temperature, although such correc-

tions are highly important in the more refined operations of a geodetic survey.

Pendulum observations (79) show that at any given place the acceleration of a falling body is constant, but it is found to have different values at different places; adopting the units of feet and seconds, it is found that very approximately

$$g = g'(1 - 0{\cdot}00256 \cos 2\phi),$$

at a station whose latitude is ϕ, where g' denotes the number 32·1724, or the value of g at lat. 45°.

Experience teaches that in all cases where a force is exerted there must be *two* bodies, between which the force acts. Newton's third law asserts that the mutual action of the two bodies is always equal and oppositely directed.

The attraction of the earth for m pounds of matter is mg, where g is the acceleration of the body. The attraction of the m pounds for the earth is Ma, where M is the mass of the earth in pounds, and a is the acceleration with which it moves towards m. According to the third law of motion

$$Ma = mg.$$

If m is a small body, like a few thousand pounds, then, since the mass of the earth is very large, the acceleration of the earth will be inappreciable. If m and M were equal, a and g would be equal. Remembering that the acceleration is the change per second in the velocity, if the two bodies move towards each other for t seconds, the initial velocities being V_1 and V_2, and the final velocities v_1 and v_2, the above expression becomes

$$\frac{Mv_1 - MV_1}{t} = \frac{mv_2 - mV_2}{t}.$$

As t divides out of this equation, it will follow that the two bodies which mutually attract each other will suffer equal changes of momenta in the same time. If the two bodies start from rest at the same instant, so that V_1 and V_2 are zero, then

$$Mv_1 = mv_2,$$

or they will have equal momenta at the same instant. The momenta of a freely-suspended rifle and of a bullet fired from it will be equal so long as the ball is in the barrel. If the rifle is supported, the supporting body must be included with the rifle in the value M.

30. **Representation of forces.**—Draw any straight line AB (fig. 6), and fix on any point O in it. We may suppose a force to act on the point O, along the line AB, either towards A or B: then O is called the *point of application* of the force, AB its line of action; if it acts towards A, its *direction* is OA, if towards B, its direction is OB. It is rarely necessary to make the distinction between the line of action and direction of a force; it being very convenient to make the convention that the statement—a force acts on a point O along the line OA—means that it acts from O to A. Let us suppose the force which acts on O along OA to contain P units of force;

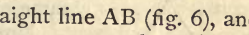

Fig. 6.

from O towards A measure ON, containing P units of length, the line ON is said to *represent* the force. The analogy between the line and the force is very complete ; the line ON is drawn from O in a given direction OA, and contains a given number of units P, just as the force acts on O in the direction OA, and contains a given number of units P. It is scarcely necessary to add, that if an equal force were to act on O in the opposite direction, it would be said to act in the direction OB, and would be represented by OM, equal in magnitude to ON.

When we are considering several forces acting along the same line we may indicate their directions by the positive and negative signs. Thus the forces mentioned above would be denoted by the symbols + P and – P respectively.

31. **Forces acting along the same line.**—If forces act on the point O in the direction OA equal to P and Q units respectively, they are equivalent to a single force R containing as many units as P and Q together—that is,

$$R = P + Q.$$

If the sign + in the above equation denote *algebraical* addition, the equation will continue true whether one or both the forces act along OA or OB. It is plain that the same rule can be extended to any number of forces, and if several forces have the same line of action, they are equivalent to one force containing the same number of units as their *algebraical* sum. Thus, if forces of 3 and 4 units act on O in the direction OA, and a force of 8 in direction OB, they are equivalent to a single force containing R units given by the equation

$$R = 3 + 4 - 8 = -1 ;$$

that is, R is a force containing one unit acting along OB. This force R is called their *resultant.* If the forces are in equilibrium R is equal to zero. In this case the forces have equal tendencies to move the point O in opposite directions.

32. **Resultant and components.**—In the last article we saw that a single force R could be found equivalent to several others ; this is by no means peculiar to the case in which all the forces have the same line of action ; in fact, when a material point, A (fig. 7), remains in equilibrium under the action of several forces, S, P, Q, it does so because any one of the forces, as S, is capable of neutralising the combined effects of all the others. If the force S, therefore, had its direction reversed, so as to act along AR, the prolongation of AS, it would produce the same effect as the system of forces P, Q.

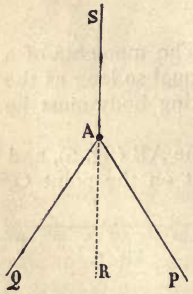

Fig. 7.

Now, a force whose effect is equivalent to the combined effects of several other forces is called their *resultant*, and with respect to this resultant, the other forces are termed *components.*

When the forces P, Q act on a point they can only have *one* resultant ; but any single force can be resolved into components in an indefinite number of ways.

If a point move from rest, under the action of any number of forces, it will begin to move in the direction of their resultant.

33. **Parallelogram of forces.**—When two forces act on a point their resultant is found by the following theorem, known as the principle of the parallelogram of forces :—*If two forces act on a point, and if lines be drawn from that point representing the forces in magnitude and direction, and a parallelogram be constructed on these lines as sides, their resultant will be represented in magnitude and direction by that diagonal which passes through the point.* Thus let P and Q (fig. 8) be two forces acting on the point A along AP and AQ respectively, and let AB and AC be taken containing the same number of units of length that P and Q contain units of force ; let the parallelogram ABDC be completed, and the diagonal AD drawn ; then the theorem states that the resultant, R, of P and Q is represented by AD ; that is to say, P and Q together are equal to a single force R acting along the line AD, and containing as many units of force as AD contains units of length.

Proofs of this theorem are given in treatises on Mechanics ; we will here give an account of a direct experimental verification of its truth ; but before doing so we must premise an account of a very simple experiment.

Let A (fig. 9) be a small pulley, and let it turn on a smooth, hard, and thin axle, with little or no friction : let W be a weight tied to the end of a fine thread which passes over the pulley ; let a spring CD be attached by one end to the end C of the thread and by the end D to another piece of thread, the other end of which is fastened to a fixed point B ; a scale CE can be fastened by one end to the point C and pass inside the spring so that

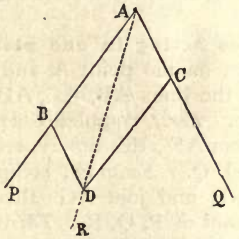

Fig. 8.

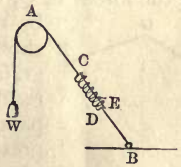

Fig. 9.

the elongation of the spring can be measured. Now it will be found on trial that with a given weight W the elongation of the spring will be the same whatever the angle contained between the parts of the string WA and BA. Also it would be found that if the whole were suspended from a fixed point, instead of passing over the pulley, the weight would in this case stretch the string to the same extent as before. This experiment shows that when care is taken to diminish to the utmost the friction of the axle of the pulley, and the imperfect flexibility of the thread, the weight of W is transmitted without sensible diminution to B, and exerts on that point a pull or force along the line BA virtually equal to W.

This being premised, an experimental proof, or illustration of the parallelogram of forces, may be made as follows :—

Suppose H and K (fig. 10) to be two pulleys with axles made as smooth and fine as possible ; let P and Q be two weights suspended from fine and

flexible threads which, after passing over H and K, are fastened at A to a third thread AL, from which hangs a weight R ; let the three weights come to rest in the positions shown in the figure. Now the point A is acted on by three forces in equilibrium—viz. P from A to H, Q from A to K, and R from A to L—consequently any one of them must be equal and opposite to the resultant of the other two. Now if we suppose the apparatus to be arranged immediately in front of a large slate, we can draw lines upon it coinciding with AH, AK, and AL. If now we measure off along AH the part AB containing as many inches as P contains pounds, and along AK the part AC containing as many inches as Q contains pounds, and complete the parallelogram ABCD, it will be found that the diagonal AD is in the same line as AL, and contains as many inches as R weighs pounds. Consequently, the resultant of P and Q is represented by AD. Of course, any other units of length and force might have been employed. Now it will be found that when P, Q, and R are changed in any way whatever, consistent with equilibrium, the same construction can be made—the point A will have different positions in the different cases ; but when equilibrium is established, and the parallelogram ABCD is constructed, it will be found that AD is vertical, and contains as many units of length as R contains units of force, and consequently it represents a force equal and opposite to R—that is, it represents the resultant of P and Q.

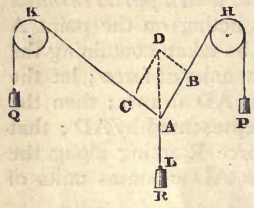

Fig. 10.

34. **Resultant of any number of forces acting in one plane on a point.**—Let the forces P, Q, R, S (fig. 11) act on the point A, and let them be represented by the lines AB, AC, AD, AE, as shown in the figure. *First*, complete the parallelogram ABFC and join AF ; this line represents the resultant of P and Q. *Secondly*, complete the parallelogram AFGD and join AG ; this line represents the resultant of P, Q, R. *Thirdly*, complete the parallelogram AGHE and join AH ; this line represents the resultant of P, Q, R, S. It is manifest that the construction can be extended to any number of forces. A little consideration will show that the line AH might be determined by the following construction :—Through B draw BF parallel to, equal to, and towards the same part as AC ; through F draw FG parallel to, equal to, and towards the same part as AD ; through G draw GH parallel to, equal to, and towards the same part as AE ; join AH, then AH represents the required resultant.

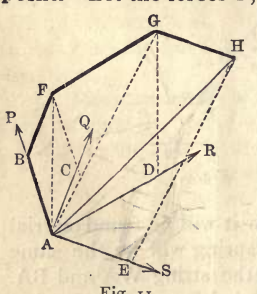

Fig. 11.

35. **Triangle of forces.**—If the resultant of the forces is zero, they have no joint tendency to move the point, and consequently are in equilibrium.

The cases of three forces acting on a point is of such importance that we may give a brief statement of it. Let P, Q, R (fig. 12) be three forces in equilibrium on the point O. From any point B draw BC parallel to and towards the same part as OP, from C draw CA parallel to and towards the

same part as OQ, and take CA such that P : Q :: BC : CA ; then, on joining AB, the third force R must act along OR parallel to and towards the same part as AB, and must be proportional in magnitude to AB. This construction is frequently called the *Triangle of Forces.* It is evident that while the sides of the triangle are severally proportional to P, Q, R, the angles A, B, C are supplementary to QOR, ROP, POQ respectively ; consequently, every trigonometrical relation existing between the sides and angles of ABC will equally exist between the forces P, Q, R, and the supplements of the angles between their directions. Thus in the triangle ABC it is known that the sides are proportional to the sines of the opposite angles ; now, since the sines of the angles are equal to the sines of their supplements, we at once conclude that *when three forces are in equilibrium, each is proportional to the sine of the angle between the directions of the other two.*

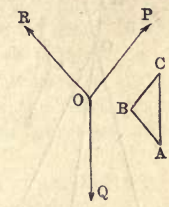

Fig. 12.

36. **Moments of forces.**—Let P (fig. 13) denote any force acting from B to P, take A any point, let fall AN a perpendicular from A on BP. The product of the number of units of force in P, and the number of units of length in AN, is called the moment of P with respect to A. Since the force P can be represented by a straight line, the moment of P can be represented by an area. In fact, if BC is the line representing P, the moment is properly represented by twice the area of the triangle ABC. The perpendicular AN is sometimes called the arm of the pressure. Now if a watch were placed with its face upwards on the paper, the force P would cause the arm AN to turn round A in the *contrary* direction to the hands of the watch. Under these circumstances, it is usual to consider the moment of P with respect to the point A to be positive. If P acted from C to B, it would turn NA in the *same* direction as the hands of the watch, and now its moment is reckoned *negative.*

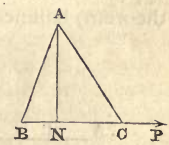

Fig. 13.

It is a simple geometrical consequence of the parallelogram of forces (33) that the moment of the resultant equals the sum of the moments of the component forces, regard being had to the *signs* of the moments.

If the point about which the moments are measured be taken in the direction of the resultant, its moment with respect to that point will be zero ; and consequently the sum of the moments with respect to such point will be zero.

37. **Composition and resolution of parallel forces.**—The case of the equilibrium of three parallel forces is merely a particular case of the equilibrium of three forces acting on a point. In fact, let P and Q be two forces whose directions pass through the points A and B, and intersect in O, fig. 14 ; let them be balanced by a third force R whose direction produced intersects the line AB in C. Now suppose the point O to move along AO, gradually receding from A, the magnitude and direction of R will continually change, and also the point C will continually change its position, but will always lie between A and B. In the limit P and Q become parallel forces, acting towards the same part balanced by a parallel force R acting towards the contrary part through a point X between A and B. The question is :—

First, in this limiting case, what is the value of R ; *secondly*, what is the position of X? Now with regard to the first point it is plain that if a triangle *abc* be drawn as in art. 35, the angles *a* and *b* in the limit will vanish, and *c* will become 180°, consequently *ab* ultimately equals *ac* + *cb* ;

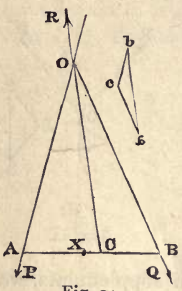

Fig. 14.

or $$R = P + Q.$$

With regard to the second point, it follows from the last article (36) that the moments of P and Q about C are always equal, whence

$$AX : XB :: Q : P,$$

a proportion which determines the position of X. Hence the following rules for the composition of any two parallel forces, viz.—

I. When two parallel forces P and Q act towards the same part, at rigidly connected points A and B, their resultant is a parallel force acting towards the same part, equal to their sum, and its direction divides the line AB into two parts AC and CB inversely proportional to the forces P and Q.

II. When two parallel forces P and Q act towards contrary parts at rigidly connected points A and B, of which P is the greater, their resultant is a parallel force acting towards the same part as P, equal to the excess of P over Q, and its direction divides BA produced in a point C such that CA and CB are inversely proportional to P and Q.

In each of the above cases if we were to apply R at the point C, in opposite directions to those shown in the figure, it would plainly (by the above theorem) balance P and Q, and therefore when it acts as shown in figs. 15

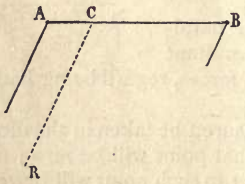

Fig. 15.

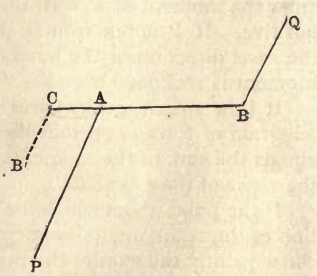

Fig. 16.

and 16 it is the resultant of P and Q in those cases respectively. It will, of course, follow that the force R acting at C can be resolved into P and Q acting at A and B respectively.

If the second of the above theorems be examined, it will be found that no force R exists equivalent to P and Q when these forces are equal. Two such forces constitute *a couple*, which may be defined to be two equal parallel forces acting towards contrary parts ; they possess the remarkable property that they are incapable of being balanced by any single force whatsoever.

In the case of more than two parallel forces the resultant of any two can

be found, then of that and a third, and so on to any number; it can be shown that however great the number of forces they will either be in equilibrium or will reduce to a single resultant or to a couple.

38. **Centre of parallel forces.**—On referring to figs. 15 and 16, it will be remarked that if we conceive the points A and B to be fixed in the directions AP and BQ of the forces P and Q, and if we suppose those directions to be turned round A and B, so as to continue parallel and to make any given angle with their original directions, then the direction of their resultant will continue to pass through C; that point is therefore called *the centre* of the parallel forces P and Q.

It appears from investigation that whenever a system of parallel forces reduces to a single resultant, those forces will have a centre; that is to say, if we conceive each of the forces to act at a fixed point, there will be a point through which the direction of their resultant will pass when the directions of the forces are turned through any equal angles round their points of application in such a manner as to retain the parallelism of their directions.

The most familiar example of a centre of parallel forces is the case in which the forces are the weights of the parts of a body; in this case the forces all acting towards the same part will have a resultant, viz. their sum; and their centre is called the *centre of gravity* of the body.

39. **Equality of action and reaction.**—We will proceed to exemplify some of the principles now laid down by investigating the conditions of equilibrium of bodies in a few simple cases; but before doing so we refer again to the law stated in art. 29, and which holds good whenever a mutual action is called into play between two bodies. *Reaction is always equal and contrary to action: that is to say, the mutual actions of two bodies on each other are always forces equal in amount and opposite in direction*, and this is equally true when the bodies are in motion as when they are at rest. A very instructive example of this law has already been given (33), in which the action on the spring CD (fig. 8) is the weight W transmitted by the spring to C, and balanced by the reaction of the ground transmitted from B to D. In these circumstances the spring is said to be stretched by a force W. If the spring were removed, and the thread were continuous from A to B, it is clear that any part of it is stretched by two equal forces, viz. an action and reaction, each equal to W, and the thread is said to sustain a tension W. When a body is urged along a smooth surface, the mutua action can only take place along the common perpendicular at the point of contact. If, however, the bodies are rough, this restriction is partially removed, and now the mutual action can take place in any direction not making an angle greater than some determinate angle with the common perpendicular. This determinate angle has different values for different substances, and is sometimes called the *limiting angle of resistance*, sometimes the *angle of repose.*

40. **The lever** is a name given to any bar, straight or curved, AB (fig. 17), resting on a fixed point or edge *c* called the *fulcrum.* The forces acting on the lever are the *weight* or resistance Q, the *power* P, and the reaction of the fulcrum. Since these are in equilibrium, the resultant P and Q must act through *c*, for otherwise they could not be balanced by the reaction. Draw *cb* at right angles to QB and *ca* to PA produced; then, observing

that $P \times ca$, and $Q \times cb$ are the moments of P and Q with respect to c, and that they have contrary signs, we have by (36),

$$P \times ca = Q \times cb ;$$

an equation commonly expressed by the rule, that *in the lever the power is to the weight in the inverse ratio of their arms.*

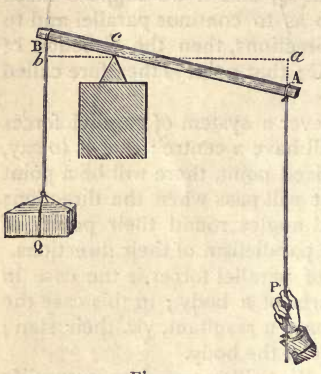

Fig. 17.

Levers are divided into three kinds, according to the position of the fulcrum with respect to the points of application of the power and the weight. In a *lever of the first kind* the fulcrum is between the power and resistance, as in fig. 17, and as in a poker and in the common steelyard; a pair of scissors and a carpenter's pincers are double levers of this kind. In a *lever of the second kind* the resistance is between the power and the fulcrum, as in a wheelbarrow, or a pair of nutcrackers, or a door; in a *lever of the third kind* the power is between the fulcrum and the resistance, as in a pair of tongs or the treadle of a lathe.

41. **Pulleys.**—The pulley is a hard circular disc of wood or of metal, in the edge of which is a groove, and which can turn freely on an axis in the centre. Pulleys are either *fixed*, as in fig. 18, where the stirrup or fork is rigidly connected with some immovable body, and where the axis rotates in the stirrup; or it may be movable, as in fig. 19, where the axis is fixed to the fork, and it passes through a hole in the centre of the disc. The rope which passes round the pulley in fig. 18 supports a weight at one end; while at the other a pull is applied to hold this weight in equilibrium.

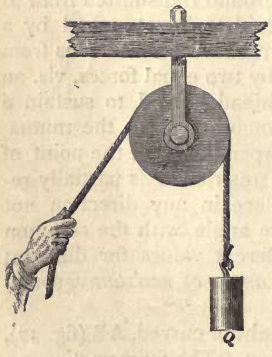

Fig. 18.

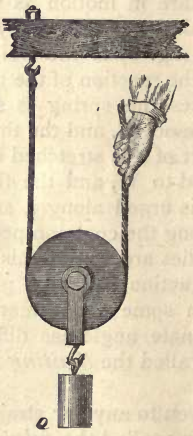

Fig. 19.

We may look upon the power and the resistance as acting at the circumference of the circle; hence as the radii are equal, if we consider the pulley as a lever, the two arms are equal, and equilibrium will prevail when the power and the resistance are equal. The fixed pulley affords thus no mechanical advantage, but is simply convenient in changing the direction of the application of a force.

In the case of the movable pulley one end of the rope is suspended to a

fixed point in a beam, and the weight is attached to the hook on which the pulley acts. The tension of the rope is everywhere the same; one portion of the weight is supported by the fixed part and the other by the power, and these are equal to each other, and are together equal to the weight, including the pulley itself; hence in this case $P = \frac{1}{2} Q$.

If several pulleys are joined together on a common axis in a special sheath, which is fixed, and a rope passes round all those, and also round a similar but movable combination of pulleys, such an arrangement, which is represented in fig. 20, is called a *block and tackle.*

If we consider the condition of the rope it will be found to have everywhere the same tension; the weight Q which is attached to the hook common to the whole system is supported by the six portions of the rope: hence each of these portions will sustain one sixth of the weight; the force which is applied at the free end of the rope which passes over the upper pulley, and which determines the tension, will have the same value; that is to say, it will support one sixth of the weight.

The relation between power and resistance in a block and tackle is expressed by the equation $P = \frac{Q}{n}$, in which P is the power, Q the weight, and n the number of cords by which the weight is supported.

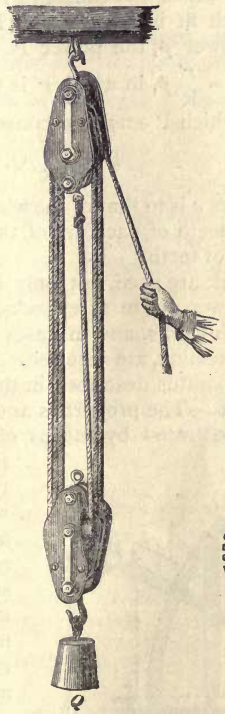

Fig. 20.

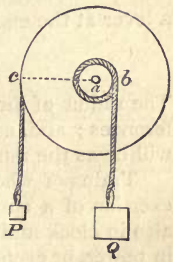

Fig. 21.

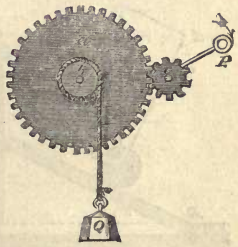

Fig. 22.

42. **The wheel and axle.** —The older form of this machine, fig. 21, is that of an axle, to which is rigidly fixed, concentric with it, a wheel of larger diameter. The power is applied tangentially on the wheel, and the resistance tangentially to the axle, as, for instance, in the treadmill and waterwheel. Sometimes, as in the case of the capstan, the power is applied to spokes fixed in the axle, which represent semi-diameters of the wheel; in other cases, as in the windlass, the handle is rigidly fixed to the axis.

In all its modifications we may regard the wheel and axle as an application of the lever, the arms of which are the radii of the wheel and axle respectively; and in all cases equilibrium exists where the power is to the

resistance as the radius of the axle is to the radius of the wheel. Thus in fig. 21, P : Q = *ab* : *ac*, or P × *ac* = Q × *ab*.

Frequent applications of wheels of different diameters are met with in which the motion of one wheel is transmitted to another, either by means of teeth fitting in each other on the circumference of the wheels, as in fig. 22, or by means of bands passing over the two wheels, as in the illustration of Ladd's Magneto-Electrical Machine (see Book viii.).

In fig. 22, which represents the essential parts of a crab winch, in order to raise the weight Q a power p must be applied at the circumference of the wheel such that $p = Q\frac{r}{R}$, in which r and R are the radii of the axle *b* and of the toothed wheel *a* respectively.

The rotation of the wheel *a* is effected by means of the smaller wheel *c* or *crab*, the teeth of which fit in those of *a*. But if this wheel *c* is to exert at its circumference a power p, the power P which is applied at the end of the handle must be $P = \frac{r'}{R'}p$, in which r' is the radius of *c*, R′ the length of a lever at the end of which P acts, and consequently

$$P = \frac{rr'}{RR'}Q.$$

The radius of the wheel *c* is to that of the wheel *a* as their respective circumferences ; and, as the teeth of each are of the same size, the circumferences will be as the number of teeth.

Trains of wheelwork are used, not only in raising great weights by the exertion of a small power, as in screwjacks, cranes, crab winches, &c., but also in clock and watch works, and in cases in which changes in velocity or in power, or even in direction, are required. Numerous examples will be met with in the various apparatus described in this work.

43. **Inclined plane.**—The properties and laws of the inclined plane may be conveniently demonstrated by means of the apparatus represented in fig. 23. RS represents the section of a smooth piece of hard wood hinged at R ; by means of a screw it can be clamped at any angle x against the arc-shaped support, by which at the same time the angle can be measured ; *a* is a cylindrical roller, to the axis of which is attached a string passing over a pulley to a scale-pan P.

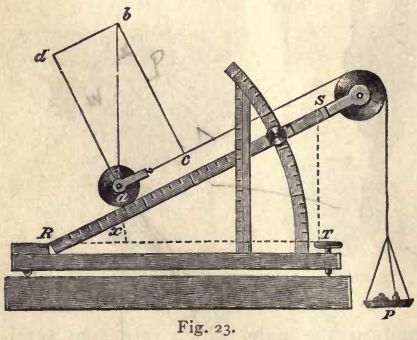

Fig. 23.

It is thus easy to ascertain by direct experiments what weights R must be placed in the pan P in order to balance a roller of any given weight, or to cause it to move with a given angle of inclination.

The line RS represents the *length*, ST the *height*, and RT the *base* of the inclined plane.

In ascertaining the theoretical conditions of equilibrium we have a useful

application of the parallelogram of forces. Let the line *ab*, fig. 23, represent the force which the weight W of the cylinder exerts acting vertically downwards; this may be decomposed into two others; one, *ad*, acting at right angles against the plane, and representing the *pressure* which the weight exerts against the plane, and which is counterbalanced by the reaction of the plane; the other, *ac*, represents the component which tends to move the weight down the plane, and this component has to be held in equilibrium by the weight P, equal to it, and acting in the opposite direction.

It can be readily shown that the triangle *abc* is similar to the triangle SRT, and that the sides *a c* and *a b* are in the same proportion as the sides ST and SR. But the line *ac* represents the power, and the line *ab* the weight; hence

$$ST : SR = P : W ;$$

that is, on an inclined plane, equilibrium obtains *when the power is to the weight as the height of the inclined plane to its length.*

Since the ratio $\frac{ST}{SR}$ is the sine of the angle x, we may also state the principle thus:

$$P = W \sin x.$$

The component *da* or *bc*, which represents the actual pressure against the plane, is equal to W cos x; that is, the pressure against the plane is to the weight as the base is to the length of the inclined plane.

In the above case it has been considered that the power acts parallel to the inclined plane. It may be applied so as to act horizontally. It will then be seen from fig. 24 that the weight W may be decomposed into two forces, one of which, *ab*, acts at right angles to the plane, and the other, *ac*, parallel to the base. It is this latter which is to be kept in equilibrium by the power. From the similarity of the two triangles *acb* and STR, $ac : bc = ST : TR = h : b$; but *bc* is equal to W, and *ac* is equal to P, hence the power which must be applied at *b* to hold the weight W in equilibrium is as the height of the inclined plane is to the base, or as the tangent of the angle of inclination x; that is, P = W tan x. The pressure upon the plane in this case may be easily shown to be $ab = \frac{bc}{\cos x}$, that is $= \frac{W}{\cos x}$. This is sometimes called the *relative weight* on the plane.

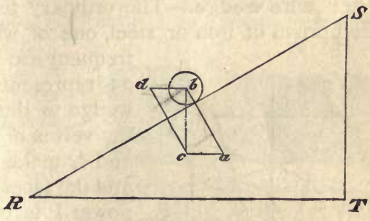

Fig. 24.

If the force P which is to counterbalance W is not parallel to the plane, but forms an angle, E, with it, this force can be decomposed into one which is parallel to it, and one which is at right angles. Of these only the first is operative, and is equal to P cos E.

In most cases of the use of the inclined plane, such as in moving carriages and waggons along roads, in raising casks into waggons or warehouses, the power is applied parallel to the inclined plane. An instance of a case in which a force acts parallel to the base is met with in the screw.

Owing to the unevenness of the surfaces in actual use, the laws of equilibrium and of motion on an inclined plane undergo modification. The *friction*, for instance, which comes into play amounts on ordinary roads to from $\frac{1}{15}$ to $\frac{1}{30}$, and on railways to from $\frac{1}{180}$ to $\frac{1}{240}$ of the relative weight. This must be looked upon as a hindrance to be continually overcome, and must be deducted from the force required to keep a body from falling down an inclined plane, or must be added to it in the case in which a body is to be moved up the plane. Hence the use of the inclined plane in unloading heavy casks into cellars, &c.

A body which cannot roll does not move on the inclined plane, provided the inclination is below a certain amount (39). The determination of this *limiting angle of resistance*, at which a body on an inclined plane just begins to move, may serve as a rough illustration of a mode of ascertaining the 'coefficient of friction.'

For in the case in which the power is applied parallel to the plane, the component of the weight which presses against the plane or the actual load, L, is W cos x; and the component which tends to move the body down the plane is equal to W sin x. If the friction, R, is just sufficient to hold this in equilibrium, the coefficient of friction will be $\frac{R}{L} = \frac{W \sin x}{W \cos x} = \tan x$.

Thus if we place on the plane a block of the same material, by gradually increasing the inclination it will begin to move at a certain angle, which will depend on the nature of the material; this angle is the limiting angle of resistance, and its tangent is the coefficient of friction for that material.

44. **The wedge.**—The ordinary form of the wedge is that of a three-sided prism of iron or steel, one of whose angles is very acute. Its most frequent use is in splitting stone, timber, &c. Fig. 25 represents in section the application of the wedge to this purpose. The side *ab* is the *back*, the vertex of the angle *acb* which the two faces *ac* and *bc* make with each other represents the *edge*, and the faces *ac* and *bc* the *sides* of the wedge. The power P is usually applied at right angles to the back; and we may look upon the cohesion between the fibres of the wood as representing the resistance to be overcome; as corresponding to what in other machines is the weight. Suppose this to act at right angles to the two faces of the wedge, and to be represented by the lines *fe* and *ge*; complete the parallelogram *gef*, then the diagonal *he* will represent the resultant of the reaction of the fibres tending to force the wedge out; the force which must be applied to hold this wedge in equilibrium must therefore be equal to *eh*. Now *efh* is similar to the triangle *acb*, therefore $ab : ac = eh : ef$; but these lines represent the pressure applied at the back of the wedge, and the pressure on the face *ac*, hence if P represent the former and Q the latter, there is equilibrium when $P : C = ab : bc$, that is, when the power is to the resistance in the same ratio as the back of

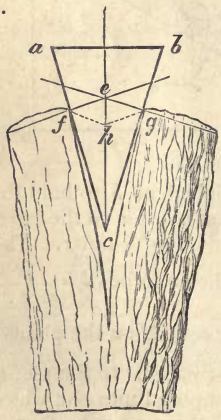

Fig. 25.

the wedge bears to one of the sides. The relation between power and resistance is more favourable the sharper the edge, that is, the smaller the angle which the sides make with each other.

The action of all sharp-cutting instruments, such as chisels, knives, scissors, &c., depends on the principle of the wedge. It is also applied when very heavy weights are to be raised through a short distance, as in launching ships, and in bracing columns and walls to the perpendicular.

45. **The screw.**—Let us suppose a piece of paper in the shape of a right-angled triangle *aee'* to be applied with its vertical side *ac'e'* against a cylinder, and parallel to the axis, and to be wrapped round the cylinder; the hypotenuse will describe a screw line or *helix* on the surface of the cylinder (fig. 26); the points *a b c d e* will occupy the positions respectively *a b'c'd'e'*.

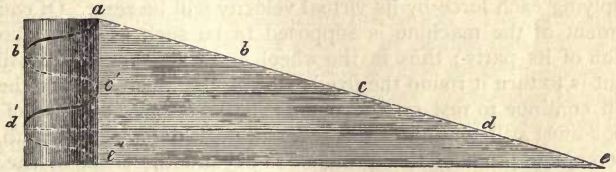

Fig. 26.

If the dimensions be so chosen that the base of the triangle, *cc'* is equal to the circumference of the cylinder, then the hypotenuse *abe* becomes an inclined plane traced on the surface of the cylinder; the distance *ac'* being the height of the plane.

An ordinary screw consists of an elevation on a solid cylinder; this elevation may be either square, as in fig. 27, or acute; and such screws are called *square* or *sharp* screws accordingly.

When a corresponding groove is cut in the hollow cylinder or nut of the same diameter as the bolt, this gives rise to an internal or *companion* screw or *nut*, fig. 28.

Fig. 27.

Fig. 28.

The vertical distance between any two threads of a screw measured parallel to the axis is called the *pitch*, and the angle *acc'* or *aee'* is called the *inclination* of the screw.

In practice, a raised screw is used with its companion in such a manner that the elevations of the one fit into, and coincide with, the depressions of the other. The screw is a modification of the inclined plane, and the conditions of equilibrium are those which obtain in the case of the plane. The resistance, which is either a weight to be raised, or a pressure to be exerted, acts in the direction of the vertical, and the power acts parallel to the base; hence we have $P : R = h : b$, and the length of the base is the circumference of the cylinder; whence $P : R = h : 2\pi r$; r being the radius of the cylinder, and h the pitch of the screw.

The power is usually applied to the screw by means of a lever, as in the bookbinders' press, the copying press, &c., and the principle of the screw may be stated to be generally that the power of the screw is to the resistance, in the same ratio as that of the pitch of the screw to the circumference of the circle through which the power acts.

46. **Virtual velocity.**—If the point of application of a force be slightly

displaced, the resolved part of the displacement in the direction of the force is termed the *virtual velocity of the force*, and is considered as positive or negative, according as it is in the same direction as the force or in the opposite direction. Thus in fig. 29 let the point of application A of the force P be displaced to A′, and draw A′*a* perpendicular to AP. Then A*a* is the virtual velocity of the force P, and being, in this case, in the direction of P, is to be considered positive.

Fig. 29.

The principle of virtual velocities asserts that if any machine or system be kept in equilibrium by any number of forces, and the machine or system then receive any *very small* displacement, the algebraic sum of the products formed by multiplying each force by its virtual velocity will be zero. Of course, the displacement of the machine is supposed to be such as not to break the connection of its parts; thus in the wheel and axle the only possible displacement is to turn it round the fixed axle; in the inclined plane the weight must still continue to rest on the plane; in the various systems of pulleys the strings must still continue stretched, and must not alter in length, &c.

The complete proof of this principle is beyond the scope of the present work, but we may easily establish its truth in any of the machines we have already considered. It will be found in every case that, if the machine receive a small displacement, the virtual velocities of P and W will be of opposite signs, and that, neglecting the signs, P × P's virtual velocity = W × W's virtual velocity. Thus to take the case of a *bent lever*, let P and Q be the forces acting at the extremities of the arms of the bent lever AFB (fig. 30), and let the lever be turned slightly round its fulcrum F, bringing A to A′, and B to B′. Draw A′*a* and B′*b* perpendicular to P and Q respectively; then A*a* is the virtual velocity of P, and B*b* that of Q, the former being positive and the latter negative. Let F*p*, F*q* be the perpendiculars from the fulcrum upon P and Q, or what we have called (art. 40) the arms of P and Q. Now, as the displacement is very small, the angles FAA′, FBB′ will be very nearly right angles; and, therefore, the right-angled triangles A*a*A′, B*b*B′ will ultimately be similar to the triangles F*p*A, F*q*B respectively, whence $\frac{Aa}{AA'}=\frac{Fp}{FA}$, and $\frac{Bb}{BB'}=\frac{Fq}{FB}$, or $\frac{Aa}{Fp}=\frac{AA'}{FA}$, and $\frac{Bb}{Fq}=\frac{BB'}{FB}$. But the triangles FAA′, FBB′ are similar, as they are both isosceles, and their vertical angles are equal, so that $\frac{AA'}{FA}=\frac{BB'}{FB}$, whence $\frac{Aa}{Fp}=\frac{Bb}{Fq}$ or, as we may put it, $\frac{P\times Aa}{P\times Fp}=\frac{Q\times Bb}{Q\times Fq}$. Now the denominators of these two equal fractions are equal if the lever be in equilibrium (art. 40). Hence the numerators are equal, or

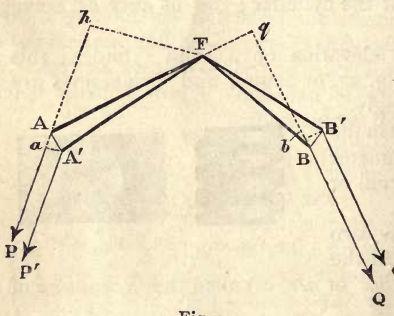

Fig. 30.

P × P's virtual velocity = Q × Q's virtual velocity.

As a further and simpler example, take the case of the block and tackle described in article 41. Suppose the weight to be raised through a space h; then the virtual velocity of the weight is h, and is negative. Now, as the distance between the block and tackle is less than before by the space h, and as the rope passes over this space n times, in order to keep the rope still tight the power will have to move through a space equal to nh. This is the virtual velocity of P, and is positive, and as $W = nP$, we see that

W × W's virtual velocity = P × P's virtual velocity.

46*a*. **Machines.**—In many machines in common use, two forces can readily be distinguished. One is a force applied in order to drive the machine, and the other is a force overcome, and is called the resistance. The force applied is usually, though improperly, called the power. In general these forces are unequal. If the machine moved without friction, these forces might be exactly balanced, in such a way that if either of them were increased in the slightest degree, the machine would begin to move with a uniformly accelerated motion. If such a machine thus balanced were to be started by an impulse which should *then* cease to act, the machine would move continuously at a uniform rate until acted upon by some other external force. If we imagine a balanced frictionless machine to become a machine with friction, then either of the two forces might be varied between certain limits, without setting the machine into motion. Hence, if the machine is to move uniformly, the force applied in driving it must be greater than would be necessary to give uniform motion to a frictionless machine. The force applied, P, and the resistance overcome, R, may be expressed in pounds weight, which may be converted into absolute units by multiplying by the value of g at the place. While P moves over a certain distance p, R moves over a distance r. These distances can be determined by measurement. The ratio of r to p can often be seen by simple inspection, since its value depends upon the gearing or construction of the machine.

If the force P is exerted over a distance p, the work applied is Pp foot-pounds. While this work is being applied to the machine, a certain amount of work, Rr, is transmitted through the machine, and is done upon the resistance. Experiment shows that the work applied Pp is always greater than the work Rr transmitted through the machine. This difference represents the work which is required to move the parts of the machine upon each other, and is called internal work. If the internal work is represented by I, the condition for uniform action of a machine is given by the equation

$$Pp = Rr + I.$$

It will be assumed that a small force P‴ is applied, sufficient to move the machine uniformly when unloaded. This value of P‴ is not included in P. In this case, the work of friction is due wholly to the load which the machine carries, and I becomes zero when R = 0. The quantity I is of the same nature as the other two quantities in the equation, being the product of a certain force of friction into a certain distance, but in general these factors cannot be determined separately. It is found that I diminishes in value as the parts of the machine in contact are made smoother, and is further

diminished by oiling the bearings—that is to say, the quantities $P\rho$ and Rr, which can be easily determined, become more nearly equal.

The equation may also be put into the following form :—

$$\frac{P}{R} = \frac{r}{\rho} + i \text{ where } i = \frac{I}{R\rho}.$$

It is evident that the ratio $\frac{r}{\rho}$ is a constant quantity, for a given machine, geared in a definite manner. Experiment shows that the ratio $\frac{P}{R}$ is also practically constant, so that the quantity i may also be considered constant for a given machine in a definite condition. It would, however, be changed by oiling the bearings, as this would make it necessary to diminish P in order to preserve uniform motion, and it also depends upon the arrangement of the machine, as will be pointed out further on.

47. **Friction.**—In the cases of the actions of machines which have hitherto been described, the resistances which are offered to motion have not been at all considered. The surfaces of bodies in contact are never perfectly smooth ; even the smoothest present inequalities which can neither be detected by the touch nor by ordinary sight ; hence when one body moves over the surface of another, the elevations of one sink into the depressions of the other, like the teeth of wheels, and thereby offer a certain resistance to motion ; this is what is called *friction*. It must be regarded as a force which continually acts in opposition to actual or possible motion.

Friction is of two kinds : *sliding*, as when one body glides over another ; this is least when the two surfaces in contact remain the same, as in the motion of an axle in its bearing ; and *rolling* friction, which occurs when one body rolls over another, as in the case of an ordinary wheel. The latter is less than the former, for by the rolling the inequalities of one body are raised over those of the other. As rolling friction is considerably less than sliding friction, it is a great saving of power to convert the latter into the former ; as is done in the case of the casters of chairs and other furniture, and also in that of friction wheels. This, however, is not always the case ; thus a sledge experiences less friction on snow than a carriage, for in this case the wheels sink and friction on the sides results. On the other hand, it is sometimes useful to change rolling into sliding friction, as when drags are placed on carriage wheels.

Friction is directly proportional to the pressure of the two surfaces against each other. That fraction of the pressure which must act as moving force merely to overcome friction is called the *coefficient of friction*.

Friction is independent of the extent of the surfaces in contact if the pressure is the same. Thus, suppose a board with a surface of a square decimetre resting on another board to be loaded with a weight of a kilogramme. If this load be distributed over a similar board of two square decimetres' surface, the total friction will be the same, while the friction per square centimetre is one-half, for the pressure on each square centimetre is one-half of what it was before. So, too, a rectangular stone experiences the same friction whether it is laid on the narrow or on the broad side. Friction is diminished by polishing and by smearing, but is increased by heat. It is

greater as a body passes from the state of rest to that of motion than during motion, but seems independent of the velocity. The coefficient of friction depends on the nature of the substance in contact; similar bodies experience in general greater friction than dissimilar ones, for with the former the inequalities fit more into one another; thus for oak upon oak it is 0·418 when the fibres are parallel, and 0·293 when they cross; for beech upon beech it is 0·36. Greasy substances, which are not absorbed by the body, diminish friction, but increase it if they are absorbed. Thus moisture and oil increase, while tallow, soap, and graphite diminish, the friction of wooden surfaces. In the sliding friction of cast iron upon bronze the coefficient was found to be 0·25 without grease; with oil it was 0·17, fat 0·11, soap 0·03, and with a mixture of fat and graphite 0·02. The coefficient of rolling friction for cast-iron wheels on iron rails as in railways is about 0·004; for ordinary wheels on an ordinary road it is 0·04, hence a horse can draw ten times as great a load on rails as on an ordinary road, and this is indeed a main use of rail and tram ways. The coefficient of steel upon smooth ice has been determined by a skater holding in his hand a spring balance (88) attached to a cord by which he was drawn along by a second skater. At starting the spiral showed a pull of 5 to 6 kilos, but during the motion this varied between 1 and 2 kilos. As the weight of the skater was 62 kilos, the coefficient of friction during the motion was $\frac{1}{62}$, to $\frac{2}{62}$, or 1·6 to 3·2 per cent.

Without friction, neither man nor animals, neither ordinary carriages nor railway carriages, could move. Friction is necessary for the transmission of power from one wheel to another by means of bands or ropes; and without friction we could hold nothing in the hands.

48. **Resistance to motion in a fluid medium.**—A body in moving through any medium, such as air or water, experiences a certain resistance; for the moving body sets in motion those parts of the medium with which it is in contact, whereby it loses an equivalent amount of its own motion.

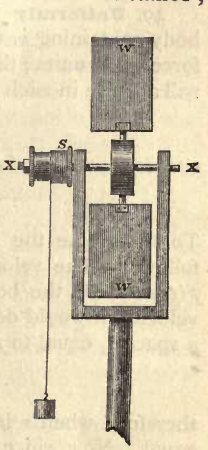

Fig. 31.

This resistance increases with the surface of the moving body; thus a soap-bubble or a snow-flake falls more slowly than does a drop of water of the same weight. It also increases with the density of the medium; in rarefied air, therefore, it is less than in air under the ordinary pressure; and in this again it is less than in water.

The influence of this resistance may be illustrated by means of the apparatus represented in fig. 31, which consists of two vanes, *w w*, fixed to a horizontal axis, *xx*, to which is also attached a bobbin *s*. The rotation of the vanes is effected by means of the falling of a weight attached to the string coiled round the bobbin. The vanes can be adjusted either at right angles or parallel to the axis. In the former position the vanes rotate rapidly when the weight is allowed to act; in the latter, however, where they press with their entire surface against the air, the resistance greatly lessens the rapidity of rotation.

The resistance increases with the velocity of the moving body, and for

moderate velocities is proportional to the square ; for, supposing the velocity of a body made twice as great, it must displace twice as much matter, and must also impart to the displaced particles twice the velocity. For high velocities the resistance in a medium increases in a more rapid ratio than that of the square, for some of the medium is carried along with the moving body, and this, by its friction against the other portions of the medium, causes a loss of velocity.

It is this resistance which so greatly increases the difficulty and cost of attaining very high speeds in steam-vessels. Use is made, on the other hand, of this resistance in parachutes (fig. 180) and in the windvanes for diminishing the velocity of falling bodies (fig. 57), the principle of which is illustrated by the apparatus, fig. 31. Light bodies fall more slowly in air than heavy ones of the same surface, for the moving force is smaller compared with the resistance. The resistance to a falling body may ultimately equal its weight; it then moves uniformly forward with the velocity which it has acquired. Thus, a rain-drop falling from a height of 3,000 feet should, when near the ground, have a velocity of nearly 440 feet in a second, or that of a musket-shot ; owing, however, to the resistance of the air, its actual velocity is probably not more than 30 feet. On railways the resistance of the air is appreciable ; with a carriage exposing a surface of 22 square feet, it amounts to 16 or 17 pounds when the speed of the train is 16 feet a second, or 11 miles an hour.

By observing the rate of diminution in the number of oscillations of a horizontal disc suspended by a thread when immersed in water, Meyer determined the coefficient of the frictional or internal resistance of water, and found that at 10° it was equal to 0·01567 gramme on a square centimetre ; and for air it was about $\frac{1}{4\frac{1}{0}}$ as much.

49. **Uniformly accelerated rectilinear motion.**—Let us suppose a body containing m units of mass to move from rest under the action of a force of F units ; the body will move in the line of action of the force, and will acquire in each second an additional velocity f given by the equation

$$F = mf;$$

consequently, if v is its velocity at the end of t seconds, we have

$$v = ft. \qquad (1)$$

To determine the space it will describe in t seconds, we may reason as follows :—The velocity at the time t being ft, that at a time $t + \tau$ will be $f(t + \tau)$. If the body moved uniformly during the time τ with the former velocity, it would describe a space s equal to $ft\tau$; if with the latter velocity a space s_1 equal to $f(t + \tau)\tau$. Consequently,

$$s_1 : s :: t + \tau : t;$$

therefore, when τ is indefinitely small, the limiting values of s and s_1 are equal. Now, since the body's velocity is continually *increasing* during the time τ, the space actually described is greater than s and less than s_1. But since the limiting values of s and s_1 are equal, the limiting value of the space described is the same as that of s or s_1. In other words, if we suppose the whole time of the body's motion to be divided into any number of equal

parts, if we determine the velocity of the body at the beginning of each of these parts, and if we ascertain the spaces described on the supposition that the body moves uniformly during each portion of time, the limiting value of the sum of these spaces will be the space actually described by the body. Draw a line AC (fig. 32), and at A construct an angle CAB, whose tangent equals f; divide AC into any number of equal parts in D, E, F,...and draw PD, QE, RF,...BC at right angles to AC; then since PD = AD × f, QE = AE × f, RF = AF × f, BC = AC × f, &c., PD will represent the velocity of the body at the end of the time represented by AD, and similarly QE, RF,...BC, will represent the velocity at the end of the times AE, AF,...AC. Complete the rectangles De, Ef, Fg... These rectangles represent the space described by the body on the above supposition during the second, third, fourth,...portions of the time. Consequently, the space actually described during the time AC is the limit of the sum of the rectangles; the limit being continually approached as the number of parts into which AC is divided is continually increased. But this limit is the area of the triangle ABC; that is ½AC × CB or ½AC × AC × f. Therefore, if AC represents the time t during which the body describes a space s, we have

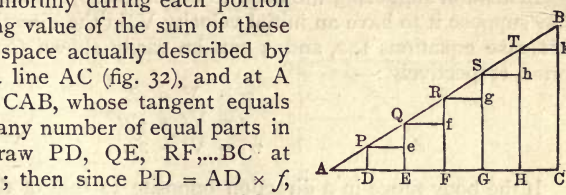

Fig. 32.

$$s = \tfrac{1}{2}ft^2. \qquad (2)$$

Since this equation can be written

$$2fs = f^2t^2$$

we find, on comparison with equation (1), that

$$v^2 = 2fs. \qquad (3)$$

To illustrate these equations, let us suppose the accelerative effect of the force to be 6; that is to say that, in virtue of the action of the force, the body acquires in each successive second an additional velocity of 6 feet per second; and let it be asked what, on the supposition of the body moving from rest, will be the velocity acquired, and the space described, at the end of 12 seconds; equations 1 and 2 enable us to answer that at that instant it will be moving at the rate of 72 feet per second, and will have described 432 feet.

The following important result follows from equation (2). At the end of the first, second, third, fourth, &c., second of the motion, the body will have described $\frac{1}{2}f$, $\frac{1}{2}f \times 4$, $\frac{1}{2}f \times 9$, $\frac{1}{2}f \times 16$, &c., feet; and consequently *during* the first, second, third, fourth, &c., second of the motion will have described $\frac{1}{2}f$, $\frac{1}{2}f \times 3$, $\frac{1}{2}f \times 5$, $\frac{1}{2}f \times 7$, &c., feet, namely spaces in arithmetical progression.

The results of the above article can be stated in the form of laws which apply to the state of a body moving from a state of rest under the action of a constant force:—

I. *The velocities are proportional to the times during which the motion has lasted.*

II. *The spaces described are proportional to the squares of the times employed in their description.*

III. *The spaces described are proportional to the squares of the velocities acquired during their description.*

IV. *The spaces described in equal successive periods of time increase by a constant quantity.*

Instead of supposing the body to begin to move from a state of rest, we may suppose it to have an initial velocity V, in the direction of the force. In this case equations 1, 2, and 3 can be easily shown to take the following forms, respectively :—

$$v = V + ft,$$
$$s = Vt + \tfrac{1}{2}ft^2,$$
$$v^2 = V^2 + 2fs.$$

If the body move in a direction opposite to that of the force, f must be reckoned negative.

The most important exemplification of the laws stated in the present article is in the case of a body falling freely *in vacuo*. Here the force causing the acceleration is that of gravity, and the acceleration produced is denoted by the letter g: it has already been stated (29) that the numerical value of g is 32·1912 at London, when the unit of time is a second and the unit of length a foot. Adopting the metre as unit of length, the value of g at London is 9·8117.

50. **Motion on an inclined plane.**—Referring to (43), suppose the force P not to act ; then the mass M is acted on by an unbalanced force M $g \sin x$, in the direction SR ; consequently the acceleration down the plane is $g \sin x$, and the motion becomes a particular case of that discussed in the last article. If it begins to move from rest, it will at the end of t seconds acquire a velocity v given by the equation

$$v = gt \sin x,$$

and will describe a length s of the plane given by the equation

$$s = \tfrac{1}{2}gt^2 \sin x.$$

Also, if v is the velocity acquired while describing s feet of the plane,

$$v^2 = 2gs \sin x.$$

Hence (fig. 23), if a body slides down the plane from S to R the velocity which it acquires at R is equal to $\sqrt{2g}$. RS sin R or $\sqrt{2g}$. ST ; that is to say, the velocity which the body has at R does not depend on the angle x, but only on the perpendicular height ST. The same would be true if for RS we substituted any smooth curve ; and hence we may state generally that when a body moves along any smooth line under the action of gravity, the change of velocity it experiences in moving from one point to another is that due to the *vertical* height of the former point above the latter.

51. **Motion of Projectiles.**—The equations given in the above article apply to the case of a body thrown vertically upwards or downwards with a certain initial velocity. We will now consider the case of a heavy body thrown in a horizontal direction. Let *a*, fig. 33, be such a body thrown with an initial velocity of v feet in a second, and let the line *ab* represent the space described in any interval ; then at the end of the 2, 3, 4... equal interval, the body, in virtue of its inertia, will have reached the points *c d e*, &c. But during all this time the body is under the influence of gravity, which, if it alone acted, would cause the body to fall through the distances repre-

sented on the vertical line ; these are determined by the successive values of $\frac{1}{2}gt^2$, which is the formula for the space described by a freely falling body (50). The effect of the combined action of the two forces is that at the end of the first interval, &c., the body will be at b', at the end of the second interval at c', of the third at d', &c., the spaces bb' cc' dd'... being proportional to the squares of ab, ac, ad, respectively, and the line joining these points represents the path of the body. By taking the intervals of time sufficiently small we get a regularly curved line of the form known as the *parabola*.

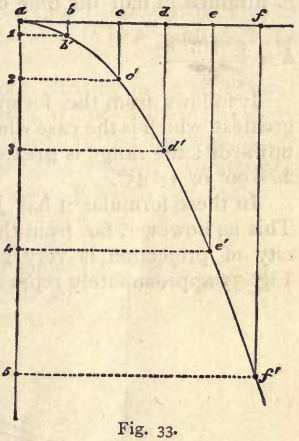

Fig. 33.

In order to demonstrate motion with horizontal and inclined direction the apparatus represented in fig. 34 may be made use of. It consists of a bottle from which a steady stream of water issues through a caoutchouc tube terminating in a jet. This can be discharged in front of a slate on which the path of the curve in each case can be chalked.

If the direction in which the body is thrown makes an angle of a with the horizon (fig. 35), then after t seconds it would have travelled a distance $ab = vt$, where v is the original velocity ; during this time, however, it will have fallen through a distance $bc = \frac{1}{2}gt^2$; the height which it will have actually reached is $= bd - bc = vt \sin a - \frac{1}{2}gt^2$; and the horizontal distance will be $ad = ab \cos a = vt \cos a$. The *range* of the body, or the greatest distance through which it is thrown, will be reached when the height is again $= 0$; that is, when $vt \sin a - \frac{1}{2}gt^2 = 0$, from which $t = \frac{2v \sin a}{g}$. Introducing this value of t into the equation for the distance, d, we have $d = \frac{2v^2 \sin a \cos a}{g}$,

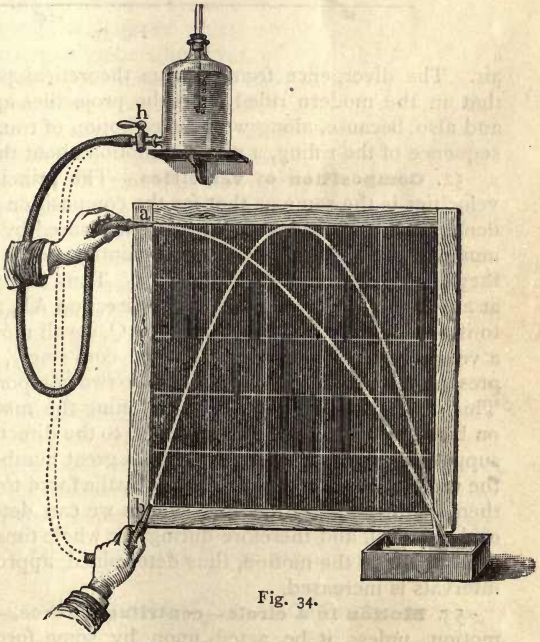

Fig. 34.

which by a trigonometrical transformation $= \frac{v^2 \sin 2a}{g}$. The greatest height is attained in half the time of flight, or when $t = \frac{v \sin a}{g}$, from which we get $h = \frac{v^2 \sin^2 a}{2g}$.

It follows from the formula that the height is greatest when sin a is greatest, which is the case when it $= 90°$, or when the body is thrown vertically upwards; the range is greatest where sin $2a$ is a maximum, that is, when $2a = 90°$ or $a = 45°$.

In these formulæ it has been assumed that the air offers no resistance. This is, however, far from the case, and in practice, particularly if the velocity of projection is very great, the path differs from that of a parabola. Fig. 35 approximately represents the path, allowing for the resistance of the

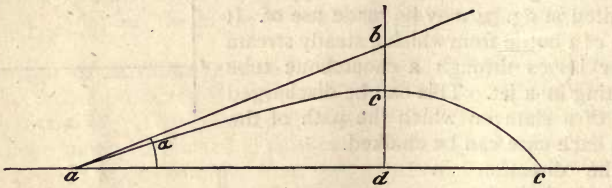

Fig. 35.

air. The divergence from the true theoretical path is affected by the fact that in the modern rifled arms the projectiles are not spherical in shape; and also because, along with their motion of translation, they have, in consequence of the rifling, a rotatory motion about their axis.

52. **Composition of velocities.**—The principle for the composition of velocities is the same as that for the composition of forces: this follows evidently from the fact that forces are measured by the momentum they communicate, and are therefore to one another in the same ratio as the velocities they communicate to the *same* body. Thus (fig. 7, art. 32), if the point has at any instant a velocity AB in the direction AP, and there is communicated to it a velocity AC in the direction AQ, it will move in the direction AS with a velocity represented by AD. And conversely, the velocity of a body represented by AD can be resolved into two component velocities AB and AC. This suggests the method of determining the motion of a body when acted on by a force in a direction transverse to the direction of its velocity; namely, suppose the time to be divided into a great number of intervals, and suppose the velocity actually communicated by the force to be communicated at once; then by the composition of velocities we can determine the motion during each interval, and therefore during the whole time; the actual motion is the limit to which the motion, thus determined, approaches when the number of intervals is increased.

53. **Motion in a circle—centrifugal force.**—When a body is once in motion, unless it be acted upon by some force, it will move uniformly forward in a straight line with unchanged velocity (26). If, therefore, a body moves uniformly in any other path than a straight line—in a circle, for

instance—this must be because some constant force is at work which continuously deviates it from this straight line.

We have already seen an example of this in the case of the motion of projectiles (51), and will now consider it in the case of central motion or motion in a circle, of which we have an example in the motion of the celestial bodies, or in the motion of a *sling*.

In the latter case, if the string is cut, the stone, ceasing to be acted upon by the tension of the string, will move in a straight line with the velocity which it already possesses—that is, in the direction of the tangent to the curve at the point where the stone was when the string was cut. The tension of the string, the effect of which is to pull the stone towards the centre of the circle and to cause the stone to move in its circular path, is called the *centripetal* or *central* force; the reaction of the stone upon the string, which is equal and opposite to this force, is called the *centrifugal* force. The amount of the forces may be arrived at as follows :—

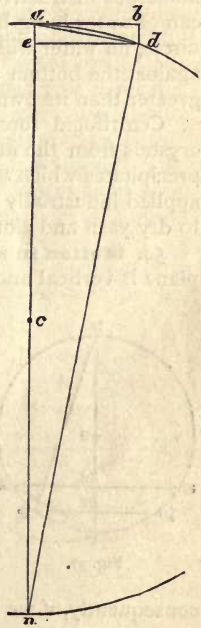

Fig. 36.

Let us suppose a body moving in a circle with given uniform velocity to be at the point *a* (fig. 36); then, had it not been acted on by a force in the direction *ac*, it would, in a small succeeding interval of time *t*, have continued to move in the direction of the tangent at *a*, and have passed through a distance which we will represent by *ab*. In consequence, however, of this force, it has not followed this direction, but has arrived at the point *d* on the curve; hence the force has made it traverse the distance $bd = ae$ in this interval. If *f* be the acceleration with which the body is drawn towards the centre $ae = \frac{1}{2}ft^2$, and if *ad* be very small, it may be taken as equal to *ab* or *vt*, where *v* is the velocity of the moving body. Now if *an* is the diameter of the circle, the triangle *adn* is inscribed in a semicircle and is right-angled, whence $ad^2 = ae \times an = ae \times 2r$. Substituting their values for *ad* and *ae* in this equation, we find that $v^2t^2 = \frac{1}{2}ft^2 \times 2r$, from which $f = \frac{v^2}{r}$; that is, in order that a body with a certain velocity may move in a circle, it must be drawn to the centre by a force which is directly as the square of the velocity with which the body moves, and which is inversely as the radius of the circle. In order to express this in the ordinary units of weight, we must multiply the above expression by the mass, which gives $F = \frac{mv^2}{r}$ or $\frac{Wv^2}{gr}$. To keep the body in a circle, an attraction towards the centre is needed, which is constantly equal to $\frac{mv^2}{r}$, and this attraction is constantly neutralised by the centrifugal force.

The above expression may be put in a form which is sometimes more convenient. If T be the time in seconds required to traverse the circum-

ference $2\pi r$ with the velocity v, then $v^2 = \frac{4\pi^2 r^2}{T^2}$, from which $F = \frac{4m\pi^2 r}{T^2}$ $= \frac{4W\pi^2 r}{gT^2}$.

If a rigid body rotates about a fixed axis, all parts of the body describe circumferences of various diameters, but all in the same time. The velocity of the motion of individual particles increases with the distance from the axis of rotation. By *angular velocity* is understood the velocity of a point at unit distance from the axis of rotation. If this is denoted by ω, the velocity v of a point at a distance from the axis is ωr, from which $\omega = \frac{v}{r} = \frac{2\pi}{T}$ and $F = r\omega^2$.

The existence of centrifugal force may be demonstrated by means of numerous instructive experiments, such as the *centrifugal railway*. If a small can of water hung by the handle to a string be rapidly rotated in a vertical circle, no water will fall out, for, at a suitable velocity, the liquid will press against the bottom of the vessel with a force at right angles to the circle and greater than its own weight.

Centrifugal force has been used in chemical laboratories to separate crystals from the mother-liquors, and also to promote the deposition of fine precipitates which under ordinary circumstances settle very slowly; it is also applied industrially in sugar factories to purify sugar from syrup, in dyeworks to dry yarn and cloth rapidly, and in laundries.

54. **Motion in a vertical circle.**—Let ACBD (fig. 37) be a circle whose plane is vertical and radius denoted by r. Suppose a point placed at A, and allowed to slide down the curve, what velocity will it have acquired on reaching any given point P? Draw the vertical diameter CD, join CA, CP, and draw the horizontal lines AMB and PNP′. Now, assuming the curve to be smooth, the velocity acquired in falling from A to P is that due to MN, the vertical height of A above P (51); if therefore v denote the velocity of the point at P, we shall have

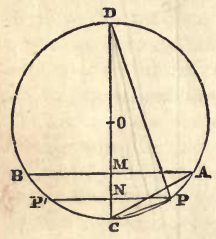

Fig. 37.

$$v^2 = 2gMN.$$

Now by similar triangles DCP, PCN, we have

$$DC : CP :: CP : CN;$$

consequently, if we denote by s the chord CP,

$$2rNC = s^2.$$

In like manner, if a denote the chord CA,

$$2rMC = a^2,$$

therefore
$$2rMN = a^2 - s^2,$$

and
$$v^2 = \frac{g}{r}(a^2 - s^2).$$

Now v will have equal values when s has the same value, whether positive or negative, and for any one value of s there are two equal values of v, one positive and one negative. That is to say, since CP′ is equal to CP, the body will have the same velocity at P′ that it has at P, and at any point the

body will have the same velocity whether it is going up the curve or down the curve. Of course it is included in this statement that if the body begins to move from B it will just ascend to a point A on the other side of C, such that A and B are in the same horizontal line. It will also be seen that at C the value of s is zero; consequently, if V is the velocity acquired by the body in falling from A to C, we have

$$V = a\sqrt{\frac{g}{r}};$$

and, on the other hand, if the body begins to move from C with a velocity V, it will reach a point A such that the chord AC or a is given by the same equation. In other words, the velocity at the lowest point is proportional to the chord of the arc described.

55. **Motion of a simple pendulum.**—By a simple pendulum is meant a heavy particle suspended by a fine thread from a fixed point, about which it oscillates without friction. So far as its changes of velocity are concerned, they will be the same as those of the point in the previous article, for the tension of the thread, acting at each position in a direction at right angles to that of the motion of the point, will no more affect its motion than the reaction of the smooth curve affects that of the point in the last article. The time of an oscillation—that is, the time in which the point moves from A to B—can be easily ascertained when the arc of vibration is small; that is, when the chord and the arc do not sensibly differ.

Thus, let AB (fig. 38) equal the arc or chord ACB (fig. 37); with centre C and radius AC or a describe a circle, and suppose a point to describe the circumference of that circle with a uniform velocity V or $a\sqrt{\frac{g}{r}}$. At any instant let the point be at Q, join CQ, draw the tangent QT, also draw QP at right angles and QN parallel to AB, then the angles NQT and CQP are equal. Now the velocity of Q resolved parallel to AB is V cos TQN or $a\sqrt{\frac{g}{r}}\cos^2 CQP$; that is, if CP equals s, the velocity of Q parallel to AB is

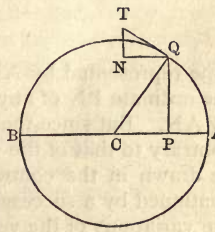

Fig. 38.

$$\sqrt{\frac{g}{r}}\,PQ \text{ or } \sqrt{\frac{g}{r}}(a^2 - s^2).$$

But if we suppose a point to move along AB in such a manner that its velocity in each position is the same as that of the oscillating body, its velocity at P would also equal $\sqrt{\frac{g}{r}}(a^2 - s^2)$; and, therefore, this point would describe AB in the same time that Q describes the semicircumference AQB. If then t be the required time of an oscillation, we have

$$t = \pi a \div a\sqrt{\frac{g}{r}} = \pi\sqrt{\frac{r}{g}}.$$

This result is independent of the length of the arc of vibration, provided its *amplitude*, that is AB, be small—not exceeding 4 or 5 degrees, for instance. It is evident from the formula that the time of a vibration is directly pro-

portional to the square root of the length of the pendulum, and inversely proportional to the square root of the accelerating force of gravity.

As an example of the use of the formula we may take the following :—It has been found that 39·13983 inches is the length of a single pendulum whose time of oscillation at Greenwich is one second ; the formula at once leads to an accurate determination of the accelerating force of gravity g ; for using feet and seconds as our units we have $t = 1$, $r = 3{\cdot}26165$, and π stands for the known number 3·14159 ; therefore the formula gives us

$$g = (3{\cdot}14159)^2 \times 3{\cdot}26165 = 32{\cdot}1912.$$

This is the value employed in (29).

Other examples will be met with in the Appendix.

56. **Graphic representation of the changes of velocity of an oscillating body.**—The changes which the velocity of a vibrating body undergoes may be graphically represented as follows :—Draw a line of indefinite length and mark off AH (fig. 39) to represent the time of one vibration, HH′ to represent the time of the second vibration, and so on. During the first vibration the velocity increases from zero to a maximum at the half-vibration, and then decreases during the second half-vibration from the maximum to zero. Consequently, a curved line or arc AQH may be drawn, whose ordinate QM at any point Q will represent the velocity of the body at the

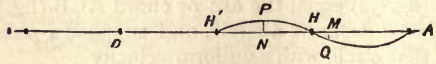

Fig. 39.

time represented by AM. If a similar curved line or arc HPH′ be drawn, the ordinate PN of any point P will represent the velocity at a time denoted by AN. But since the *direction* of the velocity in the second oscillation is contrary to that of the velocity in the first oscillation, the ordinate NP must be drawn in the contrary direction to that of MQ. If then the curve be continued by a succession of equal arcs alternately on opposite sides of AD, the variations of the velocity of the vibrating body will be completely represented by the varying magnitudes of the ordinates of successive points of the curve. The last article shows this to be the *curve of sines* for a pendulum.

57. **Impulsive forces.**—When a force acts on a body for an inappreciably short time, and yet sensibly changes its velocity, it is termed an *instantaneous* or *impulsive* force. Such a force is called into play when one body strikes against another. A force of this character is nothing but a finite though very large force, acting for a time so short that its duration is nearly, or quite, insensible. In fact, if M is the mass of the body, and the force contains Mf units, it will, in a time t, communicate a velocity ft ; now, however small t may be, Mf, and therefore f, may be so large that ft may be of sensible or even considerable magnitude. Thus if M contains a pound of matter, and if the force contains ten thousand units, though t were so short as to be only the $\frac{1}{10000}$ of a second, the velocity communicated by the force would be one of 10 feet per second. It is also to be remarked that the body will not sensibly move while this velocity is being communicated; thus in

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the case supposed, the body would only move through $\frac{1}{2}ft^2$ or the $\frac{1}{200}$ of a foot while the force acts upon it.

When one body impinges on another, it follows from the law of the equality of action and reaction (39) that whatever force the first body exerts upon the second, the second will exert an equal force upon the first in the opposite direction. Now forces are proportional to the momenta generated in the same time; consequently, these forces generate, during the whole or any part of the time of impact, in the bodies respectively, equal momenta with contrary signs; and therefore the sum of the momenta of the two bodies will remain constant during and at the end of the impact. It is of course understood that if the two bodies move in contrary directions their momenta have opposite signs, and the sum is an algebraical sum. In order to test the physical validity of this conclusion, Newton made a series of experiments, which may be thus briefly described—Two balls A and B (fig. 40) are hung from points C, D in the same horizontal line by threads in such a manner that their centres A and B are in the same horizontal line. With centre C and radius CA describe a semicircle EAF, and with centre D and radius DB describe a semicircle GBH, on the wall in front of which the balls hang. Let A be moved back to R, and be allowed to descend to A; it there impinges on B; both A and B will now move along the arcs AF and BH respectively; let A and B come to their highest points at r and k respectively. Now if V denote the velocity with which A reaches the lowest point, v and u the velocities with which A and B leave the lowest points after impact, and r the radius AC, it follows from (54) that

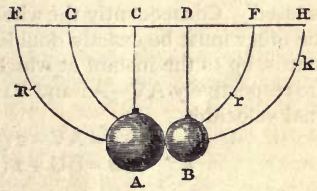


Fig. 40.

$$V = \text{chd AR}\sqrt{\frac{g}{r}}, v = \text{chd A}r\sqrt{\frac{g}{r}}, \text{ and } u = \text{chd B}k\sqrt{\frac{g}{r}};$$

therefore if A and B are the masses of the two balls, the momentum at the instant before impact was proportional to $A \times \text{chd AR}$, and the momentum after impact was proportional to $A \times \text{chd A}r + B \times \text{chd B}k$. Now when the position of the points R, r, and k had been properly corrected for the resistance of the air, it was found that these two expressions were equal to quantities so small that they could be properly referred to errors of observation. The experiment succeeded equally under every modification, whether A impinged on B at rest or in motion, and whatever the materials of A and B might be.

58. **Direct collision of two bodies.**—Let A and B be two bodies moving with velocities V and U respectively, along the same line, and let their mutual action take place in that line; if the one overtake the other, what will be their respective velocities at the instant after impact? We will answer this question in two extreme cases.

(i.) Let us suppose the bodies to be *quite inelastic*. In this case, when A touches B, it will continue to press against B until their velocities are equalised when the mutual action ceases. For whatever deformation the

bodies may have undergone, they have no tendency to recover their shapes. If, therefore, x is their common velocity after impact, we shall have $Ax + Bx$ their joint momentum at the end of impact, but their momentum before impact was $AV + BU$. Whence

$$(A + B)x = AV + BU,$$

an equation which determines x.

(ii.) Let us suppose the bodies *perfectly elastic*. In this case they recover their shapes, with a force exactly equal to that with which they were compressed. Consequently the whole momentum lost by the one and gained by the other must be exactly double of that lost while compression took place; that is, up to the instant at which their velocities were equalised. But these are respectively $AV - Ax$ and $Bx - BU$; therefore, if v and u are the required final velocities,

$$Av = AV - 2(AV - Ax) \text{ or } v = -V + 2x$$
$$Bu = BU + 2(Bx - BU) \text{ or } u = 2x - U;$$

hence $$(A + B)v = 2BU + (A - B)V,$$

and $$(A + B)u = 2AV - (A - B)U.$$

The following conclusion from these equations may be noticed: suppose a ball A, moving with a velocity V, to strike directly an equal ball B at rest. In this case $A = B$ and $U = 0$, consequently $v = 0$ and $u = V$; that is, the former ball A is brought to rest, and the latter B moves on with a velocity V. If now B strike on a third equal ball C at rest, B will in turn be brought to rest, and C will acquire the velocity V. And the same is true if there is a fourth, or fifth, or indeed any number of balls. This result may be shown with ivory balls, and is a very remarkable experiment.

59. **Work: meaning of the term.**—It has been pointed out (19, 26) that a moving body has no power of itself to change either the direction or the speed of its motion, and that, if any such change takes place, it is a proof that the body is acted upon by some external force. But although change of motion thus always implies the action of force, forces are often exerted without causing any change in the motion of the bodies on which they act. For instance, when a ship is sailing at a uniform speed, the force exerted on it by the wind causes no change in its motion, but simply prevents such a change being produced by the resistance of the water; or, when a railway-train is running with uniform velocity, the force of the engine does not change, but only maintains its motion in opposition to the forces, such as friction and the resistance of the air, which tend to destroy it.

These two classes of cases—namely, first, those in which forces cause a change of motion; and, secondly, those in which they prevent, wholly or in part, such a change being produced by other forces—include all the effects to which the action of forces can give rise. When acting in either of these ways, a force is said to *do work*: an expression which is used scientifically in a sense somewhat more precise, but closely accordant with that in which it is used in common language. A little reflection will make it evident that in all cases in which we are accustomed to speak of work being done—whether by men, horse-power, or steam-power, and however various the products may be in different cases—the physical part of the process consists

solely in producing or changing motion, or in keeping up motion in opposition to resistance, or in a combination of these actions. The reader will easily convince himself of this by calling to mind what the definite actions are which constitute the work done by (say) a navvy, a joiner, a mechanic, a weaver; that done by a horse, whether employed in drawing a vehicle or in turning a gin; or that of a steam-engine, whether it be used to drag a railway-train or to drive machinery. In all cases the work done is reducible, from a mechanical point of view, to the elements that have been mentioned, although it may be performed on different materials, with different tools, and with different degrees of skill.

It is, moreover, easy to see (comp. 53) that any possible change or motion may be represented as a gain by the moving body of an additional (positive or negative) velocity either in the direction of its previous motion, or at right angles to it; but a body which gains velocity is (27) said to be *accelerated*. Hence, what has been said above may be summed up as follows:—*When a force produces acceleration, or when it maintains motion unchanged in opposition to resistance, it is said to do* WORK.

60. **Measure of work.**—In considering how work is to be measured, or how the relation between different quantities of work is to be expressed numerically, we have, in accordance with the above, to consider, first, *work of acceleration*; and, secondly, *work against resistance*. But in order to make the evaluation of the two kinds of work consistent, we must bear in mind that one and the same exertion of force will result in work of either kind according to the conditions under which it takes place: thus, the force of gravity acting on a weight let fall from the hand causes it to move with a continually accelerated velocity until it strikes the ground; but if the same weight, instead of being allowed to fall freely through the air, be hung to a cord passing round a cylinder by means of which various degrees of friction can be applied to hinder its descent, it can be made to fall with a very small and practically uniform velocity. Hence, speaking broadly, it may be said that, in the former case, the work done by gravity upon the weight is work of acceleration only, while in the latter case it is work against resistance (friction) only. But it is very important to note that an essential condition, without which a force, however great, cannot do work either of one kind or the other, is that the thing acted on by it shall *move* while the force continues to act. This is obvious, for if no motion takes place it clearly cannot be either accelerated or maintained against resistance. The motion of the body on which a force acts being thus necessarily involved in our notion of work being done by the force, it naturally follows that, in estimating how much work is done, we should consider how much—that is to say, how far—the body moves while the force acts upon it. This agrees with the mode of estimating quantities of work in common life, as will be evident if we consider a very simple case—for instance, that of a labourer employed to carry bricks up to a scaffold: in such a case a double number of bricks carried would represent a double quantity of work done, but so also would a double height of the scaffold, for whatever amount of work is done in raising a certain number to a height of twenty feet, the same amount must be done again to raise them another twenty feet, or the amount of work done in raising the bricks forty feet is twice as great as that done when they are raised only

twenty feet. It is also to be noted that no direct reference to *time* enters into the conception of a quantity of work : if we want to know how much work a labourer has done, we do not ask how long he has been at work, but what he has done—for instance, how many bricks he has carried, and to what height ; and our estimate of the total amount of work is the same whether the man has spent hours or days in doing it.

The foregoing relations between force and work may be put into definite mathematical language as follows :—If the point of application of a force moves in a straight line, and if the part of the force resolved along this line acts in the direction of the motion, the product of that component and the length of the line is the work done by the force. If the component acts in the opposite direction to the motion, the component may be considered as a resistance, and the product is work done against the resistance. Thus, in (43), if we suppose a to move up the plane from R to S, the work done by P is P × RS : the work done against the resistance W is W sin x × RS. It will be observed that if the forces are in equilibrium during the motion, so that the velocity of a is uniform, P equals W sin x, and consequently the work done by the power equals that done against the resistance. Also, since RS sin x equals ST, the work done against the resistance equals W × ST. In other words, to raise W from R to S requires the same amount of work as to raise it from T to S.

If, however, the forces are not in equilibrium, the motion of a will not be uniform, but accelerated ; the work done upon it will nevertheless still be represented by the product of the resultant force resolved along the direction of motion into the distance through which it moves.

In order to ascertain the relation between the amount of work done and the change produced by it in the velocity of the moving mass, we must recall one or two elementary mechanical principles. Let F be the resultant force resolved along the direction of motion, and S the distance through which its point of application moves : then, according to what has been said, the work done by the force = FS. Further, it has been pointed out (29) that a constant force is measured by the momentum produced by it in a unit of time : hence, if T be the time during which the force acts, V the velocity of the mass M at the beginning of this period, and V_1 the velocity at the end, the momentum produced during the time T is $MV_1 - MV$, and consequently the momentum produced in a unit of time, or, in other words, the measure of the force, is

$$F = \frac{M(V_1 - V)}{T}.$$

The distance S through which the mass M moves while its velocity changes from the value V to the value V_1 is the same as if it had moved during the whole period T with a velocity equal to the average value of the varying velocity which it actually possesses. But a constant force acting upon a constant mass causes its velocity to change at a uniform rate ; hence, in the present case, the average velocity is simply the arithmetical mean of the actual and final velocities :

$$S = \tfrac{1}{2}(V_1 + V)T.$$

Combining this with the last equation, we get as the expression for the work done by the force F:

$$FS = \tfrac{1}{2}M(V_1^2 - V^2);$$

or, in words, *when a constant force acts on a mass so as to change its velocity, the work done by the force is equal to half the product of the mass into the change of the square of the velocity.*

The foregoing conclusion has been arrived at by supposing the force F to be constant, but it is easy to show that it holds good equally if F is the *average* magnitude of a force which varies from one part to another of the total distance through which it acts. To prove this, let the distance S be subdivided into a very great number n of very small parts, each equal to s, so that $ns = S$. Then, by supposing s to be sufficiently small, we may without any appreciable error consider the force as constant within each of these intervals, and as changing suddenly as its point of application passes from one interval to the next. Let F_1, F_2, F_3 . . . . F_n, be the forces acting throughout the 1st, 2nd, 3rd . . . . nth interval respectively, and let the velocity at the end of the same intervals be v_1, v_2, v_3, ... v_n $(= V_1)$ respectively; then, for the work done in the successive intervals, we have:

$$\begin{aligned}
F_1 s &= \tfrac{1}{2}M(v_1^2 - V^2)\\
F_2 s &= \tfrac{1}{2}M(v_2^2 - v_1^2)\\
F_3 s &= \tfrac{1}{2}M(v_3^2 - v_2^2)\\
&\cdot \qquad\qquad \cdot\\
&\cdot \qquad\qquad \cdot\\
&\cdot \qquad\qquad \cdot\\
F_n s &= \tfrac{1}{2}M(v_n^2 - v_{n-1}^2) = \tfrac{1}{2}M(V_1^2 - v_{n-1}^2),
\end{aligned}$$

or, for the total work,

$$(F_1 + F_2 + F_3 + \ldots\ldots + F_n)s = \tfrac{1}{2}M(V_1^2 - V^2);$$

where the quantity of the left-hand side of the equation may also be written $\dfrac{F_1 + F_2 + \ldots + F_n}{n}\, ns = FS$, if we put F to stand for the average (or arithmetical mean) of the forces F_1, F_2, &c.

An important special case of the application of the above formula arises when either the initial or the final velocity of the mass M is nothing; that is to say, when the effect of the force is to make a body pass from a state of rest into one of motion, or from a state of motion into one of rest. The general expression then assumes one of the following forms, namely :—

$$\begin{aligned}
FS &= \tfrac{1}{2}MV_1^2 \text{ or,}\\
-FS &= \tfrac{1}{2}MV^2;
\end{aligned}$$

the first of which denotes the quantity of work which must be done on a body of mass M in order to give to it the velocity V_1, while the second expresses the work that must be done in order to bring the same mass to rest when it is moving with the velocity V, the negative sign in the latter case showing that the force here acts *in opposition* to the actual motion, and is therefore to be regarded as a resistance.

In practice, the case which most frequently occurs is where work of acceleration and work against resistance are performed simultaneously. Thus,

recurring to the inclined plane already referred to in art. 43 ; if the force P (where P is the constant force with which the string pulls W up the plane) be greater than W sin x, the body W will move up the incline with a continually increasing velocity, and if the point of application of P be displaced from R to S, the total amount of work done, namely, P × RS, consists of a portion = W sin x RS done against the resistance of the weight W, and of a portion = (P − W sin x) RS expended in accelerating the weight. Hence, to determine the velocity v with which W arrives at the top of the incline, we have the equation

$$(P - W \sin x)\ RS = \tfrac{1}{2} W v^2 ;$$

for the portion of P which is in excess of what is required to produce equilibrium with the weight W, namely, P − W sin x, corresponds to the resultant force F supposed in the foregoing discussion, and RS to the distance through which this resultant force acts.

61. **Unit of work. Power.**—For strictly scientific purposes a unit of work is taken to be the work done by a unit of force when its point of application moves through one foot in the direction of its action ; but, as a convenient and sufficiently accurate standard for practical purposes, the quantity of work which is done in lifting 1 pound through the height of 1 foot is commonly adopted as the unit, and this quantity of work is spoken of as one 'foot-pound.' It is, however, important to observe that the foot-pound is not perfectly invariable, since the weight of a pound, and therefore the work done in lifting it through a given height, differs at different places, being a little greater near the Poles than near the Equator.

On the metrical system the *kilogrammetre* is the unit ; it is the work done when a weight of a kilogramme is raised through a height of a metre. This is equal to 7·23 foot-pounds, and one foot-pound = ·1383 of a kilogrammetre.

In estimating the usefulness of any motor it becomes necessary to know the time required by it for doing a given amount of work. The amount of work per second is the *power* of the motor. The unit of power is the power required to do a unit of work in a unit of time. For measuring the power of engines the unit used is the *horse-power*, which represents a rate of work of 33,000 foot-pounds per minute.

It is to be observed that in every case the unit is of the same denomination as the thing or quantity measured. The unit of length must be a length ; the unit of value must be a definite quantity of some valuable commodity. The numbers, to determine which is one of the objects of physical research, are to be considered as abstract numbers, representing how many times the unit is taken.

61*a*. **Systems of units.**—The units of mass, length, and time are said to be *fundamental* units, as all other units, such as those of area, velocity, acceleration, power, &c., are referred to them. These latter units are therefore called *derived* units. The magnitudes of the fundamental units are, however, arbitrary. A large class of writers use the centimetre, gramme, and second, and this system is usually called the *C.G.S. system* ; others use the foot, pound, and second. It thus becomes important to have a systematic method of reducing measurements from one system of units to another.

Let L, M, T represent respectively the magnitude or *dimensions* of the centimetre, the gramme, and the second, and L′, M′, T′ represent the dimensions of the foot, the pound, and the minute. Then, if a wire is found to be l cm. or l' ft. in length, its length may be represented either by lL or l'L′, and hence

$$l\mathrm{L} = l'\mathrm{L}', \text{ or } l = \frac{\mathrm{L}'}{\mathrm{L}} l'.$$

The ratio $\frac{\mathrm{L}'}{\mathrm{L}}$ is the length of a foot in centimetres, and has been found by direct comparison to be 30·4797. Hence any measurement, l' in feet, is converted into centimetres by multiplying l' by this number.

In a similar manner, if m and m' represent the number of units of mass in a piece of matter in the two systems,

$$m = \frac{\mathrm{M}'}{\mathrm{M}} m',$$

where the unit ratio is the number of grammes in a pound, or 453·59.

For converting a volume v' into the equivalent v,

$$(l'\mathrm{L}')^3 = (l\mathrm{L})^3, \text{ or } l^3 = \left(\frac{\mathrm{L}'}{\mathrm{L}}\right)^3 l'^3$$

or

$$v = \left(\frac{\mathrm{L}'}{\mathrm{L}}\right)^3 v';$$

For Density, $\frac{m}{l^3} = \mathrm{D}$

$$\frac{m}{l^3} \cdot \frac{\mathrm{M}}{\mathrm{L}^3} = \frac{m'}{l'^3} \cdot \frac{\mathrm{M}'}{\mathrm{L}'^3},$$

$$\mathrm{D} = \frac{\mathrm{M}'}{\mathrm{M}} \cdot \left(\frac{\mathrm{L}}{\mathrm{L}'}\right)^3 \mathrm{D}'.$$

Here the ratio $\frac{\mathrm{M}}{\mathrm{L}^3}$ is said to be a measure of the magnitude or dimensions of the unit of density, in terms of the dimensions of the fundamental units of mass and length. If a substance is said to have a unit density, then if M is the gramme and L^3 the cubic centimetre, the density of the substance would be that of water. If, however, M were the kilogramme and L^3 the cubic centimetre, the density would be a thousand times that of water. If, again, L^3 represents a cubic decimetre, and M the kilogramme, the density would again be that of water. It appears, then, that the magnitude of the unit of density is directly proportional to the magnitude of the unit of mass, and inversely as the magnitude of the unit of volume or the cube of the unit of length. As unit density is the density of a unit mass for unit volume, it is clear that $\frac{\mathrm{M}}{\mathrm{L}^3}$ measures the dimensions of the unit of density. Similar explanations apply in the succeeding cases.

For Velocity, $v = \frac{l}{t}$

$$\frac{l}{t} \cdot \frac{\mathrm{L}}{\mathrm{T}} = \frac{l'}{t'} \cdot \frac{\mathrm{L}}{\mathrm{T}'},$$

$$v = \frac{\mathrm{L}'}{\mathrm{L}} = \frac{\mathrm{T}}{\mathrm{T}'} \cdot v'$$

The ratio $\frac{T}{T'} = \frac{\text{second}}{\text{minute}} = \frac{1}{60}$.

If the units of time were the same, the unit factor $\frac{T}{T'} = 1$, and the velocity in centimetres would be

$$v = \frac{L'}{L} v',$$

where v' is the velocity in feet per second.

For Momentum, $mv = \frac{ml}{t}$

$$\frac{ml}{t} \cdot \frac{ML}{T} = \frac{m'l'}{t'} \cdot \frac{M'L'}{T'},$$

or

$$mv = \frac{M'L'T}{M\ L\ T'},\ m'v'.$$

For Acceleration, $a = \frac{v}{t} = \frac{l}{t^2}$

$$\frac{l}{t^2} \cdot \frac{L}{T^2} = \frac{l'}{t'^2} \cdot \frac{L'}{T'^2},$$

$$a = \frac{L'}{L} \left(\frac{T}{T'}\right)^2 \cdot a'$$

where a' is the acceleration in feet per minute.

For Force, $F = ma = \frac{ml}{t^2}$

$$\frac{ml}{t^2} \cdot \frac{ML}{T^2} = \frac{m'l'}{t'^2} \cdot \frac{M'L'}{T'^2},$$

$$F = \frac{M'}{M} \frac{L'}{L} \left(\frac{T}{T'}\right)^2 F'.$$

In the C.G.S. system the unit is called the *Dyne*.

For Work, $W = Fl = \frac{ml^2}{t^2}$

$$\frac{ml^2}{l^2} \cdot \frac{ML^2}{T^2} = \frac{m'l'^2}{t'^2} \cdot \frac{M'L'^2}{T'^2},$$

$$W = \frac{M'}{M} \left(\frac{L'}{L} \cdot \frac{T}{T'}\right)^2 W'.$$

In the C.G.S. system the unit of work is called the *Erg*.

Rate of Work, or Power, $P = \frac{Fl}{t} = \frac{ml^2}{t^3}$

$$\frac{ml^2}{t^3} \cdot \frac{ML^2}{T^3} = \frac{m'l'^2}{t'^3} \cdot \frac{M'L'^2}{T'^3}.$$

$$P = \frac{M'}{M} \left(\frac{L'}{L}\right)^2 \left(\frac{T}{T'}\right)^3 P'.$$

If work is expressed in foot-pounds or kilogramme-metres, the unit of

force being the weight of a pound or kilogramme, then to convert a certain number of foot-pounds into kilogramme-metres we have

$$wl \,.\, \mathrm{WL} = w'l' \,\mathrm{W'L'}.$$

$$\text{Work (kgr.-m)} = \left(\frac{\mathrm{W'}}{\mathrm{W}} \cdot \frac{\mathrm{L'}}{\mathrm{L}}\right) \text{work, foot-pounds,}$$

$$\text{where } \frac{\mathrm{W'}}{\mathrm{W}} = \frac{\text{pound}}{\text{kilogr.}} = 0{\cdot}4536$$

$$\frac{\mathrm{L'}}{\mathrm{L}} = \frac{\text{foot}}{\text{metre}} = 0{\cdot}3048,$$

the unit factor being thus 0·1383.

Similarly, to convert foot-pounds per minute into kilogr.-metres per second,

$$\mathrm{P} = \left(\frac{\mathrm{W'}}{\mathrm{W}} \frac{\mathrm{L'}}{\mathrm{L}} \frac{\mathrm{T}}{\mathrm{T'}}\right) \mathrm{P'},$$

where the conversion factor becomes 0·00230.

The units commonly used for measuring the power of engines are the *horse-power*, which is 33,000 times as great as the unit in which P′ of the last equation was measured, and the *force de cheval*, which is 75 times as great as the unit in which P was measured. Hence, if P′ is to be in horse-power, and P in *force de cheval*, the equation will become

$$\mathrm{P} = 0{\cdot}00230 \times \frac{33{,}000}{75} \mathrm{P'}$$

$$= 1{\cdot}0139 \,\mathrm{P'},$$

and hence one British horse-power = 1·0139 *force de cheval.*

These examples will be sufficient to indicate the method of converting measurements from one system of units to any other, and the treatment of other derived units may be deferred until they are needed.

62. **Energy.**—The fact that any agent is capable of doing work is usually expressed by saying that it possesses *Energy*, and the quantity of energy it possesses is measured by the amount of work it can do. For example, in the case of the inclined plane above referred to, the working power or energy of the force P is P × RS ; and if this force acts under the conditions last supposed, by the time its own energy is exhausted (in consequence of its point of application having arrived at S, the limit of the range through which it is supposed able to act), it has conferred upon the weight W a quantity of energy equal to that which has been expended ; for, in the first place, W has been raised through a vertical height equal to ST, and could by falling again through the same height do an amount of work represented by W × ST ; and, in the second place, W can do work by virtue of the velocity that has been imparted to it, and can continue moving in opposition to any given resistance R through a distance s, such that

$$\mathrm{R}s = \tfrac{1}{2}\mathrm{W}v^2.$$

The energy possessed by the mass M in consequence of having been raised from the ground is commonly distinguished as *energy of position* or *potential energy*, and is measured by the product of the force tending to cause motion into the distance through which the point of application of the force is

capable of being displaced in the direction in which the force acts. The energy possessed by a body in consequence of its velocity is commonly distinguished as *energy of motion*, or *kinetic energy* : it is measured by half the product of the moving mass into the square of its velocity.

63. **Varieties of energy.**—It will be seen, on considering the definition of *work* given above, that a force is said to do work when it produces any change in the condition of bodies ; for the only changes which, according to the definition of *force* given previously (26), a force is capable of producing, are changes in the state of rest or motion of bodies, and changes of their place in opposition to resistances tending to prevent motion or to produce motion in an opposite direction. There are, however, many other kinds of physical changes which can be produced under appropriate conditions, and the recent progress of investigation has shown that the conditions under which changes of all kinds occur are so far analogous to those required for the production of work by mechanical forces that the term *work* has come to be used in a more extended sense than formerly, and is now often used to signify the production of any sort of physical change.

Thus work is said to be done when a body at a low temperature is raised to a higher temperature, just as much as when a weight is raised from a lower to a higher level ; or, again, work is done when an electrical, magnetic, or chemical change is produced. This extension of the meaning of the term work involves a similar extension of the meaning of *energy*, which in this wider sense may be defined as the *capacity for producing physical change.*

As examples of energy in this more general sense, the following may be mentioned :—(*a*) The energy possessed by gunpowder in virtue of the mutual chemical affinities of its constituents, whereby it is capable of doing work by generating heat or by acting on a cannon-ball so as to change its state of rest into one of rapid motion ; (*b*) the energy of a charged Leyden jar, which, according to the way in which the jar is discharged, can give rise to changes of temperature, to changes of chemical composition, to mechanical changes, or to changes of magnetic or electrical condition ; (*c*) the energy of a red-hot ball, which, amongst other effects it is capable of producing, can raise the temperature and increase the volume of bodies colder than itself, or can change ice into water or water into steam ; the energy of the stretched string of a bow : here work has been consumed in stretching the string ; when it is released the work reappears in the velocity imparted to the arrow.

64. **Transformation of energy.**—It has been found by experiment that when one kind of energy disappears or is expended energy of some other kind is produced, and that, under proper conditions, the disappearance of any one of the known kinds of energy can be made to give rise to a greater or less amount of any other kind. One of the simplest illustrations that can be given of this transformation of energy is afforded by the oscillations of a pendulum. When the pendulum is at rest in its lowest position it does not possess any energy, for it has no power of setting either itself or other bodies in motion, or of producing in them any kind of change. In order to set the pendulum oscillating, work must be done upon it, and it thereafter possesses an amount of energy corresponding to the work that has been expended.

When it has reached either end of its path, the pendulum is for an instant at rest; but it possesses energy by virtue of its position, and can do an amount of work while falling to its lowest position, which is represented by the product of its weight into the vertical height through which its centre of gravity descends. When at the middle of its path, the pendulum is passing through its position of equilibrium, and has no power of doing work by falling lower; but it now possesses energy by virtue of the velocity which it has gained, and this energy is able to carry it up on the second side of its lowest position to a height equal to that from which it has descended on the first side. By the time it reaches this position the pendulum has lost all its velocity, but it has regained the power of falling; this, in its turn, is lost as the pendulum returns again to its lowest position, but at the same time it regains its previous velocity. Thus, during every quarter of an oscillation the energy of the pendulum changes from potential energy of position into actual energy or energy of motion, or *vice versâ*.

A more complex case of the transformation of energy is afforded by a thermo-electric pile, the terminals of which are connected by a conducting wire: the application of energy in the form of heat to one face of the pile gives rise to an electric current in the wire, which, in its turn, reproduces heat, or by proper arrangements can be made to produce chemical, magnetic, or mechanical effects, such as those described below in the chapters on Electricity.

It has also been found that the transformations of energy always take place according to fixed proportions. For instance, when coal or any other combustible is burned, its chemical energy, or power of combining with oxygen, vanishes, and heat or thermal energy is produced, and the quantity of heat produced by the combustion of a given amount of coal is fixed and invariable. If the combustion takes place under the boiler of a steam-engine, mechanical work can be obtained by the expenditure of part of the heat produced, and here again the quantitative relation between the heat expended and the work gained in place of it is perfectly constant.

65. **Conservation of energy.**—Another result of great importance, which has been arrived at by experiment, is that the total amount of energy possessed by any system of bodies is unaltered by any transformations arising from the action of one part of the system upon another, and can only be increased or diminished by effects produced on the system by external agents. In this statement it is of course understood that in reckoning the sum of the energy of various kinds which the system may possess, those amounts of the different forms of energy which are mutually convertible into each other are taken as being numerically equal; or, what comes virtually to the same thing, the total energy of the system is supposed to be reduced—either actually, or by calculation from the known ratio of transformation of the various forms of energy—to energy of some one kind; then the statement is equivalent to this: that the total energy of any one form to which the energy of a given system of bodies is reducible is unalterable so long as the system is not acted on from without. Practically it is always possible, in one way or another, to convert the whole of the energy possessed by any body or system of bodies into heat, but it cannot be all converted without loss into any other form of energy; hence the principle stated at the beginning of this article can be

enunciated in the closest conformity with the direct results of experiment by saying that, so long as any system of bodies is not acted on from without, the total quantity of heat that can be obtained from it is unalterable by any changes which may go on within the system itself. For instance, a quantity of air compressed into the reservoir of an air-gun possesses energy which is represented partly by the heat which gives to it its actual temperature above the absolute zero (460), and partly by the work which the air can do in expanding. This latter portion can be converted into heat in various ways, as, for example, by allowing the air to escape through a system of capillary tubes so fine that the air issues from them without any sensible velocity; if, however, the expanding air be employed to propel a bullet from the gun, it produces considerably less heat than in the case previously supposed, the deficiency being represented for a time by the energy of the moving bullet, but reappearing in the form of heat in the friction of the bullet against the air, and, when the motion of the bullet is destroyed, by striking against an inelastic obstacle at the same level as the gun. But whatever the mode and however numerous the intermediate steps by which the energy of the compressed air is converted into heat, the total quantity of heat finally obtainable from it is the same.

BOOK II

GRAVITATION AND MOLECULAR ATTRACTION

CHAPTER I

GRAVITY. CENTRE OF GRAVITY. THE BALANCE

66. **Universal attraction: its laws.**—*Universal attraction* is a force in virtue of which the material particles of all bodies tend incessantly to approach each other; it is a mutual action, however, which all bodies, at rest or in motion, exert upon one another, no matter how great or how small the space between them may be, or whether this space be occupied or unoccupied by other matter.

A vague hypothesis of the tendency of the matter of the earth and stars to a common centre was adopted even by Democritus and Epicurus. Kepler assumed the existence of a mutual attraction between the sun, the earth, and the other planets. Bacon, Galileo, and Hooke also recognised the existence of universal attraction. But Newton was the first who established the law, and the universality of gravitation.

Since Newton's time the attraction of matter by matter was experimentally established by Cavendish. This eminent English physicist succeeded, by means of a delicate torsion balance (89), in rendering visible the attraction between a large leaden and a small copper ball.

The attraction between any two bodies is the resultant of the attractions of each molecule of the one upon every molecule of the other according to the law of Newton, which may be thus expressed: *the attraction between two material particles is directly proportional to the product of their masses and inversely proportional to the square of their distances asunder.* To illustrate this, we may take the case of two spheres, which, owing to their symmetry, attract each other just as if their masses were concentrated in their centres. If without other alteration the mass of one sphere were doubled, tripled, &c., the attraction between them would be doubled, tripled, &c. If, however, the mass of one sphere being doubled, that of the other were increased three times, the distance between their centres remaining the same, the attraction would be increased six times. Lastly, if, without altering their masses, the distance between their centres were *increased* from 1 to 2, 3, 4 . . . . units, the attraction would be *diminished* to the 4th, 9th,

16th . . . . part of its former intensity. In short, if we define the unit of attraction as that which would exist between two units of mass whose distance asunder was the unit of length, the attraction of two molecules having the masses m and m', at the distance r, would be expressed by $\frac{mm'}{r^2}$.

67. **Terrestrial gravitation.**—The tendency of any body to fall towards the earth is due to the mutual attraction of that body and the earth, or to terrestrial gravitation, and is, in fact, merely a particular case of universal attraction.

At any point of the earth's surface, the direction of gravity—that is, the line which a falling body describes—is called the *vertical* line. The vertical lines drawn at different points of the earth's surface converge very nearly to the earth's centre. For points situated on the same meridian the angle contained between the vertical lines equals the difference between the latitudes of those points.

The directions of the earth's attraction upon neighbouring bodies, or upon different molecules of one and the same body, must, therefore, be considered as parallel, for the two vertical lines form the sides of a triangle whose vertex is near the earth's centre, about 4,000 miles distant, and whose base is the small distance between the molecules under consideration.

A plane or line is said to be *horizontal* when it is perpendicular to the vertical line.

The vertical line at any point of the globe is generally determined by the *plumb-line* (fig. 41), which consists of a weight attached to the end of a string. It is evident that the weight cannot be in equilibrium unless the direction of the earth's attraction upon it passes through the point of support, and therefore coincides with that of the string.

Fig. 41.

The horizontal plane is also determined with great ease, since it coincides, as will be afterwards shown, with the *level* surface of every liquid when in a state of equilibrium.

When the mean figure of the earth has been approximately determined, it becomes possible to compare the direction of the plumb-line at any place with that of the normal to the mean figure at that place. When any difference in these directions can be detected, it constitutes a *deviation* of the plumb-line, and is due to the attraction of some great mass of matter in the neighbourhood, such as a mountain. Thus, in the case of the mountain of Schehallien, in Perthshire, it was found by Dr. Maskelyne that the angle between the directions of two plumb-lines, one at a station to the north, and the other to the south, of the mountain was greater by 11″·6 than the angle between the normals of the mean surface of the earth at those points; in other words, each plumb-line was deflected by about 6″ towards the mountain. By calculating the volume and mass of the mountain, it was inferred from this observation that the mean density of the mountain was to that of the earth in the ratio of 5 : 9, and that the mean density of the earth is about five times that of water—a

result agreeing pretty closely with that deduced from Cavendish's experiment referred to in the last article.

68. **Centre of gravity; its experimental determination.**—Into whatever position a body may be turned with respect to the earth, there is a certain point, invariably situated with respect to the body, through which the resultant of the attracting forces between the earth and its several molecules always passes. This point is called the *centre of gravity*; it may be within or without the body, according to the form of the latter; its existence, however, is easily established by the following considerations: let $m\,m'm''\,m'''$. . . . (fig. 42) be molecules of any body. The earth's attraction upon these molecules will constitute a system of parallel forces, having a common vertical direction, whose resultant will be found by seeking first the resultant of the forces which act on any two molecules, m and m', then that of this resultant and a third force acting on m'', and so on until we arrive at the final resultant W, representing the weight of the body and applied at a certain point G. If the body be now turned into the position shown in fig. 43, the molecules m, m', m''. . . . will continue to be acted on by the

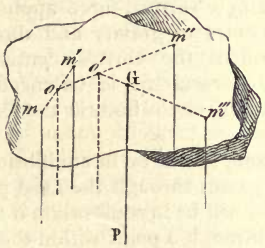


Fig. 42.

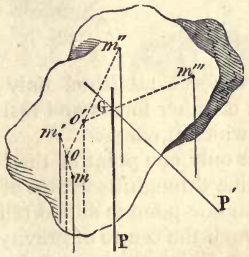


Fig. 43.

same forces as before, the resultant of the forces on m and m' will pass through the same point o in the line mm', the following resultant will again pass through the same point o' in om'', and so on up to the final resultant P, which will still pass through the same point C, which is the *centre of gravity*.

To find the centre of gravity of a body is a purely geometrical problem; in many cases, however, it can be at once determined. For instance, the centre of gravity of a right line of uniform density is the point which bisects its length; in the circle and sphere it coincides with the geometrical centre; in cylindrical bars it is the middle point of the axis. The centre of gravity of a plane triangle is in the line which joins any vertex with the middle of the opposite side, and at a distance from the vertex equal to two-thirds of this line: in a cone or pyramid it is in the line which joins the vertex with the centre of gravity of the base, and at a distance from the vertex equal to three-fourths of this line. These rules, it must be remembered, presuppose that the several bodies are of uniform density.

In order to determine experimentally the centre of gravity of a body, it is suspended by a string in two different positions, as shown in figs. 44 and 45; the point where the directions AB and CD of the string in the two

experiments intersect each other is the centre of gravity required. For, the resultant of the earth's attraction being a vertical force applied at the centre of gravity, the body can only be in equilibrium when the point lies vertically under the point of suspension ; that is, in the prolongation of the suspended string. But the centre of gravity, being in AB as well as in CD, must coincide with the point of intersection of these two lines.

Fig. 44.

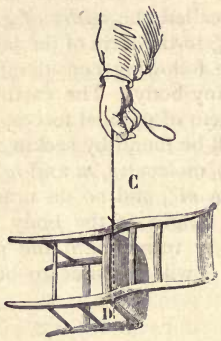


Fig. 45.

The centre of gravity of a thin piece of cardboard of irregular shape, for instance, may be found by balancing it in two positions on a knife-edge ; the centre of gravity will then lie in the intersection of the two lines.

69. **Equilibrium of heavy bodies.**—Since the action of gravity upon a body reduces itself to a single vertical force applied at the centre of gravity and directed towards the earth's centre, equilibrium will be established only when this resultant is balanced by the resultant of other forces and resistances acting on the body at the fixed point through which it passes.

When only one point of the body is fixed, it will be in equilibrium if the vertical line through its centre of gravity passes through the fixed point. If more than one point is supported, the body will be in equilibrium if a vertical line, through the centre of gravity, passes through a point within the polygon formed by joining the points of support.

The Leaning Tower of Pisa continues to stand because the vertical line drawn through its centre of gravity passes within its base.

It is easier to stand on our feet than on stilts, because in the latter case the smallest motion is sufficient to cause the vertical line through the centre of gravity of our bodies to pass outside the supporting base, which is here reduced to a mere line joining the feet of the stilts. A man carrying a load on his back must lean forward : if he carries it in the left hand he must incline the upper part of his body to the right, for otherwise the centre of gravity of the body and of the load would fall outside the line joining the feet and he would fall. Again, it is impossible to stand on one leg if we keep one side of the foot and head close to a vertical wall, because the latter prevents us from throwing the body's centre of gravity vertically above the supporting base.

70. **Different states of equilibrium.**—Although a body supported by a fixed point is in equilibrium whenever its centre of gravity is in the vertical line through that point, the fact that the centre of gravity tends incessantly to occupy the lowest possible position leads us to distinguish between three states of equilibrium—*stable*, *unstable*, *neutral*.

A body is said to be in *stable equilibrium* if it tends to return to its first position after the equilibrium has been slightly disturbed. Every body is in

this state when its position is such that the slightest alteration of the same elevates its centre of gravity; for the centre of gravity will descend again when permitted, and after a few oscillations the body will return to its original position.

The pendulum of a clock continually oscillates about its position of stable equilibrium, and an egg on a level table is in this state when its long axis is horizontal. We have another illustration in the toy represented in the adjoining fig. 46. A small figure cut in ivory is made to stand on one foot at the top of a pedestal by being loaded with two leaden balls, *a*, *b*, placed sufficiently low to throw the centre of gravity, *g*, of the whole compound body below the foot of the figure. After being disturbed, the little figure oscillates like a pendulum, having its point of suspension at the toe, and its centre of gravity at a lower point, *g*.

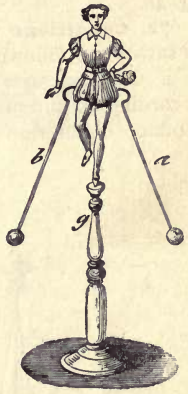


Fig. 46.

A body is said to be in *unstable equilibrium* when, after the slightest disturbance, it tends to depart still more from its original position. A body is in this state when its centre of gravity is vertically above the point of support, or higher than it would be in any adjacent position of the body. An egg standing on its end, or a stick balanced upright on the finger, is in this state.

Lastly, if in any adjacent position a body still remains in equilibrium, its state of equilibrium is said to be *neutral*. In this case an alteration in the position of the body neither raises nor lowers its centre of gravity. A perfect sphere resting on a horizontal plane is in this state.

Fig. 47 represents three cones, A, B, C, placed respectively in stable, unstable, and neutral equilibrium upon a horizontal plane. The letter *g* in each shows the position of the centre of gravity.

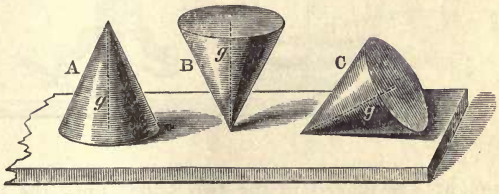


Fig. 47.

71. **The balance.**—The balance is an instrument for determining the relative weights or masses of bodies. There are many varieties.

The ordinary balance (fig. 48) consists of a lever of the first kind, called the *beam*, AB, with its fulcrum in the middle; at the extremities of the beam are suspended two scale-pans, C and D, one intended to receive the object to be weighed, and the other the counterpoise. The fulcrum consists of a steel prism, *n*, commonly called a *knife-edge*, which passes through the beam, and rests with its sharp edge, or *axis of suspension*, upon two supports; these are formed of agate, in order to diminish the friction. A needle or pointer is fixed to the beam, and oscillates with it in front of a graduated arc, *a*: when the beam is perfectly horizontal the needle points to the zero of the graduated arc.

Since by (40) two equal forces in a lever of the first kind cannot be in

equilibrium unless their leverages are equal, the length of the arms *n*A and *n*B ought to remain equal during the process of weighing. To secure this the scales are suspended from hooks, whose curved parts have sharp edges, and rest on similar edges at the ends of the beam. In this manner the scales are in effect supported on mere points, which remain unmoved during the oscillations of the beam. This mode of suspension is represented in fig. 48.

72. **Conditions to be satisfied by a balance.**—A good balance ought to satisfy the following conditions :—

i. *The two arms of the beam ought to be precisely equal*; otherwise, according to the principle of the lever, unequal weights will be required to produce equilibrium. To test whether the arms of the beam are equal,

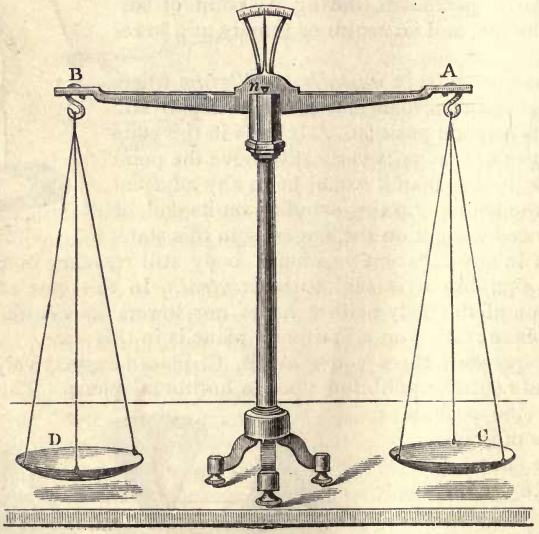


Fig. 48.

weights are placed in the two scales, until the beam becomes horizontal : the contents of the scales being then interchanged, the beam will remain horizontal if its arms are equal, but if not, it will descend on the side of the longer arm.

ii. *The balance ought to be in equilibrium when the scales are empty*, for otherwise unequal weights must be placed in the scales in order to produce equilibrium. It must be borne in mind, however, that the arms are not necessarily equal, even if the beam remains horizontal when the scales are empty; for this result might also be produced by giving to the longer arm the lighter scale.

iii. *The beam being horizontal, its centre of gravity ought to be in the same vertical line with the edge of the fulcrum, and a little below the latter*, for otherwise the beam would not be in stable equilibrium (70).

The effect of changing the position of the centre of gravity may be shown by means of a beam (fig. 49), whose fulcrum, being the nut of a screw, *a*, can be raised or lowered by turning the screw-head, *b*.

When the fulcrum is at the top of the groove *c*, in which it slides, the centre of gravity of the beam is below its edge, and the latter oscillates

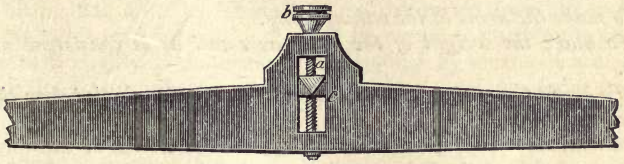


Fig. 49.

freely about a position of stable equilibrium. By gradually lowering the fulcrum its edge may be made to pass through the centre of gravity of the beam when the latter is in neutral equilibrium; that is to say, it no longer oscillates, but remains in equilibrium in all positions. When the fulcrum is lowered still more, the centre of gravity passes above its edge, the beam is in a state of unstable equilibrium, and is overturned by the least displacement.

73. **Delicacy of the balance.**—A balance is said to be *delicate* when a very small difference between the weights in the scales causes a perceptible deflection of the pointer.

Let A and B (figs. 50 and 51) be the points from which the scale-pans are suspended, and C the axis of suspension of the beam. A, B, and C are

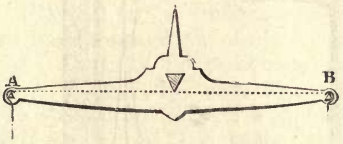


Fig. 50.

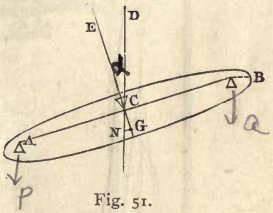


Fig. 51.

assumed to be in the same straight line, according to the usual arrangement. Suppose weights P and Q to be in the pans, suspended from A and B respectively, and let G be the centre of gravity of the beam; then the beam will come to rest in the position shown in the figure, where the line DCN is vertical, and ECG is the direction of the pointer. According to the above statement, the greater the angle ECD for a given difference between P and Q, the greater is the delicacy of the balance. Draw GN at right angles to CG.

Let W be the weight of the beam; then from the properties of the lever (40) it follows that measuring moments with respect to C, the moment of P equals the sum of the moments of Q and W—a condition which at once leads to the relation

$$(P - Q)AC = W \times GN.$$

Now it is clear that for a given value of CG the angle GCN (that is, ECD, which measures the delicacy) is greater as GN is greater; and from the formula it is clear that for a given value of P – Q we shall have GN greater as AC is greater, and as W is less. Again, for a given value of GN the angle GCN is greater as GC is less. Hence the means of rendering a balance delicate are—

i. *To make the arms of the balance long.*

ii. *To make the weight of the beam as small as is consistent with its rigidity.*

iii. *To bring the centre of gravity of the beam a very little below the point of support.*

Moreover, since friction will always oppose the action of the force that tends to preponderate, the balance will be rendered more delicate by diminishing friction. To secure this advantage, the edges from which the beam and scales are suspended are made as sharp and as hard as possible, and the supports on which they rest are very smooth and hard. This is effected by the use of agate knife-edges. And, further, the pointer is made long, since its elongation renders a given deflection more perceptible by increasing the arc which its end describes.

The sensitiveness of a balance is expressed by the ratio of the smallest weight, which will produce a measurable deflection of the pointer, to the load.

74. **Physical and chemical balances.**—Fig. 52 represents one of the accurate balances ordinarily used for chemical analysis. Its sensitiveness is

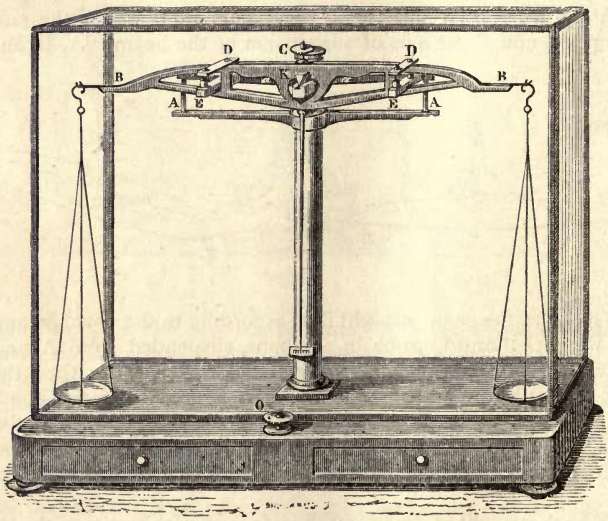


Fig. 52.

such that when charged with a kilogramme (1,000 grms.) in each scale an excess of a tenth of a milligramme ($\frac{1}{10000}$ of a grm.) in either scale produces a very perceptible deflection of the index.

In order to protect the balance from air currents, dust, and moisture, it is always, even when weighing, surrounded by a glass case, whose front slides up and down, to enable the operator to introduce the objects to be weighed. Where extreme accuracy is desired the case is constructed so that the space may be exhausted, and the weighing made *in vacuo*.

In order to preserve the edge of the fulcrum as much as possible, the whole beam, BB, with its fulcrum K, can be raised from the support on which the latter rests by simply turning the button O outside the case.

The horizontality of the beam is determined by means of a long index, which points downwards to a graduated arc near the foot of the supporting pillar. Lastly, the button C serves to alter the sensitiveness of the balance; by turning it, the centre of gravity of the beam can be made to approach or recede from the fulcrum (69).

75. **Method of double weighing.**—Even if a balance be not perfectly accurate, the true weight of a body may still be determined by its means. To do so, the body to be weighed is placed in one scale, and shot or sand poured into the other until equilibrium is produced; the body is then replaced by known weights until equilibrium is re-established. The sum of these weights will necessarily be equal to the weight of the body, for, acting under precisely the same circumstances, both have produced precisely the same effect.

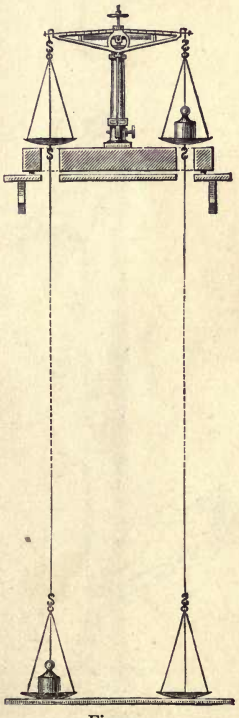


Fig. 53.

The exact weight of a body may also be determined by placing it successively in the two pans of a balance, and then deducing its true weight.

For having placed in one pan the body to be weighed, whose true weight is x, and in the other the weight p, required to balance it, let a and b be the arms of levers corresponding to x and p. Then from the principle of the lever (40) we have $ax = pb$. Similarly, if p_1 is the weight when the body is placed in the other pan, then $bx = ap_1$. Hence $abx^2 = abpp_1$, from which $x = \sqrt{pp_1}$. This method was invented by Père Amiot, but is ordinarily known as *Borda's Method.*

Jolly made use of a very delicate balance to determine the constant of gravity. The balance (fig. 53) was placed in a room in the tower of the University of Munich, and to each of the scale-pans was attached, by a wire 21 metres in length, a second scale-pan. A mass of mercury of 5 kilogrammes contained in a glass vessel was first counterpoised in the upper scale-pans; it was then moved to the lower one, and it was found necessary to add 31·683 mgr. to the upper pan in order to counterbalance the increase in attractiveness due to the greater force in the lower pan.

Taking the radius of the earth at Munich at 6,365,722 metres, the number

calculated from the formula in (82) is 33 mgr.; a sufficiently close result when the difficulties of the experiments are taken into account.

A large lead sphere was then placed immediately below the mass in the lower pan, and produced a measurable attraction. From the attraction thus produced by the known mass of the lead it was possible to deduce the mass and the mean density of the earth (67); the number obtained was 5·69. Similar experiments have been made by Prof. Poynting and have led to the same number.

CHAPTER II

LAWS OF FALLING BODIES. INTENSITY OF TERRESTRIAL GRAVITY THE PENDULUM

76. **Laws of falling bodies.**—Since a body falls to the ground in consequence of the earth's attraction on *each* of its molecules, it follows that, everything else being the same, all bodies, great and small, light and heavy, ought to fall with equal rapidity, and a lump of sand without cohesion should during its fall retain its original form as perfectly as if it were compact stone. The fact that a stone falls more rapidly than a feather is due solely to the unequal resistances opposed by the air to the descent of these bodies; *in a vacuum all bodies fall with equal rapidity.* To demonstrate this by experiment a glass tube about two yards long (fig. 54) may be taken, having one of its ends completely closed, and a brass cock fixed to the other. After having introduced bodies of different weights and densities (pieces of lead, paper, feather, &c.) into the tube, the air is withdrawn from it by an air-pump, and the cock closed. If the tube be now suddenly reversed, all the bodies will fall equally quickly. On introducing a little air and again inverting the tube, the lighter bodies become slightly retarded, and this retardation increases with the quantity of air introduced.

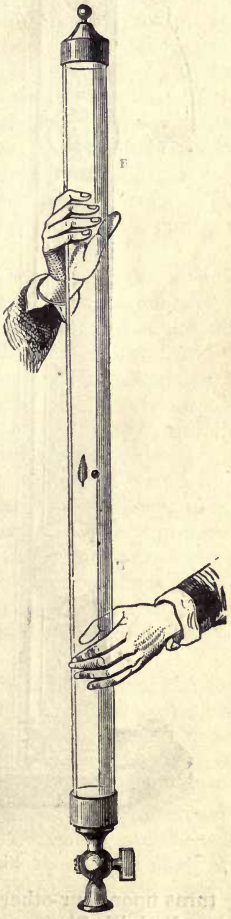


Fig. 54.

The resistance opposed by the air to falling bodies is especially remarkable in the case of liquids. The Staubbach in Switzerland is a good illustration; an immense mass of water is seen falling over a high precipice, but before reaching the bottom it is shattered by the air into the finest mist. In a vacuum, however, liquids fall like solids without separation of their molecules. The *water-hammer* illustrates this: the instrument consists of a thick glass tube about a foot long, half filled with water, the air having been expelled by ebullition previous to closing one extremity with the blow-pipe. When such a tube is suddenly inverted, the water falls in one undivided mass against the

other extremity of the tube, and produces a sharp dry sound, resembling that which accompanies the shock of two solid bodies.

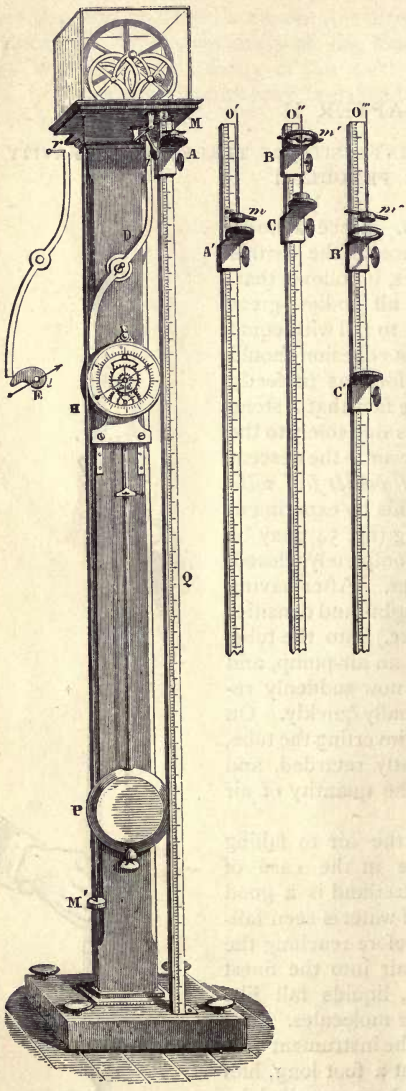


Fig. 55.

From Newton's law (66) it follows that when a body falls to the earth the force of attraction which causes it to do so increases as the body approaches the earth. Unless the height from which the body falls, however, be very great, this increase will be altogether inappreciable, and the force in question may be considered as constant and continuous. If the resistance of the air were removed, therefore, the motion of all bodies falling to the earth would be uniformly accelerated, and would obey the laws already explained (49).

77. **Atwood's machine.**— Several instruments have been invented for illustrating and experimentally verifying the laws of falling bodies. Galileo, who discovered these laws in the early part of the seventeenth century, illustrated them by means of bodies falling down inclined planes. The great object of all such instruments is to diminish the rapidity of the fall of bodies without altering the character of their motion, for by this means their motion may not only be better observed, but it will be less modified by the resistance of the air (48).

The most convenient instrument of this kind is that invented by Atwood at the end of the last century, and represented in fig. 55. It consists of a stout pillar of wood, about 2½ yards high, at the top of which is a brass pulley, whose axle rests and turns upon four other wheels, called *friction wheels*, inasmuch as they serve to diminish friction. Two equal weights, M and M′, are attached to the

extremities of a fine silk thread, which passes round the pulley ; a timepiece, H, fixed to the pillar, is regulated by a seconds pendulum, P, in the usual way ; that is to say, the oscillations of the pendulum are communicated to a ratchet, whose two teeth, as seen in the figure, fit into those of the ratchet wheel. The axle of this wheel gives motion to the seconds hand of the dial, and also to an eccentric behind the dial, as shown at E by a separate figure. This eccentric plays against the extremity of a lever, D, which it pushes until the latter no longer supports the small plate *i* ; and thus the weight M, which at first rested on this plate, is suddenly exposed to the free action of gravity. The eccentric is so constructed that the little plate *i* falls precisely when the hand of the dial points to zero.

The weights M and M′, being equal, hold each other in equilibrium ; the weight M, however, is made to descend slowly by putting a small bar or overweight *m* upon it ; and, to measure the spaces which it describes, the rod or scale Q is divided into feet and inches, commencing from the plate *i*. To complete the instrument there are a number of plates, A, A′, C, C′, and a number of rings, B, B′, which may be fixed by screws at any part of the scale. The plates arrest the descending weight M, the rings only arrest the bar or overweight *m*, which was the cause of motion, so that after passing through them the weight M, in consequence of its inertia, will move on uniformly with the velocity it had acquired on reaching the ring. The several parts of the apparatus being described, a few words will suffice to explain the method of experimenting.

Let the hand of the dial be placed behind the zero point, the lever D adjusted to support the plate *i*, on which the weight M with its overweight *m* rests, and the pendulum put in motion. As soon as the hand of the dial points to zero the plate *i* will fall, the weights M and *m* will descend, and by a little attention and a few trials it will be easy to place a plate A so that M may reach it exactly as the dial indicates the expiration of one second. To make a second experiment let the weights M and *m*, the plate *i*, and the lever D be placed as at first ; remove the plate A, and in its place put a ring, B, so as to arrest the overweight *m* just when the weight M would have reached A ; on putting the pendulum in motion again it will be easy, after a few trials, to put a plate, C, so that the weight M may fall upon it precisely when the hands of the dial point to two seconds. Since the overweight *m* in this experiment was arrested by the ring B at the expiration of one second, the space BC was described by M in one second purely in virtue of its own inertia, and consequently by (24) BC will indicate the velocity of the falling mass at the expiration of one second

Proceeding in the same manner as before, let a third experiment be made in order to ascertain the point B′ at which the weights M and *m* arrive after the lapse of two seconds, and putting a ring at B′, ascertain by a fourth experiment the point C′ at which M arrives alone, three seconds after the descent commenced ; B′C′ will then express the velocity acquired after a descent of two seconds. In a similar manner, by a fifth and sixth experiment, we may determine the space OB″ described in three seconds, and the velocity B″C″ acquired during those three seconds, and so on ; we shall find that B′C′ is twice, and B″C″ three times as great as BC—in other words, that the velocities BC, B′C′, B″C″ increase in the same proportion as the

times (1, 2, 3, . . . seconds) employed in their acquirement. By the definition (49), therefore, the motion is uniformly accelerated. The same experiments will also serve to verify and illustrate the four laws of uniformly accelerated motion as enunciated in (49). For example, the spaces OB, OB′, OB″, ... described from a state of rest in 1, 2, 3, ... seconds, will be found to be proportional to the numbers 1, 4, 9 . . . ; that is to say, to the squares of those numbers of seconds, as stated in the third law.

Lastly, if the overweight *m* be changed, the acceleration or velocity BC acquired per second will also be changed, and we may easily verify the assertion in (27), that force is proportional to the product of the mass moved, into the acceleration produced in a given time. For instance, assuming the pulley to be so light that its inertia can be neglected, then if *m* weighed half an ounce, and M and M′ each $15\frac{3}{4}$ ounces, the acceleration BC would be found to be six inches ; whilst if *m* weighed one ounce, and M and M′ each $63\frac{1}{2}$ ounces, the acceleration BC would be found to be three inches.

Now in these cases the forces producing motion, that is the overweights, are in the ratio of 1 : 2 ; while the products of the masses and the accelerations are in the ratio of $(\frac{1}{2} + 15\frac{3}{4} + 15\frac{3}{4}) \times 6$ to $(1 + 63\frac{1}{2} + 63\frac{1}{2}) \times 3$; that is, they are also in the ratio 1 : 2. Now the same result is obtained in whatever way the magnitudes of *m*, M, and M′ are varied, and consequently in all cases the ratio of the forces producing motion equals the ratio of the momenta generated.

78. **Morin's apparatus.**—The principle of this apparatus, the original idea of which is due to General Poncelet, is to make the falling body trace its own path. Fig. 56 gives a view of the whole apparatus, and fig. 57 gives the details. The apparatus consists of a wooden framework, about 7 feet high, which holds in a vertical position a very light wooden cylinder, M, which can turn freely about its axis. This cylinder is coated with paper divided into squares by equidistant horizontal and vertical lines. The latter measure the path traversed by the body falling along the cylinder, while the horizontal lines are intended to divide the duration of the fall into equal parts.

The falling body is a mass of iron, P, provided with a pencil, which is pressed against the paper by a small spring. The iron is guided in its fall by two light iron wires which pass through guide-holes on the two sides. The top of this mass is provided with a tipper which catches against the end of a bent lever, AC. This being pulled by the string K attached at A, the weight falls. If the cylinder M were fixed, the pencil would trace a straight line on it ; but if the cylinder moves uniformly, the pencil traces the line *mn*, which serves to deduce the law of the fall.

The cylinder is rotated by means of a weight, Q, suspended to a cord which passes round the axle G. At the end of this is a toothed wheel, *c*, which turns two endless screws, *a* and *b*, one of which turns the cylinder, and the other two vanes, *x* and *x′* (fig. 57). At the other end is a ratchet wheel, in which fits the end of a lever, B ; by pulling at a cord fixed to the other end of B, the wheel is liberated, the weight Q descends, and the whole system begins to turn. The motion is at first accelerated, but as the air offers a resistance to the vanes (48), which increases as the rotation becomes more rapid, the resistance finally equals the acceleration which gravity tends

to impart. From this time the motion becomes uniform. This is the case when the weight Q has traversed about three-quarters of its course ; at this moment the weight P is detached by pulling the cord K, and the pencil then traces the curve *mn*.

If, by means of this curve, we examine the double motion of the pencil on the small squares which divide the paper, we see that, for displacements

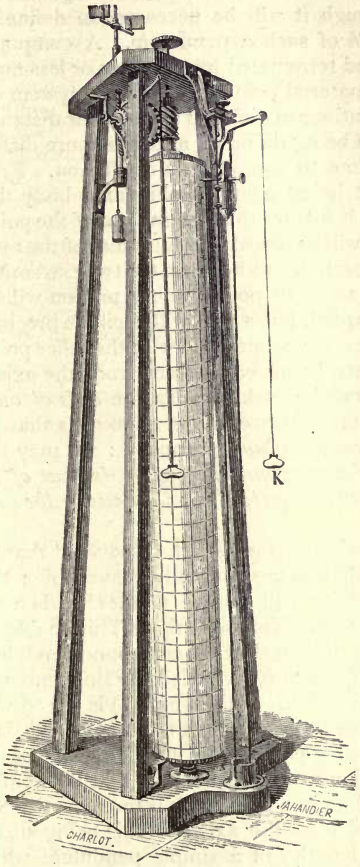


Fig. 56.

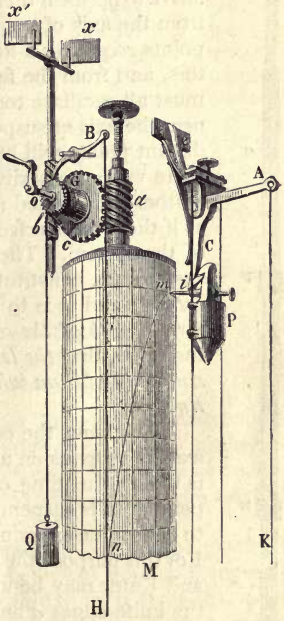


Fig. 57.

1, 2, 3 . . . . in a horizontal direction, the displacements are 1, 4, 9 . . . , in a vertical direction. This shows that the paths traversed in the direction of the fall are directly as the squares of the lines in the direction of the rotation, which verifies the second law of falling bodies.

From the relation which exists between the two dimensions of the curve *mn*, it is concluded that this curve is a *parabola* (51).

79. **The length of the compound pendulum.**—The formula deduced in article (55), and the conclusions which follow therefrom, refer to the case of the simple or mathematical pendulum ; that is, to a single heavy point suspended by a thread without weight. Such a pendulum has only an imaginary existence, and any pendulum which does not realise these conditions is called a *compound* or *physical* pendulum. The laws for the time of vibration of a compound pendulum are the same as those which regulate the motion of the simple pendulum, though it will be necessary to define accurately what is meant by the *length* of such a pendulum. A compound pendulum being formed of a heavy rod terminated by a greater or less mass, it follows that the several material points of the whole system will strive to perform their oscillations in different times, their distances from the axis of suspension being different, and the more distant points requiring a longer time to complete an oscillation. From this, and from the fact that being points of the same body they must all oscillate together, it follows that the motion of the points near the axis of suspension will be retarded, whilst that of the more distant points will be accelerated, and between the two extremities there will necessarily be a series of points whose motion will be neither accelerated nor retarded, but which will oscillate precisely as if they were perfectly free and unconnected with the other points of the system. These points, being equidistant from the axis of suspension, constitute a parallel axis known as the *axis of oscillation* ; and it is to the distance between these two axes that the term *length of the compound pendulum* is applied : we may say, therefore, that *the length of a compound pendulum is that of the simple pendulum which would describe its oscillations in the same time.*

Fig. 58.

Huyghens, the celebrated Dutch physicist, discovered that the axes of suspension and oscillation are mutually convertible ; that is to say, the time of oscillation will remain unaltered when the pendulum is suspended from its axis of oscillation. This enables us to determine experimentally the length of the compound pendulum. For this purpose the *reversible pendulum* devised by Bohnenberger and Kater may be used. One form of this (fig. 58) is a rod with the knife-edges *a* and *b* turned towards each other. W and V are lens-shaped masses the relative positions of which may be varied. By a series of trials a position can be found such that the number of oscillations of the pendulum in a given time is the same whether it oscillates about the axis *a* or the axis *b*. This being so, the distance *ab* represents the length *l* of a simple pendulum which has the same time of oscillation. From the value of *l*, thus obtained, it is easy to determine the length of the seconds pendulum.

The length of the *seconds* pendulum—that is to say, of the pendulum which makes one oscillation in a second—varies, of course, with the force of gravity. The following table gives its value at the sea-level at various places as determined by observation. The accelerative effect of gravity at these places, according to formula (55), is obtained in feet and

metres, by multiplying the length of the seconds pendulum, reduced to feet and metres respectively, by the square of 3·14159 or 9·87.

	Latitude	Length of Pendulum in inches	Acceleration of Gravity in	
			Feet	Metres
Hammerfest . .	70°·40′N.	39·1948	32·2364	9·8258
Aberdeen . .	57.9	39·1550	32·2066	9·8164
Königsberg . .	54.42	39·1507	32·2002	9·8142
Manchester . .	53.29	39·1466	32·1968	9·8134
Dublin . . .	53.21	39·1461	32·1968	9·8132
Berlin . . .	52.30	39·1439	32·1945	9·8124
Greenwich . .	51.29	39·1398	32·1912	9·8115
Paris . . .	48.50	39·1285	32·1819	9·8039
Rome . . .	41.54	39·1145	32·1712	9·8053
New York . .	40.43	39·1012	32·1594	9·8019
Washington . .	38.54	39·0968	32·1558	9·8006
Madras . . .	13.4	39·0268	32·0992	9·7836
Ascension . .	7.56	39·0242	32·0939	9·7817
St. Thomas . .	0.25	39·0207	32·0957	9·7826
Cape of Good Hope	33.55 S.	39·0780	32·1404	9·7962

Consequently, $\frac{1}{2}g$ or the space described in the first second of its motion by a body falling *in vacuo* from a state of rest (49) is

16·0478 feet or 4·891 metres at St. Thomas,
16·0956 „ „ 4·905 „ at London, and
16·1182 „ „ 4·913 „ at Hammerfest.

In all calculations, which are merely used for the sake of illustration, we may take 32 feet, or 9·8 metres, as the accelerative effect due to gravity.

From observations of this kind, after applying the necessary corrections, and taking into account the effect of rotation (82), the form of the earth can be deduced.

80. **Verification of the laws of the pendulum.**—In order to verify the laws of the simple pendulum (55) we are compelled to employ a compound one, whose construction differs as little as possible from that of the former. For this purpose a small sphere of a very dense substance, such as lead or platinum, is suspended from a fixed point by means of a very fine metal wire. A pendulum thus formed oscillates almost like a simple pendulum, whose length is equal to the distance of the centre of the sphere from the point of suspension.

In order to verify the isochronism of small oscillations, it is merely necessary to count the number of oscillations made in equal times, as the amplitudes of these oscillations diminish from 3 degrees to a fraction of a degree ; this number is found to be constant.

That the time of vibration is proportional to the square root of the length is verified by causing pendulums, whose lengths are as the numbers 1, 4, 9, ... to oscillate simultaneously. The corresponding numbers of oscillations in a given time are then found to be proportional to the fractions

1, $\frac{3}{4}$, $\frac{1}{2}$, &c., . . which shows that the times of oscillation increase as the numbers 1, 2, 3, ... &c.

By taking several pendulums of exactly equal length, B, C, D (fig. 59), but with spheres of different substances—lead, copper, ivory—it is found that, neglecting the resistance of the air, these pendulums oscillate in equal times, thereby showing that the accelerative effect of gravity on all bodies is the same at the same place.

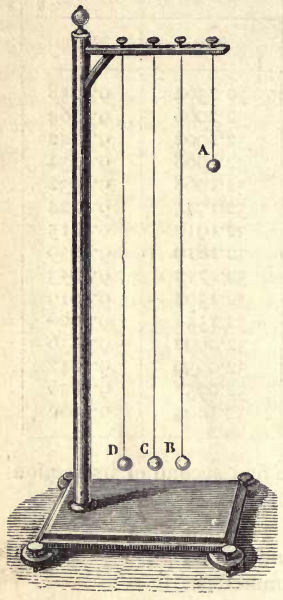


Fig. 59.

By means of an arrangement resembling the above, Newton verified the fact that the *masses* of bodies are determined by the balance; which, it will be remarked, lies at the foundation of the measure of force (28). For it will be seen on comparing (54) and (55) with (49) that the law of the time of a small oscillation is obtained on the supposition that the force of gravity on all bodies is represented by Mg, in which M is determined by the balance. In order to verify this, he had made two round equal wooden boxes; he filled one with wood, and as nearly as possible in the centre of oscillation of the other he placed an equal weight of gold. He then suspended the boxes by threads eleven feet long, so that they formed pendulums exactly equal so far as weight, figure, and resistance of the air were concerned. Their oscillations were performed in exactly the same time. The same results were obtained when other substances were used, such as silver, lead, glass, sand, salt, wood, water, corn. Now all these bodies had equal weights, and, being contained in the same boxes, they experienced the same resistance by the air, and if the inference that therefore they had equal masses had been erroneous by as little as the one-thousandth part of the whole, the experiment would have detected it.

81. **Application of the pendulum to clocks.**—The regulation of the motion of clocks is effected by means of pendulums, that of watches by *balance-springs*. Pendulums were first applied to this purpose by Huyghens in 1658, and in the same year Hooke applied a spiral spring to the balance of a watch. The manner of employing the pendulum is shown in fig. 60. The pendulum rod passing between the prongs of a fork, *a*, communicates its motion to a rod, *b*, which oscillates on a horizontal axis, *o*. To this axis is fixed a piece, *mn*, called an *escapement* or *crutch*, terminated by two projections or *pallets*, which work alternately with the teeth of the *escapement wheel* R. This wheel being acted on by the weight tends to move continously, let us say, in the direction indicated by the arrow-head. Now, if the pendulum is at rest, the wheel is held at rest by the pallet *m*, and with it the whole of the clockwork and the weight. If, however, the pendulum moves and takes the position shown by the dotted line, *m* is raised, the wheel *escapes* from the confinement in which it was held by the pallet, the weight descends, and causes the wheel to turn until its motion is arrested by

the other pallet *n*; which, in consequence of the motion of the pendulum, will be brought into contact with another tooth of the escapement wheel. In this manner the descent of the weight is alternately permitted and arrested —or, in a word, *regulated*—by the pendulum. By means of a proper train of wheelwork the motion of the escapement is communicated to the hands of the clock; and consequently their motion, also, is regulated by the pendulum. In watches the watch-spring plays the part of the weight in clocks.

The pendulum has also been used for measuring great velocities. A large wooden box filled with sand and weighing from 3 to 5 tons is coated with iron; against this arrangement, which is known as a *ballistic pendulum*, a shot is fired, and the deflection thereby produced is observed. From the laws of the impact of inelastic bodies, and from those of the pendulum, the velocity of the ball may be calculated from the amount of this deflection.

The gun may also be fastened to a pendulum arrangement; and, when fired, the reaction causes an angular velocity, from which the pressure of the enclosed gases can be deduced, and therefrom the initial velocity of the shot.

An interesting application of the pendulum is to the *metronome*, which consists of a short rod with a fixed bob; on the rod and above the axis is a sliding weight. By raising this the rate of the pendulum is lengthened; by lowering it accelerated; and thus even with a short pendulum the beats can be made pretty long. Maelzel connected this with a clockwork arrangement so that the beats are quite audible.

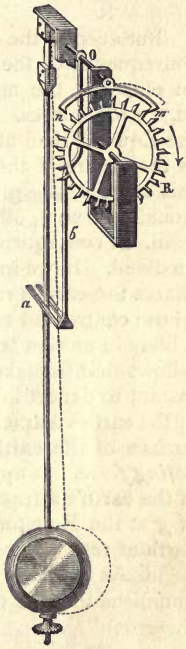


Fig. 60.

82. **Causes which modify the intensity of terrestrial gravitation.**—The intensity of the force of gravity—that is, the value of g—is not the same in all parts of the earth. It is modified by several causes, of which the form of the earth and its rotation are the most important.

i. The attraction which the earth exerts upon a body at its surface is the sum of the partial attractions which each part of the earth exerts upon that body, and the resultant of all these attractions may be considered to act from a single point—the centre. Hence, if the earth were a perfect sphere, a given body would be equally attracted at any part of the earth's surface. The attraction would, however, vary with the height above the surface. For small alterations of level the differences would be inappreciable; but for greater heights and in accurate measurements observations of the value of g must be reduced to the sea-level. The attraction of gravitation being inversely as the square of the distance from the centre (66), we shall have $g : g_h = \frac{1}{R^2} : \frac{1}{(R+h)^2}$, where g is the value of the acceleration of gravity at the sea-level, g_h its value at any height h, and R is the radius of the earth.

From this, seeing that h is very small compared with R, and that therefore its square may be neglected, we get by simple algebraical transformation

$$g_{,} = \frac{g}{1 + \frac{2h}{R}}, \text{ or } g_{,} = \frac{gR}{R + 2h}.$$

But even at the sea-level the force of gravity varies in different parts in consequence of the form of the earth. The earth is not a true sphere, but an ellipsoid, the major axis of which is 12,754,796 metres, and the minor 12,712,160 metres. The distance, therefore, from the centre being greater at the Equator than at the Poles, and as the attraction on a body is inversely as the square of these distances, calculation shows that the attraction due to this cause is $\frac{1}{578}$ greater at the Poles than at the Equator. This is what would be true if, other things being the same, the earth were at rest.

ii. In consequence of the earth's rotation, the force of gravity is further modified. If we imagine a body relatively at rest on the Equator, it really shares the earth's rotation, and describes, in the course of one day, a circle whose centre and radius are the centre and radius of the earth. Now, since a body in motion tends by reason of its inertia to move in a straight line, it follows that to make it move in a circle, a force must be employed at each instant to deflect it from the tangent (53). Consequently, a certain portion of the earth's attraction must be employed in keeping the above body on the surface of the earth, and only the remainder is sensible as *weight* or *accelerating force*. It appears from calculation that at the Equator the $\frac{1}{289}$th part of the earth's attraction on any body is thus employed, so that the magnitude of g at the Equator is less by the $\frac{1}{289}$th part of what it would be were the earth at rest.

iii. As the body goes nearer the Poles the force of gravity is less and less diminished by the effect of centrifugal force. For in any given latitude it will describe a circle coinciding with the parallel of latitude in which it is placed; but as the radii of these circles diminish, so does the centrifugal force to the Pole, where the radius is null. Further, on the Equator the centrifugal force is directly opposed to gravitation: in any other latitude only a component of the whole force is thus employed. This is seen in fig. 61, in which PP′ represents the axis of rotation of the earth, and EE′ the Equator. At any given point E on the Equator the centrifugal force is directed along CE, and acts wholly in diminishing the intensity of gravitation; but on any other point, a, nearer the Pole, the centrifugal force acting on a right line ab at right angles to the axis PP′, while gravity acts along aC, gravity is no longer directly diminished by centrifugal force, but only by its component ad, which is less the nearer a is to the Pole.

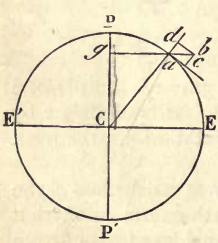


Fig. 61.

The combined effect of these two causes—the flattening of the earth at the Poles, and the centrifugal force—is to make the attraction of gravitation at the Equator less by about the $\frac{1}{192}$nd part of its value at the Poles.

CHAPTER III

MOLECULAR FORCES

83. **Nature of molecular forces.**—The various phenomena which bodies present show that their molecules are under the influence of two contrary forces, one of which tends to bring them together, and the other to separate them from each other. The first force, which is called *molecular attraction*, varies in one and the same body with the distance only. The second force is due to the *vis viva*, or moving force, which the molecules possess. It is the mutual relation between these forces, the preponderance of the one or the other, which determines the molecular state of a body (4)—whether it be solid, liquid, or gaseous.

Molecular attraction is only exerted at infinitely small distances. Its effect is inappreciable when the distance between the molecules becomes appreciable.

According to the manner in which it is regarded, molecular attraction is designated by the terms *cohesion*, *affinity*, or *adhesion*.

84. **Cohesion.**—*Cohesion* is the force which unites adjacent molecules of the same nature ; for example, two molecules of water, or two molecules of iron. Cohesion is strongly exerted in solids, less strongly in liquids, and scarcely at all in gases. Its strength decreases as the temperature increases, because then the *vis viva* of the molecules increases. Hence it is that when solid bodies are heated they first liquefy, and are ultimately converted into the gaseous state, provided that heat produces in them no chemical change.

Cohesion varies not only with the nature of bodies, but also with the arrangement of their molecules ; thus, the difference between tempered and untempered steel (94) is due to a difference in the molecular arrangement produced by tempering. Many of the properties of bodies, such as tenacity, hardness, and ductility, are due to the modifications which this force undergoes.

In large masses of liquids the force of gravity overcomes that of cohesion. Hence liquids acted upon by the former force have no special shape ; they take that of the vessel in which they are contained. But in smaller masses cohesion gets the upper hand, and liquids assume then the spheroidal form. This is seen in the drops of dew on the leaves of plants. It is also seen when a liquid is placed on a solid which it does not moisten ; as, for example, mercury upon wood. The experiment may also be made with water, by sprinkling upon the surface of the wood some light powder, such as lycopodium or lampblack, and then dropping a little water on it. The following experiment is an illustration of the force of cohesion causing a liquid to assume the spheroidal form. A saturated solution of zinc sulphate is placed in a

narrow-necked bottle (fig. 62), and a few drops of bisulphide of carbon, coloured with iodine, made to float on the surface. If pure water be now carefully added, so as to rest on the surface of the sulphate of zinc solution, the bisulphide collects in the form of a flattened spheroid, which presents the appearance of blown coloured glass, and is larger than the neck of the bottle, provided a sufficient quantity has been taken.

Fig. 62.

The force of cohesion of liquids may be illustrated and even measured as follows. A plane, perfectly smooth disc, D (fig. 63), is suspended horizontally to one scale-pan, p, of a delicate balance, and is accurately equipoised. A somewhat wide vessel of liquid is placed below, and the position of the disc regulated by means of the sliding screw S until it just touches the liquid. Weights are then carefully added to the other scale-pan until the disc is detached from the liquid. In this way it has been found that the weights required to detach the disc vary with the nature of the liquid; with a disc of 118 mm. diameter the numbers for waters, alcohol, and turpentine were 59·4, 31, and 34 grammes respectively.

The results were the same whether the disc was of glass, of copper, or of other metals, and they thus only depend on the nature of the liquid. It is a measure of the cohesion of the liquid, for a layer remains adhering to the disc; hence the weight on the other side does not separate the disc from the liquid, but separates the particles of liquid from each other.

85. **Affinity.**—*Chemical affinity*, or *chemical attraction*, is the force which is exerted between molecules not of the same kind. Thus, in water, which is composed of oxygen and hydrogen, it is affinity which unites these elements, but it is cohesion which binds together two molecules of water. In compound bodies cohesion and affinity operate simultaneously, while in simple bodies or elements cohesion has alone to be considered.

To affinity are due all the phenomena of combustion and of chemical combination and decomposition.

Those causes which tend to weaken cohesion are most favourable to affinity; for instance, the action of affinity between substances is facilitated by their division, and still more by reducing them to a liquid or gaseous state. It is most powerfully exerted by a body in its *nascent* state—that is, the state in which the body exists at the moment it is disengaged from a compound; the body is then free and ready to obey the feeblest affinity. An increase of temperature modifies affinity differently under different circumstances. In some cases by diminishing cohesion, and increasing the distance between the molecules, heat promotes combination. Sulphur and oxygen, which at the ordinary temperature are without action on each other, combine to form sulphur dioxide when the temperature is raised: in other cases heat tends to decompose compounds by imparting to their elements an unequal expansibility. Thus it is that many metallic oxides—as, for example, those of silver and mercury—are decomposed, by the action of heat, into gas and metal.

86. **Adhesion.**—The molecular attraction exerted between the *surfaces* of bodies in contact is called *adhesion*.

i. Adhesion takes place between solids. If two leaden bullets are cut with a penknife so as to form two equal and brightly polished surfaces, and

the two faces are pressed and turned against each other, until they are in the closest contact, they adhere so strongly as to require a force of more than 100 grammes to separate them. The same experiment may be made with two equal pieces of glass which are polished and made perfectly plane. When they are pressed one against the other, the adhesion is so powerful that they cannot be separated without breaking. As the experiment succeeds in vacuo, it cannot be due to atmospheric pressure, but must be attributed to a reciprocal action between the two surfaces. The attraction also increases as the contact is prolonged, and is greater in proportion as the contact is closer.

In the operation of glueing the adhesion is complete, for the pores and crevices of the fresh surfaces being filled with liquid glue, so that there is one empty space on drying, wood and glue form one compact whole. In some cases the adhesion of cemented objects is so powerful that the mass breaks more readily at other places than at the cemented parts. Both in glueing and cementing the layer should be thin.

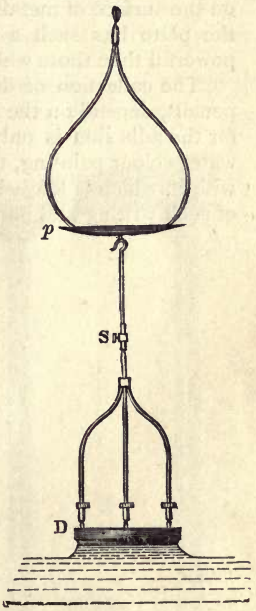


Fig. 63.

Spring exposed various powders, such as saltpetre, sawdust, fine sand, and chalk, to a pressure of 10,000 atmospheres (163). He thus obtained masses of greater hardness and tenacity than the original substances possessed, and destitute of crystalline form.

Soldering is due to cohesion ; the surface of the metals must be quite clean, which is effected by removing the layer of oxide, with which they are usually coated, by acid or by borax. The solder when it solidifies only adheres to clean metal surfaces.

There is no real difference between adhesion and cohesion ; thus when two freshly cut surfaces of caoutchouc are pressed together, they adhere with considerable force, and ultimately form one compact solid mass.

ii. Adhesion also takes place between solids and liquids. If we dip a glass rod into water, and then withdraw it a drop will be found to collect at its lower extremity, and remain suspended there. As the weight of the drop tends to detach it, there must necessarily be some force superior to this weight which maintains it there ; this force is the force of adhesion.

This is the cause why liquids when poured out of a vessel so easily run down the outside ; it is prevented by greasing the outer edge, and thus doing away with the adhesion.

The adhesion between liquids and solids is more powerful than that between solids. Thus, if in the above experiment a thin layer of oil is interposed between the plates they adhere firmly, but when pulled asunder each plate is moistened by the oil, thus showing that in separating the plates the cohesion of the plates is overcome, but not the adhesion of the oil to the metal.

In the above case the solid is wetted by the liquid ; that is, some remains adhering even when the drop falls. But liquids adhere to solids even when they are not wetted. Thus if a smooth glass plate be suspended horizontally from one arm of a balance, and be counterpoised as in fig. 63 ; on sliding a mercury level under the plate, so that they touch, a considerable weight must be placed in the other pan so as to detach the plate from the mercury. Small drops of mercury, too, adhere to the under side of a glass or porcelain plate.

iii. The force of adhesion operates, lastly, between solids and gases. If a glass or metal plate be immersed in water, bubbles will be found to appear on the surface. As air cannot penetrate into the pores of the plate, the bubbles could not arise from the air which has been expelled. It is solely due to the layer of air which covered the plate, and *moistened* it like a liquid. In many cases when gases are separated in the *nascent state* on the surface of metals—as in electrolysis—the layer of gas which covers the plate has such a density that it can produce chemical actions more powerful than those which it can bring about in the free state.

The collection of dust on walls, writing and drawing with chalks and pencils, depend on the adhesion of solids. Yet these are easily rubbed out, for the adhesion is only to the surface layer. In writing with ink, and in water-colour painting, the liquid penetrates into the pores, taking the solid with it, which is left behind as the liquid evaporates, and hence the adhesion of such writing and painting is far more complete.

CHAPTER IV

PROPERTIES PECULIAR TO SOLIDS

87. **Various special properties.**—After having described the principal properties common to solids, liquids, and gases, we shall discuss the properties peculiar to solids. They are *elasticity of traction*, *elasticity of torsion*, *elasticity of flexure*, *tenacity*, *ductility*, and *hardness*.

88. **Elasticity of traction.**—Elasticity, as a general property of matter, has been already mentioned (17), but simply in reference to the elasticity developed by pressure; in solids it may also be called into play by traction, by torsion, and by flexure. The definitions there given require some extension. In ordinary life we consider those bodies as highly elastic which, like caoutchouc, undergo considerable change on the application of only a small force. Yet the force of elasticity is greatest in many bodies, such as iron, which do not seem to be very elastic. For by *force of elasticity* is understood the force with which the displaced particles tend to revert to their original position, and which force is equivalent to that which has brought about the change. Considered from this point of view, gases have the least force of elasticity; that of liquids is considerably greater, and is, indeed, greater than that of many solids. Thus the force of elasticity of mercury is greater than that of caoutchouc, glass, wood, and stone. It is, however, less than that of the other metals, with the exception of lead.

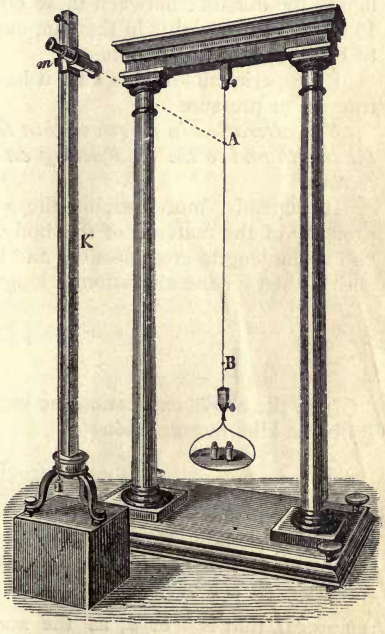


Fig. 64.

This seems discordant with ordinary ideas about elasticity; but it must be remembered that those bodies which, by the exertion of a small force, undergo a considerable change, generally have also the property of under-

going this change without losing the property of reverting completely to their original state. They have a wide *limit of elasticity* (17). Those bodies which require great force to effect a change are also, for the most part, those on which the exertion of a force produces a permanent alteration; when the force is no longer exerted, they do not completely revert to their original state.

In order to study the laws of the elasticity of traction, Savart used the apparatus represented in fig. 64. It consists of a wooden support from which are suspended the rods or wires taken for experiment. At the lower extremity there is a scale-pan, and on the wire two points, A and B, are marked, the distance between which is measured by means of the *cathetometer* before the weights are added.

The *cathetometer* consists of a strong upright brass support, K, divided into millimetres, and which can be adjusted in an exactly vertical position by means of levelling screws and the plumb-line. A small telescope, exactly at right angles to the scale, can be moved up and down, and is provided with a vernier which measures fiftieths of a millimetre. By adjusting the telescope successively on the two points A and B, as represented in the figure, the distance between these points is obtained on the graduated scale. Placing, then, weights in the pan, and measuring again the distance from A to B, the elongation is obtained.

By experiments of this kind it has been ascertained that for elasticity of traction or pressure—

The alteration in length within the limits of elasticity is in proportion to the length and to the load acting on the body, and is inversely as the cross section.

It depends, moreover, on the *specific elasticity*; that is, on a special property of the material of the body. If this coefficient be denoted by E, and if the length, cross section, and load be respectively designated by l, s, and P, then for the alteration in length, e, we have

$$e = \mathrm{E}\frac{l\mathrm{P}}{s}.$$

If in the above expression the sectional area be a square millimetre, and P be one kilogramme, then

$$e = \mathrm{E}l, \text{ from which } \mathrm{E} = \frac{e}{l},$$

which expresses by what fraction the length of a bar a square millimetre in section is altered by a load of a kilogramme. This is called the *coefficient of elasticity*; it is a very small fraction, and it is therefore desirable to use its reciprocal, that is $\frac{1}{e}$ or μ, as the *modulus of elasticity*; or the weight in kilogrammes which applied to a bar would elongate it by its own length, assuming it to be perfectly elastic. This coefficient is known as *Young's modulus*. This cannot be observed, for no body is perfectly elastic, but it may be calculated from any accurate observations by means of the above formula.

The following are the best values for some of the principal substances :—

	μ	e
Wrought-iron	20,869	0·000048
Steel-iron	18,809	0·000053
Platinum	17,044	0·000058
Copper	12,500	0·000080
Slate	11,035	0·000090
Zinc	8,734	0·000114
Brass	8,543	0·000117
Crown Glass	7,917	0·000126
Plate Glass	7,015	0·000142
Rock Salt	4,230	0·000236
Marble	2,309	0·000382
Lead	1,803	0·000555
Bone	1,635	0·000612
Acacia	1,262	0·000792
Pine	1,113	0·000890
Oak	921	0·001085
Whalebone	700	0·001428
Ice	650	0·011667
Sandstone	631	0·001521
Fir	564	0·001768
Gypsum	400	0·002500

Thus, to double the length of a wrought-iron wire a square millimetre in section would (if this were possible) require a weight of 19,000 kilogrammes; but a weight of 15 kilogrammes produces a permanent alteration in length of $\frac{1}{1354}$th, and this is the limit of elasticity. The weight, which when applied to a body of unit section just brings about an appreciable permanent change, is a measure of the *limit of elasticity*. Whalebone has only a modulus of 700, and experiences a permanent elongation by a weight of 5 kilogrammes; its limit is, therefore, relatively greater than that of iron. Steel has a high modulus, along with a wide limit.

Longitudinal stretching is accompanied by a lateral contraction, and the ratio of the contraction to the proportional stretching is known as *Poisson's coefficient*. It was taken by him to be $\frac{1}{4}$, but later experiments have found the ratio to be about $\frac{1}{3}$. When a wire is stretched by a load to within the limit of elasticity, some time often elapses before the full effect is produced, and conversely when the load is removed the wire does not at once wholly resume its original condition, but a small portion of the deformation remains, and it only reverts to its initial state after the lapse of some time. This phenomenon, which is met with in most elastic changes of form, is called the *elastic after action or effect*, or the *elastic fatigue*.

This may be illustrated by the following experiment. A piece of caoutchouc tube is closed by a glass plug at the bottom, while the open end is passed over a piece of glass tube. Coloured liquid is then poured in so that it stands at a certain height in this tube. If then a weight is suspended to the lower end of the india-rubber tube, the liquid at once sinks to a considerable distance, and afterwards very slowly a little further. On removing

the weight it rises again, but not immediately to the old height. This it only reaches after some time.

Both calculation and experiment show that when bodies are lengthened by traction their volume increases.

When weights are placed on a bar, the amount by which it is shortened, or the *coefficient of contraction*, is equal to the elongation which it would experience if the same weights were suspended to it, and is represented by the above numbers.

The influence of temperature on the elasticity of iron, copper, and brass was investigated by Kohlrausch and Loomis. They found that the alteration in the coefficient of elasticity by heat is the same as that which heat produces in the coefficient of expansion and in the refractive power; it is also much the same as the change in the permanent magnetism, and in the specific heat, while it is less than the alteration in the conductivity for electricity.

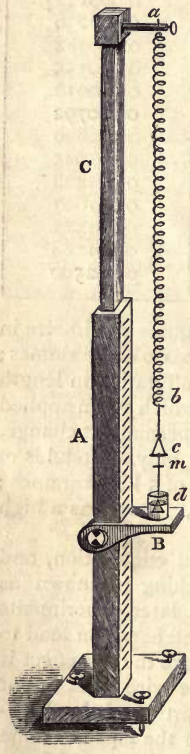


Fig. 65.

As an application of elasticity may be mentioned Jolly's *spring balance*. This consists of a long steel wire, *ab* (fig. 65), wound in the form of a spiral, which is suspended in front of an accurately graduated scale. To the lower end of the spiral two scale-pans, *c* and *d*, are hung by a thread, the lower one, *d*, dipping in a small vessel of water on an adjustable support. The instrument is graduated empirically by observing what displacement of the mark *m* is produced by putting a known weight in the scale pan *d*. Knowing then once for all the constant of the instrument, it is easy to determine the weight of a body by reading the displacement which it produces along the scale.

89. **Elasticity of torsion.**—The laws of the torsion of wires were determined by Coulomb, by means of an apparatus called the *torsion balance* (fig. 66). It consists essentially of a metal wire, clamped at one end in a support, A, and holding at the other a metal sphere, B, to which is affixed an index, C. Immediately below this there is a graduated circle, CD. If the needle is turned from its position of equilibrium through a certain angle, which is the *angle of torsion*, the force necessary to produce this effect is the *force of torsion*. When, after this deflection, the sphere is left to itself, the reaction of torsion produces its effect, the wire untwists itself, and the sphere rotates about its vertical axis with increasing rapidity until it reaches its position of equilibrium. It does not, however, rest there; in virtue of its inertia it passes this position, and the wire undergoes a torsion in the opposite direction. The equilibrium being again destroyed, the wire again tends to untwist itself, the same alterations are again produced, and the needle does not rest at zero of the scale until after a certain number of oscillations about this point have been completed.

By means of this apparatus Coulomb found that when the amplitude of

the oscillations is within certain limits, the oscillations are subject to the following laws :

I. *The oscillations are very nearly isochronous.*

II. *For the same wire, the angle of torsion is proportional to the moment of the force of torsion.*

III. *With the same force of torsion, and with wires of the same diameter, the angles of torsion are proportional to the length of the wires.*

IV. *The same force of torsion being applied to wires of the same length, the angles of torsion are inversely proportional to the fourth powers of the diameters.*

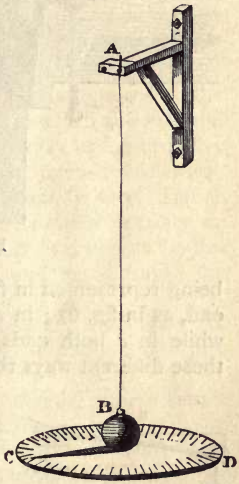


Fig. 66.

Wertheim examined the elasticity of torsion in the case of stout rods by means of a different apparatus, and found that it is also subject to these laws. He further found that, all dimensions being the same, different substances undergo different degrees of torsion for the same force, and each substance has its own coefficient of torsion, which is usually denoted by $\frac{I}{T}$ or by τ. The value of this coefficient is about $\frac{1}{5}$ that of the modulus of elasticity.

The laws of torsion may be enunciated in the formula $w = \frac{I}{T}\frac{Fl}{r^4}$; in which w is the angle of torsion, F the moment of the force of torsion, l the length of the wire, r its radius, and $\frac{I}{T}$ the specific torsion-coefficient.

As the angle of torsion is inversely proportional to the fourth power of the radius, rods of some thickness require very great force to produce even small twists. With very small diameters, such as those of a cocoon or glass thread, the proportionality between the angle of torsion and the twisting force holds even for several complete turns. We may here mention a very ingenious method of obtaining very fine threads of glass, and even of quartz and other minerals, which has been devised by Mr. Boys. It consists in attaching a stout thread of the substance in question to a small arrow of straw, melting the end so as to form a small drop. When the arrow is shot from a small cross-bow, the drop remains behind in virtue of its inertia (17), and a thread practically uniform but of excessive tenuity is spun out from it and carried along with the arrow. In this way glass threads 90 feet in length and $\frac{1}{10000}$th of an inch in diameter have been produced. By the same method melting quartz with the oxyhydrogen blowpipe, threads of this substance have been produced which are not more than 0·00001 inch in diameter. Such threads are of great value in torsion experiments, for, while they possess great tenacity, they are almost destitute of the property of *elastic fatigue* (88).

90. **Elasticity of flexure.**—A solid, when cut into a rod or thin plate, and fixed at one end, after having been more or less bent, strives to return to its original position when left to itself. This property is known as the

elasticity of flexure, and is very distinct in steel, caoutchouc, wood, and paper.

If a rectangular bar A B be clamped at one end and loaded at the other end by a weight W (fig. 67), a flexure will be produced which may be observed by the cathetometer. The amount of this flexure, λ, is represented by the formula

$$\lambda = \frac{4Pl^3k}{bh^3\mu}$$

where P is the load, l the length of the bar, b its breadth, h its depth or thickness, μ the modulus of elasticity, and k a constant which depends on the manner in which the rod is supported, the three principal cases being represented in fig. 68; *a* is that in which the rod is supported at one end, as in fig. 67; in *b* the rod rests on knife-edges, with both ends free; while in *c* both ends are rigid; if one and the same bar be fastened in these different ways the values of λ are respectively as 64:4:1.

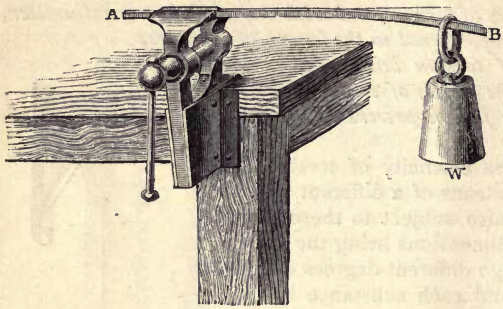


Fig. 67.

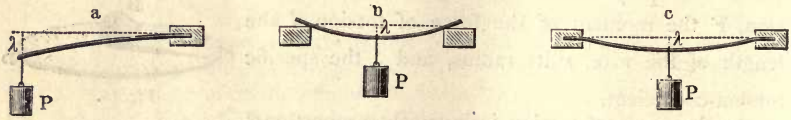


Fig. 68.

If the section of the bar is a circle of radius r, then

$$\lambda = \frac{4}{3}\frac{Pl^3k}{\pi r^4\mu}.$$

It will thus be seen that if for a given load the depression is not to be greater with a long beam than with a short one, the height must increase in the same ratio as the length.

It is clear that an accurate measurement of the flexure of a bar furnishes a means of determining its modulus of elasticity.

The elasticity of flexure is applied in a vast variety of instances—for example, in bows, watch-springs, carriage-springs; in spring balances it is used to determine weights, in dynamometers to determine the force of agents in prime movers; and, as a property of wool, hair, and feathers, it is applied to domestic uses in cushions and mattresses.

Whatever be the kind of elasticity, there is, as has been already said (88), a limit to it—that is, there is a molecular displacement beyond which bodies are broken, or at any rate do not regain their primitive form. This limit is affected by various causes The elasticity of many metals is increased by

hardening, whether by cold, by means of the draw-plate, by rolling, or by hammering. Some substances, such as steel, cast iron, and glass, become both harder and more elastic by tempering (94).

Elasticity, on the other hand, is diminished by *annealing*, which consists in raising the body to a temperature lower than that necessary for tempering, and allowing it to cool slowly. By this means the elasticity of springs may be regulated at pleasure. Glass, when it is heated, undergoes a true tempering in being rapidly cooled, and hence, in order to lessen the fragility of glass objects, they are reheated in a furnace, and are carefully allowed to cool slowly, so that the particles have time to assume their most stable position (94).

91. **Tenacity.**—*Tenacity* is the resistance which a body opposes to the total separation of its parts. According to the manner in which the external force acts, we may have various kinds of tenacity : *tenacity* in the ordinary sense, or resistance to traction ; *relative* tenacity, or resistance to fracture ; *reactive* tenacity, or resistance to crushing ; *sheering* tenacity, or resistance to displacement of particles in a lateral direction ; and *torsional* tenacity, or resistance to twisting. Ordinary tenacity is determined in different bodies by forming them into cylindrical or prismatic wires, and ascertaining the weight necessary to break them.

Mere increase in length does not influence the breaking weight, for the weight acts in the direction of the length, and stretches all parts as if it had been directly applied to them.

Tenacity is directly proportional to the breaking weight, and inversely proportional to the area of a transverse section of the wire.

Tenacity diminishes with the duration of the traction. A small force continuously applied for a long time will often break a wire, which would not at once be broken by a larger weight.

Not only does tenacity vary with different substances, but it also varies with the form of the body. Thus, with the same sectional area, a cylinder has greater tenacity than a prism. The quantity of matter being the same, a hollow cylinder has greater tenacity than a solid one ; and the tenacity of this hollow cylinder is greatest when the external radius is to the internal one in the ratio of 11 to 5. The shape has also the same influence on the resistance to crushing as it has on the resistance to traction. A hollow cylinder with the same mass, and the same weight, offers a greater resistance than a solid cylinder. Thus it is that the bones of animals, the feathers of birds, the stems of corn and other plants, offer greater resistance than if they were solid, the mass remaining the same.

Tenacity, like elasticity, is different in different directions in bodies. In wood, for example, both the tenacity and the elasticity are greater in the direction of the fibres than in a transverse direction. And this difference obtains in general in all bodies, the texture of which is not the same in all directions.

Wires by being worked acquire greater tenacity on the surface, and have therefore a higher coefficient, than even somewhat thicker rods of the same material ; and, according to some physicists, solids have a *surface tension* analogous to that of liquids (134). A strand of wires is stronger than a rod whose section is equal to the sum of the sections of the wires.

Wertheim found the following numbers representing the weight in kilo-

grammes for the limit of elasticity, and for the tenacity of wires, 1 mm. in diameter.

		Limit of Elasticity. Kilogrammes	Tenacity. Kilogrammes
Lead	drawn	0·25	2·07
	annealed	0·20	1·80
Tin	drawn	0·45	2·45
	annealed	0·20	1·70
Silver	drawn	11·25	29·00
	annealed	2·75	16·02
Copper	drawn	12·00	40·30
	annealed	3·00	30·54
Platinum	drawn	26·00	34·10
	annealed	14·50	23·50
Iron	drawn	32·50	61·10
	annealed	5·00	46·88
Steel	drawn	42·50	70·00
	annealed	15·00	40·00
Cast-steel	drawn	55·60	80·00
	annealed	5·00	65·75

The table shows that of all metals cast steel has the greatest tenacity. Yet it is exceeded by fibres of unspun silk, a thread of which 1 square millimetre in section can carry a load of 500 kilogrammes. Single fibres of cotton can support a weight of 100 to 300 grammes ; that is, millions of times their own weight.

In this table the bodies are supposed to be at the ordinary temperature. At higher temperatures the tenacity rapidly decreases. Seguin made some experiments on this point with iron and copper, and obtained the following values for the tenacity, in kilogrammes, of millimetre wire at different temperatures :—

Iron . . at 10°, 60 ; at 370°, 54 ; at 500°, 37 ;
Copper . . „ 21 ; „ 7·7 ; „ 0.

92. **Ductility.**—*Ductility* is the property in virtue of which a great number of bodies change their forms by the action of traction or pressure.

With certain bodies, such as clay, wax, &c., the application of a very little force is sufficient to produce a change ; with others, such as the resins and glass, the aid of heat is needed ; while with the metals more powerful agents must be used, such as percussion, the draw-plate, or the rolling-mill.

Malleability is that modification of ductility which is exhibited by hammering. The most malleable metal is gold, which has been beaten into leaves about the $\frac{1}{300000}$th of an inch thick.

The most ductile metal is platinum. Wollaston obtained a wire of it 0·00003 of an inch in diameter. This he effected by covering with silver a platinum wire 0·01 of an inch in diameter, so as to obtain a cylinder 0·2 inch in diameter only, the axis of which was of platinum. This was then drawn out in the form of wire as fine as possible ; the two metals were equally extended. When this wire was afterwards boiled with dilute nitric acid the silver was dissolved, and the platinum wire left intact. The wire was so fine that a mile of it would have weighed only 1·25 of a grain.

The glass threads drawn by Mr. Boys' method (89) are so fine, being under the $\frac{1}{10000}$th of an inch, that a mile would not weigh more than one-third of a grain. Such threads of quartz have a tenacity approaching that of steel wire.

93. **Hardness.**—*Hardness* is the resistance which bodies offer to being scratched or worn by others. It is only a relative property, for a body which is hard in reference to one body may be soft in reference to others. The relative hardness of two bodies is ascertained by trying which of them will scratch the other. Diamond is the hardest of all bodies, for it scratches all, and is not scratched by any. The hardness of a body is expressed by referring it to a *scale of hardness* : that usually adopted is—

1. Talc	5. Apatite	8. Topaz
2. Rock salt	6. Felspar	9. Corundum
3. Calcspar	7. Quartz	10. Diamond
4. Fluorspar		

Thus, the hardness of a body which would scratch felspar, but would be scratched by quartz, would be expressed by the number 6·5.

Huegenay determined the weight necessary to force a steel point to a depth of 10 mm., and found the order of the metals in increasing hardness as follows : lead, tin, aluminium, gold, silver, platinum, zinc, copper, iron, steel.

The pure metals are softer than their alloys. Hence it is that, for jewellery and coinage, gold and silver are alloyed with copper to increase their hardness.

The hardness of a body has no relation to its resistance to compression. Glass and diamond are much harder than wood, but the latter offers far greater resistance to the blow of a hammer. Hard bodies are often used for polishing powders ; for example, emery, pumice, and tripoli. Diamond, being the hardest of all bodies, can only be ground by means of its own powder.

A body which moves with great velocity can cut into bodies which are harder than itself. Thus a disc of wrought iron rotating with a velocity of 11 metres in a second was cut by a steel graver ; while when it rotated with a velocity of 20 metres, the edge of the disc could cut the graver, and with a velocity of 50 to 100 metres it could even cut into agate and quartz.

A *brittle* body is one in which the connection between the parts is destroyed by the application of a small force. Arsenic, bismuth, and heated zinc are examples of brittle metals ; they are easily reduced to powder.

94. **Temper.**—By sudden cooling after they have been raised to a high temperature, many bodies, more especially steel, become hard and brittle. By reheating and cooling slowly, which is called *annealing*, hard and brittle steel may be converted into a soft, flexible material, and in general, by varying the limits of temperature within which the change takes place, almost any degree of elasticity and flexibility may be given to it. This operation is called *tempering*. All cutting instruments are made of tempered steel. There are, however, some few bodies upon which tempering produces quite a contrary effect. An alloy of one part of tin and four parts of copper, called *tantam metal*, is ductile and malleable when rapidly cooled, but hard and brittle as glass when cooled slowly.

BOOK III

ON LIQUIDS

CHAPTER I

HYDROSTATICS

95. **Province of Hydrostatics.**—The science of *hydrostatics* treats of the conditions of the equilibrium of liquids, and of the pressures they exert, whether within their own mass or on the sides of the vessels in which they are contained.

96. **General characters of liquids.**—It has been already seen (4) that liquids are bodies whose molecules are displaced by the slightest force. Their fluidity, however, is not perfect; their particles always adhere slightly to each other, and when a thread of liquid moves, it attempts to drag the adjacent stationary particles with it, and conversely is held back by them. This property is called *viscosity* (147), and bodies which possess this property in a high degree are said to be *viscous*.

Gases also possess fluidity, but in a higher degree than liquids. The distinction between the two forms of matter is that liquids are almost incompressible and are comparatively inexpansible, while gases are eminently compressible and expand spontaneously.

The fluidity of liquids is seen in the readiness with which they take all sorts of shapes. Their compressibility is established by the following experiment.

97. **Compressibility of liquids.**—From the experiment of the Florentine Academicians (13), liquids were for a long time regarded as being completely incompressible. Since then researches have been made on this subject by various physicists, which have shown that liquids are really compressible.

The apparatus used for measuring the compressibility of liquids has been named the *piezometer* (πιέζω, I compress; μέτρον, measure). That shown in fig. 69 consists of a strong glass cylinder with very thick sides, and an internal diameter of about $3\frac{1}{4}$ inches. The base of the cylinder is firmly cemented into a wooden foot, and on its upper part is fitted a metal cylinder closed by a cap which can be unscrewed. In this cap there is a funnel, R, for introducing water into the cylinder, and a small barrel hermetically closed by a piston, which is moved by a screw, P.

In the inside of the apparatus there is a glass vessel, A, containing the liquid to be compressed. The upper part of this vessel terminates in a capillary tube, which dips under mercury, O. This tube has been previously divided into parts of equal capacity, and it has been determined how many of these parts the vessel A contains. The latter is ascertained by finding the weight, P, of the mercury which the reservoir A contains, and the weight, p, of the mercury contained in a certain number of divisions, n, of the capillary tube. If N be the number of divisions of the small tube contained in the whole reservoir, we have $\frac{N}{n} = \frac{P}{p}$, from which the value of N is obtained. There is further a *manometer*. This is a glass tube, B, containing air, closed at one end, and the other end of which dips under mercury. When there is no pressure on the water in the cylinder, the tube B is completely full of air; but when the water within the cylinder is compressed by means of the screw P, the pressure is transmitted to the mercury, which rises in the tube, compressing the air which it contains. A graduated scale fixed on the side of the tube shows the reduction of volume, and this reduction of volume indicates the pressure exerted on the liquid in the cylinder, as will be seen in speaking of the manometer (184).

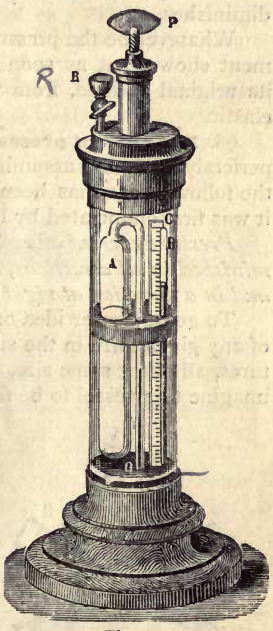


Fig. 69.

In making the experiment, the vessel A is filled with the liquid to be compressed, and the end dipped under the mercury. By means of the funnel R the cylinder is entirely filled with water. The screw P being then turned, the piston moves downwards, and the pressure exerted upon the water is transmitted to the mercury and the air; in consequence of which the mercury rises in the tube B, and also in the capillary tube. The ascent of mercury in the capillary tube shows that the liquid in the vessel A has diminished in volume, and gives the amount of its compression, for the capacity of the whole vessel A in terms of the graduated divisions on the capillary tube has been previously determined.

In his first experiments, Oersted assumed that the capacity of the vessel A remained the same, its sides being compressed both internally and externally by the liquid. But this capacity diminishes in consequence of the external and internal pressures. Colladon and Sturm made some experiments allowing for this change of capacity, and found that for a pressure equal to that of the atmosphere, mercury experiences a compression of 0·000003 part of its original volume, water a compression of 0·00005, and ether a compression of 0·000133 part of its original bulk. The compressibility of sea water is only about 0·000044: it is not materially denser even at great depths; thus at the depth of a mile its density would be only about $\frac{1}{130}$th greater. The

compressibility is greater the higher the temperature; thus that of ether at 14° is one-fourth greater than its compressibility at 0°.

It appears from recent researches that the compressibility of water diminishes with increase of temperature up to a certain limit, beyond which it increases again. This limit seems to be at about 63° C.

As the pressure increases, the average compressibility for each atmosphere diminishes.

Whatever be the pressure to which a liquid has been subjected, experiment shows that as soon as the pressure is removed the liquid regains its original volume, from which it is concluded that liquids are perfectly elastic.

98. **Equality of pressures. Pascal's law.**—By considering liquids as perfectly fluid, and assuming them to be uninfluenced by the action of gravity, the following law has been established. It is often called *Pascal's law*, for it was first enunciated by him.

Pressure exerted anywhere upon a mass of liquid is transmitted undiminished in all directions, and acts with the same force on all equal surfaces, and in a direction at right angles to those surfaces.

To get a clearer idea of the truth of this principle, let us conceive a vessel of any given form in the sides of which are placed various cylindrical apertures, all of the same size, and closed by movable pistons. Let us, further, imagine this vessel to be filled with liquid and unaffected by the action of

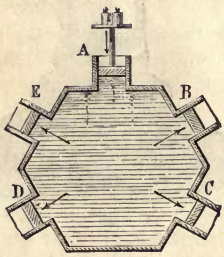


Fig. 70.

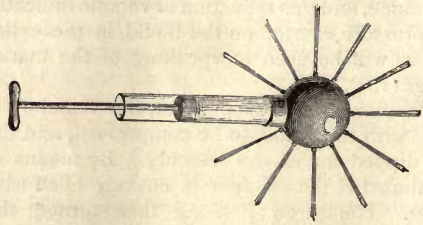


Fig. 71.

gravity; the pistons will, obviously, have no tendency to move. If now a weight of P pounds be placed upon the piston A (fig. 70), which has a surface *a*, it will be pressed inwards, and the pressure will be transmitted to the internal faces of each of the pistons B, C, D, and E, which will each be forced outwards by a pressure P, their surfaces being equal to that of the first piston. Since each of the pistons undergoes a pressure, P, equal to that on A, let us suppose two of the pistons united so as to constitute a surface $2a$, it will have to support a pressure 2 P. Similarly, if the piston were equal to $3a$, it would experience a pressure of 3 P; and if its area were 100 or 1,000 times that of *a*, it would sustain a pressure of 100 or 1,000 times P. In other words, the pressure on any part of the internal walls of the vessel would be proportional to the surface.

The principle of the equality of pressure is assumed as a consequence of the constitution of fluids. By the following experiment it can be shown that pressure is transmitted in all directions, although it cannot be shown that it

is equally transmitted. A cylinder provided with a piston is fitted into a hollow sphere (fig. 71), in which small cylindrical jets are placed perpendicular to the sides. The sphere and the cylinder being both filled with water, when the piston is moved the liquid spouts forth from all the orifices, and not merely from that which is opposite to the piston.

The reason why a satisfactory quantitative experimental demonstration of the principle of the equality of pressure cannot be given is, that the influence of the weight of the liquid and of the friction of the pistons cannot be altogether eliminated.

Yet an approximate verification may be effected by the experiment represented in fig. 72. Two cylinders of different diameters are joined by a tube and filled with water. On the surface of the liquid are two pistons, P and p, which hermetically close the cylinders, but move without friction. Let the area of the large piston, P, be, for instance, thirty times that of the smaller one, p. That being assumed, let a weight, say of two pounds, be placed upon the small piston; this pressure will be transmitted to the water and to the large piston, and as this pressure amounts to two pounds *on each portion of its surface equal to that of the small piston*, the large piston must be exposed to an upward pressure thirty times as much, or of sixty pounds. If now this weight be placed upon the large piston, both will remain in equilibrium; but, if the weight is greater or less, this is no longer the case. If S and s are the areas of the large and small piston respectively, and P and p the corresponding loads, then $\frac{P}{p} = \frac{S}{s}$; whence $P = \frac{pS}{s}$.

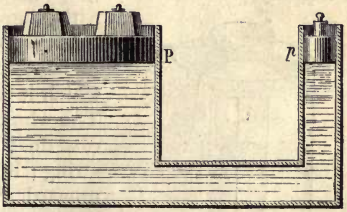


Fig. 72.

It is important to observe that in speaking of the transmission of pressures to the sides of the containing vessel, these pressures must always be supposed to be perpendicular to the sides; for any oblique pressure may be decomposed into two others, one at right angles to the side, and the other acting parallel with the side; but, as the latter has no action on the side, the perpendicular pressure is the only one to be considered.

PRESSURE PRODUCED IN LIQUIDS BY GRAVITY

99. **Vertical downward pressure: its laws.**—Any given liquid being in a state of rest in a vessel, if we suppose it to be divided into horizontal layers of the same density, it is evident that each layer supports the weight of those above it. Gravity, therefore, produces internal pressures in the mass of a liquid, which vary at different points. These pressures are submitted to the following general laws:—

I. *The pressure in each layer is proportional to the depth.*

II. *With different liquids and the same depth, the pressure is proportional to the density of the liquid.*

III. *The pressure is the same at all points of the same horizontal layer.*

The first two laws are self-evident; the third necessarily follows from the first and from Pascal's principle.

Meyer has found, by direct experiments, that pressure is transmitted through liquids contained in tubes, with the same velocity as that with which sound travels in the same circumstances.

100. **Vertical upward pressure.**—The pressure which the upper layers of a liquid exert on the lower layers causes them to exert an equal reaction in an upward direction, a necessary consequence of the principle of transmission of pressure in all directions. This upward pressure is termed the *buoyancy* of liquids; it is very sensible when the hand is plunged into a liquid, more especially one of great density, like mercury.

The following experiment (fig. 73) serves to exhibit the upward pressure of liquids. A large open glass tube, A, one end of which is ground, is fitted with a ground-glass disc, O, or still better, with a thin card or piece of mica, the weight of which may be neglected. To the disc is fitted a string, C, by which it can be held against the bottom of the tube. The whole is then immersed in water, and now the disc does not fall, although no longer held by the string; it is consequently kept in its position by the upward pressure of the water. If water be now slowly poured into the tube, the disc will only sink when the height of the water inside the tube is equal to the height outside. It follows thence that the upward pressure on the disc is equal to the pressure of a column of water, the base of which is the nternal section of the tube A, and the height the distance from the disc to the upper surface of the liquid. Hence the *upward pressure of liquids at any point is governed by the same laws as the downward pressure.*

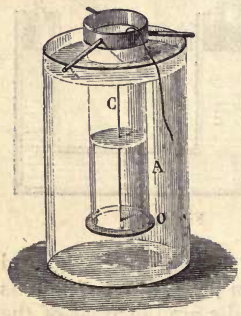


Fig. 73.

101. **Pressure is independent of the shape of the vessel.**—The pressure exerted by a liquid, in virtue of its weight, on any portion of the liquid, or on the sides of the vessel in which it is contained, depends on the depth and density of the liquid, but *is independent of the shape of the vessel and of the quantity of the liquid.*

This principle, which follows from the law of the equality of pressure, may be experimentally demonstrated by many forms of apparatus. The following is the one most frequently used, and is due to Haldat. It consists of a bent tube, A B C (fig. 74), at one end of which, A, is fitted a stop-cock, in which can be screwed two vessels, M and P, of the same height, but different in shape and capacity, the first being conical, and the other nearly cylindrical. Mercury is poured into the tube A B C, until its level nearly reaches A. The vessel M is then screwed on and filled with water. The pressure of the water acting on the mercury causes it to rise in the tube C, and its height may be marked by means of a little collar, *a*, which slides up and down the tube. The level of the water in M is also marked by means of the movable rod *o*. When this is done, M is emptied by means of the stop-cock, unscrewed, and replaced by P. When water is now poured in this, the

mercury, which had resumed its original level in the tube ABC, again rises in C ; and when the water in P has the same height as it had in M, which is indicated by the rod *o*, the mercury will have risen to the height it had before, which is marked by the collar *a*. Hence the pressure on the mercury in both cases is the same. This pressure is therefore independent of the shape of the vessels, and, consequently, also of the quantity of liquid. The base of the vessel is obviously the same in both cases ; it is the surface of the mercury in the interior of the tube A.

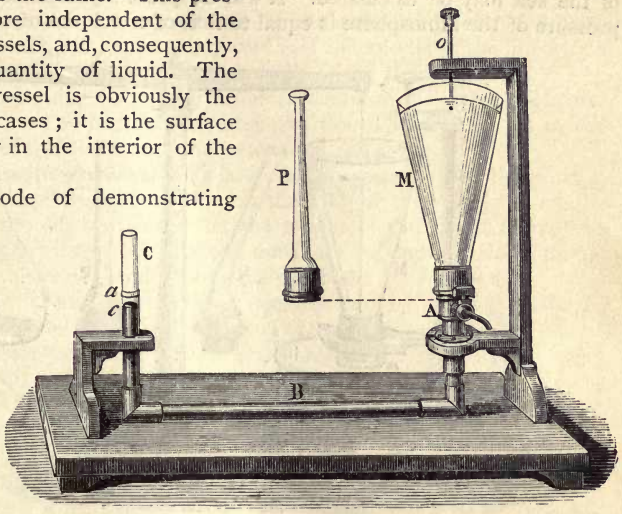


Fig. 74.

Another mode of demonstrating this principle is by means of an apparatus devised by Masson. In this (fig. 75) the pressure of the water contained in the vessel M is not exerted upon the column of mercury, as in that of Haldat, but on a small disc or stop, *a*, which closes a tubulure, *c*, on which is screwed the vessel M. The disc is now fixed to the tubulure, but is sustained by a thread attached to the end of a scale-beam. At the other end is a pan, in which weights can be placed until they counterbalance the pressure exerted by the water on the stop. The vessel M being emptied is unscrewed, and replaced by the narrow tube P. This being filled to the same height as the large vessel, which is observed by means of the mark *o*, it will be observed that to keep the disc in its place just the same weight must be placed in the pan as before ; which leads, therefore, to the same conclusion as does Haldat's experiment. The same result is obtained if, instead of the vertical tube P, the oblique tube Q be screwed to the tubulure.

From a consideration of these principles it will be readily seen that a very small quantity of water can produce considerable pressures. Let us imagine any vessel—a cask, for example—filled with water, and with a long narrow tube tightly fitted into the side. If water is poured into the tube, there will be a pressure on the bottom of the cask equal to the weight of a column of water whose base is the bottom itself, and whose height is equal to that of the water in the tube. The pressure may be made as great as we please ; by means of a narrow thread of water forty feet high Pascal succeeded in bursting a very solidly constructed cask.

The toy known as the *hydrostatic bellows* depends on the same principle, and we shall meet with a most important application of it in the hydraulic press (108).

From the principle just laid down, the pressures produced at the bottom of the sea may be calculated. It will be presently demonstrated that the pressure of the atmosphere is equal to that of a column of sea water about

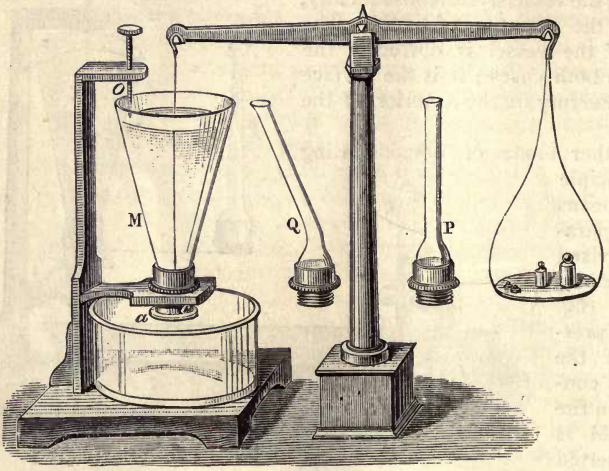


Fig. 75.

thirty-three feet high. At sea the lead has frequently descended to a depth of thirteen thousand feet; at the bottom of some seas, therefore, there must be a pressure of four hundred atmospheres.

102. **Pressure on the sides of vessels.**—Since the pressure caused by gravity in the mass of a liquid is transmitted in every direction, according to the general law of the transmission of fluid pressure, it follows that at every point of the side of any vessel a pressure is exerted, at right angles to the side, which we will suppose to be plane. The resultant of all these pressures is the total pressure on the sides. But since these pressures increase in proportion to the depth, and also in proportion to the horizontal extent of their side, their resultant can only be obtained by calculation, which shows that the total pressure on any given portion of the side *is equal to the weight of a column of liquid which has this portion of the side for its base, and whose height is the vertical distance from the centre of gravity of the portion to the surface of the liquid.* If the side of a vessel is a curved surface, the same rule gives the pressure on the surface, but the total pressure is no longer the resultant of the fluid pressures.

The point in the side supposed to be plane, at which the resultant of all the pressure is applied, is called the *centre of pressure*, and is always below the centre of gravity of the side. For if the pressures exerted at different parts of the plane side were equal, the point of application of their resultant, the centre of pressure, would obviously coincide with the centre of gravity of the side. But since the pressure increases with the depth, the centre of pressure

is necessarily below the centre of gravity. This point is determined by calculation, which leads to the following results :—

i. With a rectangular side whose upper edge is level with the water, the centre of pressure is at two-thirds of the line which joins the middle of the horizontal sides measured from the top.

ii. With a triangular side whose base is horizontal, and coincident with the level of the water, the centre of pressure is at the middle of the line which joins the vertex of the triangle with the middle of the base.

iii. With a triangular side whose vertex is level with the water, the centre of pressure is in the line joining the vertex and the middle of the base, and at three-fourths of the distance of the latter from the vertex.

103. **Hydrostatic paradox.**—We have already seen that the pressure on the bottom of a vessel depends neither on the form of the vessel nor on the quantity of the liquid, but simply on the height of the liquid above the bottom. But the pressure thus exerted must not be confounded with the pressure which the vessel itself exerts on the body which supports it. The latter is always equal to the combined weight of the liquid and the vessel in which it is contained, while the former may be either smaller or greater than this weight, according to the form of the vessel. This fact is often termed the *hydrostatic paradox*, because at first sight it appears paradoxical.

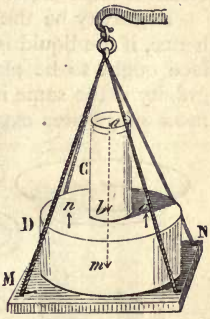


Fig. 76.

CD (fig. 76) is a vessel composed of two cylindrical parts of unequal diameters, and filled with water to *a*. From what has been said before, the bottom of the vessel CD supports the same pressure as if its diameter were everywhere the same as that of its lower part ; and it would at first sight seem that the scale MN of the balance, in which the vessel CD is placed, ought to show the same weight as if there had been placed in it a cylindrical vessel having the same height of water, and having the diameter of the part D. But the pressure exerted on the bottom of the vessel is not all transmitted to the scale MN ; for the *upward* pressure upon the surface *n o* of the vessel is precisely equal to the weight of the *extra* quantity of water which a cylindrical vessel would contain, and balances an equal portion of the *downward* pressure on *m*. Consequently the pressure on the plate MN is simply equal to the weight of the vessel CD and of the water which it contains.

CONDITIONS OF THE EQUILIBRIUM OF LIQUIDS

104. **Equilibrium of a liquid in a single vessel.**—In order that a liquid may remain at rest in a vessel of any given form, it must satisfy the two following conditions :

I. *Its surface must be everywhere perpendicular to the resultant of the forces which act on the molecules of the liquid.*

II. *Every molecule of the mass of the liquid must be subject in every direction to equal and contrary pressures.*

The second condition is self-evident ; for if, in two opposite directions,

the pressures exerted on any given molecule were not equal and contrary, the molecule would be moved in the direction of the greater pressure, and there would be no equilibrium. Thus the second condition follows from the principle of the equality of pressures, and from the reaction which all pressure causes on the mass of liquids.

To prove the first condition, let us suppose that *mp* is the resultant of all the forces acting upon any molecule *m* on the surface (fig. 77), and that this surface is inclined in reference to the force *mp*. The latter can consequently be decomposed into two forces, *mq* and *mf*; the one perpendicular to the surface of the liquid, and the other to the direction *mp*. Now the first force *mq* would be destroyed by the resistance of the liquid, while the second would move the molecule in the direction *mf*, which shows that the equilibrium is impossible.

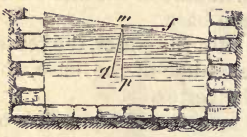


Fig. 77.

If gravity be the force acting on the liquid, the direction *mp* is vertical; hence, if the liquid is contained in a basin or vessel of small extent, the surface ought to be plane and horizontal (67), because then the direction of gravity is the same in every point. But the case is different with liquid surfaces of greater extent, like the ocean. The surface will be perpendicular to the direction of gravity; but as this changes from one point to another, and always tends towards a point near the centre of the earth, it follows that the direction of the surface of the ocean will change also, and assume a nearly spherical form.

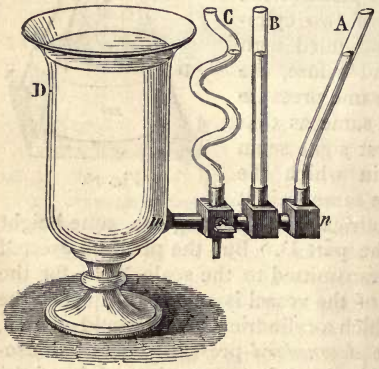


Fig. 78.

105. **Equilibrium of the same liquid in several communicating vessels.**—When several vessels of any given form communicate with each other, there will be equilibrium when the liquid in each vessel satisfies the two preceding conditions (104), and further, *when the surfaces of the liquids in all the vessels are in the same horizontal plane.*

In the vessels ABCD (fig. 78), which communicate with each other, let us consider any transverse section of the tube *mn*; the liquid can only remain in equilibrium as long as the pressures which this section supports from *m* in the direction of *n*, and from *n* in the direction of *m*, are equal and opposite. Now it has been already proved that these pressures are respectively equal to the weight of a column of water, whose base is the supposed section, and whose height is the distance from the centre of gravity of this section to the surface of the liquid. If we conceive, then, a horizontal plane, *mn*, drawn through the centre of gravity of this section, it will be seen that there will only be equilibrium as long as the height of the liquid above this plane is the same in each vessel, which demonstrates the principle enunciated.

106. **Equilibrium of superposed liquids.**—In order that there should be equilibrium when several heterogeneous liquids are superposed in the same vessel, each of them must satisfy the conditions necessary for a single liquid (104); and further, *there will be a stable equilibrium only when the liquids are arranged in the order of their decreasing densities from the bottom upwards.*

The last condition is experimentally demonstrated by means of the *phial of four elements.* This consists of a long narrow bottle containing mercury, water saturated with carbonate of potass, alcohol coloured red, and petroleum. When the phial is shaken the liquids mix, but when it is allowed to rest they separate; the mercury sinks to the bottom, then comes the water, then the alcohol, and then the petroleum. This is the order of the decreasing densities of the bodies. The water is saturated with carbonate of potass to prevent its mixing with the alcohol.

This separation of the liquids is due to the same cause as that which enables solid bodies to float on the surface of a liquid of greater density than their own. It is also on this account that fresh water, at the mouths of rivers, floats for a long time on the denser salt water of the sea; and it is for the same reason that cream, which is lighter than milk, rises to the surface.

107. **Equilibrium of two different liquids in communicating vessels.**—When two liquids of different densities, which do not mix, are contained in two communicating vessels, they will be in equilibrium when, in addition to the preceding principles, they are subject to the following: *that the heights above the horizontal surface of contact of two columns of liquid in equilibrium are in the inverse ratio of their densities.*

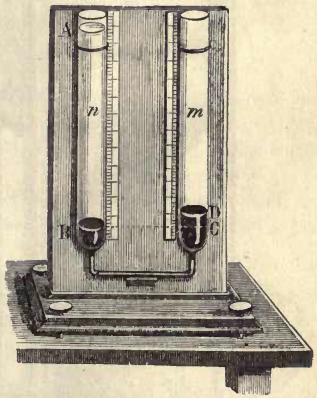


Fig. 79.

To show this experimentally, mercury is poured into a bent glass tube, *mn*, fixed against an upright wooden support (fig. 79), and then water is poured into one of the legs, AB. The column of water, AB, pressing on the mercury at B, lowers its level in the leg AB, and raises it in the other by a quantity, CD; so that if, when equilibrium is established, we imagine a horizontal plane, BC, to pass through B, the column of water in AB will balance the column of mercury CD. If the heights of these two columns are then measured by means of the scales, it will be found that the height of the column of water is about $13\frac{1}{2}$ times that of the height of the column of mercury. We shall presently see that the density of mercury is about $13\frac{1}{2}$ times that of water, from which it follows that the heights are inversely as the densities.

It may be added that the equilibrium cannot exist unless there is a sufficient quantity of the heavier liquid for part of it to remain in *both legs* of the tube.

The preceding principle may be deduced by a very simple calculation. Let d and d' be the densities of water and mercury, and h and h' their re-

spective heights, and let g be the force of gravity. The pressure on B will be proportional to the density of the liquid, to its height, and to the force of gravity; on the whole, therefore, to the product dhg. Similarly, the pressure at C will be proportional to $d'h'g$. But in order to produce equilibrium, dhg must be equal to $d'h'g$, or $dh = d'h'$. This is nothing more than an algebraical expression of the above principle; for since the two products must always be equal, d' must be as many times greater than d as h' is less than h.

In this manner the density of a liquid may be determined. Suppose one of the branches contained water and the other oil, and their heights were, respectively, 15 inches for the oil and 14 inches for the water. The density of water being taken as unity, and that of oil being called x, we shall have

$$15 \times x = 14 \times 1 \text{ ; whence } x = \frac{14}{15} \times 0{\cdot}933.$$

APPLICATIONS OF THE PRECEDING HYDROSTATIC PRINCIPLES

108. **Hydraulic press.**—The law of the equality of pressure has received a most important application in the *hydraulic press*, a machine by which

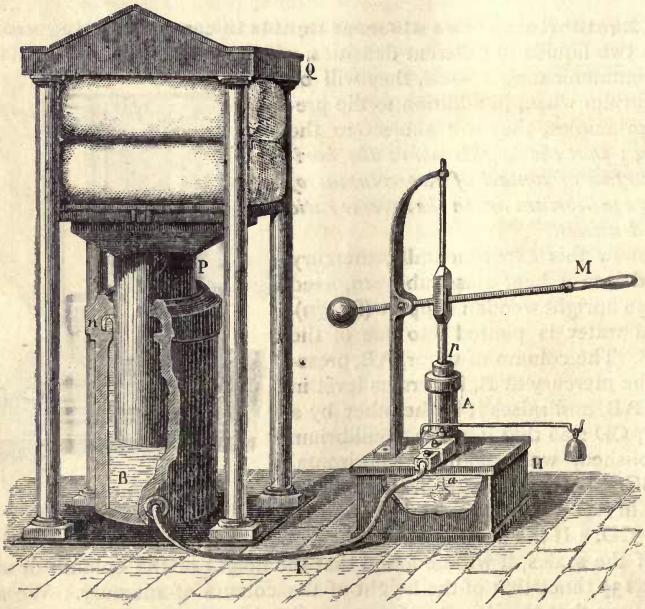


Fig. 80.

enormous pressures may be produced. Its principle is due to Pascal, but it was first constructed by Bramah in 1796.

It consists of a cylinder, B, with very strong thick sides (fig. 80), in which there is a cast-iron ram, P, working water-tight in the collar of the

cylinder. On the ram P there is a cast-iron plate on which the substance to be pressed is placed. Four strong columns serve to support and fix a second plate, Q.

By means of a leaden pipe, K, the cylinder B, which is filled with water, communicates with a small force-pump, A, which works by means of a lever, M. When the piston of this pump *p* ascends, a vacuum is produced, and the water rises in the tube *a*, at the end of which there is a rose, to prevent the entrance of foreign matters. When the piston *p* descends, it drives the water into the cylinder by the tube K.

Fig. 81 represents a section, on a larger scale, of the system of valves necessary in working the apparatus. The valve *o*, below the piston *p*, opens when the piston rises, and closes when it descends. The valve *o*, during this descent, is opened by the pressure of the water which passes by the pipe K. The valve *i* is a *safety-valve*, held by a weight which acts on it by means of a lever. By weighting the latter to a greater or less extent the pressure can be regulated, for as soon as there is an upward pressure greater than that of the weight upon it, it opens and water escapes. A screw, *r*, serves to relieve the pressure, for when it is opened it affords a passage for the efflux of the water in the cylinder B.

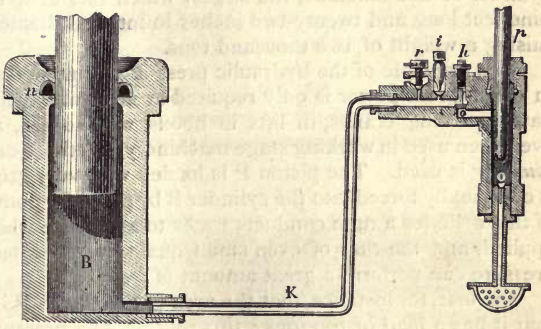


Fig 81.

A most important part is the leather collar *n*, the invention of which by Bramah removed the difficulties which had been experienced in making the large ram work water-tight when submitted to great pressures. It consists of a circular piece of stout leather (fig. 82), saturated with oil so as to be impervious to water, in the centre of which a circular hole is cut. This piece is bent so that a section of it represents a reversed U, and is fitted into a groove, *n*, made in the neck of the cylinder. This collar being concave downwards, in proportion as the pressure increases it fits the more tightly against the ram P on one side and the neck of the cylinder on the other, and quite prevents any escape of water.

Fig. 82.

The pressure which can be obtained by this press depends on the relation of the piston P to that of the piston *p*. If the former has a transverse section fifty or a hundred times as large as the latter, the upward pressure on the large piston will be fifty or a hundred times that exerted upon the small one. By means of the lever M an additional advantage is obtained. If the distance from the fulcrum to the point where the power is applied is five times

the distance from the fulcrum to the piston p, the pressure on p will be five times the power. Thus, if a man acts on M with a force of sixty pounds, the force transmitted by the piston p will be 300 pounds, and the force which tends to raise the piston P will be 30,000 pounds, supposing the section of P is a hundred times that of p.

The hydraulic press is used in all cases in which great pressures are required. It is used in pressing cloth and paper, in extracting the juice of beet-root, in compressing hay and cotton, in expressing oil from seeds, and in bending iron plates; it also serves to test the strength of cannon, of steam boilers, and of chain cables. The parts composing the tubular bridge which spans the Menai Straits were raised by means of an hydraulic press. The cylinder of this machine, the largest which has ever been constructed, was nine feet long and twenty-two inches in internal diameter; it was capable of raising a weight of two thousand tons.

The principle of the hydraulic press is advantageously employed in cases in which great power is only required at intervals, such as in opening dock gates, working cranes, in lifts in hotels, warehouses, and the like. It has even been used in working stage machinery. In these cases an hydraulic *accumulator* is used. The piston P is loaded with very great weights, and water is continually forced into the cylinder B by powerful pumps. From the bottom of this cylinder a tube conducts water to any place where the power is to be applied, and the flow of even small quantities of water which is under high pressure can perform a great amount of work.

Suppose, for instance, that the area of the piston P is four square feet, and that it has a load of 100 tons; this represents a pressure of over 370 pounds on the square inch, or more than 25 atmospheres. When the large piston sinks through the $\frac{1}{17}$th of an inch about a pint of water will flow out, and this represents a work of about 1,100 foot-pounds. In London hydraulic power is supplied by water delivered under a pressure of 750 pounds per square inch (151).

109. **The water-level.**—The *water-level* is an application of the conditions of equilibrium in communicating vessels. It consists of a metal tube

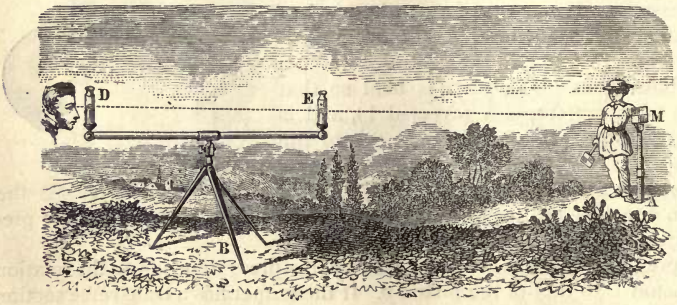


Fig. 83.

bent at both ends, in which are fitted glass tubes, D and E (fig. 83). It is placed on a tripod, and water poured in until it rises in both legs. When the

liquid is at rest, the level of the water in both tubes is the same ; that is, they are both in the same horizontal plane.

This instrument is used in levelling, or ascertaining how much one point is higher than another. If, for example, it is desired to find the difference between the heights of B and A, a *levelling-staff* is fixed on the latter place. This staff consists of a rule formed of two sliding pieces of wood, and supporting a piece of tin plate, M, in the centre of which there is a mark. This staff being held vertically at A, an observer looks at it through the level along the surfaces D and E, and directs the holder to raise or lower the slide until the mark is in the prolongation of the line DE. The height AM is then measured, and, subtracting it from the height of the level, the height of the point A above B is obtained.

110. **The spirit-level.**—The *spirit-level* is both more delicate and more accurate than the water-level. It consists of a glass tube, AB (fig. 84), very slightly curved ; that is, the tube, instead of being a true cylinder as it seems to be, is in fact slightly curved in such a manner that its axis is an arc of a circle of very large radius. It is filled with spirit with the exception of a bubble of air, which tends to occupy the highest part. The tube is placed in a brass case CD (fig. 85), which is so arranged that when it is in a perfectly horizontal position the bubble of air is exactly between the two points marked in the case.

Fig. 84.

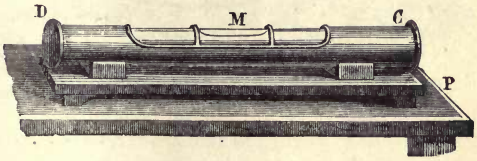


Fig. 85.

To take levels with this apparatus, it is fixed on a telescope, which can be placed in a horizontal position.

111. **Artesian wells.**—All natural collections of water exemplify the tendency of water to find its level. Thus a group of lakes, such as the great lakes of North America, may be regarded as a number of vessels in communication, and consequently the waters tend to maintain the same level in all. This, too, is the case with the source of a river and the sea, and, as the latter is on the lower level, the river continually flows down to the sea along its bed, which is, in fact, the means of communication between the two.

Perhaps the most striking instance of this class of natural phenomena is that of *artesian wells*. These wells derive their name from the province of Artois, where it has long been customary to dig them, and whence their use in other parts of France and Europe was derived. It seems, however, that at a very remote period wells of the same kind were dug in China and Egypt.

To understand the theory of these wells it must be premised that the strata composing the earth's crust are of two kinds : the one *permeable* to water, such as sand, gravel, &c. ; the other *impermeable*, such as clay. Let

us suppose, then, a geographical basin of greater or less extent, in which the two impermeable layers AB, CD (fig. 86), enclose between them a permeable layer, KK. The rain-water falling on that part of this layer which comes to the surface, and which is called the *outcrop*, will filter through it, and, following the natural fall of the ground, will collect in the hollow of the basin, whence it cannot escape owing to the impermeable strata above and below it. If, now, a vertical hole, I, be sunk down to the water-bearing stratum, the water striving to regain its level will spout out to a height which depends on the difference between the levels of the outcrop and of the point at which the perforation is made.

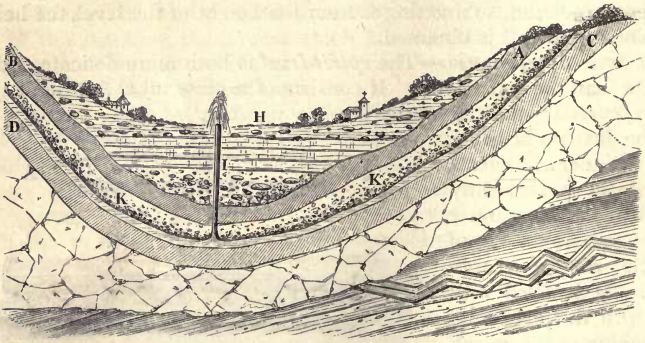


Fig. 86.

The waters which feed artesian wells often come from a distance of sixty or seventy miles. The depth varies in different places. The well at Grenelle is 1,800 feet deep ; it gives 656 gallons of water in a minute, and is one of the deepest and most abundant which have been made. The temperature of the water is 27° C. It follows from the law of the increase of temperature with the increasing depth below the surface of the ground, that, if this well were 210 feet deeper, the water would have all the year round a temperature of 32° C. ; that is, the ordinary temperature of baths.

BODIES IMMERSED IN LIQUIDS

112. **Pressure supported by a body immersed in a liquid.**—When a solid is immersed in a liquid, every portion of its surface is submitted to a perpendicular pressure which increases with the depth. If we imagine all these pressures decomposed into horizontal and vertical pressures, the first set are in equilibrium. The vertical pressures are obviously unequal, and will tend to move the body upwards.

Let us imagine a cube immersed in a mass of water (fig. 87), and that four of its edges are vertical. The pressures upon the four vertical faces being clearly in equilibrium, we need only consider the pressures exerted on the horizontal faces A and B. The first is pressed downwards by a column of water whose base is the face A, and whose height is AD ; the lower face B is pressed upwards by the weight of a column of water whose base is the

face itself, and whose height is BD (100). The cube, therefore, is urged upwards by a force equal to the difference between these two pressures, which latter is manifestly equal to the weight of a column of water having the same base and the same height as this cube. *Consequently, this upward pressure is equal to the weight of the volume of water displaced by the immersed body.*

We shall readily see from the following reasoning that every body im- [redacted] force equal to the weight of [redacted] sup- [redacted] ular [redacted] ther [redacted] thus [redacted] must [redacted] and [redacted] of [redacted] If in [redacted] solid [redacted] ume [redacted] same [redacted] lidi- [redacted] stain [redacted] ards [redacted] quid, [redacted] quid. If, as almost invariably [redacted] ntre of gravity of the displaced [redacted] rsed part of the body *supposed* [redacted] ometimes of importance: for [redacted] sphere of iron and another of [redacted] with its geometrical centre, [redacted] gravity of the displaced water [redacted] ould have the same position [redacted] orm density.

Fig. 87.

[redacted]

but if at the same time the beam is lowered so that the solid cylinder B becomes immersed in a vessel of water placed beneath it, the equilibrium will be restored. By being immersed in water the cylinder B loses a portion of its weight equal to that of the water in the cylinder A. Now, as the capacity of the cylinder A is exactly equal to the volume of the cylinder B, the principle which has been before laid down is proved.

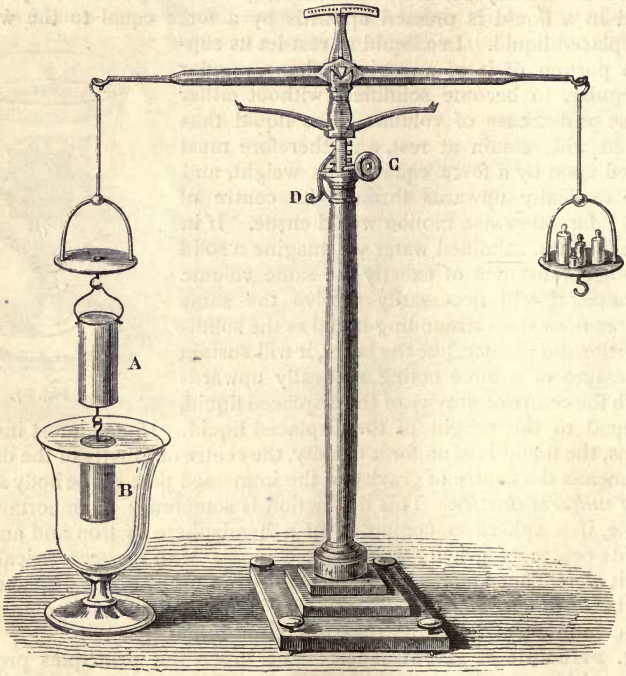


Fig. 88.

114. **Determination of the volume of a body.**—The principle of Archimedes furnishes a method for obtaining the volume of a body of any shape, provided it is not soluble in water. The body is suspended by a fine thread to the hydrostatic balance, and is weighed first in the air, and then in distilled water at 4° C. The loss of weight is the weight of the displaced water, from which the volume of the displaced water is readily calculated. But this volume is manifestly that of the body itself. Suppose, for example, 155 grammes is the loss of weight. This is consequently the weight of the displaced water. Now it is known that a gramme is the weight of a cubic centimetre of water at 4°; consequently, the volume of the body immersed is 155 cubic centimetres.

115. **Equilibrium of floating bodies.**—A body when floating is acted on by two forces, namely, its weight, which acts vertically downwards through its centre of gravity, and the resultant of the fluid pressures, which

(112) acts vertically upwards through the centre of gravity of the fluid displaced; but if the body is at rest these two forces must be equal and act in opposite directions; whence follow the conditions of equilibrium, namely,—

i. *The floating body must displace a volume of liquid whose weight equals that of the body.*

ii. *The centre of gravity of the floating body must be in the same vertical line with that of the fluid displaced.*

Thus in fig. 89, if C is the centre of gravity of the body, and G that of the displaced fluid, the line G C must be vertical, since when it is so the weight of the body and the fluid pressure will act in opposite directions along the same line, and will be in equilibrium if equal. It is convenient, with reference to the subject of the present article, to speak of the line C G produced, as the axis of the body.

Next, let it be inquired whether the equilibrium be stable or unstable. Suppose the body to be turned through a small angle (fig. 90), so that the axis takes a position inclined to the vertical. The centre of gravity of the displaced fluid will no longer be G, but some other point, G′. And since the fluid pressure acts vertically upwards through G′, its direction will cut the axis in some point M′, which will generally have different positions according as the inclination of the axis to the vertical is greater or smaller. If the angle is indefinitely small, M′ will have a definite position M, which always admits of determination, and is called the *metacentre*.

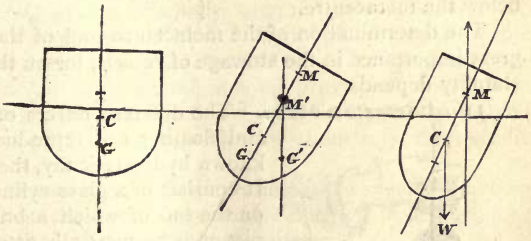


Fig. 89. Fig. 90. Fig. 91.

If we suppose M to be above C, an inspection of fig. 91 will show that when the body has received an indefinitely small displacement, the weight of the body W, and the resultant of the fluid pressures R, tend to bring the body back to its original position; that is, in this case, the equilibrium is stable (70). If, on the contrary, M is below C, the forces tend to cause the axis to deviate farther from the vertical, and the equilibrium is unstable. Hence the rule—

iii. *The equilibrium of a floating body is stable or unstable according as the metacentre is above or below the centre of gravity.*

The determination of the metacentre can rarely be effected except by means of a somewhat difficult mathematical process. When, however, the form of the immersed part of a body is spherical, it can be readily determined; for since the fluid pressure at each point converges to the centre, and continues to do so when the body is slightly displaced, their resultant must in all cases pass through the centre, which is therefore the metacentre. To illustrate this: let a spherical body float on the surface of a liquid (fig. 92); then, its centre of gravity and the metacentre both coinciding with the

geometrical centre C, its equilibrium is neutral (70). Now suppose a small heavy body to be fastened at P, the summit of the vertical diameter. The centre of gravity will now be at some point G above C. Consequently, the equilibrium is unstable, and the sphere, left to itself, will instantly turn over and will rest when P is the lower end of a vertical diameter.

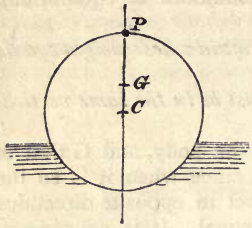


Fig. 92.

On investigating the position of the metacentre of a cylinder, it is found that, when the ratio of the radius to the height is greater than a certain quantity, the position of stable equilibrium is that in which the axis is vertical ; but if it be less than that quantity, the equilibrium is stable when the axis is horizontal. For this reason the stump of a tree floats lengthwise, but a thin disc of wood floats flat on the water. Hence, also, if it is required to make a cylinder of moderate length float with its axis vertical, it is necessary to load it at the lower end. By so doing its centre of gravity is brought below the metacentre.

The determination of the metacentre and of the centre of gravity is of great importance in the stowage of vessels, for on their relative positions the stability depends.

116. **Cartesian diver.**—The different effects of suspension, immersion, and floating are reproduced by means of a well-known hydrostatic toy, the *Cartesian diver* (fig. 93). It consists of a glass cylinder nearly full of water, on the top of which a brass cap, provided with a piston, is hermetically fitted. In the liquid there is a little porcelain figure attached to a hollow glass ball *a*, which contains air and water, and floats on the surface. In the lower part of this ball there is a little hole by which water can enter or escape, according as the air in the interior is more or less compressed. The quantity of water in the globe is such that very little more is required to make it sink. If the piston is slightly lowered the air is compressed, and this pressure is transmitted to the water of the vessel and the air in the bulb. The consequence is that a small quantity of water penetrates into the bulb, which therefore becomes heavier and sinks. If the pressure is relieved, the air in the bulk expands, expels the excess of water which had entered it, and the apparatus, being now lighter, rises to the surface. The experiment may also be simplified by replacing the brass cap and piston by a cover of sheet india-rubber, which is tightly tied over the mouth ; when this is pressed by the hand the same effects are produced.

Fig. 93.

117. **Swimming-bladder of fishes.**—Most fishes have an air-bladder below the spine, which is called the *swimming-bladder*. The fish can com-

press or dilate this at pleasure by means of a muscular effort, and produce the same effects as those just described—that is, it can either rise or sink in water.

118. **Swimming.**—The human body is lighter, on the whole, than an equal volume of water; it consequently floats on the surface, and still better in sea-water, which is heavier than fresh water. The difficulty in swimming consists, not so much in floating, as in keeping the head above water, so as to breathe freely. In man the head is heavier than the lower parts, and consequently tends to sink, and hence swimming is an art which requires to be learned. With quadrupeds, on the contrary, the head, being less heavy than the posterior parts of the body, remains above water without any effort, and these animals therefore swim naturally.

SPECIFIC GRAVITY—HYDROMETERS

119. **Determination of specific gravities.**—It has been already explained (24) that the specific gravity of a body, whether solid or liquid, is the number which expresses the relation of the weight of a given volume of this body to the weight of the same volume of distilled water at a temperature of 4°. In order, therefore, to calculate the specific gravity of a body, it is sufficient to determine its weight and that of an equal volume of water, and then to divide the first weight by the second: the quotient is the specific gravity of the body.

Three methods are commonly used in determining the specific gravities of solids and liquids. These are—1st, the method of the hydrostatic balance; 2nd, that of the hydrometer; and 3rd, the specific gravity flask. All three, however, depend on the same principle—that of first ascertaining the weight of a body, and then that of an equal volume of water. We shall first apply these methods to determining the specific gravity of solids, and then to the specific gravity of liquids.

120. **Specific gravity of solids.**—i. *Hydrostatic balance.*—To obtain the specific gravity of a solid by the hydrostatic balance (fig. 88), it is first weighed in the air, and is then suspended to the hook of the balance and weighed in water (fig. 94). The loss of weight which it experiences is, according to Archimedes' principle, the weight of a volume of water equal to its own volume; consequently, dividing the weight in air by the loss of weight in water, the quotient is the specific gravity required. If P is the weight of the body in air, P′ its weight in water, and D its specific gravity, $P - P'$ being the weight of the displaced water, we have $D = \frac{P}{P - P'}$.

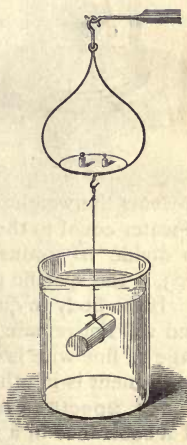


Fig. 94.

It may be observed that though the weighing is performed in air, yet, strictly speaking, the quantity required is the weight of the body *in vacuo*;

and, when great accuracy is required, it is necessary to apply to the observed weights a correction for the weights of the unequal volumes of air displaced by the substance, and the weights in the other scale-pan. The water in which bodies are weighed is supposed to be distilled water at the standard temperature.

ii. *Nicholson's hydrometer.*—The apparatus consists of a hollow metal cylinder B (fig. 95), to which is fixed a cone C, loaded with lead. The object of the latter is to bring the centre of gravity below the metacentre, so that the cylinder may float with its axis vertical. At the top is a stem terminated by a pan, in which is placed the substance whose specific gravity is to be determined. On the stem a standard point, *o*, is marked.

The apparatus stands partly out of the water, and the first step is to ascertain the weight which must be placed in the pan in order to make the hydrometer sink to the standard point *o*. Let this weight be 125 grains, and let sulphur be the substance whose specific gravity is to be determined. The weights are then removed from the pan, and replaced by a piece of sulphur which weighs less than 125 grains, and weights added till the hydrometer is again depressed to the standard *o*. If, for instance, it has been necessary to add 55 grains, the weight of the sulphur is evidently the difference between 125 and 55 grains ; that is, 70 grains. Having thus determined the weight of the sulphur in air, it is only necessary to ascertain the weight of an equal volume of water. To do this, the piece of sulphur is placed in the lower pan C at *m*, as represented in the figure. The whole weight is not changed; nevertheless, the hydrometer no longer sinks to the standard ; the sulphur, by immersion, has lost a part of its weight equal to that of the water displaced. Weights are added to the upper pan until the hydrometer sinks again to the standard. This weight, 34·4 grains, for example, represents the weight of the volume of water displaced ; that is, of the volume of water equal to the volume of the sulphur. It is only necessary, therefore, to divide 70 grains, the weight in air, by 34·4 grains, and the quotient, 2·03, is the specific gravity.

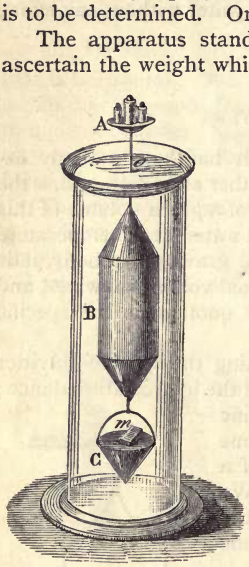


Fig. 95.

If the body in question is lighter than water, it tends to rise to the surface, and will not remain on the lower pan C. To obviate this, a small movable cage of fine wire is adjusted so as to prevent the ascent of the body. The experiment is in other respects the same.

121. **Specific gravity bottle. Pyknometer.**—When the specific gravity of a substance in a state of powder is required, it can be found most conveniently by means of the *pyknometer*, or *specific gravity bottle*. This instrument is a bottle, in the neck of which is fitted a thermometer, A, an enlargement on the stem being carefully ground for this purpose (fig. 96). In the side is a narrow capillary stem widened at the top and provided with a stopper, as shown in the figure. On this tube is a mark *m*, and the thermo-

meter stopper having been inserted, the bottle is filled with water exactly to this mark at each weighing. The bottle may conveniently have dimensions such that when the thermometer stopper is inserted and the liquid filled to the mark *m*, it represents a definite volume. This is done by filling the bottle when wholly under water, and putting in the stopper while it is immersed. The bottle and the tube are then completely filled, and the quantity of water in excess is removed by blotting-paper. To find the specific gravity, proceed as follows: Having weighed the powder, place it in one of the scale-pans, and with it the bottle filled exactly to *m*, and carefully dried. Then balance it by placing small shot, or sand, in the other pan. Next, remove the bottle and pour the powder into it, and, as before, fill it up with water to the mark *m*. On replacing the bottle in the scale-pan it will no longer balance the shot, since the powder has displaced a volume of water equal to its own volume. Place weights in the scale-pan along with the bottle until they balance the shot. These weights give the weight of the water displaced. Then the weight of the powder and the weight of an equal bulk of water being known, its specific gravity is determined as before. The thermometer gives the temperature at which the determination is made, and thus renders it easy to make a correction (124).

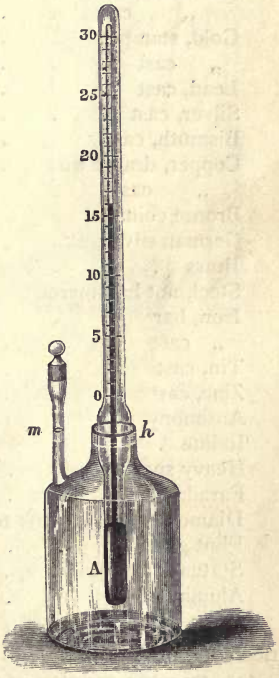


Fig. 96.

It is important in this determination to remove the layer of air which adheres to the powder, and unduly increases the quantity of water expelled. This is effected by placing the bottle under the receiver of an air-pump and exhausting. The same result is obtained by boiling the water in which the powder is placed.

122. **Specific gravity of bodies soluble in water.**—If the body, whose specific gravity is to be determined by any of these methods, is soluble in water, the determination is made in some liquid in which it is not soluble, such as oil of turpentine or naphtha, the specific gravity of which is known. The specific gravity is obtained by multiplying the number obtained in the experiment by the specific gravity of the liquid used for the determination.

Suppose, for example, a determination of the specific gravity of potassium has been made in naphtha. For equal volumes, P represents the weight of the potassium, P′ that of the naphtha, and P″ that of water; consequently, $\frac{P}{P'}$ will be the specific gravity of the substance in reference to naphtha, and $\frac{P'}{P''}$ the specific gravity of the naphtha in reference to water. The product

of these two fractions $\frac{P}{P''}$ is the specific gravity of the substance compared with water.

In determining the specific gravity of porous substances, they are varnished before being immersed in water, which renders them impervious to moisture without altering their volume.

Specific gravity of solids at zero as compared with distilled water at 4° C.

Platinum, rolled	22·069	Rock crystal	2·653
„ cast	20·337	St. Gobin glass	2·488
Gold, stamped	19·362	China porcelain	2·380
„ cast	19·258	Sèvres porcelain	2·140
Lead, cast	11·352	Native sulphur	2·043
Silver, cast	10·474	Common salt	2·220
Bismuth, cast	9·822	Ivory	1·917
Copper, drawn wire	8·878	Anthracite	1·800
„ cast	8·788	Magnesia	1·740
Bronze coinage	8·660	Sugar	1·600
German silver	8·432	Boxwood	1·330
Brass	8·383	Compact coal	1·329
Steel, not hammered	7·816	Amber	1·078
Iron, bar	7·788	Sodium	0·970
„ cast	7·207	Melting ice	0·930
Tin, cast	7·291	Paraffin	0·874
Zinc, cast	6·861	Potassium	0·865
Antimony, cast	6·712	Beech	0·852
Iodine	4·950	Oak	0·845
Heavy spar	4·430	Elm	0·800
Faraday's glass	4·360	Yellow pine	0·657
Diamond	3·531 to 3·501	Lithium	0·585
Flint glass	3·329	Common poplar	0·389
Statuary marble	2·837	Cork	0·240
Aluminium	2·680	Snow	0·183

In this table the different woods are supposed to be in the ordinary air-dried condition.

123. **Specific gravity of liquids.**—i. *Method of the hydrostatic balance.* From the pan of the hydrostatic balance a body is suspended, on which the liquid whose specific gravity is to be determined exerts no chemical action ; for example, a ball of platinum. This is then successively weighed in air, in distilled water, and in the liquid. The loss of weight of the body in these two liquids is noted. They represent respectively the weights of equal volumes of water and of the given liquid, and consequently it is only necessary to divide the second of them by the first to obtain the required specific gravity.

Let P be the weight of the platinum ball in air, P′ its weight in water, P″ its weight in the given liquid, and let D be the specific gravity sought. The weight of the water displaced by the platinum is $P - P'$, and that of the second liquid is $P - P''$, from which we get $D = \frac{P - P''}{P - P'}$.

ii. *Fahrenheit's hydrometer.*—This instrument (fig. 97) resembles Nicholson's hydrometer, but it is made of glass, so as to be used in all liquids. At its lower extremity, instead of a pan, it is loaded with a small bulb containing mercury. There is a standard mark on the stem.

The weight of the instrument is first accurately determined in air; it is then placed in water, and weights added to the scale-pan until the mark on the stem is level with the water. It follows, from the first principle of the equilibrium of floating bodies, that the weight of the hydrometer, together with the weight in the scale-pan, is equal to the weight of the volume of the displaced water. In the same manner the weight of an equal volume of the given liquid is determined, and the specific gravity is found by dividing the latter weight by the former.

Fig. 97.

Fig. 98.

Neither Fahrenheit's nor Nicholson's hydrometer gives such accurate results as the hydrostatic balance or the specific gravity bottle.

iii. *Specific gravity bottle.*—This has been already described (121). In determining the specific gravity of a liquid, a bottle of special construction is used; it consists of a cylindrical reservoir, *b* (fig. 98), to which is fused a capillary tube, *c*, and to this again a wider tube, *a*, closed with a stopper. The bottle is first weighed empty, and then successively full of water to the mark *c* on the capillary stem, and of the given liquid. If the weight of the bottle be subtracted from the two weights thus obtained, the result represents the weights of equal volumes of the liquid and of water, from which the specific gravity is obtained by division.

iv. *Specific gravity bulbs.*—The specific gravity of a liquid is often determined for technical and even scientific purposes by means of *specific gravity bulbs*; these are small hollow glass bulbs (fig. 99), which are prepared in series, loaded and adjusted so that they exactly float in a liquid of a definite specific gravity. When carefully prepared they are susceptible of considerable accuracy.

Fig. 99.

Solutions of certain metallic salts of high specific gravity have been used for the mechanical separation of individual minerals of certain rocks. Such minerals will float or sink according as their specific gravities are lower or higher than that of a given solution. A saturated solution of the double iodide of barium and mercury, the specific gravity of which is 3·58, has been used for this purpose. A saturated solution of borotungstate of cadmium has the specific gravity 3·3.

124. **On the observation of temperature in ascertaining specific gravities.**—As the volume of a body increases with the temperature, and as this increase varies with different substances, the specific gravity of any given body is not exactly the same at different temperatures; and, consequently, a certain fixed temperature is chosen for these determinations. That of water, for example, has been made at 4° C., for at this point it has

the greatest density. The specific gravities of other bodies are assumed to be taken at zero; but, as this is not always possible, certain corrections must be made, which we shall consider in the Book on Heat.

Specific gravities of liquids at zero, compared with that of water at 4° C. *as unity*

Mercury	13·598	Urine	1·020
Methylene iodide	3·342	Distilled water at 4° C.	1·000
Bromine	2·960	„ „ at 0° C.	0·999
Ethylic iodide	1·946	Claret	0·994
Sulphuric acid	1·841	Olive oil	0·915
Chloroform	1·525	Liquid oxygen	0·899
Nitric acid	1·420	Oil of turpentine	0·870
Bisulphide of carbon	1·293	Oil of lemon	0·852
Glycerine	1·260	Petroleum	0·836
Hydrochloric acid	1·240	Liquid carbonic acid	0·830
Blood	1·060	Absolute alcohol	0·793
Milk	1·029	Ether	0·713
Sea water	1·026	Pentane	0·626

125. **Use of tables of specific gravity.**—Tables of specific gravity admit of numerous applications. In mineralogy the specific gravity of a mineral is often a highly distinctive character. By means of tables of specific gravities the weight of a body may be calculated when its volume is known, and conversely the volume when its weight is known.

With a view to explaining the last-mentioned use of these tables, it will be well to premise a statement of the connection existing between the British units of length, capacity, and weight. It will be sufficient for this purpose to define that which exists between the yard, gallon, and pound avoirdupois, since other measures stand to these in well-known relations. The *yard*, consisting of 36 inches, may be regarded as the primary unit. Though it is essentially an arbitrary standard, it is determined by this, that the simple pendulum which makes one oscillation in a mean second, at London on the sea-level, is 39·13983 inches long. The *gallon* contains 277·274 cubic inches. A gallon of distilled water at the standard temperature weighs ten pounds avoirdupois or 70,000 grains troy; or, which comes to the same thing, one cubic inch of water weighs 252·5 grains.

On the French system the *metre* is a primary unit, and is so chosen that 10,000,000 metres are the length of a quadrant of the meridian from either pole to the equator. The metre contains 10 *decimetres*, or 100 *centimetres*, or 1,000 *millimetres*; its length equals 1·0936 yard. The unit of the measure of capacity is the *litre* or cubic decimetre. The unit of weight is the *gramme*, which is the weight of a cubic centimetre of distilled water at 4° C. The *kilogramme* contains 1,000 grammes, or is the weight of a decimetre of distilled water at 4° C. The *gramme* equals 15·432 grains.

If V is the number of cubic centimetres (or decimetres) in a certain quantity of distilled water at 4° C., and P its weight in grammes (or kilogrammes), it is plain that $P = V$. Now consider a substance whose specific gravity is D; every cubic centimetre of this substance will weigh as much

as D cubic centimetres of water, and therefore V centimetres of this substance will weigh as much as DV centimetres of water. Hence, if P is the weight of the substance in grammes, we have $P = DV$. If, however, V = the volume in cubic inches, and P the weight in grains, we shall have $P = 252{\cdot}5\ DV$.

As an example, we may calculate the internal diameter of a glass tube. Mercury is introduced, and the length and weight of the column at 4° C. are accurately determined. As the column is cylindrical, we have $V = \pi r^2 l$, where r is the radius and l the length of the column in centimetres. Hence, if D is the specific gravity of mercury, and P the weight of the column in grammes, we have $P = \pi r^2 l D$, and therefore

$$r = \sqrt{\frac{P}{\pi D l}}.$$

If r and l are in inches and P in grains, we shall have $P = 252{\cdot}5\pi r^2 l D$; and therefore

$$r = \sqrt{\frac{P}{252{\cdot}5\pi D l}}$$

In a similar manner, by determining the weight of a given length, the diameter of very fine metal wires can be determined with great accuracy.

126. **Hydrometers of variable immersion.**—The hydrometers of Nicholson and Fahrenheit are called *hydrometers of constant immersion but variable weight*, because they are always immersed to the same extent, but carry different weights. There are also *hydrometers of variable immersion, but of constant weight.*

127. **Beaumé's hydrometer.**—This, which was the first of these instruments, may serve as a type of them. It consists of a glass tube (fig. 100) loaded at the bottom with mercury, and with a bulb blown in the middle. The stem, the external diameter of which is as regular as possible, is hollow, and the scale is marked upon it.

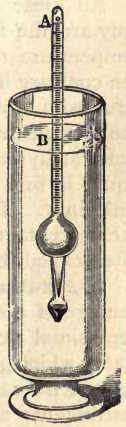


Fig. 100.

The graduation of the instrument differs according as the liquid for which it is to be used is heavier or lighter than water. In the first case, it is so constructed that it sinks in water nearly to the top of the stem, to a point A, which is marked zero. A solution of fifteen parts of salt in eighty-five parts of water is made, and the instrument immersed in it. It sinks to a certain point on the stem B, which is marked 15 ; the distance between A and B is divided into 15 equal parts, and the graduation continued to the bottom of the stem. Sometimes the graduation is on a piece of paper inside the stem.

The hydrometer thus graduated only serves for liquids of a greater specific gravity than water, such as acids and saline solutions. For liquids lighter than water a different plan must be adopted. Beaumé took for zero the point to which the apparatus sank in a solution of 10 parts of salt in 90 of water, and for 10° he took the level in distilled water. This distance he divided into 10°, and continued the division to the top of the scale.

Tweddell's hydrometer is in common use in England for testing liquids denser than water. It is graduated in such a manner that the reading or number of degrees multiplied by five and added to 1,000 gives the specific gravity with reference to water at 1,000. Thus 10° Tweddell represents the specific gravity 1050, and 90° represents 1450.

The graduation of these hydrometers is entirely conventional, and they give neither the densities of the liquids nor the quantities dissolved. But they are very useful in making mixtures or solutions in given proportions and in evaporating acids, alkaline liquids, solutions of salts, worts, syrups, and the like to a proper degree of concentration, the results they give being sufficiently near in the majority of cases.

128. **Gay-Lussac's alcoholometer.**—This instrument is used to determine the strength of spirituous liquors; that is, the proportion of pure alcohol which they contain. It differs from Beaumé's hydrometer in the graduation.

The alcoholometer is so constructed that, when placed in pure distilled water, the bottom of its stem is level with the water, and this point is zero. It is next placed in absolute alcohol, which marks 100°, and then successively in mixtures of alcohol and water containing 10, 20, 30, &c., per cent. The divisions thus obtained are not exactly equal, but their difference is not great, and they are subdivided into 10 divisions, each of which marks *one* per cent. of absolute alcohol in a liquid. Thus a brandy in which the alcoholometer stood at 48° would contain 48 per cent. of absolute alcohol, and the rest would be water.

All these determinations are made at 15° C., and for that temperature only are the indications correct. For, other things being the same, if the temperature rises the liquid expands, and the alcoholometer will sink, and the contrary if the temperature fall. To obviate this error, Gay-Lussac constructed a table which for each percentage of alcohol gives the reading of the instrument for each degree of temperature from 0° up to 30°. When the exact analysis of an alcoholic mixture is to be made, the temperature of the liquid is first determined, and then the point to which the alcoholometer sinks in it. The number in the table corresponding to these data indicates the percentage of alcohol. From its giving the percentage of alcohol, this is often called the *centesimal alcoholometer*.

129. **Salimeters.**—*Salimeters*, or instruments for indicating the percentage of a salt contained in a solution, are made on the principle of the centesimal alcoholometer. They are graduated by immersing them in pure water, which gives the zero, and then in solutions containing different percentages of the salt, and marking on the scale the corresponding points. These instruments are open to the objection that every salt requires a special instrument. Thus one graduated for common salt would give false indications in a solution of nitre.

Lactometers are similar instruments, and are based on the fact that the average density of a good natural quality of milk is 1·029. Hence if water is added to milk, it will indicate a lower specific gravity. But a common plan of adulteration is to remove cream from the milk, by which its specific gravity is increased, and then add water so as to reproduce the original density; the lactometer will not reveal a fraud of this kind. *Urino-*

meters are frequently used in medicine to test the variations in the density of urine, which accompany and characterise certain forms of disease.

130. **Densimeter.**—*Rousseau's densimeter* (fig. 101) is of great use in many scientific investigations, in determining the specific gravity of a small quantity of a liquid. It has the same form as Beaumé's hydrometer, but there is a small tube AC at the top of the stem, in which is placed the substance to be determined. A mark, A, on the side of the tube indicates a measure of a cubic centimetre.

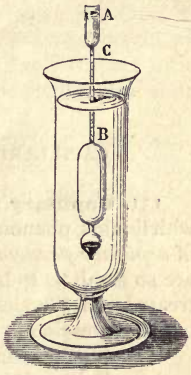


Fig. 101.

The instrument is so constructed that when AC is empty it sinks in distilled water to a point, B, just at the bottom of the stem. It is then filled with distilled water to the height measured on the tube AC, which indicates a cubic centimetre, and the point to which it now sinks is 20°. The interval between 0 and 20 is divided into 20 equal parts, and this graduation is continued to the top of the scale. As this is of uniform bore, each division corresponds to $\frac{1}{20}$ gramme, or 0·05.

To obtain the density of any liquid, bile for example, the tube is filled with it up to the mark A ; if the densimeter sinks to 20 divisions, its weight is $0{\cdot}05 \times 20{\cdot}5 = 1{\cdot}025$; that is to say, with equal volumes, the weight of water being 1, that of bile is 1·025. The specific gravity of bile is therefore 1·025.

CHAPTER II

CAPILLARITY, ENDOSMOSE, EFFUSION, AND ABSORPTION

131. **Capillary phenomena.**—When solid bodies are placed in contact with liquids, phenomena are produced which are classed under the general head of *capillary phenomena*, because they are best seen in tubes whose diameters are so small as to be comparable with that of a hair. These phenomena are treated of in Physics under the head of *capillarity* or *capillary attraction*; the latter expression is also applied to the force which produces the phenomena.

The phenomena of capillarity are very various, but may all be referred to the relation of the attraction of the liquid molecules for each other, or to the attraction between these molecules and solid bodies. The following are some of these phenomena :—

When a body is placed in a liquid which wets it—for example, a glass rod in water—the liquid, as if not subject to the action of gravitation, is raised upwards against the sides of the solid, and its surface, instead of being horizontal, becomes slightly concave (fig. 102). If, on the contrary, the solid is

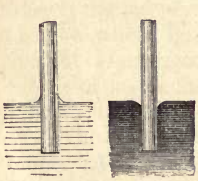


Fig. 102.

Fig. 103.

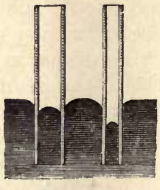


Fig. 104.

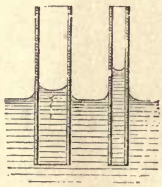


Fig. 105.

one which is not moistened by the liquid, as glass by mercury, the liquid is depressed against the sides of the solid, and assumes a convex shape, as represented in fig. 103. The surface of the liquid exhibits the same concavity or convexity against the sides of a vessel in which it is contained, according as the sides are or are not moistened by the liquid.

These phenomena are much more marked when a tube of small diameter is placed in a liquid. And according as the tubes are or are not moistened by the liquid an ascent or a depression of the liquid is produced which is greater in proportion as the diameter is less (figs. 104 and 105).

When the tubes are moistened by the liquid, its surface assumes the form of a concave hemispherical segment, called the *concave meniscus* (fig. 104); when the tubes are not moistened, there is a *convex meniscus* (fig. 105).

132. **Laws of the ascent and depression in capillary tubes.**—The most important law in reference to capillarity is known as *Jurin's law.* It is: *For the same liquid and the same temperature, the mean height of the ascent in a capillary tube is inversely as the diameter of the tube.* Thus, if water rises to a height of 30 mm. in a tube 1 mm. in diameter, it will only rise to a height of 15 mm. in a tube 2 mm. in diameter, but to a height of 300 mm. in a tube 0·1 mm. in diameter. This law has been verified with tubes whose diameters ranged from 5 mm. to 0·07 mm. It presupposes that the liquid has previously moistened the tube.

The *mean height* is the height of a cylinder with a circular base which has exactly the same volume as the liquid column raised. If h is this height and $2r$ the diameter of the tube, Jurin's law may be expressed by the equation

$$2rh = const.$$

If r, the radius, is taken at 1 mm., then the height in millimetres to which any liquid rises is a measure of the *capillary constant.*

For various liquids, and the same temperature, the mean heights raised in capillary tubes of the same diameter vary with the nature of the liquid. Of all liquids water rises the highest; thus in a glass tube 1·29 mm. in diameter, the heights of water, alcohol, and turpentine are respectively 23·16, 9·18, and 9·85 mm.

For the same liquid, and the same temperature, the mean heights are independent of the form of the capillary tube. That is to say, the shape of the tube above or below the meniscus has no effect on the phenomenon. The columns raised would be of very unequal weights, but of equal heights, h, in the tubes represented in fig. 106, all of which have the same diameter when the liquid stops. The coefficient r is the diameter which corresponds to the region of the meniscus.

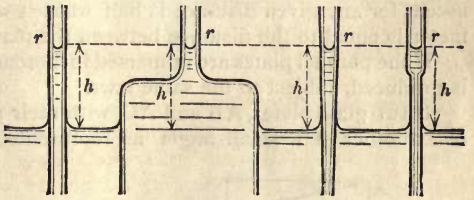


Fig. 106.

Provided the liquid moistens the tube, neither its thickness nor its nature has any influence on the height to which the liquid rises. Thus water rises to the same height in tubes of different kinds of glass, and of rock crystal, provided the diameters are the same.

The height to which a given liquid rises in a capillary tube diminishes as the temperature increases. Thus in a capillary tube in which water stood at a height of 30·7 mm. at 0°, it stood at 28·6 mm. at 35°, and at 26 mm. at 80°. This diminution of height is considerably greater than is accounted for by the diminished density of the water; for, while this is about 0·00045 for each degree between 0° and 100°, the mean diminution of the height is 0·00182, or about four times as much.

At the same time that the heights become less the menisci are *flattened*, so that from a certain temperature, which varies with different liquids, the capillary surface becomes flat and horizontal, and its level is that of the

external liquid. Working in closed vessels, Wolff found this temperature to be 191° for ether, and 500° for water.

In regard to the depression of liquids in tubes which they do not moisten, Jurin's law has not been found to hold with the same accuracy. The reason for this is probably to be found in the following circumstances :— When a liquid moistens a capillary tube, a very thin layer of liquid is formed against the sides, and remains adherent even when the liquid sinks in the tube. The ascent of the column of liquid takes place then, as it were, inside a central tube, with which it is physically and chemically identical. The ascent of the liquid is thus an act of cohesion. It is therefore easy to understand why the nature of the sides of the capillary tube should be without influence on the height of the ascent, which only depends on the diameter.

With liquids, on the contrary, which do not moisten the sides of the tube, the capillary action takes place between the sides and the liquid. The nature and structure of the sides are never quite homogeneous, and there is always, moreover, a layer of air on the inside, which is not dissolved by the liquid. These two causes undoubtedly exert a disturbing influence on the law of Jurin.

133. **Ascent and depression between parallel or inclined surfaces.**— When two bodies of any given shape are dipped in water, analogous phenomena are produced, provided the bodies are sufficiently near. If, for example, two parallel glass plates are immersed in water at a very small distance from each other, water will rise between the two plates in the inverse ratio of the distance which separates them. The height of the ascent for any given distance is half what it would be in a tube whose diameter is equal to the distance between the plates.

If the parallel plates are immersed in mercury, a corresponding depression is produced, subject to the same laws.

If two glass plates, AB and AC, with their planes vertical and inclined to one another at a small angle, as represented in fig. 107, have their ends

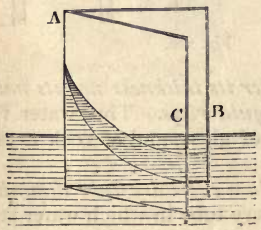


Fig. 107.

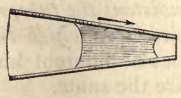


Fig. 108.

Fig. 109.

dipped into a liquid which wets them, the liquid will rise between them. The elevation will be greatest at the line of contact of the plates, and from hence gradually less, the surface taking the form of an equilateral hyperbola.

If a drop of water be placed within a conical glass tube whose angle is small, and axis horizontal, it will have a concave meniscus at each end (fig. 108), and will tend to move towards the vertex. But if the drop be of

mercury it will have a convex meniscus at each end (fig. 109), and will tend to move from the vertex.

134. **Tension of the free surface of liquids.**—The great mobility which is characteristic of the liquid state undergoes an alteration in the neighbourhood of the *free surface* of a liquid, or that which is bounded by a gas or by a vacuum. This surface has greater cohesion than any other. For, consider any particle, *a*, at the surface (fig. 110), and let the sphere represent the range through which the molecular attraction is exerted, or what is called the *radius of molecular activity*. The attractive forces of the adjacent particles, which are exerted in all directions, may be resolved into horizontal and vertical components; the attractions of the former will compensate each other. But the attractions represented by the molecules within the hemisphere beneath the surface are not so compensated, and consequently the latter will exercise a considerable pull towards the interior.

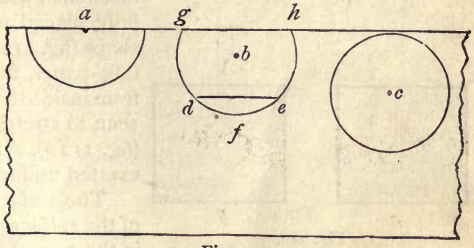


Fig. 110.

Consider, again, a particle, *b*, so much below the surface that the greater part of the sphere comes into operation. If a plane, *de*, be laid as much below *b* as the surface is above it, the attractive forces from the molecules within *ghed* will neutralise each other. But the segment *def* remains uncompensated, and exerts a pull similar to, though weaker than, that which acts on the molecule *a*.

The molecule *c* finally is surrounded uniformly by its adjacent ones, and their resultant action is zero.

The effect of these actions is to lessen the mobility of particles at or very near the surface, while those in the interior are quite mobile; the surface, as it were, is stretched by an elastic skin, the result being the same as if the surface layer exerted a pressure on the interior. This *surface tension*, as it is called, is greater, the greater the cohesion of the liquid.

When the surface of a liquid increases, more particles enter into the condition of the surface layer, to effect which a certain amount of work is required. On the other hand, when the surface is diminished, the molecules pass into the state of the internal layer, and they perform work. The work done when a square mm. of surface passes into the interior is called the *coefficient of surface tension.*

The existence of this surface tension may be illustrated by several interesting experiments. In that of Dupré (fig. 111), a quadrangular flat vessel, ABCD, is used, of which one side, CD, is movable about a hinge. By means of a string this side is pressed against a wedge, and the vessel is filled with water. On burning the wire the side CD′ reverts to its original position, CD. Now, as the hydrostatic pressure would

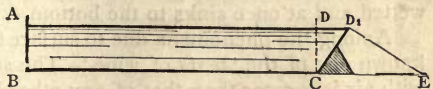


Fig. 111.

have kept it pressed against the wedge, there must be a tangential force at work restoring it to the vertical, which is an effect of the surface tension.

Another experiment by Mensbrugghe is made by means of a wire frame (fig. 112 *a*), which is immersed in solution of soap, such as is used for blowing soap bubbles. On removing this a thin film is formed. A loop of fine silk thread moistened with the liquid in question is carefully placed on the film, and assumes any shape (fig. 112 *a*). By means of a spill of blotting paper, the liquid is carefully removed from inside the loop, and the contour is then seen to stretch and assume a circular form (fig. 112 *b*), which is owing to the lateral pull exerted uniformly on the edge of the loop.

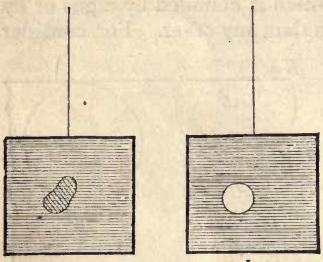


Fig. 112.

The surface tension depends on the form of the surface. Its value has been determined in the case of spheroidal bodies. If the pressure which is exerted on a *plane* surface be called P, the pressure p, on a spherical surface of radius ρ, is $p = P + \frac{2\phi}{\rho}$ for convex, and $p = P - \frac{2\phi}{\rho}$ for concave surfaces.

Hence for a spheroidal shell, the internal radius OA (fig. 113) of which is ρ, and its thickness $AB = d$, the pressure of the outer layer is $p = P + \frac{2\phi}{\rho + d}$, and of the inner layer $p_1 = P - \frac{2\phi}{\rho}$, and the resultant is their difference $= \frac{2\phi}{\rho + d} + \frac{2\phi}{\rho}$ a pressure exerted inwards, since $p > p_1$. This is well illustrated by blowing a soap bubble on a glass tube. So long as the other end of the tube is closed, the bubble remains, the elastic force of the enclosed air counterbalancing the tension of the surface; but when the tube is opened, the tension of the surface being unchecked, the bubble gradually contracts and finally disappears.

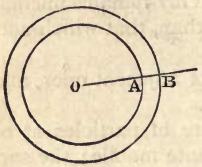


Fig. 113.

Insects can often move on the surface of water without sinking. This phenomenon is caused by the fact that, as their feet are not wetted by the water, a depression is produced and the elastic reaction of the surface layer keeps them up in spite of their weight. Similarly, a sewing-needle, gently placed on water, does not sink, because its surface, being covered with an oily layer, does not become wetted. The pressure of the needle brings about a concavity, the surface tension of which acts in opposition to the weight of the needle. But if washed in alcohol or in potash, the metal is wetted and at once sinks to the bottom.

Among the phenomena due to surface tension may be mentioned the well-known one of the 'tears of wine.' The surface tension of water in contact with air is greater than that of any other liquid except mercury. It is more than three times greater than that of alcohol. When a wine-glass is half filled with a strong wine, the wine rises up against the sides like any other liquid;

but the alcohol evaporates rapidly from the surface, the consequence of which is that the liquid layer becomes more watery. Near the surface of the liquid the strength of the liquid layer is kept up by diffusion; but higher up, owing to the increased surface tension of the more aqueous wine, it creeps up the sides and draws with it some of the stronger alcoholic liquid below, the increasing weight of which ultimately causes it to break and run down in drops.

If a thin layer of water be spread on a plate, and a drop of ether be placed upon it, the water retreats from the drop. Here, instead of the surface tension between water and air, we have that between water and ether, which is smaller; the effect is much the same as if there were a tightly stretched india-rubber skin, and a portion of it were softened or made thinner.

135. **Cause of the curvature of liquid surfaces in contact with solids.** The form of the surface of a liquid in contact with a solid depends on the relation between the attraction of the solid for the liquid, and of the mutual attraction between the molecules of the liquid.

Let *m* be a liquid molecule (fig. 114) in contact with a solid. This molecule is acted upon by three forces: by gravity, which attracts it in the direction of the vertical *m*P; by the attraction of the liquid F, which acts in

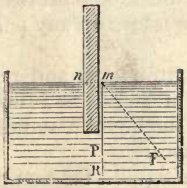


Fig. 114.

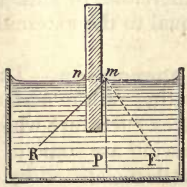


Fig. 115.

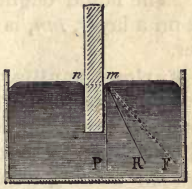


Fig. 116.

the direction *m*F; and by the attraction of the plate *n*, which is exerted in the direction *mn*. According to the relative intensities of these forces, their resultant can take three positions:—

i. The resultant is in the direction of the vertical *m*R (fig. 114). In this case the surface *m* is plane and horizontal; for, from the condition of the equilibrium of liquids, the surface must be perpendicular to the force which acts upon the molecules.

ii. If the force *n* increases or F diminishes, the resultant R is within the angle *nm*P (fig. 115); in this case the surface takes a direction perpendicular to *m*R, and becomes concave.

iii. If the force F increases or *n* diminishes, the resultant R takes the direction *m*R (fig. 116) within the angle P*m*F, and the surface, becoming perpendicular to this direction, is convex.

136. **Influence of curvature on capillary phenomena.**—The elevation or depression of a liquid in a capillary tube depends on the concavity or convexity of the meniscus. In a concave meniscus, *abcd* (fig. 117), the liquid molecules are sustained in equilibrium by the forces acting on them, and they exert no downward pressure on the inferior layers. On the contrary, in virtue of molecular attraction, they act on the nearest inferior layers, from which it follows that the pressure on any layer, *mn*, in the interior of the tube, is less

than if there were no meniscus. The consequence is that the liquid rises in the tube until the internal pressure on the layer *mn* is equal to the pressure *op*, which acts externally on a point, *p*, of the same layer.

Where the meniscus is convex (fig. 118), equilibrium exists in virtue of the molecular forces acting on the liquid; but as the molecules which

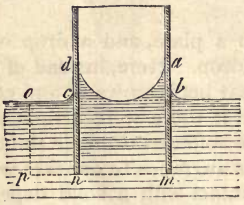


Fig. 117.

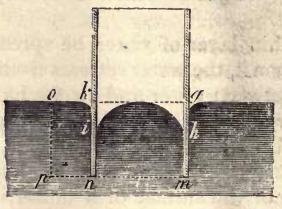


Fig. 118.

would occupy the same space *ghik*, if there were no molecular action, do not exist, they exert no attraction on the lower layers. Consequently, the pressure on any layer, *mn*, in the interior of the tube, is greater than if the space *ghik* were filled, for the molecular forces are more powerful than gravity. The liquid ought, therefore, to sink in the tube until the internal pressure on a layer, *mn*, is equal to the external pressure on any point, *p*, of this layer.

137. **Various capillary phenomena.**—The attractions and repulsions observed between bodies floating on the surface of liquids find their explanation in the concave or convex curvature which the liquid assumes in contact with the solid. The following are some of them.

When two floating balls both moistened by the liquid—for example, cork upon water—are so near that the liquid surface between them is not level, an attraction takes place. The same effect is produced when neither of the balls is moistened, as is the case with balls of wax on water.

Lastly, if one of the balls is moistened and the other not, as a ball of cork and a ball of wax in water, they repel each other if the curved surfaces of the liquid in their respective neighbourhoods intersect.

A drop of mercury on a table has a spherical shape, which, like that of the heavenly bodies, is due to attraction. The globule of mercury behaves as if its molecules had no weight, since it remains spherical. That is, the molecular attraction is far greater than the weight, which only alters the shape of the globule if the quantity of mercury is much greater; it then flattens, but always retains at its edge the convex form which molecular attraction imparts to it. A liquid immersed in another, with which it does not mix, of exactly the same specific gravity, such as olive oil in a mixture of alcohol and water, assumes the spherical form (fig. 62).

To this cause also is due the spherical form acquired by small masses of liquid which fall through great heights, such as raindrops, and molten lead in casting small shot.

When a capillary tube is immersed in a liquid which moistens it, and is then carefully removed, the column of liquid in the tube is seen to be twice as long as when the tube was immersed in the liquid. This arises from the fact that a drop adheres to the lower extremity of the tube and forms a

convex meniscus, which concurs with that of the upper meniscus to form a longer column (131).

For the same reason a liquid does not overflow in a capillary tube, although the latter may be shorter than the liquid column which would otherwise be formed in it. For when the liquid reaches the top of the tube, its upper surface, though previously concave, becomes convex, and, as the downward pressure becomes greater than if the surface were plane, the ascending motion ceases.

It is from capillarity that oil ascends in the wicks of lamps, that water rises in woods, sponge, bibulous paper, sugar, sand, and in all bodies which possess pores of a perceptible size. In the cells of plants the sap rises with great force, for here we have to do with vessels whose diameter is less than 0·01 mm. Efflorescence of salts is also due to capillarity; a solution rising against the side of a vessel, the water evaporates, and the salt forms on the

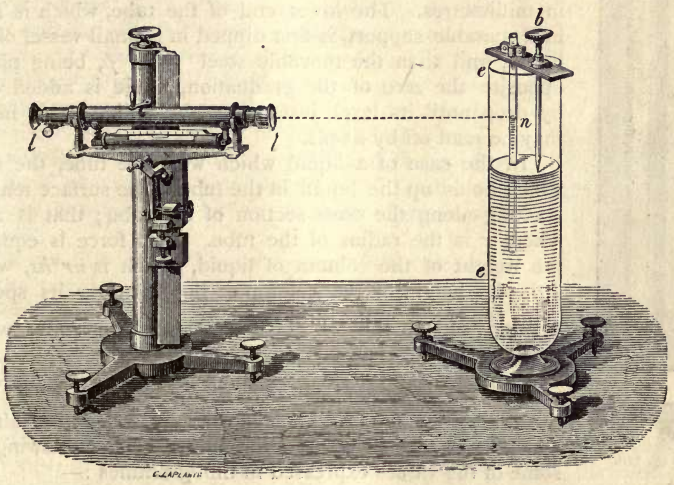


Fig. 119.

side a means of furthering still more the ascent of a liquid. Capillarity is, moreover, the cause of the following phenomenon:—When a porous substance, such as gypsum, or chalk, or even earth, is placed in a porous vessel of unbaked porcelain, and the whole is dipped in water, the water penetrates into the pores, and the air is driven inwards, with such force that it is under four or five times its usual pressure and density. Jamin has proved this by cementing a manometer into blocks of chalk, gypsum, &c., and he has shown the probability that a pressure of this kind, exerted upon the roots, promotes the ascent of sap in plants.

138. **Determination of the constant of capillarity.**—This determination may be effected in various ways, of which the simplest and perhaps the most accurate is that of measuring the ascent of a liquid in capillary tubes. For this purpose capillary tubes of glass are used, the diameters of

which are determined by introducing a thread of mercury into the tube and ascertaining the weight of a given length (125).

The height to which the liquid rises in the capillary tube may be read off by a cathetometer (fig. 119). The capillary tube is fixed to a cross-piece of wood, which is placed on the edges of a glass tube, *e e*, half filled with the liquid. In order that the liquid may properly moisten the tube, it is sucked up by means of a caoutchouc tube beyond the height at which it finally stands. The cathetometer is then raised to the level *lln* of the lowest point of the meniscus. The pointed screw *b* is then turned until its point just grazes the liquid, and the position of the point is read off. The difference of these two readings gives the desired height.

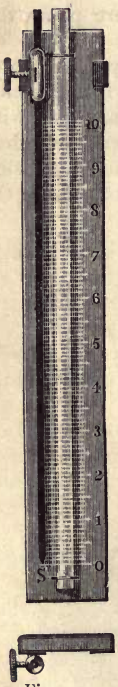


Fig. 120.

A simpler arrangement is the following (fig. 120). The capillary tube is fixed to a strip of opaque glass, graduated in millimetres. The lower end of the tube, which is fixed in a suitable support, is first dipped in a small vessel of the liquid, and then the movable steel point *p*, being placed opposite the zero of the graduation, liquid is added drop by drop until its level just grazes the point. This height may be read off by a lens.

In the case of a liquid which wets the tube, the force which holds up the liquid in the tube is the surface tension, a acting along the cross-section of the tube; that is $2\pi r a$, where r is the radius of the tube. This force is equal to the weight of the column of liquid, which is $\pi r^2 hs$, where h is the height of the column of liquid, and s its specific gravity. From this we get $a = \frac{hrs}{2}$, and for water, where s is unity, $a = \frac{hr}{2}$. This, which is known as the *capillary constant*, gives the weight supported by the unit of length, which is usually taken at a millimetre. The following are some of the values expressed in milligrammes :—

Water	7·24	Turpentine . . .	2·77
Hydrochloric acid . .	7·15	Petroleum . . .	2·57
Olive oil	3·27	Alcohol	2·27

When a small but very hot flame is directed against the point of a fine metal wire, such as gold or platinum, the metal is melted and falls in drops the weight P of which is found to be very uniform. P is the greatest weight which the melted mass can support, and is equal to $2\pi r a$, where a is the constant of capillarity and $2r$ the diameter of the wire. Quincke has applied this method of determining the constant in cases where other methods are not applicable, such as in the case of the noble metals, salts, phosphorus, &c.

139. **Osmose.**—Other phenomena are observed when two different liquids miscible with each other are separated by a porous diaphragm. This may be best illustrated by means of the apparatus represented in fig. 121, in which

a vessel open at the bottom is tied round with a bladder. In the neck a long narrow tube, *a a*, is fitted. This vessel is filled with solution of copper sulphate, so that it stands at a certain height, *r*, in the tube, and is then placed in a larger vessel containing pure water, at the level *n n*. If the temperature remains stationary, it will be seen that after some time the liquid in the tube *aa*, which was originally at the level *r*, has risen, while at the same time the level of *nn* has become somewhat lower ; it will also be seen that the outer liquid has acquired a faint bluish tinge. This process continues for some time until the liquid has attained a certain height. It thus appears that there is an interchange of the two liquids, but the quantity of water which passes into the sulphate of copper is greater than that of the solution which passes out. If the experiment be reversed, that is, if water is contained in *b*, and copper sulphate in the outer vessel, the phenomena are reversed ; that is, the level in *r* sinks, while that in *nn* rises. Dutrochet, who first investigated these phenomena, applied the term *exosmose* to the current which passes from the denser liquid to the less dense, and *endosmose* to the opposite current, and the apparatus itself he called an *endosmometer*. The phenomena are now generally known as those of *diosmose* or *osmose*.

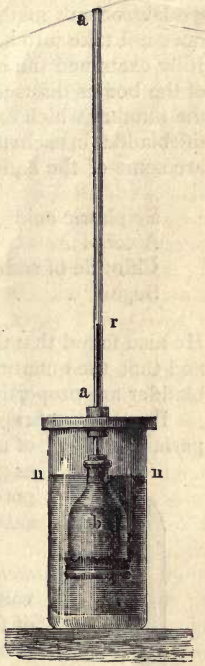


Fig. 121.

The conditions for the occurrence of osmose are that the membrane be permeable to at least one of the liquids, and that the liquids be different, but capable of mixing, such as alcohol and water ; there is none, for instance, between water and oil. Osmose may occur between two liquids of the same kind, but of different densities, such as solutions of acids or salts of different strengths ; here the current is from the weaker towards the stronger solution, and this is general, osmose usually taking place towards the denser liquid. Alcohol and ether form an exception ; although they are specifically lighter than water, they behave in this respect like liquids which are denser.

If a tube filled with water be closed at both ends by a bladder (fig. 122), and one end is placed in a vessel of water, the other being in contact with the air, the water gradually evaporates through the bladder ; it is, however, as rapidly replaced, so that, in consequence of evaporation, water moves towards the place where this occurs. Hence osmose plays a part in the motion. The evaporation from the skin of animals brings about a motion of liquids from the interior towards the evaporating surface. In like manner, the passage of water to the rootlets of plants, as well as the ascent of sap to the highest points of the trees, is favoured by evaporation from branchlets, leaves, flowers, and fruit.

The well-known fact that dilute alcohol kept in a porous vessel becomes concentrated depends on osmose. If a mixture of alcohol and water be

kept for some time in a bladder, the volume diminishes, but the alcohol becomes much more concentrated. The reason doubtless is that the bladder absorbs water more readily than alcohol, and accordingly water evaporates on the surface, and thus brings about a concentration of the residue.

Dutrochet's method is not adapted for quantitative measurements, for it does not take into account the hydrostatic pressure produced by the column. Jolly examined the endosmose of various liquids by determining the weights of the bodies diffused. He called the *endosmotic equivalent* of a substance the number which expresses how many parts by weight of water pass through the bladder in exchange for one part by weight of the substance. The following are some of the endosmotic equivalents which he determined :—

Sulphuric acid . .	0·4	Sulphate of copper . .	9·5
Alcohol	4·2	„ magnesium .	11·7
Chloride of sodium .	4·3	Caustic potass . . .	215·0
Sugar	7·1		

He also found that the endosmotic equivalent increases with the temperature, and that the quantities of substances which pass in equal times through the bladder are proportional to the strengths of the solutions.

Porous diaphragms differ very greatly in the facility with which they permit osmose ; of all substances goldbeater's skin is the best, being twice as good as vegetables, and sixty or seventy times as good as porous earthenware, which, however, is necessary in some cases, for organic membranes are apt to decompose.

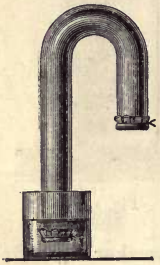


Fig. 122.

Pfeuffer has constructed what he calls *semipermeable membranes* by immersing a porous cell, such as is used for voltaic cells, in solution of copper sulphate, and then in one of ferrocyanide of potassium. By double decomposition, a coherent layer of ferrocyanide of copper is found, which is permeable to the molecules of water, but not to those of sugar, for instance. If a solution of sugar be exposed to pressure in such a vessel, pure water filters through ; the membrane acts as a *molecular sieve*.

The whole phenomena and laws of osmose have, in recent times, acquired great importance from the theoretical considerations of Van 't Hoff, on the nature of solutions, of which we may indicate the general results.

If a one-per-cent. solution of sugar be placed in the vessel *a* in such an arrangement as that represented in fig. 121, when the semi-permeable diaphragm is of ferrocyanide of copper, it will be found that after the lapse of some time the liquid will rise in the tube to a maximum height of 53·5 cm. ; this height is a measure of the *osmotic pressure*. Dealing with dilute solutions, it is found that this pressure is proportional to the concentration ; thus, with solutions of 1, 2, 4, and 6 per cent. respectively, the corresponding osmotic pressures are 53·5, 101·6, 208·2, and 307·5, respectively. We shall afterwards see (293) that we conceive a mass of gas as made up of a very large number of molecules moving in all directions with extreme velocity, and that the pressure of a gas is due to the impacts of these molecules against the sides of

the containing vessel. Now in what are called *ideal solutions*—those, that is to say, in which the dimensions of the molecules (3) of the body dissolved may be disregarded in comparison with the space in which they are contained—Van 't Hoff considers that the molecules of the body dissolved are animated by just such a motion as they possess in the case of gases.

If the pressure on a given volume of gas be gradually increased its volume will be diminished, and it is found that to reduce it to one-half, the original pressure must be doubled. For a given mass of gas, the product of pressure and volume is constant; this is what is known as Boyle's law (180). Osmotic pressure in liquids is analogous to the pressure of gases; if we double the number of molecules in a given volume of liquid, we double the pressure, just as we can force two volumes of gas into the space occupied by one if we double the pressure. This analogy between osmotic pressure is not a fanciful one, but holds good in details so far as it has been tried. It can be shown, for instance, that the osmotic pressure of sugar in solution is the same as would be exerted by the same weight of sugar if it existed in the state of gas in the same space as that occupied by the solution.

Gases, as will afterwards be shown, expand by a certain definite proportion when heated; or, if the volume be kept constant, the pressure is proportional to the increase of temperature. This is also found to hold with the osmotic pressure; it increases in direct proportion to the temperature.

140. **Diffusion of liquids.**—If oil be poured on water, no tendency to intermix is observed, and even if the two liquids be violently agitated together, two separate layers are formed on allowing them to stand. With alcohol and water the case is different; if alcohol, which is specifically lighter, be carefully poured upon water, so as to form two distinct layers, it will be seen that the heavier water rises in opposition to gravity with the lighter alcohol, which, in turn, passes into the denser liquid below, the liquids gradually intermix, in spite of the difference of their specific gravities; they *diffuse* into one another.

This point may be illustrated by the experiment represented in fig. 123. A tall jar contains water coloured by solution of blue litmus; by means of a funnel some dilute sulphuric acid is carefully poured in, so as to form a layer at the bottom; the colour of the solution is changed into red, progressing upwards, and after forty-eight hours the change is complete—a result of the action of the acid, and a proof, therefore, that it has diffused throughout the entire mass.

The laws of this diffusion, in which no porous diaphragm is used, were completely investigated by Graham. The method by which his latest experiments were made was the following:—A small wide-necked bottle, A (fig. 124), filled with the liquid whose rate of diffusion was to be examined, was closed by a thin glass disc and placed in a larger vessel, B, in which water was poured to a height of about an inch above the top of the bottle. The disc was carefully removed, and then after a given time successive layers were carefully drawn off by means of a siphon or pipette, and their contents examined.

The general results of these investigations may be thus stated:—

i. When solutions of the same substance, but of different strengths, are

taken, the quantities diffused in equal times are proportional to the strengths of the solutions.

ii. In the case of solutions containing equal weights of different substances, the quantities diffused vary with the nature of the substances. Saline substances may be divided into a number of *equidiffusive groups*, the rates of diffusion of each group being connected with the others by a simple numerical relation.

iii. The quantity diffused varies with the temperature. Thus, taking the rate of diffusion of hydrochloric acid at 15° C. as unity, at 49° C. it is 2·18.

iv. If two substances which do not combine be mixed in solution, they may be partially separated by diffusion, the more diffusive one passing out most rapidly. In some cases even chemical decomposition may be effected by diffusion. Thus, bisulphate of potassium is decomposed into free sulphuric acid and neutral sulphate of potassium.

v. If liquids be dilute, a substance will diffuse into water containing

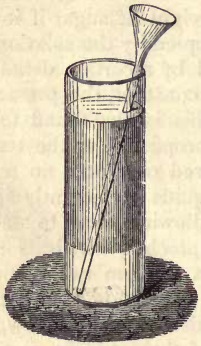


Fig. 123.

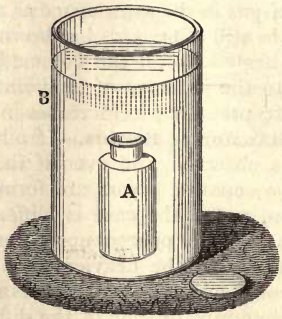


Fig. 124.

another substance dissolved, as if into pure water; but the rate is materially reduced if a portion of the same diffusing substance be already present.

The following table gives the approximate times of equal diffusion :—

Hydrochloric acid . .	1·0	Sulphate of magnesium .	7·0
Chloride of sodium . .	2·3	Albumen	49·0
Sugar	7·0	Caramel	98·0

It will be seen from the above table that the difference between the rates of diffusion is very great. Thus sulphate of magnesium, one of the least diffusible of saline substances, diffuses 7 times as rapidly as albumen and 14 times as rapidly as caramel. These last substances, like hydrated silicic acid, starch, dextrine, gum, &c., constitute a class of substances which are characterised by their incapacity for taking the crystalline form, and by the mucilaginous character of their hydrates. Considering gelatine as the type of this class, Graham called them *colloids* (κολλη, glue), in contradistinction to the far more easily diffusible *crystalloid* substances. Colloids are for the most part bodies of high molecular weight, and it is probably the

larger size of their molecules which hinders their passing through minute apertures.

Graham devised a method of separating bodies based on their unequal diffusibility, which he called *dialysis*. His *dialyser* (fig. 125) consists of a ring of gutta-percha, over which is stretched while wet a sheet of parchment paper, forming thus a vessel about two inches high and ten inches in diameter, the bottom of which is of parchment paper. After pouring in the mixed solution to be dialysed, the whole is floated on a vessel containing a

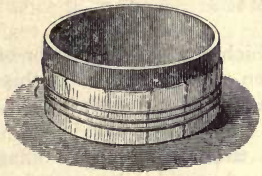


Fig. 125.

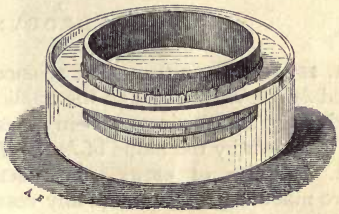


Fig. 126.

very large quantity of water (fig. 126). In the course of one or two days a more or less complete separation will have been effected. Thus a solution of arsenious acid mixed with various kinds of food readily diffuses out. The process has received important applications to laboratory and pharmaceutical purposes.

Emulsions, such as are of frequent use in medicine, are prepared by mixing intimately oil with a solution of gum, albumen, or some other colloid, and water. As stated above, the reason of difficulty with which a colloid diffuses through the membrane of another colloid is probably that its molecules are too large and too near each other—in other words, that the pores are too small. With an ordinary emulsion, the minute droplets of oil are dispersed among the large and difficult mobile particles of the colloid, which thus hinder their motion, and thereby prevent them from uniting and forming a coherent layer.

CHAPTER III

HYDRODYNAMICS

141. **Hydrodynamics.**—The science which treats of the motion of liquids is called *hydrodynamics*; and the application of the principles of this science to conducting and raising water in pipes and to the use of water as a motive power is known by the name of *hydraulics*.

142. **Velocity of efflux. Torricelli's theorem.**—Let us imagine an aperture made in the bottom of any vessel, and consider the case of a particle of liquid on the surface, without reference to those which are beneath. If this particle fell freely, it would have a velocity on reaching the orifice equal to that of any other body falling through the distance between the level of the liquid and the orifice. This, from the laws of falling bodies, is $\sqrt{2gh}$, in which g is the accelerating force of gravity, and h the height. If the liquid be maintained at the same level, for instance, by a stream of water running into the vessel, sufficient to replace what has escaped, the particles will follow one another with the same velocity, and will issue in the form of a stream. Since pressure is transmitted equally in all directions, a liquid would issue from an orifice in the side with the same velocity, provided the depth were the same.

The law of the velocity of efflux was discovered by Tŏrricelli. It may be enunciated as follows:—*The velocity of efflux is the velocity which a freely falling body would have on reaching the orifice after having started from a state of rest at the surface.* It is algebraically expressed by the formula $v = \sqrt{2gh}$.

It follows directly from this law that the velocity of efflux depends on the depth of the orifice below the surface, and not on the nature of the liquid. Through orifices of equal size and of the same depth, water and mercury would issue with the same velocity; for although the density of the latter liquid is greater, the weight of the column, and consequently the pressure, are greater too. It follows further that the velocities of efflux are directly proportional to the square roots of the depth of the orifices. Water would issue from an orifice 100 inches below the surface with ten times the velocity with which it would issue from one an inch below the surface.

The quantities of water which issue from orifices of different areas are very nearly proportional to the size of the orifice, provided the level remains constant.

143. **Direction of the jet from lateral orifices.**—From the principle of the equal transmission of pressure, water issues from an orifice in the side of a vessel with the same velocity as from an aperture in the bottom of a vessel at the same depth. Each particle of a jet issuing from the side of a

vessel begins to move horizontally with the velocity above mentioned, but it is at once drawn downward by the force of gravity in the same manner as a bullet fired from a gun, with its axis horizontal. It is well known that the bullet describes a parabola (51) with a vertical axis, the vertex being the muzzle of the gun. Now, since each particle of the jet moves in the same curve, the jet itself takes the parabolic form (123).

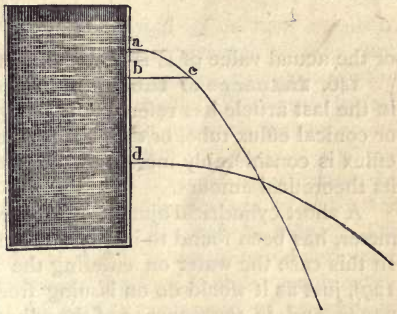


Fig. 127.

In every parabola there is a certain point called the *focus*, and the distance from the vertex to the focus fixes the magnitude of a parabola in much the same manner as the distance from the centre to the circumference fixes the magnitude of a circle. Now it can easily be proved that the focus is as much below as the surface of the water is above the orifice. Accordingly, if water issues through orifices which are small in comparison with the contents of the vessel, the jets from orifices at different depths below the surface take different forms, as shown in fig. 127. If these are traced on paper held behind the jet, then, knowing the horizontal distance and the vertical height, it is easy to demonstrate that the jet forms a parabola.

144. **Height of the jet.**—If a jet issuing from an orifice in a vertical direction has the same velocity as a body would have which fell from the surface of the liquid to that orifice, the jet ought to rise to the level of the liquid. It does not, however, reach this; for the particles which fall hinder it. But by inclining the jet at a small angle with the vertical it reaches about $\frac{98}{100}$ of the theoretical height, the difference being due to friction and to the resistance of the air. By experiments of this nature the truth of Torricelli's law has been demonstrated.

145. **Quantity of efflux. Vena contracta.**—If we suppose the sides of a vessel containing water to be thin, and the orifice to be a small circle whose area is A, we might think that the quantity of water, E, discharged in a second would be given by the expression $A\sqrt{2gh}$, since each particle has, on the average, a velocity equal to $\sqrt{2gh}$, and particles issue from each point of the orifice. But this is by no means the case. This may be explained by reference to fig. 128, in which AB represents an orifice in the bottom of a vessel—what is true in this case being equally true of an orifice in the side of the vessel. Every particle above AB endeavours to pass out of the vessel, and in so doing exerts a pressure on those near it. Those that issue near A and B exert pressures in the directions MM and NN; those near the centre of the orifice in the direction RQ, those in the intermediate parts in the directions PQ, PQ. In consequence, the water within the space PQP is unable to escape, and that which does escape, instead of assuming a cylindrical form, at first contracts, and takes the form

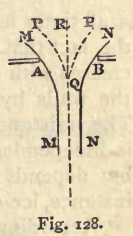


Fig. 128.

of a truncated cone. It is found that the escaping jet continues to contract until at a distance from the orifice about equal to the diameter of the orifice. This part of the jet is called the *vena contracta.* It is found that the area of its smallest section is about $\frac{8}{13}$ or 0·625 of that of the orifice. Accordingly, the true value of the efflux per second is given approximately by the formula

$$E = 0{\cdot}62A\sqrt{2gh},$$

or the actual value of E is about 0·62 of its *theoretical amount.*

146. **Influence of tubes on the quantity of efflux.**—The result given in the last article has reference to an aperture in a thin wall. If a cylindrical or conical efflux tube, or *ajutage*, is fitted to the aperture, the amount of the efflux is considerably increased, and in some cases falls but little short of its theoretical amount.

A short cylindrical ajutage, whose length is from two to three times its diameter, has been found to increase the efflux per second to about $0{\cdot}82A\sqrt{2gh}$. In this case the water on entering the ajutage forms a contracted vein (fig. 129), just as it would do on issuing freely into the air; but afterwards it expands, and, in consequence of the adhesion of the water to the interior surface of the tube, has, on leaving the ajutage, a section greater than that of the contracted vein. The contraction of the jet within the ajutage causes a partial vacuum. If an aperture is made in the ajutage, near the point of greatest contraction, and is fitted with a vertical tube, the other end of which dips into water (fig. 129), it is found that water rises in the vertical tube, thereby proving the formation of a partial vacuum.

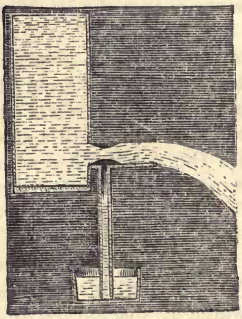


Fig. 129.

If the ajutage has the form of a conic frustum whose larger end is at the aperture, the efflux in a second may be raised to $0{\cdot}92A\sqrt{2gh}$, provided the dimensions are properly chosen. If the smaller end of a frustum of a cone of suitable dimensions be fitted to the orifice, the efflux may be still further increased, and fall very little short of the theoretical amount.

When the ajutage has more than a certain length, a considerable diminution takes place in the amount of the efflux: for example, if its length is 48 times its diameter, the efflux is reduced to $0{\cdot}63A\sqrt{2gh}$. This arises from the fact that, when water passes along cylindrical tubes, the resistance increases with the length of the tube; for a thin layer of liquid is attracted to the walls by adhesion, and the internal flowing liquid rubs against this. The resistance which gives rise to this result is called *hydraulic friction*; it is independent of the material of the tube, provided it be not roughened; but depends in a considerable degree on the viscosity of the liquid; for instance, ice-cold water experiences a greater resistance than lukewarm water.

According to Prony, the mean velocity v of water in a cast-iron pipe of the length l, and the diameter d, under the pressure p, is in metres

$$v = 26{\cdot}8\sqrt{\frac{dp}{l}}.$$

This is on the assumption that the tubes are straight. Any angle or curvature of the tube diminishes it, seeing that part of the motion is used up in pressure against the sides. Thus Venturi found the time requisite to fill a small vessel by means of a tube 38 inches in length by 3·3 in diameter was 45, 50, or 70 seconds, according as the tube was straight, curved, or bent at a right angle.

By means of hydraulic pressure Tresca submitted solids such as silver, lead, iron and steel, powders like sand, soft plastic substances such as clay, and brittle bodies like ice, to such enormous pressures as 100,000 kilogrammes, and has found that they then behave like fluid bodies. His experiments show also that these bodies transmit pressure equally in all directions when the pressure is considerable enough.

147. **Efflux through capillary tubes.**—This was investigated by Poisseuille by means of the apparatus represented in fig. 130, in which the capillary tube AB is sealed to a glass tube on which a bulb is blown. The volume of the space between the marks M and N is accurately determined, and the apparatus, having been filled with the liquid under examination by suction, is connected at the end M with a reservoir of compressed air, in which the pressure is measured by means of a mercury manometer (183). The time is then noted which is required for the level of the liquid to sink from M to N, the pressure remaining constant. It is thus found that v, the volume which flows out in a given time, is, with close approximation, represented by the formula

$$v = \frac{\pi p r^4}{8 \eta l},$$

where l is the length, and r the diameter of the tube, p the pressure, and η the *coefficient of internal friction* (48); which may be defined as the resistance to motion offered by two layers of the liquid of unit surface, at unit distance, and moving away from each other with unit velocity. Knowing the dimensions, a determination of the volume which flows out in a given time is a ready means of obtaining this coefficient. If the liquid which flows out through the tube is one which moistens it, a layer of liquid adheres to the side; and accordingly the friction which the liquid experiences is not against the sides, but is due to that of the particles against each other. The *coefficient of internal friction* is represented in the above formula by η. Bodies with a high coefficient of internal friction are said to be *viscous* (96). The liquids ether, water, sulphuric acid, linseed oil, Venice turpentine, represent, for instance, a series with increasing viscosity. The reciprocal of the viscosity

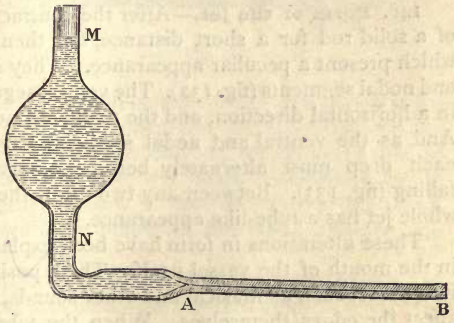


Fig. 130.

or $\frac{1}{\eta}$, is called the *fluidity*; it appears probable that this increases with the temperature in the same ratio as the conductivity for electricity (958). The coefficient of internal friction is greater in the case of solution of salts than with water, and increases with the strength of the solution. It greatly diminishes with the temperature, and, for water, at 60° is one-third what it is at zero.

A convenient apparatus for the purpose, more particularly for comparative measurements, is that given by Ostwald (fig. 131). It consists essentially of a tube, *de*, which is narrowed at *c*, and with a bulb, to which is attached the capillary *ab*, which again terminates in the wider tube *be*. The tube is filled with liquid to cover the bulb from the bottle, by aspiration through the caoutchouc tube *t*; the liquid is then allowed to flow out, and the time noted which its surface takes in falling from *e* to a mark, *a*. If the experiment be made with water, which is taken as standard, then, using the same apparatus, other liquids may be compared with it; this has the advantage of dispensing with a determination of the dimensions of the tube, and particularly of the diameter—a matter of importance, since its fourth power occurs in the formula, and thus any error in its determination greatly affects the result.

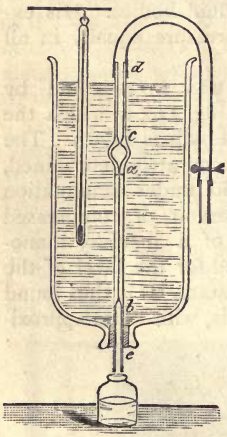


Fig 131.

If t is the time required for the flow of the given volume of water, and t' that of a liquid whose coefficient is η and sp. gr. σ, then

$$\eta = \frac{\sigma t'}{t},$$

the coefficient of water and its specific gravity being each unity.

148. **Form of the jet.**—After the contracted vein, the jet has the form of a solid rod for a short distance, but then begins to separate into drops, which present a peculiar appearance. They seem to form a series of ventral and nodal segments (fig. 132). The ventral segments consist of drops extended in a horizontal direction, and the nodal segments in a longitudinal direction. And as the ventral and nodal segments have respectively a fixed position, each drop must alternately become elongated and flattened while it is falling (fig. 133). Between any two drops there are smaller ones, so that the whole jet has a tube-like appearance.

These alterations in form have been explained as being due to vibrations in the mouth of the vessel itself. Their position is modified by extraneous influences, such as musical and other sounds, but only when these influences affect the edges themselves. When the vibrations of the vessel itself are stopped, the enlargements and contractions in the jet cease also; they are strengthened, on the contrary, if a violoncello, for instance, is sounded.

If the jet is momentarily illuminated by the electric spark, its structure is well seen; the drops appear then to be stationary, and separate from each other. If the aperture is not circular, the form of the jet undergoes curious changes.

149. **Hydraulic tourniquet.**—If water be contained in a vessel, and an aperture be made in one of the sides, the pressure at this point is removed, for it is expended in sending out the water; but it remains on the other side; and if the vessel were movable in a horizontal direction, it would move in a direction opposite to that of the issuing jet. This is illustrated by the apparatus known as the *hydraulic tourniquet* or *Barker's mill* (fig. 134). It consists of a glass vessel, M, containing water, and capable of moving about its vertical axis. At the lower part there is a tube, C, bent horizontally in opposite directions at the two ends. If the vessel were full of water and the tubes

Fig. 132.

Fig. 133.

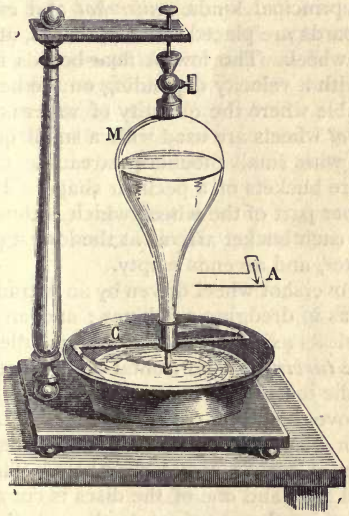


Fig. 134.

closed, the pressures on the sides of C would balance each other, being equal and acting in contrary directions; but, being open, the water runs out, and the pressure is not exerted on the open part, but only on the opposite side, as shown in the figure A. And this pressure, not being neutralised by an opposite pressure, imparts a rotatory motion in the direction of the arrow, the velocity of which increases with the height of the liquid and the size of the aperture.

The same principle may be illustrated by the following experiment. A tall cylinder containing water, and provided with a lateral stop-cock near the bottom, is placed on a light shallow dish on water, so that it easily floats. On opening the stop-cock so as to allow water to flow out, the vessel is observed to move in a direction diametrically opposite to that in which the water is issuing. Similarly, if a vessel containing water be suspended by a string, on opening an aperture in one of the sides the water will jet out, and the vessel be deflected away from the vertical in the opposite direction.

Segner's water-wheel and the reaction machine depend on this principle. So also do rotating fireworks; that is, an unbalanced reaction from the heated gases which issue from openings in them gives them motion in the opposite direction.

150. **Water-wheels. Turbines.**—When water is continuously flowing from a higher to a lower level, it may be made use of as a motive power. The motive power of water is generally utilised either by means of *water-wheels*, *turbines*, *rams*, or *hydraulic engines*.

Water-wheels are wheels provided with buckets or float-boards at the circumference, and on which the water acts either by pressure or by impact. They are made to turn in a vertical plane round a horizontal axis, and are of two principal kinds, *undershot* and *overshot*. In *undershot* wheels the float-boards are placed radially, that is, at right angles to the circumference of the wheel. The lowest float-boards are immersed in the water, which flows with a velocity depending on the height of the fall. Such wheels are applicable where the quantity of water is great, but the fall inconsiderable. *Overshot* wheels are used with a small quantity of water which has a high fall, as with small mountain streams. On the circumference of the wheel there are buckets of a peculiar shape. The water falls into the buckets on the upper part of the wheel, which is thus moved by the weight of the water, and as each bucket arrives at the lowest point of revolution it discharges all the water, and ascends empty.

An overshot wheel driven by an extraneous force may be used for raising water, as in dredging machines; and an undershot one for moving a vessel to which its axis is fixed, as in the paddles of steam-vessels.

The *turbine* is a horizontal water-wheel, and is similar in principle to the hydraulic tourniquet or reaction wheel (149). It consists of a pair of discs, one above the other, connected together by a number of specially shaped thin arms or blades, which divide the space between the discs into an equal number of curved radial chambers. The wheel works generally upon a vertical axis, and one of the discs is cut away at the centre. In an *outward flow turbine*, the water enters through the opening so made into the space between the discs, and passes outwards radially through the chambers above mentioned, causing the wheel to rotate by its reaction upon their curved walls. In order to prevent waste of energy in giving useless rotation to the water, the peripheral openings of the wheel are surrounded by a series of corresponding fixed chambers, whose sides (guide-blades) are so curved that the water when it leaves them has lost all its rotational motion, and simply flows away at right angles to the axis. In an *inward flow turbine* the water enters the peripheral opening of the wheel through the guide-blades, and leaves the wheel at the centre.

The total theoretical effect of a fall of water is never realised; for the water, after acting on the wheel, still retains some velocity, and therefore does not impart the whole of its velocity to the wheel. In many cases water flows past without acting at all; if the water acts by impact, vibrations are produced which are transmitted to the earth and lost; the same effect is produced by the friction of water over an edge of the sluice, in the channel which conveys it, or against the wheel itself, as well as by the friction of this latter against the axle. A wheel working freely in a stream, as with the

corn-mills on the Rhine near Mainz, does not utilise more than 20 per cent. of the theoretical effect. One of the most perfect forms of turbines will work up to over 80 per cent. Turbines also, when properly designed, may be made to have a very high efficiency either with high or low falls; while, on account of the great speed at which they run, they are very much smaller than water-wheels in proportion to their power. They are thus more 'efficient' motors than steam-engines, which, even if perfect, can only transform into work from 25 to 30 per cent. of the energy represented by the coal they burn, and seldom in practice utilise more than half of this percentage.

150*a*. **The Hydraulic Ram.**—If a quantity of water flow through a pipe open at one end, and if this aperture be quickly closed, a sudden impact will be exerted on the closure as well as on the sides of the pipe. Some of the energy of the falling water is thereby converted into heat, and some exerts a dangerous pressure on the pipe. The existence of this pressure may be readily observed in any town with a high-pressure water supply, by the sharp click heard if the tap through which water is flowing is suddenly closed.

The *hydraulic ram* invented by Montgolfier is an arrangement by which the energy of falling water is applied so as to raise a portion of it to a greater height than the reservoir from which it is fed.

The principle of such an arrangement is represented in fig. 135, in which E is the reservoir, A the pipe in which the water falls, B the channel, which should be long and straight, *a* and *b* the valves, C the wind-chest, and D the rising main. Water first flows out in quantity through the valve *a*, and as soon as it has acquired a certain velocity it raises that valve, and the aperture is shut. The impact thus produced, acting on the sides of the pipe and on the valve *b*, raises this valve, and a quantity of water passes into the wind-chest shutting off air, and compressing it in the space above the mouth *d* of the rising main D. This air by its elastic force closes the valve *b*, and the water which has entered is raised in the main pipe D.

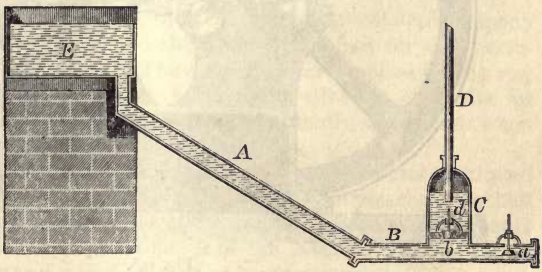


Fig. 135.

As soon as the impulsive action is over, and the water in the channel is at rest, the valve *a* falls again by its own weight, the flow begins afresh, and when it has acquired sufficient velocity the valve *b* is again closed, and the whole process is repeated.

In this way water can be raised to a height several times as great as the difference in level from E to the valve *b*. If no energy were lost in friction, and in raising the valves, the height of ascent would be to the fall as the quantity of water which flows out at *a* is to that which is raised. Thus $\frac{1}{5}$ of the water flowing out of the channel could be raised to 5 times the height of the available fall.

151. **Hydraulic engine.**—Historically, falling water was one of the earliest sources of power; but it is only in recent times that attention has been called (first by Lord Armstrong) to the advantage of using hydraulic power in towns and other places where there is no *natural* fall of water for driving certain classes of machines, in those cases more especially where the use of the machinery is only intermittent.

For this purpose the most important docks and large warehouses are now generally furnished with means of obtaining a water-supply at a very high pressure, generally about 700 pounds to the square inch. Steam-pumping engines are employed to pump water more or less continuously into what are practically large cylinders with immensely heavy pistons loaded to the required pressure. These vessels are called *accumulators*, and pipes from them are led away to the various places (lock gates, sluice valves,

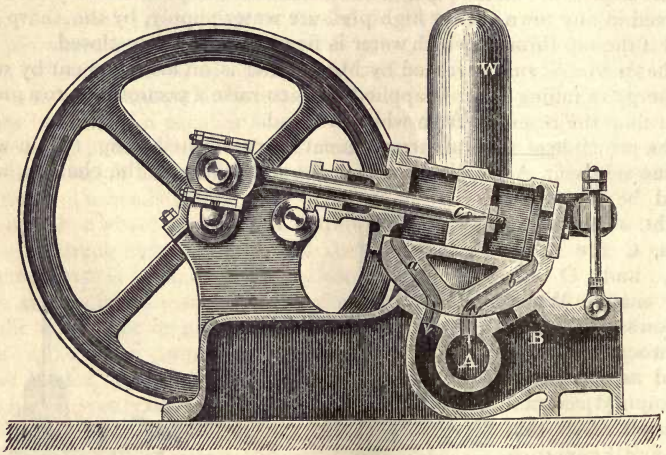


Fig. 136.

cranes, capstans, &c.) where power may be wanted. At each of these places there is some kind of hydraulic motor suitable to the particular work to be done, and this motor can be instantaneously set to work by opening the communication between it and the high-pressure water in the accumulator. The motor used is not uncommonly a small engine similar in principle to a steam-engine, and one of the best of these engines is that illustrated in fig. 136, which is the invention of Schmidt of Zürich. It consists of a cylinder fitted with a piston, *c*, whose rod is connected directly to a crank upon a horizontal shaft. The cylinder has two *ports* or passages, *a* and *b*, one at each end, both terminating below in openings upon a convex curved face, which is kept continually pressed against a similar concave face upon the framing of the engine. In this fixed face are also an inlet port or passage A, and outlet passages B. When the cylinder is in the position shown in the figure, the high-pressure water is passing through A and *b*, forcing the piston along, and driving out the already used water through *a* and B. As the piston moves and turns the crank, the cylinder oscillates on

its bearings, and by the time the piston has got to the end of its stroke, the cylinder then being horizontal, the process is just being reversed, water passing in through A and *a*, and out through *b* and B. W is an air-vessel for preventing shocks.

This motor utilises 90 per cent. of the available power; in this respect it far exceeds a steam-engine which does not utilise more than 10 per cent. of the power due to the consumption of the coal burnt.

The chief drawback about the use of water power, except where there is a large natural supply under pressure, is its expense. For each revolution of the crank shaft two complete cylinders full of water must be passed through such an engine, as, whether the power be wanted or not, the water cannot be expanded like steam.

With any given pressure it is easy to find out how much water will be required for a given power. At a pressure of 30 pounds per square inch, for instance, one horse-power will require, supposing the *efficiency* of the machine to be 70 per cent. (472), $\frac{33000 \times 60}{30 \times 144 \times 0{\cdot}7}$ = about 655 cubic feet or 4,000 gallons per hour, a quantity the cost of which would in most cases put the use of this power out of the question. The pressure in town mains generally lies between 20 and 40 pounds per square inch, and it is therefore only in cases where a special high-pressure supply is available that the power can be economically used.

In London, water is supplied to consumers by the Hydraulic Power Company under a pressure of 700 pounds; and the quantity required for one horse-power would be about 175 gallons. The cost of power supplied in this way is about fourpence per horse-power per hour, which, although expensive for continuous working, is not so when it is intermittently used, and when only the quantity actually consumed is paid for.

Water-power is usually represented by the weight of the water multiplied into the height of the available fall; or it may also be represented by half the product of the mass into the square of the velocity. Both measurements give the same result (60). The water-power of the Niagara Falls is calculated to be equal to four and a half millions of horse-power.

BOOK IV

ON GASES

CHAPTER I

PROPERTIES OF GASES. ATMOSPHERE. BAROMETERS

152. **Physical properties of gases.**—Gases are bodies which, unlike solids, have no independent shape, and, unlike liquids, have no independent volume. Their molecules possess almost perfect mobility; they are conceived as darting about in all directions, and are continually tending to occupy a greater space. This property of gases is known by the names *expansibility*, *tension*, or *elastic force*, from which they are often called *elastic fluids*.

Gases and liquids have several properties in common, and some in which they seem to differ are in reality only different degrees of the same property. Thus, in both, the particles are capable of moving; in gases with almost perfect freedom; in liquids not quite so freely, owing to a greater degree of viscosity (147). Both are compressible, though in very different degrees. If a liquid and a gas both exist under the pressure of one atmosphere, and then the pressure be doubled, the water is compressed by about the $\frac{1}{20000}$ part, while the gas is compressed by one-half. In density there is a great difference: water, which is the type of liquids, is 770 times as heavy as air, the type of gaseous bodies, while under the pressure of one atmosphere. A spiral spring only shows elasticity when it is compressed; it loses its tension when it has returned to its primitive condition. A gas has no original volume; it is always elastic, or, in other words, it is always striving to attain a greater volume; this tendency to indefinite expansion is the chief property by which gases are distinguished from liquids.

Matter assumes the solid, liquid, or gaseous form according to the relative strength of the cohesive and repulsive forces exerted between their molecules. In liquids these forces balance; in gases repulsion preponderates.

By the aid of pressure and of low temperatures, the force of cohesion may be so far increased in many gases that they are readily converted into liquids, and we know now that with sufficient pressure and cold they may all be liquefied. On the other hand, heat, which increases the *vis viva* of the molecules, converts liquids, such as water, alcohol, and ether, into the aëriform

state in which they obey all the laws of gases. The aëriform state of liquids is known by the name of *vapour*; while gases are bodies which, under ordinary temperature and pressure, remain in the aëriform state.

In describing exclusively the properties of gases we shall, for obvious reasons, refer to atmospheric air as their type.

153. **Expansibility of gases.**—This property of gases, their tendency to assume continually a greater volume, is exhibited by means of the following experiment :—A bladder, closed by a stop-cock and about half full of air, is placed under the receiver of the air-pump (fig. 137), and a vacuum is produced, on which the bladder immediately distends. This arises from the fact that the molecules of air flying about in all directions (293) press against the sides of the bladder. Under ordinary conditions, this internal pressure is counterbalanced by the air in the receiver, which exerts an equal and contrary pressure. But when this pressure is removed, by exhausting the receiver, the internal pressure becomes evident. When air is admitted into the receiver, the bladder resumes its original form.

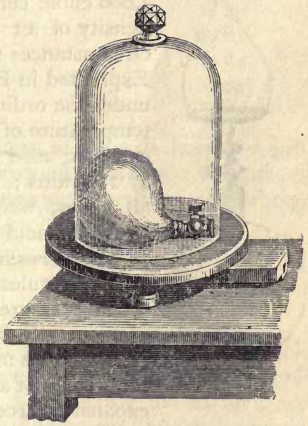


Fig 137.

154. **Compressibility of gases.**—The compressibility of gases is readily shown by the *pneumatic syringe* (fig. 138). This consists of a stout glass tube closed at one end, and provided with a tight-fitting solid piston. When the rod of the piston is pressed it moves down in the tube, and the air becomes compressed into a smaller volume; but as soon as the force is removed the air regains its original volume, and the piston rises to its former position.

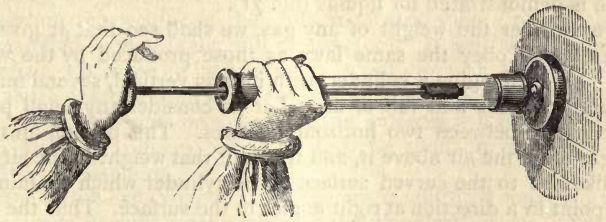


Fig. 138.

155. **Weight of gases.**—From their extreme fluidity and expansibility, gases seem to be uninfluenced by the force of gravity : they nevertheless possess weight like solids and liquids. To show this, a glass globe of 3 or 4 quarts' capacity is taken (fig. 139), the neck of which is provided with a stop-cock, which hermetically closes it, and by which it can be screwed to the plate of the air-pump. The globe is then exhausted, and its weight determined by means of a delicate balance. Air is now allowed to enter, and the globe again weighed. The weight in the second case will be found to be

greater than before, and if the capacity of the vessel is known the increase will obviously be the weight of that volume of air.

By a modification of this method, and with the adoption of certain precautions, the weight of air and of other gases has been determined. Perhaps the most accurate are those of Regnault, who found that a litre of dry air at 0° C., and under a pressure of 760 millimetres, weighs 1·293187 gramme. Since a litre of water (or 1,000 cubic centimetres) at 0° weighs ·999877 gramme, the density of air is 0·00129334 that of water under the same circumstances; that is, water is 773 times as heavy as air. Expressed in English measures, 100 cubic inches of dry air under the ordinary atmospheric pressure of 30 in. and at the temperature of 16° C. weigh 31 grains; the same volume of carbonic acid gas under the same circumstances weighs 47·25 grains; 100 cubic inches of hydrogen, the lightest of all gases, weigh 2·14 grains; and 100 cubic inches of hydriodic acid gas weigh 146 grains.

Fig. 139.

156. **Pressure exerted by gases.**—Gases exert on their own molecules, and on the sides of vessels which contain them, pressures which may be regarded from two points of view. First, we may neglect the weight of the gas; secondly, we may take account of its weight. If we neglect the weight of any gaseous mass at rest, and only consider its expansive force, it will be seen that the pressures due to this force act with the same strength on all points, both of the mass itself and of the vessel in which it is contained. For it is a necessary consequence of the elasticity and fluidity of gases that the repulsive force between the molecules is the same at all points, and acts equally in all directions. This principle of the equality of the pressure of gases in all directions may be shown experimentally by means of an apparatus resembling that by which the same principle is demonstrated for liquids (fig. 71).

If we consider the weight of any gas, we shall see that it gives rise to pressures which obey the same laws as those produced by the weight of liquids. Let us imagine a cylinder, with its axis vertical, several miles high, closed at both ends and full of air. Let us consider any small portion of the air enclosed between two horizontal planes. This portion must sustain the weight of all the air above it, and transmit that weight to the air beneath it, and likewise to the curved surface of the cylinder which contains it, and at each point in a direction at right angles to the surface. Thus the pressure increases from the top of the column to the base; at any given layer it acts equally on equal surfaces, and at right angles to them, whether they are horizontal, vertical, or inclined. The pressure acts on the sides of the vessel, and on any small surface it is equal to the weight of a column of gas whose base is this surface, and whose height its distance from the summit of the column. The pressure is also independent of the shape and dimensions of the supposed cylinder, provided the height remain the same.

For a small quantity of gas the pressures due to its weight are quite insignificant, and may be neglected; but for large quantities, like the atmosphere, the pressures are considerable, and must be allowed for.

157. **The atmosphere: its composition.**—The atmosphere is the layer of air which surrounds our globe in every part. It partakes of the rotatory motion of the globe, and would remain fixed relatively to terrestrial objects but for local circumstances, which produce winds, and are constantly disturbing its equilibrium.

It is essentially a mixture of oxygen and nitrogen gases; its average composition by volume being as follows:—

Nitrogen	78·49
Oxygen	20·63
Aqueous vapour	0·84
Carbonic acid	0·04
	100·00

The carbonic acid arises from the respiration of animals from the processes of combustion, and from the decomposition of organic substances. Boussingault estimated that in Paris the following quantities of carbonic acid are produced every 24 hours:—

By the population and by animals	11,895,000 cubic feet.
By processes of combustion	92,101,000 „
	103,996,000 „

Notwithstanding this enormous continual production of carbonic acid the composition of the atmosphere does not vary; for plants in the process of vegetation decompose the carbonic acid, assimilating the carbon, and restoring to the atmosphere the oxygen, which is being continually consumed in the processes of respiration and combustion.

158. **Atmospheric pressure.**—If we neglect the perturbations to which the atmosphere is subject, as being inconsiderable, we may consider it as a fluid sea of a certain depth, surrounding the earth on all sides, and exercising the same pressure as if it were a liquid of very small density. Consequently, the pressure on the unit of area is constant at a given level, being equal to the weight of the column of atmosphere above that level whose horizontal section is the unit of area (99). It will act at right angles to the surface, whatever be its position. It will diminish as we ascend, and increase as we descend from that level. Consequently, at the same height, the atmospheric pressures on unequal plane surfaces will be proportional to the areas of those surfaces, provided they be small in proportion to the height of the atmosphere.

In virtue of the expansive force of the air, it might be supposed that the molecules would expand indefinitely into the planetary spaces. But, in proportion as the air expands, its expansive force decreases, and is further weakened by the low temperature of the upper regions of the atmosphere, so that, at a certain height, equilibrium is established between the expansive force which separates the molecules and the action of gravity which draws them towards the centre of the earth. It is therefore concluded that the atmosphere is limited.

From the weight of the atmosphere, and its increase in density, and from the observation of certain phenomena of twilight, its height has been estimated at from 30 to 40 miles. Above that height the air is extremely rarefied,

and at a height of 60 miles it is assumed that there is a perfect vacuum. On the other hand, meteorites have been seen at a height of 200 miles, and, as their luminosity is undoubtedly due to friction against air, there must be air at such a height. This higher estimate is supported by observations made at Rio Janeiro on the twilight arc, by M. Liais, who estimated the height of the atmosphere at between 198 and 212 miles. The question as to the exact height of the atmosphere must therefore be considered as still awaiting settlement.

As it has been previously stated that 100 cubic inches of air weigh 31 grains, it will readily be conceived that the whole atmosphere exercises a considerable pressure on the surface of the earth. The existence of this pressure is shown by the following experiments.

159. **Crushing force of the atmosphere.**—On one end of a stout glass cylinder, about 5 inches high, and open at both ends, a piece of bladder is tied quite air-tight. The other end, the edge of which is ground and well greased, is pressed on the plate of the air-pump (fig. 140). As soon as the air in the vessel is rarefied by working the air-pump, the bladder is depressed by the weight of the atmosphere above it, and finally bursts with a loud report caused by the sudden entrance of the air.

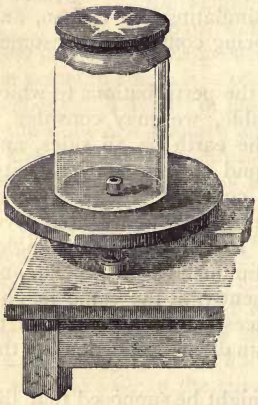


Fig. 140.

Fig. 141.

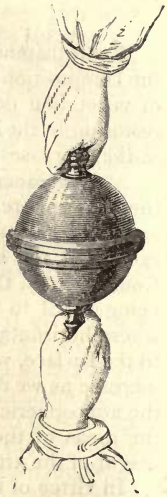


Fig. 142.

160. **Magdeburg hemispheres.**—The preceding experiment only serves to illustrate the downward pressure of the atmosphere. By means of the *Magdeburg hemispheres* (figs. 141 and 142), the invention of which is due to Otto von Guericke, burgomaster of Magdeburg, it can be shown that the pressure acts in all directions. This apparatus consists of two hollow brass hemispheres of 4 to 4½ inches diameter, the edges of which are made to fit tightly, and are well greased. One of the hemispheres is provided with a stop-cock, by which it can be screwed on to the air-pump, and on the other there is a handle. As long as the hemispheres contain air they can be separated

without any difficulty, for the external pressure of the atmosphere is counterbalanced by the elastic force of the air in the interior. But when the air in the interior is pumped out by means of the air-pump, the hemispheres cannot be separated without a powerful effort; and as this is the case in whatever position they are held, it follows that the atmospheric pressure is transmitted in all directions.

DETERMINATION OF THE ATMOSPHERIC PRESSURE. BAROMETERS

161. **Torricelli's experiment.**—The above experiments demonstrate the existence of the atmospheric pressure, but they give no precise indication as to its amount. The following experiment, which was first made in 1643, by Torricelli, a pupil of Galileo, gives an exact measure of the weight of the atmosphere.

A glass tube is taken, about a yard long and a quarter of an inch internal diameter (fig. 143). It is sealed at one end, and is quite filled with mercury. The aperture C being closed by the thumb, the tube is inverted, the open end placed in a small mercury trough, and the thumb removed. The tube being in a vertical position, the column of mercury sinks, and, after oscillating some time, it finally comes to rest at a height, A, which at the level of the sea is about 30 inches above the mercury in the trough. The mercury is raised in the tube by the pressure of the atmosphere on the mercury in the trough. There is no contrary pressure on the mercury in the tube, because it is closed; but, if the end of the tube be opened, the atmosphere will press equally inside and outside the tube, and the mercury will sink to the level of that in the trough. It has been shown in hydrostatics (107) that the heights of two columns of liquid in communication with each other are inversely as their densities; and hence it follows that the pressure of the atmosphere is equal to that of a column of mercury the height of which is 30 inches. If, however the weight of the atmosphere diminishes, the height of the column which it can sustain must also diminish.

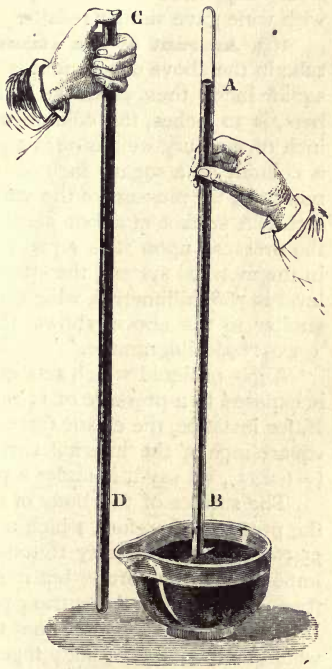


Fig. 143.

162. **Pascal's experiments.**—Pascal, who wished to ascertain whether the force which sustained the mercury in the tube was really the pressure of the atmosphere, made the following experiments. (i.) If it were the case, then the column of mercury ought to be lower in proportion as we ascend in the atmosphere. He accordingly requested one of his relatives to repeat

Torricelli's experiment on the summit of the Puy de Dôme in Auvergne. This was done, and it was found that the column of mercury was about 3 inches lower, thus proving that it is really the weight of the atmosphere which supports the mercury, since, when this weight diminishes the height of the column also diminishes. (ii.) Pascal repeated Torricelli's experiment at Rouen in 1646, with other liquids. He took a tube closed at one end nearly 50 feet long, and, having filled it with water, placed it vertically in a vessel of water, and found that the water stood in the tube at a height of 34 feet; that is, 13·6 times as high as mercury. But, since the mercury is 13·6 times as heavy as water, the height of the column of water was exactly equal to that of a column of mercury in Torricelli's experiment, and it was consequently the same force, the pressure of the atmosphere, which successively supported the two liquids. Pascal's other experiments with oil and with wine gave similar results.

163. **Amount of the atmospheric pressure.**—Let us assume that the tube in the above experiment is a cylinder, the section of which is equal to a square inch; then, since the height of the column of mercury, in round numbers, is 30 inches, the column will contain 30 cubic inches; and as a cubic inch of mercury weighs 3433·5 grains = 0·49 of a pound, the pressure of such a column on a square inch of surface is equal to 14·7 pounds. In round numbers, the pressure of the atmosphere is taken at 15 pounds on the square inch. A surface of a foot square contains 144 square inches, and therefore the pressure upon it is equal to 2,160 pounds, or nearly a ton. Expressed in the metrical system, the standard atmospheric pressure at 0° and the sea-level is 760 millimetres, which is equal to 29·9217 inches; and a calculation similar to the above shows that the pressure on a square centimetre is = 1·032896 kilogramme.

A gas or liquid which acts in such a manner that a square inch of surface is exposed to a pressure of 15 pounds is called a pressure of *one atmosphere*. If, for instance, the elastic force of the steam of a boiler is so great that each square inch of the internal surface is exposed to a pressure of 90 pounds ($= 6 \times 15$), we say it is under a pressure of six atmospheres.

The surface of the body of a man of middle size is about 16 square feet; the pressure, therefore, which a man supports on the surface of his body is 35,560 pounds, or nearly 16 tons. Such an enormous pressure might seem impossible to be borne; but it must be remembered that, in all directions, there are equal and contrary pressures which counterbalance one another. It might also be supposed that the effect of this force, acting in all directions, would be to press the body together and crush it. But the solid parts of the skeleton could resist a far greater pressure; and as to the air and liquids contained in the organs and vessels, the air has the same density as the external air, and cannot be further compressed by the atmospheric pressure; and from what has been said about liquids (97), it is clear that they are virtually incompressible. When the external pressure is removed from any part of the body, either by means of a cupping-vessel or by the air-pump, the pressure from within is seen by the distension of the surface.

164. **Different kinds of barometers.**—The instruments used for measuring the atmospheric pressure are called *barometers*. In ordinary barometers the pressure is measured by the height of a column of mercury, as in Torri-

celli's experiment ; the barometers which we are about to describe are of this kind. But there are barometers without any liquid, one of which, the aneroid (187), is remarkable for its simplicity and portability.

165. **Cistern barometer.**—The *cistern barometer* consists of a straight glass tube closed at one end, about 33 inches long, filled with mercury, and dipping into a cistern containing the same metal. In order to render the barometer more portable, and the variations of level in the cistern less perceptible when the mercury rises or falls in the tube, several different

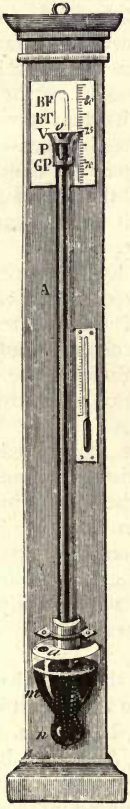


Fig. 144.

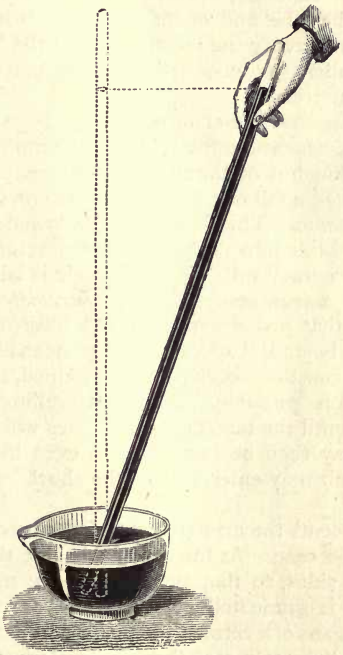


Fig. 145.

Fig. 146.

forms have been constructed. Fig. 144 represents one form of the cistern barometer. The apparatus is fixed to a mahogany stand, on the upper part of which there is a scale graduated in millimetres or inches from the level of the mercury in the cistern ; a movable index, *i*, shows on the scale the level of the mercury. A thermometer on one side of the tube indicates the temperature.

There is one fault to which this barometer is liable, in common with all

others of the same kind. The zero of the scale does not always correspond to the level of the mercury in the cistern. For, as the atmospheric pressure is not always the same, the height of the mercurial column varies ; sometimes mercury is forced from the cistern into the tube, and sometimes from the tube into the cistern, so that in the majority of cases the graduation of the barometer does not indicate the true height. If the diameter of the cistern is large, relatively to that of the tube, the error from this source, which is known as the *error of capacity*, is lessened.

The *height* of the barometer is the distance between the levels of the mercury in the tube and in the cistern. Hence the barometer should always be perfectly vertical ; for if not, the tube being inclined, the column of mercury is elongated (fig. 145), and the number read off on the scale is too great. As the pressure which the mercury exerts by its weight at the base of the tube is independent of the form of the tube and of its diameter (101), provided it is not capillary, the height of the barometer is independent of the diameter of the tube and of its shape, but is inversely as the density of the liquid. With mercury the mean height at the level of the sea is 29·92, or, in round numbers, 30 inches ; in a water barometer it would be about 34 feet, or 10·33 metres.

In *marine barometers* the error of capacity is got rid of by graduating the scale, not in the true measurements, but by an empirical correction depending on the relative diameters of the tube and cistern. Thus if a rise of 10 mm. in the tube produced a fall of 1 mm. in the cistern, the true change would not be 10 mm. but 11 mm. This is obviously allowed for by dividing the space of 10 mm. on the scale into 11 mm. The correctness of such an instrument depends on the accuracy with which the scale is laid off.

166. **Fortin's barometer.**—*Fortin's barometer* differs in the shape of the cistern from that just described. The base of the cistern is made of leather, and can be raised or lowered by means of a screw ; this has the advantage that a constant level can be obtained, and also that the instrument is made more portable. For, in travelling, it is only necessary to raise the leather until the mercury, which rises with it, quite fills the cistern ; the barometer may then be inclined, and even inverted, without any fear that a bubble of air may enter, or that the shock of the mercury may crack the tube.

Fig. 146 represents the arrangement of the barometer, the tube of which is placed in a brass case. At the top of this case there are two longitudinal slits on opposite sides, so that the level of the mercury, B, is seen. The scale on the case is graduated in millimetres. An index, A, moved by the hand, gives, by means of a vernier, the height of the mercury to $\frac{1}{10}$th of a millimetre. At the bottom of a case there is a cistern, *b*, containing mercury, *o*.

Fig. 147 shows the details of the cistern on a larger scale. It consists of a glass cylinder, *b*, through which the mercury can be seen ; this is closed at the top by a boxwood disc fitted on the under surface of the brass cover M. Through this passes the barometer tube E, which is drawn out at the end, and dips in the mercury ; the cistern and the tube are connected by a piece of buckskin, *ce*, which is firmly tied at *c* to a contraction in the tube, and at *e* to a brass tubulure in the cover of the cistern. This mode of closing prevents the mercury from escaping when the barometer is inverted, while

the pores of the leather transmit the atmospheric pressure. The bottom of the cylinder *b* is cemented on a boxwood cylinder, *zz*, on a contraction in which, *ii*, is firmly tied the buckskin, *mn*, which forms the base of the cistern. On this skin is fastened a wooden button, *x*, which rests against the end of a screw, C. According as this is turned in one direction or the other the skin *mn* is raised or lowered, and with it the mercury. In using this barometer the mercury is first made exactly level with the point *a*, which is effected by turning the screw C either in one direction or the other. The graduation of the scale is counted from this point *a*, and thus the distance of the top B of the column of mercury from *a* gives the height of the barometer. The bottom of the cistern is surrounded by a brass case, which is fastened to the cover M by screws, *k*, *k*, *k*. We have already seen (165) the importance of having the barometer quite vertical, which is effected by the following plan, known as *Cardan's suspension.*

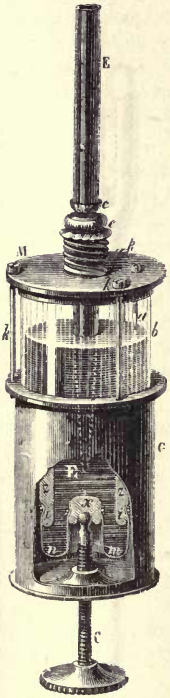


Fig. 147.

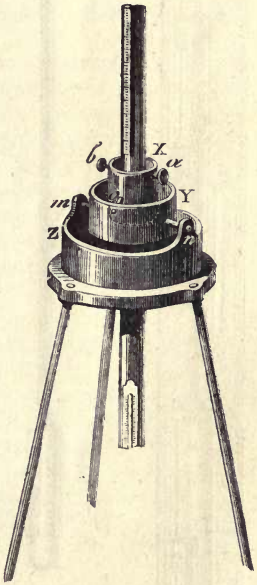


Fig. 148.

The metal case containing the barometer is fixed in a copper sheath X by two screws, *a* and *b* (fig. 148). This is provided with two axles (only one of which, *o*, is seen in the figure), which turn freely in two holes in a ring, Y. In a direction at right angles to that of the axles, *oo*, the ring has also two similar axles, *m* and *n*, resting on a support, Z. By means of this double suspension the barometer can oscillate freely about the axes *mn* and *oo*, in two directions at right angles to each other. But as care is taken that the point at which these axes cross corresponds to the tube itself, the centre of gravity of the system, which must always be lower than the axis of suspension, is below the point of intersection, and the barometer is thus perfectly vertical.

167. **Gay-Lussac's syphon barometer.**—The syphon barometer is a bent glass tube, one of the branches of which is much longer than the other. The longer branch, which is closed at the top, is filled with mercury as in the cistern barometer; while the shorter branch, which is open, serves as a cistern. The difference between the two levels is the height of the barometer.

Fig. 149 represents the syphon barometer as modified by Gay-Lussac. In order to render it more available for travelling, by preventing the entrance of air, he joined the two branches by a capillary tube (fig. 150); when the instrument is inverted (fig. 151) the tube always remains full in virtue of its capillarity, and air cannot penetrate into the longer branch. A sudden shock, however, might separate the mercury and admit some air. To avoid this, Bunten introduced an ingenious modification into the apparatus. The

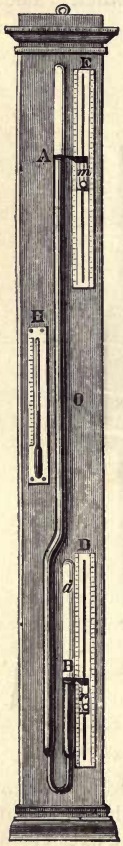


Fig. 149.

Fig. 150.

Fig. 151.

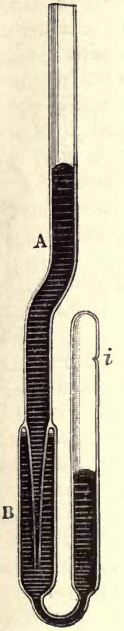


Fig. 152.

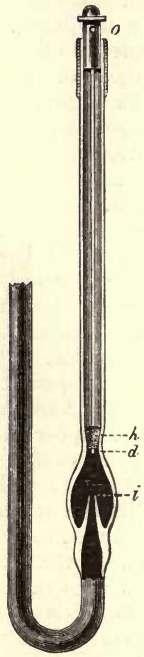


Fig. 153.

longer branch is drawn out to a fine point, and is joined to a tube, B, of the form represented in fig. 152. This arrangement forms an *air-trap;* for if air passes through the capillary tube it cannot penetrate the drawn-out extremity of the longer branch, but lodges in the upper part of the enlargement B. In this position it does not affect the observations, since the vacuum is always at the upper part of the tube; it is, moreover, easily removed.

In the syphon barometer the shorter branch is closed, but there is a

capillary aperture in the side *i*, through which the atmospheric pressure is transmitted.

The barometric height is determined by means of two scales, which have a common zero at O, towards the middle of the longer branch, and are graduated in contrary directions, the one from O to E, and the other from O to B, either on the tube itself, or on brass rules fixed parallel to the tube. Two sliding verniers, *m* and *n*, indicate tenths of a millimetre. The total height of the barometer, A B, is the sum of the distances from O to A and from O to B.

Fig. 153 represents a very convenient mode of arranging the open end of a syphon barometer for transport. The quantity of mercury is so arranged that when the Torricellian space is quite filled with mercury, by inclining the tube the enlargement is just filled to *d*. This is closed by a carefully fitted cork fixed on the end of a glass tube about a millimetre in the clear, which allows for the expansion of mercury by heat. When the barometer is to be used, the cork and tube are raised.

168. **Precautions in reference to barometers.**—In constructing barometers mercury is chosen in preference to any other liquid, for, being the densest of all liquids, it stands at the least height. When the mercurial barometer stands at 30 inches, the water barometer would stand at about 34 feet (165). It also deserves preference because it does not moisten the glass. It is necessary that the mercury be pure and free from oxide, otherwise it adheres to the glass and tarnishes it. Moreover, if it is impure, its density is changed, and the height of the barometer is too great or too small. Mercury is purified, before being used for barometers, by treatment with dilute nitric acid, and by distillation.

The space at the top of the tube (figs. 147 and 149), which is called the *Torricellian vacuum*, must be quite free from air and from aqueous vapour, for otherwise either would depress the mercurial column by its elastic force. To obtain this result, a small quantity of pure mercury is placed in the tube and boiled for some time. It is then allowed to cool, and a further quantity, previously warmed, added, which is boiled, and so on, until the tube is quite full; in this manner the moisture and the air which adhere to the sides of the tube (193) pass off with the mercurial vapour. A barometer tube should not be too narrow, for otherwise the mercury is moved with difficulty; and before reading off, the barometer should be tapped so as to get rid of the adhesion to the glass.

A barometer is free from air and moisture if, when it is inclined, the mercury strikes with a sharp metallic sound against the top of the tube. If there is air or moisture in it, the sound is deadened.

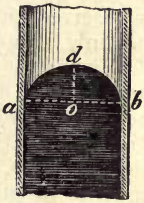


Fig. 154.

169. **Correction for capillarity.**—In cistern barometers there is always a certain depression of the mercurial column due to capillarity, unless the internal diameter of the tube exceeds 0·8 inch. To make the correction due to this depression, it is not enough to know the diameter of the tube; we must also know the height of the meniscus *od* (fig. 154), which varies according as the meniscus has been formed during an ascending or descending motion of the mercury in the tube. Consequently, the height of the meniscus must be determined by

bringing the pointer to the level *ab*, and then to the level *d*, when the difference of the readings will give the height, *od*, required. These two terms—namely, the internal diameter of the tube and the height of the meniscus—being known, the resulting correction can be taken out of the following table:

Internal diameter in inches	Height of sagitta of meniscus in inches						
	0·010	0·015	0·020	0·025	0·030	0·035	0·040
0·157	0·0293	0·0431	0·0555	0·0677	0·0780	0·0870	0·0948
0·236	0·0119	0·0176	0·0231	0·0294	0·0342	0·0398	0·0432
0·315	0·0060	0·0088	0·0118	0·0144	0·0175	0·0196	0·0221
0·394	0·0039	0·0048	0·0063	0·0078	0·0095	0·0110	0·0125
0·472	0·0020	0·0029	0·0036	0·0045	0·0053	0·0063	0·0073
0·550	0·0010	0·0017	0·0024	0·0029	0·0034	0·0039	0·0044

In the syphon barometer the two tubes are of the same diameter, so that the error caused by the depression in the one tube very nearly corrects that caused by the depression in the other. As, however, the meniscus in the one tube is formed by a column of mercury with an ascending motion, while that in the other is formed by a column with a descending motion, their heights will not be the same, and the reciprocal correction will not be quite exact.

170. **Correction for temperature.**—In all observations with barometers, whatever be their construction, a correction must be made for temperature. Mercury contracts and expands with different temperatures, hence its density changes, and consequently the length of the barometric column for this height is inversely as the density of the mercury, so that for different atmospheric pressures the mercurial column might represent the same height. Accordingly, in each observation the height observed must be reduced to a determinate temperature. The choice of this is quite arbitrary, but that of melting ice is always adopted in practice. It will be seen in the book on Heat how this correction is made.

171. **Variations in the height of the barometer.**—When the barometer is observed for several days, its height is found to vary in the same place, not only from one day to another, but also during the same day.

The extent of these variations—that is, the difference between the greatest and the least height—is different in different places. It increases from the equator towards the poles. Except under extraordinary circumstances, the greatest variations do not exceed six millimetres under the equator, 30 under the tropic of Cancer, 40 in France, and 60 at 25 degrees from the pole. The greatest variations are observed in winter.

The *mean daily height* is the height obtained by dividing the sum of 24 successive hourly observations by 24. In our latitudes the barometric height at noon corresponds to the mean daily height.

The *mean monthly height* is obtained by adding together the mean daily

heights for a month and dividing by 30. The *mean yearly height* is similarly obtained.

Under the equator, the mean annual height at the level of the sea is $0^m{\cdot}758$, or 29·84 inches. It increases from the equator, and between the latitudes 30° and 40° it attains a maximum of $0^m{\cdot}763$, or 30·04 inches. In lower latitudes it decreases, and in Paris it does not exceed $0^m{\cdot}7568$.

The general mean at the level of the sea is $0^m{\cdot}761$, or 29·96 inches.

The mean monthly height is greater in winter than in summer, in consequence of the cooler atmosphere.

Two kinds of variations are observed in the barometer :—1st, the *accidental variations*, which present no regularity ; they depend on the seasons, the direction of the winds, and the geographical position, and are common in our climates ; 2nd, the *daily variations*, which are produced periodically at certain hours of the day.

At the equator, and between the tropics, no accidental variations are observed ; but the daily variations take place with such regularity that a barometer may serve to a certain extent as a clock. The barometer sinks from midday till towards four o'clock ; it then rises, and reaches its maximum at about ten o'clock in the evening. It then again sinks, and reaches a second minimum towards four o'clock in the morning, and a second maximum at ten o'clock. In the temperate zones there are also daily variations, but they are detected with difficulty, since they occur in conjunction with accidental variations.

The hours of the maxima and minima appear to be the same in all climates, whatever be the latitude ; they merely vary a little with the seasons.

172. **Causes of barometric variations.**—It is observed that the course of the barometer is generally in the opposite direction to that of the thermometer ; that is, that when the temperature rises the barometer falls, and *vice versâ* ; which indicates that the barometric variations at any given place are produced by the expansion or contraction of the air, and therefore by its change in density. If the temperature were the same throughout the whole extent of the atmosphere, no currents would be produced, and at the same height atmospheric pressure would be everywhere the same. But when any portion of the atmosphere becomes warmer than the neighbouring parts, its specific gravity is diminished, and it rises and passes away through the upper regions of the atmosphere, whence it follows that the pressure is diminished and the barometer falls. If any portion of the atmosphere retains its temperature, while the neighbouring parts become cooler, the same effect is produced ; for in this case, too, the density of the first-mentioned portion is less than that of the others. Hence, also, it usually happens that an extraordinary fall of the barometer at one place is counterbalanced by an extraordinary rise at another place. The daily variations appear to result from the expansions and contractions which are periodically produced in the atmosphere by the heat of the sun during the rotation of the earth.

173. **Relation of barometric variations to the state of the weather.**—It has been observed that, in our climate, the barometer in fine weather is generally above 30 inches, and is below this point when there is rain, snow, wind, or storm ; and also, that for any given number of days at which the

barometer stands at 30 inches there are as many fine as rainy days. From this coincidence between the height of the barometer and the state of the weather, the following indications have been marked on the barometer, counting by thirds of an inch above and below 30 inches :—

Height	State of the weather
31 inches	Very dry.
$30\frac{2}{3}$ „	Settled weather
$30\frac{1}{3}$ „	Fine weather.
30 „	Variable.
$29\frac{2}{3}$ „	Rain or wind.
$29\frac{1}{3}$ „	Much rain.
29 „	Tempest.

In using the barometer as an indicator of the state of the weather, we must not forget that it really only serves to measure the weight of the atmosphere, and that it only rises or falls as the weight increases or diminishes ; and although a change of weather frequently coincides with a change in the pressure, they are not necessarily connected. This coincidence arises from meteorological conditions peculiar to our climate, and does not occur everywhere. That a fall in the barometer usually precedes rain in our latitudes is caused by the position of Europe. The prevailing winds here are the south-west and north-east. The former, coming to us from the equatorial regions, are warmer and lighter. They often, therefore, blow for hours or even days in the higher regions of the atmosphere before manifesting themselves on the surface of the earth. The air is therefore lighter, and the pressure lower. Hence a fall of the barometer is a probable indication of the south-west winds, which gradually extend downwards, and, reaching us, after having traversed large tracts of water, are charged with moisture, and bring us rain.

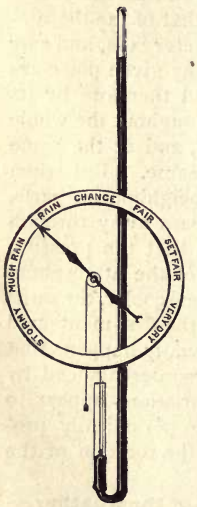


Fig. 155.

The north-east blows simultaneously above and below, but the hindrances to the motion of the current on the earth, by hills, forests, and houses, cause the upper current to be somewhat in advance of the lower ones, though not so much so as the south-west wind. The air is therefore somewhat heavier even before we perceive the north-east, and a rise of the barometer affords a forecast of the occurrence of this wind, which, as it reaches us after having passed over the immense tracts of dry land in Central and Northern Europe, is mostly dry and fine.

When the barometer rises or sinks slowly, that is, for two or three days, towards fine weather or towards rain, it has been found from a great number of observations that the indications are then extremely probable. Sudden variations in either direction indicate bad weather or wind.

174. **Wheel barometer.**—The *wheel barometer*, which was invented by Hooke, is a syphon barometer, and is especially intended to indicate good and bad weather (fig. 155). In the shorter leg of the syphon there is a float which rises and falls with the

mercury. A string attached to this float passes round a pulley, and at the other end there is a weight somewhat lighter than the float. A needle fixed to the pulley moves round a graduated circle, on which is marked *stormy*, *rain*, *set fair*, &c. When the pressure varies the float sinks or rises, and moves the needle round to the corresponding points on the scale.

The barometers ordinarily met with in houses, and which are called *weather-glasses*, are of this kind. They are, however, of little use, for two reasons. The first is, that they are neither very delicate nor very accurate in their indications. The second, which applies equally to all barometers, is that those commonly in use in this country are made in London, and the indications, if they are of any value, are only so for a place of the same level and of the same climatic conditions as London. Thus a barometer standing at a certain height in London would indicate a certain state of weather, but if removed to Shooter's Hill it would stand half an inch lower, and would indicate a different state of weather. As the pressure differs with the level and with geographical conditions, it is necessary to take these into account if exact data are wanted.

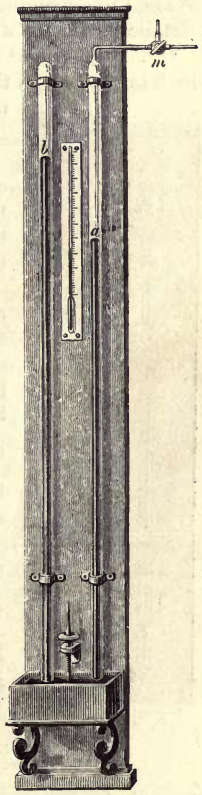


Fig. 156.

175. **Fixed barometer.**—For accurate observations Regnault uses a barometer the height of which he measures by means of a cathetometer (88). The cistern (fig. 156) is of cast iron; against the frame on which it is supported a screw is fitted, which is pointed at both ends, and the length of which has been determined, once for all, by the cathetometer. To measure the barometric height, the screw is turned until its point grazes the surface of the mercury in the bath, which is the case when the point and its image are in contact. The distance then from the top of the point to the level of the mercury in the tube *b* is measured by the cathetometer, and this, together with the length of the screw, gives the barometric height with great accuracy. This barometer has, moreover, the advantage that, as a tube an inch in diameter may be used, the influence of capillarity becomes inappreciable. Its construction, moreover, is very simple, and the position of the scale leads to no kind of error, since this is transferred to the cathetometer. Unfortunately, the latter instrument requires great accuracy in its construction, and is expensive.

176. **Glycerine barometer.**—Jordan constructed a barometer in which the liquid used is pure glycerine. This has the specific gravity 1·26, and therefore the length of the column of liquid is rather more than ten times that of mercury; hence small alterations in the atmospheric pressure produce considerable oscillations in the height of the liquid. The tube consists of ordinary composition gas-tubing about $\frac{1}{8}$ of an inch in diameter and 28 feet or so in length;

the lower end is open and dips in the cistern, which may be placed in a cellar; the top is sealed to a closed glass tube an inch in diameter, in which the fluctuations of the column are observed. This may be arranged in an upper story, and the tubing, being easily bent, lends itself to any adjustment which the locality requires.

The vapour of glycerine has very low tension at ordinary temperatures, and is, therefore, not so exposed to such back pressures, varying with the temperature, as is water. On the other hand, it readily attracts moisture from the air, whereby the density, and therewith the height of the liquid column, vary. This is prevented by covering the liquid in the cistern with a layer of paraffin oil.

The 'Philosophical Magazine,' vol. xxx. Fourth series, page 349, contains a detailed account of a method of constructing a water barometer.

177. **Huyghens' barometer.**—The desire to amplify the small variations which take place in the barometer has led to a number of contrivances, one of the best-known of which was invented by Huyghens (fig. 157).

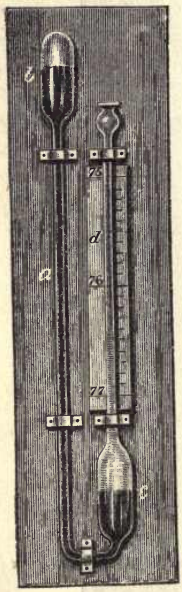


Fig. 157.

The barometer tube *a* is wider at the closed end *b*, and also at *c*, where a liquid of smaller specific gravity than mercury, such as coloured water, is poured on the mercury; it fills the rest of the tube *c* and a portion of *d*.

Suppose *b* and *c* to have the same diameter, which is n times that of *d*. When the column of mercury in *b* sinks through x millimetres, the level of the mercury in *c* rises just as much, while the coloured liquid rises nx millimetres, and therefore its level is $(n-1)x$ millimetres higher. A column of this liquid $(n-1)x$ in height has the same pressure as a column of mercury $\frac{(n-1)x}{s}$ in height, where s is the number expressing the ratio of the specific gravities of mercury and the liquid.

Accordingly, when the mercury in *b* sinks x millimetres,

$$y = 2x + \frac{n-1}{s}x$$

is the height of the column of mercury, which corresponds to the decrease of atmospheric pressure. From this we have

$$x = \frac{sy}{2s+n-1}.$$

Thus, if the section of the tubes *b* and *c* is 20 times that of *d*, and if the coloured liquid be water, we have

$$\frac{13{\cdot}6y}{27{\cdot}2+20-1} = \frac{13{\cdot}6y}{46{\cdot}2} = 0{\cdot}294y.$$

Accordingly, when an ordinary barometer sinks through y millimetres, the mercury in *b* sinks $0{\cdot}294y$ millimetre, while the coloured liquid in *d* rises

$20 \times 2{\cdot}294v = 5{\cdot}98y$. Whenever, that is, an ordinary barometer sinks or rises 1 millimetre, the coloured liquid rises or sinks 5·98 millimetres, or nearly six times as much.

Such barometers are useful in cases where the variations in the height of the barometer, rather than its actual height, are to be observed. The scale should be placed behind the tube *d*, and two points, fixed, near the top and bottom, by comparison with standard barometers ; the interval between the two is then suitably divided.

178. **Determination of heights by the barometer.**—Since the atmospheric pressure decreases as we ascend, it is obvious that the barometer will keep on falling as it is taken to a greater and greater height. On this depends a method of determining the difference between the heights of two stations, such as the base and summit of a mountain. The method may be explained as follows.

According to Boyle's law (180), if the temperature of an enclosed portion of air is constant, its volume will vary inversely as the pressure ; that is to say, if we double the pressure we shall halve the volume. But if we halve the volume we manifestly double the quantity of air in each cubic inch—that is to say, we double the density of the air ; and so on in any proportion. Consequently the law is equivalent to this :—*That for a constant temperature the density of air is proportional to the pressure which it sustains.*

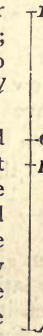


Fig. 158

Now suppose A and B (fig. 158) to represent two stations, and that it is required to determine the vertical height of B above A, it being borne in mind that A and B are not necessarily in the same vertical line. Take P, any point in AB, and Q, a point at a small distance above P. Suppose a pressure on a square inch of the atmosphere at P to be denoted by p, and at Q let it be diminished by a quantity denoted by dp. It is clear that this diminution equals the weight of the column of air between P and Q, whose section is one square inch. But, since the density of the air is directly proportional to p, the weight of a cubic inch of air will equal kgp, where k denotes a certain quantity to be determined presently, and g the accelerating force of gravity (79). Hence, if we denote PQ in inches by dx, the pressure will be diminished by $kpg \,.\, dx$, and we may represent this algebraically by the equation

$$kpg \,.\, dx = dp.$$

By a certain algebraical process this leads to the conclusion that

$$kg\mathrm{X} = \log \frac{\mathrm{P}}{\mathrm{P}_1},$$

where X denotes the height of AB, and P and P_1 the atmospheric pressures at A and B respectively, the logarithms being what are called 'Napierian logarithms.' Now, if H and H_1 are the heights of the barometer at A and B respectively, the temperature of the mercury being the same at both stations, their ratio equals that of P to P_1, and therefore

$$\mathrm{X} = \frac{1}{kg} \cdot \log \frac{\mathrm{H}}{\mathrm{H}_1}.$$

It remains to determine k and g.

(1) Since the force of gravity is different for places in different latitudes, g will depend upon the latitude (82). It is found that if g is the accelerating force of gravity in latitude ϕ, and f that force in latitude 45°, then

$$g = \frac{f}{1 + 0{\cdot}00256 \cos 2\phi},$$

where f has a definite numerical value.

(2) If ρ is the density of air at a temperature of t° C., under Q, the pressure exerted by 29·92 inches of mercury, we shall have

$$k\mathrm{Q} = \rho.$$

But it will be afterwards shown (332) that if ρ_0 is the density of air under the same pressure Q at o° C., we shall have

$$\rho = \frac{\rho_0}{1 + at},$$

where a represents the coefficient of expansion of gases. Therefore

$$k\mathrm{Q} = \frac{\rho_0}{1 + at}.$$

Now if σ is the density of mercury, and if the latitude is 45°, we shall have

$$\mathrm{Q} = 29{\cdot}92 \,.\, \sigma f;$$

and therefore

$$kf = \frac{\rho_0}{\sigma} \,.\, \frac{1}{29{\cdot}92\ (1 + at)}.$$

But $\rho_0 \div \sigma$ is the ratio which the density of dry air at a temperature o° C., in latitude 45°, under a pressure of 29·92 inches of mercury, bears to the density of mercury at o° C., and therefore $\rho_0 \div \sigma$ is a determinate number.

Substituting, we have

$$\mathrm{X} = 29{\cdot}92 \text{ in.} \,.\, \frac{\sigma}{\rho_0}(1 + 0{\cdot}00256 \cos 2\phi)\ (1 + at) \log \frac{\mathrm{H}}{\mathrm{H}_1}.$$

The value of a is 0·003665, which is nearly equal to $\frac{1}{273}$. If we substitute the proper values for $\sigma \div \rho_0$, and change the logarithms into common logarithms, and instead of t use the mean of T and T_1, the temperatures at the upper and lower stations, it will be found that

$$\mathrm{X} \text{ (in feet)} = 60346\ (1 + 0{\cdot}00256 \cos 2\phi) \left(1 + \frac{2(\mathrm{T} + \mathrm{T}_1)}{1000}\right) \log \frac{\mathrm{H}}{\mathrm{H}_1}$$

which is La Place's barometric formula. In using it, we must remember that T and T_1 are temperatures on the Centigrade thermometer, and that H and H_1 are the heights of the barometer reduced to o° C. Thus if h is the measured height of the barometer at the lower station we have

$$\mathrm{H} = h\left(1 - \frac{t}{6500}\right).$$

If the height to be measured is not great, one observer is enough. For greater heights the ascent takes some time, and in the interval the pressure

may vary. Consequently, in this case there must be two observers, one at each station, who make simultaneous observations.

Let us take the following example of the above formula :—Suppose that in latitude 65° N. at the lower of the two stations the height of the barometer was 30·025 inches, and the temperature of air and mercury 17°·32 C., while at the upper the height of the barometer was 28·230 inches, and the temperature of air and mercury was 10°·55 C. What is the height of the upper station above the lower?

(1) Find H and H_1 : viz.

$$H = 30{\cdot}025\left(1 - \frac{17{\cdot}32}{6500}\right) = 29{\cdot}945.$$

$$H_1 = 28{\cdot}230\left(1 - \frac{10{\cdot}55}{6500}\right) = 28{\cdot}184.$$

$$\text{Hence } \log \frac{H}{H_1} = 1{\cdot}4763243 - 1{\cdot}4500026 = 0{\cdot}0263217.$$

(2) Find $1 + \frac{2(T + T_1)}{1000}$ viz. 1·05574.

(3) Find $1 + 0{\cdot}00256 \cos 2\phi$.

Since $0{\cdot}00256 \cos 130° = -0{\cdot}00256 \cos 50° = -0{\cdot}001645$,

therefore $1 + 0{\cdot}00256 \cos 2\phi = -0{\cdot}998355$.

Hence the required height in feet equals

$$60346 \times 0{\cdot}998355 \times 1{\cdot}05574 \times 0{\cdot}0263217 = 1674.$$

If H and H_1 do not greatly differ, the Napierian logarithm of

$$\frac{H}{H_1} = 2\frac{H - H_1}{H + H_1}.$$

If, for instance, $H = 30$ and $H_1 = 29$ inches, the resulting error would not exceed the $\frac{1}{3000}$ part of the whole. Accordingly for heights not exceeding 2,000 ft. we may, without much error, use the formula

$$X \text{ (in feet)} = 52500\left(1 + \frac{2(T + T_1)}{1000}\right) \times \frac{H - H_1}{H + H_1}.$$

179. **Ruhlmann's observations.**—The results obtained for the difference in height of places by using the above formula often differ from the true heights as measured trigonometrically, to an extent which cannot be ascribed to errors in observation. The numbers thus found for the height of places are influenced by the time of day, and also by the season of year, at which they are made. Ruhlmann has investigated the cause of this discrepancy by a series of direct barometric and thermometric observations made at two different stations in Saxony, and also by a comparison of the continuous series of observations made at Geneva and on the St. Bernard.

Ruhlmann thus ascertained that the cause of the discrepancy is to be found in the fact that the mean of the temperatures indicated by the thermometer at the two stations is not an accurate measure of the actual mean temperature of the column of air between the two stations—a condition which is assumed in the above formula. The variations in the temperature of the column of air are not of the same extent as those indicated by the thermometer, nor do they follow them so rapidly ; they drag after them as it were.

If the mean monthly temperatures at the two fixed stations are introduced into the formula, they give in winter heights which are somewhat too low, and in summer such as are too high. The results obtained by introducing the mean yearly temperature of the two stations are very near the true ones.

This influence of temperature is most perceptible in individual observation of low heights. Thus, using the observed temperatures in the barometric formula, the error in height of the Uetliberg above Zürich (about 1,700 feet) was found to be $\frac{1}{23}$ of the total, while the height of the St. Bernard above Geneva was found within $\frac{1}{135}$ of the true height.

The reason why the thermometers do not indicate the true temperature of the air is undoubtedly because they are too much influenced by radiation from the earth and surrounding bodies. The earth is highly absorbent, and becomes rapidly heated under the influence of the sun's rays, and as rapidly cooled at night; the air, as a very diathermanous body, is but little heated by the sun's rays, and on the contrary is little cooled by radiation during the night.

CHAPTER II

MEASUREMENT OF THE ELASTIC FORCE OF GASES

180. **Boyle's law.**—The law of the compressibility of gases was discovered by Boyle in 1662, and afterwards independently by Mariotte in 1679. It is in England commonly called 'Boyle's Law,' and on the Continent 'Mariotte's Law.' It is as follows :—

The temperature remaining the same, the volume of a given quantity in gas is inversely as the pressure which it bears.

This law may be verified by means of an apparatus devised by Boyle (fig. 159). It consists of a long glass tube fixed to a vertical support; it is open at the upper part, and the other end, which is bent into a short vertical leg, is closed. On the shorter leg there is a scale which indicates equal capacities; the scale against the long leg gives the heights. The zero of both scales is in the same horizontal line.

A small quantity of mercury is poured into the tube, so that its level in both branches is at zero, which is effected without much difficulty after a few trials (fig. 159). The air in the short leg is thus under the ordinary atmospheric pressure which is exerted through the open tube. Mercury is then poured into the longer tube until the volume of the air in the smaller tube is reduced to one-half; that is, until it is reduced from 10 to 5, as shown in fig. 160. If the height of the mercurial column, CA, be measured, it will be found exactly equal to the height of the barometer at the time of the experiment. The pressure of the column CA is therefore equal to an atmosphere which, with the atmospheric pressure acting on the surface of the column at C, makes two atmospheres. Accordingly, by doubling the pressure, the volume of the gas has been diminished to one-half.

If mercury be poured into the longer branch until the volume of the air is reduced to one-third, it will be found that the distance between the level of the two tubes is equal to two barometric columns. The pressure is now three atmospheres, while the volume is reduced to one-third. Dulong and Petit have verified the law for air up to 27 atmospheres, by means of an apparatus analogous to that which has been described.

The law also holds good in the case of pressures of less than one atmosphere. To establish this, mercury is poured into a graduated tube until it is about two-thirds full, the rest being air. It is then inverted in a deep trough M containing mercury (fig. 161), and lowered until the levels of the mercury inside and outside the tube are the same, and the volume AB noted. The tube is then raised, as represented in the figure, until the volume of air AC is double that of AB (fig. 162). The height of the mercury in the tube above the mercury in the trough CD is then found to be exactly half the

height of the barometric column. The air whose volume is now doubled is now only under the pressure of half an atmosphere ; for it is the elastic force of this air which, added to the weight of the column CD, is equivalent to the atmospheric pressure. Accordingly the volume is inversely as the pressure.

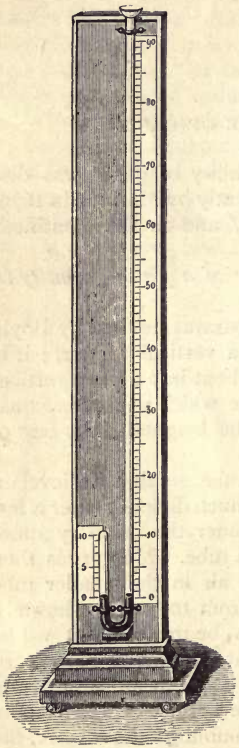


Fig. 159.

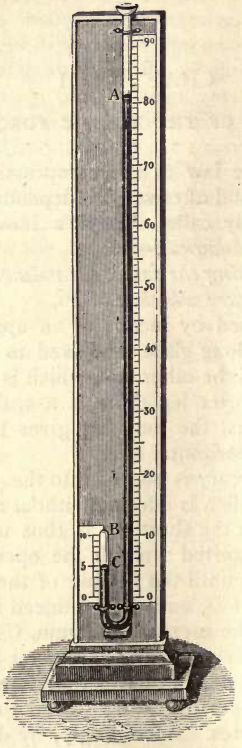


Fig. 160.

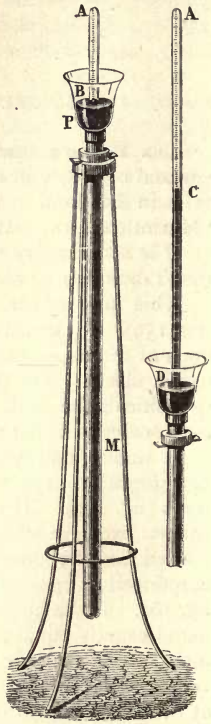


Fig. 161. Fig. 162.

In general, if V be the original volume of a gas under the pressure P, and V′ the volume of the same gas under another pressure P′, we have the ratio

$$V : V' = P' : P \text{ or } VP = V'P'.$$

This may be expressed by saying that *the temperature of a given* mass of gas being constant, the product of pressure and volume is constant ; that is,

$$PV = const.$$

In the experiment with Boyle's tube, as the mass of air remains the same, its density must obviously increase as its volume diminishes, and *vice versâ*. The law may thus be enunciated :—'*For the same temperature the density of a gas is proportional to its pressure.*' Hence, as water is 773 times as heavy as air, under a pressure of 773 atmospheres air would be as dense as water.

Boyle's law must not be understood to mean that gases of equal density have equal elastic force; different gases of various densities have the same tension when they are under the same pressure. A given volume of hydrogen under the ordinary atmospheric pressure has the same elastic force as the same volume of air, although the latter is 14 times as heavy as the former. Since, for the same volume, there are the same number of molecules in all gases, the lighter molecules must possess a greater velocity in order to exert the same pressure as the same number of molecules of greater mass.

181. **Boyle's law is only approximately true.**—Until within the last few years Boyle's law was supposed to be absolutely true for all gases at all pressures, but Despretz obtained results incompatible with the law. He took two graduated glass tubes of the same length, and filled one with air and the other with the gas to be examined. These tubes were placed in the same mercury trough, and the whole apparatus immersed in a strong glass cylinder filled with water. By means of a piston moved by a screw which worked in a cap at the top of a cylinder the liquid could be subjected to an increasing pressure, and it could be seen whether the compression of the two gases was the same or not. The apparatus resembled that used for examining the compressibility of liquids (fig. 64). In this manner Despretz found that carbonic acid, sulphuretted hydrogen, ammonia, and cyanogen are more compressible than air: hydrogen, which has the same compressibility as air up to 15 atmospheres, is then less compressible. From these experiments it was concluded that the law of Boyle was not general.

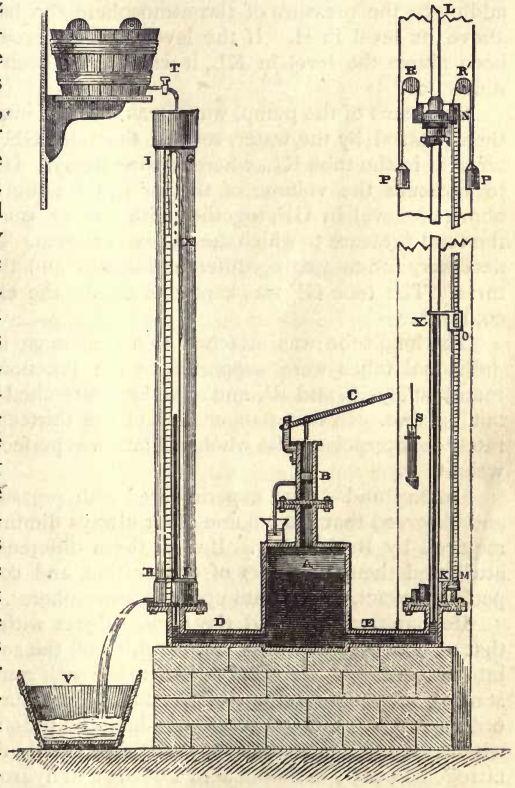


Fig. 163.

In some experiments on the elastic force of vapours, Dulong and Arago had occasion to test the accuracy of Boyle's law. The method adopted was exactly that of Boyle, but the apparatus had gigantic dimensions.

The gas to be compressed was contained in a strong glass tube, GF (fig.

163), about six feet long and closed at the top, G. The pressure was produced by a column of mercury, which could be increased to a height of 65 feet, contained in a long vertical tube, KL, formed of a number of tubes firmly joined by good screws, so as to be perfectly tight.

The tubes KL and GF were hermetically fixed in a horizontal iron pipe, DE, which formed part of a mercurial reservoir, A. On the top of this reservoir there was a force-pump, BC, by which mercury could be forced into the apparatus.

At the commencement of the experiment the volume of the air in the manometer (183) was observed, and the initial pressure determined, by adding to the pressure of the atmosphere the height of the mercury in K above its level in H. If the level of the mercury in the manometer had been above the level in KL, it would have been necessary to subtract the difference.

By means of the pump, water was injected into A. The mercury, being then pressed by the water, rose in the tube GF, where it compressed the air, and in the tube KL, where it rose freely. It was only then necessary to measure the volume of the air in GF; the height of mercury in KL above the level in GF, together with the pressure of the atmosphere, was the total pressure to which the gas was exposed. These were all the elements necessary for comparing different volumes and the corresponding temperatures. The tube GF was kept cool during the experiment by a stream of cold water.

The long tube was attached to a long mast by means of staples. The ndividual tubes were supported at the junction by cords, which passed round pulleys R and R^1, and were kept stretched by small buckets, P, containing shot. In this manner each of the thirteen tubes having been separately counterpoised, the whole column was perfectly free notwithstanding its weight.

Dulong and Arago experimented with pressures up to 27 atmospheres, and observed that the volume of air always diminished a little more than is required by Boyle's law. But as these differences were very small, they attributed them to errors of observation, and concluded that the law was perfectly exact, at any rate up to 27 atmospheres.

Regnault investigated the same subject with an apparatus resembling that of Dulong and Arago, but in which all the sources of error were taken into account, and the observations made with remarkable precision. Thus, starting with a unit volume of gas under a pressure of 1 metre of mercury in order to reduce this volume to one-half, the pressure should be two metres, whereas the following were the pressures actually required; air 1·9978 metre; nitrogen 1·9985; carbonic acid 1·9829; and hydrogen 2·011. Similar results were obtained at higher pressures; thus to reduce air to $\frac{1}{10}$ of its original volume, a pressure of 19·7199 m. was required instead of 20; and while carbonic acid only required 16·705, hydrogen required 20·269 metres.

It thus appears that with increasing pressures hydrogen has a greater, and the other gases a smaller, volume than is required by Boyle's law.

Very much higher pressures have been employed in similar experiments by Natterer and by Andrews. Cailletet used a special apparatus by which the pressure could be raised to 600 atmospheres. Amagat made

a remarkable series of experiments by a method based on Boyle's experiment. The pressure could be applied directly by means of mercury in a steel tube about 1,000 feet in length, arranged in the shaft of a deep coalpit, and suitably connected at the bottom with a carefully calibrated glass tube. In this way pressures of as much as 400 atmospheres could be applied, and the temperatures remained constant.

The general result of these experiments is to show that at high pressures the volume is greater than that required by Boyle's law, agreeing in this respect with hydrogen at ordinary pressures. This is well illustrated by the deportment of ethylene as given in the following table, where P is the pressure in metres of mercury, and PV the product of pressure into volume, which according to Boyle's law should be constant.

Pressure	24	34·8	45·1	55·4	64	72	84	134	214	303
PV	21·5	18·4	12·3	9·8	9·4	9·7	10·7	15·1	22·1	29·3

It will thus be seen that the product PV decreases with increasing pressure to a minimum, and then increases again with the pressure.

The pressure at which this *minimum of compressibility* occurs is different with different gases, as is also the extent of the deviation from the law.

At a temperature of 20° this minimum occurs at the following pressures in metres of mercury: nitrogen and carbonic oxide 50, air and ethylene 65, oxygen 100, and marsh gas 120.

182. **Van der Waals' formula.**—Under high pressures gases do not, as we have seen, follow Boyle's law with strictness. In order to account for these discrepancies Van der Waals has introduced a modification into the formula PV = *const.* (180) which is based on the following considerations. We shall afterwards see (293) that Boyle's law may be deduced from the dynamical theory of gases, which assumes that they are made up of infinitely small particles moving with great velocities; it is also assumed that these particles have no cohesion or specific attraction for each other, and further that they are mere mathematical points. Van der Waals takes account of these limitations. He considers that the cohesion a, which the particles possess, though small, has a certain value, the effect of which is to add itself to the pressure; its force will be proportional to the number of acting and attracting particles, and will be directly proportional to the square of the density, or inversely proportional to the square of the volume. The other correction is for the volume of the particles themselves, b, which, though exceedingly small, has a certain value. The pressure of a given mass of gas being due to the number of impacts which take place in a given time, it is clear that if the particles have a certain magnitude they must collide against each other more frequently than if they are mere mathematical points; the influence on the formula will be that the volume V will be diminished by an amount which represents a multiple of the molecular volume, or the space actually occupied by the particles.

The formula of Boyle's law, as thus modified by Van der Waals, becomes

$$\left(\mathrm{P} + \frac{a}{\mathrm{V}^2}\right)(\mathrm{V} - b) = const.$$

It will thus be seen that the two influences mentioned affect Boyle's law in opposite directions. With hydrogen, where the molecules have little

or no attraction, there is no cohesion, and accordingly the product PV increases continuously with the pressure, and there is no maximum of compressibility.

With other gases a has a definite value; at low pressures the product PV is less than that required by Boyle's law, and the influence of a preponderates; but as the pressure continuously increases this diminishes in comparison with the influence of b, and the product now increases, and at high pressures the gases behave as does hydrogen at low ones. Between these a maximum compressibility is seen, which varies with different gases according to the values of a and b in each case.

Van der Waals deduced from the experimental results obtained by Regnault for the condensation of various gases and for their expansion by heat, values for a and b for the respective gases, which when introduced into the formula satisfactorily represent the numbers obtained by experiment.

Thus for b he got 0·00069; this is confirmed by Budde, who obtained 0·0007 by an entirely different method.

183. **Manometers.**—*Manometers* are instruments for measuring the tension of gases or vapours. In all such instruments the unit chosen is the pressure of one atmosphere, or 30 inches of mercury at the standard temperature, which, as we have seen, is nearly 15 lbs. to the square inch.

The *open-air manometer* consists of a bent glass tube BD (fig. 164), fastened to the bottom of a reservoir AC, of the same material, containing mercury, which is connected with the closed recipient containing the gas or vapour the pressure of which is to be measured. The whole is fixed on a long plank kept in a vertical position.

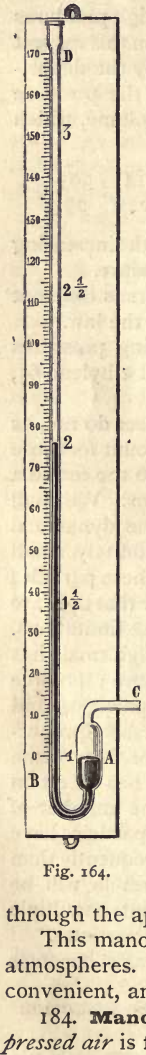


Fig. 164.

In graduating this manometer, C is left open, and the number 1 marked at the level of the mercury, for this represents one atmosphere. From this point the numbers 2, 3, 4, 5, 6, are marked at each 30 inches, indicating so many atmospheres, since a column of mercury 30 inches represents a pressure of one atmosphere. The intervals, from 1 to 2, and from 2 to 3, &c., are divided into tenths. C being then placed in connection with a boiler, for example, the mercury rises in the tube BD to a height which measures the tension of the vapour. In the figure the manometer marks 2 atmospheres, which represents a height of 30 inches, plus the atmospheric pressure exerted at the top of the column through the aperture D.

This manometer is only used where the pressures do not exceed 5 to 6 atmospheres. Beyond this, the length of tube necessary makes it very inconvenient, and the following apparatus is commonly used.

184. **Manometer with compressed air.**—The *manometer with compressed air* is founded on Boyle's law: one form is represented in fig. 165, which may be screwed into a boiler or steam-pipe where pressure is to be measured. The pressure is transmitted through the opening a into the

closed space *b*. In this is an iron vessel containing mercury, in which dips the open end of the manometer tube, which is screwed airtight in the tubulure.

In the graduation of this manometer, the quantity of air contained in the tube is such that when the aperture A communicates freely with the atmosphere, the level of the mercury is the same in the tube and in the tubulure. Consequently, at this level, the number 1 is marked on the scale to which the tube is affixed. As the pressure acting through the tubulure A increases, the mercury rises in the tube, until its weight, added to the tension of the compressed air, is equal to the external pressure. It would consequently be incorrect to mark two atmospheres in the middle of the tube; for, since the volume of the air is reduced to one-half, its tension is equal to two atmospheres, and, together with the weight of the mercury raised in the tube, is therefore more than two atmospheres. The position of the number is at such a height that the elastic force of the compressed air, together with the weight of the column of mercury in the tube, is equal to two atmospheres. The exact position of the numbers 2, 3, 4, &c., on the manometer scale can only be determined by calculation. Sometimes this manometer is made of one glass tube; the principle is obviously the same.

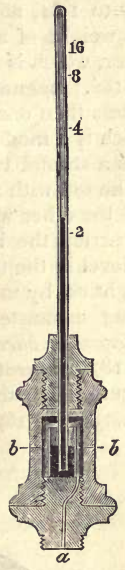


Fig 165.

185. **Volumometer.**—An interesting application of Boyle's law is met with in the *volumometer*, which is used in determinations of the specific gravity of solids which cannot be brought into contact with water or other liquids. A simple form consists of a glass tube with a cylinder G at the top (fig. 166), the edges of which are carefully ground, and which can be closed hermetically by means of a ground-glass plate D. The top being open, the tube is immersed until the level of the mercury inside and outside is the same; this is represented by the mark Z. The apparatus is then closed airtight by the plate, and is raised until the mercury stands at a height h, above the level Q in the bath. The original volume of the enclosed air V, which was under the pressure of the atmosphere, is now increased to $V + v$, since the pressure has diminished by the height of the column of mercury h. Calling the pressure of the atmosphere at the time of observation b, we shall have $V : V + v = b - h : b$.

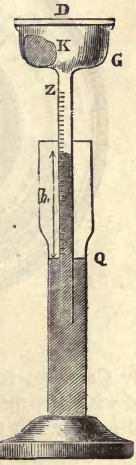


Fig. 166.

Placing now in the cylinder a body K, whose volume x is unknown, the same operations are repeated; the tube is raised until the mercury again stands at the same mark as before, but its height above the bath is now different: a second reading, h_1, is obtained, and we have $(V - x) : (V - x) + v = b - h_1 : b$. Combining and reducing, we get $x = (V + v)\left(1 - \frac{h}{h_1}\right)$. The volume $V + v$ is constant, and is determined numerically, once for all, by

making the experiment with a substance of known volume, such as a glass bulb.

This apparatus, which is also known as the *sterometer*, is of great value in determining the *gravimetrical density* of gunpowder ; this averages from 1·67 to 1·84, and is thus materially different from its *apparent density*, or the weight of a given volume compared with that of an equal volume of water, which is from 0·89 to 0·94.

186. **Regnault's barometric manometer.**—For measuring pressures of less than one atmosphere, Regnault devised the following arrangement, which is a modification of his fixed barometer (fig. 156). In the same cistern dips a second tube *a* of the same diameter, open at both ends, and provided at the top with a three-way cock, one of which is connected with an air-pump and the other with the space to be exhausted. The further the exhaustion is carried the higher the mercury rises in the tube *a*. The differences of level in the tubes *b* and *a* give the pressures. Hence, by measuring the height *ab*, by means of the cathetometer, the pressure in the space that is being exhausted is accurately given. This apparatus is also called the *differential barometer*.

187. **Aneroid barometer.**—This instrument derives its name from the circumstance that no liquid is used in its construction (*ἀ*, without ; *νηρὸς*, moist) Fig. 167 represents one of the forms of these instruments, constructed

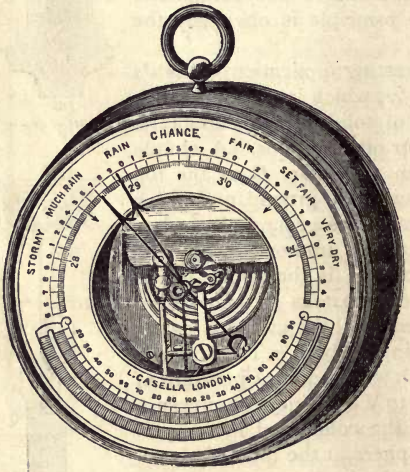


Fig. 167.

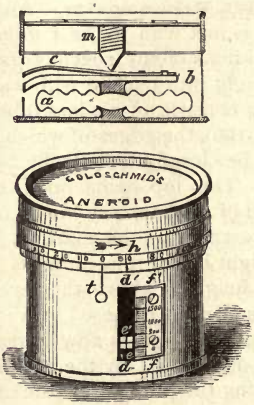


Fig. 168.

by Casella ; it consists of a cylindrical metal box, exhausted of air, the top of which is made of thin corrugated metal, so elastic that it readily yields to alterations in the pressure of the atmosphere.

When the pressure increases, the top is pressed inwards ; when, on the contrary, it decreases, the elasticity of the lid, aided by a spring, tends to move it in the opposite direction. These motions are transmitted by delicate multiplying levers to an index which moves on a scale. The instrument is

graduated empirically by comparing its indications, under different pressures, with those of an ordinary mercurial barometer.

The aneroid has the advantage of being portable, and can be constructed of such delicacy as to indicate the difference in pressure between the height of an ordinary table and the ground. It is hence much used in determining heights in mountain ascents. But it is somewhat liable to get out of order, especially when it has been subjected to great variations of pressure; and its indications must from time to time be compared with those of a standard barometer.

The errors arising from the use of the aneroid are mainly due to the transmission of the motion of the lid by the multipying arrangement. Goldschmid of Zürich devised a form in which the motion of the lid is directly observed.

Like that of other aneroids, the lid of a box a (fig. 168), in which the alterations of pressure are determined, is of fine corrugated sheet metal. To this is fixed a horizontal metal strip b, on the front end of which is a small square e, acting as index. This rises and falls with the movement of the lid, and indicates on a scale ff', on the sides of the slit dd', alterations of pressure in centimetres. To this strip a second and more delicate one, c, is fixed, on the front end of which is also fixed an index e'. Before making an observation, the horizontal line of this index is made to coincide with that of e; this is effected by means of a micrometer screw m, which is raised or lowered by the movable ring h; on the corresponding scale millimetres and tenths of a millimetre are read off. To do this the instrument is provided with a lens, not represented in the figure. There is also a small thermometer t; from its indications a correction is made for temperature according to an empirical scale specially constructed for each instrument.

188. **Laws of the mixture of gases.**—If a communication is opened between two closed vessels containing gases, they at once begin to mix whatever be their density, and in a longer or shorter time the mixture is complete, and will continue so unless chemical action is set up. The laws which govern the mixture of gases may be thus stated:—

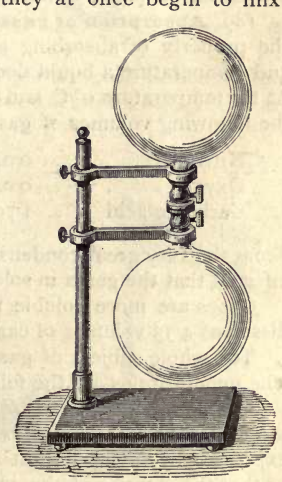


Fig. 169.

I. *The mixture takes place rapidly and is homogeneous; that is, each portion of the mixture contains the two gases in the same proportion.*

II. *If the gases severally and the mixture have the same temperature, and if the gases severally and the mixture occupy the same volume, then the pressure exerted by the mixture will equal the sum of pressures exerted by the gases severally.*

From the second law a very convenient formula can be easily deduced.

Let v_1, v_2, v_3 . . . . be the volumes of several gases under pressure of p_1, p_2, p_3 . . . . respectively. Suppose these gases when mixed to have a volume V, under a pressure P, the temperatures being the same

By Boyle's law we know that v_1 will occupy a volume V under a pressure p_1' provided that

$$Vp' = v_1 p_1 \text{ ; similarly, } Vp_2' = v_2 p_2$$

and so on. But from the above law

$$P = p_1' + p_2' + \ldots$$

therefore

$$VP = v_1 p_1 + v_2 p_2 + v_3 p_3 + \ldots$$

It obviously follows that if the pressures are all the same, the volume of the mixture equals the sum of the separate volumes.

The first law was shown experimentally by Berthollet, by means of an apparatus represented in fig. 169. It consists of two glass globes provided with stopcocks, which can be screwed one on the other. The upper globe was filled with hydrogen, and the lower one with carbonic acid, which has 22 times the density of hydrogen. The globes, having been fixed together, were placed in the cellars of the Paris Observatory and the stopcocks then opened, the globe containing hydrogen being uppermost. Berthollet found after some time that the pressure had not changed, and that, in spite of the difference in density, the two gases had become uniformly mixed in the two globes. Experiments made in the same manner with other gases gave the same results, and it was found that the diffusion was more rapid in proportion as the difference between the densities was greater.

The second law may be demonstrated by passing into a graduated tube, over mercury, known volumes of gas at known pressures. The pressure and volume of the whole mixture are then measured, and found to be in accordance with the law.

Gaseous mixtures follow Boyle's law, like simple gases, as has been proved for air (180), which is a mixture of nitrogen and oxygen.

189. **Absorption of gases by liquids.**—Water and many liquids possess the property of absorbing gases. Under the same conditions of pressure and temperature a liquid does not absorb equal volumes of different gases. At the temperature 0° C. and pressure 760 mm., one volume of water dissolves the following volumes of gas :—

Nitrogen	0·020	Sulphuretted hydrogen	4·37
Oxygen	0·041	Sulphurous acid	79·79
Carbonic acid	1·79	Ammonia	1046·63

From the very great condensation, to which the latter correspond, it may be inferred that the gases in solution are in the liquid state.

Gases are more soluble in alcohol than in water ; thus at 0° C. alcohol dissolves 4·33 volumes of carbonic acid gas.

The whole subject of gas absorption has been investigated by Bunsen. The general rules are the following :—

I. *For the same gas, the same liquid, and the same temperature, the weight of gas absorbed is proportional to the pressure.* This may also be expressed by saying that at all pressures the volume dissolved is the same ; or that the density of the gas absorbed is in constant relation with that of the external gas which is not absorbed.

Accordingly, when the pressure diminishes, the quantity of dissolved gas

decreases. If a solution of gas be placed under the air-pump and a vacuum created, the gas obeys its expansive force, and escapes with effervescence.

II. *The quantity of gas absorbed decreases with increase of the temperature* ; that is to say, when the elastic force of the gas is greater. Thus at 15° water absorbs only 1·00 of carbonic acid.

III. *The quantity of gas which a liquid can dissolve is independent of the nature and of the quantity of other gases which it may already hold in solution.*

This absorption of gases may be determined by the absorptiometer represented in fig. 170, which consists of a graduated measuring tube, A, connected by a caoutchouc tube with a tube of equal diameter, B. The absorption vessel, C, is connected with A by means of a thin flexible capillary lead tube ; *a* and *b* are three-way stopcocks, and *c* an ordinary one. The vessel C is fitted with air-free liquid, and A with the gas, which, by means of the two three-way stopcocks, is easily effected. The tube B is raised or lowered until the level of the mercury is the same as in A, and the volume of gas is read off. A is now put in connection with C, and the stopcock *c*, having been opened, B is raised so that a determinate volume of liquid runs out. An equal volume of the gas then passes into C, and the absorption proceeds, C being constantly shaken. In order to work at constant temperature, A and C may be surrounded by water.

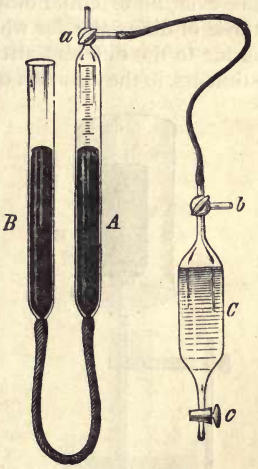


Fig. 170.

In every gaseous mixture each gas exercises the same pressure as it would if its volume occupied the whole space ; and the total pressure is equal to the sum of the individual pressures. When a liquid is in contact with a gaseous mixture, it absorbs a certain part of each gas, but less than

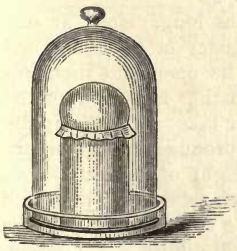


Fig. 171.

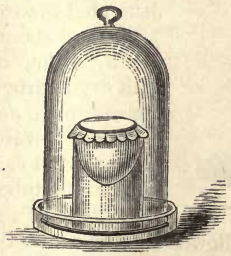


Fig. 172.

Fig. 173.

it would if the whole space were occupied by each gas. The quantity of each gas dissolved is proportional to the pressure which the unabsorbed gas

exercises alone. For instance, oxygen forms only about $\frac{1}{5}$ the quantity of air ; and water, under ordinary conditions, absorbs exactly the same quantity of oxygen as it would if the atmosphere were entirely formed of this gas under a pressure equal to $\frac{1}{5}$ that of the atmosphere.

190. **Diffusion of gases.**—Phenomena analogous to those of endosmose (140) are seen in a high degree in the case of gases. When two different gases are separated by a porous diaphragm, an interchange takes place between them, and ultimately the composition of the gas on both sides of the diaphragm is the same ; but the rapidity with which different gases *diffuse* into each other under these circumstances varies considerably. There is, however, an essential difference between the phenomena of endosmose and those of diffusion ; for while the inequality in the currents in the former case is due to the different attraction of the material of the diaphragm for the constituents, in the diffusion of gases the nature of this material has no influence ; from he smallness of the pores the actions are molecular, and not molar, and the rate of interchange depends only on the size of the molecules, that is, on the specific gravities of the gases. The laws of the diffusion of gases were investigated by Graham. Numerous experiments illustrate it, some of the most interesting of which are the following :—

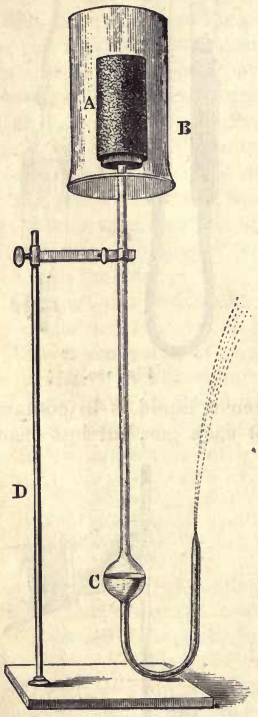


Fig. 174.

A glass cylinder closed at one end is filled with carbonic acid gas, its open end tied over with a bladder, and the whole placed under a jar of hydrogen. Diffusion takes place between them through the porous diaphragm, and after the lapse of a certain time hydrogen has passed through the bladder into the cylindrical vessel in much greater quantity than the carbonic acid which has passed out, so that the bladder becomes very much distended outwards (fig. 171). If the cylinder be filled with hydrogen and the bell-jar with carbonic acid, the reverse phenomenon will be produced—the bladder will be distended inwards (fig. 172).

A tube about 12 inches long, closed at one end by a plug of dry plaster of Paris, is filled with dry hydrogen, and its open end then immersed in a mercury bath. Diffusion of the hydrogen towards the air takes place so rapidly that a partial vacuum is produced, and mercury rises in the tube to a height of several inches (fig. 173). If several such tubes are filled with different gases, and allowed to diffuse into the air in a similar manner, in the same time, different quantities of the various gases will diffuse, and Graham found that the law regulating these diffusions is that *the force of diffusion is inversely as the square roots of the densities of gases.* Thus, if two vessels of equal capacity, containing oxygen and hydrogen, be separated by a porous

plug, diffusion takes place ; and after the lapse of some time, for every one part of oxygen which has passed into the hydrogen, four parts of hydrogen have passed into the oxygen. Now, the density of hydrogen being 1, that of oxygen is 16; hence the force of diffusion is inversely as the square roots of these numbers. It is four times as great in the one which has $\frac{1}{16}$ the density of the other.

Let the stem of an ordinary tobacco pipe be cemented, so that its ends project, in an outer glass tube, which can be connected with an air-pump and thus exhausted. On allowing then a slow current of air to enter one end of the pipe, its nitrogen diffuses more rapidly on its way through the porous pipe than the heavier oxygen, so that the gas which emerges at the other end of the porous pipe, and which can be collected, is richer in oxygen, and by repeating the operation on the gas which has passed through, the proportion of oxygen is so much increased that the gas can relight a semi-extinguished taper. To this process, in which one gas can be separated from another by diffusion, the term *atmolysis* is given.

Fig. 174 is an excellent illustration of the action of diffusion. A porous pot, A, such as is used for voltaic cells, is fixed by means of a cork to the glass tube, which contains water up to the bulb, C, the upper part containing air. When a beaker containing hydrogen, B, is placed over the pot, the diffusion of the hydrogen into it is so rapid that the water is at once driven down and jets out. When the beaker is removed, the gas inside the pot being richer in hydrogen, now diffuses out with great rapidity, and the water rises in the tube much higher than its original level.

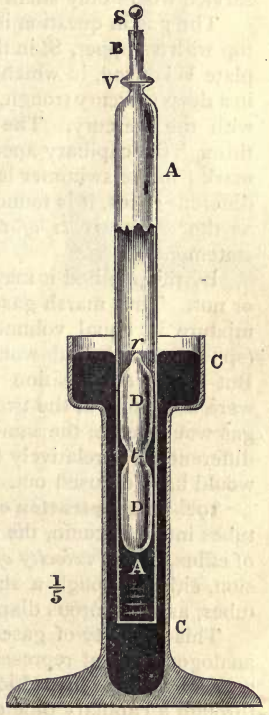


Fig. 175.

191. **Effusion of gases.**—A gas can only flow from one space to another space occupied by the same gas when the pressure in the one is greater than in the other. *Effusion* is the term applied to the phenomenon of the passage of gases into vacuum, through a minute aperture not much more or less than 0·013 millimetre in diameter, in a thin plate of metal or of glass ; for in a tube we are dealing with masses of gases, and friction comes into play, and in a larger aperture the particles would strike against one another, and form eddies and whirlpools. The velocity of the efflux is measured by the formula $v = \sqrt{2gh}$, in which h represents the pressure under which the gas flows, expressed in terms of the height of a column of the gas which would exert the same pressure as that of the effluent gas. Thus for air under the ordinary pressure flowing into a vacuum the pressure is equivalent to a column of mercury 76 centimetres high ; and as mercury is approximately 10,500 times as dense as air, the equivalent column

of air will be $76 \times 10{,}500 = 7{,}980$ metres. Hence the velocity of efflux of air into vacuum is $= \sqrt{2 \times 9{\cdot}8 \times 7980} = 395{\cdot}5$ metres. This velocity into vacuum only holds, however, for the first moment, for the space contains a continually increasing quantity of air, so that the velocity becomes continually smaller, and is null when the pressure on each side is the same. If the height of the column of air hh, corresponding to the external pressure, is known, the velocity may be calculated by the formula $v = \sqrt{2g(h-h_1)}$.

For gases lighter than air a greater height must be inserted in the formula, and for heavier gases a lower height; and this change must be inversely as the change of density. Hence *the velocities of efflux of various gases must be inversely as the square roots of their densities.* A simple inversion of this statement is that the *densities of two gases are inversely as the squares of their velocities of effusion.* On this law Bunsen has based an interesting method of determining the densities of gases and vapours, which is of great service where only small quantities of the substances are available.

The gas in question is contained (fig. 175) in a glass tube A, closed at the top with a stopper, S, in the neck, B. In a little enlargement here a platinum plate V is fixed, in which is a fine capillary aperture. The tube is inserted in a deep mercury trough, CC, so that the top r of a glass swimmer D is level with the mercury. The stopper S having been removed, the gas issues through the capillary aperture, and the time is noted which elapses until a mark t in the swimmer is level with the mercury. Working in this way with different gases, it is found that the ratios of the times of effusion are directly as the *square roots of the densities*, which is another form of the above statement.

By this method it may often be ascertained whether a gas is a mixture or not. Thus marsh gas (CH_4) has the same specific gravity (0·554) as a mixture in equal volumes of dimethyl (C_2H_6 sp. gr. 1·039) and hydrogen (sp. gr. 0·069), and would furnish the same results on chemical analysis. But if the composition of the gas which had been subjected to diffusion were examined in the two cases, it would be found that the residual marsh gas would retain the same composition, while that of the mixture would be different, for a relatively larger volume of the specifically lighter hydrogen would have diffused out.

192. **Transpiration of gases.**—If gases issue through long, fine capillary tubes into a vacuum, the phenomenon is called *transpiration*; and the rate of efflux, or the *velocity of transpiration*, is not the same as the rate of diffusion, either through a single aperture or through a series of fine capillary tubes, as in a porous diaphragm.

This property of gases may be investigated by means of an apparatus analogous to that represented in fig. 131, and which consists essentially of an arrangement by which gas under known pressure is allowed to flow through a capillary tube of known length and diameter.

The volume which flows out in a given time, at the rate of transpiration, is represented by a formula which is identical with that for liquids, namely,

$$v = \frac{\pi(p-p_1)r^4}{8\eta l}$$

when p is the pressure of the gas on entering, and p_1 that on leaving the

capillary tube ; r is the diameter, and l the length of the tube, and η is the coefficient of the internal friction of gases.

This furnishes an easy method of determining the value of η in this formula, as all the other magnitudes are capable of direct accurate measurement. This is a most important physical constant, as it occurs in many formulæ by which molecular magnitudes are determined, such as the length of the mean free path of gases (293), the number of impacts in a second, and even the dimensions of the molecules themselves. Expressed in CGS units, the value of η for air is $0{\cdot}0_3 18$.

193. **Absorption of gases by solids.**—The surfaces of all solid bodies exert an attraction on the molecules of gases with which they are in contact, of such a nature that they become covered with a more or less thick layer of *condensed gas*. When a porous body, such as a piece of charcoal, which consequently presents an immensely increased surface in proportion to its size, is placed in a vessel of ammonia gas over mercury (fig. 176), the great diminution of volume which ensues indicates that considerable quantities of gas are absorbed.

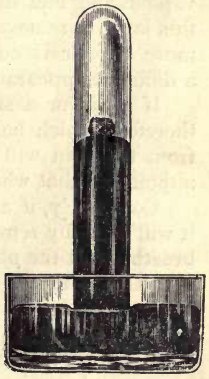


Fig. 176.

Now, although there is no absorption such as arises from chemical combination between the solid and the gas (as with phosphorus and oxygen), still the quantity of gas absorbed is not entirely dependent on the physical conditions of the solid body; it is influenced in some measure by the chemical nature both of the solid and the gas. Boxwood charcoal has very great absorptive power. The following table gives the volumes of gas, under standard conditions of temperature and pressure, absorbed by one volume of boxwood charcoal and of meerschaum respectively :—

	Charcoal	Meerschaum
Ammonia	90	15
Hydrochloric acid	85	—
Sulphurous acid	65	—
Sulphuretted hydrogen	55	11
Carbonic acid	35	5·3
Carbonic oxide	9·4	1·2
Oxygen	9·2	1·5
Nitrogen	7·4	1·6
Hydrogen	1·75	0·5

The absorption of gases is in general greatest in the case of those which are most easily liquefied.

Cocoa-nut charcoal is even more highly absorbent; it absorbs 171 of ammonia, 73 of carbonic acid, and 108 of cyanogen at the ordinary pressure; the amount of absorption increases with the pressure. The absorptive power of pine charcoal is about half as much as that of boxwood. The charcoal made from cork wood, which is very porous, is not absorbent; neither is graphite. Platinum, in the finely divided form known as platinum

sponge, is said to absorb 250 times its volume of oxygen gas. Many other porous substances, such as meerschaum, gypsum, silk, &c., are also highly absorbent.

If a coin be laid on a plate of glass or metal, after some time, when the plate is breathed on, an image of the coin appears. If a figure is traced on a glass plate with the finger, nothing appears until the plate is breathed on, when the figure is at once seen. Indeed, the traces of an engraving which has long lain on a glass plate may be produced in this way.

These phenomena are known as *Moser's images*, for he first investigated them, although he explained them erroneously. The correct explanation was given by Waidele, who ascribed them to alterations in the layer of gas, vapour, and fine dust which is condensed on the surface of all solids. If this layer is removed by wiping, on afterwards breathing against the surface more vapour is condensed on the marks in question, which then present a different appearance from the rest.

If a die or a stamp is laid on a freshly polished metal plate, and one therefore which has been deprived of its atmosphere, the layer of vapour from the coin will diffuse on to the metal plate, which thereby becomes altered; so that when this is breathed on an impression is seen.

Conversely, if a coin be polished and placed on an ordinary glass plate, it will partially remove the layer of gas from the parts in contact, so that on breathing on the plate the image is visible.

194. **Occlusion of gases.**—Graham found that at a high temperature platinum and iron allow hydrogen to traverse them even more readily than does caoutchouc in the cold. Thus while a square metre of caoutchouc 0·014 millimetre in thickness allowed 129 cubic centimetres of hydrogen at 20° to traverse it in a minute, a platinum tube 1·1 millimetre in thickness and of the same surface allowed 489 cubic centimetres to traverse it at a bright red heat.

This is probably connected with the property which some metals, though destitute of physical pores, possess of absorbing gases either on their surface or in their mass, and to which Graham has applied the term *occlusion*. It is best observed by allowing the heated metal to cool in contact with the gas. The gas cannot then be extracted by the air-pump, but is disengaged on heating. In this way Graham found that platinum occluded four times its volume of hydrogen; iron wire 0·44 its volume of hydrogen, and 4·15 volumes of carbonic oxide; silver, reduced from the oxide, absorbed about seven volumes of oxygen, and nearly one volume of hydrogen when heated to dull redness in these gases. This property is most remarkable in palladium, which absorbs hydrogen not only in cooling after being heated, but also in the cold. When, for instance, a palladium electrode is used in the decomposition of water, one volume of the metal can absorb 980 times its volume of the gas. This gas is again driven out on being heated, in which respect there is a resemblance to the solution of gases in liquids. By the occlusion of hydrogen the volume of palladium is increased by 0·09827 of its original amount, from which it follows that the hydrogen, which under ordinary circumstances has a density of 0·000089546 that of water, has here a density nearly 9,868 times as great, or about 0·88 that of water. Hence the hydrogen must be in the liquid or even solid state; it probably forms thus an alloy with palladium, like a true metal—a view of this gas which is

strongly supported by independent chemical considerations. The physical properties, too, in so far as they have been examined, support this view of its being an alloy.

The phenomenon of occlusion may be illustrated by the following experiment (fig. 177). A platinum wire, *bc*, is stretched between supports on a glass plate; one end of a palladium wire, *fg*, is also fixed, the other end of which is attached to the short arm of a light lever movable about *o*, the long arm of which is loaded with a weight (not represented in the figure) to keep the wire tight. The platinum wire is connected with the positive pole *a*, and the palladium with the negative pole *d*, of a voltaic battery, and the apparatus is partially immersed in acidulated water; the water is thereby decomposed into its constituent gases; oxygen is liberated in bubbles from the platinum wire, but there is no visible disengagement at the palladium. It becomes longer, however, as is seen by the lever moving downwards. If the current is reversed, the wire again contracts, and the lever resumes its original position.

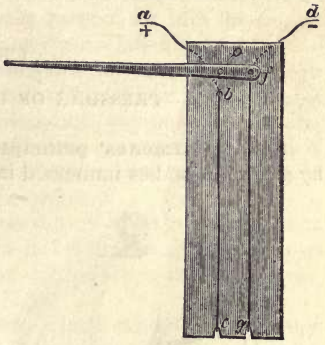


Fig. 177.

CHAPTER III

PRESSURE OF BODIES IN AIR. BALLOONS

195. **Archimedes' principle applied to gases.**—The pressure exerted by gases on bodies immersed in them is transmitted equally in all directions, as has been shown by the experiment with the Magdeburg hemispheres. It therefore follows that all which has been said about the equilibrium of bodies in liquids applies to bodies in air; they lose a part of their weight equal to that of the air which they displace.

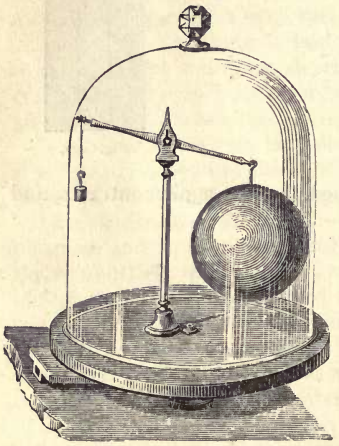


Fig. 178.

The loss of weight in air is demonstrated by means of the *baroscope*, which consists of a scalebeam, at one end of which a small leaden weight is supported, and at the other there is a hollow copper sphere (fig. 178). In the air they exactly balance each other; but when they are placed under the receiver of an air-pump, and a vacuum is produced, the sphere sinks, thereby showing that in reality it is heavier than the smaller leaden weight. Before the air is exhausted each body is buoyed up by the weight of the air which it displaces. But as the sphere is much the larger of the two, its weight undergoes most apparent diminution, and thus, though in reality the heavier body, it is balanced by the small leaden weight. It may be proved by means of the same apparatus that this loss is equal to the weight of the displaced air. Suppose the volume of the sphere is 10 cubic inches. The weight of this volume of air is 3·1 grains. If now this weight be added to the leaden weight, it will overbalance the sphere in air, but will exactly balance it in vacuo.

The principle of Archimedes is true for bodies in air; all that has been said about bodies immersed in liquids applies to them; that is, that when a body is heavier than air it will sink, owing to the excess of its weight over the buoyancy. If it is as heavy as air, its weight will exactly counterbalance the buoyancy, and the body will float in the atmosphere. If the body is lighter than air, the buoyancy of the air will prevail, and the body will rise in the atmosphere until it reaches a layer of the same density as its own.

The force of the ascent is equal to the excess of the buoyancy over the weight of the body. This is the reason why smoke, vapours, clouds, and air-balloons rise in the air.

AIR-BALLOONS

196. **Air-balloons.**—*Air-balloons* are hollow spheres made of some light impermeable material, which, when filled with heated air, with hydrogen gas, or with coal gas, rise in the air by virtue of their relative lightness.

They were invented by the brothers Montgolfier of Annonay, and the first experiment was made at that place in June 1783. Their balloon was a sphere of forty yards in circumference, and weighed 500 pounds. At the lower part there was an aperture, and a sort of boat was suspended, in which fire was lighted to heat the internal air. The balloon rose to a height of 2,200 yards, and then descended without any accident.

Charles, a professor of physics in Paris, substituted hydrogen for hot air. He himself ascended in a balloon of this kind in December 1783. The use of hot-air balloons was entirely given up in consequence of the serious accidents to which they were liable.

Since then the art of ballooning has been greatly extended, and many ascents have been made. That which Gay-Lussac made in 1804 was the most remarkable for the facts with which it has enriched science, and for the height which he attained—23,000 feet above the sea-level. At this height the barometer sank to 12·6 inches, and the thermometer, which was 31° C. on the ground, was 9 degrees below zero.

In these high regions the dryness was such on the day of Gay-Lussac's ascent, that hygrometric substances, such as paper, parchment, &c., became dried and crumpled as if they had been placed near the fire. The respiration and circulation of the blood were accelerated in consequence of the great rarefaction of the air. Gay-Lussac's pulse made 120 pulsations in a minute instead of 66, the normal number. At this great height the sky had a very dark blue tint, and an absolute silence prevailed.

One of the most remarkable of ascents was made by Mr. Glaisher and Mr. Coxwell, in a large balloon belonging to the latter. This was filled with 90,000 cubic feet of coal gas (sp. gr. 0·37 to 0·33); the weight of the load was 600 pounds. The ascent took place at 1 P.M. on September 5, 1861; at 1.28 they had reached a height of 15,750 feet, and in eleven minutes after a height of 21,000 feet, the temperature being – 10·4°; at 1.50 they were at 26,200 feet, with the thermometer at – 15·2°. At 1.52 the height attained was 29,000 feet, and the temperature – 16° C. At this height the rarefaction of the air was so great, and the cold so intense, that Mr. Glaisher fainted and could no longer observe. According to an approximate estimation the lowest barometric height they attained was 7 inches, which would correspond to an elevation of from 36,000 to 37,000 feet.

197. **Construction and management of balloons.**—A balloon (fig. 179) is made of long bands of silk sewed together and covered with caoutchouc varnish, which renders it air-tight. At the top there is a safety-valve closed by a spring, which the aëronaut can open at pleasure by means of a cord. A light wickerwork boat is suspended by means of cords to a network which entirely covers the balloon.

A balloon of the ordinary dimensions, which can carry three persons, is about 16 yards high, 12 yards in diameter, and its volume, when it is quite full, is about 680 cubic yards. The balloon itself weighs 200 pounds; the accessories, such as the rope and boat, 100 pounds.

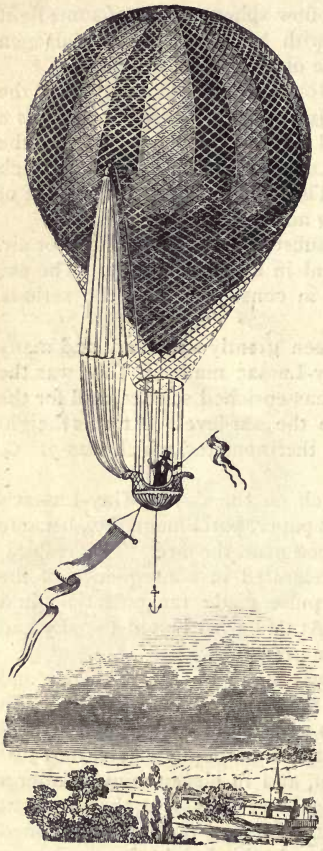


Fig. 179.

The balloon is filled either with hydrogen or with coal gas. Although the latter is heavier than the former, it is generally preferred, because it is cheaper and more easily obtained. It is passed into the balloon from the gas reservoir by means of a flexible tube. It is important not to fill the balloon quite full, for the atmospheric pressure diminishes as it rises, and the gas inside, expanding in consequence of its elastic force, tends to burst it. It is sufficient for the ascent if the weight of the displaced air exceeds that of the balloon by 8 or 10 pounds. And this force remains constant so long as the balloon is not quite distended by the dilatation of the air in the interior. If the atmospheric pressure, for example, has diminished to one-half, the gas in the balloon, according to Boyle's law, has doubled its volume. The volume of the air displaced is therefore twice as great; but since its density has become only one-half, the weight, and consequently the upward buoyancy, are the same. When once the balloon is completely dilated, if it continues to rise, the force of the ascent decreases, for the volume of the displaced air remains the same, but its density diminishes, and a time arrives at which the buoyancy is equal to the weight of the balloon. The balloon can now only take a horizontal direction, carried by the currents of air which prevail in the atmosphere. The aëronaut knows by the barometer whether he is ascending or descending, and by the same means he determines the height which he has reached. A long flag fixed to the boat would indicate, by the position it takes either above or below, whether the balloon is descending or ascending.

When the aëronaut wishes to descend, he opens the valve at the top of the balloon by means of the cord, which allows gas to escape, and the balloon sinks. If he wants to descend more slowly, or to rise again, he empties out bags of sand, of which there is an ample supply in the car. The descent is facilitated by means of a grappling-iron fixed to the boat. When

once this is fixed to any obstacle, the balloon is lowered by pulling the cord.

The only practical applications which air-balloons have hitherto had have been in military reconnoitring. At the battle of Fleurus, in 1794, a captive balloon—that is, one held by a rope—was used, in which there was an observer who reported the movements of the enemy by means of signals. At the battle of Solferino the movements and dispositions of the Austrian troops were watched from a captive balloon; and in the war in America balloons were frequently used, while their importance during the siege of Paris will not have been forgotten. The whole subject of military ballooning was treated in two papers by Col. Grover and by Col. Beaumont, in a volume of the Professional Papers of the Royal Engineers; and experiments are in progress at Woolwich and at Aldershot, with a view of ascertaining the most practical means of inflating balloons, and the best form and equipment for service in the field. It has been proposed to use captive balloons for observations on the changes of temperature in the air, &c. Air-balloons can only be truly useful when they can be guided, and as yet all attempts made with this view have completely failed. There is no other course at present than to rise in the air until there is a current which has more or less the desired direction. Unfortunately, the currents in the higher regions of the atmosphere are variable and irregular.

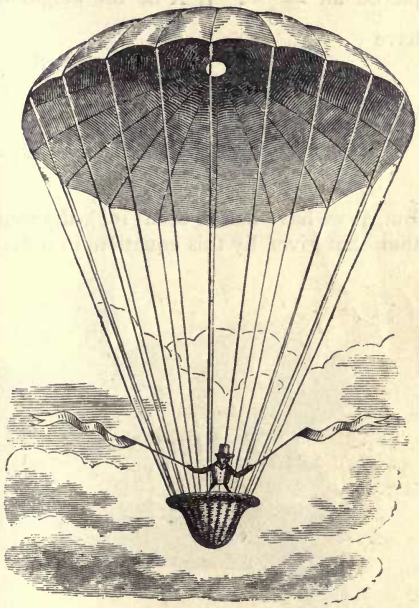


Fig. 180.

198. **Parachute.**—The object of the parachute is to allow the aëronaut to leave the balloon, by giving him the means of lessening the rapidity of his descent. It consists of a large circular piece of cloth (fig. 180), about 16 feet in diameter, and which by the resistance of the air spreads out like a gigantic umbrella. In the centre there is an aperture through which the air compressed by the rapidity of the descent makes its escape; for otherwise oscillations might be produced, which, when communicated to the boat, would be dangerous.

In fig. 179 there is a parachute attached to the network of the balloon by means of a cord which passes round a pulley, and is fixed at the other end to the boat. When the cord is cut the parachute sinks, at first very rapidly but more slowly as it becomes distended, as represented in the figure.

199. **Calculation of the weight which a balloon can raise.**—To calculate the weight which can be raised by a balloon of given dimensions, let us suppose it perfectly spherical, and premise that the formulæ which express the volume and the superficies in terms of the radius are $V = \frac{4\pi R^3}{3}$ $S = 4\pi R^2$; π being the ratio of the circumference to the diameter. The radius R being measured in feet, let p be, in pounds, the weight of a square foot of the material of which the balloon is constructed; let P be the weight of the car and the accessories, a the weight in pounds of a cubic foot of air at zero, and under the pressure $0{\cdot}76^m$, and a' the weight of the same volume, under the same conditions, of the gas with which the balloon is inflated (155). Then the total weight of the envelope in pounds will be $4\pi R^2 p$; that of the gas will be $\frac{4\pi R^3 a'}{3}$; and that of the displaced air $\frac{4\pi R^3 a}{3}$. If X be the weight which the balloon can support, we have

$$X = \frac{4\pi R^3 a}{3} - \frac{4\pi R^3 a'}{3} - 4\pi R^2 p - P.$$

Whence

$$X = \frac{4\pi R^3}{3}(a - a') - 4\pi R^2 p - P.$$

But, as we have before seen (197), the weight must be less by 8 or 10 pounds than that given by this equation, in order that the balloon may rise.

CHAPTER IV

APPARATUS WHICH DEPEND ON THE PROPERTIES OF AIR

200. **Air-pump.**—The air-pump is an instrument by which a vacuum can be produced in a given space, or rather by which air can be greatly rarefied, for an absolute vacuum cannot be produced by its means. It was invented

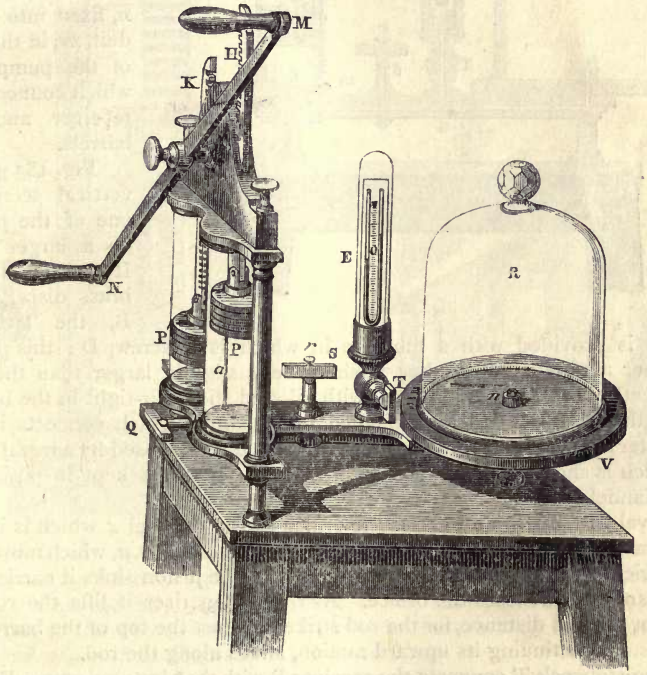


Fig. 181.

by Otto von Guericke in 1650, a few years after the invention of the barometer.

The air-pump, as now usually constructed, may be described as follows. Fig. 181 represents a general view; fig. 182 a section, and figs. 183–188 various parts; the letters in all the figures having everywhere the same meaning.

The base VGL is of stout metal, and is firmly fixed on a table. At one

end two glass cylinders or *barrels* are firmly cemented, and the two leather pistons P and P′ work air-tight in them. To these pistons are attached racks H, K, and by means of a handle M N, working about a pinion X, the pistons P and P′ are moved alternately up and down. On the plate V is fitted a thick glass plate with a very true surface. In its centre is a screw tubulure, *n*, fixed into a conduit, *nc*, in the base of the pump, and which connects the receiver and the barrels.

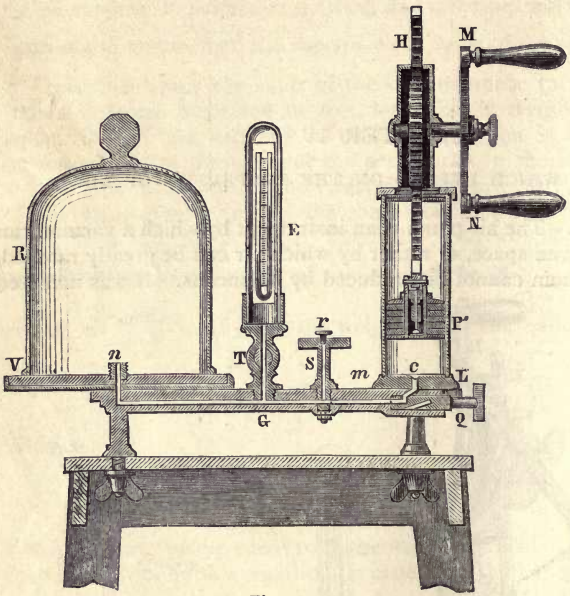


Fig. 182.

Fig. 183 gives a vertical section of one of the pistons on a larger scale It consists of two brass discs, A and B, the latter of which is provided with a tubulure in which is a screw, D ; this presses together a number of leather washers, very slightly larger than the disc. The leather is thoroughly soaked with oil, and slides air-tight in the barrels, but with slight friction. D is pierced by a channel which connects it with the outer air. In the centre of the disc B is a hole, *i*, closed by a metal valve, Z, which is shod with cork, and by means of a rod, *e*, is kept in position in the channel.

A valve, *s*, opens and closes the orifice of the channel *c* which is in connection with the receiver. It is fixed to the end of a rod, *a*, which moves, but with friction, through the piston. Then when the piston sinks it carries with it the rod *a*, and closes the orifice. As the piston rises it lifts the rod, but only for a small distance, for the rod strikes against the top of the barrel, and the piston, continuing its upward motion, slides along the rod.

The stopcock T connects the receiver R with the air-pump gauge E (201), while S connects the receiver with the barrels. When the receiver has been exhausted S is turned through a quarter, and the vacuum is thus preserved. Air can be admitted by opening a screw, *r*, at the top of a channel in the stopcock itself.

The piston P′ being at the bottom of the barrel (fig. 184), as the handle is worked the piston rises, and with it the rod *a* and the valve *s*, while Z is closed by its own weight and the pressure of the air. A partial vacuum is created under the piston, but the valve *s* having opened up con-

nection with the receiver R, the air in this expands and fills both the receiver and the barrel. When P′ begins to descend, the valve *s* is closed by the descent of the rod *a*, the rarefied air in the barrel can no longer return to the receiver, it gets more and more condensed, and its elastic force is ultimately so great as to open the valve Z, and the air under the piston escapes by the channel D into the outer air, and thus the rarefaction produced in the receiver is permanent. At the second stroke of the piston the same phenomenon is repeated, until a limit is reached at which, although there is air in the receiver, its elastic force is insufficient to raise the valve Z.

It is clear that when the rarefaction has proceeded to a considerable extent, the atmospheric pressure on the top of P will be very great, but it will

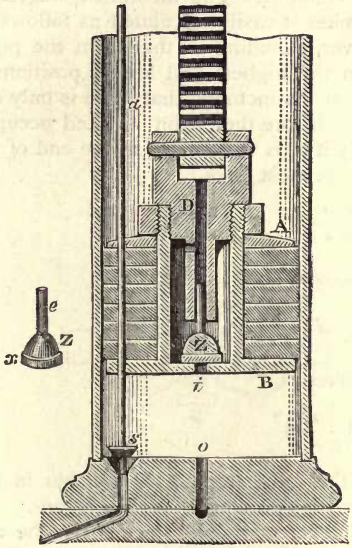


Fig. 183.

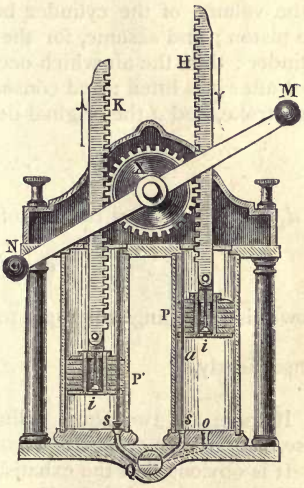


Fig. 184.

be very nearly balanced by the atmospheric pressure on the top of the other piston. Consequently, the experimenter will have to overcome only the difference of the two pressures. This is the reason why two barrels are employed, a plan first adopted by Hawksbee.

201. **Air-pump gauge.**—When the pump has been worked some time, the pressure in the receiver is indicated by the difference of level of the mercury in the two legs of a glass tube bent like a syphon, one of which is opened, and the other closed like the barometer. This little apparatus, which is called the *gauge*, is fixed to an upright scale and placed under a small bell-jar, E, which communicates with the receiver R by a stopcock, T, inserted in the tube leading from the orifice G to the cylinders (fig. 182).

Before commencing to exhaust the air in the receiver, its elastic force exceeds the weight of the column of mercury which is in the closed branch

and which consequently remains full. But as the pump is worked the elastic force soon diminishes, and is unable to support the weight of the mercury, which sinks and tends to stand at the same level in both legs. If an absolute vacuum could be produced, they would be exactly on the same level, for there would be no pressure either on the one side or the other. But with the very best machines the level is always about a thirtieth of an inch higher in the closed branch, which indicates that the vacuum is not absolute, for the elastic force of the residue is equal to the pressure of a column of mercury of that height.

Theoretically an absolute vacuum is impossible; for since the volume of each cylinder is, say, $\frac{1}{20}$ that of the receiver, only $\frac{1}{21}$ of the air in the receiver is extracted at each stroke of the piston, and consequently it is impossible to exhaust all the air which it contains. The theoretical degree of exhaustion after a given number of strokes is easily calculated as follows:— Let a denote the volume of the receiver, including in that term the pipe; b the volume of the cylinder between the highest and lowest positions of the piston; and assume, for the sake of distinctness, that there is only one cylinder: then the air which occupied a before the piston is lifted occupies $a+b$ after it is lifted; and consequently if d_1 is the density at the end of the first stroke, and d the original density, we must have

$$d_1 = d\frac{a}{a+b}$$

If d_2 is the density at the end of the second stroke, we have

$$d_2 = d_1\frac{a}{a+b} = d\left(\frac{a}{a+b}\right)^2$$

Now this reasoning will apply to n strokes;

consequently,
$$d_n = d\left(\frac{a}{a+b}\right)^n$$

If there are two equal cylinders, the same formula holds; but in this case, in counting n, upstrokes and downstrokes equally reckon as *one*.

It is obvious that the exhaustion is never complete, since d can be zero only when n is infinite. However, no very great number of strokes is required to render the exhaustion virtually complete, even if a is several times greater than b. Thus if $a = 10b$ a hundred strokes will reduce the density from d to $0{\cdot}00007d$; that is, if the initial pressure is 30 inches, the pressure at the end of 100 strokes is 0·021 of an inch.

Practically, however, a limit is placed on the rarefaction that can be produced by any given air-pump; for, as we have seen, the air becomes ultimately so rarefied that, when the pistons are at the bottom of the cylinder, its elastic force cannot overcome the pressure in the valves on the inside of the piston; they therefore do not open, and there is no further action of the pump.

202. **Double-exhaustion stopcock.**—By means of this device the exhaustion of the air can be carried to a very high degree. Fig. 185 gives a horizontal section of the stopcock Q, which by means of a central channel and two lateral ones forms a communication with the receiver and the

barrels. When the working ceases, that is, when Z no longer rises, a quarter-turn is given to Q (fig. 187). The connections are now altered, as is seen from the horizontal sections in figs. 185 and 187, and the vertical sections in figs. 187 and 188. The new channels correspond now with those of the base, and the right barrel is *alone* connected with the receiver by the channel *nmc*, while the left is connected by an oblique channel in the stopcock with a central aperture, *s*, in the base of the right barrel.

The right piston as it rises exhausts air from the receiver; but when it sinks the exhausted air is drawn into the left barrel by the apertures *o* and *d*, this latter being always open, for the corresponding conical valve is raised.

Fig. 185.

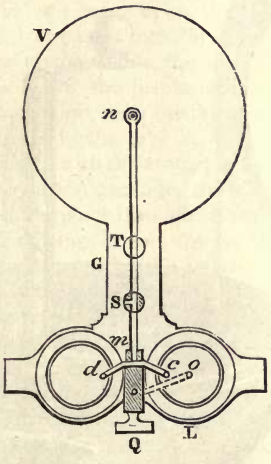
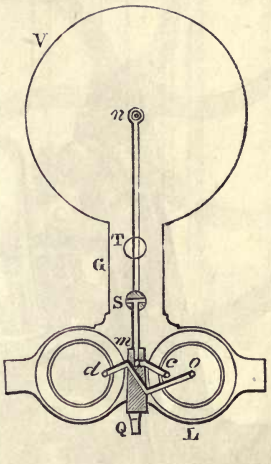


Fig. 186.



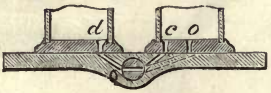


Fig. 187.

Fig. 188.

When the right piston rises that of the left sinks; but the air below does not return to the right barrel, for the orifice is now closed by the conical valve. As the right cylinder continues to exhaust the air in the receiver, and to force it into the left cylinder, the air accumulates here, and ultimately acquires sufficient pressure to raise the valve of the piston P′, which was impossible before the stopcock was turned, for it is only when the valves in the piston no longer open that a quarter of a turn is given to the stopcock. In this way a rarefaction of half a millimetre has been attained.

203. **Bianchi's air-pump.**—Bianchi invented an air-pump which has several advantages. It is made entirely of iron, and it has only one cylinder, which oscillates on a horizontal axis fixed at its base, as seen in fig. 189. A horizontal shaft, with heavy fly-wheel V, works in a frame, and is turned

by a handle, M. A crank, *m*, which is joined to the top of the piston rod, is fixed to the same shaft, and consequently at every revolution of the wheel the cylinder makes two oscillations.

In some cases, as in that shown in the figure, the crank and the fly-wheel are on parallel axes connected by a pair of cog-wheels. The modification in the action produced by this arrangement is as follows: If the cog-wheel on the former axis has twice as many teeth as that on the latter axis,

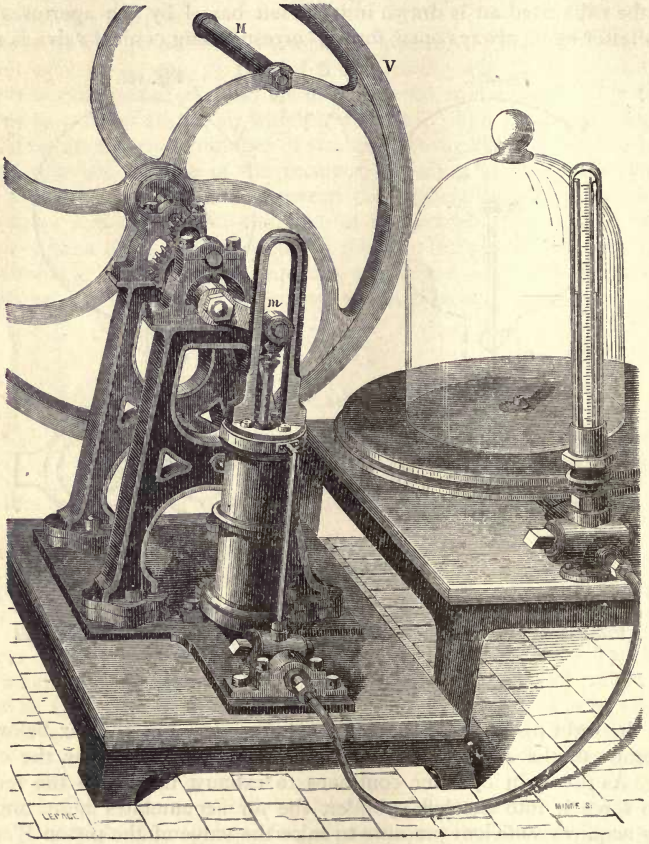


Fig. 189.

the pressure which raises the piston is doubled; an advantage which is counterbalanced by the inconvenience that now the piston will make one oscillation for one revolution of the fly-wheel.

The machine is double-acting; that is, the piston PP (fig. 190) produces a vacuum, both in ascending and descending. This is effected by the following arrangements:—In the piston there is a valve, *b*, opening upwards

as in the ordinary machine. The piston rod AA is hollow, and in the inside there is a copper tube, X, by which the air makes its escape through the valve *b*. At the top of the cylinder there is a second valve, *a*, opening upwards. An iron rod, D, works with gentle friction in the piston, and terminates at its ends in two conical valves, *s* and *s'*, which fit into the openings of the tube BB leading to the receiver.

Let us suppose the piston descends. The valve *s'* is then closed, and, the valve *s* being open, the air of the receiver passes into the space above the piston, while the air in the space below the piston undergoes compression, and, raising the valve, escapes by the tube X, which communicates with the atmosphere. When the piston ascends, the exhaustion takes place through *s'*, and the valve *s* being closed, the compressed air escapes by the valve *a*.

The machine has a stopcock for double exhaustion, similar to that already described (202). It is also oiled in an ingenious manner. A cup, E, round the rod is filled with oil, which passes into the annular space between the rod AA and the tube X; it passes then into a tube *oo* in the piston, and, forced by the atmospheric pressure, is uniformly distributed on the surface of the piston.

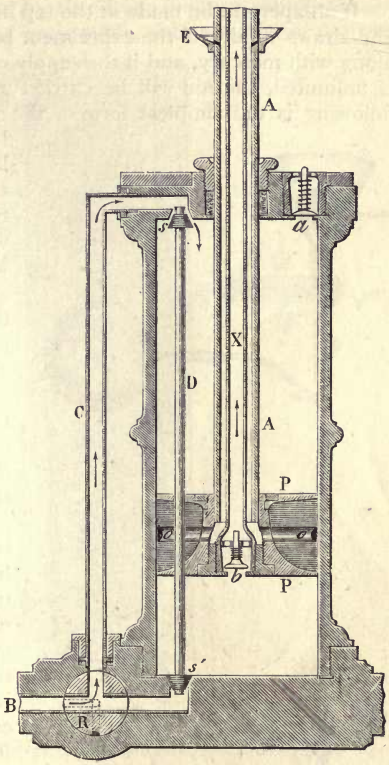


Fig. 190.

The apparatus, being of iron, may be made of much greater dimensions than the ordinary air-pump. A vacuum can also be produced with it in far less time and in apparatus of greater size than usual.

204. **Deleuil's air-pump.**—In this air-pump the main peculiarity is its piston, which is of considerable length, and consists of a series of accurately constructed metal discs bolted together. This works easily and smoothly in the barrel, and no packing or lubricator is used; or rather, the lubricator is the air in the space between the piston and the barrel. The internal friction of the air in this narrow space is so great that the rate at which it leaks into the barrel is far inferior to the rate at which the pump is exhausting air from the receiver. And Maxwell showed that the internal friction is not diminished even when its density is greatly reduced. Hence the pump works very satisfactorily up to a considerable degree of exhaustion—to a millimetre of mercury, for instance.

205. **Sprengel's air-pump.**—Sprengel has devised a form of air-pump which depends on the principle of converting the space to be exhausted into a Torricellian vacuum.

If an aperture be made in the top of a barometer tube, the mercury sinks and draws in air; if the experiment be so arranged as to allow air to enter along with mercury, and if the supply of air be limited while that of mercury is unlimited, the air will be carried away and a vacuum produced. The following is the simplest form of the apparatus in which this action is realised. In fig. 191, *cd* is a glass tube longer than a barometer, open at both ends, and connected by means of india-rubber tubing with a funnel, A, filled with mercury and supported by a stand. Mercury is allowed to fall in this tube at a rate regulated by a clamp at *c*; the lower end of the tube *cd* fits in the flask B, which has a spout at the side a little higher than the lower end of *cd*; the upper part has a branch at *x*, to which a receiver, R, can be tightly fixed. When the clamp at *c* is opened, the first portions of mercury which run out close the tube and prevent air from entering below. As the mercury is allowed to run down the exhaustion begins, and the whole length of the tube from *x* to *d* is filled with cylinders of air and mercury having a downward motion. Air and mercury escape through the spout of the flask B which is above the basin H, where the mercury is collected. It is poured back from time to time into the funnel A, to be re-passed through the tube until the exhaustion is complete. As this point is approached, the enclosed air between the mercury cylinders is seen to diminish, until the lower part of *cd* forms a continuous column of mercury about 30 inches high. Towards this stage of the process a noise is heard like that of a water-hammer when shaken; the operation is completed when the column of mercury encloses no air, and a drop of mercury falls on the top of the column without enclosing the slightest air-bubble. The height of the column then represents the height of the column of mercury in the barometer; in other words, it is a barometer whose Torricellian vacuum is the receiver R. This apparatus has been used with great success in experiments in which a very complete exhaustion is required, as in the preparation of Geissler's tubes and in incandescent electrical lamps. It

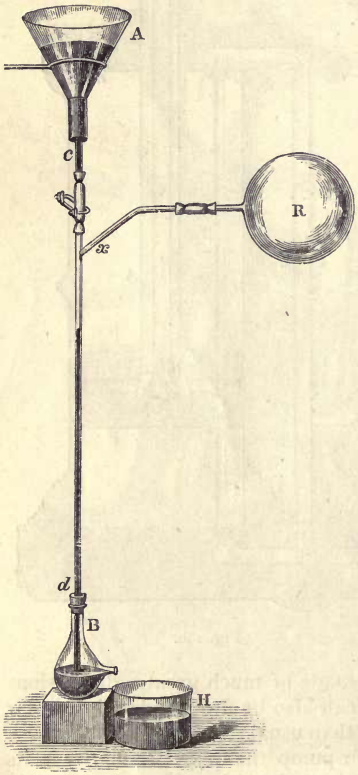


Fig. 191.

may be advantageously combined with an exhausting syringe, which first removes the greater part of the air, the exhaustion being then completed as above.

The most perfect vacua are obtained by absorbing the residual gas, after the exhaustion has been pushed as far as possible, either mechanically or by some substance with which it combines chemically. Thus Dewar has produced a vacuum which he estimates at $\frac{1}{350}$ of a millimetre, by heating charcoal to redness, in a vessel from which air had been exhausted by the Sprengel pump, and then allowing it to cool. Finkener filled a vessel with oxygen, then exhausted as far as possible, and finally heated to redness some copper contained in the vessel. This absorbed the minute quantity of gas left, with the formation of cupric oxide. In some of his experiments Crookes obtained by chemical means a vacuum of $\frac{1}{2000000}$ of a millimetre. In these highly rarefied gases the pressure is so low that it is very difficult to measure minute differences. For such cases McLeod has devised a very valuable gauge, the principle of which is to condense a measured volume of the highly rarefied gas to a much smaller volume, and then to measure its pressure under the new conditions.

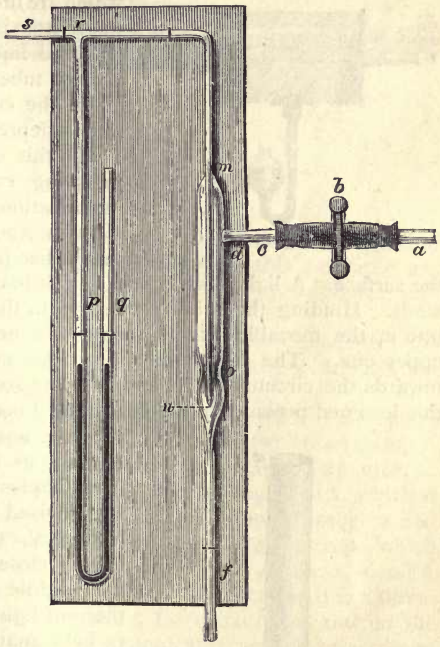


Fig. 192.

206. **Bunsen's Sprengel pump.**—This is a very convenient arrangement for producing a vacuum in cases where a good supply of water is available, as in laboratories. A composition tube, *a* (fig. 192), connected with the service-pipe of a water-supply, is joined by means of a caoutchouc tube to a glass tube, *cdf*, to which is attached at *f* a leaden tube about 10 to 12 yards long. The tube *sr* is connected with the space to be exhausted. The water enters by *a*, and in falling down the tube carries with it air from the space to be exhausted. The supply of water, and therewith the rate of exhaustion, can be regulated by the stopcock *b* ; the bent tube *pq*, which contains mercury, measures the degree of exhaustion, which may be reduced to a pressure of 10 to 15 millimetres.

207. **Aspirating action of currents of air.**—When a jet of liquid or of a gas passes through air, it carries the surrounding air along with it, fresh air rushes in to supply its place, comes also in contact with the jet, and is in like manner carried away. Thus, then, there is a continual rarefaction

of the air round the jet, in consequence of which it exerts an aspiratory action.

This phenomenon may be well illustrated by means of an apparatus represented in fig. 193, the analogy of which to the experiment described (146) will be at once evident. It consists of a wide glass tube, in the two ends of which are fitted two small tubes, *nd* and B ; in the bottom is a manometer tube containing a coloured liquid. On blowing through the narrow tube the liquid at *o* is seen to rise. If, on the contrary, the wide tube is blown into, a depression is produced at *o*.

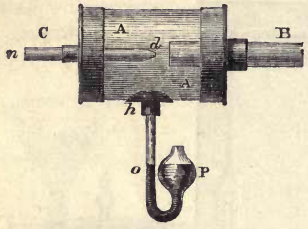


Fig 193

To this class of phenomena belongs the following experiment, which is a simple modification of one originally described by Clement and Desormes. A tube is fixed in a metal disc (fig. 194), its end being flush with the surface. A light disc is held at a little distance by means of three metal studs. Holding the tube vertically with the discs downwards, and blowing into it, the movable disc is seen to rise until it comes in contact with the upper one. The current of air spreads out from the centre of the plate towards the circumference, and in doing so it is rarefied ; in consequence of this lessened pressure in the space, the lower disc is lifted by the external pressure against the upper one, where it remains as long as the blowing continues. The simplest plan of making this experiment was devised by Faraday. Holding one hand horizontal, the palm downwards and the fingers closed, the space between the index and middle finger is blown through. If a piece of light paper, of 2 or 3 square inches, is held against the aperture, it does not fall as long as the blowing continues.

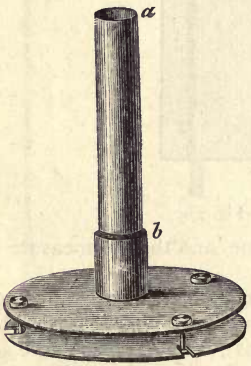


Fig. 194.

The old *water-bellows*, still used in mountainous places where there is a continuous fall, is a further application of the principle. Water falling from a reservoir down a narrow tube divides and carries air along with it ; and, if there are apertures in the sides through which air can enter, this also is carried along, and becomes accumulated in a reservoir placed below, from which by means of a lateral tube it can be directed into the hearth of a forge.

This may be illustrated by the simple apparatus represented in fig. 195, the construction of which from glass tubes and corks will be readily intelligible. It may be remarked that the outer tube at *b* is represented in section, and that the part of the tubes *ofd* and *ghk* outside the cork are relatively much longer horizontally and vertically than is here represented.

If the vertical tube *fd* is fitted to a vessel of boiling water, as soon as steam issues through *o*, it not only raises water from a vessel in which the

bottom of the tube *gh* dips, but drives it through the aperture *o*. And if a bent tube, with a narrow opening like *o*, be fitted at *n*, and directed upwards, a continuous jet of water is produced, often reaching to the ceiling.

This apparatus serves well to illustrate the principle of *Giffard's injector*, an extremely ingenious and important apparatus by which steam-boilers are kept supplied with water.

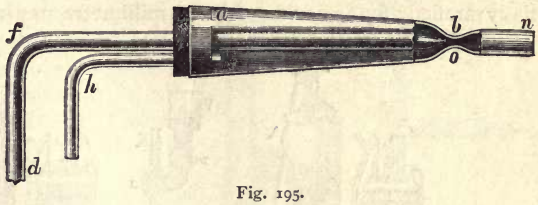


Fig. 195.

The principle is also applied in a series of machines for moving and lifting liquids, and even solids such as corn; in pumping, in blowers, exhausters, air-pumps, &c. An interesting application is that of the well-known *spray producer;* this principle has further been utilised by Sprengel in supplying water to sulphuric acid chambers.

By the *locomotive steam-pipe* a jet of steam entering the chimney of the locomotive carries the air away, so that fresh air must arrive through the fire, and thus the draught be kept up.

208. **Morren's mercury pump.**—Figs. 196 and 197 represent a mercury air pump, constructed by Alvergniat. It consists of two reservoirs, A and B, connected by a barometer tube, T, and a long caoutchouc tube, C. The reservoir B and the tube T are fixed to a vertical support, A, which is movable and open, and can be alternately raised and lowered through a distance of nearly 4 feet. This is effected by means of a long wire rope, which is fixed at one end to the reservoir A, and passes over two pulleys, *a* and *b*, the latter of which is turned by a handle. Above the reservoir B is a three-way cock, *n*; to this is attached a tube, *d*, for exhaustion, and on the left is an ordinary stopcock, *m*, which communicates with a reservoir of mercury, *v*, and with the air. The exhausting tube *d* is not in direct communication with the receiver to be exhausted; it is first connected with a reservoir, *o*, partially filled with sulphuric acid, and designed to dry the gases which enter the apparatus. A caoutchouc tube, *c*, makes communication with the receiver which is to be exhausted. On the reservoir *o* is a small mercury manometer, *p*.

These details being understood, suppose the reservoir A at the top of its course (fig. 196), the stopcock *m* open, and the stopcock *n* turned as seen in Z; the caoutchouc tube C, the tube T, the reservoir B, and the tube above are filled with mercury as far as *v*; closing then the stopcock *m*, and lowering the reservoir A (fig. 197), the mercury sinks in the reservoir B, and in the tube T, until the difference of levels in the two tubes is equal to the barometric height, and there is a vacuum in the reservoir B. Turning now the stopcock *n*, as shown in fig. X, the gas from the space to be exhausted passes into the barometer chamber B by the tubes *c* and *d*, and the level again sinks in the tube T. The stopcocks are now replaced in the first position (fig. Z), and the reservoir A is again lifted, the excess of pressure of mercury in the caoutchouc tube expels, through the stopcocks *n* and *m*, the gas which

had passed into the chamber B, and, if a few droplets of mercury are carried along with them, they are collected in the vessel *v*. The process is repeated until the mercury is virtually at the same level in both legs.

Like Sprengel's pump, this is very slow in its working, and, like it, is best employed in completing the exhaustion of a space which has already been partially rarefied; for a vacuum of $\frac{1}{10}$ of a millimetre may be obtained by its means.

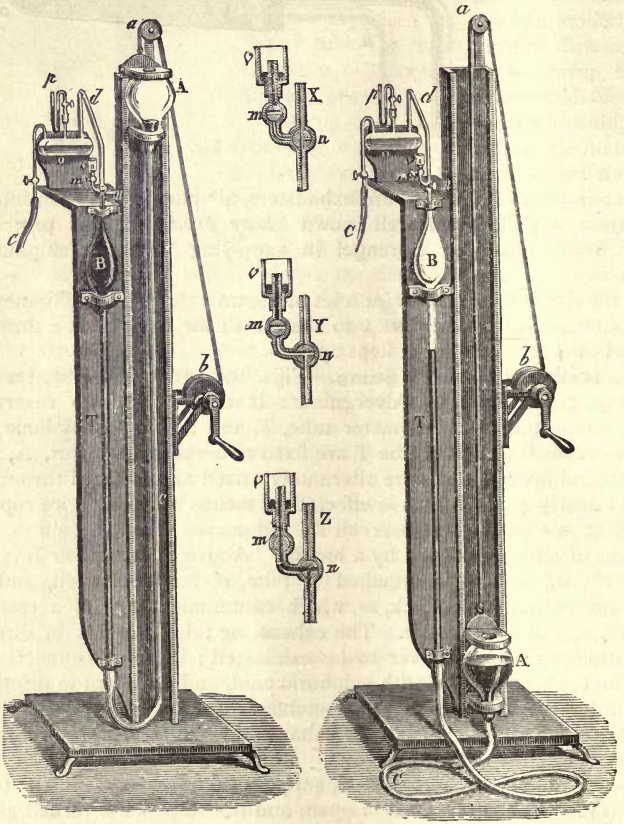


Fig. 196. Fig. 197.

209. **Condensing pump.**—The condensing pump is an apparatus for compressing air or any other gas. The form usually adopted is the following:—In a cylinder, A, of small diameter (fig. 199), there is a solid piston, the rod of which is moved by the hand. The cylinder is provided with a screw which fits into the receiver K. Fig. 198 shows the arrangement of the valves, which are so constructed that the lateral valve *o* opens from the outside, and the lower valve *s* from the inside.

When the piston descends the valve *o* closes, and the elastic force of the

compressed air opens the valve *s*, which thus allows the compressed air to pass into the receiver. When the piston ascends, *s* closes and *o* opens, and permits the entrance of fresh air, which in turn becomes compressed by the descent of the piston, and so on. This apparatus is chiefly used for charging liquids with gases. For this purpose the stopcock B is connected with a reservoir of the gas by means of the tube D. The pump exhausts this gas, and forces it into the vessel K, in which the liquid is contained. Artificial gaseous waters are made by means of analogous apparatus.

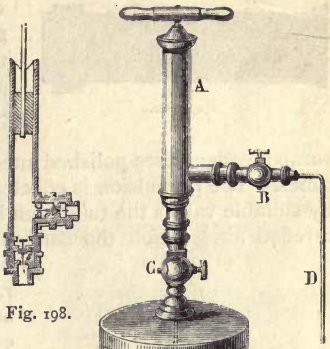


Fig. 198.

The applications of condensed air are both numerous and important. In a certain sense condensed air plays the part of a metal spring in which is stored up a greater or less provision of work, and which can then be utilised by expanding the air at a given moment, and at a given point in the most favourable condition for its being applied. In some cases the expansion is sudden and intermittent, as in the air-gun, the pneumatic post, or in air-brakes, and in some cases slow, gradual, and continuous, as in boring machines.

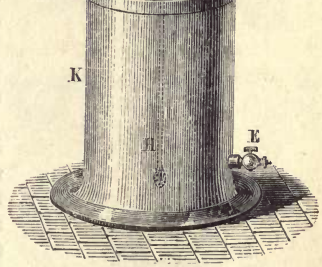


Fig. 199.

One of the most important applications is that to the larger boring machines used in tunnelling through the Alps and elsewhere. There, where steam power would be objectionable owing to the steam produced, compressed air is of great service, for it not only supplies the power, but it ventilates the underground spaces.

The principal parts of such machines, which were first employed on a large scale in the Mont Cenis tunnel, are as follows :—A sheaf of borers or iron rods with punches on the ends are mounted on a framework. Each of these borers is susceptible of three simultaneous motions, one backward and forward producing repeated shocks against the rock ; a second analogous to that of a gimlet ; while a third moves the whole framework backwards and forwards.

This triple motion is effected by a machine like a steam engine, but driven by compressed air ; the first motion by a piston, the action of which is regulated by a slide valve (469) ; the other two motions are effected by means of a separate machine. The air is under a pressure of five atmospheres, the compression being effected by special machines worked by water power. The air by which all this is effected on expanding serves to cool and ventilate the mine.

The *pneumatic post* is of great service in London and other large towns in forwarding the actual written telegraphic messages from the several

receiving stations to a central telegraph station. The messages are placed in a *carrier* (fig. 200), which is a gutta-percha cylinder 7 in. long by 2 in. in diameter, closed at one end; it is covered with felt, and there is a welt of that material at one end; the felt projects at the other, so that it can be folded down and held in position by an india-rubber band, in order to keep the contents in their place.

Fig. 200.

Such carriers move air-tight in carefully turned leaden tubes polished internally and protected by being incased in iron tubes. The propulsion is effected either by pressure or by exhaustion; and by suitable valves the tubes can be placed in connection with compressed or rarefied air, so that the carriers may either be shot in one direction by compressed air, or drawn in the other by rarefied air. The compression and rarefaction are produced by means of powerful steam engines to a pressure of about ten pounds, or a vacuum of eight pounds to the inch. By this means a speed of nearly a mile in a minute may be obtained in tubes not more than a mile in length.

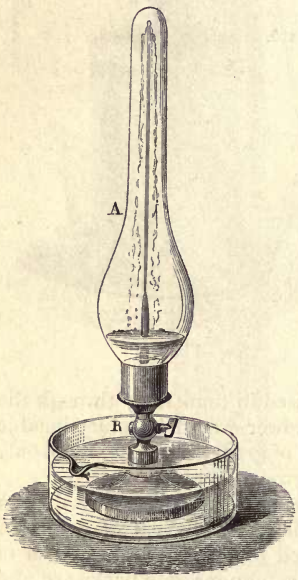


Fig. 201.

Other applications of compressed air are in the small pumps used by plumbers for testing and for clearing gas-pipes, in ventilating mines, in supplying air to blast-furnaces, in the air-brakes used in railway trains, as motive power in torpedoes, and so forth.

Compressed air has long been used in Paris for the synchronised working of clocks, and this has led to its successful application on a large scale for the transmission of power from a central station for working small motors. It is distributed in tubes a foot in diameter under a pressure of 6 atmospheres, and in this way it is possible to work small air motors at a distance of four miles from the source of power with an efficiency of 50 per cent.

210. **Uses of the air-pump.**—A great many experiments with the air-pump have been already described. Such are the mercurial rain (13), the fall of bodies in vacuo (76), the bladder (153), the bursting of a bladder (159), the Magdeburg hemispheres (160), and the baroscope (195).

The fountain in vacuo (fig. 201) is an experiment made with the air-pump, and shows the elastic force of the air. It consists of a glass vessel, A, provided at the bottom with a stopcock, and a tubulure which projects into the interior. Having screwed this apparatus to the air-pump, it is exhausted, and, the stopcock being closed, it is placed in a vessel of water, R. Opening then the stopcock, the atmospheric pressure upon the water in the vessel

makes it jet through the tubulure into the interior of the vessel, as shown in the drawing.

Fig. 202 represents an experiment illustrating the effect of atmospheric pressure on the human body. A glass vessel, open at both ends, being placed on the plate of the machine, the upper end of the cylinder is closed by the hand, and a vacuum is made. The hand then becomes pressed by the weight of the atmosphere, and can only be taken away by a great effort. And as the elasticity of the fluids contained in the organs is not counterbalanced by the weight of the atmosphere, the palm of the hand swells, and blood tends to escape from the pores.

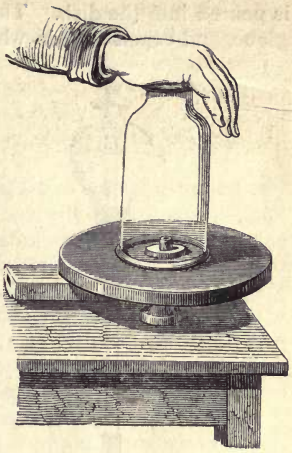


Fig. 202.

By means of the air-pump it may be shown that air, by reason of the oxygen it contains, is necessary for the support of combustion and of life. For we place a lighted taper under the receiver, and begin to exhaust the air, the flame becomes weaker as rarefaction proceeds and is finally extinguished. Similarly, an animal faints and dies if a vacuum is formed in a receiver under which it is placed. Mammalia and birds soon die in vacuo. Fish and reptiles support the loss of air for a much longer time. Insects can live several days in vacuo.

Substances liable to ferment may be kept in vacuo for a long time without alteration, as they are not in contact with oxygen, which is necessary for fermentation. Food kept in air-tight cases, from which the air had been exhausted, has been found as fresh after years as on the first day.

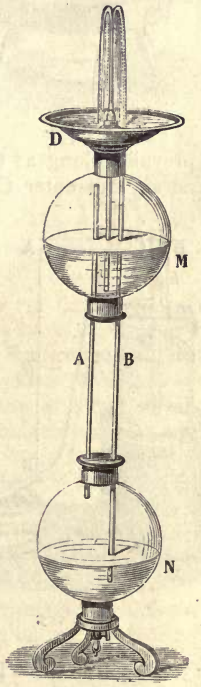


Fig. 203.

211. **Hero's fountain.**—Hero's fountain, which derives its name from its inventor, Hero, who lived at Alexandria, 120 B.C., depends on the elasticity of the air. It consists of a brass dish, D (fig. 202), and of two glass globes, M and N. The dish communicates with the lower part of the globe N by a long tube, B ; and another tube, A, connects the two globes. A third tube passes through the dish D to the lower part of the globe M. This tube having been taken out, the globe M is partially filled with water ; the tube is then replaced and water

is poured into the dish. The water flows through the tube B into the lower globe, and expels the air, which is forced into the upper globe ; the air, thus compressed, acts upon water, and makes it jet out as represented in the figure. If it were not for the resistance of the atmosphere and friction, the liquid would rise to a height above the water in the dish equal to the difference of the level in the two globes.

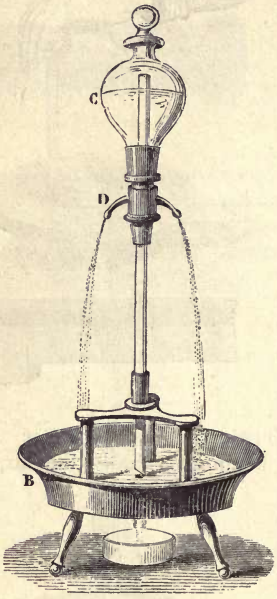


Fig. 204.

212. **Intermittent fountain.**—The *intermittent fountain* consists of a stoppered glass globe (C, fig. 204), provided with two or three capillary tubulures, D. A glass tube open at both ends reaches at one end to the upper part of the globe C ; the other end terminates just above a little aperture in the dish B which supports the whole apparatus.

The water with which the globe C is nearly two-thirds filled runs out, by the tubes D, as shown in the figure, the internal pressure at D being equal to the atmospheric pressure together with the weight of the column of water CD, while the external pressure at that point is only that of the atmosphere. These conditions prevail so long as the lower end of the glass tube is open ; that is, so long as air can enter C and keep the air in C at the same density as the external air ; but the apparatus is arranged so that the orifice in the dish B does not allow so much water to flow out as it receives from the tubes D, in consequence of which the level gradually rises in the dish, and closes the lower end of the glass tube. As the external air cannot now enter the globe C, the air becomes rarefied in proportion as the flow continues, until the pressure of the column of water CD, together with that of the air contained in the globe, is equal to this external pressure at D ; the flow consequently stops. But as water continues to flow out of the dish B, the tubes D become open again, air enters, and the flow recommences, and so on, as long as there is water in the globe C.

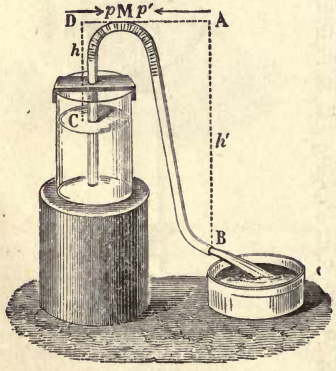


Fig. 205.

213. **The siphon.**—The siphon is a bent tube open at both ends, and with unequal legs (fig. 205.) It is used in transferring liquids in the following

manner :—The siphon is filled with some liquid, and, the two ends being closed, the shorter leg is dipped in the liquid, as represented in fig. 205 ; or, the shorter leg having been dipped in the liquid, the air is exhausted by applying the mouth at B. A vacuum is thus produced ; the liquid in C rises and fills the tube in consequence of the atmospheric pressure. It will then run out through the siphon as long as the shorter end dips in the liquid.

To explain this flow of water from the siphon, let us suppose it filled and the short leg immersed in the liquid. The pressure then acting on C, and tending to raise the liquid in the tube, is the atmospheric pressure minus the height of the column of liquid DC. In like manner, the pressure on the end of the tube B is the weight of the atmosphere less the pressure of the column of liquid AB. But as this latter column is longer than CD, the force acting at B is less than the force acting at C, and consequently a flow takes place proportional to the difference between these two forces. The flow will therefore be more rapid in proportion as the difference of level between the aperture B and the surface of the liquid in C is greater.

It follows from the theory of the siphon that it would not work in vacuo, nor if the height CD were greater than that of a column of liquid which counterbalances the atmospheric pressure.

214. **The intermittent siphon.**—In the *intermittent* siphon the flow is not continuous. It is arranged in a vessel, so that the shorter leg is near the bottom of the vessel, while the longer leg passes through it (fig. 206). Being fed by a constant supply of water, the level gradually rises both in the vessel and in the tube to the top of the siphon, which it fills, and water begins to flow out. But the apparatus is arranged so that the flow of the siphon is more rapid than that of the tube which supplies the vessel, and consequently the level sinks in the vessel until the shorter branch no longer dips in the liquid ; the siphon is then empty, and the flow ceases. But as the vessel is continually fed from the same source the level again rises, and the same series of phenomena is reproduced.

Fig. 206.

The theory of the intermittent siphon explains the natural intermittent springs which are found in many countries, and of which there is an excellent example near Giggleswick in Yorkshire. Many of these springs furnish water for several days or months, and then, after stopping for a certain interval, again recommence. In others the flow stops and recommences several times in an hour.

These phenomena are explained by assuming that there are subterranean fountains, which are more or less slowly filled by springs, and which are then emptied by fissures so occurring in the ground as to form an intermittent siphon.

215. **Different kinds of pumps.**—*Pumps* are machines which serve to raise water either by suction, by pressure, or by both efforts combined ; they are consequently divided into *suction or lift pumps*, *force pumps*, and *suction and forcing pumps.*

The various parts entering into the construction of a pump are the barrel, the piston, the valves, and the pipes. The *barrel* is a cylinder of metal or of wood, in which is the *piston*. The latter is a metal or wooden cylinder wrapped with tow, and working with gentle friction the whole length of the barrel.

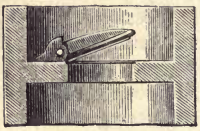


Fig. 207.

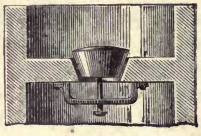


Fig. 208.

The valves are discs of metal or leather, which alternately close the apertures which connect the barrel with the pipes. The most usual valves are the *clack valve* (fig. 207) and the *conical valve* (fig. 208). The former is a metal disc fixed to a hinge on the edge of the orifice to be closed. In order more effectually to close it, the lower part of the disc is covered with thick leather. Sometimes the valve consists merely of a leather disc, of larger diameter than the orifice, nailed on the edge of the orifice. Its flexibility enables it to act as a hinge.

The conical valve consists of a metal cone fitting in an aperture of the same shape. Below this is an iron hoop, through which passes a bolt-head fixed to the valve. The object of this is to limit the play of the valve when it is raised by the water, and to prevent its removal.

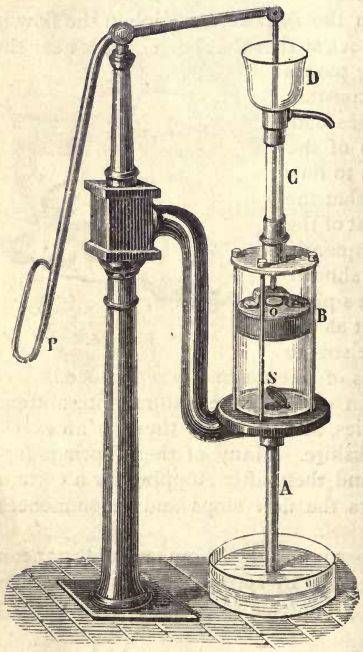


Fig. 209.

216. **Suction-pump.**—Fig. 209 represents a model of a suction-pump such as is used in lectures, but which has essentially the same arrangement as the pumps in common use. It consists, 1st, of a *glass cylinder*, B, at the bottom of which is a valve, S, opening upwards; 2nd, of a *suction-tube*, A, which dips into the reservoir from which water is to be raised; 3rd, of a *piston*, which is moved up and down by a rod worked by a handle, P. The piston is perforated by a hole; the upper aperture is closed by a valve, O, opening upwards.

When the piston rises from the bottom of the cylinder B, a vacuum is produced below, and the valve O is kept closed by the atmospheric pressure, while the air in the pipe A, in consequence of its elasticity, raises the valve S, and partially passes into the cylinder. The air being thus rarefied, water rises in the pipe until the pressure of the liquid column, together with the pressure of the rarefied air

which remains in the tube, counterbalances the pressure of the atmosphere on the water of the reservoir.

When the piston descends, the valve S closes by its own weight, and prevents the return of the air from the cylinder into the tube A. The air compressed by the piston opens the valve O, and escapes into the atmosphere by the pipe C. With a second stroke of the piston the same series of phenomena is produced, and after a few strokes the water reaches the cylinder. The effect is now somewhat modified; during the descent of the piston the valve S closes, and the water raises the valve O, and passes above the piston by which it is lifted into the upper reservoir D. There is now no more air in the pump, and the water forced by the atmospheric pressure rises with the piston, provided that when it is at the summit of its course it is not more than 34 feet above the level of the water in which the tube A dips, for we have seen (163) that a column of water of this height is equal to the pressure of the atmosphere.

In practice the height of the tube A does not exceed 26 to 28 feet; for although the atmospheric pressure can support a higher column, the vacuum produced in the barrel is not perfect, owing to the fact that the piston does not fit exactly on the bottom of the barrel. But when the water has passed the piston, it is the ascending force of the latter which raises it, and the height to which it can be brought depends on the power which works the piston.

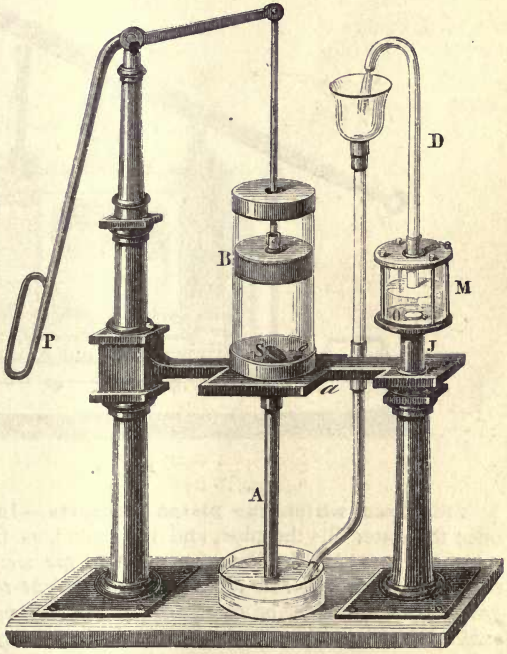


Fig. 210.

217. **Suction and force pump.**—The action of this pump, a model of which is represented in fig. 210, depends both on exhaustion and on pressure. At the base of the barrel, where it is connected with the tube A, there is a valve, S, which opens upwards. Another valve, O, opening in the same direction, closes the aperture of a conduit, which passes from a hole, *o*, near the valve S, into a vessel, M, which is called the *air-chamber*. From this chamber there is another tube, D, up which the water is forced.

At each ascent of the piston B, which is solid, the water rises through the tube A into the barrel. When the piston sinks the valve S closes, and the

water is forced through the valve O into the reservoir M, and thence into the tube D. The height to which it can be raised in this tube depends solely on the motive force which works the pump.

If the tube D were a prolongation of the tube J*ao*, the flow would be intermittent; it would take place when the piston descended, and would cease as soon as it ascended. But between these tubes there is an interval, which, by means of the air in the reservoir M, ensures a continuous flow. The water forced into the reservoir M divides into two parts, one of which, rising in D, presses on the water in the reservoir by its weight; while the other, in virtue of this pressure, rises in the reservoir above the lower orifice of the tube D, compressing the air above. Consequently, when the piston ascends, and no longer forces the water into M, the air of the reservoir, by the pressure it has received, reacts on the liquid, and raises it in the tube D, until the piston again descends, so that the jet is continuous.

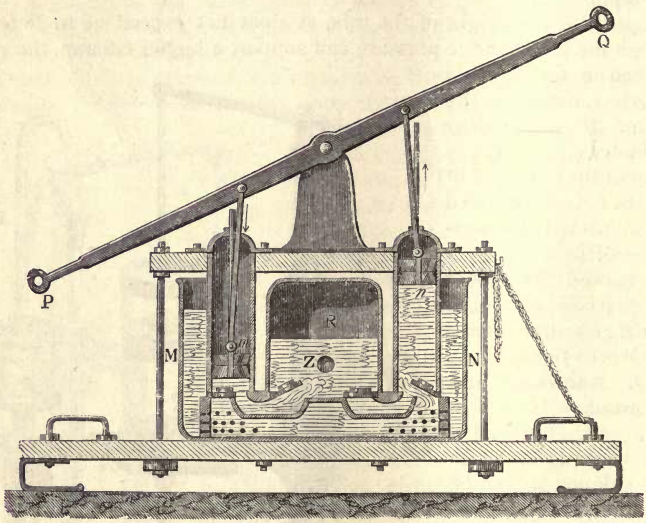


Fig. 211.

218. **Load which the piston supports.**—In the suction-pump, when once the water fills the pipe, and the barrel, as far as the spout, *the effort necessary to raise the piston is equal to the weight of a column of water the base of which is this piston, and the height the vertical distance in the spout from the level of the water in the reservoir; that is, the height to which the water is raised.* For if H is the atmospheric pressure, h the height of the water above the piston, and h' the height of the column which fills the suction tube A (fig. 210), and the lower part of the barrel, the pressure above the piston is obviously $H + h$, and that below is $H - h'$, since the weight of the column h' tends to counterbalance the atmospheric pressure. But as the pressure $H - h'$ tends to raise the piston, the effective resistance is equal to the excess of $H + h$ over $H - h'$, that is to say, to $h + h'$.

In the suction and force pump it is readily seen that the pressure which the piston supports is also equal to the weight of a column of water the base of which is the section of the piston, and the height that to which the water is raised.

219. **Fire-engine.**—The *fire-engine* is a force-pump in which a steady jet is obtained by the aid of an air-chamber, and also by two pumps working alternately (fig. 211). The two pumps *m* and *n*, worked by the same lever, PQ, are immersed in a tank, which is kept filled with water as long as the pump works. From the arrangement of the valves it will be seen that when one pump, *n*, draws water from the tank, the other, *m*, forces it into the *air-chamber* R ; whence, by an orifice, Z, it passes into the delivery tube, by which it can be sent in any direction.

Without the air-chamber the jet would be intermittent. But as the velocity of the water on entering the reservoir is less than on emerging, the level of the water rises above the orifice Z, compressing the air which fills the reservoir. Hence, whenever the piston stops, the air thus compressed, reacting on the liquid, forces it out during its momentary stoppage, and thus keeps up a constant flow.

BOOK V

ON SOUND

CHAPTER I

PRODUCTION, PROPAGATION, AND REFLECTION OF SOUND

220. **Province of acoustics.**—The study of sounds and that of the vibrations of elastic bodies forms the province of the science of *sounds*, or *acoustics*.

Music considers sounds with reference to the pleasurable feeling they are calculated to excite. Acoustics is concerned with the questions of the production, transmission, and comparison of sounds ; to which may be added the physiological question of the perception of sounds.

221. **Sound and noise.**—*Sound* is the peculiar sensation excited in the organ of hearing by the vibratory motion of bodies, when this motion is transmitted to the ear through an elastic medium.

Sounds are distinguished from *noises*. Sound properly so called, or *musical sound*, is that which produces a continuous sensation, and the musical value of which can be estimated ; while noise is either a sound of too short a duration to be determined, like the report of a cannon ; or else it is a confused mixture of many discordant sounds, like the rolling of thunder or the noise of the waves. Nevertheless, the difference between sound and noise is by no means precise ; Savart showed that there are relations of height in the case of noise, as well as in that of sound ; and there are said to be certain ears sufficiently well organised to determine the musical value of the sound produced by a carriage rolling on the pavement.

222. **Cause of sound.**—Sound is always the result of rapid oscillations imparted to the molecules of elastic bodies, when the state of equilibrium of these bodies has been disturbed either by a shock or by friction. Such bodies tend to regain their first position of equilibrium, but only reach it after performing, on each side of that position, very rapid vibratory movements, the amplitude of which quickly decreases. A body which produces a sound is called a *sonorous* or sounding body.

As understood in England and Germany, a vibration comprises a motion to *and* fro ; in France, on the contrary, a vibration means a movement to *or*

fro. The French vibrations are with us semi-vibrations, an *oscillation* or *vibration* is the movement of the vibrating molecule in only one direction; a *double* or *complete vibration* comprises the oscillation both backwards and forwards. Vibrations of sounding bodies are very readily observed. If a light powder is sprinkled on a body which is in the act of yielding a musical sound, a rapid motion is imparted to the powder, which renders visible the vibrations of the body; and, in the same manner, if a stretched cord be smartly pulled and let go, its vibrations are apparent to the eye.

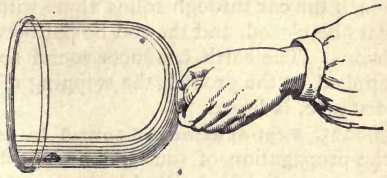


Fig. 212.

A bell-jar is held horizontally in one hand (fig. 212), and made to vibrate by being struck with the other; if then a piece of metal is placed in it, it is rapidly raised by the vibrations of the side; touching the bell-jar with the hand, the sound ceases, and with it the motion of the metal.

223. **Sounds not propagated in vacuo.**—The vibrations of elastic bodies can only produce the sensation of sound in us by the intervention of a medium interposed between the air and the sonorous body and vibrating with it. This medium is usually the air; but all gases, vapours, liquids and solids, also transmit sounds.

The following experiment shows that the presence of a ponderable medium is necessary for the propagation of sound. A small metal bell, which is continually struck by a small hammer by means of clockwork, or else an ordinary musical box, is placed under the receiver of an air-pump (fig. 213). So long as the receiver is full of air at the ordinary pressure the sound is transmitted; but in proportion as the air is exhausted the sound becomes feebler, and cannot be heard in a vacuum.

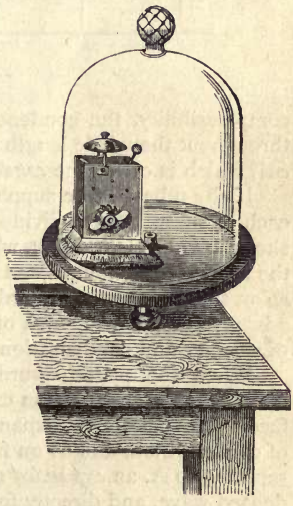


Fig. 213.

To ensure the success of the experiment, the bellwork or the musical box must be placed on wadding; for otherwise the vibrations would be transmitted to the air through the plate of the pump.

224. **Sound is propagated in all elastic bodies.**—If, in the above experiment, any vapour or gas be admitted after the vacuum has been made, the sound of the bell will be heard, showing that sound is propagated in this medium as in air.

Sound is also propagated in liquids. When two stones are struck against each other under water, the shock is distinctly heard; and a diver at the bottom of the water can hear the sound of voices on the bank. The sound

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is, however, enfeebled, as a considerable portion is reflected at the boundary of the two media.

The conductibility of solids is such that the faint scratching of a pen or the ticking of a watch at one end of a long horizontal wooden rod is heard much more distinctly when the ear is directly applied against the other end of the rod than when it is at the same distance in the air. Sound may even reach the ear through solids alone without passing through the air; for if the ears be closed, and the rod be put between the teeth, the ticking is distinctly heard. The earth conducts sound so well that at night, when the ear is applied to the ground, the stepping of horses, or any other noise at a great distance, is heard.

225. **Propagation of sound in air.**—In order to simplify the theory of the propagation of sound in air, we shall first consider the case in which it is propagated in a cylindrical tube of indefinite length. Let MN (fig. 214) be a tube filled with air at a constant pressure and temperature, and let P be a piston oscillating rapidly from A to *a*. When the piston passes from A to *a* it compresses the air in the tube. But in consequence of the great

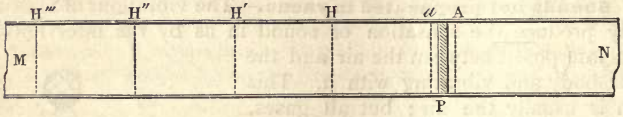


Fig. 214.

compressibility, the condensation of the air does not take place at once throughout the whole length of the tube, but solely within a certain length, *a*H, which is called the *condensed wave*.

If the tube MN be supposed to be divided into lengths equal to *a*H, and each of these lengths divided into layers parallel to the piston, it may be shown by calculation that, when the first layer of the wave *a*H comes to rest, the motion is communicated to the first layer of the second wave HH′, and so on from layer to layer in all parts of H′H″,H″H‴. The condensed wave advances in the tube, each of its parts having successively the same degree of velocity and condensation.

When the piston returns in the direction *a*A, a vacuum is produced behind it, which causes an expansion of the air in contact with its posterior face. The next layer, expanding in turn, brings the first to its original state of condensation, and so on from layer to layer. Thus when the piston has returned to A, an *expanded wave* is produced of the same length as the condensed wave, and directly following it in the tube where they are propagated together, the corresponding layers of the two waves possessing equal and contrary velocities.

The whole of a condensed and expanded wave forms an *undulation*; that is, an undulation comprehends that part of the column of air affected during the backward and forward motion of the piston. The *length of an undulation* is the space which sound traverses during a complete vibration of the body which produces it. This length is less in proportion as the vibrations are more rapid.

It is important to remark that if we consider a single row of particles, which when at rest occupy a line parallel to the axis of the cylinder—for instance, those along AH″ (fig. 214)—we shall find they will have respectively at the same instant all the various velocities which the piston has had successively while oscillating from A to *a* and back to A. So that if in fig. 39 AH′ represents the length of one undulation, the curved line H′PQA will represent the various velocities which all the points in the line AH′ have *simultaneously* ; for instance, at the instant the piston has returned to A, the particle at M will be moving to the right with a velocity represented by QM ; the particle at N will be moving to the left with a velocity represented by PN, and so on of the other particles.

When an undulatory motion is transmitted through a medium, the motions of any two particles are said to be in the *same phase* when those particles move with equal velocities in the same direction ; the motions are said to be in *opposite phases* when the particles move with the same velocities in opposite directions. It is plain from an inspection of fig. 39 that when any two particles are separated by a distance equal to half an undulation, their motions are always in opposite phases, but if their distance equals the length of a complete undulation their motions are in the same phase. A little consideration will show that in the *condensed wave* the condensation will be greatest at the middle of the wave, and likewise that the *expanded wave* will be most rarefied at its middle.

It is an easy transition from the explanation of the motion of sound-waves in a cylinder to that of their motion in an unenclosed medium. It is simply necessary to apply in all directions to each molecule of the vibrating body what has been said about a piston movable in a tube. A series of spherical waves alternately condensed and rarefied is produced around each centre of disturbance. As these waves are contained within two concentrical spherical surfaces, whose radii gradually increase, while the length of the undulation remains the same, their mass increases with the distance from the centre of disturbance, so that the amplitude of the vibration of the molecules gradually lessens, and the intensity of the sound diminishes.

It is these spherical waves, alternately condensed and expanded, which in being propagated transmit sound. If many points are disturbed at the same time, a system of waves is produced around each point. But all these waves are transmitted one through the other without modifying either their lengths or their velocities. Sometimes condensed or expanded waves coincide with others of the same nature to produce an effect equal to their sum ; sometimes they meet and produce an effect equal to their difference. If the surface of still water is disturbed at two or more points, the co-existence of waves becomes sensible to the eye.

226. **Causes which influence the intensity of sound.**—Many causes modify the force or the *intensity* of sound. These are the distance of the sounding body, the amplitude of the vibrations, the density of the air at the place where the sound is produced, the direction of the currents of air, and, lastly, the neighbourhood of other sounding bodies.

i. *The intensity of sound is inversely as the square of the distance of the sonorous body from the ear.* This law has been deduced by calculation, but it may be also demonstrated experimentally. Let us suppose several sounds

of equal intensity—for instance, bells of the same kind, struck by hammers of the same weight, falling from equal heights. If four of these bells are placed at a distance of 20 yards from the ear, and one at a distance of 10 yards, it is found that the single bell produces a sound of the same intensity as the four bells struck simultaneously. Consequently, for double the distance the intensity of the sound is only one-fourth. A method of comparing the intensities of different sounds will be described afterwards (289).

The distance at which sounds can be heard depends on their intensity. The report of a volcano at St. Vincent was heard at Demerara, 300 miles off, and the firing at Waterloo was heard at Dover.

ii. *The intensity of the sound increases with the amplitude of the vibrations of the sonorous body.* The connection between the intensity of the sound and the amplitude of the vibrations is readily observed by means of vibrating cords. For, if the cords are somewhat long, the oscillations are perceptible to the eye, and it is seen that the sound is feebler in proportion as the amplitude of the oscillations decreases.

iii. *The intensity of sound depends on the density of the air in the place in which it is produced.* As we have already seen (222), when an alarum moved by clockwork is placed under the bell-jar of an air-pump, the sound becomes weaker in proportion as the air is rarefied.

In hydrogen, which is about $\frac{1}{14}$ the density of air, sounds are much feebler, although the pressure is the same. In carbonic acid, on the contrary, whose density is 1·529, sounds are more intense. On high mountains, where the air is much rarefied, it is necessary to speak with some effort in order to be heard, and the discharge of a gun produces only a feeble sound. The ticking of a watch is heard in water at a distance of 23 feet, in oil of 16½, in alcohol of 13, and in air of only 10 feet.

iv. *The intensity of sound is modified by the motion of the atmosphere and the direction of the wind.* In calm weather sound is always better propagated than when there is wind ; in the latter case, for an equal distance sound is more intense in the direction of the wind than in the contrary direction.

v. Lastly, *sound is strengthened by the neighbourhood of a sonorous body.* A string made to vibrate in free air has but a very feeble sound ; but when it vibrates above a sounding-box, as in the case of the violin, guitar, or violoncello, its sound is much stronger. This arises from the fact that the box and the air which it contains vibrate in unison with the string. Hence the use of sounding-boxes in stringed instruments.

Attempts have been made to get a measure of the loudness of sound which should serve as a standard, by allowing leaden pellets to fall from various heights on an iron plate of some size. It appears that within certain limits the loudness is nearly proportional to the square root of the height from which the pellet falls, and not to the height itself. It thus appears that only a portion of the energy of the falling body is expended in producing vibrations of the plate.

227. **Apparatus to strengthen sound.**—The apparatus represented in fig. 214 was used by Savart to show the influence of boxes in strengthening sound. It consists of a hemispherical brass vessel, A, which is set in vibration by means of a violin bow. Near it there is a hollow cardboard cylinder

B, closed at the further end. By means of a handle this cylinder can be turned on its support, so as to be inclined at any given degree towards the vessel. The cylinder is fixed on a slide, C, by which means it can be placed at any distance from A. When the vessel is made to vibrate, the strengthening of the sound is very remarkable. But the sound loses almost all its intensity if the cylinder is turned away, and it becomes gradually weaker when the cylinder is removed to a greater distance, showing that the strengthening is due to the vibration of the air in the cylinder.

Fig. 215.

The cylinder B is made to vibrate in unison with the brass vessel by adjusting it to a certain depth, which is effected by making one part slide into the other.

Vitruvius states that in the theatres of the ancients resonant brass vessels were placed to strengthen the voices of the actors.

228. **Influence of tubes on the transmission of sound.**—The law that the intensity of sound decreases in proportion to the square of the distance does not apply to the case of tubes, especially if they are straight and cylindrical. The sound-waves in that case are not propagated in the form of increasing concentrical spheres, and sound can be transmitted to a great distance without any perceptible alteration. Biot found that in one of the Paris water-pipes, 1,040 yards long, the voice lost so little of its intensity that a conversation could be kept up at the ends of a tube in a very low tone. The weakening of sound becomes, however, perceptible in tubes of large diameter, or where the sides are rough. This property of transmitting sounds was first used in England for *speaking tubes*. They consist of caoutchouc or metal tubes of small diameter passing from one room to another. If a person speaks at one end of the tube, he is distinctly heard by a person with his ear at the other end.

From Biot's experiments it is evident that a communication might be made between two towns by means of speaking tubes. The velocity of sound is 1,125 feet in a second at 16°·6 C., so that a distance of 50 miles would be traversed in four minutes.

229. **Regnault's experiments.**—Theoretically, a sound-wave should be propagated in a straight cylindrical tube with a constant intensity. Regnault found, however, that in these circumstances the intensity of sound gradually diminishes with the distance, and that the distance at which it ceases to be audible is nearly proportional to the diameter of the tube.

He reproduced sound-waves of equal strength by means of a small pistol charged with a gramme of powder, and fired at the open ends of tubes of various diameters ; and he then ascertained the distance at which the sound could no longer be heard, or at which it ceased to act on what he calls a *sensitive membrane*. This was a very flexible membrane which could be fixed across the tube at various distances, and was provided with a small metal disc in its centre. When the membrane began to vibrate, this disc struck against a metallic contact, and thereby closed a voltaic circuit, which traced on a chronograph the exact moment at which the membrane received the sound-wave.

Experimenting in this manner, Regnault found that the report of a pistol charged as stated is no longer audible at a distance of

1,159 metres	in a tube of	. . . . .	$0^m{\cdot}108$ diameter
3,810 „	„	. . . . .	$0^m{\cdot}300$ „
9,540 „	„	. . . . .	$1^m{\cdot}100$ „

These numbers represent the limit of distance at which the sound-wave is no longer heard, but it still acts on the membrane at the distances of 4,156, 11,430, and 19,851 metres respectively.

According to Regnault the principal cause of this diminution of intensity is the loss of *vis viva* against the sides of the tube ; he found also that sounds of high pitch are propagated in tubes less easily than those of low ones : a bass would be heard at a greater distance than a treble voice.

230. **Velocity of sound in air.**—Since the propagation of sound-waves is gradual, sound requires a certain time for its transmission from one place to another, as is seen in numerous phenomena. For example, the sound of thunder is only heard some time after the flash of lightning has been seen, although both the sound and the light are produced simultaneously; and in like manner we see a mason in the act of striking a stone before hearing the sound.

The velocity of sound in air has often been the subject of experimental determination. The most accurate of the direct measurements was made by Moll and Van Beck in 1823. Two hills, near Amsterdam, Kooltjesberg and Zevenboomen, were chosen as stations : their distance from each other as determined trigonometrically was 57,971 feet, or nearly eleven miles. Cannons were fired at stated intervals simultaneously at each station, and the time which elapsed between seeing the flash and hearing the sound was noted by chronometers. This time could be taken as that which the sound required to travel between the two stations ; for it will be subsequently seen that light takes an inappreciable time to traverse the above distance. Introducing corrections for the barometric pressure, temperature, and hygrometric state, and eliminating the influence of the wind, Moll and Van Beck's results as recalculated by Schröder van der Kolk gave 1,092·78 feet as the velocity of sound in one second in dry air at 0° C. and under a pressure of 760 mm. Kendall, in a North Pole expedition, found that the velocity of sound at a temperature of −40° was 314 metres. Stone's determinations, made at the Cape of Good Hope with very great care, gave 1090·57 or 332·4 metres, as the velocity of sound at 0°.

The velocity of sound at zero may be taken at 1,093 feet, or 333 metres.

This velocity increases with the increase of temperature; it may be calculated for a temperature $t°$ from the formula

$$v = 1{,}093\sqrt{(1 + 0{\cdot}003665t)}$$

where 1,093 is the velocity in feet at 0° C., and 0·003665 the coefficient of expansion for 1° C. This amounts to an increase of nearly two feet for every degree Centigrade. For the same temperature it is independent of the density of the air, and consequently of the pressure. It is the same for the same temperature with all sounds, whether they be strong or weak, deep or acute. Biot found, in his experiments on the conductivity of sound in tubes, that when a well-known air was played on a flute at one end of a tube 1,040 yards long, it was heard without alteration at the other end, from which he concluded that the velocity of different sounds is the same. For the same reason the tune played by a band is heard at a great distance without alteration, except in intensity, which could not be the case if some sounds travelled more rapidly than others.

This cannot, however, be admitted as universally true. Earnshaw, by a mathematical investigation of the laws of the propagation of sound, concludes that the velocity of a sound depends on its strength; and, accordingly, that a violent sound ought to be propagated with greater velocity than a gentler one. This conclusion is confirmed by an observation made by Captain Parry on his Arctic expedition. During artillery practice it was found, by persons stationed at a considerable distance from the guns, that the report of the cannon was heard before the command to fire given by the officer. And, more recently, Mallet made a series of experiments on the velocity with which sound is propagated in rocks, by observing the times which elapsed before blastings, made at Holyhead, were heard at a distance. He found that the larger the charge of gunpowder, and therefore the louder the report, the more rapid was the transmission. With a charge of 2,000 pounds of gunpowder the velocity was 967 feet in a second, while with a charge of 12,000 it was 1,210 feet in the same time.

Jacques made a series of experiments by firing different weights of powder from a cannon, and observing the velocity of the report at different distances from the gun by means of an electrical arrangement. He thus found that, nearest the gun, the velocity is least, increasing to a certain maximum which is considerably greater than the average velocity. The velocity is also greater with the heavier charge. Thus with a charge of 1½ pound the velocity was 1,187, and with a charge of ½ pound it was 1,032 at a distance of from 30 to 50 feet; while at a distance of 70 to 80 it was 1,267 and 1,120: and at 90 to 100 feet it was 1,262 and 1,114 respectively.

Bravais and Martins found, in 1844, that sound travelled with the same velocity from the base to the summit of the Faulhorn as from the summit to the base.

A laboratory method of determining the velocity of sounds consists in using a metronome (81) which is beating slowly, and is approached to a wall until a position is found at which the echo of one beat coincides with the sound of another heard directly. The distance from the wall is then half the distance, which sound traverses in the interval between two beats of the metronome.

231. **Calculation of the velocity of sound in gases.**—From theoretical considerations Newton gave a rule for calculating the velocity of sound in gases, which may be represented by the formula

$$v = \sqrt{\frac{e}{d}}$$

in which v represents the velocity of the sound, or the distance it travels in a second, e the elasticity of the gas, and d its density.

This formula expresses that the velocity of the propagation of sound in *gases is directly as the square root of the elasticity of the gas, and inversely as the square root of its density.* It follows that the velocity of sound is the same under any pressure; for although the elasticity increases with increased pressure, according to Boyle's law, the density increases in the same ratio. At Quito, where the mean pressure is only 21·8 inches, the velocity is the same as at the sea-level, provided the temperature is the same.

Now the measure of the elasticity of a gas is the pressure to which it is subjected; hence, if g be the force of gravity, h the barometric height reduced to the temperature zero, and δ the density of mercury, also at zero, then for a gas under the standard atmospheric pressure and for zero, $e = gh\delta$; Newton's formula accordingly becomes

$$v = \sqrt{\frac{gh\delta}{d}}.$$

Now, if we suppose the temperature of a gas to increase from 0° to $t°$, its volume will increase from unity at zero to $1 + at$ at t, a being the coefficient of expansion of the gas. But the density varies inversely as the volume, therefore d becomes $d \div (1 + at)$. Hence

$$v = \sqrt{\frac{gh\delta}{d}(1 + at)}.$$

Substituting in this formula the values in centimetres and grammes, $g = 981$, $h = 76$, $d = 0{\cdot}001293$, we get for the value v a number 29,795 centimetres = 297·95 metres, which is about one-sixth less than the experimental result. Laplace assigned as a reason for this discrepancy the heat produced by pressure in the condensed waves; and, by considerations based on this idea, Poisson and Biot found that Newton's formula ought to be written $v = \sqrt{\frac{gh\delta}{d}(1 + at)\frac{c}{c'}}$; c being the specific heat of the gas for a constant pressure, and c' its specific heat for a constant volume (460). The average value of the constant $\frac{c}{c'}$ is 1·41, and if the formula be modified by the introduction of the value $\sqrt{1{\cdot}41}$ or 1·1875, the calculated numbers agree with the experimental results.

The physical reason for introducing the constant $\sqrt{\frac{c}{c'}}$ into the equation for the velocity of sound may be understood from the following considerations:—We have already seen (225) that sound is propagated in air by a series of alternate condensations and rarefactions of the layers. At each condensation heat is evolved, and this heat increases the elasticity, and thus the rapidity with which each condensed layer acts on the next; but in the

rarefaction of each layer the same amount of heat disappears as was developed by the condensation, and its elasticity is diminished by the cooling. The effect of this diminished elasticity of the cooled layer is the same as if the elasticity of an adjacent wave had been increased, and the rapidity with which this latter would expand upon the dilated wave would be greater. Thus, while the average temperature of the air is unaltered, both the heating which increases the elasticity and the chilling which diminishes it concur in increasing the velocity.

Knowing the velocity of sound, we can calculate approximately the distance at which it is produced. Light travels with such velocity that the flash or the smoke accompanying the report of a gun may be considered to be seen simultaneously with the occurrence of the explosion. Counting then the number of seconds which elapse between seeing the flash and hearing the sound, and multiplying this number by 1,125, we get the distance in feet at which the gun is discharged. In the same way the distance of thunder may be estimated.

232. **Velocity of sound in various gases.**—Approximately the same results have been obtained for the velocity of sound in air by another method, by which the velocity in other gases could be determined. As the wave-length λ is the distance which sound travels during the time of one oscillation, that is, $\frac{1}{n}$ of a second, the velocity of sound or the distance traversed in a second is $v = n\lambda$. Now the length of an open pipe is half the wave-length of the fundamental note of that pipe; and that of a closed pipe is a quarter of the wave-length (275). Hence, if we know the number of vibrations of the note emitted by any particular pipe, which can be easily ascertained by means of a sirene, and we know the length of this pipe, we can calculate v. Taking the temperature into account, Wertheim found in this way 1,086 feet for the velocity of sound in air at zero.

Further, since in different gases which have the same elasticity, but differ in density, the velocity of sound varies inversely as the square root of the density, knowing the velocity of sound in air, we may calculate it for other gases; thus in hydrogen it will be

$$\frac{1093}{\sqrt{0{\cdot}0688}} = 4168 \text{ feet.}$$

This number cannot be universally accurate, for the coefficient $\frac{c}{c'}$ differs somewhat in different gases. And when pipes were sounded with different gases, and the number of vibrations of the notes multiplied with twice the length of the pipe, numbers were obtained which differed for those calculated by the above formula. When, however, the calculation was made, introducing for each gas its special valve of $\frac{c}{c'}$, the theoretical results agreed very well with the observed ones.

By the above method the following values have been obtained:—

Chlorine	677 feet in a second.
Carbonic acid	856 „

Oxygen	1040 feet in a second.
Air	1093 „
Carbonic oxide	1106 „
Hydrogen	4163 „

233. **Doppler's principle.**—When a sounding body approaches the ear, the note perceived is somewhat higher than the true one; but if the source of sound recedes from the ear, the note perceived is lower. The truth of this, which is known as *Doppler's principle*, will be apparent from the following considerations:—When the source of sound and the ear are at rest, the ear receives n waves in a second; but if the ear approaches the sound, or the sound approaches the ear, it receives more; just as a ship meets more waves when it ploughs through them than if it is at rest. Conversely, the ear receives a smaller number when it recedes from the source of sound. The effect in the first case is as if the sounding body emitted more vibrations in a second than it really does, and in the second case fewer. Hence in the first case the note appears higher; in the second case lower.

If the distance which the ear traverses in a second towards the source of sound (supposed to be stationary) is s feet, and the wave-length of the particular note is λ feet, then there are $\frac{s}{\lambda}$ waves in a second; or also $\frac{ns}{c}$, for $\lambda = \frac{c}{n}$, where c is the velocity of sound (230). Hence the ear receives not only the n original waves, but also $\frac{ns}{c}$ in addition. Therefore the number of vibrations which the ear actually receives is

$$n' = n + \frac{ns}{c} = n\left(1 + \frac{s}{c}\right)$$

for an ear which approaches a note; and by similar reasoning it is

$$n' = n - \frac{ns}{c} = n\left(1 - \frac{s}{c}\right)$$

for an ear receding from a tone.

To test Doppler's theory Buys Ballot stationed trumpeters on the Utrecht railways and also upon locomotives, and had the height of the approaching or receding notes compared with stationary ones by musicians. He thus found both the principle and the formula fully confirmed. Similar conclusive experiments were made by Scott Russell on English railways. The observation may often be made as a fast train passes a station in which an electrical alarum is sounding. Independently of the difference in loudness, an attentive ear can detect a difference in pitch on approaching or on leaving the station. A speed of about 40 miles an hour sharpens the note of the whistle of an approaching train by a semitone, and flattens it to that extent as the train recedes.

Doppler's principle may also be established by direct laboratory experiments. Rollmann fixed a long rod on a turning machine, at the end of which was a large glass bulb with a slit in it, which sounded like a humming-top when a tangential current of air was blown against the slit.

The uniform and sufficiently rapid rotation of the sphere developed such a current, and produced a steady note, the pitch of which was higher or lower in each rotation according as the bulb came nearer, or receded from, the observer.

The principle may also be illustrated by means of a tuning-fork with wide branches, and producing a very high note of 2,046 vibrations. When this is loudly sounded, and, being held in front of a smooth wall, is moved towards it with a velocity of a metre in a second, the direct note and that reflected from the wall undergo opposite changes, so that an observer hears distinctly twelve beats in a second (262).

234. **Velocity of sound in liquids.**—The velocity of sound in water was experimentally determined in 1827 by Colladon and Sturm. They moored two boats at a known distance in the Lake of Geneva. The first supported a bell immersed in water, and a bent lever provided at one end with a hammer which struck the bell, and at the other with a lighted wick, so arranged that it ignited some powder the moment the hammer struck the bell. To the second boat was affixed an ear-trumpet, the bell of which was in water, while the mouth was applied to the ear of the observer, so that he could measure the time between the flash of light and the arrival of sound by the water. By this method the velocity was found to be 4,708 feet in a second at the temperature 8°·1, or four times as great as in air.

The velocity of sound, which is different in different liquids, can be calculated by a formula analogous to that given above (230) as applicable to gases, that is, $v = \sqrt{\frac{g h \delta}{\mu d}}$; in which g, h, and δ have their previous significance; while μ is the coefficient of the compressibility for the liquid in question (97), that is, its diminution in volume by a pressure of one atmosphere—and d is the density. In this way were obtained the numbers given in the following table. As in the case of gases, the velocity varies with the temperature, which is therefore appended in each case.

River water (Seine)	13° C.	= 4714 feet in a second.	
„ „	30	= 5013	„
Artificial sea water	20	= 4761	„
Mercury	10	= 4866	„
Solution of common salt	18	= 5132	„
Absolute alcohol	23	= 3854	„
Turpentine	24	= 3976	„
Ether		= 3801	„

It will be seen how close is the agreement between the two values for the velocity of sound in water, the only case in which they have been directly compared. There is considerable uncertainty about the values for other liquids, owing to the doubt as to the values for their compressibility.

235. **Velocity of sound in solids.**—As a general rule, the elasticity of solids, as compared with the density, is greater than that of liquids, and consequently the propagation of sound is more rapid.

The difference is well seen in an experiment by Biot, who found that when a bell was struck by a hammer, at one end of an iron tube 3,120 feet long,

two sounds were distinctly heard at the other end. The first of these was transmitted by the tube itself with a velocity, x; and the second by the enclosed air with a known velocity, a. The interval between the sounds was 2·5 seconds. The value of x obtained from the equation

$$\frac{3120}{a} - \frac{3120}{x} = 2{\cdot}5$$

shows that the velocity of sound in the tube is nearly 9 times as great as that in air.

That the report of the firing of cannon is heard at far greater distances than peals of thunder, is doubtless owing to the fact that the sound in the former case is mainly transmitted through the earth.

To this class of phenomena belongs the fact that if the ear is held against a rock in which a blasting is being made at a distance, two distinct reports are heard—one transmitted through the rock to the ear, and the other transmitted through the air. The conductivity of sound in solids is also well illustrated by the fact that in manufacturing telegraph wires the filing at any particular part can be heard at distances of miles by placing one end of the wire in the ear. The toy telephone also is based on this fact.

The velocity of sound in wires has also been determined theoretically by Wertheim and others, by the formula $v = \sqrt{\frac{\mu}{d}}$, in which μ is the modulus of elasticity (89), while d is the mass in unit volume, which is equal to the specific gravity, or the weight of unit volume divided by the acceleration of gravity, or $\frac{s}{g}$.

This may be illustrated from a determination by Wertheim of the velocity of sound in a specimen of annealed steel wire, the specific gravity s of which was 7·631 and its modulus 21,000 (88). That is, a weight of 21,000 kilogrammes would double the unit length of a wire 1 sq. mm. in cross section, if this were possible without exceeding the limit of elasticity. This is equal to 2,100,000,000, or 21×10^8 grammes on a wire 1 sq. cm. in cross section. Hence

$$v = \sqrt{\frac{2100000000 \times 981}{7{\cdot}63}} = 519581 \text{ cm.} = 17047 \text{ feet.}$$

The following table gives the velocity in various bodies, expressed in feet per second, mostly from the experimental determinations of Wertheim and of Stefan :—

Caoutchouc	100 to 200	Oak	12622
Tallow	1180	Cedar	13120
Wax	2394	Elm	13516
Paraffin	4250	Ash	15314
Lead	4653	Fir	15316
Gold	7021	Walnut	15744
Silver	8806	Glass	16057
Pine	10900	Steel wire	16336
Copper	12194	Wrought iron and steel	16498

The numbers for caoutchouc are of the same order of magnitude as those for the propagation of a nervous impulse, and suggest that this impulse is transmitted by longitudinal vibrations (281) like those of sound.

In the case of wood these velocities are in the directions of the fibres, and are considerably greater than across the rings or along the rings; thus with fir the velocities are 4,382 and 2,572 for these directions respectively.

From a recent determination of the elasticity of ice, Trowbridge and Macrae have deduced the velocity of sound in it to be 9,600 feet per second, 2,900 metres, or about 9 times that of air.

Mallet investigated the velocity of the transmission of sound in various rocks, and found that it is as follows:—

Wet sand	825	feet in a second.
Contorted, stratified quartz and slate rock .	1088	,,
Discontinuous granite	1306	,,
Solid granite	1664	,,

A direct experimental method of determining the velocity of sound in solids, gases, and vapours will be described subsequently (277).

If a medium through which sound passes is heterogeneous, the waves of sound are reflected on the different surfaces, and the sound becomes rapidly enfeebled. Thus a soft earth conducts sound badly, while a hard ground which forms a contact mass conducts it well. So also we hear badly through air spaces which are filled with porous materials, such as shavings, sawdust, cinders, and the like.

236. **Reflection of sound.**—So long as sound-waves are not obstructed in their motion they are propagated in the form of concentric spheres; but when they meet with an obstacle they follow the general law of elastic bodies; that is, they return upon themselves, forming new concentric waves, which seem to emanate from a second centre on the other side of the obstacle. This phenomenon constitutes the reflection of sound.

Fig. 216 represents a series of incident waves reflected from an obstacle, PQ. Taking for example the incident wave MCDN, emitted from the

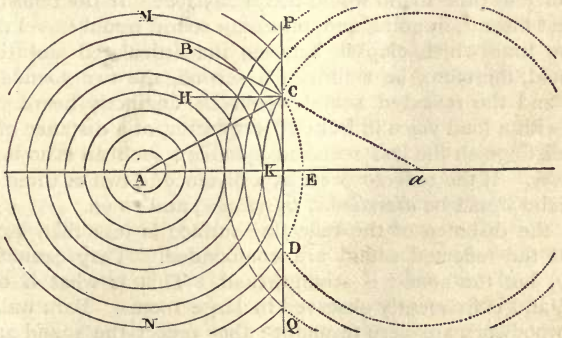


Fig. 216.

centre A, the corresponding reflected wave is represented by the arc CKD, of a circle whose centre *a* is as far behind the obstacle PQ as A is before it.

If any point, C, of the reflecting surface be joined to the centre of sound, and if the perpendicular CH be let fall on the surface of this body, the angle ACH is called the *angle of incidence*, and the angle BCH, formed by the prolongation of *a*C, is the *angle of reflection.*

The reflection of sound is subject to the two following laws :—

I. *The angle of reflection is equal to the angle of incidence.*

II. *The incident sonorous ray and the reflected ray are in the same plane perpendicular to the reflecting surface.*

From these laws it follows that the wave, which in the figure is propagated in the direction AC, takes the direction CB after reflection, so that an observer placed at B hears a second sound, which appears to come from C, besides the sound proceeding from the point A.

The laws of the reflection of sound are the same as those for light and radiant heat, and may be demonstrated by similar experiments. One of the simplest of these is made with conjugate mirrors (see chapter on Radiant Heat) ; if in the focus of one of these mirrors, which should be rather large, a watch is placed, the ear placed in the focus of the second mirror hears the ticking very distinctly even when the mirrors are at a distance of 12 or 13 yards. The mirrors should be large, so that the head does not prevent too large a proportion of the waves from the first mirror from falling on the other. With smaller mirrors the bell of an ear trumpet is held at the focus, and the tube end is placed in the ear, which is held on one side of the mirror.

In like manner, the explosion of fulminating mercury in the focus of one mirror causes that of iodide of nitrogen placed in that of the other.

237. **Echoes and resonances.**—An *echo* is the repetition of a sound in the air, caused by its reflection from some obstacle.

A very sharp quick sound can produce an echo when the reflecting surface is 55 feet distant : but for articulate sounds at least double that distance is necessary, for it may be easily shown that no one can pronounce or hear distinctly more than five syllables in a second. Now, as the velocity of sound at ordinary temperatures may be taken at 1,125 feet in a second, in a fifth of that time sound would travel 225 feet. If the reflecting surface is 112·5 feet distant, in going and returning sound would travel through 225 feet. The time which elapses between the articulated and the reflected sound would, therefore, be a fifth of a second, the two sounds would not interfere, and the reflected sound would be distinctly heard. A person speaking with a loud voice in front of a reflector, at a distance of 112·5 feet, can only distinguish the last reflected syllable : such an echo is said to be *monosyllabic.* If the reflector were at a distance of two or three times 112·5 feet, the echo would be *dissyllabic*, *trisyllabic*, and so on.

When the distance of the reflecting surface is less than 112·5 feet, the direct and the reflected sound are confounded. They cannot be heard separately, but the sound is strengthened. This is what is often called *resonance*, and is frequently observed in large rooms. Bare walls and particularly woodwork are very resonant ; they reflect the sound and add to it the effect of their own vibrations, so that the sound is prolonged and enforced. In a large meeting-room this may considerably aid a speaker's voice ; too great resonance, however, hinders the distinct perception of the

words. Tapestry and hangings, on the contrary, which are bad reflectors, deaden the sound. To control or eliminate the effects of resonance is a difficult problem in the acoustics of the building art.

Multiple echoes are those which repeat the same sound several times ; this is the case when two opposite surfaces (for example, two parallel walls) successively reflect sound. There are echoes which repeat the same sound 20 or 30 times. An echo in the château of Simonetta, in Italy, repeats a sound 30 times. At Woodstock there is one which repeats from 17 to 20 syllables.

As the laws of reflection of sound are the same as those of light and heat, curved surfaces produce *acoustic foci* like the luminous and calorific foci produced by concave reflectors. If a person standing under the arch of a bridge speaks with his face turned towards one of the piers, the sound is reproduced near the other pier with such distinctness that a conversation can be kept up in a low tone, which is not heard by anyone standing in the intermediate spaces.

There is a square room with an elliptical ceiling on the ground floor of the Conservatoire des Arts et Métiers, in Paris, which presents this phenomenon in a remarkable degree to persons standing in the two foci of the ellipse.

Whispering galleries are formed of smooth walls having a continuous curved form. The mouth of the speaker is presented at one point, and the ear of the hearer at another and distant point. In this case, the sound is successively reflected from one point to the other until it reaches the ear.

In the whispering gallery of St. Paul's, the faintest sound is thus conveyed from one side to the other of the dome, but it is not heard at any intermediate points. Placing himself close to the upper wall of the Colosseum, a circular building 130 feet in diameter, Wheatstone found a word to be repeated a great many times. A single exclamation sounded like a peal of laughter, while the tearing of a piece of paper resembled the patter of hail.

It is not merely by solid surfaces, such as walls, rocks, ships' sails, &c., that sound is reflected. It is also reflected by clouds, and it has even been shown by direct experiment that a sound in passing from a gas of one density into another is reflected at the surface of separation as it would be against a solid surface. Now, different parts of the earth's surface are unequally heated by the sun, owing to the shade of trees, evaporation of water, and other causes, so that in the atmosphere there are numerous ascending and descending currents of air of different density. Whenever a sound wave passes from a medium of one density into another it undergoes partial reflection, which, though not strong enough to form an echo, distinctly weakens the direct sound. This is doubtless the reason, as Humboldt remarked, why sound travels further at night than at daytime, even in the South American forests, where the animals, which are silent by day, fill the atmosphere at night with thousands of confused sounds. To this may be added that at night and in repose, when other senses are at rest, that of hearing becomes more acute. This is the case with persons who have become blind.

It has generally been considered that fog in the atmosphere is a great deadener of sound ; it being a mixture of air and globules of water, at each of the innumerable surfaces of contact a portion of the vibration is lost.

The evidence as to the influence of this property is conflicting; recent researches of Tyndall show that a white fog, or snow, or hail, is not an important obstacle to the transmission of sound, but that aqueous vapour is. Experiments made on a large scale, in order to ascertain the best form of fog signals, gave some remarkable results.

On some days, which optically were quite clear, certain sounds could not be heard at a distance far inferior to that at which they could be heard even during a thick haze. Tyndall ascribes this result to the presence in the atmosphere of aqueous vapour, which forms in the air innumerable striæ that do not interfere with its optical clearness, but render it acoustically turbid, the sound being reflected by this invisible vapour just as light is by the visible cloud.

These conclusions, first drawn from observations, have been verified by laboratory experiments. Tyndall has shown that a medium consisting of alternate layers of light and heavy gas, such as coal gas and carbonic acid, deadens sound, and also that a medium consisting of alternate strata of heated and ordinary air exerts a similar influence. The same is the case with an atmosphere containing the vapours of volatile liquids. So long as the continuity of air is preserved, sound has great power of passing through the interstices of solids; thus it will pass through twelve folds of a dry silk handkerchief, but is stopped by a single layer if it is wetted.

238. **Refraction of sound.**—It will be found afterwards (536) that *refraction* is the change of direction which light and heat experience on passing from one medium to another. It has been shown by Hajech that the laws of the refraction of sound are the same as those for light and heat: he used tubes filled with various gases and liquids, and closed by membranes; the membrane at one end was at right angles to the axis of the tube, while the other made an angle with it. When these tubes were placed in an aperture in the wall between two rooms, a sound produced in front of the tube in one room, that of a tuning-fork for instance, was heard in directions in the other varying with the inclination of the second membrane, and with the nature of the substance with which the tube was filled. Accurate measurements showed that the law held that the sines of the angle of incidence and of refraction are in a constant ratio, and that this ratio is equal to that of the velocity of sound in the two media.

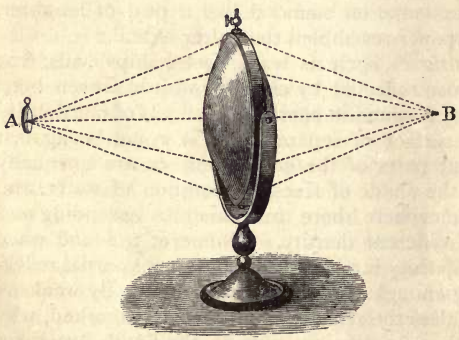


Fig. 217.

Thus the velocity of sound in water is not very different from that in hydrogen, and the deviations produced are nearly equal.

Sondhauss confirmed the analogy of the refraction of sound-waves to those of light and heat. He constructed lenses of gas by cutting equal

segments out of a large collodion balloon, and fastening them on the two sides of a sheet-iron ring a foot in diameter, so as to form a double convex lens about 4 inches thick in the centre (fig. 212). This was filled with carbonic acid, and a watch A was placed in the direction of the axis ; the point was then sought on the other side of the lens at which the sound was most distinctly heard. It was found that when the ear was removed from the axis the sound was scarcely perceptible, but that at a certain point B on the axial line it was very distinctly heard. Consequently, the sound-waves in passing from the lens had converged towards the axis ; their direction had been changed ; in other words, they had been refracted.

The refraction of sound may be easily demonstrated by means of one of the very thin india-rubber balloons used as children's toys, inflated by carbonic acid. If the balloon be filled with hydrogen, no focus is detected ; it acts like a concave lens, and the divergence of the rays is increased, instead of their being converged to the ear.

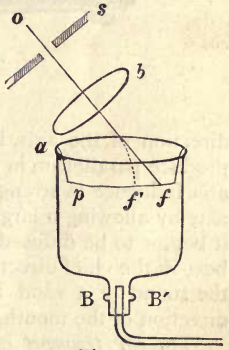


Fig. 218.

A direct proof of the refraction of sound is given by the experiments of Schellbach and Böhm. The source of sound was a film of collodion stretched across a ring, *ab* (fig. 218), and which was put in vibration by electrical sparks at *o*. A disc of paper, *p*, sprinkled with fine charcoal powder, was suspended in the vessel BB′. When this vessel contained air, rings of dust were formed, the centre of which was at *f* in the direction of the propagation of the sound. But if the vessel was filled with carbonic acid the centre of the rings was found to be at *f′*, showing that the sound had been refracted towards the perpendicular on passing from air into the denser medium ; and measurements showed that the position of the point *f′* was in accordance with the law of refraction for light. Experiments suitably modified showed that, when hydrogen was substituted for carbonic acid, the sound was bent away from the perpendicular.

It has long been known that sound is propagated in a direction against that of the wind with less velocity than with the wind. This is probably due to a refraction of sound on a large scale. The velocity of wind along the ground is always considerably less than at a greater height ; thus, the velocity at a height of 8 feet has been observed to be double what it is at a height of one foot above the ground. Hence the front of a condensed wave (fig. 213), which was originally vertical, becomes tilted upwards and with the lower part forward ; and, as the direction of the wave-motion is at right angles to the front of the wave, the effect of the coalescence of a number of these rays, thus directed upwards, is to produce an increase of the sound in the higher regions. The rays which travel with the wind will, for similar reasons, be refracted downwards, and thus the sound be better heard.

239. **Speaking trumpet. Ear trumpet.**—These instruments depend both on the reflection of sound and on its conductibility in tubes.

The *speaking trumpet*, as its name implies, is used to render the voice

audible at great distances, more especially on board ship. It consists of a slightly conical tin or brass tube (fig. 219), very much wider at one end (which is called the *bell*), and provided with a mouthpiece at the other. It is as much as 7 feet in length, the bell being 1 foot in diameter.

The larger the dimensions of this instrument the greater is the distance at which the voice is heard. Its action is usually ascribed to the successive reflections of sound-waves from the sides of the tube, by which the waves tend more and more to pass in a direction parallel to the axis of the instrument. It has, however, been objected to this explanation that the sounds emitted by the speaking trumpet are not stronger solely in the

Fig. 219.

direction of the axis, but in all directions ; that the bell would not tend to produce parallelism in the sound-wave, whereas it certainly exerts considerable influence in strengthening the sound. According to Hassenfratz the bell acts by allowing a large mass of air to be set in consonant vibration before it begins to be diffused. This is probably also the reason why sound travels best in the chief direction of the sounding body ; thus the report of a cannon, the sound of a wind instrument in the line of the tube, the voice in the direction of the mouth, &c.

The *ear trumpet* is used by persons who are hard of hearing. It is essentially an inverted speaking trumpet, and consists of a conical metallic tube, one of whose extremities, terminating in a *bell*, receives the sound, while the other end is introduced into the ear. This instrument is the reverse of the speaking trumpet. The bell serves as a mouthpiece ; that is, it receives the sound coming from the mouth of the person who speaks. These sounds are transmitted by a series of reflections to the interior of the trumpet, so that the waves, which would become greatly diffused, are concentrated on the ear, and produce a far greater effect than divergent waves would have done.

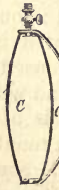


Fig. 220.

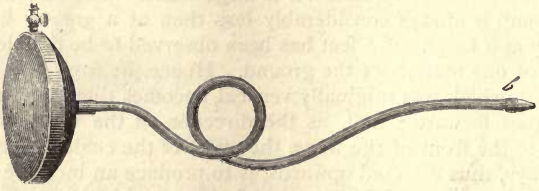


Fig. 221.

240. **Stethoscope.**—One of the most useful applications of acoustical principles is the *stethoscope*. Figs. 220, 221 represent an improved form of this instrument devised by König. Two sheets of caoutchouc, *c* and *a*, are

fixed to the circular edge of a hollow metal hemisphere; the edge is provided with a stopcock, so that the sheets can be inflated, and then present the appearance of a double convex lens, as represented in section in fig. 220. To a tubulure on the hemisphere is fixed a caoutchouc tube terminated by horn or ivory, *b*, which is placed in the ear (fig. 221).

When the membrane *c* of the stethoscope is applied to the chest of a sick person the beating of the heart and the sounds of respiration are transmitted to the air in the chamber *a*, and from thence to the ear by means of the flexible tube. If several tubes are fixed to the instrument, as many observers may simultaneously auscultate the same patient.

CHAPTER II

MEASUREMENT OF THE NUMBER OF VIBRATIONS

241. **Savart's apparatus.**—*Savart's toothed wheel*, so called from the name of its inventor, is an apparatus by which the absolute number of vibrations corresponding to a given note can be determined. It consists of a solid oak frame in which there are two wheels, A and B (fig. 222); the larger wheel, A, is connected with the toothed wheel by means of a strap and a multiplying wheel, thereby causing the toothed wheel to revolve with great velocity; a card, E, is fixed on the frame, and, in revolving, the toothed wheel strikes against it and causes it to vibrate. The card, being struck by each tooth, makes as many vibrations as there are teeth. At the side of the apparatus there is an indicator, H, which gives the number of revolutions of the wheel, and consequently the number of vibrations in a given time.

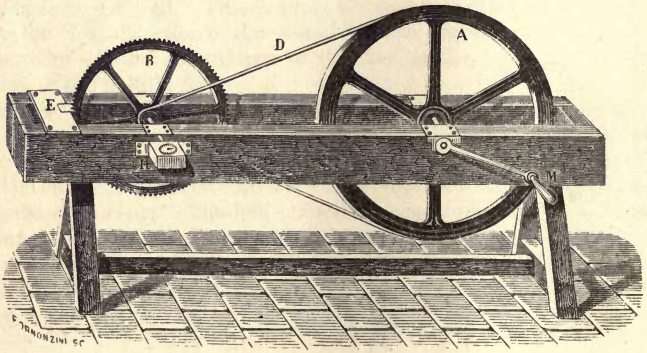


Fig. 222.

When the wheel is moved slowly, the separate shocks against the card are distinctly heard; but if the velocity is gradually increased, the sound becomes higher and higher. Having obtained the sound whose number of vibrations is to be determined, the revolution of the wheel is continued with the same velocity for a certain number of seconds. The number of turns of the toothed wheel B is then read off on the indicator, and this multiplied by the number of teeth in the wheel gives the total number of vibrations. Dividing this by the corresponding number of seconds, the quotient gives the number of vibrations per second for the given sound.

242. **Sirene.**—The *sirene* is an apparatus which, like Savart's wheel, is used to measure the number of vibrations of a body in a given time. The

name 'sirene' was given to it by its inventor, Cagniard Latour, because it yields sounds under water.

It is made entirely of brass. Fig. 223 represents it fixed on the table of a bellows, by which a continuous current of air can be sent through it. Figs. 224 and 225 show the internal details. The lower part consists of a cylindrical box, O, closed by a fixed plate, B. On this plate a vertical rod, T, rests, to which is fixed a disc, A, moving with the rod. In the plate B there are equidistant circular holes, and in the disc A are an equal number of holes of the same size, and the same distance from the centre as those of the plate. These holes are not perpendicular to the disc; they are all inclined to the same extent in the same direction in the plate, and to the same extent in the opposite direction in the disc, so that when they are opposite each other they have the appearance represented in *mn*, fig. 224. Consequently, when a current of air from the bellows reaches the hole *m*, it strikes obliquely

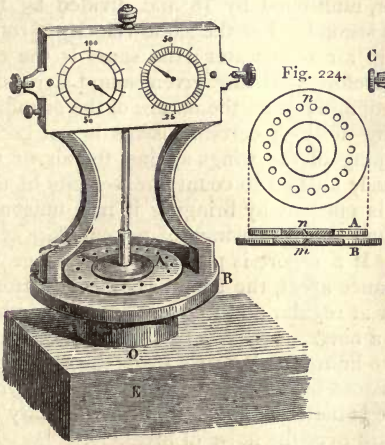


Fig. 223.

Fig. 224.

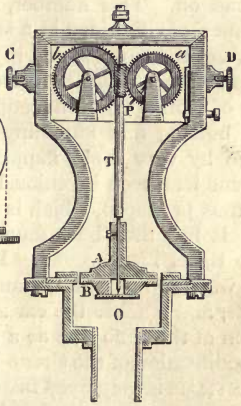


Fig. 225.

against the sides of the hole *n*, and imparts to the disc A a rotary motion in the direction *n*A.

For the sake of simplicity, let us first suppose that in the movable disc A there are eighteen holes, and in the fixed plate B only one, which faces one of the upper holes. The wind from the bellows striking against the sides of the latter, the movable disc begins to rotate, and the space between two of its consecutive holes closes the hole in the lower plate. But as the disc continues to turn from its acquired velocity, two holes are again opposite each other, a new impulse is produced, and so on. During a complete revolution of the disc the lower hole is eighteen times open and eighteen times closed. A series of effluxes and stoppages is thus produced, which makes the air vibrate, and ultimately produces a sound when the successive impulses are sufficiently rapid. If the fixed plate, like the moving disc, had eighteen holes, each hole would separately produce the same effect as a

separate one, the sound would be eighteen times as intense, but the number of vibrations would not be increased.

In order to know the number of vibrations corresponding to the sound produced, it is necessary to know the number of revolutions of the disc A in a second. For this purpose an endless screw on the rod T transmits the motion to a wheel, *a*, with 100 teeth. On this wheel, which moves by one tooth for every turn of the disc, there is a catch, P, which at each complete revolution moves one tooth of a second wheel, *b* (fig. 225). On the axis of these wheels there are two needles, which move round dials represented in fig. 223. One of these indices gives the number of turns of the disc A, the other the number of hundreds of turns. By means of two screws, D and C, the wheel *a* can be uncoupled from the endless screw.

Since the pitch of the sound rises in proportion to the velocity of the disc A, the wind is forced until the desired sound is produced. The same current is kept up for a certain time—two minutes, for example—and the number of turns read off. This number, multiplied by 18 and divided by 120, gives the number of vibrations in a second. For the same velocity of rotation the sirene gives the same sound in air as in water; the same is the case with all gases; and it appears, therefore, that any given sound depends on the number of vibrations produced, and not on the nature of the sounding body.

The buzzing and humming noise of certain insects is not vocal, but is produced by very rapid flapping of the wings against the air or the body. The sirene has been ingeniously applied to count the velocity of the undulations thus produced, which is effected by bringing it into unison with the sound. It has thus been found that the wings of a gnat flap at the rate of 1,500 times in a second. If a report is produced in a space with two parallel walls at no great distance apart, the sound is reflected from one to the other, and reaches the ear at regular and frequent intervals; that is, the repetition of the echo acts as a note.

A modification of the sirene known as Brown's *steam-horn*, in which high-pressure steam is employed instead of compressed air, is used as a *fog-signal*. Its shrill and penetrating note is better adapted than an ordinary fog-horn, or even cannon, for being heard over the noise of breakers.

243. **Bellows.**—In acoustics a *bellows* is an apparatus by which wind instruments, such as the sirene and organ-pipes, are worked. Between the four legs of a table there is a pair of bellows, S (fig. 226), which is worked by means of a pedal, P. D is a reservoir of flexible leather, in which is stored the air forced in by the bellows. If this reservoir is pressed by means of weights on a rod, T, moved by the hand, the air is driven through a pipe, E, into a chest, C, fixed on the table. In this chest there are small holes closed by leather valves, which can be opened by pressing on keys in front of the box. The sirene or sounding pipe is placed in one of these holes.

244. **Limit of perceptible sounds.**—Previous to Savart's researches, physicists assumed that the ear could not perceive a sound when the number of vibrations was below 16 for deep sounds, or above 9,000 for acute sounds. But he showed that these limits were too close, and that the faculty of perceiving sounds depends rather on their intensity than on their height; so that when extremely acute sounds are not heard, it arises from the fact that

they have not been produced with sufficient intensity to affect the organ of hearing.

By increasing the diameter of the toothed wheel, and consequently the amplitude and intensity of the vibrations, Savart pushed the limit of acute sounds to 24,000 vibrations in a second.

For deep sounds he substituted for the toothed wheel an iron bar about two feet long, which revolved on a horizontal axis between two thin wooden plates, about 0·08 of an inch from the bar. As often as the bar passed a grave sound was produced, due to the displacement of the air. As the motion was accelerated the sound became continuous, very grave, and deafening. By this means Savart found that, with 7 to 8 vibrations in a second, the ear perceived a distinct but very deep sound.

Despretz, however, who investigated the same subject, disputed Savart's results as to the limits of deep sounds, and considers that no sound is audible that is made by less than 16 vibrations per second. Helmholtz holds that the perception of a sound begins at 30 vibrations, and only has a definite musical value when the number is more than 40. Below 30 the impression of a number of separate beats is produced. On the other hand, acute sounds are audible up to those corresponding to 38,000 vibrations in a second. Such sounds, however, are far from pleasurable: they affect the ear as if it had been pricked with a pin or needle.

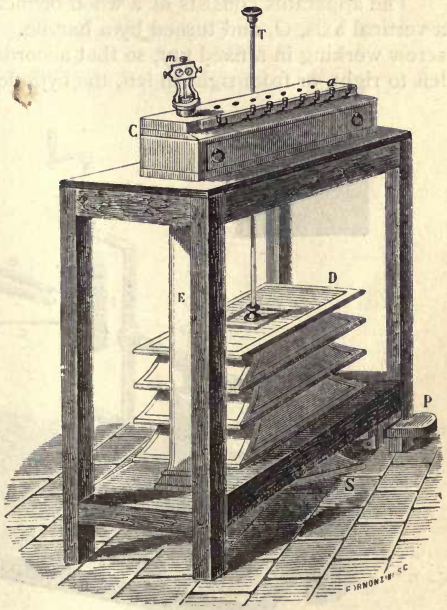


Fig. 226.

The discordant results obtained by these and other observers for the limit of audibility of higher notes are no doubt due to the circumstance that different observers have different capacities for the perception of sounds. Preyer has investigated this subject by means of experimental methods of greater precision than any that have hitherto been applied for this purpose. The minimum limit for the normal ear he found to lie between 16 and 24 single vibrations in a second; the maximum limit reached 41,000; but many persons with average powers of hearing were found to be absolutely deaf to notes of 16,000, 12,000, or even fewer vibrations.

It appears that the limit of audibility for any particular ear is increased with the strength of the sound. Paucher examined this by sounding a powerful sirene by steam; he found that with steam of $\frac{1}{2}$ atmosphere pres-

sure the upper limit was at 48,000 vibrations, with 1½ atmosphere it was 60,000, while with steam of 2½ atmospheres it had not been attained with 72,000 vibrations.

245. **Duhamel's graphic method.**—When the sirene or Savart's wheel is used to determine the exact number of vibrations corresponding to a given note, it is necessary to bring the sounds which they produce into unison with the given note, and this cannot be done exactly unless the experimenter has a practised ear. Duhamel's graphic method is very simple and exact, and free from this difficulty. It consists in fixing a fine point to the body emitting the note, and causing it to trace the vibrations on a properly prepared surface.

The apparatus consists of a wood or metal cylinder, A (fig. 227), fixed to a vertical axis, O, and turned by a handle. The lower part of the axis is a screw working in a fixed nut, so that according as the handle is turned from left to right, or from right to left, the cylinder is raised or depressed. Round

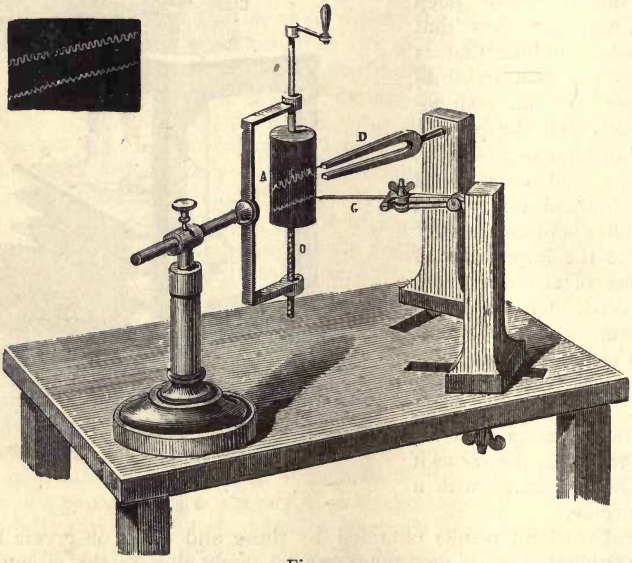


Fig. 227.

the cylinder is rolled a sheet of paper covered with an inadhesive film of lampblack. On this film the vibrations register themselves. This is effected as follows. Suppose the body emitting the note to be a steel rod. It is held firmly at one end, and carries at the other a fine point which grazes the surfaces of the cylinder. If the rod is made to vibrate and the cylinder is at rest, the point would describe a short line; but if the cylinder is turned, the point produces an *undulating line*, containing as many undulations as the point has made vibrations. Consequently, the number of vibrations can be counted. It remains only to determine the time in which the vibrations were made.

There are several ways of doing this. The simplest is to compare the curve traced by the vibrating rod with that traced by a tuning-fork (251), which gives a known number of vibrations per second—for example, 500. The prong of the fork is furnished with a point, which is placed in contact with the lampblack. The fork and the rod are then set vibrating together, and each produces its own undulating trace. When the paper is unrolled, it is easy, by counting the number of vibrations each has made in the same distance, to determine the number of vibrations made per second by the elastic rod. Suppose, for instance, that the tuning-fork made 150 vibrations, while the rod made 165 vibrations. Now we already know that the tuning-fork makes one vibration in the $\frac{1}{500}$ part of a second, and therefore 150 vibrations in $\frac{150}{500}$ of a second. But in the same time the rod makes 165 vibrations; therefore it makes one vibration in the $\frac{150}{500 \times 165}$ of a second, and hence it makes per second $\frac{500 \times 165}{150}$, or 550 vibrations.

CHAPTER III

THE PHYSICAL THEORY OF MUSIC

246. **Properties of musical notes.**—A simple musical note results from continuous rapid isochronous vibrations, provided the number of the vibrations falls within the very wide limits mentioned in the last chapter (244). Musical notes are in most cases compound. The distinction between a simple and a compound musical note will be explained later in the chapter. The tone yielded by a tuning-fork furnished with a proper resonance-box is *simple*; that yielded by a wide-stopped organ pipe, or by a flute, is *nearly simple*; that yielded by a musical string is *compound.*

Musical notes have three leading qualities, namely, *pitch*, *intensity*, and *timbre* or *quality.*

i. The *pitch* of a musical note is determined by the number of vibrations per second yielded by the body producing the note.

ii. The *intensity* of the note depends on the *extent* of the vibrations. It is greater when the extent is greater, and less when it is less. It is, in fact, proportional to the square of the extent, or amplitude of the vibrations which produce the note.

iii. The *timbre* or stamp or *quality* is that peculiar property of note which distinguishes a note when sounded on one instrument from the same note when sounded on another, and which by some is called the *colour.* Thus when the C of the treble stave is sounded on a violin and on a flute, the two notes will have the same pitch; that is, they are produced by the same number of vibrations per second, and they may have the same intensity, and yet the two notes will have very distinct qualities; that is, their timbre is different. The cause of the peculiar timbre of notes will be considered later in the chapter.

247. **Musical intervals.**—Let us suppose that a musical note, which for the sake of future reference we will denote by the letter C, is produced by m vibrations per second; and let us further suppose that any other musical note, X, is produced by n vibrations per second, n being greater than m; then the interval from the note C to the note X is the ratio $n : m$, the interval between two notes being obtained by *division*, not by *subtraction.* Although two or more notes may be separately musical, it by no means follows that when sounded together they produce a pleasant sensation. On the contrary, unless they are *concordant*, the result is harsh, and usually unpleasing. We have, therefore, to inquire what *notes* are fit to be sounded together. Now, when musical notes are compared, it is found that if they are separated by an interval of 2 : 1, 4 : 1, &c., they so closely resemble one another that they may for most purposes of music be considered as the same note. Thus, suppose c to stand for a musical note produced by $2m$ vibrations per second, then C and c so closely resemble each other as to be called in music by

the same name. The interval from C to *c* is called an *octave*, and *c* is said to be an octave above C, and conversely C an octave below *c*. If we now consider musical sounds that do not differ by an octave, it is found that if we take three notes, X, Y, and Z, resulting respectively from p, q, and r vibrations per second, these three notes when sounded together will be concordant if the ratio of $p : q : r$ equals $4 : 5 : 6$. Three such notes form a *harmonic triad*, and if sounded with a fourth note, which is the octave of X, constitute what is called in music a *major chord*. Any of the notes of a chord may be altered by one or more octaves without changing its distinctive character; for instance, C, E, G, and *c* are a chord, and C, *c*, *e*, *g* form the same chord.

If, however, the ratio $p : q : r$ equals $10 : 12 : 15$, the three sounds are slightly dissonant, but not so much so as to disqualify them from producing a pleasing sensation. When these three notes and the octave to the lower are sounded together, they constitute what in music is called a *minor chord*.

248. **The musical scale.**—The series of sounds which connects a given note C with its octave *c* is called the *diatonic* scale or *gamut*. The notes composing it are indicated by the letters C, D, E, F, G, A, B. The scale is then continued by taking the octaves of these notes, namely, *c*, *d*, *e*, *f*, *g*, *a*, *b*, and again the octaves of these last, and so on.

The notes are also known by names, viz., *do* or *ut*, *re*, *mi*, *fa*, *sol*, *la*, *si*, *do*. The relations existing between the notes are these:—C, E, G form a major *triad*, G, B, *d* form a major *triad*, and F, A, *c* form a major *triad*. C, G, and F have, for this reason, special names, being called respectively the *tonic*, *dominant*, and *sub-dominant*, and the three triads the *tonic*, *dominant*, and *sub-dominant* triads or chords respectively. Consequently, the numerical relations between the notes of the scale will be given by the three proportions—

$$\begin{aligned} C : E : G &:: 4 : 5 : 6 \\ G : B : 2D &:: 4 : 5 : 6 \\ F : A : 2C &:: 4 : 5 : 6 \end{aligned}$$

Hence, if m denotes the number of double vibrations corresponding to the note C, the number of vibrations corresponding to the remaining notes will be given by the following table—

do	*re*	*mi*	*fa*	*sol*	*la*	*si*	*do*
C	D	E	F	G	A	B	*c*
m	$\frac{9}{8}m$	$\frac{5}{4}m$	$\frac{4}{3}m$	$\frac{3}{2}m$	$\frac{5}{3}m$	$\frac{15}{8}$	$2m$

The intervals between the successive notes being respectively—

C to D	D to E	E to F	F to G	G to A	A to B	B to *c*
$\frac{9}{8}$	$\frac{10}{9}$	$\frac{16}{15}$	$\frac{9}{8}$	$\frac{10}{9}$	$\frac{9}{8}$	$\frac{16}{15}$

It will be observed here that there are three kinds of intervals, $\frac{9}{8}$, $\frac{10}{9}$, and $\frac{16}{15}$; of these the first two are called a *tone*, the last a *semitone*, because it is about half as great as the interval of a tone. The two tones, however, are not identical, but differ by an interval of $\frac{81}{80}$, which is called a *comma*. Two notes which differ by a *comma* can be readily distinguished by a trained

ear. The interval between the tonic and any note is denominated by the position of the latter note in the scale ; thus the interval from C to G is a *fifth*. The scale we have now considered is called the *major* scale, as being formed of *major* triads. If the minor triad were substituted for the major, a scale would be formed that could be strictly called a *minor* scale. As scales are usually written, however, the *ascending* scale is so formed that the tonic bears a minor triad, the dominant and sub-dominant bear *major* triads, while in the *descending* scale they all bear *minor* triads. Practically, in musical composition, the dominant triad is always *major*. If the ratios given above are examined, it will be found that in the major scale the interval from C to E equals $\frac{5}{4}$, while in the minor scale it equals $\frac{6}{5}$. The former interval is called a *major* third, the latter a *minor* third. Hence the major third exceeds the minor third by an interval of $\frac{25}{24}$. This interval is called a semitone, though very different from the interval above called by that name.

249. **On semitones and on scales with different key-notes.**—It will be seen from the last article that the term 'semitone' does not denote a constant interval, being in one case equivalent to $\frac{16}{15}$ and in another to $\frac{25}{24}$. It is found convenient for the purposes of music to introduce notes intermediate to the seven notes of the gamut ; this is done by raising or lowering these notes by an interval of $\frac{25}{24}$. When a note (say C) is increased by this interval, it is said to be *sharpened*, and is denoted by the symbol C♯, called 'C sharp' ; that is, C♯ ÷ C = $\frac{25}{24}$. When it is lowered by the same interval, it is said to be *flattened*, and is represented thus—B♭, called 'B flat' ; that is, B ÷ B♭ = $\frac{25}{24}$. If the effect of this be examined, it will be found that the number of notes in the scale from C up to *c* has been increased from seven to twenty-one notes, all of which can be easily distinguished by the ear. Thus, reckoning C to equal 1, we have—

C	C♯	D♭	D	D♯	E♭	E	&c.
1	$\frac{25}{24}$	$\frac{27}{25}$	$\frac{9}{8}$	$\frac{75}{64}$	$\frac{6}{5}$	$\frac{5}{4}$	&c.

Hitherto we have made the note C the tonic or *key-note*. Any other of the twenty-one distinct notes above mentioned, *e.g.*, G, or F, or C♯, &c., may be made the key-note, and a scale of notes constructed with reference to it. This will be found to give rise in each case to a series of notes, some of which are identical with those contained in the series of which C is the key-note, but most of them different. And of course the same would be true for the minor scale as well as for the major scale, and indeed for other scales which may be constructed by means of the fundamental triads.

250. **On musical temperament.**—The number of notes that arise from the construction of the scales described in the last article is so great as to prove quite unmanageable in the practice of music ; and particularly for music designed for instruments with fixed notes, such as the pianoforte or harp. Accordingly, it becomes practically important to reduce the number of notes, which is done by slightly altering their just proportions. This process is called *temperament*. By tempering the notes, however, more or less dissonance is introduced, and accordingly several different systems of temperament have been devised for rendering this dissonance as slight as possible. The system usually adopted is called the system of *equal tempera-*

ment. It consists in retaining the octaves pure, and in substituting between C and *c* eleven notes at equal intervals, each interval being, of course, the twelfth root of 2, or 1·05946. By this means the distinction between the semitones is abolished, so that, for example, C♯ and D♭ become the same note. The scale of twelve notes thus formed is called the *chromatic scale.* It of course follows that major triads become slightly dissonant. Thus, in the diatonic scale, if we reckon C to be 1, E is denoted by 1·25000, and G by 1·50000. On the system of equal temperament, if C is denoted by 1, E is denoted by 1·25992, and G by 1·49831.

If individual intervals are made pure while the errors are distributed over the others, such a system is called that of *unequal temperament.* Of this class is *Kirnberger's*, in which nine of the tones are pure.

Although the system of equal temperament has the advantage of affording the greatest variety of tones with as small a number of notes as possible, yet it has the drawback that no chord of an equally tempered instrument, such as the piano, is perfectly pure. And as musical education mostly has its basis on the piano, even singers and instrumentalists usually give equally-tempered intervals. Only in the case of string quartet players, who have freed themselves from school rules, and in that of vocal quartet singers, who sing much without accompaniment, does the natural pure temperament assert itself, and thus produce the highest musical effect.

251. **The number of vibrations producing each note. The tuning-fork.**—Hitherto we have denoted the number of vibrations corresponding to the note C by *m*, and have not assigned any numerical value to that symbol. In the theory of music it is frequently assumed that the middle C corresponds to 256 double vibrations in a second. This is the note which, on a pianoforte of seven octaves, is produced by the white key on the left of the two black keys close to the centre of the keyboard. This number is convenient as being continually divisible by two, and is therefore frequently used in numerical illustrations. It is, however, arbitrary. An instrument is in tune provided the intervals between the notes are correct, when *c* is yielded by any number of vibrations per second not differing much from 256. Moreover, two instruments are in tune with one another if, being separately in tune, they have any one note, for instance C, yielded by the same number of vibrations. Consequently, if two instruments have one note in common, they can then be brought into tune jointly by having their remaining notes separately adjusted with reference to the fundamental note. A *tuning-fork* or *diapason* is an instrument yielding a constant sound, and is used as a standard for tuning musical instruments. It consists of an elastic steel rod, bent as represented in fig. 228. It is made to vibrate either by drawing a bow across the ends, or by striking one of

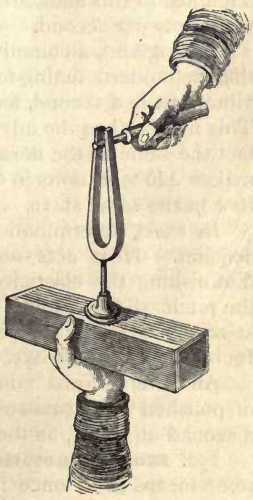


Fig. 228.

the legs against a small hammer covered with leather, or by rapidly separating the two prongs by means of a steel rod as shown in the figure. The vibration produces a note which is always the same for the same tuning-fork. The note is strengthened by fixing the tuning-fork on a box open at one end, called a *sounding* or *resonance box*, adjusted so as to strengthen the special note of the tuning-fork. The length of this column of air enclosed in the box is a quarter that of the wave-length of the note which the tuning-fork emits. The vibrations of the air produce the same note as the fork itself; the vibrations of the tuning-fork, being communicated to the column of air in the box, set it in vibration, by which a strong and pure note is obtained (255).

The standard tuning-fork in any country represents its accepted concert pitch.

It has been remarked for some years that not only has the pitch of the tuning-fork been getting higher in the large theatres of Europe, but also that it is not the same in London, Paris, Berlin, Vienna, Milan, &c. This is a source of great inconvenience both to composers and singers, and a commission was appointed in 1859 to establish in France a tuning-fork of uniform pitch, and to prepare a standard which would serve as an invariable type. In accordance with the recommendations of that body, a *normal tuning-fork* has been established, which is compulsory on all musical establishments in France, and a standard has been deposited in the Conservatory of Music in Paris. It performs 437·5 double vibrations per second, and gives the standard note *a* or *la*, or the *a* in the treble stave (252). Consequently, with reference to this standard, the middle *c* or *do* would result from 261 double vibrations per second.

In England, a committee, appointed by the Society of Arts, recommended that a standard tuning-fork should be one constructed to yield 528 double vibrations in a second, and that this should represent *c′* in the treble stave. This number has the advantage of being divisible by 2 down to 33, and is in fact the same as the normal tuning-fork adopted in Stuttgart in 1834, which makes 440 vibrations in the second, and, like the French one, corresponds to *a* in the same stave.

In exact determinations of pitch the temperature must be taken into account. Heat acts on the tuning-fork by expanding it, and also by diminishing the elasticity of the metal. Both effects concur in lowering the pitch. Thus König found that a tuning-fork which made 512 vibrations at 20° C. varied by 0·0572 for each degree Centigrade. Stone and McLeod found the number 0·055.

An international conference at Vienna in 1885 adopted a tuning-fork of polished mild cast-steel with prismatic prongs, making 435 vibrations in a second at 15° C., as the standard *a* note.

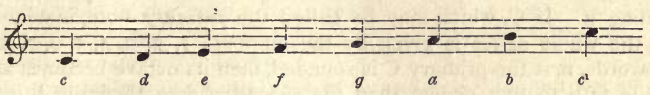
252. **Musical notation. Musical range.**—It is convenient to have some means of at once naming any particular note in the whole range of musical sounds other than by stating its number of vibrations. Perhaps a convenient practice is to call the octave, of which the C is produced by an eight-foot organ pipe, by the capital letters C, D, E, F, G, A, B; the next higher octave by the corresponding small letters, *c*, *d*, *e*, *f*, *g*, *a*, *b*; and to designate the octaves higher than this by the index placed over the letter, thus, *c′*, *d′*, *e′*, *f′*, *g′*, *a′*, *b′*, and the higher series in a similar manner. The

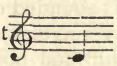
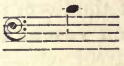
same principle may be applied to the notes below C; thus the octave below C is $C_{,}$, and the next lower one $C_{,,}$.

Hence we have the series

$$C_{,,}\ C_{,}\ C\ c\ c'\ c''\ c'''\ c^{iv}.$$

In musical writing the notes are expressed by signs which indicate the length of time during which the note is to be played or sung, and are written on a series of lines called *a stave*. Thus



stands for the octave in the treble clef, of which the top note is the standard *c'* and the bottom is the middle *c*. When the five lines are insufficient they are continued above and below the stave by what are called *leger* lines. In order to avoid confusion, a bass clef is used for the lower notes; and it may be remarked that  and  stand for the same note (251), which is the middle *c*.

The deepest note of orchestral instruments is the $E_{,}$ of the double bass, which makes 41⅛ vibrations, taking the key-note as making 440 vibrations in a second. Some organs and pianofortes go as low as $C_{,,,}$ with 32 vibrations in a second, some grand pianos even as low as $A_{,,,}$ with 27½ vibrations. But the musical character of all these notes below $E_{,}$ is imperfect, for we are near the limit at which it is possible for the ear to combine the separate vibrations to a musical note (244). These notes can only be used musically with their next higher octave, to which they impart a certain character of depth and richness.

In the other direction, pianofortes go to a^{iv} with 3,520 or even c^{v} with 4,224 vibrations in a second. The highest note of the orchestra is probably the d^{v} of the piccolo flute, which makes 4,752 vibrations. Although the ear can distinguish sounds which are still higher, they have no longer a pleasurable character. And while the notes which are distinguishable by the ear range between 16 and 38,000 vibrations, or 11 octaves, those which are musically available range from about 40 to 4,000 vibrations, or within 7 octaves.

253. **Wave-length of a given note. Amplitude of oscillation.**—Knowing the number of vibrations which a sounding body makes in a second, the corresponding wave-length is easily calculated. For since sound travels at about 1,120 feet in a second, if a body only made one vibration in a second its wave-length would be 1,120 feet; if it made two, the wave-length would be half of 1,120 feet; if it made three, the third, and so on—that is, *the wave-length of any note is the quotient obtained by dividing the velocity of sound by the number of vibrations;* and this whatever the height of the sound, since the velocity is the same for high and low notes.

Hence, calling v the velocity of sound, l the wave-length, n the number of vibrations in a second, we have $v = ln$, from which $n = \frac{v}{l}$; that is, the number of vibrations is inversely as the wave-length.

The amplitude of oscillation which is required for the production of audible sounds is very small. Lord Rayleigh determined it in the case of the waves due to a pipe which sounded the note f^{iv}, and which could be heard at a distance of 820 metres. He found that the amplitude of the oscillation of these waves could not be greater than 0·06 of a millimetre.

254. **On compound musical tones and harmonics.**—When any given note (say C) is sounded on most musical instruments, not that note alone is produced, but a series of notes, each being of less intensity than the one preceding it. If C, which may be called the *primary* note, is denoted by unity, the whole series is given by the numbers 1, 2, 3, 4, 5, 6, 7, &c.; in other words, first the primary C is sounded, then its octave becomes audible, then the fifth to that octave, then the second octave, then the third, fifth, and a note between the sixth and seventh to the second octave, and so on. These secondary notes are called the *harmonics* of the primary *note*. Though feeble in comparison with the primary note, they may, with a little practice, be heard when the primary note is produced on most musical instruments; when, for instance, one of the lower notes is sounded on the pianoforte.

254*a*. **Consonance and resonance.**—A singular property of bodies in a state of vibration is that of setting in vibration bodies at rest. Thus, if two tuning-forks, tuned so as to give accurately the same note, be at some distance from each other, and one of them be sounded, the other will be set in vibration and emit the same note. But, if one of the forks be put slightly out of tune with the other, by attaching a piece of wax to one prong, for instance, then the excitation of either one will have no effect on the other.

It is remarkable that the successive action of a series of impulses of small mechanical force should, as in this case, be able to set a relatively very heavy body—such as a tuning-fork—in vibration; but for this there are many purely mechanical analogies. Thus, if a series of pulls be exerted in regular intervals on the rope of a large church bell, the superposition of these small motions will ultimately set the bell swinging. A regiment of soldiers marching in step over an iron bridge at Angers set it in such powerful oscillation as to endanger its stability. In like manner, the position of a ship in the trough of the sea is very dangerous, when the period of vibration of the waves coincides with that of its own vibration.

This phenomenon, that a body in a state of vibration has the power of causing an independent body at rest to vibrate in the same period, is called *consonance.*

If a metal wire freely suspended in the air be tightly stretched and then be set in vibration, the note which it emits will be feeble, seeing that from its small surface it can set in vibration only small masses of air. So, too, a tuning-fork when sounded gives but a feeble note; but if its stem be held on a table the note becomes far louder.

The reinforcement of a sound by attaching the sounding body to a large, dry, elastic wooden plate, called a sound-board, or to a wooden box enclosing a mass of air, is called *resonance;* the vibrations of the sounding body are transmitted to the sound-board, which, being set in vibration, communicates its motion to large masses of air.

Although the terms consonance and resonance are sometimes used indiscriminately, there are distinctions between them.

Consonance is the excitation of an independent body to vibrate in unison with the sounding body; it begins later than the sounding body, and continues after it has become silent. Resonance begins and ends with the sound of the exciting body. A sound-board strengthens and imparts a general sonority to a complex series of notes. The more a body diverges from the form of a plate and approaches that of a rod, the more is its resonance limited to strengthening one or two notes.

In resonance, however, there is a certain amount of tuning. For the loud and deep notes of the 'cello a large resonance-box is used, and a smaller one for the higher notes of the violin. Small enclosed volumes of air also strengthen one note in preference.

255. **Von Helmholtz's analysis of sound.**—For the purpose of experimentally proving the presence of the harmonics as distinct tones, Von Helmholtz devised an instrument which he called a *resonance-globe*. This may be shown by the following experiment, which is an illustration of what has been said in the previous article, and is indeed analogous in principle with that described in article 227:—If an empty glass cylinder be taken, and a vibrating tuning-fork be held over the mouth of the vessel, the air will not be set in vibration unless it be of a certain definite length; such, indeed, that

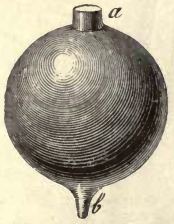


Fig. 229.

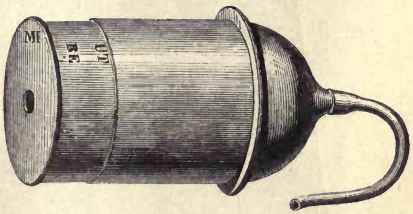


Fig. 230.

the wave-length of the fundamental note corresponds to the wave-length of the note produced by the tuning-fork. Now, by pouring in water we can regulate the length of the column of air, and by trial can hit off the exact length; when this is attained the note of the tuning-fork will be heard to be powerfully reinforced (227). A resonance-globe (fig. 229) is a glass globe tuned to a particular note, furnished with two openings, one of which, *a*, is turned towards the origin of the sound, and the other, *b*, by means of an india-rubber tube, is applied to the ear. If the note proper to the resonance-globe exists among the harmonics of the compound note that is sounded, it is strengthened by the globe, and thereby rendered distinctly audible. Further, other things being the same, the note proper to a given globe depends on the diameter of the globe and that of the uncovered opening. Consequently, by means of a series of such globes, the whole series of harmonics in a given compound tone can be rendered distinctly audible, and their existence put beyond a doubt.

König, the eminent acoustical-instrument maker, has made an important modification in the resonance-globe, to which he has given the form represented in fig. 230. The resonator is cylindrical, and the end which receives the sound can be drawn out, so that the volume may be increased at pleasure.

As the sound thereby becomes deeper, the same resonator may be tuned to a variety of notes. On the tubulure fits a caoutchouc tube by which the vibrations may be transmitted in any direction.

256. **König's apparatus for the analysis of sound.**—As the successive application to the ear of various resonators is both slow and tedious, König devised a remarkable apparatus in which a series of resonators act on manometric flames (288); the sounds thus, as it were, become visible, and may be shown to a large auditory.

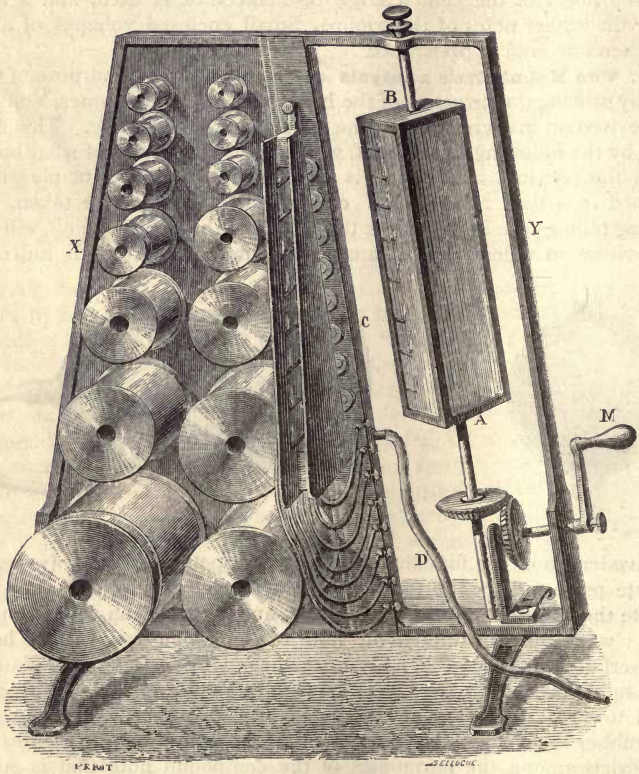


Fig 231.

It consists of an iron frame (fig. 231) on which are fixed in two parallel lines fourteen resonators tuned so as to give the notes from F, to *c*‴—that is to say, four octaves and a half; or notes of which the highest give the lower harmonics of the primary. On the right is a chamber, C, which is supplied with coal gas by the caoutchouc tube D, and on which are placed eight gas-jets, each provided with a manometric capsule (288). Each jet is connected with the chamber C by a special caoutchouc tube, while behind the apparatus a second tube connects the same jet to one of the resonators.

On the right of the jets is a system of rotating mirrors identical with that described in article 288.

These details being understood, suppose the largest resonator on the right tuned to resound with the note 1, and seven others with the harmonics of this note. Let the sound 1 be produced in part of this apparatus; if it is simple, the lower resonator alone answers, and the corresponding flame is alone dentated; but if the fundamental note is accompanied by one or more of its harmonics, the corresponding resonators speak at the same time, which is recognised by the dentation of their flames; and thus the constituents of each sound may be detected.

257. **Synthesis of sounds.**—Not only did Von Helmholtz succeed in decomposing sounds into their constituents; he also verified the result of his analysis by performing the reverse operation, the synthesis; that is, he

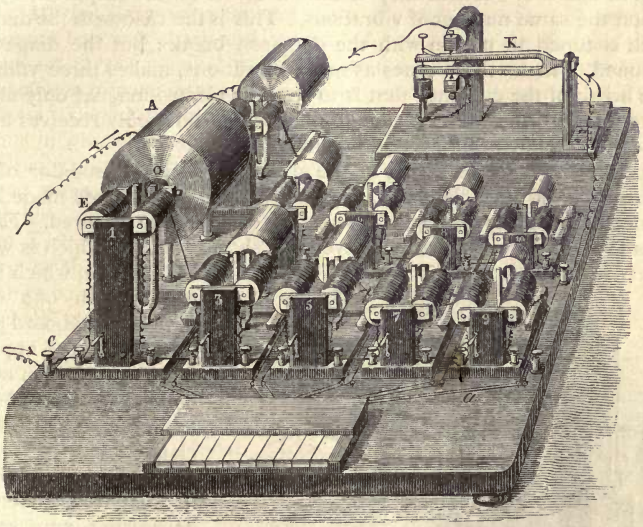


Fig. 232.

reproduced a given sound by combining the individual sounds of which his resonators had shown that it was composed. The apparatus which he used for this purpose consists of eleven tuning-forks, the first of which yields the fundamental note of 256 vibrations, or C, nine others its harmonics, while the eleventh serves as make and break to cause the diapasons to vibrate by means of electro-magnets. Each diapason has a special electro-magnet, and moreover a resonator, which strengthens it.

All these diapasons and their accessories are arranged in parallel lines of five (fig. 232), the first comprising the fundamental note and its uneven harmonics, 3, 5, 7, and 9; the second the even harmonics, 2, 4, 6, 8, and 10; beyond, there is the diapason break, K, arranged horizontally. One of its prongs is provided with a platinum point which grazes the surface of mercury

contained in a small cup, the bottom of which is connected by a copper wire with an electro-magnet placed in front of the diapason.

The apparatus being thus arranged, a wire from a voltaic battery is connected with the binding-screw, *c*, and this with the electro-magnet, E ; which in turn is connected with those of the nine following diapasons, and then with the diapason, K, itself. So long as the diapason does not vibrate the current does not pass, for the platinum point does not dip in the mercury cup which is connected with the other pole of the battery. But when the diapason is made to vibrate by means of a bow, the current passes. Owing to their elasticity, the limbs of the tuning-fork soon revert to their original position, the point is no longer in the mercury, the current is broken, and so on at each double vibration of the diapason. The intermittence of the current being transmitted to all the other electro-magnets, they are alternately active and inactive. Hence they communicate to all the diapasons by their attraction the same number of vibrations. This is the case with the diapason 1, which is tuned in unison with the diapason break ; but the diapason 3, being tuned to make three times as many vibrations, makes three vibrations at each break of the current ; that is to say, the electro-magnet only attracts it at every third vibration ; in like manner, diapason 5 only receives a fresh impulse every five vibrations, and so on.

The following is the working of the apparatus :—The resonator of each diapason is closed by a clapper, O (fig. 233), so that the sounds made by the diapasons are scarcely perceptible when the clappers are lowered. Each of these is fixed to the end of a bent lever, the shorter arm of which is worked by a cord, *a*, which is connected with one of the keys of a keyboard placed in front of the apparatus (fig. 232). When a key is depressed, the cord moves the lever, which raises the clapper, and the resonator then acts by strengthening its diapason. Hence by depressing any key we may add to the fundamental sounds any of the nine primary harmonics, and thus reproduce the sounds, the composition of which has been determined by analysis. Thus by depressing all the keys at once we obtain the sound of an open pipe in unison with the deepest diapason. By depressing the key of the fundamental note and those of its uneven harmonics, we obtain the sound of a closed pipe.

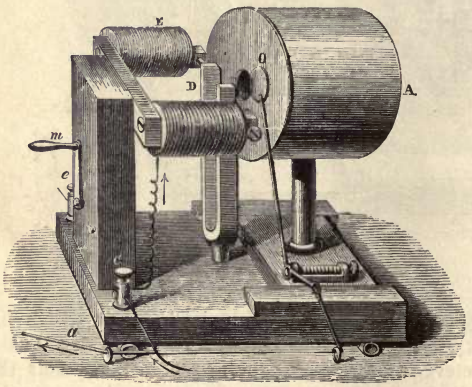


Fig. 233.

258. **Results of Von Helmholtz's researches.**—By both his analytical and synthetical investigations into sounds of the most varied kinds—those from various musical instruments, the human voice, and even noises—Von Helmholtz has fully succeeded in explaining the different *timbre* or quality of

sounds. It is due to the different intensities of the harmonics which accompany the primary tones of these sounds. The leading results of these researches into the colour (246) of sounds may be thus stated :—

i. Simple notes, as those produced by a tuning-fork with a resonance-box, and by wide covered pipes, are soft and agreeable without any roughness, but weak, and in the deeper notes dull.

ii. Musical sounds accompanied by a series of harmonics, say up to the sixth, in moderate strength, are full and musical. In comparison with simple tones they are grander, richer, and more sonorous. Such are the sounds of open organ-pipes, of the pianoforte, &c.

iii. If only the uneven harmonics are present, as in the case of narrow stopped pipes, of pianoforte strings struck in the middle, clarionets, &c., the sound becomes indistinct ; and, when a greater number of harmonics is audible, the sound acquires a nasal character.

iv. If the harmonics beyond the sixth and seventh are very distinct, the sound becomes sharp and rough. If less strong, the harmonics are not prejudicial to the musical usefulness of the notes. On the contrary, they are useful as imparting character and expression to the music. Of this kind are most stringed instruments, and most pipes furnished with tongues, &c. Sounds in which harmonics are particularly strong acquire thereby a peculiarly penetrating character ; such are those yielded by brass instruments.

259. **Production of vocal sounds.**—The *trachea* or *windpipe* is a tube which terminates at one end in the lungs, and at the other in the *larynx*, which is the true organ of vocal sound. Fig. 234 represents a horizontal section of this organ. It consists of a number of cartilaginous structures, *bb*, which are connected by various muscles, by which great variety and control in the motions are attainable. These muscles are connected with, and move, two elastic membranes or bands with broad bases fixed to the larynx, and with sharp edges, *cc* ; these are called the *vocal chords*. According to the pressure of the muscles, these chords are more or less tightly stretched, and the space between them, the *vocal slit*, is narrower or wider accordingly. In ordinary breathing, air passes through the triangular aperture *o* ; but when in singing this is closed, the vocal chords are stretched and are put in vibration by the current of air, and produce tones which are higher the more tightly the chords are stretched and the narrower is the vocal slit. These changes can be effected with surprising rapidity, so that in this respect the human voice far exceeds anything that can be made artificially.

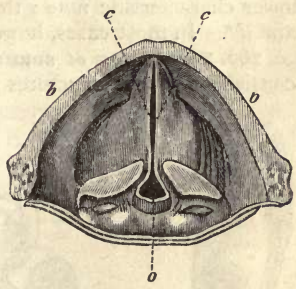


Fig. 234.

The notes produced by men are deeper than those of women or boys, because in them the larynx is longer and the vocal cords larger and thicker ; hence, though equally elastic, they vibrate less swiftly. The vocal chords are 18 millimetres long in men, and 12 millimetres long in women. Chest notes are due to the fact that the whole membrane vibrates, while the falsetto is produced by a vibration of the extreme edges only. The ordinary

compass of the individual voice is within two octaves, though this is exceeded by some celebrated singers. Catalani, for instance, is said to have had a range of $3\frac{1}{2}$ octaves.

The wave-length of the sounds emitted by a man's voice in ordinary conversation is from 8 feet to 12 feet, and that of a woman's voice is from 2 feet to 4 feet.

The vowel sounds can be produced in any pitch, and the difference in them arises from the fact that to form a given vowel sound one or more characteristic notes, which are always the same, must be added. These change with the syllable pronounced, but depend neither on the height of the note nor on the person who emits them.

The form and cavity of the mouth can be greatly modified by the extent to which it is opened, by the altered position of the tongue, and so forth. It thus forms a resonator which can be quickly and completely controlled. When the mouth is adjusted so as to produce the broad A, as in *father*, it has then a sort of funnel shape, with the wide part outward; for O, as in *more*, the effect is like that of a bottle with a wide neck; and for U, as in *poor*, it is that of a similar bottle with a narrow neck. For the other vowels, such as A, E, and I, the effect is as if the bottle were prolonged by a tube, formed by contracting the tongue against the palate.

If now, while the mouth is adjusted for the position in which it could utter the vowel U, different vibrating tuning-forks are successively held in front of it, only that emitting the note f will be found to be reinforced by the enclosed column of air vibrating in unison with it. This is accordingly the characteristic note of that vowel; in like manner, b' is the note for O, and b'' that for A. The other vowel sounds, such as I, have a higher and lower characteristic note; thus those of A, as in *day*, are d and a''', of I, f and d^{iv}. In most cases, however, the deeper notes have but little influence.

260. **Perception of sounds. The ear.**—The organ of hearing in man consists of several structures; there is first the outer ear (fig. 235) by which the sound is collected and transmitted through the auditory passage, *a*, to the *drum* or *tympanum*, *t*. This is a delicate tightly stretched membrane or skin which separates the outer ear from the middle ear or *tympanic cavity*, which is a cavity in the temporal bone in which are several small bones whose dimensions are considerably exaggerated in the figure. One of these, the *hammer*, *d*, is attached at one end to the *drum*, and at the other is jointed to the *anvil*, *e*; the latter is connected by means of the *stirrup bone*, *f*, to the *oval window*, an aperture closed by a

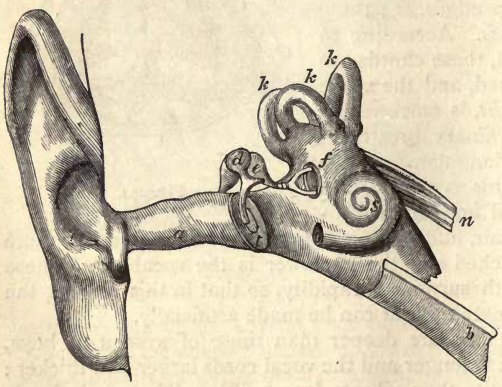


Fig. 235.

fine membrane, and which separates the tympanic cavity from the *labyrinth*. The tympanic cavity is also connected by the *Eustachian tube*, *b*, with the cavity of the mouth, so that the air in it is always under the same pressure.

The labyrinth is a complicated structure filled with fluid; it is entirely of bone, with the exception of the oval window already mentioned and the *round window*, *o*. The labyrinth consists of three parts: the *vestibule*, which is closed by the oval window; the three semicircular canals, *k*; and the spiral-shaped *cochlea*, or snail shell, *s*. This is separated throughout its entire length by a division partly of bony projection and partly of membrane; the upper part of this division is connected with the vestibule, and therefore with the oval window, while the lower part is connected with the round window. In the labyrinthine fluid of this part the termination of the auditory nerve is spread, the other end leading to the brain.

The membranous part of this diaphragm is lined with about 3,000 extremely minute fibres, which are the terminations of the acoustic nerve *n*. Each of these, which are called *Corti's fibres*, seems to be tuned for a particular note as if it were a small resonator. Thus when the vibrations of any particular note reach these fibres, through the intervention of the stirrup bone and the fluid of the labyrinth, one fibre or set of fibres only vibrates in unison with this note, and is deaf for all others. Hence each simple note only causes one fibre to vibrate, while compound notes cause several; just as when we sing with a piano, only the fundamental note and its harmonics vibrate. Thus, however complex external sounds may be, these microscopic fibres can analyse them and reveal the constituents of which they are formed.

261. **Interference of sound.**—If two waves of sound of the same length proceed in the same direction, and if they coincide in their phases, they strengthen one another; if, however, their phases differ by half a wave-length, they neutralise each other, and silence is the result. This is called the *interference of sound*.

It may be illustrated by a number of experiments, of which that represented in fig. 236 is one of the simplest and most convenient. Two T-shaped glass tubes, *obac* and *nedf*, are connected at one end by a short india-rubber tube *ad*, while at the other ends they are connected by a long india-rubber tube, *cqf*. The end *o* provided with a caoutchouc tube is held in one ear, the other ear being closed, and a tuning-fork is sounded in front of the long free tube, *nrs*. If the length of the india-rubber tube *cqf* be half the wave-length of the note produced by the fork, the sounds will reach the ear in completely opposite phases; they will accordingly neutralise each other and no sound will be heard. But if this india-rubber tube is closed by pinching it, the note is at once heard. If the tuning-fork gives the note *c*, the note it produces makes 528 vibrations in a second, and the length of the tube should be 34 centimetres.

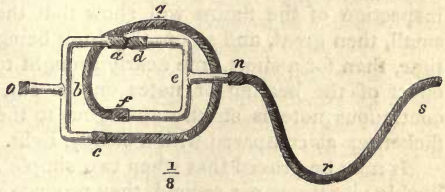
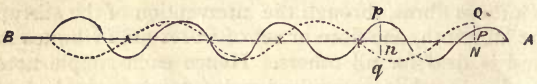


Fig. 236.

262. **Beats.**—If the notes are different and are not quite in the same phase, they alternately weaken and strengthen each other; they are said to *beat* with one another. This may be explained as follows:—Suppose AB, in fig. 237, to be a row of particles transmitting the sound: suppose the vibrations producing the one note to be indicated by the continuous curved line; then, on the one hand, the ordinates of the different points of AB give the velocities with which those points are *simultaneously* moving, and, on the other hand, each point will have *successively* the different velocities represented by the successive ordinates. In like manner, let the dotted line show the vibrations which produce the second note. And, for the sake of distinctness, suppose the number of vibrations in a second producing the former note to be to that producing the latter in the ratio of 3 : 2. Now, let us consider any point which when at rest occupies the position N; draw the ordinate, cutting the former curve in P and the latter in Q. If the notes were sounded separately, the velocity of N at a given distance produced by the former note would be PN, and that of N at the same instant produced by the latter note would be QN. Consequently, as they are sounded together, the actual velocity of N at the given instant is the sum of these, or PN + QN. If at the same instant we consider the point *n*, its velocity will consist of *pn* and *nq* jointly; but, as these are in opposite directions, its actual amount will be *pn* − *nq*. Hence the actual velocity resulting from the coexistence of the two notes will be indicated by the curve in fig. 238, whose ordinates equal the (algebraical) sum of the corresponding ordinates of the two curves in fig. 237; that is, if AN, A*n*, . . . represent equal distances in both figures, the curve is described by taking RN equal to PN + QN, *rn* equal to *pn* − *qn*, and so on. This curve shows by its successive ordinates the simultaneous velocities of the different particles of AB, and the successive velocities communicated to the drum of the ear. An inspection of the figure will show that the velocities are first great, then small, then great, and so on, the drum being first moved rapidly for a short time, then for a short time nearly brought to rest, and so on. In short, the effect of the beating of notes on the ear, as compared with that of a continuous note, is strictly analogous to the effect produced on the eye by a flickering as compared with a steady, light.

Fig. 237.



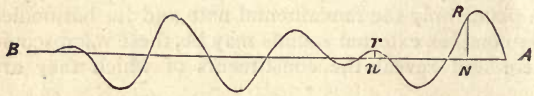


Fig. 238.

It may be proved that when two simple notes are produced by *m* and *n* double vibrations per second, they produce *m* − *n* beats per second; thus, if C is produced by 128, and D by 144, double vibrations per second, on being sounded together they will produce 16 beats per second. It has been ascertained that the beats produced by two notes are not audible unless the ratio *m* : *n* is less than the ratio 6 : 5. Hence, in the case represented by fig. 237, though the alternations of intensity exist, they would not be audible. Also, if the notes have very different intensities, the intensity of the beat is very much disguised.

It is found that when beats are fewer than 10 per second or more than 70 per second they are disagreeable, but not to the extent of producing discord. Beats from 10 to 70 per second may be regarded as the source of all discord in music, the maximum of dissonance being attained when about 30 beats are produced in a second. For example, if *c* and B are sounded together the effect is very discordant, the interval between those notes being 16 : 15, so that the beats are audible, and the number of beats per second being 16. On the other hand, if C, E, and G are sounded together there is no dissonance : but if C, E G, B are sounded together the discord is very marked, since C produces *c*, which is discordant with B. It will be remarked that C, E, G is a major triad, while E, G, B is a minor triad.

A compound musical note, being composed of simple notes represented by 1, 2, 3, 4, 5, 6, 7, &c., does not give rise to any simple notes capable of producing an audible beat up to the seventh—the sixth and seventh are the first that produce an audible beat. It is for this reason that there is no trace of roughness in a compound note, unless the seventh harmonic be audible.

If we were to represent graphically a compound note, we should proceed to construct a curve out of simple notes of different intensities in the same manner as fig. 238 is constructed from two simple notes of equal intensity represented by fig. 237. It is evident that the resulting curve will take different *forms* according to the presence or absence of different harmonics and their different intensities ; in other words, the quality or timbre of the notes produced by different instruments will depend upon the *form* of the vibrations producing the sound.

Beats not too fast to be readily counted arise between adjacent notes in the lower octaves of large organs. They are also met with in the sounds of church bells, and in those emitted by telegraph wires when vibrating powerfully in a strong wind. They are heard very distinctly in the latter case by pressing one ear against a telegraph-post and closing the other.

By means of beats, the notes emitted by two musical instruments may be brought into very accurate unison, by continuing the tuning until the beats disappear. In order to make tuning-forks produce the normal number of 440 vibrations, on auxiliary tuning-fork is used which makes 436 vibrations ; each of the forks under experiment must then make with this 4 beats in a second, which can be controlled with very great accuracy.

263. **Combinational notes.**—Besides the beats produced when two musical notes are sounded together, there is another and distinct phenomenon, which may be thus described :—Suppose two simple notes to be simultaneously produced by *n* and *m* vibrations per second. It has been shown by Von Helmholtz that they generate a series of other notes. The principal one of these, which may be called the *differential note*, is produced by *n*—*m* vibrations per second. Its intensity is usually very small, but it is distinctly audible in beats. It has been called the *grave harmonic*, as its pitch is generally much lower than that of the notes by which it is generated. It has been supposed to be caused by the beats becoming too numerous to be distinguished, and coalescing into a continuous sound, and this supposition was countenanced by the fact that its pitch is the same as the beat number. The supposition is shown to be erroneous, first by the existence of the

differential tones for intervals that do not beat; and, secondly, by the fact that, under certain circumstances, both the beats and the differential tones may be heard together.

264. **The physical constitution of musical chords.**—Let us suppose two compound notes to be sounded together, say C and G; then we obtain two series of notes each consisting of a primary and its harmonics, namely, denoting C by 4, the two series, 4, 8, 12, 16. . . . and 6, 12, 18, 24, &c. Now, if, instead of producing the two notes C and G, we had sounded the octave below C, we should have produced the series, 2, 4, 6, 8, 10, 12, 14, 16, 18, &c. It is plain that the two former series when joined differ from the last in the following respects :—(*a*) The primary note 2 is omitted. (*b*) In the case of the last series, the consecutive notes continually decrease in intensity; whereas in the two former series, 4 and 6 are of the same intensity, 8 is of lower intensity, but the two 12's will strengthen each other, and so on. (*c*) Certain of the harmonics of the primary 2 are omitted; for example, 10, 14, &c., do not occur in either of the two former series. In spite of these differences, however, the two compound notes affect the ear in a manner very closely resembling a single compound note; in short, they coalesce into a single note with an artificial colour. It may be added that in the case above taken C and G produce as a combination note 2 (that is, $6-4$); so that, strictly speaking, the 2 is not wanted in the series produced by C and G, only it exists in very diminished intensity. The same explanation will apply to all possible chords; for example, in the case of the major chord, C, E, G, we have a note of artificial colour expressed by the series of simple tones, 4, 5, 6, 8, 10, 12, 15, 16, 18, &c., together with the combination notes, 1, 1, 2. It will be remarked that in the whole of this series there are no dissonant notes introduced, except 15, 16, and 16, 18, and this dissonance will be inappreciably slight, since 15 is the third harmonic of 5, and 16 the fourth harmonic of 4, so that their intensities will be different, as also will be the intensities of 16 and 18. On the other hand, nearly all the notes which form a *natural* compound note are present, namely, there are 1, 2, 4, 5, 6, 8, 10, 12, &c., in place of 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, &c. In short, the major triad differs only from a *natural* compound note in that it consists of a series of simple notes of different intensities, and omits those which, by beating with the neighbouring note, would produce dissonance; for example, 7, which would beat with 6 and 8; 9, which would beat with 8 and 10; and 11, which would beat with 10 and 12. It is this circumstance which renders the major chord of such great importance in harmony. If the constituents of the minor chord are similarly discussed, namely, three compound tones whose primaries are proportional to 10, 12, 15, it will be found to differ from the major chord in the following principal respects :—(*a*) The primary of the natural tone to which it approximates is very much deeper than that of the corresponding major chord. (*b*) It introduces the *differential* notes, 2, 3, 5, which form a major chord. Now it has already been remarked that when a major and minor chord are sounded together they are distinctly dissonant; for example, when C, E, G, A are sounded together. Accordingly, the fact of the differential notes forming a major chord shows that an elementary dissonance exists in every minor chord.

CHAPTER IV

VIBRATIONS OF STRETCHED STRINGS AND OF COLUMNS OF AIR

265. **Vibrations of strings.**—By a *string* is meant the string of a musical instrument, such as a violin, which is stretched by a certain force, and is commonly of catgut, or is a metal wire. The vibrations which strings experience may be either *transverse* or *longitudinal*; but practically the former are alone important. *Transverse vibrations* may be produced by drawing a bow across the string, as in the case of the violin; or by striking the string, as in the case of the pianoforte; or by pulling it transversely, and then letting it go suddenly, as in the case of the guitar and harp.

266. **Sonometer.**—The *sonometer* is an apparatus by which the transverse vibrations of strings may be studied. It is also called the *monochord*,

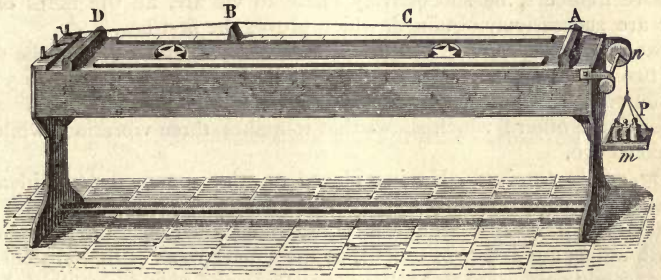


Fig. 239.

because it has often only one string. In addition to the string, it consists of a box of thin wood, which has the effect of strengthening the sound; this it does by presenting a far larger area to the air that the string itself. On this there are two fixed bridges, A and D (fig. 239), over which and over the pulley n passes the string, which is usually a metal wire. This is fastened at one end, and stretched at the other by weights, P, which can be increased at will. By means of a third movable bridge, B, the length of that portion of the wire which is to be put in vibration can be altered at pleasure.

267. **Laws of the transverse vibrations of strings.**—If l be the length of a string—that is, the vibrating part between two bridges, A and B (fig. 239)—r the radius of the string, d its density, P the stretching weight, and n the number of vibrations per second, it is found by calculation that $n = \frac{1}{2rl}\sqrt{\frac{Pg}{\pi d}}$; π being the ratio of the circumference to the diameter, g the acceleration of gravity.

This formula expresses the following laws :—

I. *The stretching weight or tension being constant, the number of vibrations in a second is inversely as the length.*

II. *The number of vibrations in a second is inversely as the diameter of the string.*

III. *The number of vibrations in a second is directly as the square root of the stretching weight or tension.*

IV. *The number of vibrations of a string in a second is inversely as the square root of its density.*

These laws are applied in the construction of stringed instruments, in which the length, diameter, tension, and material of the strings are so chosen that given notes may be produced from them.

268. **Experimental verification of the laws of the transverse vibration of strings.**—*Law of the lengths.* In order to prove this law, we may call to mind that the relative numbers of vibrations of the notes of the gamut are

C	D	E	F	G	A	B	c
1	$\frac{9}{8}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{5}{3}$	$\frac{15}{8}$	2

If now the entire length of the sonometer be made to vibrate, and then, by means of the bridge B, the lengths $\frac{8}{9}$, $\frac{4}{5}$, $\frac{3}{4}$, $\frac{2}{3}$, $\frac{3}{5}$, $\frac{8}{15}$, $\frac{1}{2}$, which are the inverse of the above numbers, be successively made to vibrate, all the notes of the gamut are successively obtained, which proves the first law.

Law of the diameters. This law is verified by stretching upon the sonometer two cords of the same material, the diameters of which are as 3 to 2, for instance. When these are made to vibrate, the second cord gives the *fifth* above the other; which shows that it makes three vibrations while the first makes two.

Law of the tensions. Having placed on the sonometer two identical strings, they are stretched by weights which are as 4 : 9. The second now gives the fifth of the first, from which it is concluded that the numbers of their vibrations are as 2 : 3; that is, as the square roots of the tensions. If the two weights are as 16 to 25, the major third or $\frac{5}{4}$ would be obtained.

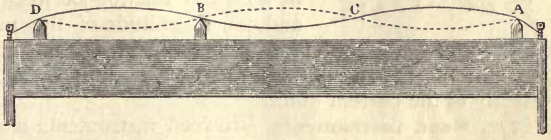
Law of the densities. Two strings of the same radius, but different densities, are fixed on the sonometer. Having been subjected to the same stretching weight, the position of the movable bridge on the denser one is altered until it is in unison with the other string. If then d and d' are the densities of the two strings, and l and l' the lengths which vibrate in unison, we find $\frac{l}{l'} = \frac{\sqrt{d}}{\sqrt{d'}}$. But as we know from the first law that $\frac{l}{l'} = \frac{n'}{n}$, we have $\frac{n}{n'} = \frac{\sqrt{d'}}{\sqrt{d}}$, which verifies this law. Thus, if a copper wire whose density is 9, and a catgut string of the density 1, are of equal length and diameter, and are stretched by the same weight, the vibrations of the copper wire will be one-third as rapid as those of the string.

The laws of vibrating strings presuppose that they are long, flexible, and tightly stretched; but if they are short, stout, and but little stretched, the *rigidity* of the string comes into play, and the number of vibrations they make is higher than the theoretical number; the effect of the rigidity is the same as if a constant weight were added to the stretching weight.

269. **Nodes and loops.**—Let us suppose the string AD (fig. 239) to begin vibrating, the ends A and D being fixed, and, while it is doing so, let a point B be brought to rest by a stop, and let us suppose DB to be one-third part of AD. The part DB must now vibrate about B and D as fixed points in the manner indicated by the continuous and dotted lines (fig. 240); now all parts of the same string tend to make a vibration in the same time; accordingly, the part between A and B will not perform a single vibration, but will divide into two at the point C, and vibrate in the manner shown in the figure. If BD were one-fourth part of AD (fig. 241), the part AB would be subdivided at C and C′ into three vibrating portions each equal to BD. The points B, C, C′ are called *nodes* or *nodal points*; the middle point of the part of the string between any two consecutive nodes is called a *loop* or *ventral segment.* It will be remarked that the ratio of BD : BA must be that of some two whole numbers, for example; 1 : 2, 1 : 3, 2 : 3, &c.; otherwise the nodes cannot be formed, since the two portions of the string cannot then be made to vibrate at the same time, and the vibrations will interfere with and soon destroy one another.

If now we refer to fig. 240, the existence of the node at C can be easily proved by bending some light pieces of paper and placing them as riders on the string, say three pieces, one at C and the others respectively midway between B and C and between C and A. The one at C experiences only a very slight motion, and remains in its place, thereby proving the existence of a node at C; the other two are violently shaken, and in most cases thrown off the string.

Fig. 240.



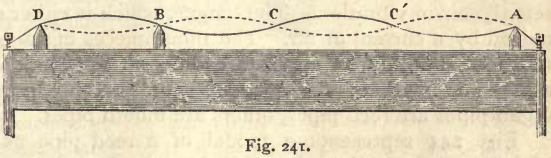


Fig. 241.

When a musical string vibrates between fixed points, A and B, its motion is not quite so simple as might be inferred from the above description. In point of fact, partial vibrations are soon produced, and superimposed upon the primary vibrations. The partial vibrations correspond to the half, third, fourth, &c., parts of the string. It is by these partial vibrations that the harmonics are produced which accompany the fundamental note due to the primary vibrations; they are usually, however, so feeble as to be imperceptible to ordinary ears.

270. **Wind instruments.**—In the cases hitherto considered, the sound results from the vibrations of solid bodies, and the air only serves as a vehicle for transmitting them. In wind instruments, on the contrary, when the sides of the tube are of adequate thickness, the enclosed column of air is the sounding body. In fact, the substance of the tubes is without influence on the fundamental note; with equal dimensions, it is the same whether the tubes are of glass, of wood, or of metal. These different materials simply do no

more than give rise to different harmonics, and thereby impart a different quality to the compound tone produced.

In reference to the manner in which the air in tubes is made to vibrate, wind instruments are divided into *mouth* instruments and *reed* instruments.

271. **Mouth instruments.**—In mouth instruments all parts of the mouthpiece are fixed. Fig. 243 represents the mouthpiece of an organ pipe, and fig. 242 that of a whistle, or of a flageolet. In both figures, the aperture *ib* is called the mouth ; it is here that air enters the pipe ; *b* and *o* are the *lips*, the upper one of which is bevelled. The mouthpiece is fixed at one end of a tube, the other end of which may be either opened or closed. In fig. 243 the tube can be fitted on a wind-chest by means of the foot P.

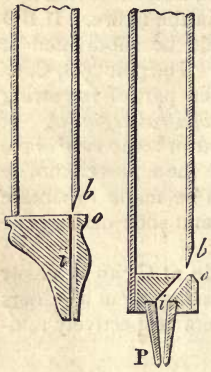


Fig. 242. Fig. 243.

When a rapid current of air enters by the mouth, it strikes against the upper lip, and a shock is produced which causes the air to issue from *bo* in an intermittent manner. In this way pulsations are produced which, transmitted to the air in the pipe, make it vibrate, and a sound is the result. In order that a pure note may be produced, there must be a certain relation between the form of the lips and the magnitude of the mouth ; the tube also ought to have a great length in comparison with its diameter. The number of vibrations depends in general on the dimensions of the pipe and the velocity of the current of air.

272. **Reed instruments.**—In reed instruments a simple elastic tongue sets the air in vibration. The tongue, which is either of metal or of wood, is moved by a current of air. The mouthpieces of the oboe, the bassoon, the clarionet, the child's trumpet, are different applications of the reed, which, it may be remarked, is seen in its simplest form in the Jew's-harp. Some organ pipes are reed pipes, others are mouth pipes.

Fig. 244 represents a model of a reed pipe as commonly shown in lectures. It is fixed on the wind-chest Q of a bellows, and the vibrations of the reed can be seen through a glass plate, E, fitting into the sides. A wooden horn, H, strengthens the sound.

Fig. 245 shows the reed out of the pipe. It consists of four pieces : 1st, a rectangular wooden tube closed below and open above at *o* ; 2nd, a copper plate, *cc*, forming one side of the tube, and in which there is a longitudinal aperture, through which air passes from the tube MN to the orifice *o* ; 3rd, a thin elastic plate, *i*, called the *tongue*, which is fixed at its upper end, and which grazes the edge of the longitudinal aperture, nearly closing it ; 4th, a curved wire, *r*, which presses against the tongue, and can be moved up and down. It thus regulates the length of the tongue and determines the pitch of the note. It is by this wire that reed pipes are tuned. The reed being replaced in the pipe MN, when a current of air enters by the foot P, the tongue is compressed, it bends inwards, and affords a passage to air, which escapes by the orifice *o*. But, being elastic, the tongue regains its original position, and, performing a series of oscillations, successively opens and closes

the orifice. In this way sound waves result and produce a note whose pitch increases with the velocity of the current.

In this reed the tongue vibrates alternately before and behind the aperture, and just escapes grazing the edges, as is seen in the harmonium, concertina, &c.; such a reed is called a *free reed.* But there are other reeds, called *beating* or *striking reeds*, in which the tongue, which is larger than the orifice, strikes against the edges at each oscillation, closing it like a flap. The reed of the clarionet, represented in fig. 246, is an example of this; it is kept in its place by the pressure of the lips. The reeds of the oboe and bassoon are also of this kind.

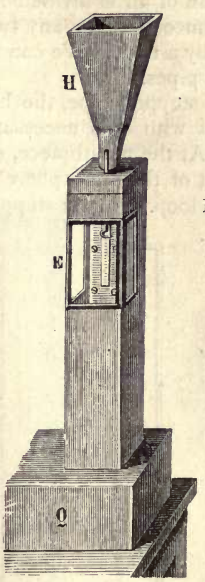


Fig. 244.

Fig. 245.

Fig. 246.

273. **Of the notes produced by the same pipe.**—Daniel Bernouilli discovered that the same organ pipe can be made to yield a succession of notes by properly varying the force of the current of air. The results he arrived at may be thus stated :—

i. If the pipe is open at the end opposite to the mouthpiece, then, denoting the fundamental note by 1, we can, by gradually increasing the force of the current of air, obtain successively the notes 2, 3, 4, 5, &c.; that is to say, all the *harmonics* of the primary note.

ii. If the pipe is closed at the end opposite to the mouthpiece, then, denoting the fundamental note by 1, we can, by gradually increasing the force of the current of air, obtain successively the notes 3, 5, 7, &c.; that is to say, only the uneven *harmonics* of the primary note.

A closed and an open pipe yield the same fundamental note if the closed pipe is half the length of the open pipe, and if in other respects they are the same; or, what is an equivalent statement, with a closed and an open pipe of the same length the former gives a note an octave higher than the latter.

In any case, it is impossible to produce from the given pipe a note not included in the above series respectively.

Although the above laws are enunciated with reference to an organ pipe, they are true of any other pipe of uniform section.

274. **On the nodes and loops of an organ pipe.**—The vibrations of the air producing a musical note take place in a direction parallel to the axis of the pipe—not transversely, as in the case of the portions of a vibrating string. In the former case, however, as well as in the latter, the phenomena of *nodes* and *loops* may be produced. But now by a *node* must be understood a section of the column of air contained in the pipe, where the particles

remain at rest, but where there are rapid alternations of *condensation* and *rarefaction*. By a *loop* or *ventral segment* must be understood a section of the column of air contained in the pipe where the vibrations of the particles of air have the greatest amplitudes, and where there is no change of density. The sections of the column of air are, of course, made at right angles to its axis. When the column of air is divided into several vibrating portions, it is found that the distance between any two consecutive loops is constant, and that it is bisected by a node. We can now consider separately the cases of the open and closed pipes.

1. In the case of a stopped pipe, the bottom is always a node, for the layer of air in contact with it is necessarily at rest, and only undergoes variations in density. At the mouthpiece, on the contrary, where the air has a constant density (that of the atmosphere), and the vibration is at its maximum, there is always a loop. In any stopped pipe there is at least one node

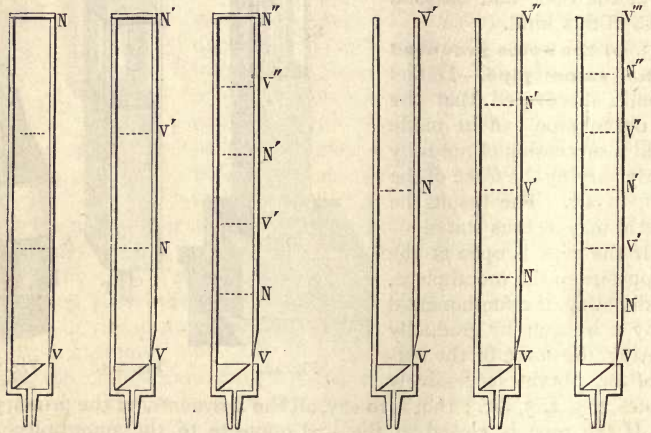


Fig. 247. Fig. 248. Fig. 249. Fig. 250. Fig. 251. Fig. 252.

and one loop (fig. 247); the pipe then yields its fundamental note, and the distance VN from the loop to the node is equal to half a condensed or rarefied wave-length.

If the current of air be forced, the mouthpiece always remains a loop, and the bottom a node, the column divides into three equal parts (fig. 248), and an intermediate node and loop are formed. The sound produced is the first harmonic. When the second harmonic (5) is produced, there are two intermediate nodes and two loops, and the tube is then subdivided into five equal parts (fig. 249), and so on.

ii. In the case of the open pipe, whatever note it produces, there must be a loop at each end, since the enclosed column of air is in contact with the external air at those points. When the primary note is produced, there will be a loop at each end, and a node at the middle section of the pipe, the nodes and loops dividing the column into *two* equal parts (fig. 250). When the first harmonic (2) is produced, there will be a loop at each end, and a loop

in the middle, the column being divided into *four* equal parts by the alternate loops and nodes (fig. 251). When the second harmonic (3) is produced, the column of air will be divided into *six* equal parts by the alternate nodes and loops, and so on (fig. 252). It will be remarked that the successive modes of division of the vibrating column are the only ones compatible with the alternate recurrence at equal intervals of nodes and loops, and with the occurrence of a loop at each end of the pipe.

There are several experiments by which the existence of nodes and loops can be shown.

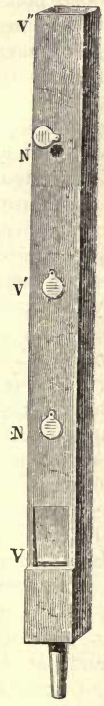


Fig. 253.

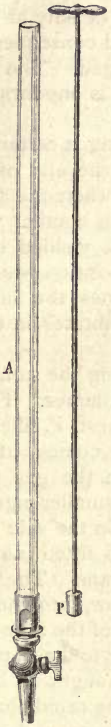


Fig. 254.

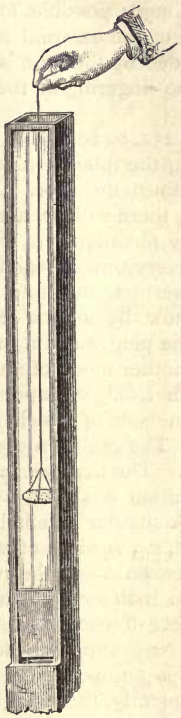


Fig. 255.

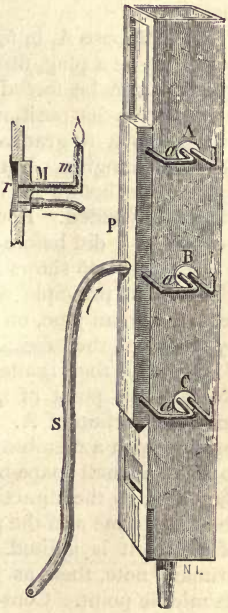


Fig. 256.

(*a.*) If a fine membrane is stretched over a pasteboard ring, and has sprinkled on it some fine sand, it can be gradually let down a tube, as shown in fig. 255. Now, suppose the tube to be producing a musical note. As the membrane descends, it will be set in vibration by the vibrating air. But when it reaches a node it will cease to vibrate, for there the air is at rest. Consequently, the grains of sand, too, will be at rest, and their quiescence will indicate the position of the node. On the other hand, when the membrane reaches a loop—that is, a point where the amplitude of the vibrations of the air attains a maximum—it will be violently agitated, as will be shown

by the agitation of the grains of sand. And thus the positions of the loops can be rendered manifest.

(*b.*) Again, suppose a pipe to be constructed with holes bored in one of its sides, and these covered by little doors which can be opened and shut, as shown in fig. 253. Let us suppose the little doors to be shut and the pipe to be caused to produce such a note that the nodes are at N and N′, and the loops at V, V′, V″. At the latter points the density is that of the external air, and consequently if the door at V′ is opened no change is produced in the note. At the former points, N and N′, condensation and rarefaction are alternately taking place. If now the door at N′ is opened, this alternation of density is no longer possible, for the density at this open point must be the same as that of the external air, and consequently N′ becomes a loop, and the note yielded by the tube is changed. The change of notes produced by changing the fingering of the flute is one form of this experiment.

(*c.*) Suppose A, in fig. 254, to be a pipe emitting a certain note, and suppose P to be a plug, fitting the tube, fastened to the end of a long rod by which it can be forced down the tube. Now, when the plug is inserted, whatever be its position, there will be a node in contact with it. Consequently, as it is gradually forced down, the note yielded by the pipe will keep on changing. But every time it reaches a position which was occupied by a node before its insertion, the note becomes the same as the note originally yielded. For now the column of air vibrates in exactly the same manner as it did before the plug was put in.

(*d.*) Fig. 256 shows another mode of illustrating the same point, which is identical in principle with König's manometric flames. The figure represents an organ pipe, on one side of which is a chest, P, filled with coal gas, by means of the tube S. The gas from the chest comes out in three jets, A, B, C, and is then ignited. The manner in which the gas passes from the chest to the point of ignition is shown in the smaller figure, which is an enlarged section of A. A circular hole is bored in the side of the pipe and covered with a membrane, *r*. A piece of wood is fitted into the hole so as to leave a small space between it and the membrane. The gas passes from the chest, in the direction indicated by the arrow, into the space between the membrane and the piece of wood, and so out of the tube *m*, at the mouth of which it is ignited. Now suppose the pipe to be caused to yield its primary note, then, as it is an open pipe, there ought to be a node at B, its middle point. Consequently, there ought to be rapid changes of density at B; these would cause the membrane *r* to vibrate, and thereby blow out the flame *m*, and this is what actually happens. If by increasing the force of the wind the octave to the primary note is produced, B will be a loop, and A and C nodes. Consequently, the flames at A and C will now be extinguished, as is, in point of fact, the case. But at B, there being no change of density, the membrane is unmoved, and the flame continues to burn steadily.

By each and all of these experiments it is shown that in a given pipe, whether open or closed, there are always a certain number of nodes, and midway between any two consecutive nodes there is always a *loop* or *ventral segment*.

275. **Formulæ relative to the number of vibrations produced by a musical pipe.**—It follows from what has been said that the column of air in stopped pipes is always divided by the nodes and loops into an uneven number of parts which are equal to each other, and each of which is a quarter of a complete vibration (figs. 247, 248, and 249), while in an open pipe it is divided into an even number of such parts (figs. 250, 251, 252). If L be the length of the pipe, l the wave-length of the sound which it emits, and p any whole number, then for stopped pipes we have $L = (2p + 1)\frac{l}{4}$; and for open pipes $L = 2p\frac{l}{4} = \frac{pl}{2}$. Replacing in each of these formulæ l by its value $\frac{v}{n}$ (253), we have $L = (2p + 1)\frac{v}{4n}$ and $L = \frac{pv}{2n}$; from which for stopped pipes we have $n = \frac{(2p + 1)v}{4L}$, and for open ones $n = \frac{pv}{2L}$.

The laws connecting the length of pipes with the note produced only hold for narrow pipes—those, for instance, whose length is not less than 12 times their diameter; for shorter pipes organ-builders have various empirical rules. Within wide limits the formula holds, $L' = L - \frac{5}{6}d$, where L is the theoretical length, L′ the length sought, while d is the diameter of the round pipe.

If, in the first formula, we give to p the successive values 0, 1, 2, 3, 4, &c., we have $n = \frac{v}{4L}, \frac{3v}{4L}, \frac{5v}{4L}$, that is, the fundamental sound and all its uneven harmonics; and in the formula for the open pipe we get similarly $\frac{v}{2L}, \frac{2v}{2L}, \frac{3v}{2L}$, &c., that is, the fundamental note and all its harmonics even and uneven.

276. **Explanation of the existence of nodes and loops in a musical pipe.**—The existence of nodes and loops is to be explained by the co-existence in the same pipe of two equal waves travelling in contrary directions.

Let A (fig. 257) be a point from which a series of waves sets out towards B, and let the length of these waves, whether of condensation or rarefaction,

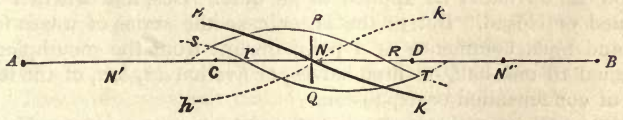


Fig. 257.

be AC, CD, DB. And let B be the point from which the series of exactly equal waves sets out towards A. It must be borne in mind that in the case of a wave of condensation originating at A the particles move in the direction A to B, but in a wave of condensation originating at B they move in the direction B to A. Now let us suppose that condensation at C, caused by the wave from A, begins at the same instant that condensation caused by the wave from B begins at D. Consequently, restricting our attention to the particles in the line CD, at any instant the velocities of the particles in CD due to the former wave will be represented by the ordinates of the curve

SPRT, while those due to the wave from B will be represented by the co-ordinates of the curve TQ*r*S. Then, since the waves travel with the same velocity, and are at C and D respectively at the same instant, we must have, for any subsequent instant, CR equal to D*r*. If, therefore, N is the middle point between C and D, we must have *r*N equal to RN, and consequently PN equal to QN ; that is to say, if the particle at N transmitted only one vibration, its motion at each instant would be in the opposite phase to that of its motion if it transmitted only the other vibration. In other words, the particle N will at every instant tend to be moved with equal velocity in opposite directions by the two waves, and therefore will be *permanently* at rest. That point is therefore a *node*. In like manner, there is a node at N′ midway between A and C, and also at N″ midway between B and D. In regard to the motion of the remaining particles, it is plain that their respective velocities will be the (algebraical) sum of the velocities they would at each instant receive from the waves separately. Hence, at the instant indicated by the diagram, they are given by the ordinates of the curve HNK. This curve will change from instant to instant, and at the end of the time occupied by the passage of a wave of condensation (or of rarefaction) from C to D will occupy the position shown by the dotted line *h*N*k*. It is evident therefore that particles near N have but small changes of velocity, whilst those near C and D experience large changes of velocity.

If the curve HK were produced both ways, it would always pass through N′ and N″ ; the part, however, between N and N′ would sometimes be on one side, and sometimes on the other side of AB. Hence all the particles between N′ and N have simultaneously, first a motion in the direction A to B, and then a motion in the direction B to A, those particles near C having the greatest amplitude of vibrations. Accordingly near N and N′ there will be alternately the greatest condensation and rarefaction.

This explanation applies to the case in which AB is the axis of an open organ pipe, A being the end where the mouthpiece is situated. The waves from B have their origin in the reflections of the series of waves from A. In the particular case considered, the note yielded by the pipe is that indicated by 3 ; that is, the fifth above the octave to the primary note. A similar explanation can obviously be applied to all other cases, and whether the end be opened or closed. But in the latter case the series of waves from the closed end must commence at a point distant from the mouthpiece by a space equal to one half, or three halves, or five halves, &c., of the length of a wave of condensation or expansion.

277. **Kundt's determination of the velocity of sound.**—Kundt has devised a method of determining the velocity of sound in solids and in gases which can be easily performed by means of simple apparatus, and is capable of great accuracy. A glass tube, BB′, about two yards long (fig. 258) and two inches in internal diameter, is closed at one end by a movable stopper, *b* ; the other end is fitted with a cork, KK, which tightly grasps a glass tube, AA′, the same length, but of smaller diameter. This is closed at one end by a piston, *a*, which moves with gentle friction in the outer tube, BB′. Then by rubbing the free end of the tube, AA, with a wet cloth, it produces longitudinal vibrations, and these transmit their motion to the air in the tube *ab*. If the tube *ab* contain some lycopodium powder, or, still

better, powdered cork, this is set in active vibration and then arranges itself in small patches in a certain definite order, as represented in the figure, the nature and arrangement of which depend on the vibrating part of the rod and the tube.

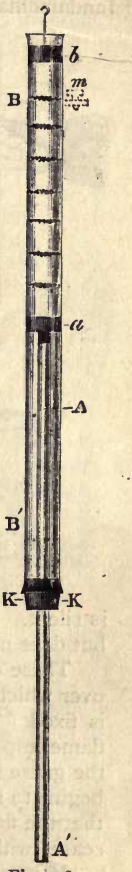


Fig. 258.

These heaps represent the nodes, and the mean distance d between them can be measured with great accuracy; it represents the distance between two nodes, or, half a wave-length; that is, the wave-length of the sound in air is $2d$. If the rod has the length s and is grasped in the middle by the cork KK, by the law of the longitudinal vibrations of rods (281), the wave-length of the sound it then emits is twice its length, or $2s$. That is, the wave-length of the vibrating column of air is to that in the rod as $2d : 2s$. As the velocity of sound in any body is equal to the wave-length in that body multiplied by the number of vibrations in a second; and since the number of vibrations is here the same in both cases, for the note is the same, the velocity of sound in the glass is to the velocity of sound in air as $2sn : 2dn$, that is, as $s : d$. Thus when the glass tube was clamped in the middle by KK, so that the length ab was equal to half the length of the tube AA′, the number of the ventral segments was found to be eight. This corresponds to a ratio of wave-length of 1 to 16; in other words, the velocity of sound in glass is 16 times greater than that in air.

The method is capable of great extension. By means of the stopcock m, different gases could be introduced instead of air, and corresponding differences found for the length of the ventral segments; from which, by a simple calculation, the corresponding velocities were found. Thus the velocities of sound in carbonic acid, coal gas, and hydrogen were found to be respectively 0·8, 1·56, and 3·56 that of air, or nearly as the inverse squares of the densities.

So, also, by varying the material of the rod AA′, different velocities are obtained. Thus the velocity in steel was found to be 15·24, and in brass 10·87 that of air.

Kundt's figures may likewise be obtained by providing glass tubes a yard or two in length with lycopodium powder, as in the above experiment, and hermetically sealing them at both ends. The tubes are then put into longitudinal vibrations; instead of air they may be filled with hydrogen or any other gas.

Using this method, with iron filings instead of lycopodium, Kundt and Lehmann determined the velocity of sound in water contained in glass tubes of various diameters and thicknesses; the thicker the tubes and the smaller their diameter, the more nearly do the results agree with those required by theory and with those obtained by Colladon and Sturm (234).

By surrounding the tube with a jacket which can be raised to various constant temperatures by the vapours of different liquids, the velocity of sound in such vapours may be determined.

278. **Chemical harmonicon.**—The air in an open tube may be made to give a sound by means of a luminous jet of hydrogen, coal gas, &c. When

a glass tube about 12 inches long is held over a lighted jet of hydrogen (fig. 259), a note is produced, which, if the tube is in a certain position, is the fundamental note of the tube. The sounds are considered to arise from the successive, exceedingly rapid explosions produced by the periodic combinations of the atmospheric oxygen with the issuing jet of hydrogen. The apparatus is called the *chemical harmonicon.*

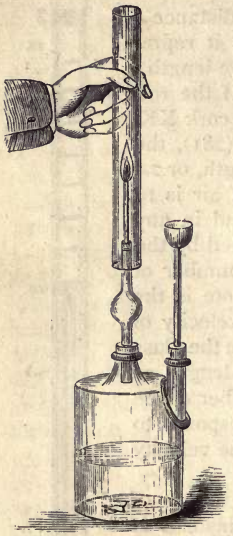


Fig. 259.

The note depends on the size of the flame and the length of the tube : with a long tube, by varying the position of the jet in the tube, the series of notes, in the ratio 1 : 2 : 3 : 4 : 5, is obtained.

If, while the tube emits a certain sound, the voice or the sirene (242) be gradually raised to the same height, as soon as the note is nearly in unison with the harmonicon, the flame becomes agitated, jumps up and down, and is finally steady when the two sounds are in unison. If the note of the sirene is gradually heightened the pulsations again commence ; they are the optical expressions of the beats (262) which occur near perfect unison.

If, while the jet burns in the tube and produces a note, the position of the tube is slightly altered, a point is reached at which no sound is heard. If now the voice, or the sirene, or the tuning-fork, be pitched at the note produced by the jet, it begins to sing, and continues to sing even after the sirene is silent. A mere noise, or shouting at an incorrect pitch, agitates the flame, but does not cause it to sing.

These effects may also be conveniently studied by means of a gas-burner, over which, at a distance of four inches, a ring covered with fine wire gauze is fixed. The gas is lighted above the gauze, and forms a very sensitive flame, especially when a moderately wide tube is held over the gauze. If the gauze is raised with the tube, the flame becomes duller and smaller, but begins to sound with a uniform loud tone. If now the gauze is lowered so that the flame is just silent, it begins at once when a sound is produced, but ceases with the sound.

If a metal tube 4 cm. wide and 15 to 20 cm. high, closed at the bottom by a wire gauze, is held vertically over a Bunsen's jet, an acute sound is heard, almost as loud as the whistle of a locomotive, on lighting the gas inside the tube.

Such sensitive flames are very conveniently produced by means of coal gas stored in pressure bags, connected by means of a caoutchouc tube with a slender conical brass tube, ending in a steatite burner having a fine aperture. Pressure is applied to the bags and the supply of gas is regulated by a tap. In this way the gas when ignited may be made to burn with a flame A, a couple of feet in length, which is highly sensitive ; the slightest noise in the room, the crumpling off paper for example, causes this form to change to that of B ; the rattling of keys or the sound of a whistle causes it to shrink still more.

279. **Stringed instruments.**—Stringed musical instruments depend on the production of transverse vibrations. In some, such as the piano, the sounds are *constant*, and each note requires a separate string; in others, such as the violin and guitar, the sounds are *varied* by the fingering, and can be produced by fewer strings.

In the piano the vibrations of the strings are produced by the stroke of the *hammer*, which is moved by a series of bent levers communicating with the keys. The sound is strengthened by the vibrations of the air in the sounding board on which the strings are stretched. Whenever a key is struck, a *damper* is raised which falls when the finger is removed from the key, and stops the vibrations of the corresponding string. By means of a *pedal* all the dampers can be simultaneously raised, and the vibrations then last for some time.

The *harp* is a sort of transition from the instruments with constant to those with variable sounds. Its strings correspond to the natural notes of the scale; by means of the pedals the length of the vibrating parts can be changed, so as to produce sharps and flats. The sound is strengthened by the sounding-box, and by the vibrations of all the strings harmonic with those played.

Fig. 260.

In the violin and guitar each string can give a great number of sounds according to the length of the vibrating part, which is determined by the pressure of the fingers of the left hand while the right hand plays the bow, or twitches the strings themselves. In both these instruments the vibrations are communicated to the upper face or *belly* of the sounding-box by means of the bridge over which the strings pass. These vibrations are communicated from the upper to the lower face or *back* of the box either by the sides or by an intermediate piece called the *sound-post*. The air in the interior is set in vibration by both faces, and the strengthening of the sound is produced by all these simultaneous vibrations. The value of the instrument consists in the perfection with which all possible sounds are intensified, which depends essentially on the quality of the wood, the mellowness of which increases with age, and on the relative arrangement of the parts.

The number and strength of the harmonics produced in a twitched or stroked string varies with the manner in which it is sounded, and with the nature of the string. The sharper the edge of the exciting body the shorter and broader are the waves, and therefore the higher and stronger are the harmonics and the shriller the clang; if the strings are struck with a metal rod the harmonics are so predominant that the fundamental note is scarcely heard, and thus what is called a hollow sound is produced. The tone is fullest when struck with the finger, and somewhat less so with a soft hammer, as in the piano. The deeper harmonics are often stronger than the fundamental note, so that the note is not so strong, but is richer; all the harmonics, whose nodes are in the place struck, are wanting. If a string is struck in the middle, none of the even harmonics are produced, and therefore all the octaves of the fundamental note are wanting; the tone is nasal and hollow.

This is the characteristic of a note which is wanting in the harmonics nearer and most allied to the fundamental note. If the string is struck near one end, the clang has a jingling character. Instrument makers, led by practised ears, have long found it advantageous that the piano be struck at about one-seventh of the length of the string ; the reason for this advantage lies in the fact that in this way the seventh and ninth harmonics, which are unharmonic with each other, are deadened, while the deeper harmonics—the octaves, fifths, thirds—preponderate, and the clang is rich and harmonious.

The higher harmonics fade away in gut-strings more rapidly than in metal wires ; hence the guitar and the harp are not so jingling as the zither.

280. **Wind instruments.**—All wind instruments may be referred to the different types of sounding tubes which have been described. In some, such as the organ, the notes are *fixed*, and require a separate pipe for each note, in others the notes are *variable*, and are produced by only one tube : the flute, horn, &c. are of this class.

In the *organ* the pipes are of various kinds ; namely, mouth pipes, open and stopped, and reed pipes with apertures of various shapes. By means of *stops* the organist can produce any note by both kinds of pipe.

In the *flute*, the mouthpiece consists of a simple lateral circular aperture ; the current of air is directed by means of the lips, so that it grazes the edge of the aperture. The holes at different distances are closed either by the fingers or by keys ; when one of the holes is opened, a loop is produced in the corresponding layer of air, which modifies the distribution of nodes and loops in the interior, and thus alters the note. The whistling of a key is similarly produced.

The *pandæan pipe* consists of stopped pipes of different lengths corresponding to the different notes of the gamut.

In the trumpet, the horn, the trombone, cornet-à-piston, and ophicleide, the lips form the reed, and vibrate in the mouthpiece. In the *horn*, different notes are produced by altering the distance of the lips. In the *trombone*, one part of the tube slides within the other, and the performer can alter at will the length of the tube, and thus produce higher or lower notes. In the *cornet-à-piston*, the tube forms several convolutions ; pistons placed at different distances can, when closed, cut off communication with other parts of the tube, and thus alter the length of the vibrating column of air.

CHAPTER V

VIBRATION OF RODS, PLATES AND MEMBRANES

281. **Vibration of rods.**—The term *rods* is applied in acoustics to solids whose length is considerable in proportion to their breadth and thickness; they are, nevertheless, so broad and thick that, while they have not the flexibility of strings, they are yet elastic enough to vibrate without being stretched like strings. They are ordinarily of wood, glass, metal, and more particularly of tempered steel. Like strings, they have two kinds of vibrations, *longitudinal* and *transverse*. Longitudinal vibrations are produced by fixing the rod at any part, and rubbing it lengthwise with a piece of cloth sprinkled with resin.

Fig. 261.

The point at which the rod is clamped is a node; if it is clamped in the middle and put in longitudinal vibrations, it gives the fundamental note, its wave-length being twice its own length. This is thus analogous to the case of an open pipe. If the rod is clamped at one quarter its length it gives the octave of the fundamental note.

If the same rod is clamped at both ends, and made to vibrate longitudinally, its wave-length is four times its own length; the case is analogous to that of a stopped pipe.

The elongation due to longitudinal vibration is as great as would be due to a pull of several tons. If a glass rod (fig. 261) is made to vibrate longitudinally by rubbing it with a wetted cloth, the glass is seen to split off at the ends in a number of ring-shaped fragments. Here the effect is due to the superposition of small vibrations; a small effect frequently repeated may give rise to effects far exceeding those due to only one application.

Transverse vibrations are produced by clamping one end of a rod, or otherwise holding it firmly, and then passing a bow across the free part.

It is shown by calculation that *the number of transverse vibrations made in a given time by rods and thin plates of the same material is directly as their thickness and inversely as the square of their length.* The width of the plate does not affect the number of vibrations. A wide plate, however, requires a greater force to set it in motion than a narrow one.

The laws of the longitudinal vibrations of strings are expressed in the formula $n = \frac{1}{2rl}\sqrt{\frac{g\mu}{\pi d}}$, in which n, r, l, d, and g have all the same meaning as in the formula for the transverse vibrations, while μ is the modulus of elasticity of the string, the number which expresses the weight by which it must be stretched in order to elongate by its own length (88).

Fig. 262 represents an instrument invented by Marloye, and known as *Marloye's harp*, based on the longitudinal vibration of rods. It consists of a solid wooden pedestal, in which are fixed twenty thin deal rods, some coloured and others white. They are of such a length that the white rods give the diatonic scale, while the coloured ones give the semitones and complete the chromatic scale. The instrument is played by rubbing the rods in the direction of their length between the finger and thumb, which have been previously covered with powdered resin. The notes produced resemble those of a pandæan pipe.

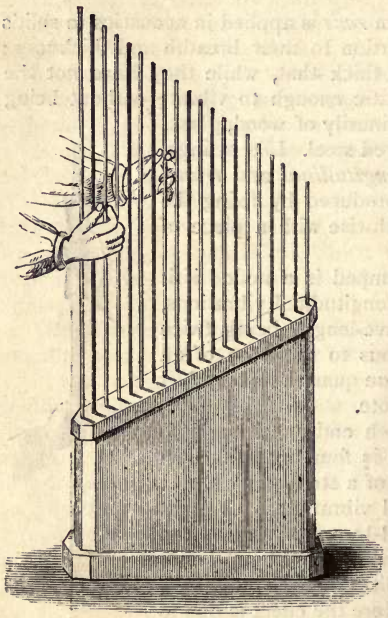


Fig. 262.

The *tuning-fork*, the *triangle*, and *musical boxes* are examples of the transverse vibrations of rods. In musical boxes, small plates of steel of different dimensions are fixed on a rod, like the teeth of a comb. A cylinder whose axis is parallel to this rod, and whose surface is studded with steel teeth, arranged in a certain order, is placed near the plates. By means of a clockwork motion, the cylinder rotates, and the teeth striking the steel plates set them in vibration, producing a tune, which depends on the arrangement of the teeth on the cylinder.

The velocity of sound in any solid may be determined experimentally by clamping it at one end and putting it in longitudinal vibrations. The length of a stopped pipe which gives the same note is next ascertained. The velocity of sound in the material in question is thus to its velocity in air in the same ratio as the length of the rod to the length of the stopped pipe. Thus a rod of alder a metre in length was found to give the same longitudinal note as a stopped pipe 7 cm. in length; the velocities are accordingly as 100 : 7, or the velocity of sound in this wood is 14·3 times that in air.

Stefan has determined the velocity of sound in soft bodies by attaching them, in the form of rods, to long glass or wooden rods. The compound rod was made to vibrate and the number of vibrations of the note was determined. Knowing this, and also the velocity of sound in the longer rod,

the velocity in the shorter rod was at once obtained. By this method some of the numbers in the table in article 235 were obtained.

Scratching and scraping sounds are produced by moving a rod over a smooth surface; the rod is thereby put in vibration; the vibrations are regular for a short interval, but frequently change their period during the motion.

282. **Vibration of plates.**—In order to make a plate vibrate, it is fixed in the centre (fig. 263), and a bow rapidly drawn across one of the edges; or else it is fixed at any point of its surface, and caused to vibrate by rapidly drawing a string covered with resin against the edges of a central hole (fig. 264).

Vibrating plates contain nodal lines (269), which vary in number and position according to the form of the plates, their elasticity, the mode of excitation, and the number of vibrations. These nodal lines may be made visible by covering the plate with fine sand, before it is made to vibrate. As soon the vibasrations commence, the sand leaves the vibrating parts and accumulates on the nodal lines, as seen in figs. 263 and 264.

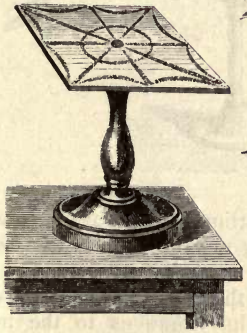


Fig. 263.

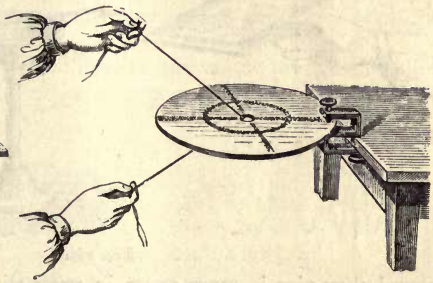


Fig. 264.

The position of the nodal lines may be determined by touching the points at which it is desired to produce them. Their number increases with the number of vibrations; that is, as the note given by the plates is higher. The nodal lines always possess great symmetry of form, and the same form is always produced on the same plate under the same conditions. They were discovered by Chladni, and the plates are known as Chladni's plates.

The vibrations of plates are governed by the following law:—*In plates of the same kind and shape, and giving the same system of nodal lines, the number of vibrations in a second is directly as the thickness of the plates, and inversely as their area.*

Gongs and *cymbals* are examples of instruments in which sounds are produced by the vibration of metal plates. The *glass* and the steel *harmonicon* depend on the vibrations of glass and of steel plates respectively.

Bells, which are to be regarded as curved plates, never vibrate as a whole but when they give their fundamental note in four equal parts which are

separated by nodal lines. This can be shown by suspending pith balls by silk threads from the ends of glass rods arranged crosswise, so that the pith balls just rest against the rim of a bell jar held vertically with the mouth upwards. When this is made to sound by drawing a bow across the edge, the balls are powerfully repelled from the ventral segments, but with far less force from the nodes.

Bells are also capable of vibrating in 6, 8, 10, or 12 parts, producing thus a corresponding series of over-tones. The note of a bell is higher in proportion as the surface is smaller and the substance thicker.

If water is poured into a bell jar which is made to vibrate by means of a violin bow, the surface of the water forms a series of nodes and segments, and water is projected in the form of spray from the ventral segments. If alcohol or ether be used instead of water, a number of droplets form and group themselves into beautiful starlike figures.

283. **Vibration of membranes.**—In consequence of their flexibility, membranes cannot vibrate unless they are stretched, like the skin of a drum.

Fig. 265.

The sound they give is more acute in proportion as they are smaller and more tightly stretched. To obtain vibrating membranes, Savart fastened gold-beater's skin on wooden frames.

In the *drum*, the skins are stretched on the ends of a cylindrical box. When one end is struck, it communicates its vibrations to the internal column of air, and the sound is thus considerably strengthened. The cords stretched against the lower skin strike against it when it vibrates, and produce the sound characteristic of the drum.

Membranes either vibrate by direct percussion, as in the drum, or they may be set in vibration by the vibrations of the air, as Savart had observed, provided these vibrations are sufficiently intense. Fig. 265 shows a membrane vibrating under the influence of the vibrations in the air caused by a sounding bell. Fine sand strewn on the membrane shows the formation of nodal lines just as upon plates.

Membranes are eminently fitted for taking up the vibrations of the air, on account of their small mass, their large surface, and the readiness with which they subdivide. With a pretty strong whistle, nodal lines may be produced in a membrane stretched on a frame, even at the distant end of a large room.

The phenomenon so easily produced in easily-moved bodies is also found in larger and less elastic masses ; the pillars and walls of a church vibrate more or less while the bells are being rung.

CHAPTER VI

GRAPHICAL METHOD OF STUDYING VIBRATORY MOTIONS

284. **Lissajous' method of making vibrations apparent.**—The method of Lissajous exhibits the vibratory motion of bodies either directly or by projection on a screen. It has also the great advantage that the vibratory motions of two sounding bodies may be compared *without the aid of the ear*, so as to obtain the exact relation between them.

This method, which depends on the persistence of visual sensations on the retina (625), consists in fixing a small mirror on the vibrating body, so as to vibrate with it, and impart to a luminous ray a vibratory motion similar to its own.

Lissajous uses tuning-forks, and fixes to one of the prongs a small metal mirror, *m* (fig. 266), and to the other a counterpoise, *n*, which is necessary

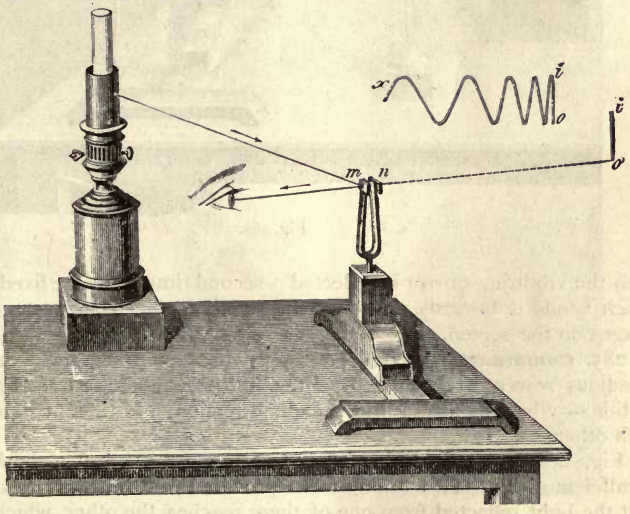


Fig. 266.

to make the tuning-fork vibrate regularly for a long time. At a few yards' distance from the mirror there is a lamp surrounded by a dark chimney, in which is a small hole giving a single luminous point. The tuning-fork being at rest, the eye is placed so that the luminous point is seen at *o*. The tuning-fork is then made to vibrate, and the image elongates so as to form a persistent image, *oi*, which diminishes in proportion as the

amplitude of the oscillation decreases. If, during the oscillation of the mirror, it is made to rotate by rotating the tuning-fork on its axis, a sinuous line, *oix*, is produced instead of the straight line *oi*. These different effects are explained by the successive displacements of the luminous pencil, and by the duration of these luminous impressions on the eye after the cause has ceased—a phenomenon to which we shall revert in treating of vision.

If, instead of viewing these effects directly, they are projected on a screen, the experiment is arranged as shown in fig. 267 ; the pencil reflected

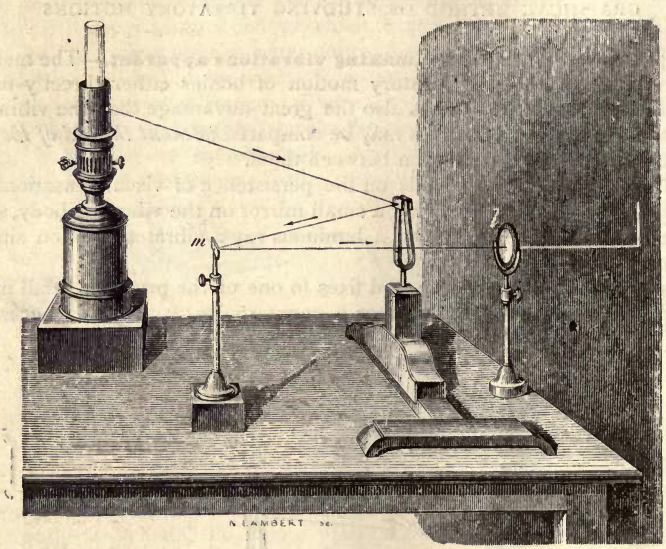


Fig. 267.

from the vibrating mirror is reflected a second time from the fixed mirror, *m*, which sends it towards an achromatic lens, *l*, placed so as to project the images on the screen.

285. **Combination of two vibratory motions in the same direction.**—Lissajous resolved the problem of the optical combination of two vibratory motions—vibrating at first in the same direction, and then at right angles to each other.

Fig. 268 represents the experiment as arranged for combining two parallel motions. Two tuning-forks provided with mirrors are so arranged that the light reflected from one of them reaches the other, which is almost parallel to it, and is then sent towards a screen after having passed through a lens.

If now the first tuning-fork alone vibrates, the image on the screen is the same as in fig. 268 ; but if they both vibrate, supposing they are very nearly in unison, the elongation increases or diminishes according as the simultaneous motions imparted to the image by the vibrations of the mirrors do or do not coincide.

If the tuning-forks pass their position of equilibrium in the same time and in the same direction, the image attains its maximum; and the image is at its minimum when they pass at the same time but in opposite directions. Between these two extreme cases, the amplitude of the image varies according to the time which elapses between the exact instant at which the tuning-forks pass through their position of rest respectively. The ratio of

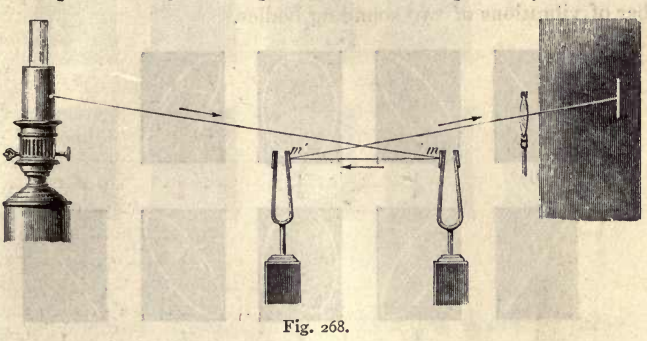


Fig. 268.

this time to the time of a double vibration is called a *difference of phase* of the vibration.

If the tuning-forks are exactly in unison, the luminous appearance on the screen experiences a gradual diminution of length in proportion as the amplitude of the vibration diminishes; but if the pitch of one is very little altered, the magnitude of the image varies periodically, and, while the beats resulting

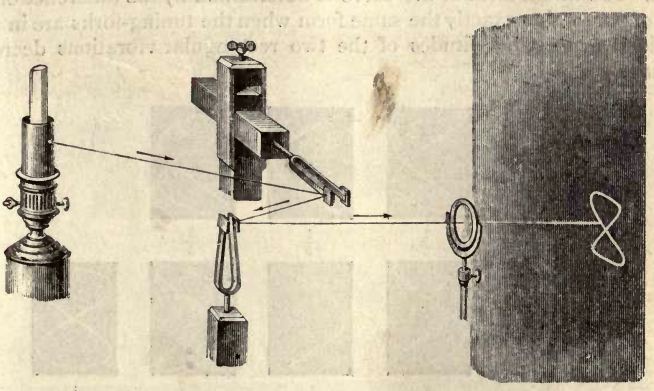


Fig. 269.

from the imperfect harmony are distinctly heard, the eye sees the concomitant pulsations of the image.

286. **Optical combination of two vibratory motions at right angles to each other.**—The optical combination of two rectangular vibratory motions is effected as shown in fig. 269; that is, by means of two tuning-forks, one of which is horizontal and the other vertical, and both provided

with mirrors. If the horizontal fork first vibrates alone, a horizontal luminous outline is seen on the screen, while the vibration of the other produces a vertical image. If both tuning-forks vibrate simultaneously, the two motions combine, and the reflected pencil describes a more or less complex curve, the form of which depends on the number of vibrations of the two tuning-forks in a given time. This curve gives a valuable means of comparing the number of vibrations of two sounding bodies.

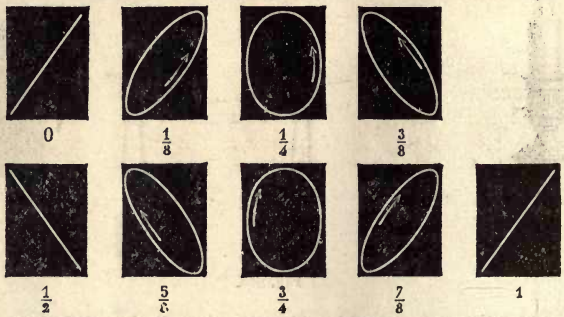


Fig. 270

Fig. 270 shows the luminous image on the screen when the tuning-forks are in unison; that is, when the number of vibrations is equal.

The fractions below each curve indicate the differences of phase between them. The initial form of the curve is determined by the difference of phase. The curve retains exactly the same form when the tuning-forks are in unison, provided that the amplitudes of the two rectangular vibrations decrease in the same ratio.

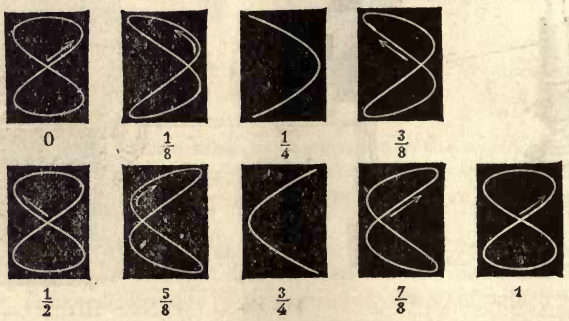


Fig. 271.

If the tuning-forks are not quite in unison, the initial difference of phase is not preserved, and the curve passes through all its variations.

Fig. 271 represents the different appearances of the luminous image when the difference between the tuning-forks is an octave; that is, when the

numbers of their vibrations are as 1 : 2 ; and fig. 272 gives the series of curves when the numbers of the vibrations are as 3 : 4.

It will be seen that the curves are more complex when the ratios of the

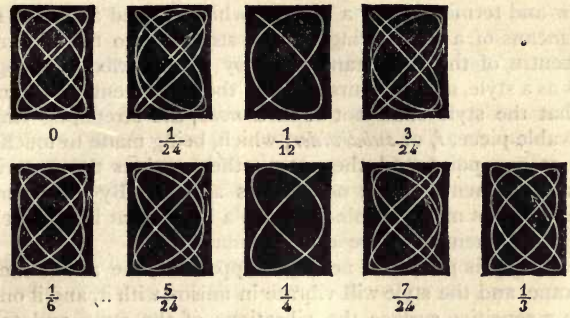


Fig. 272.

numbers of vibrations are less simple. Lissajous examined these curves theoretically, and has calculated their general equations.

When these experiments are made with the electric light, instead of an ordinary lamp, the phenomena are remarkably brilliant.

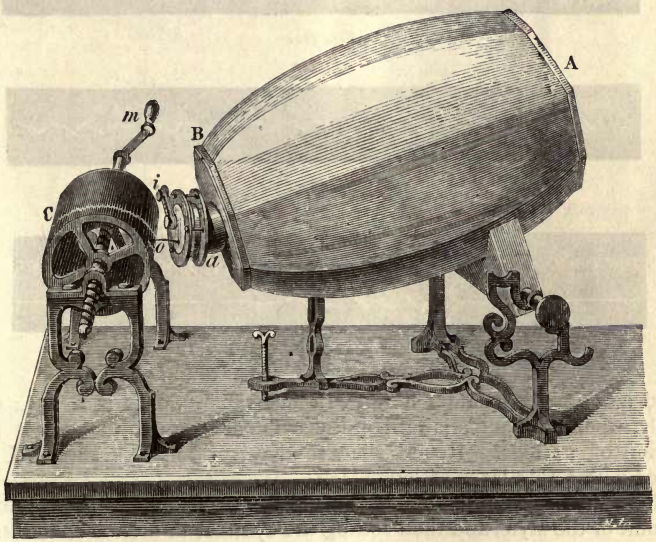


Fig. 273.

287. **Léon Scott's Phonautograph.**—This apparatus registers not only the vibrations produced by solid bodies, but also those produced by wind instruments, by the voice in singing, and even by any noise whatsoever ; for

instance, that of thunder, or the report of a cannon. It consists of an ellipsoidal barrel, AB, about a foot and a half long and a foot in its greatest diameter, made of plaster of Paris. The end A is open, but the end B is closed by a solid bottom, to the middle of which is fixed a brass tube *a*, bent at an elbow and terminated by a ring, on which is fixed a flexible membrane which, by means of a second ring, can be stretched to the required extent. Near the centre of the membrane, fixed by sealing-wax, is a hog's bristle, which acts as a style, and, of course, shares the movements of the membrane. In order that the style shall not be at a *node*, the stretching ring is fitted with a movable piece, *i*, or *subdivider*, which, being made to touch the membrane first at one point and then at another, enables the experimenter to alter the arrangements of the nodal lines at will. By means of the subdivider, the point is made to coincide with a loop ; that is, a point where the vibrations of the membrane are at a maximum.

When a sound is produced near the apparatus, the air in the ellipsoid, the membrane, and the style will vibrate in unison with it, and it only remains to trace on a sensitive surface the vibrations of the style, and to fix them. For this purpose there is placed in front of the membrane a brass cylinder, C, turning round a horizontal axis by means of a handle, *m*. On the pro-

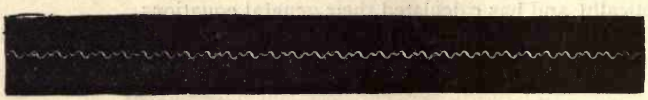


Fig. 274.

Fig. 275.

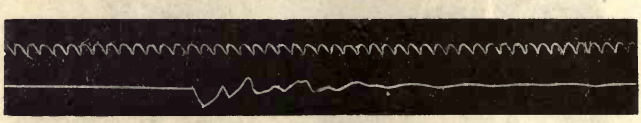


Fig. 276.

longed axis of the cylinder a screw is cut which works in a nut ; consequently, when the handle is turned, the cylinder gradually advances in the direction of its axis. Round the cylinder is wrapped a sheet of paper covered with a thin layer of lampblack.

The apparatus is used by bringing the prepared paper into contact with the point of the style, and then setting the cylinder in motion round its axis. So long as no sound is heard, the style remains at rest, and merely removes the lampblack along a line which is a helix on the cylinder, but which becomes straight when the paper is unwrapped. But when a sound is heard, the membrane and the style vibrate in unison, and the line traced out is no longer straight, but undulates, each undulation corresponding to a double

vibration of the style. Consequently, the figures thus obtained faithfully denote the number, amplitude, and isochronism of the vibrations.

Fig. 274 shows the trace produced when a simple note is sung, and strengthened by means of an upper octave. The latter note is represented by the curve of lesser amplitude. Fig. 275 represents the sound produced jointly by two pipes whose notes differ by an octave. The lower line of fig. 276 represents the rolling sound of the letter R when pronounced with a ring.

The upper line of fig. 276 represents the perfectly isochronous vibrations of a tuning-fork placed near the ellipsoid. This line was traced by a fine point on one branch of the fork, which was thus found to make exactly 500 vibrations per second. Hence, each undulation of the upper line corresponds to the $\frac{1}{500}$ part of a second; and thus these lines become very exact means of measuring short intervals of time. For example, in fig. 276 each of the separate shocks producing the rolling sound of the letter R corresponds to about 18 double vibrations of the tuning-fork, and consequently lasts about $\frac{18}{500}$ or about $\frac{1}{28}$ of a second.

288. **König's manometric flames.**—König's method consists in transmitting the motion of the waves which form a sound to gas flames, which, by their pulsations, indicate the nature of the sounds. For this purpose a

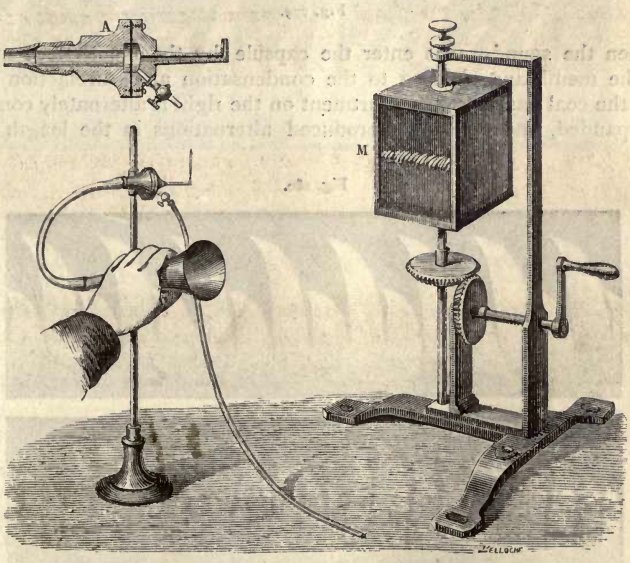


Fig. 277.

metal capsule, represented in section at A, fig. 277, is divided into two compartments by a thin membrane of caoutchouc; on the right of the figure is a gas jet, and below it a tube conveying coal gas; on the left is a tubulure, to which may be attached a caoutchouc tube. The other end of this

may be placed at the node of an organ pipe (274), or it terminates in a mouthpiece in front of which a given note may be sung: this is the arrangement represented in fig. 277.

Fig. 278.

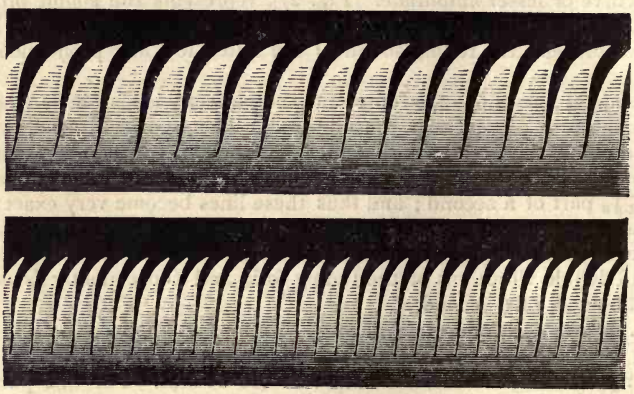


Fig. 279.

When the sound-waves enter the capsule by the mouthpiece and the tube, the membrane yielding to the condensation and rarefaction of the waves, the coal gas in the compartment on the right is alternately contracted and expanded, and hence are produced alternations in the length of the

Fig. 280.

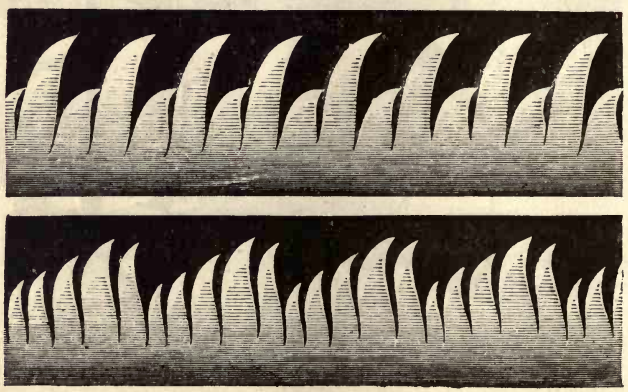


Fig. 281.

flame, which are, however, scarcely perceptible when the flame is observed directly. But to render them distinct they are received on a mirror with four faces, M, which may be turned by two cog-wheels and a handle. As

long as the flame burns steadily, there appears in the mirror, when turned, a continuous band of light. But, if the capsule is connected with a sounding-tube yielding the fundamental note, the image of the flame takes the form represented in fig. 278, and that of figure 279 if the sound yields the octave. If the two sounds reach the capsule simultaneously, the flame has the appearance of fig. 280; in that case, however, the tube leading to the capsule must be connected by a T-pipe with two sounding-tubes, one giving the fundamental note, and the other the octave. If one gives the fundamental note and the other the third, the flame has the appearance of fig. 281.

Fig. 282.

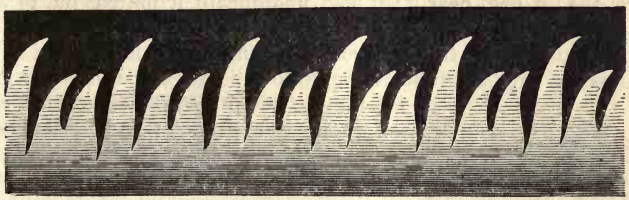


Fig. 283.

If the vowel E be sung in front of the mouthpiece first upon *c*, and then upon *c'*, the rotating mirror gives the flames represented in figs. 282 and 283.

289. **Determination of the intensity of sounds.**—Meyer has devised a plan by which the intensities of two sounds of the same pitch may be directly compared. The two sounds are separated from each other by a medium impervious to sound, and in front of each of them is a resonance globe (255) accurately tuned to the sound. Each of these resonance globes is attached by means of caoutchouc tubes of equal length to the two ends of a U-tube, in the middle of the bend which is a third tube provided with a manometric capsule.

If the resonance globes are each at the same distance from the sounding bodies, and if the note of only one of them is produced, the flame vibrates. If both sounds are produced, and they are of the same intensity, and in the same phase, they interfere completely in the tube, so that the flame of the manometric capsule is quite stationary, and appears in the turning mirror as a straight luminous band.

If, however, the sounds are not of the same intensity, the interference will be incomplete, and the luminous band will be jagged at the edge. The distance of one of the sounds from the resonance globes is altered until the

flame is stationary. The intensities of the two sounds are then directly as the squares of their distances from the resonators.

290. **Acoustic attraction and repulsion.**—It was observed by Guyot, and afterwards independently by Guthrie and by Schellbach, that a sounding body, one in a state of vibration therefore, exercises an action on a body in its neighbourhood which is sometimes one of attraction and sometimes of repulsion. The vibrations of an elastic medium attract bodies which are specifically heavier than itself, and repel those which are specifically lighter. Thus a balloon of goldbeater's skin filled with carbonic acid is attracted towards the opening of a resonance-box on which is a vibrating tuning-fork ; while a similar balloon filled with hydrogen and tied down by a thread is repelled. This result always follows, even when the hydrogen balloon is made heavier than air by loading it with wax.

A light piece of cardboard suspended and held near a tuning-fork moves towards it when the fork is made to vibrate. If the tuning-fork is suspended and is then made to vibrate, it moves towards the card if the latter is fixed. Two suspended tuning-forks in a state of vibration move towards each other. The flame of a candle placed near the end of a sounding tuning-fork is repelled if held near it; if held underneath it flattens out to a disc. A gas flame near the end of the tuning-fork divides into two arms.

Guthrie found that, when one prong of a tuning-fork is enclosed in a tube provided with a capillary tube dipping into a liquid, and is set in vibration by bowing the free prong, the air around the enclosed prong is expanded ; and he thence concluded that the approach, above described, of a suspended body to the sounding-fork is due to the diminution of the pressure of the air between the fork and the body below that on the other side of the body.

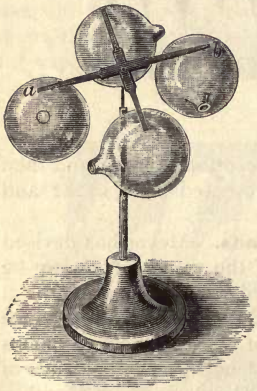


Fig. 284.

A cylindrical resonator of stiff drawing-paper is fastened to a strip of wood, which is provided with a glass cap and counterpoise, and thus can be made to turn on a needle point. If the open end of the sounding-box of a tuning-fork vibrating in unison with the resonator is brought near this, it is repelled even at a distance of some inches. When a small mill with four arms (fig. 284), each provided with a small resonator, is placed near the open end of the sounding-box, the repulsion is so strong as to produce a uniform rotation.

These phenomena do not seem to be due to the aspirating action of currents of air, nor are they caused by any heating effect ; and it must be confessed that the phenomena require further elucidation ; they are of special interest as furnishing a possible clue to the solution of the problem of attraction in general.

291. **Phonograph. Graphophone.**—In the year 1877 Edison devised the apparatus known as the *phonograph* for recording and reproducing sound, which is equally remarkable for the simplicity of its construction and for the striking character of the results which it produces.

This instrument is illustrated in fig. 285, and it consists generally of a cylinder C, mounted on a horizontal axis AA′, which can be rotated beneath a mouthpiece E, by means of a winch-handle M, the speed of rotation being controlled by a fly-wheel attached to one end of the spindle AA′, and the whole is supported by a base-board L. Upon the cylindric surface of C is cut a helical groove, and one end of the spindle A′ is formed into a screw, the pitch of which is equal to that of the groove upon the cylinder. This screw works in a correspondingly screwed bearing, so that on turning the handle the cylinder not only rotates upon its axis, but also travels from end to end in a direction parallel to its axis.

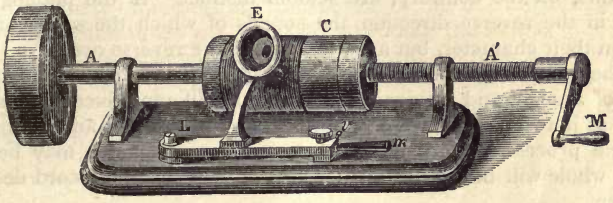


Fig. 285.

The mouthpiece is closed with a diaphragm or membrane P, to the centre of which is attached, by means of a caoutchouc tube, a small style S, directed towards the cylinder, and which is caused to vibrate longitudinally by the vibratory action of the diaphragm P, and the position of the mouthpiece is so adjusted that the point of the style is always directed to the centre of the helical groove in the cylinder. On this grooved cylinder is stretched a sheet of tinfoil which bridges over the grooves, being supported by the ridges and the position of the mouthpiece, and its distance from the cylinder is adjusted by the handle *m*, which can be fixed in its place by the set screw *v*. Their position and distance are so adjusted that when the apparatus is at rest the point of the style is within the groove and a little lower than the top of the ridge.

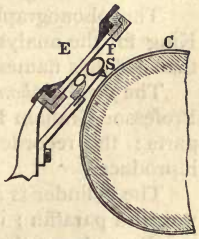


Fig. 286.

If, while the cylinder is being rotated, sounds or words be uttered into the mouthpiece, the diaphragm attached thereto will be set in vibration, and will cause the style to indent on the foil a groove of varying depth, the bottom of which is a mechanical record of the vibrations of the diaphragm, and therefore of the sounds by which those vibrations were set up, and as the tinfoil is a very imperfectly elastic material, it is able to retain the record so made.

If now this record be passed again beneath the style, the varying indentations on the foil will cause the style to vibrate as it did when it produced the indentations, and the diaphragm will be similarly set in vibration, and will reproduce the sound by which it was in the first instance set into vibration.

In this way sound may be reproduced so as to be audible to a large audience; the articulation is distinct though feeble; it reproduces the voice of a person who speaks into it, but with a nasal intonation. Speech

may thus be stored up on a sheet of tinfoil and kept for an indefinite period, and the sound may be reproduced more than once from the same record, but after a second reproduction the clearness is greatly diminished.

If the velocity of rotation be greater than before, the pitch of the sound is raised; and if it be not uniform, then, in the case of a song, the reproduction is incorrect. In order to produce a uniform velocity the instrument may with advantage be driven by clockwork.

There is a great difference in the distinctness with which the various consonants and vowels are reproduced; the most distinct are words containing the vowels A, O, and U, and the consonants *t*, *k*, and *r*; the *s* and similar consonants, on the contrary, are seldom distinct. If the phonograph be rotated in the reverse direction, the sounds of which the words are made up retain their character, but are produced in the reverse order.

If the instrument be reset to the starting-point of the phonographic record of a song, and be again sung into, it will reproduce both series of sounds, as if two persons were singing at the same time; and, by repeating the same process, a third or fourth succession of sounds may be added, and the whole will be heard together and without the one record destroying the other.

The impressions on the tinfoil appear at first sight as a series of successive points or dots, but when examined under a microscope they are seen to have a distinct form of their own. When a cast is taken by means of fusible metal and a longitudinal section made, the outline closely resembles the jagged edge of a König's flame. Mr. Edison states that as many as 40,000 words can be registered on a space not exceeding 10 square inches.

The phonograph has been used with great advantage by Jenkins and King for the analysis of vocal sounds, for which purpose it is better suited than König's flames.

The *graphophone*, invented by Mr. Sumner Tainter, in conjunction with Professor Graham Bell and Dr. Chichester Bell, consists essentially of three parts: the recorder, the cylinder on which the record is made, and the reproducer.

The cylinder is a hollow cone of cardboard coated with a composition of wax and paraffin; it is mounted horizontally and is rotated by means of a treadle underneath the table, which supports the whole apparatus. Between the treadle and the cylinder is interposed a very ingenious governor by which the speed of rotation of the cylinder may be regulated to perfect uniformity, the force required for this rotation being very small.

On a bar parallel to and in front of the cylinder is clamped the recorder, which consists of an exceedingly minute cutting point, or rather chisel, fixed to a mica diaphragm. This diaphragm is at the end of a flexible tube provided with a mouthpiece. If this be spoken into, the diaphragm vibrates with a to and fro motion, and if at the same time the cylinder rotates at a uniform speed the style cuts or carves out a groove in the surface of the wax, forming a very irregular outline which is the exact reproduction of the sound-wave. Therein lies the difference between the graphophone and the phonograph, for in the latter the record is produced by a process of indentation, while in the former the record of the sound-waves is engraved in a waxy material. The grooves are so excessively minute that their variations in depth cannot

be recognised by the naked eye ; they are not more than the $\frac{3}{1000}$ of an inch in diameter, and there are 160 to the inch.

The reproducer consists of a light ebonite tube, at one end of which is the enlargement containing the diaphragm, which, like that of the recorder, is of mica, but is somewhat smaller. The diaphragm is connected by means of a fine waxed silk thread with a fine steel point or hook which rocks on a pivot at the end of the tube. There is an arrangement by which this reproducer can be clamped in front of the recorder, so that when the cylinder is rotated the reproducer travels at a proportionate speed, allowing the small point to rest in the groove forming the sound record, and along which it rides and vibrates ; and these vibrations are transmitted to the mica diaphragm, and, being communicated to the ear, faithfully reproduce the sound.

Notwithstanding what appears the very yielding character of the wax, the sounds, and even elaborate pieces of music, are reproduced with great fidelity, and it is stated that the same record will reproduce the original sound some thousand times.

BOOK VI

ON HEAT

CHAPTER I

PRELIMINARY IDEAS. THERMOMETERS

292. **Heat. Hypothesis as to its nature.**—In ordinary language the term *heat* is used not only to express a particular sensation, but also to describe that particular state or condition of matter which produces this sensation. Besides producing this sensation, heat acts variously upon bodies ; it melts ice, boils water, makes metals red-hot, produces electrical currents, decomposes compound bodies, and so forth.

Two theories as to the cause of heat have been propounded : these are, the *theory of emission*, and the *theory of undulation.*

On the first theory, heat is caused by a subtle imponderable fluid, which surrounds the molecules of bodies, and which can pass from one body to another. These *heat atmospheres*, which thus surround the molecules, exert a repelling influence on each other, in consequence of which heat acts in opposition to the force of cohesion. The entrance of this substance into our bodies produces the sensation of warmth, its egress the sensation of cold.

On the second hypothesis the heat of a body is caused by an extremely rapid oscillating or vibratory motion of its molecules ; and the hottest bodies are those in which the vibrations have the greatest velocity and the greatest amplitude. At any given time the whole of the molecules of a body possess a sum of *vis viva*, which is the heat they contain. To increase their temperature is to increase their *vis viva* ; to lower their temperature is to decrease their *vis viva.* Hence, on this view, heat is not a substance but a *condition of matter*, and a condition which can be transferred from one body to another. When a heated body is placed in contact with a cooler one, the former cedes more molecular motion than it receives ; but the loss of the former is the equivalent of the gain of the latter.

It is also assumed that there is an imponderable elastic ether, which pervades all matter and infinite space. A hot body sets this in rapid vibration, and the vibrations of this ether being communicated to material objects set them in more rapid vibration ; that is, increase their temperature. Here we have an analogy with sound ; a sounding body is in a state of vibration, and

its vibrations are transmitted by atmospheric air to the auditory apparatus in which is produced the sensation of sound.

This hypothesis as to the nature of heat is now adopted by the most distinguished physicists. It affords a better explanation of all the phenomena of heat than any other theory, and it reveals an intimate connection between heat and light. It will be subsequently seen that by the friction of bodies against each other an indefinite quantity of heat is produced. Experiment has shown that there is an exact equivalence between the motion thus destroyed and the heat produced. These and many other facts are utterly inexplicable on the assumption that heat is a substance, and not a form of motion.

In what follows, however, the phenomena of heat will be considered, as far as possible, independently of either hypothesis; but we shall subsequently return to the reason for the adoption of the latter hypothesis.

Assuming that the heat of bodies is due to the motion of their particles, we may admit the following explanation as to the nature of this motion in the various forms of matter:—

In *solids* the molecules of even the most rigid bodies have a kind of vibratory motion about certain fixed positions. This motion is probably very complex; the constituents of the molecule may oscillate about each other, besides the molecule oscillating as a whole; and this latter again may be a to-and-fro motion, or it may be a rotatory motion about the centre. In cases in which external forces, such as violent shocks, act upon the body, the molecules may permanently acquire fresh positions.

In the *liquid* state the molecules have no fixed positions. They can rotate about their centres of gravity, and the centre of gravity itself may move. But the repellent action of the motion, compared with the mutual attraction of the molecules, is not sufficient to separate the molecules from each other. A molecule no longer adheres to particular adjacent ones; but it does not spontaneously leave them except to come into the same relation to fresh ones as to its previous adjacent ones. Thus in a liquid there is a vibratory, rotatory, and progressive motion.

In the *gaseous* state the molecules are entirely without the sphere of their mutual attraction. They fly forward in straight lines according to the ordinary laws of motion, until they impinge against other molecules or against a fixed envelope which they cannot penetrate, and then return in an opposite direction, with, in the main, their original velocity. If the molecules were in space, where no external force could act upon them, they would fly apart, and disappear in infinity. But if contained in any vessel, the molecules continually impinge in all directions against the sides, and thus arises the pressure which a gas exerts on its vessel.

The perfection of the gaseous state, or what may be regarded as an *ideal* gas, implies that the space actually occupied by the molecules of the gas is infinitely small compared with the entire volume of the gas; that the time occupied by the impact of a molecule either against another molecule, or against the sides of the vessel, is infinitely small in comparison with the interval between any two impacts; and that the influence of molecular attraction is infinitely small. When these conditions are not fulfilled the gas partakes more or less of the nature of a liquid, and exhibits certain

deviations from Boyle's law (180). This is the case with all gases ; to a very slight extent with the less easily condensable gases, but to a far greater extent with vapours and the more condensable gases, especially near their points of liquefaction. These are now explained by the modification which Van der Waals has introduced into the equation for gases (182).

293. **Dynamical theory of gases.**—We have seen that in the gaseous condition the particles are assumed to fly about in right lines in all possible directions. A rough illustration of this condition of matter is afforded by imagining the case of a number of bees enclosed in a box.

Let us suppose a cubical vessel to be filled with air under standard conditions of temperature and pressure. Let the length of the sides be a. We will for the present suppose that each particle moves freely in the space without striking against another particle. All possible motions may be conceived to be resolved into motions in three directions which are parallel to the faces of the cube. Conceive any single particle, of mass m ; it will strike against one face with such a velocity, u, as not only to annul its own motion, but to cause it to rebound in the opposite direction with the same velocity ; hence the measure of the momentum with which it strikes against the side will be $2mu$. Now, by their rapid succession and their uniform distribution, the total action of these separate impacts is to produce a pressure against the sides of the vessel which is the elastic force of the gas ; and, to measure the pressure on the side, we must multiply the momentum of each individual impact by the total number of such impacts.

Since the length of the side is a, if there are n molecules in the unit of space, there will be na^3 in the volume of the cube, of which $\frac{na^3}{3}$ will be moving in a direction parallel to each one of the sides. To get the number of impacts on one face, we must remember that they succeed each other, after the interval of time required for a particle to fly to the opposite side and back again. Hence, u being the velocity, the number of impacts which each particle makes in the unit of time, a second will be $\frac{u}{2a}$, and the number of all such which strike against one side will be $\frac{1}{3}na^3\frac{u}{2a} = \frac{1}{6}na^2u$.

Now, since each one exerts a pressure represented by $2mu$, we shall have for the total pressure p on the surface a^2

$$pa^2 = \tfrac{1}{3}a^2nmu^2,$$

and therefore the pressure on the unit of surface will be

$$p = \tfrac{1}{3}nmu^2.$$

Again, if N is the number of molecules in the volume v, $N = nv$, and therefore

$$p = \tfrac{1}{3}\frac{N}{v}mu^2 \text{ ; that is, } pv = \tfrac{1}{3}Nmu^2.$$

But, for any given mass of gas, N, m, and u are constant quantities, and the product pv must therefore also be constant ; this, however, is only one form of expressing Boyle's law (180).

We may put the above expression in the form $pv = \frac{2}{3}N\frac{mu^2}{2}$, so that for equal volumes of two different gases under the same conditions of temperature and pressure, $\frac{2}{3}N\frac{mu^2}{2} = \frac{2}{3}N'\frac{m_1u_1^2}{2}$. The latter part of the expression represents the *vis viva* of the gas in each case. But under the conditions stated these are equal ; accordingly we have $N = N'$, that is to say that equal volumes of two different gases under the same conditions of temperature and pressure contain the same number of molecules. This, which is known as *Avogadro's law*, was deduced by him from other considerations and forms one of the most important bases of theoretical chemistry.

294. **Molecular velocity.**—In the formula $p = \frac{1}{3}nmu^2$, nm represents the mass in unit volume which we may designate as the density ρ, of the gas referred to that of water, and which can be directly measured ; and, since the pressure p is also capable of direct measurement, we can calculate the third magnitude u in absolute measure.

The pressure p on a gas is equal to the action of gravity on a column of mercury of given height h ; so that, if δ is the density of mercury = 13·596, and g the acceleration of gravity, $p = \delta gh$ and

$$u^2 = \frac{3\delta gh}{\rho}.$$

Now, if σ be the specific gravity of the gas as compared with air, which is $\frac{1}{773·3}$ lighter than water, $\rho \times 773·3 = \sigma$, or $\rho = \frac{\sigma}{773·3}$,

$$u^2 = \frac{3 \times 13·596 \times 0·76 \times 9·8115 \times 773·3}{\sigma}$$

which gives $u = \frac{485}{\sqrt{\sigma}}$; that is, for atmospheric air the mean velocity of the particles is 485 metres in a second. For other gases we have, expressed in the same units, O = 461, N = 492, H = 1844.

It follows from the above equation that

$$u : u_1 = \sqrt{\sigma_1} : \sqrt{\sigma}$$

that is, *the molecular velocities are inversely as the square roots of the densities or the molecular weights.* This is confirmed by experiments on diffusion (190).

In a gas the velocities of the particles are unequal ; since, even supposing that they were all originally the same, it is not difficult to see that they would soon alter. For imagine a particle to be moving parallel to one side, and to be struck centrically by another moving at right angles to the direction of its motion, the particle struck would proceed on its new path with increased velocity, while the striking particle would rebound in a different direction with a smaller velocity.

Notwithstanding the accidental character of the velocity of any individual particle in such a mass of gas as we have been considering, there will, at any one given time, be a certain average distribution of velocities. Now, from considerations based on the theory of probabilities, Maxwell inferred that

some velocities will be more probable than others—that there will, indeed, be one velocity which is more probable than any other. This is called the *most probable velocity*. The *mean velocity* of the particle, as deduced above, is not this, nor is it the same as the arithmetical mean of all the velocities; it may be defined to be that velocity which, if all the molecules possessed it, would give rise to the same mean energy of the molecular impacts against the side as that which actually exists. This mean velocity is about $\frac{1}{1\frac{1}{2}}$ greater than the arithmetical mean velocity, and is $\frac{5}{4}$ that of the most probable single velocity.

Theoretical as well as experimental observations render it possible to determine with great probability the average length of the path which one molecule of a gas traverses before it encounters another, which is known as the *free path*. This is not a constant number in one and the same gas; that is, the paths which the molecules travel between two impacts are not equal, and the average of these is known as the *mean free path*. The length of this depends on the number of molecules in unit volume of a gas, being inversely as the density; for it is obvious that as the density increases the number of molecules increases also, and therewith the path which one molecule travels before it meets another will be so much the smaller. The mean free path in different gases will be shorter the larger are the molecules. In nitrogen measured under standard conditions it has been determined to be $0{\cdot}00000986$ or $0{\cdot}0_5986$ cm.; in hydrogen $0{\cdot}0_41855$, and in carbonic acid $0{\cdot}0_368$. The frequency of the impacts has also been determined; in the case of hydrogen this is 9,480 millions, and of nitrogen and air 8,000 millions.

It has been urged against the kinetic theory of gases that with its enormous molecular velocity the diffusion of one gas with another ought to take place instantaneously, and be at once perceived in the case of gases with strong odours; this, as we know, is not the case, the velocity is the rate between each impact, and billions of such impacts must take place before a molecule passes to any great distance.

The magnitudes of the molecules themselves have been calculated by several observers from different methods based on various physical phenomena (3). Loschmidt found, for instance, that the diameter of the molecule of hydrogen was 41, oxygen 7, and nitrogen 8 hundred millionths of a centimetre. The results of other estimates, made by various methods, agree remarkably with these.

295. **General effects of heat.**—The general effects of heat upon bodies may be classed under three heads. One portion is expended in raising the temperature of the body; that is, in increasing the *vis viva* of its molecules. In the second place, the molecules of bodies have a certain attraction for each other, to which is due their relative position; hence a second portion of heat is consumed in augmenting the amplitude of the oscillations, by which an increase of volume is produced, or in completely altering the relative positions of the molecules, by which a change of state is effected. These two effects are classed as *internal work*. Thirdly, since bodies are surrounded by atmospheric air which exerts a certain pressure on their surface, this has to be overcome or lifted through a certain distance. The heat or work required for this is called the *external work*.

If Q units of heat are imparted to a body, and if A be the quantity of heat which is equivalent to the unit of work; then if W is the amount of heat which serves to increase the temperature, I that required to alter the position of the molecules, and if L be that expended in external work, then

$$Q = A\,(W + I + L).$$

296. **Expansion.**—All bodies expand by the action of heat. As a general rule, gases are the most expansible, then liquids, and lastly solids.

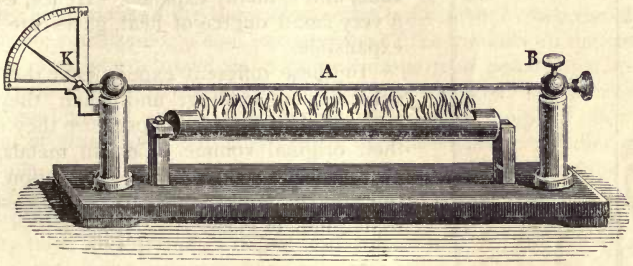


Fig. 287.

In solids which have definite figures, we can either consider the expansion in one dimension, or the *linear* expansion; in two dimensions, the *superficial* expansion; or in three dimensions, the *cubical* expansion or the expansion of volume, although one of these never takes place without the other. As liquids and gases have no definite figures, the expansions of volume have in them alone to be considered.

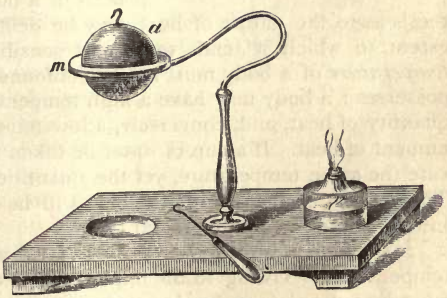


Fig. 288.

To show the linear expansion of solids, the apparatus represented in fig. 287 may be used. A metal rod, A, is fixed at one end by a screw, B, while the other end presses against the short arm of an index, K, which moves on a scale. Below the rod there is a sort of cylindrical lamp in which alcohol is burned. The needle K is at first at the zero point, but as the rod becomes heated it expands, and moves the needle along the scale.

The cubical expansion of solids is shown by a *Gravesande's* ring. This consists of a brass ball, *a* (fig. 288), which at the ordinary temperature passes freely through a ring, *m*, almost of the same diameter. But when the ball has been heated, it expands, and no longer passes through the ring.

In order to show the expansion of liquids, a large glass bulb provided with a capillary stem is used (fig. 289). If the bulb and a part of the stem contain some coloured liquid, the liquid rapidly rises in the stem when heat

is applied, and the expansion thus observed is far greater than in the case of solids.

The same apparatus may be used for showing the expansion of gases. Being filled with air, a small thread of mercury is introduced into the capillary tube to serve as index (fig. 290). When the globe is heated in the slightest degree, even by approaching the hand, the expansion is so great that the index is driven to the end of the tube, and is finally expelled. Hence, even for a very small degree of heat, gases are highly expansible.

Fig. 289.

Fig. 290.

In these different experiments the bodies contract on cooling, and when they have attained their former temperature they resume their original volume. Certain metals, however, especially zinc, form an exception to this rule, and it appears also to be the case with some kinds of glass.

MEASUREMENT OF TEMPERATURE. THERMOMETRY.

297. **Temperature.**—The *temperature* or *hotness* of a body, independently of any hypothesis as to the nature of heat, may be defined as being the greater or less extent to which it tends to impart sensible heat to other bodies. The *temperature* of a body must not be confounded with the *quantity of heat* it possesses: a body may have a high temperature and yet have a very small quantity of heat, and, conversely, a low temperature and yet possess a large amount of heat. If a cup of water be taken from a bucketful, both will indicate the same temperature, yet the quantities they possess will be different. This subject of the quantity of heat will be afterwards more fully explained in the chapter on Specific Heat.

298. **Thermometers.**—*Thermometers* are instruments for measuring temperatures. Owing to the imperfections of our senses we are unable to measure temperatures by the sensation of heat or cold which they produce in us, and for this purpose recourse must be had to the physical actions of heat on bodies. These actions are of various kinds, but the expansion of bodies has been selected as the easiest to observe. But heat also produces electrical phenomena in bodies; and on these the most delicate methods of observing temperatures have been based, as we shall see in a subsequent chapter.

Liquids are best suited for the construction of thermometers—the expansion of solids being too small, and that of gases too great. Mercury and alcohol are the only liquids used—the former because it only boils at a very high temperature, and the latter because it does not solidify at the greatest known cold.

The mercurial thermometer is the most extensively used. It consists of

a capillary glass tube, at the end of which is blown the *bulb*, a cylindrical or spherical reservoir. Both the bulb and a part of the stem are filled with mercury, and the expansion is measured by a scale graduated either on the stem itself, or on a frame to which it is attached.

Besides the manufacture of the bulb, the construction of the thermometer comprises three operations : the *calibration* of the tube, or its division into parts of equal capacity ; the introduction of the mercury into the reservoir ; and the graduation.

299. **Division of the tube into parts of equal capacity. Calibration.** —As the indications of the thermometer are only correct when the divisions of the scale correspond to equal expansions of the mercury in the reservoir, the scale must be graduated, so as to indicate parts of equal capacity in the tube. If the tube were quite cylindrical, and of the same diameter throughout, it would only be necessary to divide it into equal lengths. But as the diameter of glass tubes is usually greater at one end than another, parts of equal capacity in the tube are represented by unequal lengths of the scale.

In order, therefore, to select a tube of uniform bore it is *calibrated* ; for this purpose, a thread of mercury about an inch long is introduced into the capillary tube, and moved in different positions in the tube, care being taken to keep it at the same temperature. If the thread is of the same length in every part of the tube, it shows that the capacity is everywhere the same ; but if the thread occupies different lengths the tube is rejected, and another one sought.

300. **Filling the thermometer.**—In order to fill the thermometer with mercury, a small funnel, C (fig. 291), is blown on the top, and is filled with mercury ; the tube is then slightly inclined, and the air in the bulb expanded by heating it with a spirit lamp. The expanded air partially escapes by the funnel, and, on cooling, the air which remains contracts, and a portion of the mercury passes into the bulb D. The bulb is then again warmed, and allowed to cool, a fresh quantity of mercury enters, and so on, until the bulb and part of the tube are full of mercury. The mercury is then heated to boiling ; the mercurial vapours in escaping carry with them the air and moisture which remain in the tube. The tube, being full of the expanded mercury and of mercurial vapour, s hermetically sealed at one end. When the thermometer is cold, the mercury ought to fill the bulb and a portion of the stem.

Fig. 291.

301. **Graduation of the thermometer.**—The thermometer being filled, it requires to be graduated ; that is, to be provided with a scale to which variations of temperature can be referred. And, first of all, two points must be fixed which represent identical temperatures, and which can always be easily reproduced.

Experiment has shown that ice constantly melts at the same temperature, whatever be the degree of heat, and that distilled water under the same pressure and in a vessel of the same kind always boils at the same temperature. Consequently, for the first fixed point, or zero, the temperature of melting ice has been taken ; and for a second fixed point, the temperature of boiling water in a metal vessel under the normal atmospheric pressure of 760 millimetres.

This interval of temperature—that is, the range from zero to the boiling point—is taken as the unit for comparing temperatures : just as a certain length, a foot or a metre for instance, is used as a basis for comparing lengths.

302. **Determination of the fixed points.**—To obtain zero, snow or pounded ice is placed in a vessel in the bottom of which is an aperture by which water escapes (fig. 292). The bulb and a part of the stem of the thermometer are immersed in this for about a quarter of an hour, and a mark made at the level of the mercury, which represents zero.

Fig. 292.

According to Bunsen it is doubtful whether a very accurate determination is obtained by placing a thermometer in melting ice, for some slight admixtures lower the freezing point considerably. The best plan is to let water, in which is the thermometer, be over-cooled (345) and then made to freeze by shaking ; the point to which the mercury rises is the true melting point.

The second fixed point is determined by means of the apparatus represented in figs. 293 and 294, of which fig. 294 represents a vertical section. In both, the same letters designate the same parts. The whole of the apparatus is of metal. A central tube, A, open at both ends, is fixed on a cylindrical vessel containing water ; a second tube, B, concentric with the first, and surrounding it, is fixed on the same vessel, M. In this second cylinder, which is closed at both ends, there are three tubulures, *a*, E, D. A cork, in which is the thermometer *t*, fits in *a*. To E, a glass tube, containing mercury, is attached, which serves as a manometer for measuring the pressure of the vapour in the apparatus. D is an escape tube for the vapour and condensed water.

The apparatus is placed on a furnace and heated till the water boils ; the vapour produced in M rises in the tube A, and, passing through the two tubes in the direction of the arrows, escapes by the tubulure D. The thermometer *t* being thus surrounded with vapour, the mercury expands, and, when it has become stationary, the point at which it stops is marked. This is the point sought for. The object of the second case, B, is to avoid the cooling of the central tubulure by its contact with the air.

The determination of the point 100 (see next Article) would seem to require that the height of the barometer during the experiment should be 760 millimetres ; for, when the barometric height is greater or less than this

quantity, water boils either above or below 100 degrees. But the point 100 may always be exactly obtained, by making a suitable correction. For every 27 millimetres difference in height of the barometer there is a difference in the boiling point of 1 degree. If, for example, the height of the barometer is 778—that is, 18 millimetres, or two-thirds of 27, above 760—water would boil at 100 degrees and two-thirds. Consequently $100\frac{2}{3}$ would have to be marked at the point at which the mercury stops.

Gay-Lussac observed that water boils at a somewhat higher temperature in a glass than in a metal vessel ; and as the boiling point is raised by any salts which are dissolved, it has been assumed that it was necessary to use a metal vessel and distilled water in fixing the boiling point. Rudberg showed, however, that these latter precautions are superfluous. The nature of the vessel and salts dissolved in ordinary water influence the temperature

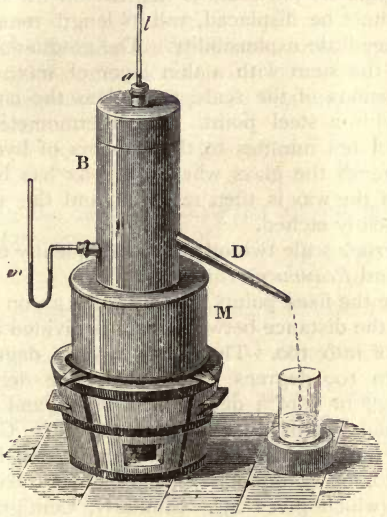


Fig. 293

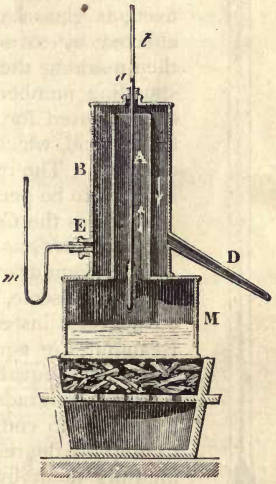


Fig. 294.

of boiling water, but not that of the vapour which is formed. That is to say, if the temperature of boiling water from any of the above causes is higher than 100 degrees, the temperature of the vapour does not exceed 100, provided the pressure is not more than 760 millimetres. Consequently, the higher point may be determined in a vessel of any material, if the thermometer is quite surrounded by vapour, and does not dip in the water.

Even with distilled water, the bulb of the thermometer must not dip in the liquid ; for, strictly speaking, it is only the upper layer that really has the temperature of 100 degrees, since the temperature increases from layer to layer towards the bottom, in consequence of the increased pressure.

303. **Construction of the scale.**—Just as the foot-rule which is adopted as the unit of comparison for length, is divided into a number of equal

divisions called inches for the purpose of having a smaller unit of comparison, so likewise the unit of comparison of temperatures, the range from zero to the boiling point, must be divided into a number of parts of equal capacity called *degrees*. On the Continent, and more especially in France, this space is divided into 100 parts, and this division is called the *Centigrade* or *Celsius* scale; the latter being the name of the inventor. The Centigrade thermometer is almost exclusively adopted in foreign scientific works, and, as its use is gradually extending in this country, it has been and will be adopted in this book.

Fig. 295.

The degrees are designated by a small cypher placed a little above on the right of the number which marks the temperature, and to indicate temperatures below zero the minus sign is placed before them. Thus, $-15°$ signifies 15 degrees below zero.

In accurate thermometers the scale is marked on the stem itself (fig. 295). It cannot be displaced, and its length remains fixed, as glass has very little expansibility. The graduation is effected by covering the stem with a thin layer of wax, and then marking the divisions of the scale, as well as the corresponding numbers, with a steel point. The thermometer is then exposed for about ten minutes to the vapours of hydrofluoric acid, which attacks the glass where the wax has been removed. The rest of the wax is then removed, and the stem is found to be permanently etched.

Besides the *Centigrade* scale two others are frequently used —*Fahrenheit's scale* and *Réaumur's scale*.

In Réaumur's scale the fixed points are the same as on the Centigrade scale, but the distance between them is divided into 80 degrees, instead of into 100. That is to say, 80 degrees Réaumur are equal to 100 degrees Centigrade; one degree Réaumur is equal to $\frac{100}{80}$ or $\frac{5}{4}$ of a degree Centigrade, and one degree Centigrade equals $\frac{80}{100}$ or $\frac{4}{5}$ of a degree Réaumur. Consequently, to convert any number of Réaumur degrees into Centigrade degrees (20, for example), it is merely necessary to multiply them by $\frac{5}{4}$ (which gives 25). Similarly, Centigrade degrees are converted into Réaumur by multiplying them by $\frac{4}{5}$.

The thermometric scale invented by Fahrenheit in 1714 is still much used in England, and also in Holland and North America. The higher fixed point is, like that of the other scales, the temperature of boiling water; but the null point of zero is the temperature obtained by mixing equal weights of sal-ammoniac and snow, and the interval between the two points is divided into 212 degrees. The zero was selected because the temperature was the lowest then known, and was thought to represent absolute cold. When Fahrenheit's thermometer is placed in melting ice it stands at 32 degrees, and therefore 100 degrees on the Centigrade scale are equal to 180 degrees on the Fahrenheit scale, and thus 1 degree Centigrade is equal to $\frac{9}{5}$ of a degree Fahrenheit, and, conversely, 1 degree Fahrenheit is equal to $\frac{5}{9}$ of a degree Centigrade.

If it be required to convert a certain number of Fahrenheit degrees (95, for example) into Centigrade degrees, the number 32 must first be subtracted in order that the degrees may count from the same part of the scale. The remainder in the example is thus 63, and, as 1 degree Fahrenheit is equal to $\frac{5}{9}$ of a degree Centigrade, 63 degrees are equal to $63 \times \frac{5}{9}$ or 35 degrees Centigrade.

If F be the given temperature in Fahrenheit degrees and C the corresponding temperature in Centigrade degrees, the former may be converted into the latter by means of the formula

$$(F - 32)\tfrac{5}{9} = C,$$

and, conversely, Centigrade degrees may be converted into Fahrenheit by means of the formula

$$\tfrac{9}{5}C + 32 = F.$$

The formulæ are applicable to all temperatures of the two scales, provided the signs are taken into account. Thus, to convert the temperature of 5 degrees Fahrenheit into Centigrade degrees, we have

$$(5 - 32)\tfrac{5}{9} = \frac{-27 \times 5}{9} = -15 \text{ C.}$$

In like manner we have, for converting Réaumur into Fahrenheit degrees, the formula

$$\tfrac{9}{4}R + 32 = F,$$

and, conversely, for changing Fahrenheit into Réaumur degrees, the formula

$$(F - 32)\tfrac{4}{9} = R.$$

304. **Displacement of zero.**—Thermometers, even when constructed with the greatest care, are subject to a source of error which must be taken into account; this is, that in course of time the zero tends to rise, the displacement sometimes extending to as much as two degrees; so that when the thermometer is immersed in melting ice it no longer sinks to zero.

This is generally attributed to a diminution of the volume of the bulb and also of the stem, occasioned by the pressure of the atmosphere. It is usual with very accurate thermometers to fill them two or three years before they are graduated. Joule once observed that even after twenty-five years a delicate thermometer indicated a displacement of zero.

Besides this slow displacement, there are often variations in the position of the zero, when the thermometer has been exposed to temperatures above 60°, caused by the fact that the bulb and stem do not contract on cooling to their original volume (296); these differences are greater the thicker the glass sides, and hence it is necessary from time to time to verify the position of zero when a thermometer is used for delicate determinations.

Regnault noticed that some mercurial thermometers, which agree at 0° and at 100°, differ between these points, and that these differences frequently amount to several degrees. Regnault ascribed this to the unequal expansion of different kinds of glass.

305. **Limits to the employment of mercurial thermometers.**—Of all thermometers in which liquids are used, the one with mercury is the most useful, because this liquid expands most regularly, and is easily obtained pure, and because its expansion between $-36°$ and 100° is *regular*; that is, proportional to the degree of heat. It also has the advantage of having a very low specific heat. But for temperatures below $-36°$ C. the alcohol thermometer must be used, since mercury solidifies at $-40°$ C. Above 100 degrees the coefficient of expansion increases, and the indications of the mercurial thermometer are only approximate, the error rising sometimes to several degrees. Mercury thermometers also cannot be used for temperatures above 350°, for this is the boiling point of mercury.

306. **Alcohol thermometer.**—The *alcohol thermometer* differs from the mercury thermometer in being filled with coloured alcohol. But as the expansion of liquids is less regular in proportion as they are near the boiling point, alcohol, which boils at 78° C., expands very irregularly. Hence, alcohol thermometers are usually graduated by placing them in baths at different temperatures together with a standard mercurial thermometer, and marking on the alcohol thermometer the temperature indicated by the mercury thermometer. In this manner the alcohol thermometer is comparable with the mercury one; that is to say, it indicates the same temperatures under the same conditions. The alcohol thermometer is especially used for low temperatures, for it does not solidify except at extreme degrees of cold (343).

307. **Conditions of the delicacy of a thermometer.**—A thermometer may be delicate in two ways:—1. When it indicates very small changes of temperature. 2. When it quickly assumes the temperature of the surrounding medium.

The first object is attained by having a very narrow capillary tube and a very large bulb; the expansion of the mercury on the stem is then limited to a small number of degrees, from 10 to 20 or 20 to 30 for instance, so that each degree occupies a great length on the stem, and can be subdivided into very small fractions. The second kind of delicacy is obtained by making the bulb very small, for then it rapidly assumes the temperature of the liquid in which it is placed.

A good mercury thermometer should answer to the following tests:—When its bulb and stem, to the top of the column of mercury, are immersed in melting ice, the top of the mercury should exactly indicate 0° C.; and when suspended with its bulb and scale immersed in the steam of water boiling in a metal vessel (as in fig. 293), the barometer standing at 760 mm., the mercury should be stationary at 100° C. When the instrument is inverted, the mercury should fill the tube, and fall with a metallic click, thus showing the complete exclusion of air. The value of the degrees should be uniform; to ascertain this a little cylinder of mercury may be detached from the column by a slight jerk, and on inclining the tube it may be made to pass from one portion of the bore to another. If the scale be properly graduated, the column will occupy an equal number of degrees in all parts of the tube.

308. **Differential thermometer.**—Sir John Leslie constructed a thermometer for showing the difference of temperature of two neighbouring

places, from which it has received the name of the *differential thermometer.*

A modified form of it is that devised by Matthiessen (fig. 296), which has the advantage of being available for indicating the temperature of liquids. It consists of a bent glass tube, each end of which is bent twice, and terminates in a bulb ; the bulbs being pendent can be readily immersed in

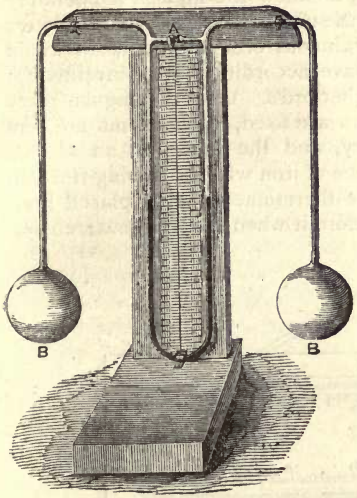


Fig. 296.

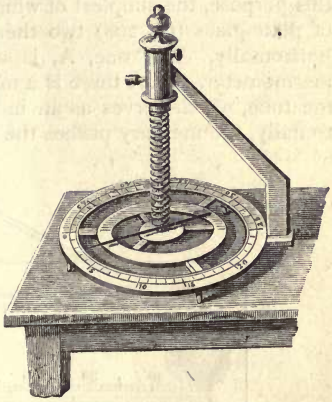


Fig. 297.

a liquid. The bend contains some coloured liquid, and in a tube which connects the two limbs is a stopcock, by which the liquid in each limb is easily brought to the same level. The whole is supported by a frame.

When one of the bulbs is at a higher temperature than the other, the liquid in the stem is depressed and rises in the other stem. The instrument is now only used as a *thermoscope* ; that is, to indicate a difference of temperature between the two bulbs, and not to measure its amount.

309. **Breguet's metallic thermometer.**—Breguet invented a thermometer of considerable delicacy, which depends on the unequal expansion of metals. It consists of three strips of platinum, gold, and silver, which are passed through a rolling mill so as to form a very thin metallic ribbon. This is then coiled in a spiral form, as seen in fig. 297, and, one end being fixed to a support, a light needle is fixed to the other, which is free to move round a graduated scale.

Silver, which is the most expansible of the metals, forms the inner face of the spiral, and platinum the outer. When the temperature rises, the silver expands more than the gold or platinum, the spiral unwinds itself, and the needle moves from left to right of the above figure. The contrary effect is produced when the temperature sinks. The gold is placed between the other two metals because its expansibility is intermediate between that of the silver and the platinum. Were these two metals employed alone, their

rapid unequal expansion might cause a fracture. Breguet's thermometer is empirically graduated in Centigrade degrees, by comparing its indications with those of a standard mercury thermometer.

On this principle depend several forms of pocket thermometers, and it is also applied in some registering thermometers.

310. **Rutherford's maximum and minimum thermometers.**—It is necessary, in meteorological observations, to know the highest temperature of the day and the lowest temperature of the night. Ordinary thermometers could only give these indications by a continuous observation, which would be impracticable. Several instruments have accordingly been invented for this purpose, the simplest of which is Rutherford's. On a rectangular piece of plate-glass (fig. 298) two thermometers are fixed, whose stems are bent horizontally. The one, A, is a mercury, and the other, B, an alcohol thermometer. In A there is a minute piece of iron wire, A, moving freely in the tube, which serves as an index. The thermometer being placed horizontally, the mercury pushes the index before it when the temperature rises.

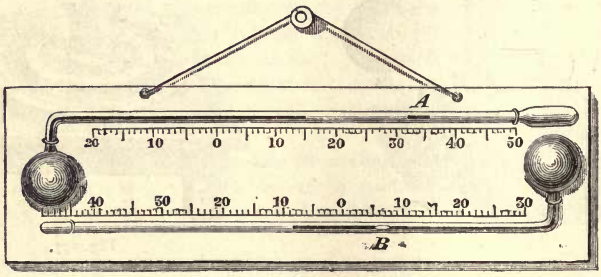


Fig. 298.

But as soon as the mercury contracts, the index remains in that part of the tube to which it has been moved, for there is no adhesion between the iron and the mercury. In this way the index registers the highest temperature which has been attained ; in the figure this is 32°. In the minimum thermometer there is a small hollow glass tube which serves as index. When it is at the end of the column of liquid, and the temperature falls, the column contracts, and carries the index with it, in consequence of adhesion, until it has reached the greatest contraction. When the temperature rises the alcohol expands, and, passing between the sides of the tube and the index, does not displace B. The position of the index gives therefore the lowest temperature which has been reached ; in the figure this is 9·5 degrees below zero.

311. **Pyrometers.**—The name *pyrometers* is given to instruments for measuring temperatures so high that mercurial thermometers could not be used. The older contrivances for this purpose—Wedgwood's, Daniell's (which in principle resembled the apparatus in fig. 287), Brongniart's, &c.—have gone entirely out of use. None of them give an exact measure of temperature. The arrangements now used for the purpose are either based on the expansion of gases and vapours, on the specific heat of solids, or on the electrical properties of bodies, and will be subsequently described.

312. **Different remarkable temperatures.**—The following table gives some of the most remarkable points of temperature. It may be observed that it is easier to produce very high temperatures than very low degrees of cold.

Greatest artificial cold produced by a bath of bisulphide of carbon and liquid nitrous acid		−140° C.
Greatest cold produced by ether and liquid carbonic acid		−110
Greatest natural cold recorded in Arctic expeditions		− 58·7
Mercury freezes		− 39·4
Mixture of snow and salt		− 20
Ice melts		0
Greatest density of water		+4
Mean temperature of London		9·9
Blood heat		36·6
Water boils		100
Highest temperature of a Turkish Bath		125
Mercury boils		350
Sulphur boils		440
Red heat (just visible)	(Daniell)	526
Silver melts	„	1000
Zinc boils	„	1040
Cast iron melts	„	1530
Highest heat of wind furnace	„	1800
Platinum melts		2000
Iridium „		2700

CHAPTER II

EXPANSION OF SOLIDS

313. **Linear expansion and cubical expansion. Coefficients of expansion.**—It has been already explained that in solid bodies the expansion may be according to three dimensions—linear, superficial, and cubical.

The *coefficient of linear expansion* is the increase of the unit of length of a body when its temperature rises from zero to 1 degree; the *coefficient of superficial expansion* is the increase of the surface in being heated from zero to 1 degree, and the *coefficient of cubical expansion* is the increase of the unit of volume under the same circumstances.

These coefficients vary with different bodies, but for the same body *the coefficient of cubical expansion is three times that of the linear expansion*, as is seen from the following considerations:—Suppose a cube, the length of whose side is 1 at zero. Let k be the elongation of this side in passing from zero to 1 degree, its length at 1 degree will be $1 + k$, and the volume of the cube, which was 1 at zero, will be $(1 + k)^3$, or $1 + 3k + 3k^2 + k^3$. But as the elongation k is always a very small fraction (see table, Art. 316), its square, k^2, and still more its cube, k^3, are so small that they may be neglected, and the value at 1 degree becomes very nearly $1 + 3k$. Consequently, the increase of volume is $3k$, or thrice the coefficient of linear expansion.

In the same manner it may be shown that the coefficient of superficial expansion is double the coefficient of linear expansion.

314. **Measurement of the coefficient of linear expansion. Lavoisier and Laplace's method.**—The apparatus used by Lavoisier and Laplace for determining the coefficients of linear expansion (fig. 299) consists of a brass

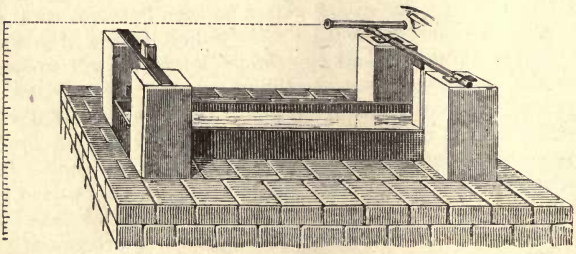


Fig. 299.

trough, placed on a furnace between four stone supports. On the two supports on the right hand there is a horizontal axis, at the end of which is a telescope; on the middle of this axis, and at right angles to it, is fixed a glass rod, turning with it, as does also the telescope. The other two supports

are joined by a cross-piece of iron, to which another glass rod is fixed, also at right angles. The trough, which contains oil or water, is heated by a furnace not represented in the figure, and the bar whose expansion is to be determined is placed in it.

Fig. 300 represents a section of the apparatus ; G is the telescope, KH the bar, whose ends press against the two glass rods F and D. As the rod F is fixed, the bar can only expand in the direction KH, and in order to eliminate the effects of friction it rests on two glass rollers. Lastly, the telescope has a cross-wire in the eye-piece, which, when the telescope moves, indicates the depression by the corresponding number of divisions on a vertical scale, AB, at a distance of 220 yards.

The trough is first filled with ice, and the bar being at zero, the division on the scale AB, corresponding to the wire of the telescope, is read off. The ice having been removed, the trough is filled with oil or water, which is heated to a given temperature. The bar then expands, and when its temperature has become stationary, which is determined by means of thermometers, the division of the scale, seen through the telescope, is read off.

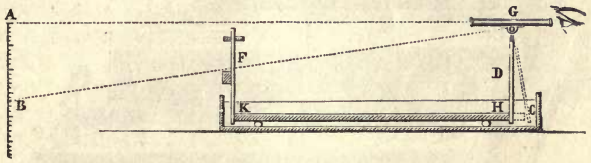


Fig. 300.

From these data the elongation of the bar is determined; for since it has become longer by a quantity, CH, and the optical axis of the telescope has become inclined in the direction GB, the two triangles, GHC and ABG, are similar, for they have the sides at right angles each to each, so that $\frac{HC}{AB} = \frac{GH}{AG}$. In the same way, if HC′ were another elongation, and AB′ a corresponding deviation, there would still be $\frac{HC'}{AB'} = \frac{GH}{AG}$; from which it follows that the ratio between the elongation of the bar and the deflection of the telescope is constant, for it is always equal to $\frac{GH}{AG}$. A preliminary measurement has shown that this ratio was $\frac{1}{744}$. Consequently, $\frac{HC}{AB} = \frac{1}{744}$, whence $HC = \frac{AB}{744}$; that is, the total elongation of the bar is obtained by dividing the length on the scale traversed by the cross-wire by 744. Dividing this elongation by the length of the bar, and then by the temperature of the bath, the quotient is the dilatation for the unit of length and for a single degree—in other words, the coefficient of linear dilatation.

315. **Roy and Ramsden's method.**—Lavoisier and Laplace's method is founded on an artifice which is frequently adopted in physical determinations, and which consists in amplifying by a known amount dimensions which, in themselves, are too small to be easily measured. Unfortunately, this plan is

often more fallacious than profitable, for it is first necessary to determine the ratio of the motion measured to that on which it depends. In the present case, it is necessary to know the lengths of the arms of the lever in the apparatus. But this preliminary operation may introduce errors of such importance as partially to counterbalance the advantage of great delicacy. The following method, used by General Roy in 1787, and which was devised by Ramsden, depends on another principle. It measures the elongations directly, and without amplifying them ; but it measures them by means of a micrometric telescope, which indicates very small displacements.

The apparatus (fig. 301) consists of three parallel metal troughs about 6 feet long. In the middle one there is a bar of the body whose expansion is to be determined, and in the two others are cast-iron bars of exactly the same length as this bar. Rods are fixed vertically on both ends of these

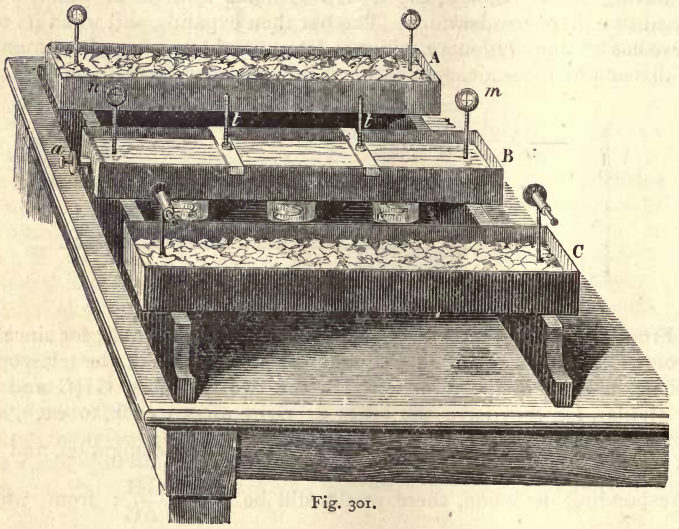


Fig. 301.

three bars. On the rods in the troughs A and B there are rings with cross-wires like those of a telescope. On the rods in the trough C are small telescopes, also provided with cross-wires.

The troughs being filled with ice, and all three bars at zero, the points of intersection of the wires in the disc, and of the wires in the telescope, are all in a line at each end of the bar. The temperature in the middle trough is then raised to 100° C. by means of spirit lamps placed beneath the trough ; the bar expands, but as it is in contact with the end of the screw, *a*, fixed on the side, all the elongation takes place in the direction *nm*, and, as the cross-wire *n* remains in position, the cross-wire *m* is moved towards B by a quantity equal to the elongation. But since the screw *a* is attached to the bar, by turning it slowly from right to left, the bar is moved in the direction *mn*, and the cross-wire *m* regains its original position. To effect this, the screw has been turned by a quantity exactly equal to the elongation of the bar,

and, as this advance of the screw is readily deduced from the number of turns of its *thread* (11), the total expansion of the bar is obtained, which, divided by the temperature of the bath, and this quotient by the length of the bar at zero, gives the coefficient of linear expansion.

316. **Coefficients of linear expansion.**—By one or the other method the following results have been obtained :—

Coefficients of linear expansion for 1° *between* 0° *and* 100° C.

Diamond	0·000001180	Silver	0·000019097
Pine	0·000006080	Tin	0·000021730
Graphite	0·000007860	Aluminium	0·000023130
Marble	0·000008490	Lead	0·000028575
White glass	0·000008613	Zinc	0·000029417
Platinum	0·000008842	Sodium chloride	0·000040390
Untempered steel	0·000010788	Mercury	0·000060000
Cast iron	0·000011250	Ice	0·000064000
Sandstone	0·000011740	Sal ammoniac	0·000063000
Wrought iron	0·000012204	Sulphur	0·000064130
Tempered steel	0·000012395	Sodium	0·000072000
Gold	0·000014660	Potassium	0·000083000
Copper	0·000017182	Ebonite (17° to 35°)	0·000080600
Bronze	0·000018167	Paraffin	0·000278540
Brass	0·000018782	Guttapercha	0·000598000

From what has been said about the linear expansion (313), the coefficients of cubical expansion of solids are obtained by multiplying those of linear expansion by 3, and conversely those of linear expansion may be deduced from the cubical if we divide by 3.

The coefficients of the expansion of the metals vary with their physical condition, being different for the same metal according as it has been cast or hammered and rolled, hardened or annealed. As a general rule, operations which increase the density increase also the rate of expansion. But even for substances in apparently the same condition, different observers have found very unequal amounts of expansion ; this may arise in the case of compound substances, such as glass, brass, or steel, from a want of uniformity in chemical composition, and in simple bodies from slight differences of physical state.

The expansion of amorphous solids, and of those which crystallise in the regular system, is the same for all dimensions, unless they are subject to a strain in some particular direction. A fragment of such a substance varies in bulk, but retains the same shape. Crystals not belonging to the regular system when heated, exhibit an unequal expansion in the direction of their different axes, in consequence of which the magnitude of their angles, and therefore their form, is altered. In the dimetric system the expansion is the same in the direction of the two equal axes, but different in the third. In crystals belonging to the hexagonal system the expansion is the same in the direction of the three secondary axes, but different from that according to the principal one. In the trimetric system it is different in all three directions.

To the general law that all bodies expand by heat there is an important exception in the case of iodide of silver, which contracts somewhat when heated. Between $-60°$ and $+142°$ C. it has a negative coefficient of expansion, the value of which is 0·00000139 for 1° C.

Fizeau determined the expansion of a great number of crystallised bodies by an optical method. He placed thin plates of the substance on a glass plate and let yellow light pass through them. He thus obtained alternately yellow and dark Newton's rings (*q.v.*) On heating, the plate of the substance expanded, the thin layer of air became thinner, and the position of the rings was altered. From the alteration in their position the amount of the expansion could be deduced. Among the results he has obtained is the curious one that certain crystallised bodies, such as diamond, emerald, and cuprous oxide, contract on being cooled to a certain temperature, but as the cooling is continued below this temperature they expand. They have thus a temperature of maximum density, as is the case with water (330). In the case of emerald and cuprous oxide this temperature is at $-4·2°$, in the case of diamond at $-42·3°$.

317. **The coefficients of expansion increase with the temperature.**—According to Matthiessen, who determined the expansion of some metals and alloys by weighing them in water at different temperatures, the coefficients of expansion are not quite regular between 0° and 100°. He found the following values for the linear expansion between 0° and 100° :—

Zinc . . .	$L_t = L_0 (1 + 0·00002741\ t + 0·0000000235\ t^2)$
Lead . . .	$L_t = L_0 (1 + 0·00002716\ t + 0·0000000074\ t^2)$
Silver . . .	$L_t = L_0 (1 + 0·00001809\ t + 0·0000000135\ t^2)$
Copper . .	$L = L_0 (1 + 0·00001408\ t + 0·0000000264\ t^2)$
Gold . . .	$L_t = L_0 (1 + 0·00001358\ t + 0·0000000112\ t^2)$

Matthiessen further found that the coefficients of expansion of an alloy are very nearly equal to the mean of the coefficients of expansion of the volumes of the metals composing it.

318. **Formulæ relative to the expansion of solids.**—Let l be the length of a bar at zero, l' its length at the temperature $t°$ C., and a its coefficient of linear expansion. The tables usually give the expansion for 1° between 0° and 100° as in Art. 316, or for 100° ; in this latter case a is obtained by dividing the number by 100.

The elongation corresponding to t is t times a or at for a single unit of length, or atl for l units. The length of the bar which is l at zero is $l + atl$ at t, consequently,

$$l' = l + atl = l(1 + at).$$

This formula gives the length of a body l' at $t°$, knowing its length l at zero, and the coefficient of expansion a ; and by simple algebraical transformations we can obtain from it formulæ for the length at zero, knowing the length l' at $t°$, and also for finding a, the coefficient of linear expansion, knowing the lengths l' and l at $t°$ and zero respectively.

The formulæ for cubical expansion are entirely analogous to the preceding.

The following are examples of the application of these formulæ :—

(i.) A metal bar has a length l' at $t'°$; what will be its length l at $t°$?

From the above formula we first get the length of the given bar at zero, which is $\frac{l'}{1+at'}$; by means of the same formula we pass from zero to t'° in multiplying by $1+at$, which gives for the desired length the formula

$$l=\frac{l'(1+at)}{1+at'}.$$

(ii.) The density of a body being d at zero, required its density d' at t°.

If 1 be the volume of the body at zero, and D its coefficient of cubical expansion, the volume at t will be $1+Dt$; and as the density of a body is in inverse ratio of the volume which the body assumes in expanding, we get the inverse proportion,

$$d' : d = 1 \quad : \quad 1+Dt$$

$$\frac{d'}{d}=\frac{1}{1+Dt}; \text{ or } d'=\frac{d}{1+Dt}.$$

Consequently, when a body is heated from o to t°, its density, and therefore its weight for an equal volume, is inversely as the expression $1+Dt$.

319. **Applications of the expansion of solids.**—In the arts we meet with numerous examples of the influence of expansion. (i.) The bars of furnaces must not be fitted tightly at their extremities, but must, at least, be free at one end, otherwise in expanding they would split the masonry. (ii.) In making railways a small space is left between the successive rails, for, if they touched, the force of expansion would cause them to curve or would break the chairs. (iii.) Water-pipes are fitted to one another by means of telescope joints, which allow room for expansion. (iv.) If a glass vessel is heated or cooled too rapidly, it cracks, especially if it be thick; this arises from the fact that, since glass is a bad conductor of heat, the sides become unequally heated, and consequently unequally expanded, which causes a fracture. (v.) The cracking off of a portion of a glass tube by red-hot charcoal is due to the expansion of the heated parts, which detach themselves from the rest.

When bodies have been heated to a high temperature, the force produced by their contraction on cooling is very considerable; it is equal to the force which is needed to compress or expand the material to the same extent by mechanical means. According to Barlow, a bar of malleable iron a square inch in section is stretched $\frac{1}{10000}$th of its length by a weight of a ton; the same increase is experienced by about 9° C. A difference of 45° C. between the cold of winter and the heat of summer is not unfrequently experienced in this country. In that range, a wrought-iron bar ten inches long will vary in length by $\frac{1}{2000}$th of an inch, and will exert a strain, if its ends are securely fastened, of fifty tons. It has been calculated from Joule's data that the work done by heat in expanding a pound of iron between o° and 100°, during which it increases about $\frac{1}{850}$ of its bulk, is equal to 16,000 foot-pounds; that is, it could raise a weight of over 7 tons through a height of one foot.

(i.) An application of this contractile force is seen in the mode of securing tires on wheels. The tire being made red-hot, and thus considerably expanded, is placed on the circumference of the wheel and then cooled. The tire, when cold, embraces the wheel with such force as not only to

secure itself on the rim but also to press home the joints of the spokes into the felloes and nave. (ii.) Another interesting application was made in the case of a gallery at the Conservatoire des Arts et Métiers in Paris, the walls of which had begun to bulge outwards. Iron bars were passed across the building and screwed into plates on the outside of the walls. Each alternate bar was then heated by means of lamps, and when the bar had expanded it was screwed up. The bars being then allowed to cool contracted, and in so doing drew the walls together. The same operation was performed on the other bars.

320. **Compensation pendulum.**—An important application of the expansion of metals has been made in the *compensation pendulum*. This is a pendulum in which the elongation, when the temperature rises, is so compensated that the distance between the centre of suspension and the centre of oscillation (79) remains constant, which, from the laws of the pendulum (80), is necessary for isochronous oscillations, and in order that the pendulum may be used as a regulator of clocks.

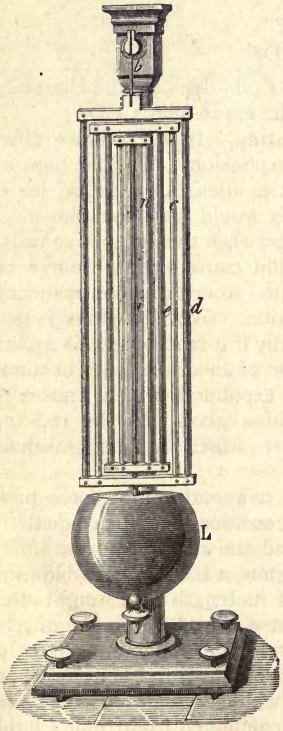


Fig. 302.

In fig. 302, which represents the *gridiron* pendulum, one of the commonest forms of compensation pendulum, the ball, L, instead of being supported by a single rod, is supported by a framework, consisting of alternate rods of steel and brass. In the figure, the shaded rods represent steel; including a small steel rod, *b*, which supports the whole of the apparatus, there are six of them. The rest of the rods, four in number, are of brass. The rod *i*, which supports the ball, is fixed at its upper end to a horizontal cross-piece; at its lower end it is free, and passes through the two circular holes in the lower horizontal cross-pieces.

Now it is easy to see from the manner in which the vertical rods are fixed to the cross-pieces, that the elongation of the steel rods can only take place downward, and that of the brass rods upward. Consequently, in order that the pendulum may remain of the same length, it is necessary that the elongation of the brass rods shall tend to make the ball rise, by exactly the same quantity that the elongation of the steel rods tend to lower it; a result which is attained when the sum of the lengths of the steel rods A is to the sum of the lengths of the brass rods B in the inverse ratio of the coefficients of expansion of steel and brass, a and b; that is, in the proportion $A : B = b : a$.

The elongation of the rod may also be compensated for by means of *compensating strips*. These consist of two blades of copper and iron soldered together and fixed to the pendulum rod, as represented in fig. 303.

The copper blade, which is more expansible, is below the iron. When the temperature sinks, the pendulum rod becomes shorter, and the ball rises. But at the same time the compensating strips become curved, as seen in fig. 304, in consequence of the copper contracting more than the iron, and two metal balls at their extremities become lower. If they have the proper size

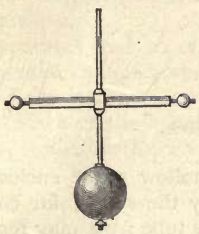


Fig. 303.

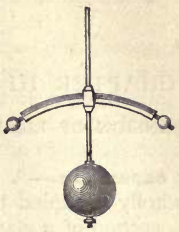


Fig. 304.

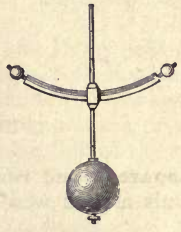


Fig. 305.

in reference to the pendulum ball, the parts which tend to approach the centre of suspension compensate those which tend to remove from it, and the centre of oscillation is not displaced. If the temperature rises, the pendulum ball descends ; but at the same time the small balls ascend, as shown in fig. 305, so that there is always compensation.

One of the most simple compensating pendulums is the *mercury pendulum*, invented by an English watchmaker, Graham. The ball of the pendulum, instead of being solid, consists of a glass cylinder, containing pure mercury, which is placed in a sort of stirrup, supported by a steel rod. When the temperature rises the rod and stirrup become longer, and thus lower the centre of gravity ; but at the same time the mercury expands, and, rising in the cylinder, produces an inverse effect, and as mercury is much more expansible than steel, a compensation may be effected without making the mercurial vessel of undue dimensions.

The same principle is applied in the *compensating balances* of chronometers (fig. 306). The motion here is regulated by a *balance* or wheel, furnished with a spiral spring not represented in the figure, and the time of the chronometer depends on the force of the spring, the mass of the balance, and on its circumference. Now when the temperature rises the circumference increases, and the chronometer goes slower : and to prevent this part of the mass must be brought nearer the axis. The circumference of the balance consists of compensating strips BC, of which the more expansible metal is on the outside, and towards the end of these are small masses of metal D, which play the same part as the balls in the above case. When the radius is expanded by heat, the small masses are brought nearer the centre in consequence of the curvature of the strips ; and as they can be fixed in any position, they are easily arranged so as to compensate for the expansion of the balance. It may, however, here be observed that the chief action of heat on chronometers is to expand and soften the spring, and thereby lessen its elasticity ; this action produces five times the effect on the rate that the expansion of the balance-wheel does.

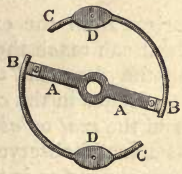


Fig. 306.

CHAPTER III

EXPANSION OF LIQUIDS.

321. **Apparent and real expansion.**—A hollow space enclosed by a solid expands as if it were wholly occupied by the solid; for consider a section of a glass tube; we may regard this as made up of a series of innumerable concentric circles; when the tube is heated each of these glass circles becomes larger, and in doing so must press outwards, and these expansions are the same whether there is another circle within it or not; the hollow space will become larger just as if it were a solid glass rod. This may be illustrated by the following experiment. If a flask of thin glass, provided with a narrow stem, the flask and part of the stem being filled with some coloured liquid, be immersed in hot water (fig. 307), the column of liquid in the stem at first sinks from *b* to *a*, but then immediately after rises, and continues to do so until the liquid inside has the same temperature as the hot water. The first sinking of the liquid is not due to its contraction; it arises from the expansion of the glass, which becomes heated before the heat can reach the liquid; but the expansion of the liquid soon exceeds that of the glass, and the liquid then ascends.

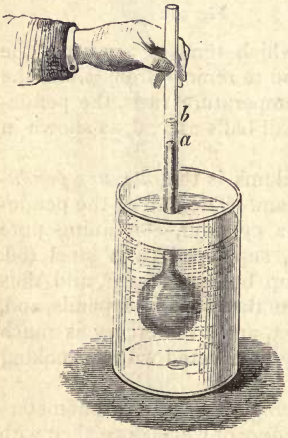


Fig. 307.

Hence in the case of liquids we must distinguish between the *apparent* and the *real* or *absolute* expansion. The apparent expansion is that which is actually observed when liquids contained in vessels are heated; the *absolute* expansion is that which would be observed if the vessel did not expand; or, as this is never the case, it is the apparent expansion corrected for the simultaneous expansion of the containing vessel.

As has been already stated, the cubical expansion of liquids is alone considered; and as in the case of solids, the *coefficient of expansion* of a liquid is the increase of the unit of volume for a single degree; but a distinction is here made between the *coefficient of absolute expansion* and the *coefficient of apparent expansion.* Of the many methods which have been employed for determining these two coefficients, we shall describe that of Dulong and Petit.

322. **Coefficient of the absolute expansion of mercury.**—In order to determine the coefficient of the absolute expansion of mercury, the influence of the envelope must be eliminated. Dulong and Petit's method depends on the hydrostatical principle that in two communicating vessels, the heights of two columns of liquid in equilibrium are inversely as their densities (107), a principle independent of the diameters of the vessels, and therefore of their expansions.

The apparatus consists of two glass tubes, A and B (fig. 308), joined by a capillary tube and kept vertical on an iron support, KM, the horizontality of which is adjusted by means of two levelling screws and two spirit levels, *m* and *n*. Each of the tubes is surrounded by a metal case, of which the smaller, D, is filled with ice; the other, E, containing oil, can be heated by

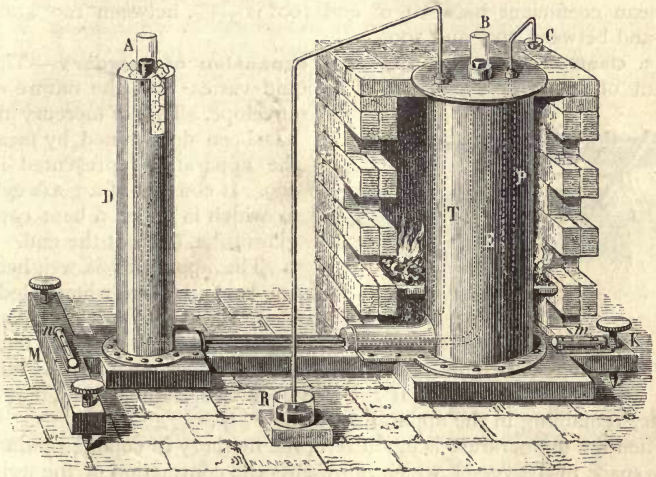


Fig. 308.

the furnace, which is represented in section so as to show the case. Mercury is poured into the tubes A and B; it remains at the same level in both, as long as they are at the same temperature, but rises in B in proportion as it is heated, and expands.

Let h and d be the height and density of the mercury in the leg A, at the temperature zero, and h' and d' the same quantities in the leg B. From the hydrostatical principle previously cited we have $hd = h'd'$. Now from the problem in Art. 318, $d' = \frac{d}{1 + Dt}$, D being the coefficient of absolute expansion of mercury; substituting this value of d' in the equation, we have $\frac{h'd}{1 + Dt} = hd$, from which we get $D = \frac{h' - h}{ht}$.

The coefficient of absolute expansion of mercury is obtained from this formula, knowing the heights h' and h, and the temperature t of the bath in which the tube B is immersed. In Dulong and Petit's experiment this tem-

perature was measured by a weight thermometer, P (323), the mercury of which overflowed into the basin, C, and by means of an air thermometer, T (334); the heights h' and h were measured by a cathetometer, K (88).

Dulong and Petit found by this method that the coefficient of absolute expansion of mercury between 0° and 100° C. is $\frac{1}{5550}$. But they found that the coefficient increased with the temperature. Between 100° and 200° it is $\frac{1}{5425}$, and between 200° and 300° it is $\frac{1}{5300}$. The same observation has been made in reference to other liquids, showing that their expansion is not regular. It has been found that this expansion is less regular in proportion as liquids are near a change in their state of aggregation; that is, approach their freezing or boiling points. Dulong and Petit found that the expansion of mercury between $-36°$ and 100° is practically quite uniform.

Regnault, who determined this important physical constant, found that the mean coefficient between 0° and 100° is $\frac{1}{5508}$, between 100° and 200°, $\frac{1}{5374}$, and between 200° and 300°, $\frac{1}{5218}$.

323. **Coefficient of the apparent expansion of mercury.**—The coefficient of apparent expansion of a liquid varies with the nature of the envelope. That of mercury in glass has been determined by means of the apparatus represented in fig. 309. It consists of a glass cylinder to which is joined a bent capillary glass tube, open at the end.

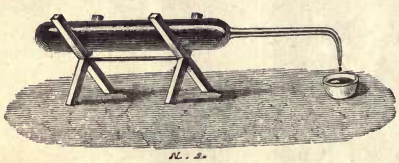


Fig. 309.

The apparatus is weighed first empty, and then when filled with mercury at zero: the difference gives the weight of the mercury, P. It is then raised to a known temperature, t; the mercury expands, a certain quantity passes out, which is received in the capsule and weighed. If the weight of this mercury be p, that of the mercury remaining in the apparatus will be $P-p$.

When the temperature is again zero, the mercury in cooling produces an empty space in the vessel, which represents the contraction of the weight of mercury $P-p$, from $t°$ to zero, or, what is the same thing, the expansion of the same weight from 0 to $t°$; that is, the weight p represents the expansion of the weight $P-p$, for $t°$. If this weight expands in glass by a quantity p for $t°$, a single unit of weight would expand $\frac{p}{(P-p)}$ for $t°$, and $\frac{p}{(P-p)t}$ for a single degree; consequently, for D', the coefficient of apparent expansion of mercury in glass, we have $D' = \frac{p}{(P-p)t}$. Dulong and Petit found the coefficient of apparent expansion of mercury in glass to be $\frac{1}{6480}$.

324. **Weight thermometer.**—The apparatus represented in fig. 309 is called the *weight thermometer*, because the temperature can be deduced from the weight of mercury which overflows.

The above experiments have placed the coefficient of apparent expansion at $\frac{1}{6480}$; we have therefore the equation $\frac{p}{(P-p)t} = \frac{1}{6480}$, from which we get

$t = \frac{6480p}{P-p}$, a formula which gives the temperature t when the weights P and p are known.

325. **Coefficient of the expansion of glass.**—As the absolute expansion of a liquid is the apparent expansion, *plus* the expansion due to the envelope, the coefficient of the cubical expansion of glass is obtained by taking the difference between the coefficient of absolute expansion of mercury in glass and that of its apparent expansion. That is, the coefficient of cubical expansion of glass is

$$\frac{1}{5508} - \frac{1}{6408} = \frac{1}{38700} = 0{\cdot}00002584.$$

Regnault found that the coefficient of expansion varies with different kinds of glass, and further with the shape of the vessel. For ordinary chemical glass tubes, the coefficient is 0·0000254.

326. **Coefficients of expansion of various liquids.**—The coefficient of apparent expansion of liquids may be determined by means of an application of the principle of the weight thermometer, and the absolute expansion is obtained by adding to this coefficient the expansion of the glass.

Mean coefficients of absolute expansion of liquids for 1° C.

Ether	0·00015	Alcohol	0·00104
Mercury	0·00018	Bromine	0·00104
Water saturated with common salt	0·00050	Nitric acid	0·00110
		Chloroform	0·00111
Sulphuric acid	0·00063	Bisulphide of carbon	0·00114
Fixed oils	0·00080	Benzole	0·00125
Oil of turpentine	0·00090		

The numbers here given only hold for moderate temperatures. The coefficient of expansion of almost all liquids increases gradually from zero, and can only be expressed with accuracy by a somewhat complicated formula,

$$V_t = V_0 (1 + at + \beta t^2 + \gamma t^3)$$

in which t is the temperature, and a, β, and γ are constants specially determined for each liquid. The expansion of mercury is practically constant between $-36°$ and 100° C., while water contracts from zero to 4° (330), and then expands.

For many physical experiments a knowledge of the exact expansion of water is of great importance. This physical constant was determined with great care by Matthiessen, who found that between 4° and 30° it may be expressed by the formula

$$V_t = 1 - 0{\cdot}00000253\ (t-4) + 0{\cdot}00000008389\ (t-4)^2 + 0{\cdot}00000007173\ (t-4)^3;$$

and between 30 and 100 by $V_t = 0{\cdot}999695 + 0{\cdot}00000547 24t^2 + 0{\cdot}00000001126t^3$. Many liquids, with low boiling points, especially condensed gases, have very high coefficients of expansion. Thilorier found that liquid carbonic acid expands four times as much as air. Drion confirmed this observation and has obtained analogous results with chloride of ethyle, liquid sulphurous acid, and liquid hyponitrous acid.

327. **Correction of the barometric height.**—It has been already explained under the barometer (164) that, in order to make the indications of this instrument comparable in different places and at different times, they must be reduced to a uniform temperature, which is that of melting ice. The correction is made in the following manner :—

Let H be the barometric height at $t°$, and h its height at zero, d the density of mercury at zero, and d' its density at $t°$. The heights H and h are inversely as the densities d and d'; that is, $\frac{h}{H} = \frac{d'}{d}$. If we call one the volume of mercury at zero, its volume at $t°$ will be $1 + Dt$, D being the coefficient of absolute expansion of mercury. But these volumes, $1 + Dt$ and 1, are inversely as the densities d and d'; that is, $\frac{d'}{d} = \frac{1}{1 + Dt}$. Consequently, $\frac{h}{H} = \frac{1}{1 + Dt}$, whence $h = \frac{H}{1 + Dt}$. Replacing D by its value $\frac{1}{5508}$, we have

$$h = \frac{H}{1 + \frac{t}{5508}} = \frac{5508\ H}{5508 + t}.$$

In this calculation, the coefficient of absolute expansion of mercury is taken, and not that of apparent expansion ; for the value H is the same as if the glass did not expand, the barometric height being independent of the diameter of the tube, and therefore of its expansion.

328. **Correction of thermometric readings.**—If the whole column of mercury of a thermometer is not immersed in the space whose temperature is to be determined, it is necessary to make a correction, which in the accurate determination of boiling points, for instance, is of great importance, in order to arrive at the true temperature which the thermometer should show. That part of the stem which projects will have a temperature which must be estimated, and which may roughly be taken as something over that of the surrounding air.

Supposing, for instance, the actual reading is 160° and that the whole of the part over 80° is outside the vessel, while the temperature of the surrounding air is 15°. We will assume that the mean temperature of the stem is 25°, and that a length of $160° - 80°$ is to be heated through $160 - 25 = 135°$; this gives $80 \times \frac{135}{6480} = 1{\cdot}66$ (taking the coefficient of apparent expansion of mercury); so that the true reading is 161·66.

329. **Force exerted by liquids in expanding.**—The force which liquids exert in expanding is very great, and equal to that which would be required in order to bring the expanded liquid back to its original volume. Now we know what an enormous force is required to compress a liquid to even a very small extent (97). Thus between 0° and 10°, mercury expands by 0·0015790 of its volume at 0° ; its compressibility is 0·00000295 of its volume for one atmosphere ; hence a pressure of more than 600 atmospheres would be requisite to prevent mercury expanding when it is heated from 0° to 10°. In like manner a pressure of 140 atmospheres would be required to prevent water from expanding when its temperature was raised from 4° to 14°.

330. **Maximum density of water.**—Water presents the remarkable phenomenon that when its temperature sinks it contracts up to 4°; but from that point, although the cooling continues, it expands up to the freezing point, so that 4° represent the point of greatest contraction of water.

Many methods have been used to determine the maximum density of water. Hope made the following experiment:—He took a deep vessel with two apertures in the sides, in which he fixed thermometers, and having filled the vessel with water at 0°, he placed it in a room at a temperature of 15°. As the layers of liquid at the sides of the vessel became heated they sank to the bottom, and the lower thermometer marked 4° while the upper one was still at zero. Hope then made the inverse experiment; having filled the vessel with water at 15°, he placed it in a room at zero. The lower thermometer having sunk to 4° remained stationary for some time, while the upper one cooled down until it reached zero. Both these experiments prove that water is heavier at 4° than at 0°, for in both cases it sinks to the lower part of the vessel.

This last experiment may be adapted for lecture illustration by using a cylinder containing water at 15° C., partially surrounded by a jacket containing bruised ice (fig. 310).

Hallström made a determination of the maximum density of water in the following manner:—He took a glass bulb, loaded with sand, and weighed it in water of different temperatures. Allowing for the expansion of glass, he found that 4·1° was the temperature at which it lost most weight, and consequently this was the temperature of the maximum density of water.

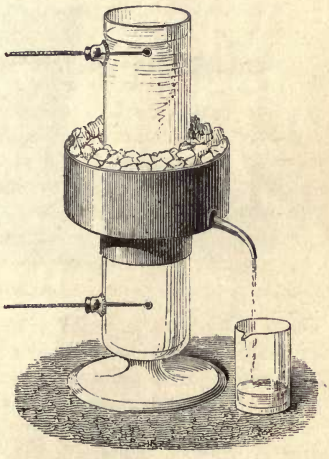


Fig. 310.

Despretz arrived at the temperature 4° by another method. He took a water thermometer—that is to say, a bulbed tube containing water—and, placing it in a bath, the temperature of which was indicated by an ordinary mercury thermometer, found that the water contracted to the greatest extent at 4°, and that this therefore is the point of greatest density.

This phenomenon is of great importance in the economy of nature. In winter the temperature of lakes and rivers falls, from being in contact with the cold air and from other causes, such as radiation. The cold water sinks to the bottom, and a continual series of currents goes on until the whole has a temperature of 4°. The cooling on the surface still continues, but the cooled layers being lighter remain on the surface, and ultimately freeze. The ice formed thus protects the water below, which remains at a temperature of 4°, even in the most severe winters, a temperature at which fish and other inhabitants of the water are not destroyed.

Salt dissolved in water lowers the temperature of the maximum density

so that sea water exhibits such a maximum. According to Rosetti, this temperature is between 3·2° and 3·9° in the Adriatic.

The following table of the density of pure water at various temperatures is based on several sets of observations :—

Density of water between 0° *and* 30°

Temperatures	Densities	Temperatures	Densities	Temperatures	Densities
0	0·99988	12	0·99955	24	0·99738
1	0·99993	13	0·99943	25	0·99714
2	0·99997	14	0·99930	26	0·99689
3	0·99999	15	0·99915	27	0·99662
4	1·00000	16	0·99900	28	0·99635
5	0·99999	17	0·99884	29	0·99607
6	0·99997	18	0·99870	30	0·99579
7	0·99994	19	0·99847	40	0·99226
8	0·99988	20	0·99827	50	0·98820
9	0·99982	21	0·99806	60	0·98232
10	0·99974	22	0·99785	70	0·97796
11	0·99965	23	0·99762	80	0·97191

CHAPTER IV

EXPANSION AND DENSITY OF GASES

331. **Gay-Lussac's method.**—Gases are the most expansible of all bodies, and at the same time the most regular in their expansion. The coefficients of expansion, too, of the several gases differ only by very small quantities. The cubical expansion of gases need alone be considered.

Gay-Lussac first determined the coefficient of the expansion of gases by means of the apparatus represented in fig. 311.

In a rectangular metal bath, about 16 inches long, was fitted an air thermometer, which consisted of a capillary tube, AB, with a bulb, A, at one end. The tube was divided into parts of equal capacity, and the contents of the bulb ascertained in terms of these parts. This was effected by weighing the bulb and tube full of mercury at zero, and then heating slightly to expel a small quantity of mercury, which was weighed. The apparatus being again cooled down to zero, the vacant space in the tube corresponded to the weight of mercury which had overflowed ; the volume of mercury remaining in the apparatus, and consequently the volume of the bulb was determined by calculations analogous to those made for the piezometer (97).

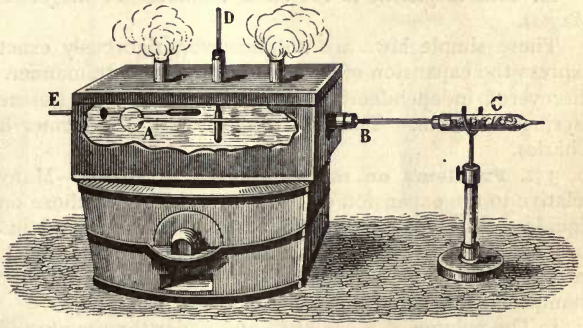


Fig. 311.

In order to fill the thermometer with dry air it was first filled with mercury, which was boiled in the bulb itself. A tube, C, filled with chloride of calcium, was then fixed on to its end by means of a cork. A fine platinum wire having then been introduced into the stem AB, through the tube C, and the apparatus being slightly inclined and agitated from time to time, air entered, having been previously well dried by passing through the chloride of calcium tube. The whole of the mercury was displaced, with the exception of a small thread, which remained in the tube AB as an index.

The air thermometer was then placed in the box filled with melting ice, the index moved towards A, and the point was noted at which it became

stationary. This gave the volume of air at zero; for the capacity of the bulb was known. Water or oil was then substituted for the ice, and the bath successively heated to different temperatures. The air expanded and moved the index from A towards B. The position of the index in each case was noted, and the corresponding temperature was indicated by means of the thermometers D and E.

Assuming that the atmospheric pressure did not vary during the experiment, and neglecting the expansion of the glass as being small in comparison with that of the air, the total expansion of the air is obtained by subtracting from its volume at a given temperature its volume at zero. Dividing this by a given temperature, and then by the number of units contained in the volume at zero, the quotient is the coefficient of expansion for a single unit of volume and a single degree; that is, the *coefficient of expansion.* It will be seen further on how corrections for pressure and temperature may be introduced.

By this method Gay-Lussac found that the coefficient of expansion of air was 0·00375; the two following laws hold in reference to the expansion of gases:—

I. *All gases have the same coefficient of expansion as air.*

II. *This coefficient is the same whatever be the pressure supported by the gas.*

These simple laws are not, however, rigorously exact (333); they only express the expansion of gases in an approximate manner. These laws were discovered independently by Dalton and by Gay-Lussac, and are usually ascribed to them. The first discoverer of the former law was, however, Charles.

332. **Problems on the expansion of gases.**—Many of the problems relative to the expansion of gases are similar to those on the expansion of liquids. With obvious modifications, they are solved in a similar manner. In most cases the pressure of the atmosphere must be taken into account in considering the expansion of gases. The following is an example of the manner in which this correction is made:—

i. The volume of a gas at $t°$, and under the pressure H, is V': what will be the volume V of the same gas at zero, and under the normal pressure 760 millimetres?

Here there are two corrections to be made; one relative to the temperature, and the other to the pressure. It is quite immaterial which is taken first. If a be the coefficient of cubical expansion for a single degree, by reasoning similar to that in the case of linear expansion (318), the volume of the gas at zero, but still under the pressure H, will be $\frac{V'}{1+at}$. This pressure is reduced to the pressure 760 in accordance with Boyle's law (180), by putting $V \times 760 = \frac{V'}{1+at} \times H$; whence $V = \frac{V'H}{760\,(1+at)}$.

ii. A volume of gas weighs P' at $t°$; what will be its weight at zero?

Let P be the desired weight, a the coefficient of expansion of the gas, d' its density at $t°$, and d its density at zero. As the weights of equal volumes are proportional to the densities, we have $\frac{P'}{P} = \frac{d'}{d}$. If 1 be the

volume of a gas at zero, its volume at t will be $1 + at$: but as the densities are inversely as the volumes, $\frac{d'}{d} = \frac{1}{1+at}$,

and therefore $\frac{P'}{P} = \frac{1}{1+at}$; whence $P = P'(1+at)$.

From this equation we get $P' = \frac{P}{1+at}$, which gives the weight at t, knowing the weight at zero, and which further shows that the weight P′ is inversely as the binomial of expansion $1 + at$.

333. **Regnault's method.**—Regnault used successfully four different methods for determining the expansion of gases. In some of them the pressure was constant and the volume variable, as in Gay-Lussac's method; in others the volume remained the same while the pressure varied. The first method will be described. It is the same as that used by Rudberg and Dulong, but is distinguished by the care with which all sources of error are avoided.

The apparatus consisted of a pretty large cylindrical reservoir, B (fig. 312), terminating in a bent capillary tube. In order to fill the reservoir with

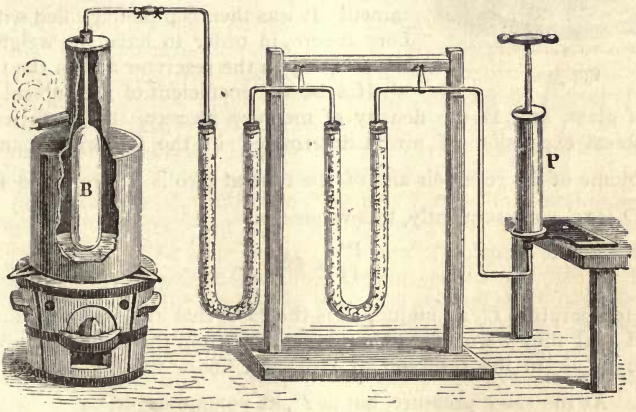


Fig. 312.

dry air, it was placed in a hot-water bath, and the capillary tube connected by a caoutchouc tube with a series of drying tubes. These tubes were joined to a small air-pump, P, by which a vacuum could be produced in the reservoir while at a temperature of 100°. The reservoir was first exhausted, and air afterwards admitted slowly; this operation was repeated a great many times, so that the air in the reservoir became quite dry, for the moisture adhering to the sides passed off in vapour at 100°, and the air which entered became dry in its passage through the U tubes.

The reservoir was then kept for half an hour at the temperature of boiling water; the air-pump having been detached, the drying tubes were then disconnected, and the end of the tube hermetically sealed, the height H of the barometer being noted. When the reservoir B was cool, it was placed

in the apparatus represented in fig. 313. It was there quite surrounded with ice, and the end of the tube dipped in the mercury bath, C. After the air in the reservoir B had sunk to zero, the point *b* was broken off by means of a forceps; the air in the interior became condensed by atmospheric pressure, the mercury rising to a height *o*G. In order to measure the height of this column, G*o*, which will be called *h*, a movable rod, *go*, was lowered until its point, *o*, was flush with the surface of the mercury in the bath; the distance between the point *o* and the level of the mercury G was measured by means of the cathetometer. The point *b* was finally closed with wax by means of the spoon *a*, and the barometric pressure noted at this moment. If this pressure be H′, the pressure in the reservoir is H′−*h*.

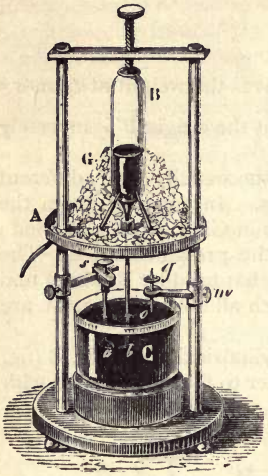


Fig. 313.

The reservoir was now weighed to ascertain P, the weight of the mercury which it contained. It was then completely filled with mercury at zero, in order to have the weight P′ of the mercury in the reservoir and in the tube.

If δ be the coefficient of the cubical expansion of glass, and D the density of mercury at zero, the coefficient *a* of the cubical expansion of air is determined in the following manner:—

The volume of the reservoir and of the tube at zero is $\frac{P'}{D}$, from the formula P = VD (125); consequently, this volume is

$$\frac{P'}{D}(1+\delta t) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

at the temperature *t*°, assuming, as is the case, that the reservoir and tube expand as if they were solid glass (321). But from the formula P = VD, the volume of air in the reservoir at zero, and under the pressure H′−*h*, is $\frac{P'-P}{D}$. At the same pressure, but at *t*°, its volume would be

$$\frac{P'-P}{D}(1+at)$$

and by Boyle's law (180), at the pressure H, at which the tube was sealed, this volume must have been

$$\frac{(P'-P)(1+at)(H'-h)}{DH} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Now the volumes represented by these formulæ (1) and (2) are each equal to the volume of the reservoir and the tube at *t*°; they are therefore equal. Removing the denominators, we have

$$P'(1+\delta t)H = (P'-P)(1+at)(H'-h) \quad . \quad . \quad . \quad . \quad . \quad (3)$$

from which the value of *a* is deduced.

The means of a great number of experiments between zero and 100° and for pressure between 300 millimetres and 500 millimetres, gave the following numbers for the coefficients of expansion for a single degree :—

Air	0·003667	Carbonic acid. .	0·003710
Hydrogen . .	0·003661	Nitrous oxide. .	0·003719
Nitrogen . . .	0·003661	Cyanogen . .	0·003877
Carbonic oxide .	0·003667	Sulphurous acid .	0·003903

These numbers, with which the results obtained by Magnus closely agree, show that the coefficients of expansion of the permanent gases differ very little ; but that they are somewhat greater in the case of the more easily condensable gases, such as carbonic and sulphurous acids. Regnault has further found that, at the same temperature, the coefficient of expansion of any gas increases with the pressure which it supports. Thus, while the coefficient of expansion of air under a pressure of 110 mm. is 0·003648, under a pressure of 3655 mm., or nearly five atmospheres, it is 0·003709.

The number found by Regnault for the coefficient of the expansion of air, 0·003667, is equal to $\frac{1}{272·9} = \frac{1}{273}$ nearly ; and if we take the coefficient of expansion at 0·0036666 . . . it may be represented by the fraction $\frac{11}{3000}$, which is convenient for many purposes of calculation.

The small differences in the expansibility of various gases may be ascribed to the circumstance that when a gas is heated the relative positions of the atoms in the molecules are thereby altered ; and a certain amount of internal work is required for this, which is different for different gases.

334. **Air thermometer.**—The *air thermometer* is based on the expansion of air. When it is used to measure small differences of temperature, it has the same form as the tube used by Gay-Lussac in determining the expansion of air (fig. 311), that is, a capillary tube with a bulb at the end. The reservoir being filled with dry air, an index of coloured sulphuric acid or a drop of mercury is passed into the tube ; the apparatus is then graduated in Centigrade degrees by comparing the positions of the index with the indications of a mercurial thermometer. Of course the end of the tube must remain open ; otherwise, the air above the index, condensing or expanding at the same time as that in the bulb, the index would remain stationary. A correction must be made at each observation for the atmospheric pressure.

When considerable variations of temperature are to be measured, the tube has a form like that used in Regnault's experiments (figs. 312 and 313). By experiments made as described in Art. 333, P, P′, H, H′, and h may be found, and the coefficients a and δ being known, the temperature t to which the tube has been raised is readily reduced from the equation (3).

Regnault found that the air and the mercurial thermometer agree up to 260°, but above that point mercury expands relatively more than air. In cases where very high temperatures are to be measured, the reservoir is made of platinum. The use of an air thermometer is seen in Dulong and Petit's experiment (322) ; it was by such an apparatus that Pouillet measured the temperature corresponding to the colours which metals take when heated in a fire, and found them to be as follows :—

Incipient red . .	525° C.	Dark orange . . .	1100° C.
Dull red . . .	700	White	1300
Cherry red . .	900	Dazzling white . .	1500

In the measurement of high temperatures Deville and Troost used with advantage the vapour of iodine instead of air, and, as platinum has been found to be permeable to gases at high temperatures, they employed porcelain instead of that metal.

The expansion of gases has been determined by Jolly by means of a form of apparatus which is also a convenient form of air thermometer (fig. 314). A quadrangular post rests on a tripod ; on one side of this post is a graduated glass scale, while in the two others are grooves in which screw-blocks A and A′ can be slid up and down and adjusted at any height.

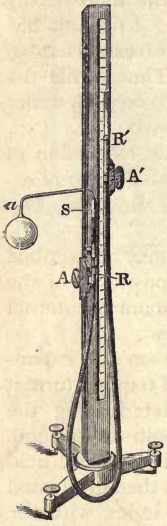


Fig. 314.

A glass bulb *a* is prolonged in a tube bent twice, the end of which is provided with a stopcock, not shown in the figure, and in which can be fitted a glass tube R supported by the block A. This again is fitted to a flexible india-rubber tube, at the other end of which is an open glass tube R′ fixed to the block A′. This tube contains mercury.

The bulb *a* having been filled with dry air, the stopcock is closed, the tube R fixed, and the stopcock opened. The bulb *a* is then immersed to the stem in melting ice, and when it is supposed that the temperature is stationary, the tube R′ is moved up and down until the mercury in the other limb is at a mark S. The difference between the levels of the mercury at S and at R′ is noted. If the latter is higher the difference is added to, and if lower subtracted from, the barometric height at the time, to give the pressure h in the vessel *a*.

The bulb *a* is then placed in a space at any constant temperature, and the same operation repeated to get the pressure h_1. From the ratio of the total pressures in the two cases we get the coefficient of expansion α from the formula $h : h_1 = 1 + at : 1 + at'$. By means of this apparatus Jolly found 0·00366957 for the value of α.

335. **Density of gases.**—The relative *density* of a gas, or its *specific gravity*, is the ratio of the weight of a certain volume of the gas to that of the same volume of air ; both the gas and the air being at zero and under a pressure of 760 millimetres.

In order, therefore, to find the specific gravity of a gas, it is necessary to determine the weight of a certain volume of this gas at a pressure of 760 millimetres, and a temperature of zero, and then the weight of the same volume of air under the same conditions. For this purpose a large globe of about two gallons capacity is used, the neck of which is provided with a stopcock, which can be screwed to the air-pump. The globe is first weighed empty, and then full of air, and afterwards full of the gas in question. The weights of the gas and of the air are obtained by subtracting the weight of the exhausted globe from the weight of the globes filled, respectively, with

air and gas. The quotient, obtained by dividing the latter by the former, gives the specific gravity of the gas. It is difficult to make these determinations at the same temperature and pressure, and therefore all the weights are reduced to zero and the normal pressure of 760 millimetres.

The gases are dried by causing them to pass through drying tubes before they enter the globe, and air must also be passed over potash to free it from carbonic acid. And as even the best air-pumps never produce a perfect vacuum, it is necessary to exhaust the globe until the manometer in each case marks the same pressure.

The globe having been exhausted, dried air is allowed to enter, and the process is repeated several times until the globe is perfectly dried. It is then finally exhausted until the residual pressure in millimetres is e. The weight of the exhausted globe is p. Air, which has been dried and purified by passing through potash and chloride of calcium tubes, is then allowed to enter slowly. The weight of the globe full of air is P. If H is the barometric height in millimetres, and $t°$ the temperature at the time of weighing, $P-p$ is the weight of the air in the globe at the temperature t, and the pressure $H-e$.

To reduce this weight to the pressure 760 millimetres and the temperature zero, let a be the coefficient of the expansion of air, and δ the coefficient of the cubical expansion of glass. From Boyle's law the weight, which is $P-p$ at $t°$ and a pressure of $H-e$, would be $\frac{(P-p)\,760}{H-e}$ under the pressure 760 millimetres and at the same temperature $t°$. If the temperature is 0°, the capacity of the globe will diminish in the ratio $1+\delta t$ to 1, while the weight of the gas increases in the ratio $1 : 1+at$, as follows from the problems in Art. 332. Consequently, the weight of the air in the globe at 0° and at the pressure 760 millimetres will be

$$(P-p)\,\frac{760\,(1+at)}{(H-e)\,(1+\delta t)} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Further, let a' be the coefficient of expansion of the gas in question ; let P′ be the weight of the globe full of gas at the temperature t' and the pressure H′, and let p' be the weight of the globe when it is exhausted to the pressure e ; the weight of the gas in the globe at the pressure 760 and the temperature zero will be

$$(P'-p')\,\frac{760\,(1+a't')}{(H'-e)\,(1+\delta t')} \quad . \quad . \quad . \quad . \quad (2)$$

Dividing the latter formula by the former we obtain the density

$$D=\frac{(P'-p')\,(H-e)\,(1+a't')\,(1+\delta t)}{(P-p)\,(H'-e)\,(1+at)\,(1+\delta t')}$$

If the temperature and the pressure do not vary during the experiment, $H=H'$ and $t=t'$; whence $D=\frac{(P'-p')(1+a't)}{(P-p)\,(1+at)}$, and, if $a=a'$, $D=\frac{P'-p'}{P-p}$.

336. **Regnault's method of determining the density of gases.**— Regnault so modified the above method that many of the corrections may be dispensed with. The globe in which the gas is weighed is suspended

from one pan of a balance, and is counterpoised by means of a second globe of the same dimensions, and hermetically sealed, suspended from the other. These two globes, expanding at the same time, always displace the same quantity of air, and consequently variations in the temperature and pressure of the atmosphere do not influence the weighing. The globe, too, is filled with the air or with the gas, at the temperature of zero. This is effected by placing it in a vessel full of ice, as shown in fig. 315. It is then connected with a three-way cock, A, by which it may be connected either with an air-pump, or with the tubes M and N, which are connected with the reservoir of gas. The tubes M and N contain substances which by their action on the gas dry and also purify it.

The stopcock A being so turned that the globe is only connected with the air-pump, a vacuum is produced; by means of the same cock, the connection with the pump being cut off, but established with M and N, the

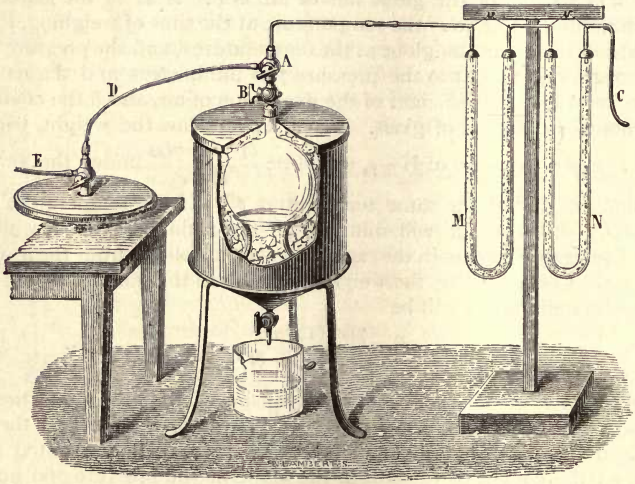


Fig. 315.

gas soon fills the globe. But, as the exhaustion could not have been complete, and some air must have been left, the globe is again exhausted and the gas allowed to enter, and the process is repeated until it is thought all air is removed. The vacuum being once more produced, a differential barometer (fig. 156), connected with the apparatus by the tube E, indicates the pressure of the residual rarified gas *e*. Closing the cock B and detaching A, the globe is removed from the ice, and after being cleaned is weighed.

This gives the weight of the empty globe p; it is again replaced in the ice, the stopcock A adjusted, and the gas allowed to enter, care being taken to leave the stopcocks open long enough to allow the gas in the globe to acquire the pressure of the atmosphere, H, which is marked by the barometer. The stopcock A is then closed, A removed, and the globe weighed with the same precautions as before. This gives the weight P' of the gas and globe.

The same operations are then repeated on this globe with air, and two corresponding weights p and P are obtained. The only correction necessary is to reduce the weights in the two cases to the standard pressure by the method described in the preceding paragraph. The correction for temperature is not needed, as the gas is at the temperature of melting ice. The ratio of the weight of the gas to that of the air is thus obtained by the formula

$$D = \frac{P' - p'}{P - p}.$$

337. **Density of gases which attack metals.**—For gases which attack the ordinary metals, such as chlorine, a metal stopcock cannot be used, and vessels with ground-glass stoppers are substituted. The gas is introduced by a bent glass tube, the vessel being held either upright or inverted, according as the gas is heavier or lighter than air; when the vessel is supposed to be full, the tube is withdrawn, the stopper inserted, and the weight taken. This gives the weight of the vessel and gas. If the capacity of the vessel be measured by means of water, the weight of the air which it contains is deduced, for the density of air at 0° C. and 760 millimetres pressure is $\frac{1}{770}$ that of distilled water under the same circumstances. The weight of the vessel full of air, less the weight of the contained air, gives the weight of the vessel itself. From these three data—the weight of the vessel full of the gas, the weight of the air which it contains, and the weight of the vessel alone—the specific gravity of the gas is readily deduced, the necessary corrections being made for temperature and pressure.

Density of gases at zero and at a pressure of 760 millimetres, that of air being taken as unity

Air	1·0000	Sulphuretted hydrogen	1·1912
Hydrogen	0·0693	Hydrochloric acid	1·2540
Ammoniacal gas	0·5367	Protoxide of nitrogen	1·5270
Marsh gas	0·5590	Carbonic acid	1·5291
Carbonic oxide	0·9670	Cyanogen	1·8600
Nitrogen	0·9714	Sulphurous acid	2·2474
Binoxide of nitrogen	1·0360	Chlorine	3·4400
Oxygen	1·1057	Hydriodic acid	4·4430

Regnault made the following determinations of the weight of a litre of the most important gases at 0° C. and 760 mm.:—

Air	1·293187 grms.	Nitrogen	1·256157 grms.
Oxygen	1·429802 „	Carbonic acid	1·977414
Hydrogen	0·089578 „		

CHAPTER V

CHANGES OF CONDITION. VAPOUR

338. **Fusion. Its laws.**—The only phenomena of heat with which we have hitherto been engaged have been those of expansion. In the case of solids it is easy to see that this expansion is limited. For in proportion as a body absorbs a larger quantity of heat, the *vis viva* of the molecules is increased, and ultimately a point is reached at which the molecular attraction is not sufficient to retain the body in the solid state. A new phenomenon is then produced; *melting* or *fusion* takes place; that is, the body passes from the solid into the liquid state.

Some substances, however, such as paper, wood, wool, and certain salts, do not fuse at a high temperature, but are decomposed. Many bodies have long been considered *refractory*—that is, incapable of fusion; but, in proportion as it has been possible to produce higher temperatures, their number has diminished. Gaudin succeeded in fusing rock crystal by means of a lamp fed by a jet of oxygen; and Despretz, by combining the effects of the sun, the voltaic battery, and the oxy-hydrogen blowpipe, melted alumina and magnesia, and softened carbon so as to be flexible, which is a condition near that of fusion.

It has been found experimentally that the fusion of bodies is governed by the two following laws:—

I. *Every substance begins to fuse at a certain temperature, which is invariable for each substance, if the pressure be constant.*

II. *Whatever be the intensity of the source of heat, from the moment fusion begins, the temperature of the body ceases to rise, and remains constant until the fusion is complete.*

Melting points of certain substances

Ethylene	−169°	Phosphorus	44°
Ammonia	−75	Spermaceti	49
Mercury	−38·8	Potassium	55
Oil of turpentine	−27	Margaric acid	57
Bromine	−12	Stearine	60
Ice	0	White wax	65
Nitrobenzene	+3·0	Wood's fusible metal	68
Formic acid	8·5	Stearic acid	70
Acetic acid	17	Sodium	90
Butter	33	Rose's fusible metal	94
Rubidium	39	Sulphur	114

Benzoic acid	120°	Magnesium	750°
Indium	176	Aluminium	850
Tin	228	Silver	954
Bismuth	246	Gold	1035
Cadmium	321	Copper	1054
Lead	335	Iron	1500
Zinc	422	Platinum	1775
Antimony	450	Iridium	1950
Arsenic	500		

Some substances pass from the solid to the liquid state without showing any definite melting point; for example, glass and iron become gradually softer and softer when heated, and pass by imperceptible stages from the solid to the liquid condition. This intermediate condition is spoken of as the state of *vitreous fusion*. Such substances may be said to melt at the lowest temperature at which perceptible softening occurs, and to be fully melted when the further elevation of temperature does not make them more fluid; but no precise temperature can be given as their melting points.

The determination of the melting point of a body is a matter of considerable importance in fixing the identity of many chemical compounds, and is moreover a point of frequent practical application in determining the commercial value of tallow and other fats.

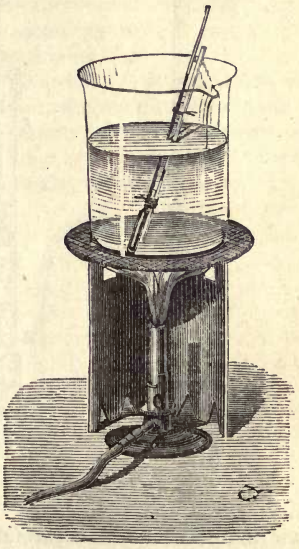


Fig. 316.

It is done as follows:—A portion of the substance is melted in a watch-glass, and a small quantity of it sucked into a fine capillary tube, which is then placed in a bath of clear water (fig. 316) attached to a thermometer, and the temperature of the bath is gradually raised until the substance is completely melted, which from its small mass is very easily observed. The bath is then allowed to cool, and the solidifying point noted; and the mean of the two is taken as the true melting point.

339. **Influence of pressure on the melting point.**—Lord Kelvin and Clausius have deduced from the principles of the mechanical theory of heat that, with an increase of pressure, the melting point of a body must be raised. All bodies which expand on passing from the solid to the liquid state have to perform external work—namely, to raise the pressure of the atmosphere by the amount of this expansion. Under ordinary circumstances, the amount of external work which solids and liquids thus perform is so small that it may be neglected. But, if the external pressure be increased, the power of overcoming it can only be obtained by an increase of *vis viva* of the molecules. The increase can do more work; the temperature of fusion

and the heat of fusion are both increased. Bunsen examined the influence of pressure on the melting point by means of the apparatus represented in fig. 317, in which *acb* is a thick tube about the thickness of a straw in the clear, in the parts *ca* and the bent part *b*. The whole tube having been filled with mercury, it was sealed at *a*, and then a small quantity was driven out at *b* and some of the substance introduced ; the end *b* was then sealed and *a* opened, and the whole tube gently warmed so as to expel some mercury, upon which *a* was again hermetically sealed.

When the tube was placed in a bath of warm water a little above the melting point of the body, the mercury expanded and a pressure resulted which could be accurately measured from the diminution in volume of the

Fig. 317.

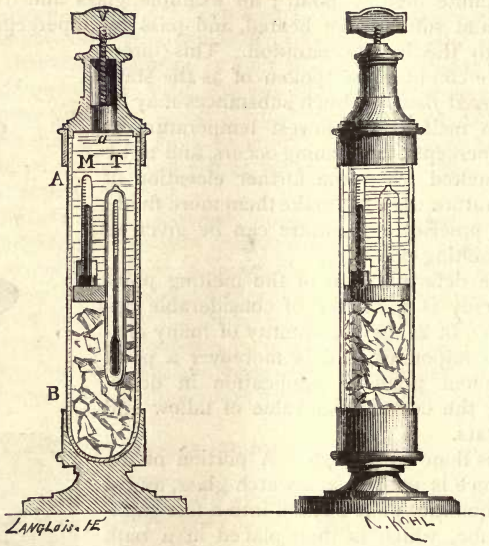


Fig. 318.

air in *ca*, which was carefully calibrated for this purpose. By carefully raising or lowering the instrument in the water, the pressure could be increased or diminished at will. It only then remained to observe the temperature at which the substance solidified, and the corresponding pressure at that moment. In this way Bunsen found that spermaceti, which melts at 48° under a pressure of 1 atmosphere, melts at 51° under a pressure of 156 atmospheres. Hopkins found that spermaceti melted at 60° under a pressure of 519 atmospheres, and at 80° under 792 atmospheres ; the melting point of sulphur under these pressures was respectively 135° and 141°.

But with regard to those bodies which contract on passing from the solid to the liquid state, and of which water is the best example, the reverse is the case. Melting ice has no external work to perform, since it has no external pressure to raise ; on the contrary, in melting, it absorbs external work, which, transformed into heat, renders a smaller quantity of heat necessary ; the external work acts in the same direction as the internal

heat—namely, in breaking up the crystalline aggregates. Yet these differences of temperature must be but small, for the molecular forces in solids preponderate far over the external pressure ; the internal work is far greater than the external.

Lord Kelvin found that increase of pressure lowered the melting point of ice. The apparatus consisted of a piezometer (fig. 319) ; a stout leaden ring divided the vessel into two compartments, the upper one of which contained water and the lower one crushed ice, which was thus prevented from rising. This also served to support a thermometer enclosed in a very stout tube, and a manometer with compressed air. The pressures were exerted by means of a screw piston V.

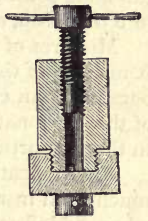


Fig. 319.

Lord Kelvin thus found that pressures of 8·1 and 16·8 atmospheres lowered the melting point of ice by 0·059° and 0·126° respectively. These results justify the theoretical previsions of his brother, Prof. J. Thomson, according to which an increase of pressure of n atmospheres lowers the melting point of ice by $0{\cdot}0074n$° C., so that a pressure of 135 atmospheres, or about 2,000 pounds to the square inch, would lower the melting point 1° C.

This lowering of the melting point is also shown by the experiment of Mousson (fig. 319). The apparatus consists of a stout steel tube closed at one end by a screw and with a screw piston at the other (fig. 319). The tube is filled with water and a metal bullet introduced. When the apparatus is closed it is inverted so that the bullet rests on the piston, and placed thus in a freezing mixture ; the water freezes and presses the ball against the piston. This is then turned again, and pressure is gradually applied by turning the handle of the screw. When the lower screw is opened the copper ball falls out, and is followed by a thick cylinder of ice which must have been formed at the moment of opening. Hence the ice must, by a pressure estimated at 13,000 atmospheres, have been converted into water at about – 18° C.

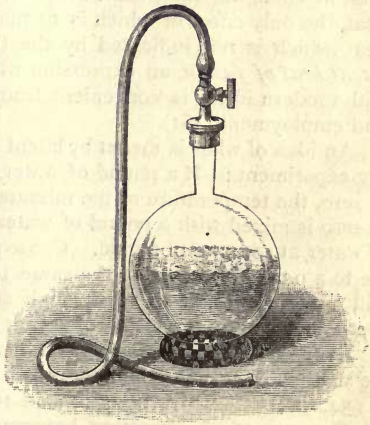


Fig. 320.

This influence is likewise readily demonstrated by an experiment of Von Helmholtz (fig. 320). Water is boiled in a flask until all air is expelled, and it is then closed. It is afterwards placed in a freezing mixture so that some ice forms inside. This is then allowed to melt again in great part, and the flask is placed in a vessel of water containing lumps of ice. It is then found that the still unfrozen water inside the flask freezes while that of the outside is melting.

340. **Alloys. Fluxes.**—Alloys are generally more fusible than any of the metals of which they are composed ; for instance, an alloy of 5 parts of

tin and 1 of lead fuses at 194°. The alloy known as *Rose's fusible metal*, which consists of 4 parts of bismuth, 1 part of lead, and 1 of tin, melts at 94°, and an alloy of 1 or 2 parts of cadmium with 2 parts of tin, 4 parts of lead, and 7 or 8 parts of bismuth, known as *Wood's fusible metal*, melts between 66° and 71° C. An alloy of potassium and sodium in equivalent proportions is liquid at the ordinary temperature. Fusible alloys are of extended use in soldering and in taking casts. Steel melts at a lower temperature than iron, though it contains carbon, which is almost completely infusible.

Mixtures of the fatty acids melt at lower temperatures than the pure acids. A mixture of the chlorides of potassium and of sodium fuses at a lower temperature than either of its constituents ; the same is the case with a mixture of the carbonates of potassium and sodium, especially when they are mixed in the proportion of their chemical equivalents.

An application of this property is met with in the case of *fluxes*, which are much used in metallurgical operations. They consist of substances which, when added to an ore, partly by their chemical action, help the reduction of the substance to the metallic state, and, partly, by presenting a readily fusible medium, promote the agglomeration of the individual particles with the formation of a mass of metal or *regulus*.

341. **Latent heat.**—Since, during the passage of a body from the solid to the liquid state, the temperature remains constant until the fusion is complete, whatever be the intensity of the source of heat, it must be concluded that, in changing their condition, bodies absorb a considerable amount of heat, the only effect of which is to maintain them in the liquid state. This heat, which is not indicated by the thermometer, is called *latent heat or latent heat of fusion*, an expression which, though not in strict accordance with modern ideas, is convenient from the fact of its universal recognition and employment (461).

An idea of what is meant by latent heat may be obtained from the following experiment :—If a pound of water at 80° is mixed with a pound of water at zero, the temperature of the mixture is 40°. But if a pound of pounded *ice* at zero is mixed with a pound of water at 80°, the ice melts and two pounds of water at zero are obtained. Consequently the mere change of a pound of ice to a pound of water at the same temperature requires as much heat as will raise a pound of water through 80°. This quantity of heat represents the latent heat of the fusion of ice, or the latent heat of water.

Every liquid has its own latent heat, and in the chapter on Calorimetry we shall show how this is determined.

342. **Solution.**—A body is said to *dissolve* when it becomes liquid in consequence of an attraction between its molecules and those of a liquid. Gum arabic, sugar, and most salts dissolve in water. The weight dissolved usually increases with the temperature, as is seen from the following table:—

	Common Salt	Nitre	Sodium Sulphate	Copper Sulphate	Lime Sulphate
0°	35	13	—	32	115
20	37	21	53	42	131
100	40	247	42	203	654

When a liquid has dissolved as much as it can at a particular temperature, it is said to be *saturated.*

When a salt dissolves in water it may be supposed that the vibrations of its bounding molecules, which are in contact with the solvent, possibly owing to the attraction of the solvent, or owing to capillarity, increase their amplitude so that they get beyond the sphere of action of the other molecules of the salt, and thereby assume a progressive motion like the molecules of a gas. Like them they then exert a pressure against the sides of the containing vessel, and which is called *osmotic pressure* (139).

During solution, as well as during fusion, a certain quantity of heat always becomes latent, and hence it is that the solution of a substance usually produces a diminution of temperature. In certain cases, however, instead of the temperature being lowered, it actually rises, as when caustic potash is dissolved in water. This depends upon the fact that two simultaneous and contrary phenomena are produced. The first is the passage from the solid to the liquid condition, which always lowers the temperature. The second is the *chemical* combination of the body dissolved with the liquid, and which, as in the case of all chemical combinations, produces an increase of temperature. Consequently, as the one or the other of these effects predominates, or as they are equal, the temperature either rises or sinks, or remains constant.

343. **Solidification.**—*Solidification* or *congelation* is the passage of a body from the liquid to the solid state. This phenomenon is regulated by the two following laws :—

I. *Every body, under the same pressure, solidifies at a fixed temperature, which is the same as that of fusion.*

II. *From the commencement to the end of the solidification, the temperature of a liquid remains constant.*

Certain bodies, more especially some of the fats, present an exception to the first law, in so far that by repeated fusions they seem to undergo a molecular change which alters their melting point.

The second law is the consequence of the fact that the latent heat absorbed during fusion becomes free at the moment of solidification.

The application of the very low temperatures which can now be so readily procured has lessened the number of those liquids which it was formerly thought could not be solidified. By allowing liquid ethylene (382) to boil in a vacuum, Wroblewski and Olszewski obtained a temperature of $-136°$. They observed that carbon disulphide solidified at $-116°$ and fused again at about $-110°$. Absolute alcohol became viscid at $-129°$ and solidified at $-130{\cdot}5°$. Pure ether solidifies at $-129°$.

Water containing a salt dissolved always solidifies below zero ; the depression of the freezing point is proportional to the weight of salt dissolved, at any rate for weak solutions. This is known as *Blagden's law.*

If several salts which have no chemical action on each other be dissolved in a given weight of water, the lowering of the freezing point is the sum of the depressions which each of them would produce separately if dissolved in the same quantity of water.

When the numbers observed in any experiment of this kind do not agree with those calculated, this points to the occurrence of some chemical action

between the substances dissolved, and the observation of such deviations has been of great use in questions of chemical statics.

The elaborate researches of Raoult on the temperature of solidification of solutions of bodies in water and other solvents have led to important conclusions. The temperature at which a solution solidifies, or its freezing point, is always lower than that of the pure solvent. If P be the weight in grammes of any substance dissolved in 100 grammes of a solvent, and C be the depression in the freezing point observed, then $\frac{C}{P} = A$ is the depression which would be produced by dissolving *one* gramme of the substance in 100 grammes, and is known as the *coefficient of depression.*

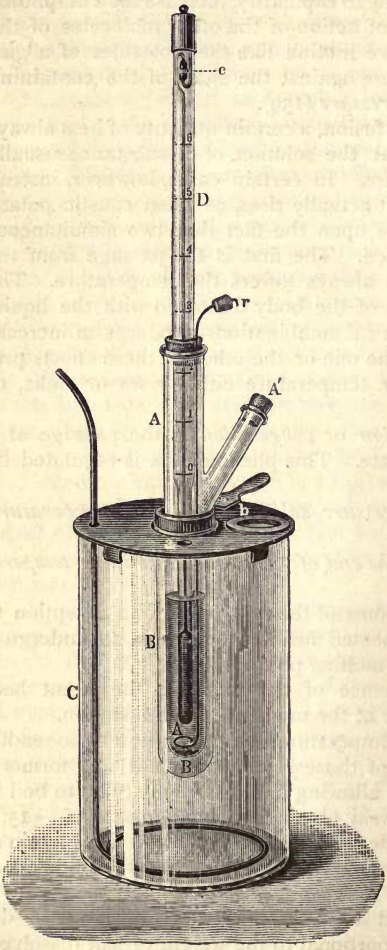


Fig. 321.

A comparison of the values for A for various substances and the same solvent shows that they differ considerably; this is not so if we compare the depressions produced by molecular weights of the substances. That is, if we multiply the value of A in the above equation by M, the molecular weight of the substance dissolved, we obtain the depression which would be produced by dissolving one molecule of a body in 100 grammes of the solvent, or the *coefficient of molecular depression*; this is called T, and we have $T = \frac{CM}{P}$.

Now it is found that in a very large number of cases the value of T, for one and the same solvent, is a constant number; it has the value 19 for water, 39 for glacial acetic acid, and 49 for benzene.

This relation makes it possible to calculate the molecular weight of a solid in solution by means of a simple determination of the freezing point of a solution, which is effected by means of the apparatus represented in fig. 321. The solvent is contained in the vessel A, and the substance to be investigated is introduced by the lateral aperture A′. A is surrounded by a wide glass tube B containing air, and this again is placed in a wider vessel C which contains the freezing mixture; for experiments with benzene or glacial

acetic acid as a solvent, this is bruised ice, and with water a mixture of ice and salt. The liquid from these may be drawn off by a syphon placed through *b*. In A is a platinum stirrer *r*, and a delicate thermometer D indicating the $\frac{1}{100}$ of a degree. C is also a stirrer.

Since C and P are known, M is determined from the formula

$$M = \frac{PT}{C},$$

where T is the constant for the particular solvent employed, which is ordinary glacial acetic acid in the majority of cases.

344. **Crystallisation.**—Generally speaking, bodies which pass slowly from the liquid to the solid state assume regular geometrical forms, such as the cube prisms, rhombohedra, &c.; these are called *crystals*. If the crystals are formed from a body in fusion, such as sulphur or bismuth, the crystallisation is said to take place by the *dry way*. The crystallisation is said to be by the *moist way* when it takes place owing to the slow evaporation of a solution of a salt, or when a solution saturated at a higher temperature is allowed to cool slowly. Snow, ice, and many salts present examples of crystallisation.

345. **Retardation of the point of solidification.**—The freezing point of pure water can be diminished by several degrees, if the water be previously freed from air by boiling and be then kept in a perfectly still place. In fact, it may be cooled to $-15°$ C., and even lower, without freezing. But when it is slightly agitated, the liquid at once solidifies. This may be conveniently shown by means of the apparatus represented in fig. 322, which consists of a delicate thermometer, round the bulb of which is a wider one containing some water. Before sealing at *a* the whole outside bulb was filled with water, which was then boiled out and sealed so that over the water the space is quite empty. This is clamped in a retort stand, and ether is dropped on it, that which has dropped off, and become colder, being used over and over again. In this way the temperature may soon be reduced to $-6°$, and if then the bulb be shaken part of the water freezes and the temperature rises to zero. The smaller the quantity of liquid, the lower is the temperature to which it can be cooled, and the greater the mechanical disturbance it supports without freezing. Fournet has observed the frequent occurrence of mists formed of particles of liquid matter suspended in an atmosphere whose temperature was 10° or even 15° below zero.

Fig. 322.

A very rapid agitation also prevents the formation of ice. The same is the case with all actions which, hindering the molecules in their movements, do not permit them to arrange themselves in the conditions necessary for the solid state. Despretz was able to lower the temperature of water contained in fine capillary tubes to $-20°$ without their solidifying. This experiment shows how it is that plants in many cases do not become frozen even during severe cold, as the sap is contained in very fine capillary vessels.

If water contains salts, or other foreign bodies, its freezing point is

lowered. Sea water freezes at $-2{\cdot}5°$ to $-3°$ C.; the ice which forms is quite pure, and a saturated solution remains. In Finland advantage is taken of this property to concentrate sea water for the purpose of extracting salt from it. If water contains alcohol, precisely analogous phenomena are observed; the ice formed is pure, and practically all the alcohol is contained in the residue.

Dufour has observed some very curious cases of liquids cooled out of contact with solid bodies. His mode of experimenting was to place the liquid in another of the same specific gravity but of lower melting point, and in which it is insoluble. Drops of water, for instance, suspended in a mixture of chloroform and oil, usually solidified between $-4°$ and $-12°$, while still smaller globules cooled down to $-18°$ or $-20°$. Contact with a fragment of ice immediately set up congelation. Globules of sulphur (which solidifies at 115°) remained liquid at 40°; and globules of phosphorus (solidifying point 42°) at 20°.

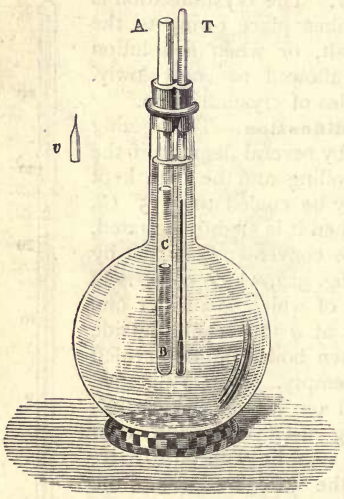


Fig. 323.

The superfusion of phosphorus may be illustrated by the experiment represented by fig. 323. A long test-tube containing phosphorus, A, and covered with a layer of water, is fixed along with a thermometer T in a large flask containing water. This flask is raised to a temperature of about 44°, at which the phosphorus fuses, and is then withdrawn from the source of heat; as its mass is considerable, it cools very slowly, and the phosphorus remains liquid even at ordinary temperatures. A glass rod may even be dipped into it without change; but if the rod be rubbed along solid phosphorus so as to detach a small particle, it at once brings about solidification if dipped in the melted mass.

When a liquid solidifies after being cooled below its normal freezing point, the solidification takes place very rapidly, and is accompanied by a disengagement of heat, which is sufficient to raise its temperature from the point at which solidification begins up to its ordinary freezing point. This is well seen in the case of hyposulphite of sodium, which melts in its own water of crystallisation at 45°, and when carefully cooled will remain liquid at the ordinary temperature of the atmosphere. If it then be made to solidify by agitation, or by adding a small fragment of the solid salt, the rise of temperature is distinctly felt by the hand. In this case the heat, which had become latent in the process of liquefaction, again becomes free, and a portion of the substance remains melted; for it is kept liquid by the heat of solidification of that which has solidified.

346. **Change of volume on solidification and liquefaction.**—The rate of expansion of bodies generally increases as they approach their

melting points, and is in most cases followed by a further expansion at the moment of liquefaction, so that the liquid occupies a greater volume than the solid from which it is formed. The apparatus represented in fig. 324 is well adapted for exhibiting this phenomenon. It consists of a glass tube, *ab*, containing water or some other suitable liquid, to which is carefully fitted a cork with a graduated glass tube *c*. This forms, in fact, a thermometer, and the values of the degrees on the tube *c* are determined in terms of the capacity of the whole apparatus. A known volume of the substance is placed in the tube *aa* and the cork inserted ; the apparatus is then placed in a space at a temperature very little below the melting point of the body in question, until it has acquired its temperature, and the position of the liquid in *c* is noted. The temperature is then allowed to rise slowly, and the position noted when the melting is complete. Knowing then the difference in the two readings and the volume of the substance under experiment, and making a correction for the expansion of the liquid and of the glass, it is easy to deduce the increase due to the melting alone. Phosphorus, for instance, increases about 3·4 per cent. on liquefaction ; that is, 100 volumes of solid phosphorus at 44° (the melting point) become 103·4 at the same temperature when melted. Sulphur expands about 5 per cent. on liquefying, and stearic acid about 11 per cent.

Fig. 324.

Water presents a remarkable exception ; it expands at the moment of solidifying, or contracts on melting, by about 10 per cent. One volume of ice at 0° gives 0·9178 of water at 0°, or 1 volume of water at 0° gives 1·102 of ice at the same temperature. In consequence of this expansion, ice floats on the surface of water. According to Dufour, the specific gravity of ice is 0·9178 ; Bunsen found for ice which had been made from water freed from air by boiling the somewhat smaller number 0·91674.

The increase of volume in the formation of ice is accompanied by an expansive force which sometimes produces powerful mechanical effects, of which the bursting of water-pipes and the breaking of jugs containing water are familiar examples. The splitting of stones, rocks, and the swelling up of moist ground during frost, are caused by the fact that water penetrates into the pores and there becomes frozen ; in short, the great expansion of water on freezing is the most active and powerful agent of disintegration on the earth's surface.

The expansive force of ice was strikingly shown by some experiments of Major Williams, in Canada. Having quite filled a 13-inch iron bomb-shell with water, he firmly closed the touch-hole with an iron plug weighing three pounds and exposed it in this state to the frost. After some time the iron plug was forced out with a loud explosion, and thrown to a distance of 415 feet, and a cylinder of ice 8 inches long issued from the opening. In another case the shell burst before the plug was driven out, and in this case a sheet of ice spread out all round the crack (fig. 325). It is probable that under the great pressure some of the water still remained liquid up to the time at which the resistance was overcome ; that it then issued from the

shell in a liquid state, but at a temperature below o°, and therefore instantly began to solidify when the pressure was removed, and thus retained the shape of the orifice whence it issued.

Fig. 325.

Cast-iron, bismuth, and antimony expand on solidifying, like water, and can thus be used for casting; but gold, silver, and copper contract, and hence coins of these metals cannot be cast, but must be stamped with a die.

An iron tube filled with molten bismuth, and closed by a screw, is broken as the bismuth cools.

This increase of volume when liquids solidify, and the correlated decrease on melting again, in the case of water and some other crystalline substances such as bismuth, are probably due to the fact that such bodies are aggregates of small crystalline masses, which are grouped in such a way that small interstices are formed. When the liquid melts these interstices fill up owing to the mobility of the molecules, and, notwithstanding the greater space which each individual group takes up, owing to expansion, there is a total decrease of volume.

347. **Freezing mixtures.**—The absorption of heat in the passage of bodies from the solid to the liquid state has been used to produce artificial cold. This is effected by mixing together bodies which have an affinity for each other, and of which one at least is solid, such as water and a salt, ice and a salt, or an acid and a salt. Chemical affinity accelerates the fusion: the portion which melts robs the rest of the mixture of a large quantity of sensible heat, which thus becomes latent. In many cases a very considerable diminution of temperature is produced.

The following table gives the names of the substances mixed, their proportions, and the corresponding diminutions of temperature:—

Substances	Parts by weight	Reduction of temperature
Sulphate of sodium	8	+ 10° to − 17°
Hydrochloric acid	5	
Pounded ice or snow	2	+ 10° to − 18°
Common salt	1	
Sulphate of sodium	3	+ 10° to − 19°
Dilute nitric acid	2	
Sulphate of sodium	6	+ 10° to − 26°
Nitrate of ammonium	5	
Dilute nitric acid	4	
Phosphate of sodium	9	+ 10° to − 29°
Dilute nitric acid	4	

If the substances taken be themselves previously cooled down, a still more considerable diminution of temperature is occasioned.

Freezing mixtures are frequently used in chemistry, in physics, and in domestic economy. One form of the portable ice-making machines which have come into use during the last few years consists of a cylindrical metallic vessel divided into four concentric compartments. In the central one is placed the water to be frozen; in the next there is the freezing

mixture, which usually consists of sulphate of sodium and hydrochloric acid; 6 pounds of the former and 5 of the latter will make 5 to 6 pounds of ice in an hour. The third compartment also contains water, and the outside one contains some badly conducting substance, such as cotton, to cut off the influence of the external temperature. The best effect is obtained when pretty large quantities (2 or 3 pounds) of the mixture are used, and when the ingredients are intimately mixed. It is also advantageous to use the machines for a succession of operations.

348. **Guthrie's researches.**—It appears from the experiments of the late Dr. Guthrie that what are called freezing mixtures may be divided into two classes—namely, those in which one of the constituents is liquid and those in which both are solid. The temperature indicated by the thermometer placed in a freezing mixture is, of course, due to the loss of heat by the thermometer to the liquefying freezing mixture, and is measured by the rate of such loss. The quantity of heat absorbed by the freezing mixture is obviously the heat required to melt the constituents, together with (±) the heat of combination of the constituents. When one constituent is liquid, as when hydrochloric acid is added to ice, then a lower temperature is got by previously cooling the hydrochloric acid. There is no advantage in cooling the ice. But when both constituents are solid, as in the case of the ice-salt freezing mixture, there is no advantage to be gained by cooling one or both constituents. Within very wide limits it is also in the latter case a matter of indifference as to the ratio between the constituents. Nor does it matter whether the ice is finely powdered as snow or in pieces as large as a pea.

The different powers of various salts when used in conjunction with ice as freezing mixtures appear to have remained unexplained until Guthrie showed that, with each salt, there is always a minimum temperature below which it is impossible for an aqueous solution of any strength of that salt to exist in the liquid form; that there is a certain strength of solution for each salt which resists solidification the longest, that is, to the lowest temperature. Weaker solutions give up ice on being cooled, stronger solutions give up the salt either in the anhydrous state or in combination with water. That particular strength of a particular salt which resists solidification to the lowest temperature, is called by Guthrie a *cryohydrate*. It is of such a strength that when cooled below 0° C. it solidifies as a whole; that is, the ice and the salt solidify together and form crystals of constant composition and constant melting and the same solidifying temperatures. The liquid portion of a freezing mixture, as long as the temperature is at its lowest, is, indeed, a melted cryohydrate. The slightest depression of temperature below this causes solidification of the cryohydrate, and hence the temperature can never sink below the solidifying temperature of the cryohydrate.

Guthrie also showed that colloid bodies, such as gum and gelatine, neither raise the boiling point of water nor depress the solidifying point, nor can they act as elements in freezing mixtures.

VAPOURS. MEASUREMENT OF THEIR TENSION

349. **Vapours.**—We have already seen (152) that *vapours* are the aëriform fluids into which volatile substances, such as ether, alcohol, water, and mercury, are changed by the absorption of heat. *Volatile liquids* are those

which thus possess the property of passing into the aëriform state, and *fixed liquids* are those which do not form vapour at any temperature without undergoing chemical decomposition, such as the fatty oils. Ice and snow volatilise in closed spaces, forming crystals on the cooled parts. The formation of vapour is thus not restricted to the liquid state, and in some bodies, such as arsenic, the boiling point is below the freezing point. As the boiling point is raised by pressure, it is possible to liquefy such bodies also, by applying sufficient pressure.

Iodine melts at 104° and boils at 175° under ordinary pressure. It therefore evaporates after melting; but at a pressure of 250 mm. its boiling point is below its melting point, and it then evaporates without melting. Even at ordinary temperatures a considerable quantity volatilises without melting.

Vapours are transparent, like gases, and generally colourless; there are only a few coloured liquids which also give coloured vapours.

350. **Vaporisation.**—The passage of a liquid into the gaseous state is designated by the general term *vaporisation*; the term *evaporation* especially refers to the slow production of vapour at the free surface of a liquid, and *boiling* to its rapid production in the mass of the liquid itself. We shall presently see (363) that at the ordinary atmospheric pressure, ebullition, like fusion, takes place at a definite temperature. This is not the case with evaporation, which occurs even with the same liquid at very different temperatures, although the formation of a vapour seems to cease below a certain point. Mercury, for example, gives no vapour below −10°, nor sulphuric acid below 30°.

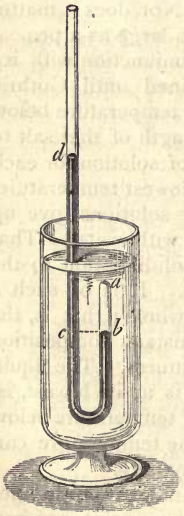


Fig. 326.

351. **Elastic force of vapour.**—Like gases, vapours have a certain elastic force, in virtue of which they exert pressures on the sides of vessels in which they are contained. The elastic force of vapour may be demonstrated by the following experiment:—A quantity of mercury is placed in a bent glass tube (fig. 326), the shorter leg of which is closed; a few drops of ether are then passed into the closed leg and the tube is immersed in a water bath at a temperature of about 45°. The mercury then sinks slowly in the short branch, and the space *ab* is filled with a gas which has all the appearance of air, and whose elastic force counterbalances the pressure of the column of mercury *cd*, and the atmospheric pressure on *d*. This gas is the vapour of ether. If the water be cooled, or if the tube be removed from the bath, the vapour which fills the space *ab* disappears, and the drop of ether is reproduced. If, on the contrary, the bath be heated still higher, the level of the mercury descends below *b*, indicating an increase in the elastic force of the vapour.

352. **Formation of vapour in a vacuum.**—In the previous experiment the liquid changed very slowly into the vaporous condition; this occurs also when a liquid is freely exposed to the air. In both cases the atmosphere is an obstacle to the vaporisation. In a vacuum there is no resistance, and the

formation of vapour is instantaneous, as is seen in the following experiment: —Four barometer tubes, filled with mercury, are immersed in the same trough, fig. 327. One of them, A, serves as a barometer, and a few drops of water, alcohol, and ether are respectively introduced into the tubes B, C, D. When the liquids reach the vacuum, a depression of the mercury is at once produced. And as this depression cannot be caused by the weight of the liquid, which is an extremely small fraction of the weight of the displaced mercury, it must be due to the formation of some vapour whose elastic force has depressed the column of mercury.

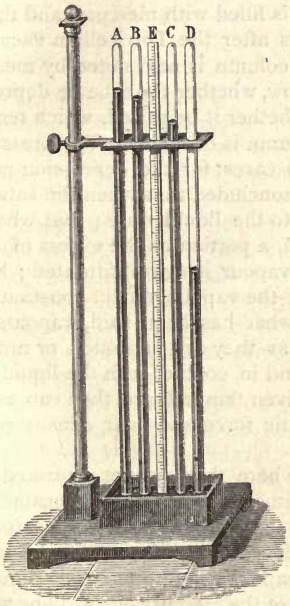


Fig. 327.

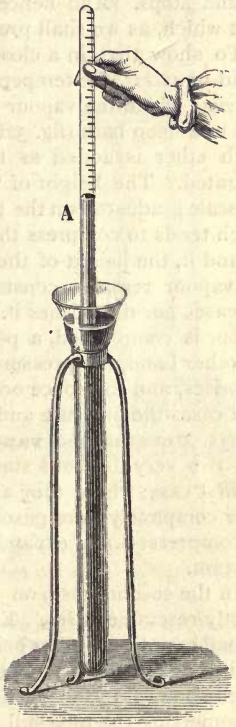


Fig. 328.

The experiment also shows that the depression is not the same in all the tubes; it is greater in the case of alcohol than of water, and greater with ether than with alcohol. We consequently obtain the two following laws of the formation of vapours :—

I. *In a vacuum all volatile liquids are instantaneously converted into vapour.*

II. *At the same temperature the vapours of different liquids have different pressures.*

For example, at 20° the pressure of ether vapour is 25 times as great as that of aqueous vapour.

353. **Saturated vapour. Maximum of pressure.**—When a very small quantity of a volatile liquid, such as ether, is introduced into a barometer tube, it is at once completely vaporised, and the mercurial column is not depressed to its full extent; for if some more ether be introduced the depression increases. By continuing the addition of ether, it finally ceases to vaporise, and remains in the liquid state. There is, therefore, for a certain temperature, a limit to the quantity of vapour which can be formed in a given space. This space is accordingly said to be *saturated.* Further, when the vaporisation of the ether ceases, the depression of the mercurial column stops. And hence there is a limit to the pressure of the vapour, a limit which, as we shall presently see (354), varies with the temperature.

To show that, in a closed space, saturated with vapour and containing liquid *in excess*, the temperature remaining constant, there is a *maximum of pressure* which the vapour cannot exceed, a barometric tube is used which dips in a deep bath (fig. 328). This tube is filled with mercury, and then so much ether is added as to be in excess after the Torricellian vacuum is saturated. The height of the mercurial column is next noted by means of the scale graduated on the tube itself. Now, whether the tube be depressed, which tends to compress the vapour, or whether it be raised, which tends to expand it, the height of the mercurial column is constant. The pressure of the vapour remains constant in the two cases, for the depression neither increases nor diminishes it. Hence it is concluded that when the saturated vapour is compressed, a portion returns to the liquid state; that when, on the other hand, the pressure is diminished, a portion of the excess of liquid vaporises, and the space occupied by the vapour is again saturated; but in both cases the pressure and the density of the vapour remain constant.

354. **Unsaturated vapours.**—From what has been said, vapours present two very different states, according as they are saturated or not. In the first case, where they are saturated and in contact with the liquid, they differ completely from gases, since for a given temperature they can neither be compressed nor expanded; their elastic force and their density remain constant.

In the second case, on the contrary, where they are not saturated, they exactly resemble gases. For if the experiments (fig. 329) be repeated, only a small quantity of ether being introduced, so that the vapour is not saturated, and if the tube be then slightly raised, the level of the mercury is seen to rise, which shows that the elastic force of the vapour has diminished. Similarly, by immersing the tube still more, the level of the mercury sinks. The vapour consequently behaves just as a gas would do, its tension diminishes when the volume increases, and *vice versâ*; and as in both cases the volume of the vapour is inversely as the pressure, it is concluded that *unsaturated vapours obey Boyle's law.*

When an unsaturated vapour is heated, its volume increases like that of a gas; and the number 0·00366, which is the coefficient of the expansion of air, may be taken for that of vapours.

Hence we see that the physical properties of unsaturated vapours are comparable with those of gases, and that the formulæ for the compressibility and expansibility of gases (182 and 332) also apply to unsaturated vapours.

355. **Pressure of aqueous vapour below zero.**—In order to measure

the elastic force of aqueous vapour below zero, Gay-Lussac used two barometer tubes filled with mercury, and placed in the same bath (fig. 330). The straight tube, A, serves as a barometer ; the other, C, is bent, so that part of the Torricellian vacuum can be surrounded by a freezing mixture, B (347). When a little water is admitted into the bent tube, the level of the mercury sinks below that in the tube A, to an extent which varies with the temperature of the freezing mixture.

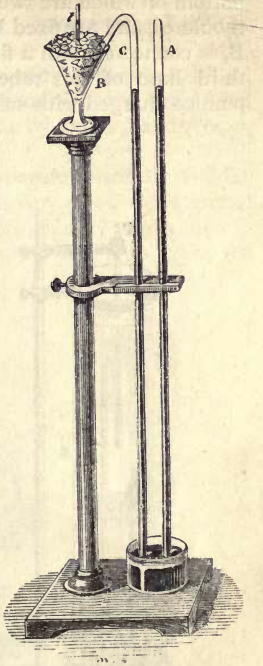


Fig. 329.

At	0°	the depression is	. 4·54	millimetres
„	− 1°	„ „	. 4·25	„
„	− 3°	„ „	. 3·63	„
„	− 5°	„ „	. 3·11	„
„	− 7°	„ „	. 2·67	„
„	−10°	„ „	. 2·08	„
„	−20°	„ „	. 0·84	„
„	−30°	„ „	. 0·36	„

These depressions, which must be due to the pressure of aqueous vapour in the space BC, show that even at very low temperatures there is always some aqueous vapour in the atmosphere.

Although in the above experiment the part B and the part C are not both immersed in the freezing mixture, we shall presently see that when two communicating vessels are at different temperatures, the tension of the vapour is the same in both, and always corresponds to that of the lower temperature.

That water evaporates even below zero follows from the fact that wet linen exposed to the air during frost becomes first stiff and then dry, showing that the particles of water evaporate even after the latter has been converted into ice.

356. **Pressure of aqueous vapour between zero and one hundred degrees.**—i. *Dalton's method.* Dalton measured the elastic force of aqueous vapour between 0° and 100° by means of the apparatus represented in fig. 331. Two barometer tubes, A and B, are filled with mercury, and inverted in an iron bath full of mercury, which is placed on a furnace. The tube A contains a small quantity of water. The tubes are supported in a cylindrical vessel full of water, the temperature of which is indicated by the thermometer. The bath being gradually heated, the water in the cylinder becomes heated too ; the water which is in the tube A vaporises, and in proportion as the tension of its vapour increases, the mercury sinks. The depressions of the mercury corresponding to each degree of the thermometer are indicated on the scale E, and in this manner a table of the elastic forces between zero and 100° has been constructed.

ii. *Regnault's method.*—Dalton's method is wanting in precision, for the liquid in the cylinder has not everywhere the same temperature, and con-

sequently the exact temperature of the aqueous vapour is not shown. Regnault's apparatus is a modification of that of Dalton. The cylindrical vessel is replaced by a large cylindrical zinc drum, MN (fig. 331), in the bottom of which are two tubulures. The tubes A and B pass through these tubulures, and are fixed by caoutchouc collars. The tube containing vapour, B, is connected with a flask, *a*, by means of a brass three-way tube, O. The third limb of this tube is connected with a drying tube, D, containing pumice charged with sulphuric acid, which is connected with the air-pump.

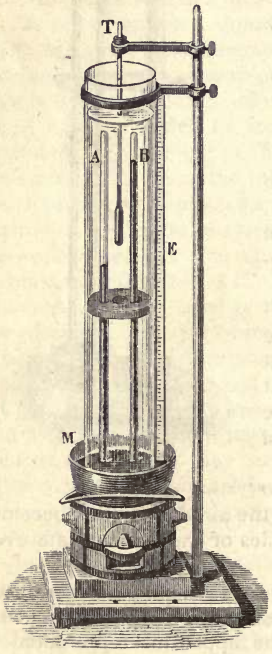


Fig. 330.

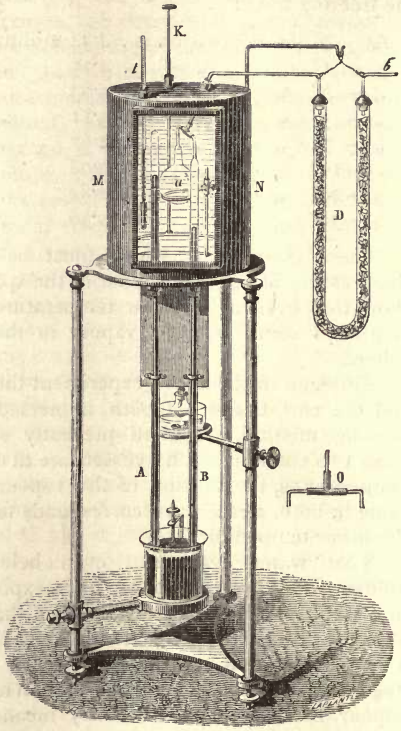


Fig. 331.

When the flask *a* contains some water, a small portion is distilled into B by gently heating the flask. Exhausting, then, by means of the air-pump, the water distils continuously from the flask and from the barometric tube towards D, which condenses the vapour. After having vaporised some quantity of water, and when it is thought that the air in the tube is withdrawn, the capillary tube which connects B with the three-way tube is sealed. The tube B being thus closed, it is experimented with as in Dalton's method.

The drum, MN, being filled with water, is gently heated by a spirit lamp, which is screened from the tubes by a wooden board. By means of a

stirrer, K, all parts of the liquid are kept at the same temperature. In the side of the drum is a glass window, through which the height of the mercury in the tubes can be read off by means of a cathetometer; from the difference in these heights, reduced to zero, the tension of vapour is deduced. By means of this apparatus, the elastic force of vapour between 0° and 50° has been determined with accuracy.

357. **Pressure of aqueous vapour above one hundred degrees.**—Two methods have principally been employed for determining the tension of aqueous vapour at temperatures above 100°; the one by Dulong and Arago, in 1830, and the other by Regnault, in 1844.

Fig. 332 represents a vertical section of the apparatus used by Dulong and Arago. It consisted of a copper boiler, *k*, with very thick sides, and of about 20 gallons capacity. Two gun-barrels, *a*, of which only one is seen in the drawing, were firmly fixed in the sides of the boiler, and plunged in the

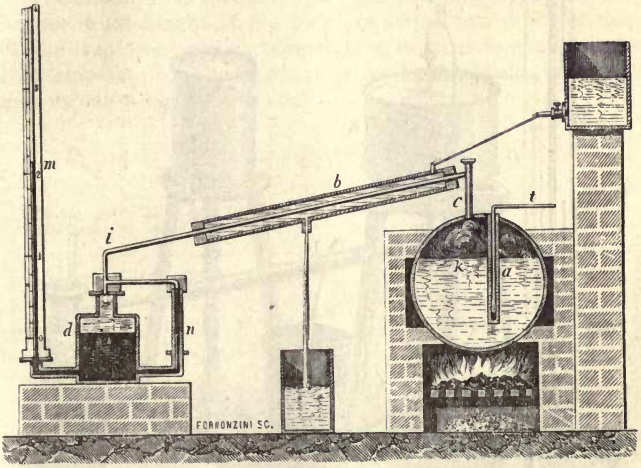


Fig. 332.

water. The gun-barrels were closed below, and contained mercury, in which were placed thermometers, *t*, indicating the temperature of the water and of the vapour. The pressure of the vapour was measured by means of a manometer with compressed air, *m*, previously graduated (184) and fitted into an iron vessel, *d*, filled with mercury. In order to see the height of the mercury in the vessel, it was connected above and below with a glass tube, *n*, in which the level was always the same as in the bath. A copper tube, *i*, connected the upper part of the vessel, *d*, with a vertical tube, *c*, fitted in the boiler. The tube *i* and the upper part of the bath *d* were filled with water, which was kept cool by means of a current of cold water flowing from a reservoir, and circulating through the tube *b*.

The vapour which was disengaged from the tube *c* exerted a pressure on the water of the tube *i*; this pressure was transmitted to the water and to the mercury in the bath *d*, and the mercury rose in the manometer. By

noting on the manometer the pressures corresponding to each degree of the thermometer, Dulong and Arago were able to make a direct measurement of the pressure up to 24 atmospheres, and the pressure to 50 atmospheres was determined by calculation.

358. **Pressure of vapour below and above one hundred degrees.**— Regnault devised a method by which the pressure of vapour may be measured at temperatures either below or above 100°. It depends on the principle that when a liquid boils, the pressure of the vapour is equal to the pressure it supports (363). If, therefore, the temperature and the corresponding pressure are known, the question is solved, and the method merely consists in causing water to boil in a vessel under a given pressure, and measuring the corresponding temperature.

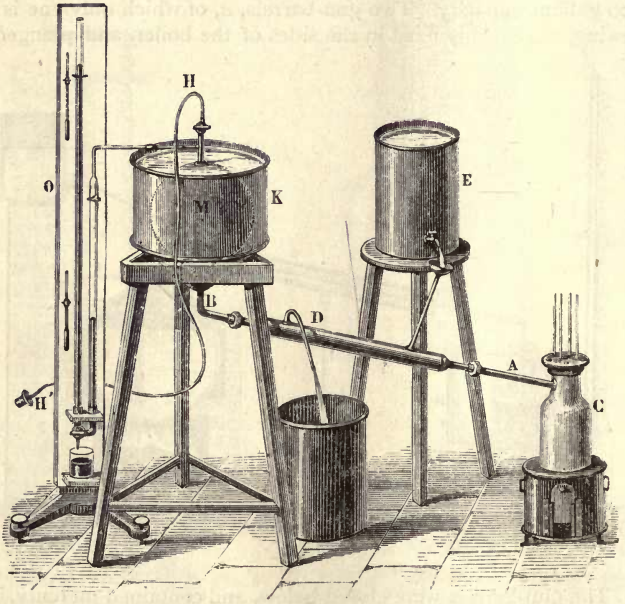


Fig. 333.

The apparatus consists of a copper retort, C (fig. 333), hermetically sealed and about two-thirds full of water. In the cover there are four thermometers, two of which just dip into the water, and two descend almost to the bottom. By means of a tube, AB, the retort C is connected with a glass globe, M, of about 6 gallons capacity, and full of air. The tube AB passes through a metal cylinder, D, through which a current of cold water is constantly flowing from the reservoir E. To the upper part of the globe a tube with two branches is attached, one of which is connected with a manometer, O ; the other tube, HH′, which is of lead, can be attached either to an exhausting or a condensing air-pump, according as the air in the globe is to be rarefied or condensed. The reservoir K, in which is the globe, contains water at the temperature of the surrounding air.

If the elastic force of aqueous vapour below 100° is to be measured, the end H′ of the lead pipe is connected with the plate of the air-pump, and the air in the globe M, and consequently that in the retort C, is rarefied. The retort being gently heated, the water begins to boil at a temperature below 100°, in consequence of the diminished pressure. And since the vapour is condensed in the tube AB, which is always cool, the pressure originally indicated by the manometer does not increase, and therefore the pressure of the vapour during ebullition remains equal to the pressure on the liquid.

A little air is then allowed to enter; this alters the pressure, and the liquid boils at a new temperature; both these are read off, and the experiment repeated as often as desired up to 100°.

In order to measure the pressure above 100°, the tube H′ is connected with a condensing pump, by means of which the air in the globe M and that in the vessel C are exposed to successive pressures, higher than the atmosphere. The ebullition is retarded (367), and it is only necessary to observe the difference in the height of the mercury in the two tubes of the manometer O, and the corresponding temperature, in order to obtain the pressure for a given temperature. The following tables by Regnault give the pressure of aqueous vapour from $-10°$ to 104°:—

Pressures of aqueous vapour from $-10°$ *to* 104° *C.*

Temperatures	Pressure in millimetres	Temperatures	Pressure in millimetres	Temperatures	Pressure in millimetres	Temperatures	Pressure in millimetres
− 10 °	2·078	12°	10·457	29°	29·782	90°	525·45
8	2·456	13	11·062	30	31·548	91	545·78
6	2·890	14	11·906	31	33·405	92	566·76
4	3·387	15	12·699	32	35·359	93	588·41
2	3·955	16	13·635	33	37·410	94	610·74
0	4·600	17	14·421	34	39·565	95	633·78
+ 1	4·940	18	15·357	35	41·827	96	657·54
2	5·302	19	16·346	40	54·906	97	682·03
3	5·687	20	17·391	45	71·391	98	707·26
4	6·097	21	18·495	50	91·982	98·5	720·15
5	6·534	22	19·659	55	117·479	99·0	733·91
6	6·998	23	20·888	60	148·791	99·5	746·50
7	7·492	24	22·184	65	186·945	100·0	760·00
8	8·017	25	23·550	70	233·093	100·5	773·71
9	8·574	26	24·998	75	288·517	101·0	787·63
10	9·165	27	26·505	80	354·643	102·0	816·17
11	9·792	28	28·101	85	433·41	104·0	875·69

In the following table the numbers were obtained by direct observation up to 24 atmospheres; the others were calculated by the aid of a formula of interpolation.

This table and the one next following show that the elastic force increases much more rapidly than the temperature. It has been attempted to express the relation between them by formulæ, but none of the formulæ seems to have the simplicity which characterises a true law.

Pressures in atmospheres from 100° *to* 230·9°

Temperatures	Number of atmospheres	Temperatures	Number of atmospheres	Temperatures	Number of atmospheres	Temperatures	Number of atmospheres
100·0°	1	170·8°	8	198·8°	15	217·9°	22
112·2	1½	175·8	9	201·9	16	220·3	23
120·6	2	180·3	10	204·9	17	222·5	24
133·9	3	184·5	11	207·7	18	224·7	25
144·0	4	188·4	12	210·4	19	226·8	26
152·2	5	192·1	13	213·0	20	228·9	27
156·2	6	195·5	14	215·5	21	230·9	28
165·3	7						

359. **Pressure of the vapours of different liquids.**—Regnault determined the elastic force, at various temperatures, of a certain number of liquids which are given in the following table :—

Liquids	Temperatures	Pressures in millimetres	Liquids	Temperatures	Pressures in millimetres
Mercury .	0°	0·02	Ether . .	−20°	68
	50	0·11		0	182
	100	0·74		60	1728
Alcohol .	0	13		100	4950
	50	220	Sulphurous acid	−20	479
	100	1695		0	1165
Bisulphide of carbon	−20	43		60	8124
	0	132	Ammonia .	−30	876
	60	1164		0	3163
	100	3329		30	8832

360. **Pressure of the vapours of mixed liquids.**—Regnault's experiments on the pressure of the vapour of mixed liquids prove that (i.) when two liquids exert no solvent action on each other—such as water and *bisulphide of carbon*, or water and *benzole*—the pressure of the vapour which rises from them is nearly equal to the sum of the pressures of the two separate liquids at the same temperature ; (ii.) with water and *ether*, which partially dissolve each other, the tension of the mixture is much less than the sum of the pressures of the separate liquids, being scarcely equal to that of the ether alone ; (iii.) when two liquids dissolve in all proportions, as ether and bisulphide of carbon, or water and alcohol, the pressure of the vapour of the mixed liquids is intermediate between the pressures of the separate liquids.

The pressure of the vapour from liquids which contain salts or other bodies in solution is less than that of the pure liquid. This may in principle be most simply demonstrated by the apparatus represented in fig. 330, in which AA is a barometer with the vacuum in which the solution of a salt of known strength is introduced, while B is an ordinary barometer.

With solutions of one and the same salt, Wüllner found that at a given

temperature the diminution of pressure is proportional to the quantity of salt dissolved, provided the solutions are dilute.

If f is the pressure of the pure solvent, and f' that of the solution, and g is the percentage strength, then $\frac{f-f'}{f} = kg$, where k is a constant. For $g = 1$, this equals $\frac{f-f'}{f}$, and is the relative diminution of pressure for one per cent.

This property is of great importance from the connection which has been established with osmotic pressure (139). The depression is different for different salts dissolved in the same solvent, but it is found that if these various solutions are *equimolar*—that is, when the weights dissolved in the same quantity of solvent are as their molecular weights—then the depression is constant. This may also be expressed by saying that the diminution in pressure stands in the same ratio to the pressure of the solvent, as the number of molecules in the dissolved body does to the total molecules in the liquid; that is

$$\frac{f-f'}{f} = \frac{n}{N+n},$$

where N and n are the number of molecules in the solvent and the salt respectively. Since $N = \frac{P}{M}$ and $n = \frac{p}{m}$, where P and p are the weights, and M and m the molecular weights, of the solvent and salt respectively, the above equation gives

$$\frac{f-f'}{f} = \frac{pM}{pM + Pm},$$

so that it is easy to deduce the molecular weight m of the body dissolved.

The above relations hold for ideal solutions, or those which are so dilute that the volume of the substance is infinitely small in comparison with that of the solvent.

361. **Pressure in two communicating vessels at different temperatures.**—When two vessels containing the same liquid, but at different temperatures, are connected with each other, the elastic force is not that corresponding to the mean of the two temperatures, as would naturally be supposed. Thus, if there are two globes (fig. 334), one, A, containing water kept at zero by means of melting ice, the other, B, containing water at 100°, the tension, as long as the globes are not con-

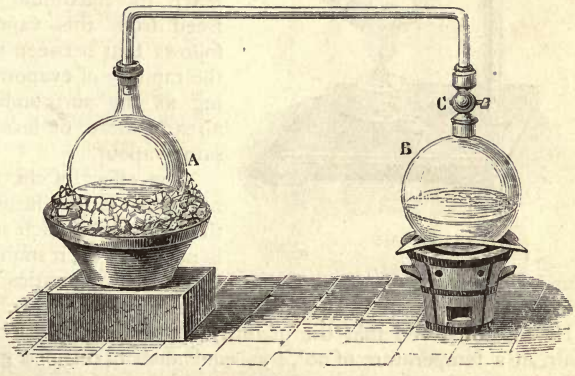


Fig. 334.

nected, is 4·6 millimetres in the first, and 760 millimetres in the second. But when the globes are connected by opening the stopcock C, the vapour in the globe B, from its greater pressure, passes into the other globe, and is there condensed, so that the vapour in B can never reach a higher pressure than that in the globe A. The liquid simply distils from B towards A without any increase of pressure.

From this experiment the general principle may be deduced that *when two vessels containing the same liquid, but at different temperatures, are connected, the pressure is identical in both vessels, and is the same as that corresponding to the lower temperature.* An application of this principle has been made by Watt in the condenser of the steam-engine.

362. **Evaporation. Causes which accelerate it.**—*Evaporation*, as has been already stated (350), is the slow production of vapour at the surface of a liquid. It is in consequence of this evaporation that wet clothes dry when exposed to the air, and that open vessels containing water become empty. The vapours which, rising in the atmosphere, condense, and, becoming clouds, fall as rain, are due to the evaporation from seas, lakes, rivers, and the earth.

Four causes influence the rapidity of the evaporation of a liquid : i. the temperature ; ii. the quantity of the same vapour in the surrounding atmosphere ; iii. the renewal of this atmosphere ; iv. the extent of the surface of evaporation.

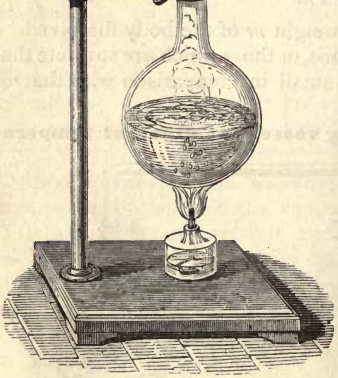


Fig. 335.

Increase of temperature accelerates the evaporation by increasing the elastic force of the vapours.

In order to understand the influence of the second cause, it is to be observed that no evaporation could take place in a space already saturated with vapour of the same liquid, and that it would reach its maximum in air completely freed from this vapour. It therefore follows that between these two extremes, the rapidity of evaporation varies according as the surrounding atmosphere is already more or less charged with the same vapour.

The effect of the renewal of this atmosphere is similarly explained ; for if the air or gas, which surrounds the liquid, is not renewed, it soon becomes saturated, and evaporation ceases. Dalton found that the ratios of the evaporation in a feeble, medium, and strong draught were respectively as 270 : 347 : 424. He also observed that the quantity evaporated in perfectly dry, almost still air, at a temperature of 20°, was equivalent to 0·1 of a gramme on a square decimetre of surface in a minute.

The effect of the fourth cause is self-evident.

Vegetation exercises a great influence on evaporation. Schübler found that the evaporation from a space covered with meadow grass, in the most

vigorous stage of its growth, was thrice as rapid as that from an adjacent surface of water. As the plants ripened the evaporation diminished.

363. **Laws of ebullition.**—*Ebullition*, or *boiling*, is the rapid production of elastic bubbles of vapour in the mass of a liquid itself.

When a liquid, water for example, is heated at the lower part of a vessel, the first bubbles are due to the disengagement of air which had previously absorbed. Small bubbles of vapour then begin to rise from the heated parts of the sides, but as they pass through the upper layers, the temperature of which is lower, they condense before reaching the surface. The formation and successive condensation of these first bubbles occasion the *singing* noticed in liquids before they begin to boil. Lastly, large bubbles rise and burst on the surface, and this constitutes the phenomenon of ebullition (fig. 334).

The laws of ebullition have been determined experimentally, and are as follows :—

I. *The temperature of ebullition or the boiling point increases with the pressure.*

II. *For a given pressure ebullition begins at a certain temperature, which varies in different liquids, but which, for equal pressures, is always the same in the same liquid.*

III. *Whatever be the intensity of the source of heat, as soon as ebullition begins the temperature of the liquid remains stationary.*

In order to determine the boiling points of liquids the apparatus represented in fig. 336 may be used, to the neck of which is fused the tube *b*; *a* can be connected either with a reflex or an ordinary condenser; the condensed vapours collect in the bent part *b*, and flow thence into the bulb. The temperature is indicated by a delicate thermometer *k* in the cork, in which there is also a capillary glass tube β; this serves to admit a few bubbles of air from time to time, which materially prevents bumping, which is likewise prevented by a few scraps of platinum foil. When the boiling point is to be determined under diminished pressure, *a* can be connected with an air-pump.

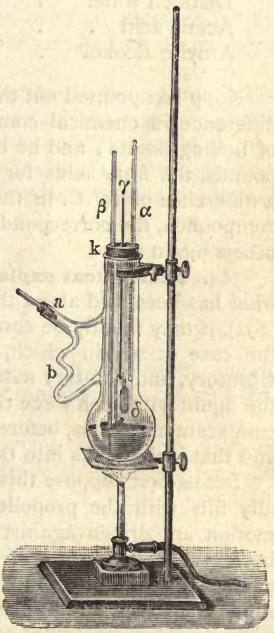


Fig. 336.

This apparatus in suitable dimensions may also be used to determine the expansion of liquids at their boiling points. For this purpose a small pyknometer (121) is suspended from a wire, which passes through the cork. This is weighed after having been filled with the liquid at a given temperature, and is then kept for some time in the vapour of the boiling liquid at the temperature t', and then when cooled is weighed again; the difference in the two weights represents the expansion between the two temperatures t and t'.

Boiling points under the pressure of 760 millimetres

Oxygen	−181·4°	Propionic acid	137°
Nitrous oxide	−92	Butyric acid	156
Carbonic acid	−80	Turpentine	157
Ammonia	−39	Aniline	182
Chloride of methyle	−23	Iodine	200
Cyanogen	−20	Naphthaline	217
Sulphurous acid	−10	Benzoic acid	261
Chloride of ethyle	+11	Phosphorus	290
Aldehyde	21	Diphenylamine	310
Ether	37	Strong sulphuric acid	318
Bisulphide of carbon	47	Phenanthrene	340
Acetone	56	Mercury	358
Bromine	58	Phosphate of phenyl	407
Methylic alcohol	66	Arsenic	437
Alcohol	78	Sulphur	448
Benzole	80	Phosphorus pentasulphide	530
Distilled water	100	Selenium	665
Acetic acid	117	Cadmium	746
Amylic alcohol	131	Zinc	940

Kopp has pointed out that in homologous chemical compounds the same difference in chemical composition frequently involves the same difference of boiling points ; and he has shown that in a very extensive series of compounds, the fatty acids for instance, the difference of CH^2 is attended by a difference of 19° C. in the boiling point. In other series of homologous compounds, the corresponding difference in the boiling point is 30°, and in others again 24°.

364. **Theoretical explanation of evaporation and ebullition.**—From what has been said about the nature of the motion of the molecules in liquids (292), it may readily be conceived that in the great variety of these motions, the case occurs in which, by a fortuitous concurrence of the progressive, vibratory, and rotatory motions, a molecule is projected from the surface of the liquid with such force that it overleaps the sphere of the action of its circumjacent molecules, before, by their attraction, it has lost its initial velocity ; and that it then flies into the space above the liquid.

Let us first suppose this space limited and originally vacuous ; it gradually fills with the propelled molecules, which act like a gas and in their motion are driven against the sides of the envelope. One of these sides, however, is the surface of the liquid itself, and a molecule when it strikes against this surface will not in general be repelled, but will be retained by the attraction which the adjacent ones exert. Equilibrium will be established when as many molecules are dispersed in the surrounding space as, on the average, impinge against the surface and are retained by it in the unit of time. This state of equilibrium is not, however, one of rest, in which evaporation has ceased, but a condition in which evaporation and condensation, which are equally strong, continually compensate each other.

The density of a vapour depends on the number of molecules which are

repelled in a given time, and this manifestly depends on the motion of the molecules in the liquid, and therefore on the temperature.

What has been said respecting the surface of the liquid clearly applies to the other sides of the vessel within which the vapour is formed: some vapour is condensed, this is subject to evaporation, and a condition ultimately occurs in which evaporation and condensation are equal. The quantity of vapour necessary for this depends on the density of vapour in the closed space, on the temperature of the vapour and of the sides of the vessel, and on the force with which this attracts the molecules. The maximum will be reached when the sides are covered with a layer of liquid, which then acts like the free surface of a liquid.

In the interior of a liquid it may happen that the molecules repel each other with such force as momentarily to destroy the coherence of the mass. The small vacuous space which is thereby formed is entirely surrounded by a medium which does not allow of the passage of the repelled molecules. Hence it cannot increase and maintain itself as a bubble of vapour, unless so many molecules are projected from the inner sides that the internal pressure which thereby results can balance the external pressure which tends to condense the bubble. The expansive force of the enclosed vapour must therefore be so much the greater, the higher the external pressure on the liquid, and we can thus understand the influence of pressure on the temperature of boiling.

365. **Influence of substances in solution on the boiling point.**—The ebullition of a liquid is the more retarded the greater the quantity of any substance it may contain in solution, provided that the substance be not volatile, or, at all events, be less volatile than the liquid itself. Water, which boils at 100° when pure, boils at the following temperatures when saturated with different salts :—

Water saturated with	common salt	. .	boils at 102°
„ „	nitrate of potassium	.	„ 116
„ „	carbonate of potassium		„ 135
„ „	chloride of calcium	.	„ 179

Acids in solution present analogous results; but substances merely mechanically suspended, such as earthy matters, bran, wooden shavings, &c., do not affect the boiling point.

Absorbed air exerts a very marked influence on the boiling point of water. Deluc first observed that water freed from air by ebullition, and

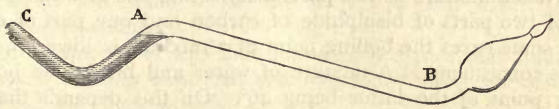


Fig. 337.

placed in a flask with a long neck, could be raised to 112° without boiling. M. Donny examined this phenomenon by means of the apparatus depicted in figure 337. It consists of a glass tube CAB, bent at one end and closed at C, while the other is blown into a pear-shaped bulb, B, drawn out to a

point. The tube contains water which is boiled until all air is expelled, and the open end is hermetically sealed. By inclining the tube the water passes into the bent end CA; this end being placed in a bath of chloride of calcium, the temperature may be raised to 130° without any signs of boiling. At 138° the liquid is suddenly converted into steam and the water is thrown over into the bulb, which is smashed if not sufficiently strong.

Boiled-out water, covered with a layer of oil, may be raised to 120° without boiling, but above this temperature it suddenly begins to boil, and with almost explosive violence.

When a liquid is suspended in another of the same specific gravity, but of higher boiling point, with which it does not mix, it may be raised far beyond its boiling point without the formation of a trace of vapour. Dufour has made a number of valuable experiments on this subject; he used in the case of water a mixture of oil of cloves and linseed oil, and placed in it globules of water, and then gradually heated the oil; in this way ebullition rarely set in below 110° or 115°; very commonly globules of 10 millimetres diameter reached a temperature of 120° or 130°, while very small globules of 1 to 3 millimetres reached the temperature of 175°, a temperature at which the tension of vapour on a free surface is 8 or 9 atmospheres.

At these high temperatures the contact of a solid body, or the production of gas bubbles in the liquid, occasioned a sudden vaporisation of the globule, accompanied by a sound like the hissing of a hot iron in water.

Saturated aqueous solutions of sulphate of copper, chloride of sodium, &c., remain liquid at a temperature far beyond their boiling point, when immersed in melted stearic acid. In like manner, globules of chloroform (which boils at 61°), suspended in a solution of chloride of zinc, could be heated to 97° or 98° without boiling.

It is a disputed question as to what is the temperature of the vapour from boiling saturated saline solutions. It has been stated by Rudberg to be that of pure water boiling under the same pressure. The most recent experiments of Magnus seem to show, however, that this is not the case, but that the vapour of boiling solutions is hotter than that of pure water; and that the temperature rises as the solutions become more concentrated, and therefore boil at higher temperatures. Nevertheless, the vapour was always found somewhat cooler than the mass of the boiling solution, and the difference was greater at high than at low temperatures.

The boiling point of a liquid is usually lowered when it is mixed with a more volatile liquid than itself, but raised when it contains one which is less volatile. Thus a mixture of two parts alcohol and one of water boils at 83°, a mixture of two parts of bisulphide of carbon and one part of ether boils at 38°. In some cases the boiling point of a mixture is lower than that of either of its constituents. A mixture of water and bisulphide boils at 43°, the boiling point of the latter being 46°. On this depends the following curious experiment. If water and bisulphide of carbon, both at the temperature 45°, are mixed together, the mixture at once begins to boil briskly.

366. **Influence of the nature of the vessel on the boiling point.**—Gay-Lussac observed that water in a glass vessel required a higher temperature for ebullition than in a metal one. Taking the temperature of boiling water in a copper vessel at 100°, its boiling point in a glass vessel was

found to be 101°; and if the glass vessel had been previously cleaned by means of sulphuric acid and of potass, the temperature would rise to 105°, or even to 106°, before ebullition commenced. A piece of metal placed in the bottom of the vessel was always sufficient to lower the temperature to 100°, and at the same time to prevent the violent concussions which accompany the ebullition of saline or acid solutions in glass vessels. Whatever be the boiling point of water, the temperature of its vapour is uninfluenced by the substance of the vessels.

367. **Influence of pressure on the boiling point.**—We see from the table of tensions (358) that at 100°, the temperature at which water boils under a pressure of 760 millimetres, which is that of the atmosphere, aqueous vapour has a tension exactly equal to this pressure. This principle is general, and may be thus enunciated: *A liquid boils when the tension of its vapour is equal to the pressure it supports.* Consequently, as the pressure increases or diminishes, the tension of the vapour, and therefore the temperature necessary for ebullition, must increase or diminish. Hence a liquid has, strictly speaking, an indefinite number of boiling points.

In order to show that the boiling point is lower under diminished pressure, a small dish containing water at 30° is placed under the receiver of an air-pump, which is then exhausted. The liquid soon begins to boil, the vapour formed being pumped out as rapidly as it is generated.

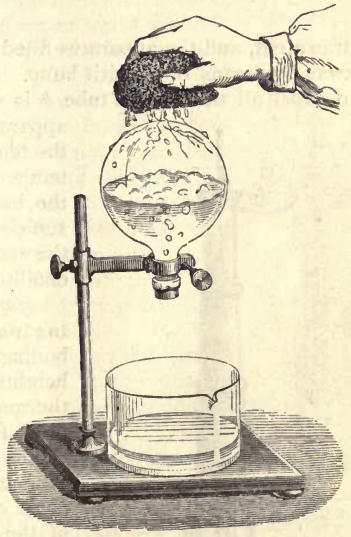


Fig. 338.

A paradoxical but very simple experiment also well illustrates the dependence of the boiling point on the pressure. In a glass flask, water is boiled for some time, and when all air has been expelled by the steam, the flask is closed by a cork and inverted, as shown in fig. 338. If the bottom is then cooled by a stream of cold water from a sponge, the water begins to boil again. This arises from the condensation of the steam above the surface of the water, by which a partial vacuum is produced.

It is in consequence of this diminution of pressure that liquids boil on high mountains at lower temperatures. On Mont Blanc, for example, water boils at 84°, and at Quito at 90°.

On the more rapid evaporation of water under feeble pressures is based the use of the air-pump in concentrating those solutions which either cannot bear a high degree of heat, or which can be more cheaply evaporated in an exhausted space. Howard made a most important and useful application of this principle in the manufacture of sugar. The syrup, in his method, is enclosed in an air-tight vessel, which is exhausted by a steam-engine. The

evaporation consequently goes on at a lower temperature, which secures the syrup from injury. The same plan is adopted in evaporating the juice of certain plants used in preparing medicinal extracts.

On the other hand, boiling is retarded by increasing the pressure; under the pressure of two atmospheres, for example, water only boils at 120°·6.

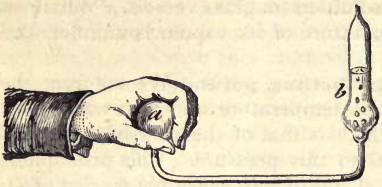


Fig. 339.

368. **Franklin's experiment.**—The influence of pressure on boiling may further be illustrated by means of an experiment originally made by Franklin. The apparatus consists of a bulb, *a*, and a tube, *b*, joined by a tube of smaller dimensions (fig. 339). The tube *b* is drawn out, and the apparatus filled with water, which is then in part boiled away by means of a spirit lamp. When it has been boiled sufficiently long to expel all the air, the tube *b* is sealed. There is then a vacuum in the apparatus, or rather there is a pressure due to the tension of aqueous vapour, which at ordinary temperatures is very small. Consequently, if the bulb, *a*, be placed in the hand, the heat is sufficient to produce a pressure which drives the water into the tube, *b*, and causes a brisk ebullition.

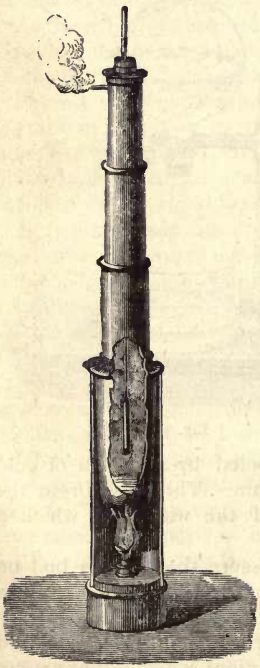


Fig. 340.

369. **Measurement of heights by the boiling point.**—From the connection between the boiling point of water and the pressure, the heights of mountains may be measured by the thermometer instead of by the barometer. Suppose, for example, it is found that water boils on the summit of a mountain at 90°, and at its base at 98°; at these temperatures the elastic force or tension of the vapour is equal to that of the pressure on the liquid; that is, to the pressure of the atmosphere at the two places respectively. Now, the tensions of aqueous vapour for various temperatures have been determined, and accordingly the tensions corresponding to the above temperatures are sought in the tables. These numbers represent the atmospheric pressures at the two places; in other words, they give the barometric heights, and from these the height of the mountain may be calculated by the method already given (178). An ascent of about 1080 feet produces a diminution of 1°C. in the boiling point.

The instruments used for this purpose are called *thermo-barometers* or *hypsometers*, and were first applied by Wollaston. They consist essentially

of a small metallic vessel for boiling water (fig. 341), fitted with very delicate thermometers, which are only graduated from 80° to 100°; so that, as each degree occupies a considerable space on the scale, the 10ths, and even the 100ths, of a degree may be estimated, and thus it is possible to determine the height of a place by means of the boiling point to within about 10 feet.

370. **Formation of vapour in closed tubes.**—We have hitherto considered vapours as being produced in an indefinite space, or where they could expand freely, and it is only under this condition that boiling can take place. In a closed vessel the vapours produced finding no issue, their tension and their density increase with the temperature, but that rapid disengagement of vapour which constitutes boiling is impossible. Hence, while the temperature of a liquid in an open vessel can never exceed that of boiling, in a closed vessel it may be much higher. The liquid state has, nevertheless, a limit; for, according to experiments by Cagniard-Latour, if either water, alcohol, or ether be placed in strong glass tubes, which are hermetically sealed after the air has been expelled by boiling, and if then these tubes are exposed to a sufficient degree of heat, a moment is reached at which the liquid suddenly disappears, and is converted into vapour at 200°. With ether this occurs at 200°; the vapour then occupies a space less than double its volume in the liquid state, its pressure being 38 atmospheres.

Alcohol which half fills a tube is converted into vapour at 207° C. If a glass tube about half full of water, in which some carbonate of soda has been dissolved, to diminish the action of the water on the glass, be heated, it is completely vaporised at about the temperature of melting zinc.

When chloride of ethyle is heated in a very stout sealed tube, the upper surface ceases to be distinct at 170°, and is replaced by an ill-defined nebulous zone. As the temperature rises this zone increases in width in both directions, becoming at the same time more transparent; after a time the liquid is completely vaporised, and the tube becomes transparent and seemingly empty. On cooling, the phenomena are reproduced in opposite order. Similar appearances are observed on heating ether in a sealed tube at 190°.

Andrews made a series of observations on the behaviour of condensed gases at different temperatures, by means of an apparatus, the principal features of which are represented in fig. 341.

Fig. 341.

The pure and dry gas is contained in a tube *g*, which is sealed at one end, and the gas is shut in by a thread of mercury. The tube is inserted in a brass end-piece, E, which is firmly screwed on a strong copper tube, R. At the other end is a similar piece, in which a steel screw works, perfect tightness being ensured by good packing. The tube is full of water, so that by turning this screw the pressure on the enclosed gas can be increased up to 500 atmospheres. In some cases the projecting capillary tube is bent downwards, so that it can be placed in a freezing mixture.

Andrews found on raising liquid carbonic acid in such a tube to a temperature of 31° C. that the surface of demarcation between the liquid and the gas became fainter, lost its curvature, and gradually disappeared. The space was then occupied by a homogeneous fluid, which, when the pressure was suddenly diminished, or the temperature slightly lowered, exhibited a peculiar appearance of moving or flickering striæ throughout its whole mass. Above 30° no apparent liquefaction of carbonic anhydride, or separation into two distinct forms of matter, could be effected, not even when the pressure of 400 atmospheres was applied.

From similar observations made with other substances it seems that there exists for every liquid a temperature, the *critical point* or *critical temperature*. While below this critical point a sudden transition from gas to liquid is accompanied by a sudden diminution of volume, and liquid and gas are separated by a sharp line of demarcation, above this critical point the change is connected with a gradual diminution of volume, and is quite imperceptible. The condensation can, indeed, only be recognised by a sudden ebullition when the pressure is lessened. Hence, ordinary condensation is only possible at a temperature below the critical point, and it is not surprising, therefore, that mere pressure, however great, should have failed to liquefy many of the gases.

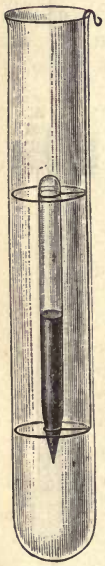


Fig. 342.

The phenomenon of the critical temperature may also be conveniently illustrated by the following arrangement (fig. 342), which is also well adapted for projection on a screen by means of a magic-lantern for lecture purposes. A stout glass tube about 2·5mm wide and 40mm long, contains liquid sulphurous acid, and is supported, with the drawn-out end downwards, in a test-tube by means of a wire frame. Pure melted paraffin is added to about 10cm above the inner tube. The whole arrangement is suspended in a retort-holder, and heat applied with a spirit lamp. With careful manipulation there is no danger, and the course of the phenomenon is readily seen through the clear paraffin.

The boiling point of a body may be defined as the temperature above which a body passes into the state of gas, not only on the surface but in the body of the liquid ; this temperature is therefore different for different pressures, and is accordingly a *relative* magnitude. The *absolute boiling point* is the temperature at which a body is converted into gas, whatever be the pressure ; it is identical with the critical temperature. Mendelejeff found that a relation existed between the absolute temperature and the capillarity of liquids. Increase of temperature diminishes the cohesion, and therefore the capillarity of liquids. The capillarity ultimately vanishes, and the temperature at which this takes place is the absolute boiling point. Some of them are very low ; that of air, for instance, is −158°.

The *critical pressure* is that at which condensation takes place at the critical temperature, and the volume of the saturated vapour at the critical temperature, and under the critical pressure, is called the *critical volume*.

A *vapour* may be defined as being a gas at any temperature below its

critical point. Hence a vapour can be converted into a liquid by pressure alone, and can therefore exist in the pressure of its own liquid, while a *gas* requires cooling as well as pressure to convert it into a liquid; that is, to alter its arrangement in such a manner that a liquid can be seen to be separated from a gas by a distinctly bounded surface.

371. **Papin's digester.**—Papin appears to have been the first to investigate the effects of the production of vapour in closed vessels. The apparatus which bears his name consists of a cylindrical iron vessel M (fig. 343), provided with a cover, which is firmly fastened down by the screw B. In order to close the vessel hermetically, sheet lead is placed between the edges of the cover and the vessel. At the bottom of a cylindrical cavity, which traverses a cylinder, and a tubulure, the cover is perforated by a small orifice in which there is a rod *u*. This rod presses against a lever *a b*, movable at *a*, and the pressure may be regulated by means of a weight *p*, movable on this lever. The lever is so weighted that when the pressure in the interior is equal to six atmospheres, for example, the valve rises and the vapour escapes. The destruction of the apparatus is thus avoided, and this mechanism has hence received the name of *safety-valve*. The digester is filled about two-thirds with water, and is heated on a furnace. The water may thus be raised to a temperature far above 100°, and the pressure of the vapour increased to several atmospheres, according to the weight on the lever.

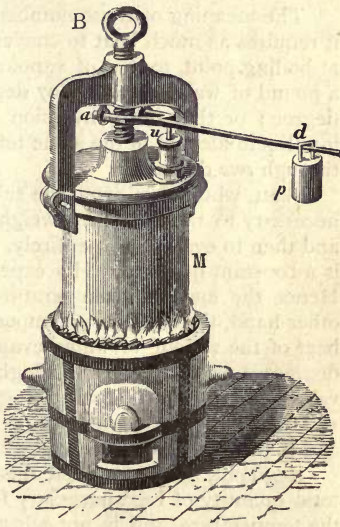


Fig. 343.

We have seen that water boils at much lower temperatures on high mountains (367); the temperature of water boiling in open vessels in such localities is not sufficient to soften animal fibre completely and extract the nutriment, and hence Papin's digester is used in the preparation of food.

Papin's digester is used in extracting gelatine. When bones are digested in this apparatus they are softened, so that the gelatine which they contain is dissolved: the part through which the screw B passes is made of such elasticity that it yields, and the lid opens when the pressure of the vapour becomes dangerous.

372. **Latent heat of vapour.**—As the temperature of a liquid remains constant during boiling, whatever be the source of heat (363), it follows that a considerable quantity of heat becomes absorbed in boiling, the only effect of which is to transform the body from the liquid to the gaseous condition. And, conversely, when a saturated vapour passes into the state of liquid, it gives out a definite amount of heat.

These phenomena were first observed by Black, and he described them by saying that during vaporisation a quantity of sensible heat became latent,

and that the latent heat again became free during condensation. The quantity of heat which a liquid must absorb in passing to the gaseous state, and which it gives out in passing from the state of vapour to that of liquid, is spoken of as the *latent heat of evaporation.*

The analogy of these phenomena to those of fusion will be at once seen; the modes of determining them will be described in the chapter on Calorimetry; but the following results, which have been obtained for the latent heats of evaporation at 0°, may be here given :—

Water	607	Bisulphide of carbon	90
Alcohol	236	Turpentine	74
Benzole	109	Chloroform	67
Acetic acid	102	Bromine	49
Ether	94	Iodine	24

The meaning of these numbers is, in the case of water, for instance, that it requires as much heat to convert a pound of water from the state of liquid at boiling point, to that of vapour at the same temperature, as would raise a pound of water through 607 degrees, or 607 pounds of water through one degree; or that the conversion of one pound of vapour of alcohol at 0° into liquid alcohol of the same temperature would heat 236 pounds of water through *one* degree.

Watt, who investigated the subject, held that the whole quantity of heat necessary to raise a given weight of water from zero to any temperature, and then to evaporate it entirely, or what is called *the heat of evaporation*, is a constant quantity. His experiments showed that this quantity is 640. Hence the lower the temperature the greater the latent heat, and, on the other hand, the higher the temperature the less the latent heat. The latent heat of the vapour of water evaporated at 100° would be 540, while at 50 degrees it would be 590. At higher temperatures the latent heat of aqueous vapour would go on diminishing. Water evaporated under a pressure of 15 atmospheres at a temperature of 200° would have a latent heat of 440, and if it could be evaporated at 640° it would have no latent heat at all.

Regnault, who examined this question with great care, found that the total quantity of heat necessary for the evaporation of water increases with the temperature, and is not constant, as Watt had supposed. It is represented by the formula

$$Q = 606{\cdot}5 + 0{\cdot}305t,$$

in which Q is the total quantity of heat, and t the temperature of the water during evaporation, while the numbers are constant quantities. The total quantity of heat necessary to evaporate water at 100° is $606{\cdot}5 + (0{\cdot}305 \times 100) = 637$; at 120° it is 643; at 150° it is 651; and at 180° it is 661.

Thus the heat required to raise a pound of water from zero and convert it into steam at 100° represents a mechanical work of 885430 units, which would be sufficient to raise a ton weight through a height of nearly 400 feet.

The total heat of the evaporation of ether is expressed by a formula similar to that of water, namely, $Q = 94 + 0{\cdot}045t$; and that for chloroform $Q = 67 + 0{\cdot}1375t$.

The heat which is expended simply in evaporating a liquid, and which is

spoken of as the latent heat, produces no rise of temperature, and only appears as doing the work of a change of state. One portion of this work is expended in overcoming the cohesion of the particles in the liquid state, and enabling them to assume the gaseous form—this is the *internal work*, and is by much the greater; the other, the *external work*, is expended in overcoming the external pressure on the vapour formed, and which is much larger than in the original liquid state, for the volume is greatly increased.

Knowing the increase of volume, and the pressure, the external work may readily be calculated; for if the volumes of unit weight of the substance in the state of liquid and of vapour are respectively s and σ, and the pressure for unit surface is p, then the external work is $Ap(\sigma - s)$, A being the mechanical equivalent of heat. So that, if r is the total heat of evaporation,

$$r = \rho + Ap(\sigma - s)$$

in which ρ is the internal work. From the values of r and of $Ap(\sigma - s)$, it is easy to deduce that of ρ, and it is found that this value decreases as the temperature increases.

Thus for the temperatures 0, 50, 100, and 150° the values are 576, 536, 496, and 457 respectively; that is, when water at 0° is converted into vapour, a greater internal work is required to overcome the cohesion than at 100° for instance.

373. **Cold due to evaporation. Mercury frozen.**—Whatever be the temperature at which a vapour is produced, an absorption of heat always takes place. If, therefore, a liquid evaporates, and does not receive from without a quantity of heat equal to that which is expended in producing the vapour, its temperature sinks, and the cooling is greater in proportion as the evaporation is more rapid.

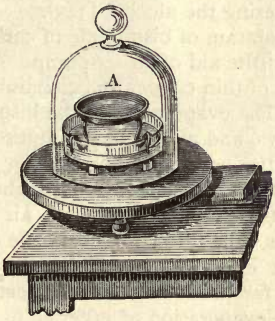


Fig. 344.

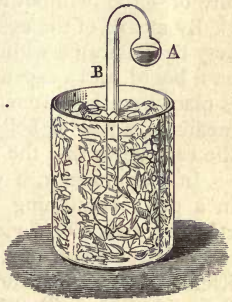


Fig. 345.

Leslie succeeded in freezing water by means of rapid evaporation. Under the receiver of the air-pump is placed a vessel containing strong sulphuric acid, and above it a thin metal capsule, A (fig. 344), containing a small quantity of water. By exhausting the receiver the water begins to boil (360), and since the vapour is absorbed by the sulphuric acid as fast as it is formed, a rapid evaporation is produced, which quickly effects the freezing of the water.

This experiment is best performed by using, instead of a thin metal dish, a watch-glass coated with lampblack and resting on a cork. The advantage of this is twofold: firstly, the lampblack is a very bad conductor; and, secondly, it is not moistened by the liquid, which remains in the form of a

globule not in contact with the glass. A small porous dish may also advantageously be used.

The same result is obtained by means of Wollaston's *cryophorus* (fig. 345), which consists of a bent glass tube provided with a bulb at each end. The apparatus is prepared by introducing a small quantity of water, which is then boiled so as to expel all air. It is then hermetically sealed, so that on cooling it contains only water and the vapour of water. The water being introduced into the bulb A, the other bulb is immersed in a freezing mixture. The vapour in the tube is thus condensed; the water in A rapidly yields more. But this rapid production of vapour requires a large amount of heat, which is abstracted from the water in A, and its temperature is so much reduced that it freezes.

By using liquids more volatile than water, more particularly liquid sulphurous acid, which boils at $-10°$, or, still better, chloride of methyle, which is now prepared industrially in large quantities, a degree of cold is obtained sufficiently low to freeze mercury. This experiment may be made on a small scale by covering the bulb of a thermometer with cotton wool, and, after having moistened it with the liquid in question, placing it under the receiver of the air-pump. When a vacuum is produced the mercury is quickly frozen.

By passing a current of air, previously cooled, through liquid chloride of methyle, temperatures of from $-23°$ to $-70°$ C. may be maintained with great constancy for several hours. Thilorier, by directing a jet of liquid carbonic acid on the bulb of an alcohol thermometer, obtained a temperature of $-100°$ without freezing the alcohol (343).

By means of the evaporation of bisulphide of carbon the formation of ice may be illustrated without the aid of an air-pump. A little water is dropped on a board, and a capsule of thin copper foil, containing bisulphide of carbon, is placed on the water. The evaporation of the bisulphide is accelerated by means of a pair of bellows, and after a few minutes the water freezes round the capsule so that the latter adheres to the wood.

In like manner, if water be placed in a test-tube, which is then dipped in a glass containing ether, and a current of air be blown through the ether by means of a glass tube fitted to the nozzle of a pair of bellows, the rapid evaporation of the ether very soon freezes the water in the tube. Richardson's apparatus for producing local anæsthesia also depends on the cold produced by the evaporation of ether.

The cold produced by evaporation is used in hot climates to cool water by means of *alcarrazas*. These are porous earthen vessels, through which water percolates, so that on the outside there is a continual evaporation, which is accelerated when the vessels are placed in a current of air. For the same reason wine is cooled by wrapping the bottles in wet cloths and placing them in a draught.

In Harrison's method of making ice artificially, a steam-engine is used to work an air-pump which produces a rapid evaporation of some ether, in which is immersed the vessel containing the water to be frozen. The apparatus is so constructed that the vaporised ether can be condensed and used again.

The cooling effect produced by a wind or draught does not necessarily

arise from the wind being cooler, for it may, as shown by the thermometer, be actually warmer, but arises from the rapid evaporation it causes from the surface of the skin. We have the feeling of oppression even at moderate temperatures, when we are in an atmosphere saturated by moisture, in which no evaporation takes place.

374. **Carré's apparatus for freezing water.**—We have already seen that when any liquid is converted into vapour it absorbs a considerable quantity of sensible heat; this furnishes a source of cold which is more abundant the more volatile the liquid, and the greater its heat of vaporisation.

This property of liquids has been utilised by M. Carré, in freezing water by the distillation of ammonia. The apparatus consists of a cylindrical boiler C (figs. 346, 347), and of a slightly conical vessel A, which is the

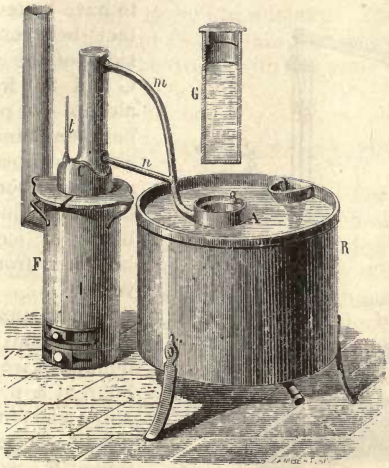


Fig. 346.

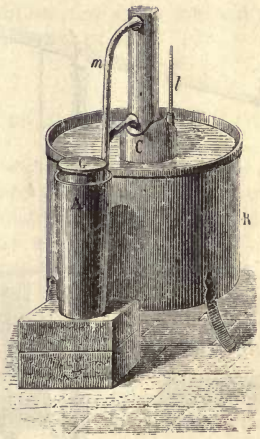


Fig. 347.

freezer. These two vessels are connected by a tube, *m*, and a brace, *n*, binds them firmly. They are made of strong galvanised iron plate, and can resist a pressure of seven atmospheres.

The boiler C, which holds about two gallons, is three parts filled with a strong solution of ammonia. In a tubulure in the upper part of the boiler some oil is placed, and in this a thermometer *t*. The freezer A consists of two concentric envelopes, arranged in such a manner that, the centre being hollow, a metal vessel, G, containing the water to be frozen, can be placed in this space. Hence only the annular space between the sides of the freezer is in communication with the boiler by means of the tube *m*. In the upper part of the freezer there is a small tubulure which can be closed by a metal stopper, and by which the solution of ammonia is introduced.

The formation of ice comprises two distinct operations. In the first, the boiler is placed in a furnace F, and the freezer in a bath of cold water of

about 12°. The boiler being heated to 130°, the ammoniacal gas dissolved in the water of the boiler is disengaged, and, in virtue of its own pressure, is liquefied in the freezer A, along with about a tenth of its weight of water. This distillation of C towards A lasts about an hour and a quarter, and when it is finished the second operation commences; this consists in placing the boiler in the cold-water bath (fig. 347), and the freezer A outside, care being taken to surround it with dry flannel. The vessel G, about three-quarters full of water, is placed in the freezer. As the boiler cools, the ammoniacal gas with which it is filled is again dissolved; the pressure thus being diminished, the ammonia which has been liquefied in the freezer is converted into the gaseous form, and now distils from A towards C, to redissolve in the water which has remained in the boiler. During this distillation the ammonia which is gasified absorbs a great quantity of heat, which is withdrawn from the vessel G and the water it contains. Hence it is that this water freezes. In order to have better contact between the sides of the vessel G and the freezer, alcohol is poured between them. In about an hour and a quarter a perfectly compact cylindrical block of ice can be taken from the vessel G.

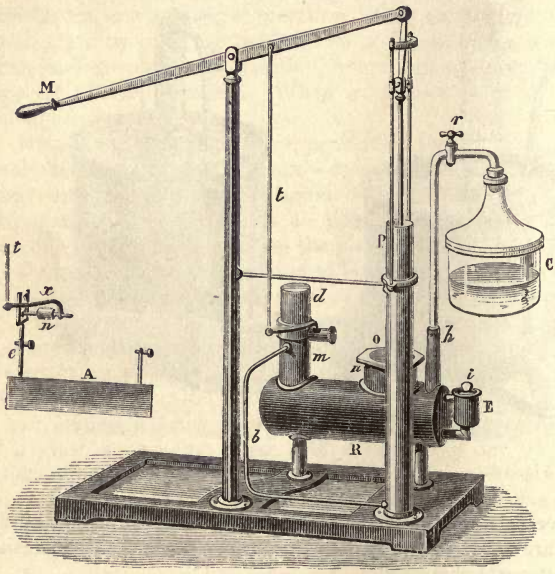


Fig. 348.

This apparatus gives about four pounds of ice in an hour, at a price of about a farthing per pound; large continuously working apparatus have, however, been constructed, which produce as much as 800 pounds of ice in an hour.

Carré has constructed an ice-making machine which is an industrial application of Leslie's experiment (373), and by which considerable quantities of water may be frozen in a short time. It consists of a cylinder, R, about 15 inches long by 4 in diameter, made of an alloy of lead and antimony (fig. 348). At one end is a funnel E, by which strong sulphuric acid can be introduced; at the other is a tubulure *m*, to which is screwed a dome *d* that supports a series of obstacles intended to prevent any sulphuric acid from spirting into *m* and *b*. There are, moreover, on the receiver a wide tube, *u*, closed by a thick glass disc O, and a long tube *h*, to the top of which is fitted the bottle C

containing water to be frozen. The dome *d*, the disc O, and the stopper *i* of the funnel E are all sealed with wax.

On the side of the receiver is an air-pump P, connected with it by a tube *b*, and worked by a handle M. To this handle is attached a rod *t*, which, by the mechanism represented on the left of the figure, works a stirrer A in the sulphuric acid. A lever *x* connected with a horizontal axis which traverses a small stuffing-box *n*, transmits its backward and forward motion to the rod *e* and to the stirrer. This and the stuffing-box *n* are fitted in a tubulure on the side of the tubulure *m*.

The smallest size which Carré makes holds 2·5 kilogrammes of sulphuric acid, and the water-bottle about 400 grammes, when it is one-third full. After about 70 strokes of the piston the water begins to boil ; the acid being in continued agitation, the vapour is rapidly absorbed by it, and the pump is worked until freezing begins. For this purpose it is merely necessary to give a few strokes every five minutes. The rate of freezing depends on the strength of the acid ; when this gets very dilute it requires renewal ; but 12 water-bottles can be frozen with the same quantity of acid.

LIQUEFACTION OF VAPOURS AND GASES

375. **Liquefaction of vapours.**—The *liquefaction* or *condensation* of vapours is their passage from the aëriform to the liquid state. Condensation may be due to three causes—cooling, compression, or chemical action. For the first two causes the vapours must be saturated (353), while the latter produces the liquefaction of the most rarefied vapours. Thus, a large number of salts absorb and condense the aqueous vapour in the atmosphere, however small its quantity.

When vapours are condensed, their latent heat becomes free ; that is, it affects the thermometer. This is readily seen when a current of steam at 100° is passed into a vessel of water at the ordinary temperature. The liquid becomes rapidly heated, and soon reaches 100°. The quantity of heat given up in liquefaction is equal to the quantity absorbed in producing the vapour.

376. **Distillation. Stills.**—*Distillation* is an operation by which a volatile liquid may be separated from substances which it holds in solution or by which two liquids of different volatilities may be separated. The operation depends on the transformation of liquids into vapour by the action of heat, and on the condensation of this vapour by cooling.

The apparatus used in distillation is called a *still*. Its form may vary greatly, but it consists essentially of three parts ; 1st, the *body* A (fig. 349), a copper vessel containing the liquid, the lower part of which fits in the furnace ; 2nd, the *head*, B, which fits on the body, and from which a lateral tube, C, leads to—3rd—the *worm*, S, a long spiral tin or copper tube placed in a cistern kept constantly full of cold water. The object of the worm is to condense the vapour by exposing a greater extent of cold surface.

To free ordinary water from the many impurities which it contains, it is placed in a still and heated. The vapours disengaged are condensed in the worm, and the distilled water arising from the condensation is collected in

the receiver D. The vapours in condensing rapidly heat the water in the cistern, which must, therefore, be constantly renewed. For this purpose a continual supply of cold water passes into the bottom of the cistern, while

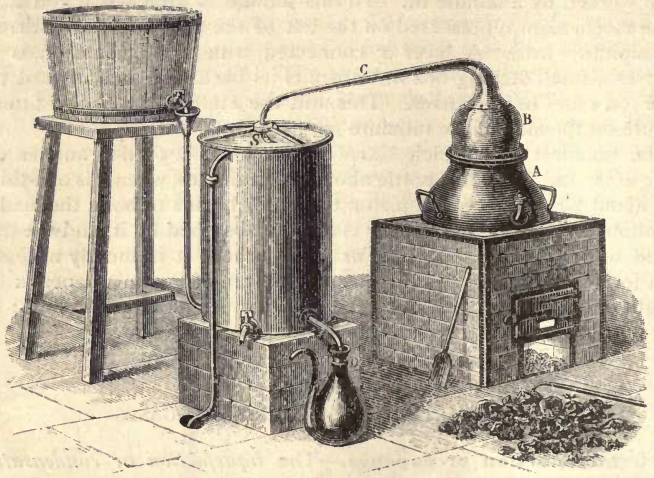


Fig. 349.

the lighter heated water rises to the surface and escapes by a tube in the top of the cistern.

377. **Liebig's Condenser.**—In distilling small quantities of liquids, or in taking the boiling point of a liquid, so as not to lose any of it, the apparatus

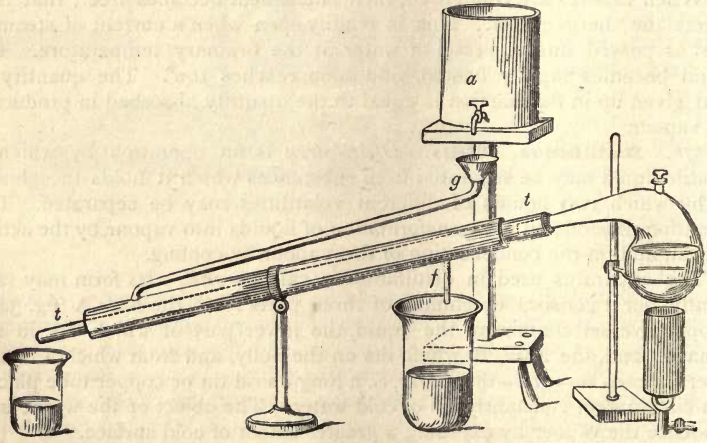


Fig. 350.

known as *Liebig's Condenser* is extremely useful. It consists of a glass tube, *tt* (fig. 350), about thirty inches long, fitted in a copper or tin tube

by means of perforated corks. A constant supply of cold water from the vessel *a* passes into the space between the two tubes, being conveyed to the lower part of the condenser by a funnel and tube *g*, flowing out from the upper part of the tube *f*. The liquid to be distilled is contained in a retort, the neck of which is placed in the tube ; the condensed liquid drops quite cold into a vessel placed to receive it at the other end of the condensing tube.

378. **Apparatus for determining the alcoholic value of wines.**—One of the forms of this apparatus consists of a glass flask resting on a tripod, and heated by a spirit lamp (fig. 351). By means of a caoutchouc tube this is connected with a worm placed in a copper vessel filled with cold water, and below which is a test glass for collecting the distillate. On this are three divisions, one *a*, which measures the quantity of wine taken ; the two others indicating one-half and one-third of this volume.

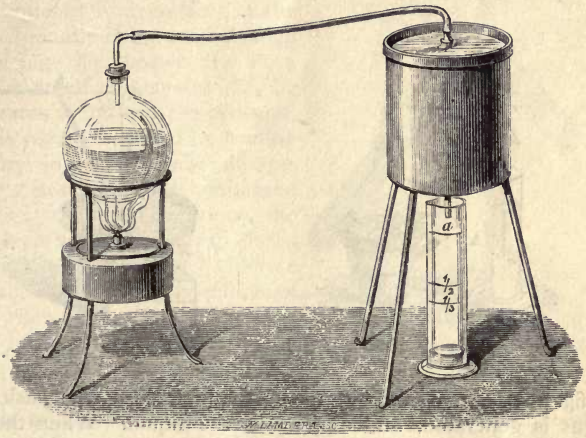


Fig. 351.

The test glass is filled with the wine up to *a* ; this is then poured into the flask, which having been connected with the worm, the distillation is commenced. The liquid which distils over is a mixture of alcohol and water ; for ordinary wines, such as clarets and hocks, about one-third is distilled over, and for wines richer in spirit, such as sherries and ports, one-half must be distilled ; experiment has shown that under these circumstances practically all the alcohol passes over in the distillate. The measure is then filled up with distilled water to *a* ; this gives the mixture of alcohol and water of the same volume as the wine taken, free from all solid matters, such as sugar, colouring matter, and acid, but containing all the alcohol. The specific gravity of this distillate is then taken by means of an alcoholometer (128), and the number thus obtained corresponds to a certain strength of alcoho as indicated by the tables.

379. **Safety-tube.**—In preparing gases and collecting them over mercury or water, it occasionally happens that these liquids rush back into the generating vessel, and destroy the operation. This arises from an excess of atmospheric pressure over the elastic force in the vessel. If a gas—sulphurous acid for example—be generated in the flask *m* (fig. 352), and be passed into water in the vessel A, as long as the gas is given off freely, its elastic force exceeds the atmospheric pressure, and the weight of the column of water.

on, so that the water in the vessel cannot rise in the tube, and absorption is impossible. But if the tension decreases, either through the flask becoming cooled or the gas being disengaged too slowly, the external pressure prevails, and when it exceeds the internal tension by more than the weight of the column of water *co*, the water rises into the flask, and the operation is spoiled. This accident is prevented by means of *safety-tubes*.

These are tubes which prevent absorption by allowing the air to enter in proportion as the internal pressure decreases. The simplest is a tube C

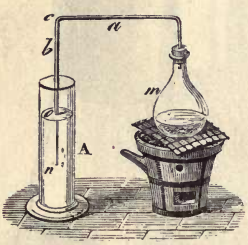


Fig. 352.

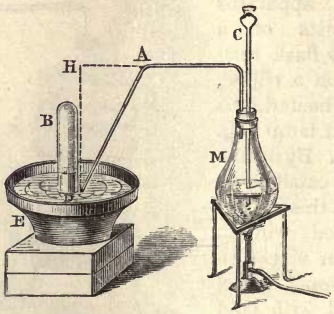


Fig. 353.

(fig. 353), passing through the cork which closes the flask M, in which the gas is generated, and dipping in the liquid. When the pressure of the gas diminishes in M, the atmospheric pressure on the water in the bath E causes it to rise to a certain height in the tube DA; but this pressure, acting also on the liquid in the tube C, depresses it to the same depth, assuming that the liquid has the same density as the water in E. Now, as this depth is less than the height DH, air enters by the aperture, before the water in the bath can rise to A, and no absorption takes place.

380. **Liquefaction of gases.**—We have already seen that a saturated vapour, the temperature of which is constant, is liquefied by increasing the pressure, and that, the pressure remaining constant, it is brought into the liquid state by diminishing the temperature.

Unsaturated vapours behave in all respects like gases. For the gaseous form is accidental, and is not inherent in the nature of the substance. At ordinary temperatures sulphurous anhydride is a gas, while in countries near the poles it is a liquid; in temperate climates ether is a liquid, at a tropical heat it is a gas. And just as unsaturated vapours may be brought to the state of saturation, and then liquefied, by suitably diminishing the temperature or increasing the pressure, so by the same means gases may be liquefied. But as they are mostly very far removed from this state of saturation, great cold and pressure are required. Some of them may indeed be liquefied either by cold or by pressure; for the majority, however, both agencies must be simultaneously employed. The recent researches of Cailletet and of Pictet (382) have shown that the distinction *permanent* gas no longer exists, now that all are liquefied.

We have seen that there is for each gas a *critical* temperature (370), so

that no pressure however great can liquefy a gas which is above this temperature. If a gas is below this point, then the nearer it is to it the greater is the pressure required ; conversely, if the temperature is very low, the pressure required to liquefy it may be low too.

Faraday was the first to liquefy some of the gases. His method consists in enclosing in a bent glass tube (fig. 354) substances by whose chemical action the gas to be liquefied is produced, and then sealing the shorter leg. In proportion as the gas is disengaged its pressure increases, and it ultimately liquefies and collects in the shorter leg, more especially if its condensation is assisted by placing the shorter leg in a freezing mixture. A small manometer may be placed in the apparatus to indicate the pressure.

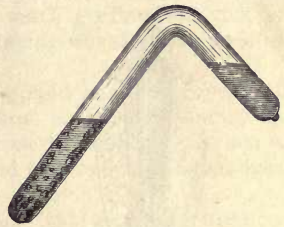


Fig. 354.

Cyanogen gas is readily liquefied by heating cyanide of mercury in a bent tube of this description ; other gases have been condensed by taking advantage of special reactions, the consideration of which belongs rather to chemistry than to physics. For example, chloride of silver absorbs about 200 times its volume of ammoniacal gas ; when the compound thus formed is placed in the long leg of a bent tube and gently heated, while the shorter leg is immersed in a freezing mixture, a quantity of liquid ammoniacal gas speedily collects in the shorter leg.

381. **Apparatus to liquefy and solidify gases.**—Thilorier first constructed an apparatus by which considerable quantities of carbonic acid gas could be liquefied. Its principle is the same as that used by Faraday in working with glass tubes ; the gas is generated in an iron cylinder, and passes through a metal tube into another similar cylinder, where it condenses. The use of this apparatus is not free from danger ; many accidents have already happened with it, and it has been superseded by an apparatus constructed by Natterer, of Vienna, which is both convenient and safe.

A perspective view of the apparatus, as modified by Bianchi, is represented in fig. 356, and a section on a larger scale in fig. 355. It consists of a wrought-iron reservoir A, of something less than a quart capacity, which can resist a pressure of more than 600 atmospheres. A small force-pump is screwed on the lower part of this reservoir. The piston rod *t* is moved by the crank-rod E, which is worked by the handle M. As the compression of the gas and the friction of the piston produce a considerable disengagement of heat, the reservoir A is surrounded by a copper vessel, in which ice or a freezing mixture is placed. The water arising from the melting of the ice passes by a tube *m* into a cylindrical copper case C, which surrounds the force-pump, from whence it escapes through the tube *n* and the stopcock *o*. The whole arrangement rests on an iron frame, PQ.

The gas to be liquefied is previously collected in airtight bags R, from whence it passes into a bottle V, containing some suitable drying substance ; it then passes into the condensing pump through the vulcanised india-rubber tube H. After the apparatus has been worked for some time the reservoir A can be unscrewed from the pump without any escape of the liquid, for it is

closed below by a valve S (fig. 355). In order to collect some of the liquid gas, the reservoir is inverted, and on turning the stopcock r the liquid escapes by a small tubulure x.

When carbonic acid has been liquefied and is allowed to escape into the air, a portion only of the liquid volatilises; in consequence of the heat absorbed by this evaporation, the rest is so much cooled as to solidify in white flakes like snow or anhydrous phosphoric acid. This may be collected by placing a stout woollen bag like a tobacco pouch over a pipe attached to the

Fig. 355

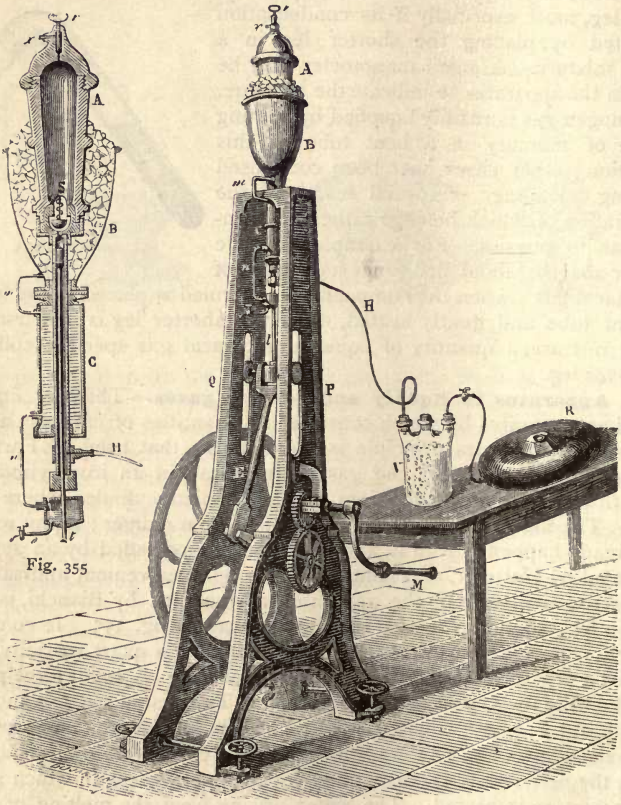


Fig. 356.

tube x; if the porous mass is compressed or hammered in stout wooden cylinders, sticks of solid carbonic acid are obtained, very like chalk in appearance.

Solid carbonic acid evaporates very slowly. By means of an alcohol thermometer its temperature has been found to be about $-90°$. A small quantity placed on the hand does not produce the sensation of such great cold as might be expected. This arises from the imperfect contact. But if

the solid be mixed with ether the cold produced is so intense that when a little is placed on the skin all the effects of a severe burn are produced. A mixture of these two substances solidifies four times its weight of mercury in a few minutes. When a tube containing liquid carbonic acid is placed in this mixture, the liquid becomes solid and looks like a transparent piece of ice.

The most remarkable liquefaction obtained by this apparatus is that of nitrous oxide. The gas once liquefied only evaporates slowly, and produces a temperature of 88° below zero. Mercury placed in it in small quantities instantly solidifies. The same is the case with water; it must be added drop by drop, otherwise, its latent heat being much greater than that of mercury, the heat given up by the water in solidifying would be sufficient to cause an explosion of the nitrous oxide.

Nitrous oxide is readily decomposed by heat, and has the property of supporting the combustion of bodies with almost as much brilliancy as oxygen; and even at low temperatures it preserves this property. When a piece of incandescent charcoal is thrown on liquid nitrous oxide, it continues to burn with a brilliant light.

The cold produced by the evaporation of ether (373) has been used by Loir and Drion in the liquefaction of gases. By passing a current of air from a blowpipe bellows through several tubes into a few ounces of ether, a temperature of $-34°$ C. can be reached in five or six minutes, and may be kept up for fifteen or twenty minutes. By evaporating liquid sulphurous acid in the same manner a great degree of cold, $-50°$ C., is obtained. At this temperature ammoniacal gas may be liquefied. By rapidly evaporating liquid ammonia under the air-pump, in the presence of sulphuric acid, a temperature of $-87°$ is attained, which is found sufficient to liquefy carbonic acid under the ordinary pressure of the atmosphere.

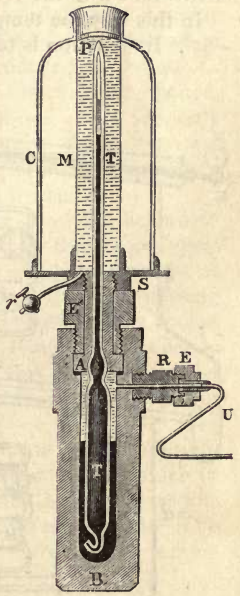


Fig. 357.

382. **Cailletet's and Pictet's researches.**— Cailletet and Pictet, working independently, but simultaneously, have effaced the old distinction between permanent and non-permanent gases, by effecting the liquefaction of oxygen and hydrogen, and other gases which it was supposed could not be condensed. This has been accomplished by means of powerful material appliances directed with great skill and ingenuity. The critical temperature of these gases is mostly below $-100°$, while their critical pressure is somewhat less than that of carbonic acid, excepting hydrogen, which is over 100 atmospheres.

The essential parts of Cailletet's apparatus are represented in fig. 357. The gas to be condensed is contained in the tube TP, which is fitted by means of a bronze screw A, into a strong wrought-iron mercury bath B. By means of a screw RE, and a tube U, this is connected with a hydraulic or a screw press not represented in the figure. The

capillary part P of the tube T is placed in a vessel M, in which it can be surrounded by a freezing mixture, and this again is surrounded by a stout safety bell-jar C.

When a pressure of 250 to 300 atmospheres is applied by means of the hydraulic press, after waiting until the heat due to the compression has disappeared, if a screw arranged in the press is suddenly opened, the pressure being diminished, the cold produced by the sudden expansion of the gas in the tube TP is so great as to liquefy a portion of the rest, as is shown by the production of a mist.

This observation was first made with nitric oxide, but similar results have been obtained with marsh gas, carbonic acid, and oxygen.

The principle of Pictet's method is that of liberating the gas under great pressure, combined with the application of great degrees of cold. The essential parts of the apparatus are the following:—Two double-acting pumps, A and B (fig. 358), are so coupled together that they cause the evaporation of liquid sulphurous acid contained in the annular receiver C. By the play of the pumps the gas thus evaporated is forced into the receiver D, where it is cooled by a current of water, and again liquefied under a pressure of three atmospheres. Thence it passes again by the narrow tube *d* to the receiver C, to replace that which is evaporated.

In this way the temperature of the liquid sulphurous acid is reduced to −65°. Its function is to produce a sufficient quantity of liquid carbonic acid, which is then submitted to a perfectly analogous process of rarefaction and condensation. This is effected by means of two similar pumps, E and F. The carbonic acid gas, perfectly pure and dry, is drawn from a reservoir through a tube not represented in the figure, and is forced into the condenser K, which is cooled by the liquid sulphurous acid to a temperature of −65°, and is there liquefied.

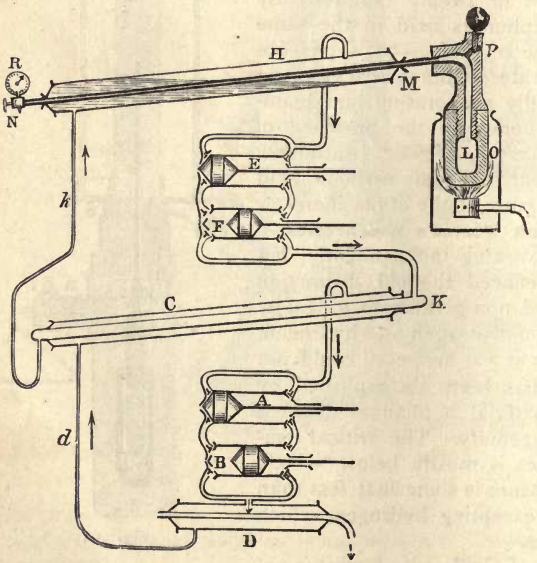


Fig. 358.

H is a tube of stout copper in connection with the condenser K by a narrow tube *k*. When a sufficient quantity of carbonic acid has been liquefied, the connection with the gasholder is cut off, and by working the pumps E and F a vacuum is created over the liquid carbonic acid in H, which produces so great a cold as to solidify it.

L is a stout wrought-iron retort capable of standing a pressure of 1,500 atmospheres. In it are placed the substances by whose chemical actions the gas is produced: potassium chlorate in the case of oxygen. This retort is closed by a strong copper tube in which the actual condensation is effected, near the end of which is a specially constructed manometer R, and which is closed by a stopcock N.

When the four pumps are set in action, for which a steam-engine of 15 horse-power is required, heat is applied to the retort. Oxygen is liberated in a calculated quantity, the temperature of the retort being about 485°. Towards the close of the decomposition the manometer indicates a pressure of 500 atmospheres, and then sinks to 320. This diminution is due to the condensation of gas, and at this stage the tube contains liquefied oxygen. If the cock N is opened, the gas issues with violence, having the appearance of a dazzling white pencil. This lasts three or four seconds. On closing the stopcock the pressure, which had diminished to 400 atmospheres, now rises, and again becomes stationary, proving that the gas is once more being condensed. The density of liquid oxygen has been found to be 0·9.

The phenomena presented by the jet of oxygen when viewed by the electric light showed that the light it emits was partially polarised, indicating a probable transient crystallisation of the gas.

For hydrogen the gas was disengaged by heating a mixture of potassic formate and hydrate, and liquid protoxide of nitrogen was used instead of carbonic acid by which the temperature could be reduced to −140° C. When the pressure had reached 650 atmospheres, and the cock was opened, a steel-blue jet issued from the aperture with a brisk noise. This suddenly became intermittent, and resembled a shower of hailstones. As the separate granules struck the ground they produced a loud noise, and Pictet considers that in all probability the hydrogen in the interior was frozen.

In some later experiments, the details of which are too complicated to give here, Cailletet has produced very low temperatures by the use of liquid ethylene gas. This gas can be liquefied by a pressure of 45 atmospheres at a temperature of 1°. By promoting the evaporation of this liquid, by passing through it a current of air or of hydrogen which has been previously cooled by the rapid evaporation of chloride of methyle, the temperature is easily reduced to −120°. When oxygen gas is cooled to this temperature the application of pressure is sufficient to resolve it into a colourless, transparent liquid, sharply separated from the gas by a meniscus.

By surrounding the gas under experiment with concentric tubes containing liquid oxygen that boils under the atmospheric pressure at −181°, which in turn is surrounded by liquid ethylene, Olszewski obtained temperatures low enough to solidify nitrogen, carbonic oxide, marsh gas and nitric oxide. The evaporation of solid nitrogen under a pressure of 4^{mm} produces a temperature of −225°.

MIXTURE OF GASES AND VAPOURS

383. **Laws of the mixture of gases and vapours.**—Every mixture of a gas and a vapour obeys the two following laws:—

I. *The pressure, and, consequently, the quantity, of vapour which saturates a given space are the same for the same temperature, whether this space contains a gas or is a vacuum.*

II. *The pressure of the mixture of a gas and a vapour is equal to the sum of the pressures which each would possess if it occupied the same space alone.*

These are known as *Dalton's laws*, from their discoverer, and are demonstrated by the following apparatus, which was invented by Gay-Lussac:—

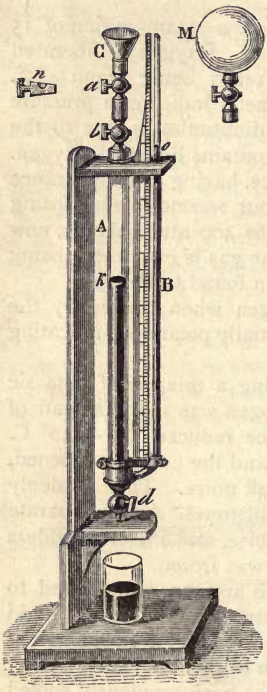


Fig. 359.

It consists of a glass tube A (fig. 359), to which two stopcocks, *b* and *d*, are cemented. The lower stopcock is provided with a tubulure which connects the tube A with a tube B of smaller diameter. A scale between the two tubes serves to measure the heights of the mercurial columns in these tubes.

The tube A is filled with mercury, and the stopcocks *b* and *d* are closed. A glass globe M, filled with dry air or any other gas, is screwed on by means of a stopcock in the place of the funnel C. All three stopcocks are then opened, and a little mercury is allowed to escape, which is replaced by the dry air of the globe. The stopcocks are then closed, and as the air in the tube expands on leaving the globe, the pressure on it is less than that of the atmosphere. Mercury is accordingly poured into the tube B until it is at the same level in both tubes. The globe is then removed, and replaced by the funnel C, provided with a stopcock *a* of a peculiar construction. It is not perforated, but has a small cavity, as represented in *n*, on the left of the figure. Some of the liquid to be vaporised is poured into C, and the height of the mercury *k* having been noted, the stopcock *b* is opened, and *a* turned so that its cavity becomes filled with liquid; being again turned, the liquid enters the space A and vaporises. The liquid is allowed to fall drop by drop until the air in the tube is saturated, which is the case when the level *k* of the mercury ceases to sink (353).

As the pressure of the vapour produced in the space A is added to that of the air already present, the total volume of gas is increased. It may easily be restored to its original volume by pouring mercury into B. When the mercury in the large tube has been raised to the level *k*, there is a difference B*o* in the level of the mercury in the two tubes, which obviously represents the pressure of the vapour; for as the air has resumed its original volume, its pressure has not changed. Now, if a few drops of the same liquid be passed into the vacuum of a barometric tube, a depression exactly equal to B*o* is produced, which proves that, for the same temperature, the pressure of a saturated vapour is the same in a gas as in a vacuum; from which it is concluded that at the same temperature the quantity of vapour is also the same.

The second law is likewise proved by this experiment, for, when the mercury has regained its level, the mixture supports the atmospheric pressure on the top of the column B, in addition to the weight of the column of mercury B*o*. But of these two pressures, one represents that of the dry air, and the other that of the vapour. The second law is, moreover, a necessary consequence of the first.

Experiments can only be made with this apparatus at ordinary temperatures ; but Regnault, by means of an apparatus which can be used at different temperatures, investigated the pressures of the vapours of water, ether, bisulphide of carbon, and benzole, both in a vacuum and in air. He found that the pressure in air is less than it is in a vacuum, but the differences are so small as not to invalidate Dalton's law. Regnault was even inclined to consider this law as theoretically true, attributing the differences which he observed to the hygroscopic properties of the sides of the tubes.

384. **Problems on mixtures of gases and vapours.**—i. A volume of dry air V, at the pressure H, being given, what will be its volume V′, when it is saturated with vapour, the temperature and the pressure remaining the same?

If F be the elastic force of the vapour which saturates the air, the latter, in the mixture, only supports a pressure equal to H − F (381). But by Boyle's law the volumes V and V′ are inversely as their pressures, consequently

$$\frac{V'}{V} = \frac{H}{H-F}, \text{ whence } V' = \frac{VH}{H-F}.$$

ii. Let V be a given volume of saturated air at the pressure H, and the temperature t ; what will be its volume V′, also saturated, at the pressure H′ and the temperature t'?

If f be the maximum pressure of aqueous vapour at $t°$, and f' its maximum pressure at $t'°$, the air alone in each of the mixtures V and V′ will be respectively under the pressures $H-f$ and $H'-f'$; consequently, assuming first that the temperature is constant, we obtain

$$\frac{V'}{V} = \frac{H-f}{H'-f'}.$$

But as the volumes V′ and V of air, at the temperatures t' and t, are in the ratio of $1+at'$ to $1+at$, a being the coefficient of the expansion of air, the equation becomes

$$\frac{V'}{V} = \frac{H-f}{H'-f'} \times \frac{1+at'}{1+at}.$$

iii. What is the weight P of a volume of air V, saturated with aqueous vapour at the temperature t and pressure H?

If F be the maximum pressure of the vapour at $t°$, the pressure of the air alone will be H − F, and the problem reduces itself to finding : 1st, the weight of V cubic inches of dry air at t, and under the pressure H − F ; and 2nd, the weight of V cubic inches of saturated vapour at $t°$ under the pressure F.

To solve the first part of the problem, we know that a cubic inch of dry

air at 0° and the pressure 760 millimetres weighs 0·31 grain, and that at $t°$, and the pressure H – F, it weighs $\frac{0{\cdot}31\ (H-F)}{(1+at)\ 760}$ (332); consequently V cubic inches of dry air weigh

$$\frac{0{\cdot}31\ (H-F)\ V}{(1+at)\ 760} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

To obtain the weight of the vapour, the weight of the same volume of dry air at the same temperature and pressure must be sought, and this is to be multiplied by the relative density of the vapour. Now as V cubic inches of dry air at $t°$, and the pressure F, weigh $\frac{0{\cdot}31 \times VF}{(1+at)\ 760}$, V cubic inches of aqueous vapour, whose density is $\frac{5}{8}$ that of air (385), weigh

$$\frac{0{\cdot}31 \times VF}{(1+at)\ 760} \times \frac{5}{8} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

and as the weight P is equal to the sum of the weights (1) and (2) we have

$$P = \frac{0{\cdot}31 \times V\ (H-F)}{(1+at)\ 760} + \frac{0{\cdot}31 \times VF}{(1+at)\ 760} \times \frac{5}{8} = \frac{0{\cdot}31 \times V}{(1+at)\ 760}(H - \tfrac{3}{8}\ F).$$

SPHEROIDAL STATE

385. **Leidenfrost's phenomena. Boutigny's experiments.**—When liquids are thrown upon incandescent metal surfaces they present remarkable phenomena, which were first observed by Leidenfrost a century ago, and have been named after their discoverer. They have since then been studied by other physicists, and more especially by Boutigny.

Figure 360 represents an interesting method of illustrating this. F is a small copper flask which is heated to dull redness over a spirit lamp, and a small quantity of boiling hot water is carefully introduced; a cork C having been loosely fitted, the lamp is removed, and in a short time steam is formed rapidly with such explosive violence as to drive out the cork.

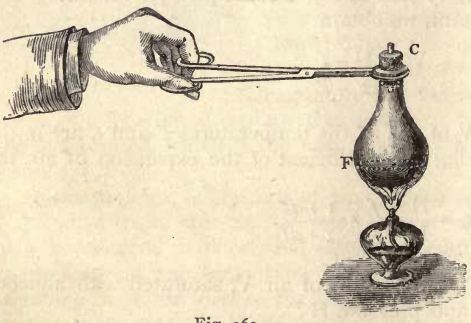


Fig. 360.

When a tolerably thick silver or platinum dish is heated to redness, and a little water, previously warmed, is dropped into the dish by means of a pipette, the liquid does not spread itself out on the dish, and does not moisten it, as it would at the ordinary temperature, but assumes the form of a flattened globule, which fact Boutigny expressed by saying that it has passed into the *spheroidal state.* It rotates rapidly round on the bottom of the dish, taking sometimes the form of a star,

and not only does it not boil, but its evaporation is only about one-fiftieth as rapid as if it boiled. As the dish cools, a point is reached at which it is not hot enough to keep the water in the spheroidal state; it is accordingly moistened by the liquid, and a violent ebullition suddenly ensues.

All volatile liquids can assume the spheroidal condition; the lowest temperature at which it can be produced varies with each liquid, and is more elevated the higher the boiling point of the liquid. For water, the dish must have at least a temperature of 200°; for alcohol, 134°; and for ether, 61°.

The temperature of a liquid in the spheroidal state is always below its boiling point. This temperature has been measured by Boutigny by means of a very delicate thermometer; but his method is not free from objections, and it is probable that the temperatures he obtained were too high. He found that of water to be 95°; alcohol, 75°; ether, 34°; and liquid sulphurous acid, – 11°. But the temperature of the vapour which is disengaged appears to be as high as that of the vessel itself.

This property of liquids in the spheroidal state remaining below their boiling point was applied by Boutigny in a remarkable experiment, that of freezing water in a red-hot crucible. He heated a platinum dish to bright redness, and placed a small quantity of liquid sulphurous acid in it. It immediately assumed the spheroidal condition, and its evaporation was remarkably slow. Its temperature, as has been stated, was about – 11° and when a small quantity of water was added, it immediately solidified, and a small piece of ice could be thrown out of the red-hot crucible. In a similar manner Faraday, by means of a mixture of solid carbonic acid and ether, succeeded in freezing mercury in a red-hot crucible.

In the spheroidal state the liquid is not in contact with the vessel. Boutigny proved this by heating a silver plate placed in a horizontal position and dropping on it a little dark-coloured water. The liquid assumed the spheroidal condition, and the flame of a candle placed at some distance could be distinctly seen between the drop and the plate (fig. 361). If a plate perforated by several fine holes be heated, a liquid will assume the spheroidal state when projected upon it. This is also the case with a flat helix of platinum wire pressed into a slightly concave shape. An experiment of another class, due to Professor Church, also illustrates the same fact. A polished silver dish is made red-hot, and a few drops of a solution of sulphide of sodium are projected on it. The liquid passes into the spheroidal condition, and the silver undergoes no alteration. But if the dish is allowed to cool, the liquid instantly moistens it, producing a dark

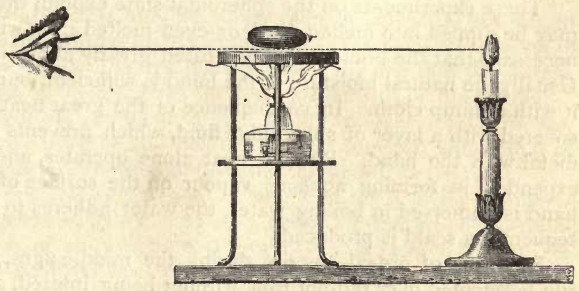


Fig. 361.

spot, due to the formation of sulphide of silver. In like manner nitric acid assumes the spheroidal state when projected on a heated silver plate, and does not attack the metal so long as the plate remains hot.

An analogous phenomenon is observed when potassium is placed on water. Hydrogen is liberated, and burns with a yellow flame ; hydrate of potassium, which is formed at the same time, floats on the surface without touching it, owing to its high temperature. In a short time it cools down, and the globule, coming in contact with water, bursts with an explosion.

Similarly, liquids may be made to roll upon liquids, and solid bodies which vaporise without becoming liquid also assume a condition analogous to the spheroidal state of liquids when they are placed on a surface whose temperature is sufficiently high to vaporise them rapidly. This is seen when a piece of carbonate of ammonium is placed in a red-hot platinum crucible.

The phenomena of the spheroidal state seem to prove that the liquid globule rests upon a sort of cushion of its own vapour, produced by the heat radiated from the hot surface against its under side. As fast as this vapour escapes from under the globule, its place is supplied by a fresh quantity formed in the same way, so that the globule is constantly buoyed up by it, and does not come in actual contact with the heated surface. When, however, the temperature of the latter falls, the formation of vapour at the under surface becomes less and less rapid, until at length it is not sufficient to prevent the globule touching the hot metal or liquid on which it rests. As soon as contact occurs, heat is rapidly imparted to the globule, it enters into ebullition and quickly boils away.

This explanation is confirmed by the experiments of Budde, who found that in an exhausted receiver water passes into the spheroidal state, even when the temperature of the support is not more than 80° or 90° ; for then the vapour has only to support the drop, and not the atmospheric pressure also.

These experiments on the spheroidal state explain the fact that the hand may be dipped into melted lead, or even melted iron, without injury. It is necessary that the liquid metal be heated greatly above its solidifying point. Usually the natural moisture of the hand is sufficient, but it is better to wipe it with a damp cloth. In consequence of the great heat the hand becomes covered with a layer of spheroidal fluid, which prevents the contact of the metal with the hand. Radiant heat alone operates, and this is principally expended in forming aqueous vapour on the surface of the hand. If the hand is immersed in boiling water, the water adheres to the flesh, and consequently a scald is produced.

The tales of ordeals by fire during the middle ages, of men who could run barefooted over red-hot iron without being injured, are possibly true in some cases, and would find an explanation in the preceding phenomena.

DENSITY OF VAPOURS

386. **Gay-Lussac's method.**—The *density of a vapour* is the relation between the weight of a given volume of this vapour and that of the same volume of air at the same temperature and pressure.

The older methods used in determining the density of vapours are :

Gay-Lussac's, which serves for liquids that boil at about 100°, and Dumas', which can be used up to 350°.

Fig. 362 represents the apparatus used by Gay-Lussac. It consists of an iron vessel containing mercury, in which there is a glass cylinder M. This is filled with water or oil, and the temperature is indicated by the thermometer T. In the interior of the cylinder is a graduated gas jar C, which at first is filled with mercury.

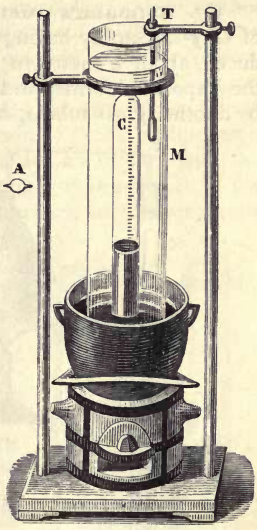


Fig. 362.

The liquid whose vapour-density is to be determined is placed in a small glass bulb A, represented on the left of the figure. The bulb is then sealed and weighed ; the weight of the liquid taken is obviously the weight of the bulb when filled, minus its weight while empty. The bulb is then introduced into the jar C, and the liquid in M gradually heated somewhat higher than the boiling point of the liquid in the bulb. In consequence of the expansion of this liquid the bulb breaks, and the liquid becoming converted into vapour, the mercury is depressed, as represented in the figure. The bulb must be so small that all the liquid in it is vaporised. The volume of the vapour is given by the graduation on the jar. Its temperature is indicated by the thermometer T, and the pressure is shown by the difference between the height of the barometer at the time of the observation and the height of the column of mercury in the gas jar. It is only necessary then to calculate the weight of a volume of air equal to that of the vapour under the same conditions of temperature and pressure. The quotient, obtained by dividing the weight of the vapour by that of the air, gives the required density of the vapour.

Let p be the weight of the vapour in grains, v its volume in cubic inches, and t its temperature ; if H be the height of the barometer, and h that of the mercury in the gas jar, the pressure on the vapour will be $H - h$.

It is required to find the weight p' of a volume of air v, at the temperature t, and under a pressure $H - h$. At zero, under a pressure of 760 millimetres, a cubic inch of air weighs 0·31 grain ; consequently, under the same conditions, v cubic inches will weigh $0{\cdot}31v$ grain. And therefore the weight of v cubic inches of air, at $t°$ and the pressure 760 millimetres, is

$$\frac{0{\cdot}31v}{1 + at} \text{ grain [332, prob. ii.]}$$

As the weight of a volume of air is proportional to the pressure, the above weight may be reduced to the pressure $H - h$ by multiplying by $\frac{H - h}{760}$, which gives

$$\frac{0{\cdot}31v\,(H - h)}{(1 + at)\,760}$$

for the weight p' of the volume of air v, under the pressure $H - h$ and at $t°$. Consequently, for the desired density we have

$$D = \frac{p}{p'} = \frac{\phi\,(1 + at)\,760}{0{\cdot}31v\,(H - h)}.$$

387. **Hofmann's method.**—Hofmann materially improved the method of Gay-Lussac by having the mercury tube *fb*, in which the vapour is produced, about a metre in length (fig. 363); it is, in fact, a barometer, and the vapour is formed in the Torricellian vacuum. This tube is surrounded by another glass tube *a*, which is connected, by a bent tube *c*, with a canister *e*, so that water, amylic alcohol, or aniline, or, indeed, any substance with a constant boiling point, may be distilled through the tube *a*, and the vapour issues by the tube *d*, which is connected with a condensing arrangement not represented in the figure. In this way more constancy in the temp ratures is ensured than with the use of a mercury bath. The liquid is contained in very minute stoppered tubes, *h*, holding from 20 to 100 milligrammes of water; the stoppers come out in the vacuum, and the tubes can be used over again.

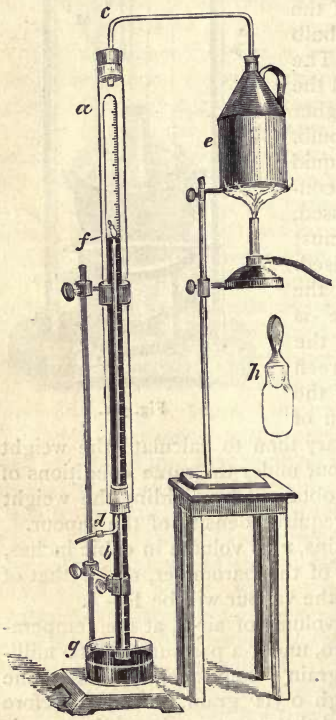


Fig. 363.

As, under the above conditions, the liquid vaporises into a vacuum, the vapour is formed under a very much lower pressure than that of the atmosphere, and therefore at a temperature much below its ordinary boiling point. Thus, the vapour-density of a body which only boils at a temperature of 150° can be determined at the temperature of boiling water. This is of great use in the case of those bodies which decompose at their boiling point under the ordinary atmospheric pressure.

388. **Dumas' method.**—The original method of Gay-Lussac cannot be applied to liquids whose boiling point exceeds 150° or 160°. In order to raise the oil in the cylinder to this temperature it would be necessary to heat the mercury to such a degree that its vapour would be dangerous to the operator. And, moreover, the pressure of the mercurial vapour in the graduated jar would add itself to that of the vapour of the liquid, and so far vitiate the result.

The following method, devised by Dumas, can be used up to the temperature at which glass begins to soften; that is, about 400°. A glass globe is used with the neck drawn out to a fine point (fig. 364). The globe,

having been dried externally and internally, is weighed, the temperature t and barometric height h being noted. This weight, W, is the weight of the glass G in addition to p, that of the air it contains. The globe is then gently warmed and its point immersed in the liquid whose vapour-density is to be determined ; on cooling, the air contracts, and a quantity of liquid enters the globe. The globe is then immersed in a bath, either of oil or fusible metal, according to the temperature to which it is to be raised. In order to keep the globe in a vertical position a metal support, on which a movable rod slides, is fixed on the side of the vessel. This rod has two rings, between which the globe is placed, as shown in the figure. There is another rod, to which a weight thermometer, D (324), is attached.

The globe and thermometer having been immersed in the bath, the latter is heated until slightly above the boiling point of the liquid in the globe. The vapour which passes out by the point expels all the air in the interior. When the jet of vapour ceases, which is the case when all the liquid has been converted into vapour, the point of the globe is hermetically sealed, the temperature of the bath t', and the barometric height h', being noted. When the globe is cooled it is carefully cleaned and again weighed. This weight, W′, is that of the glass G, plus p', the weight of the vapour which fills the globe at the temperature t', and pressure h', or $W' = G + p'$. To obtain the weight of the glass alone, the weight p of air must be known, which is determined in the following manner :—The point of the globe is placed under mercury and the end broken off with a small pair of pincers : the vapour being condensed, a vacuum is produced, and mercury rushes up, completely filling the globe, if, in the experiment, all the air has been entirely expelled. The mercury is then poured into a carefully graduated measure, which gives the volume of the globe.

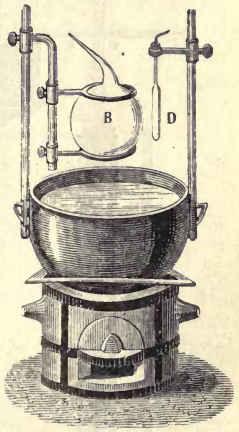


Fig. 364.

From this result, the volume of the globe at the temperature t' may be easily calculated, and consequently the volume of the vapour. From this determination of the volume of the globe, the weight p of the air at the temperature t and pressure h is readily calculated, and this result subtracted from W gives G, the weight of the glass. Now the weight of the vapour p' is $W' - G$. We now know the weight p' of a given volume of vapour at the temperature t' and pressure h', and it is only necessary to calculate the weight p'' of the same volume of air under the same conditions, which is easily accomplished. The quotient $\frac{p'}{p''}$ is the required density of the vapour.

Deville and Troost modified Dumas' method so that it can be used for determining the vapour density of liquids with very high boiling points. The globe is heated in an iron cylinder in the vapour of mercury or of sulphur, the temperatures of which are constant respectively at 350° and 440°. In other respects the determination is the same as in Dumas' method.

For determinations at higher temperatures, Deville and Troost employed

the vapour of zinc, the temperature of which is 940°. As glass vessels are softened by this heat, they used porcelain globes with finely-drawn out necks, which are sealed by means of the oxyhydrogen flame.

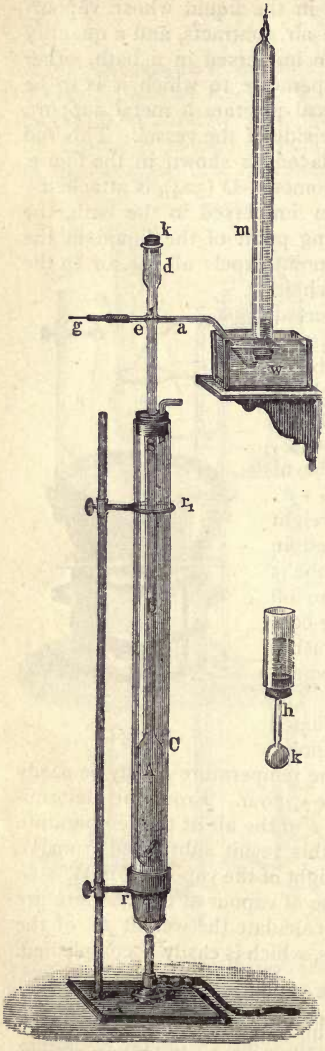


Fig. 365.

In the case of substances having a high boiling point, Victor Meyer has advantageously used a non-volatile substance, Wood's fusible alloy, which melts at 70°, instead of mercury. Habermann has introduced into Dumas' method Hofmann's modification of Gay-Lussac's, by connecting the open end of the vessel B (fig. 364) with a space in which a partial vacuum is made. Thus the vapour-density can be determined for temperatures far below the boiling point.

A method of determining vapour-density, much in use, is that devised by Victor Meyer. A long glass tube *b* with an enlargement *d* at the top is fused to the cylindrical vessel C, which has a capacity of about 100 cc. Near the top of *b* a gas delivery tube, *a*, is fused, and opposite this another tube, *e*, closed by a caoutchouc tube in which a metal rod, *g*, can be moved airtight. On this rests a small thin vessel such as *h* (fig. 365), containing the substance to be determined. For very volatile liquids small bulbs, *k*, of very thin glass are used; they are fitted by means of a cork in a tube in which is placed the liquid. By warming, air is driven out of *k*, and on cooling is filled with liquid and the end *f* is fused.

The cork *k* being inserted, the whole apparatus is suspended in the long glass vessel C, which contains a liquid of constant boiling point such as aniline or diphenyle. This is heated until it boils constantly, which is known when no air bubbles issue from the delivery tube *a*. When this is attained a graduated tube, *m*, filled with water is pushed over the open end of the tube *a*. The rod *g* is then withdrawn and the small vessel falls and is broken, some asbestos being placed at the bottom to prevent a possible breakage of the vessel A itself. The vapours from C issue through the tube, and are connected with a condensing arrangement not shown in the figure. When the substance vaporises a corresponding volume of air issues and is collected in the tube. When no more issues the tube is placed in

a cylinder of water and is depressed until the level inside and outside is the same. The volume v is read off, and also the temperature of the water t, which is also that of the room, and the barometric height H. These data, together with the weight of the substance p, and h the pressure of aqueous vapour at $t°$, enable us to calculate the density from the formula

$$D = \frac{p}{p'} = \frac{p \times 760\,(273+t)}{v\,(0{\cdot}001293)\,(H-h)\,273} = \frac{p\,(273+t)\,2153}{v\,(H-h)}$$

Thus neither the capacity of the vessel b nor the temperature of the vapour need be known, unless it be desired to investigate in what respect the density varies with the temperature; the volume of the vapour is obtained in the form of an equal volume of air measured at the temperature of the room.

389. **Relation of vapour-density to molecular weight. Dissociation.**—The densities of vapours, determined at temperatures a few degrees above their boiling points, and when they may be considered as perfect gases, are governed by a simple but very important law, that *the densities of vapours are proportional to their molecular weights.* If both densities and molecular weights are referred to the same standard, that of hydrogen being taken as 2 for instance, the *vapour-densities are equal to the molecular weights.* If the density of air is taken at 1, that of hydrogen is $0{\cdot}0693 = \frac{1}{28{\cdot}86}$, and hence for all other gases and superheated vapours the density is $\frac{1}{28{\cdot}86}$ of the molecular weight.

This law is of great importance in chemistry and in fixing the molecular weights of bodies, more especially in organic chemistry. In some cases exceptions are met with; these, when small, may be ascribed to imperfection of the gaseous state. A more important cause is the following:—When sal-ammoniac, NH_4Cl, for instance, is strongly heated, it is resolved into ammonia NH_3, and hydrochloric acid, HCl, and it then occupies a volume double that required by the law. But there is a partial decomposition even at lower temperatures, so that the vapour consists of molecules of sal-ammoniac, mixed with molecules of free hydrochloric acid and of free ammonia. In such cases the vapour-density is said to be *abnormal*; and this partial decomposition, in which there is a mixture of undecomposed and of decomposed molecules, is spoken of as *dissociation.* Thus, sulphuric acid, SO_4H_2, at 325°, consists of about one half undecomposed molecules, while the other moiety decomposes into sulphuric anhydride, SO_3, and water, H_2O. The dissociation of water begins at 1200° C., and is complete at 2500°. Stannic chloride, $SnCl_2$, has the theoretical density 6·53. At 600° the density corresponds to the formula Sn_2Cl_4; as the temperature rises this diminishes, and at 1000° it represents the formula $SnCl_2$. It appears thus that the vapour exists in two forms: at 600° as Sn_2Cl_4, and at 1000° as $SnCl_2$; as the temperature rises from 600° the molecule Sn_2Cl_4 is more and more dissociated, until at 1000° it is completely resolved into 2 $SnCl_2$.

Dissociation does not take place suddenly, but gradually; it increases with the temperature, and is limited by the tendency of the components to recombine; for each temperature the quantity dissociated is in a constant ratio to the whole. As the temperature sinks, the bodies again recombine, and at the initial temperature the body is in its original state. In this

respect dissociation differs from decomposition. The temperature at which the decomposition is half complete is taken as that of dissociation.

Dissociation is also met with in elementary bodies; thus at a temperature of 500° C. sulphur has the vapour density 96 (H = 1), representing a molecular weight of 192; as the temperature increases this becomes less, and from 1000° it is constant, being then 32, which is normal, corresponding to a molecular weight of 64. At the lower temperature the molecule is considered to be an aggregate consisting of six atoms or three molecules, while at higher temperatures this complex splits up, and at 1000° consists of the normal diatomic molecule. In like manner the density of iodine vapour, which up to 600° is 8·716, is only 4·5, or about half as much, at 1500°, but then remains constant. This probably represents a dissociation of the iodine molecule, I_2, into two atoms.

Densities of vapours

Air	1·0000	Vapour of phosphorus	4·3256
Vapour of water	0·6225	„ turpentine	5·0130
„ alcohol	1·6138	„ anthracene	6·0100
„ acetic acid	2·0800	„ sulphur	6·6542
„ carbon bisulphide	2·4476	„ mercury	6·9760
„ ether	2·5860	„ chrysene	8·1200
„ benzole	2·7290	„ iodine	8·7160
„ aniline	3·3100	„ perchloriphenyle	17·4300

The density of aqueous vapour, when a space is saturated with it, is at all temperatures $\frac{5}{8}$, or, more accurately, 0·6225, of the density of air at the same temperature and pressure.

390. **Relation between the volume of a liquid and that of its vapour.**—The density of vapour being known, we can readily calculate the ratio between the volume of a vapour in the saturated state at a given temperature and that of its liquid at zero. We may take as an example the relation between water at zero and steam at 100°.

The ratio between the weights of equal volumes of air at zero, and the normal barometric pressure, and of water under the same circumstances, is as 1 : 773. But from what has been already said (332), the density of air at zero is to its density at 100° as $1 + at$: 1. Hence the ratio between the weights of equal volumes of air at 100° and water at 0° is

$$\frac{1}{1 + 0{\cdot}003665 \times 100} : 773, \text{ or } 0{\cdot}73178 : 773.$$

Now from the above table the density of steam at 100° C., and the normal pressure, compared with that of air under the same circumstances, is as 0·6225 : 1. Hence the ratio between the weights of equal volumes of steam at 100° and water at 0° is

$$0{\cdot}73178 \times 0{\cdot}6225 : 773, \text{ or } 0{\cdot}4555 : 773, \text{ or } 1 : 1698.$$

Therefore, as the volumes of bodies are inversely as their densities, one volume of water at zero expands into 1,698 volumes of steam at 100° C. The practical rule, that a cubic inch of water yields a cubic foot of steam, though not quite accurate, expresses the relation in a convenient form.

CHAPTER VI

HYGROMETRY

391. **Province of hygrometry.**—The province of *hygrometry* is to determine the quantity of aqueous vapour contained in a given volume of air. This quantity is very variable ; but the atmosphere is seldom or never completely saturated with vapour, even in our climate. Nor is it ever completely dry ; for if *hygrometric substances*—that is to say, substances with a great affinity for water, such as chloride of calcium, sulphuric acid, &c.—be at any time exposed to the air, they absorb aqueous vapour.

392. **Hygrometric state.**—As the air is, in general, never saturated, the ratio of the quantity of aqueous vapour actually present in the atmosphere to that which it would contain if it were saturated, the temperature remaining the same, is called the *hygrometric state*, or *degree of saturation.*

The *absolute moisture* is measured by the weight of water actually present in the form of vapour in the unit of volume.

We say the 'air is dry' when water evaporates and moist objects dry rapidly ; and the 'air is moist' when they do not dry rapidly, and when the least lowering in temperature brings about deposits of moisture. The air is dry or moist according as it is more or less distant from its point of saturation. Our judgment is, in this respect, independent of the absolute quantity of moisture in the air. Thus, if in summer, at a temperature of 25° C., we find that each cubic metre of air contains 13 grammes of vapour, we say it is very dry, for at this temperature it could contain 22·5 grammes. If, on the other hand, in water we find that the same volume contains 6 grammes, we call it moist, for it is nearly saturated with vapour, and the slightest diminution of temperature produces a deposit. When a room is warmed, the quantity of moisture is not diminished, but the humidity of the air is lessened, because its point of saturation is raised. The air may thus become so dry as to be injurious to the health, and hence it is usual to place vessels of water on the stoves used for heating.

As Boyle's law applies to non-saturated vapours as well as to gases (354), it follows that, with the same temperature and volume, the weight of vapour in an unsaturated space increases with the pressure, and therefore with the pressure of the vapour itself. Instead, therefore, of the ratio of the quantities of vapour, that of the corresponding pressures may be substituted, and it may be said that the hygrometric state is *the ratio of the elastic force of the aqueous vapour which the air actually contains, to the elastic force of the vapour which it would contain at the same temperature if it were saturated.*

If f is the actual pressure of aqueous vapour in the air, and F that of

saturated vapour at the same temperature, and E the hygrometric state, we have $E = \frac{f}{F}$; whence $f = F \times E$.

As a consequence of this second definition, it is important to notice that, the temperature having varied, the air may contain the same quantity of vapour and yet not have the same hygrometric state. For, when the temperature rises, the pressure of the vapour which the air would contain, if saturated, increases more rapidly than the pressure of the vapour actually present in the atmosphere, and hence the ratio between the two forces—that is to say, the hygrometric state—becomes smaller.

Jamin proposes to replace this ratio $\frac{f}{F}$, which expresses the relative moisture, by the ratio $\frac{f}{H-f}$, in which H is the barometric height; he calls this the *hygrometric richness*, and contends that it brings out changes in the quantity of moisture present in the air with greater distinctness.

It will presently be explained (401) how the weight of the vapour contained in a given volume of air may be deduced from the hygrometric state.

393. **Different kinds of hygrometers.**—*Hygrometers* are instruments for measuring the hygrometric state of the air. There are numerous varieties of them—chemical hygrometers, condensing hygrometers, and psychrometers.

394. **Chemical hygrometer.**—The method of the chemical hygrometer consists in passing a known volume of air over a substance which readily

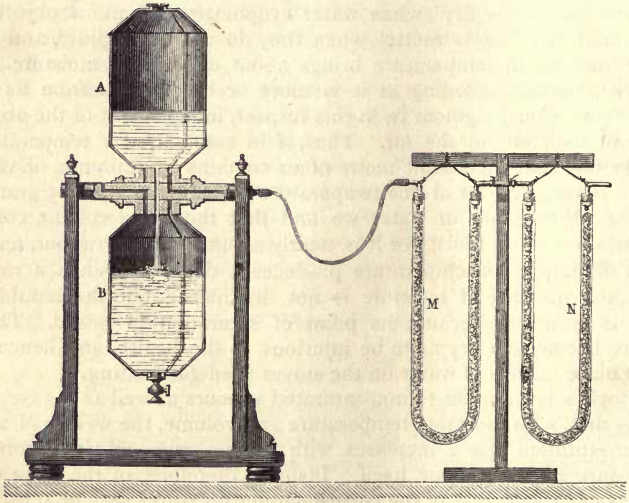


Fig. 366.

absorbs moisture—chloride of calcium, for instance. The substance having been weighed before the passage of air, and then afterwards, the increase in weight represents the amount of aqueous vapour present in the air. By means of the apparatus represented in fig. 366 it is possible to examine any

given volume of air. Two brass reservoirs, A and B, of the same size and construction, act alternately as aspirators, by being fixed to the same axis, about which they can turn. They are connected by a central tubulure, and by means of two tubulures in the axis the lower reservoir is always in connection with the atmosphere, while the upper one, by means of a caoutchouc tube, is connected with two tubes M and N, filled either with chloride of calcium, or with pumice-stone impregnated with sulphuric acid. The first absorbs the vapours in the air drawn through, while the other, M, stops any vapour which might diffuse from the reservoirs into the tube N.

The lower reservoir being full of water, and the upper one of air, the apparatus is inverted so that the liquid flows slowly from A to B. A partial vacuum being formed in A, air enters by the tubes NM, in the first of which all the vapour is absorbed. When all the water is run into B it is inverted; the same flow recommences, and the same volume of air is drawn through the tube N. Thus, if each reservoir holds a gallon, for example, and the apparatus has been turned five times, 6 gallons of air have traversed the tube N, and have been dried. If then, before the experiment, the tube with its contents has been weighed, the increase of weight gives the weight of aqueous vapour present in 6 gallons of air at the time of the experiment.

Edelmann has devised a new form of hygrometer, the principle of which is to enclose a given volume of air, and then to absorb the aqueous vapour present by means of strong sulphuric acid; in this way a diminution in the pressure is produced which is determined, and which is a direct measure of the pressure f of the aqueous vapour previously present.

Similar apparatus have been devised by Rudorff and by Neesen.

395. **Condensing hygrometers.**—When a body gradually cools in a moist atmosphere—as, for instance, when a lump of ice is placed in water contained in a polished metal vessel—the layer of air in immediate contact with it cools also, and a point is ultimately reached at which the vapour present is just sufficient to saturate the air; the least diminution of temperature then causes a precipitation of moisture on the vessel in form of dew. When the temperature rises again, the dew disappears. The mean of these two temperatures is taken as the *dew-point*, and the object of *condensing* hygrometers is to observe this point. Daniell's and Regnault's hygrometers belong to this class.

396. **Daniell's hygrometer.**—This consists of two glass bulbs at the extremities of a glass tube bent twice (fig. 367). The bulb A is two-thirds full of ether, and a very delicate thermometer plunged in it; the rest of the space contains nothing but the vapour of ether, the ether having been boiled before the bulb B was sealed. The bulb B is covered with muslin, and ether is dropped upon it. The ether in evaporating cools the bulb, and the vapour contained in it is condensed. The internal pressure being thus diminished, the ether in A forms vapour which condenses in the other bulb B. In proportion as the liquid distils from the lower to the upper bulb, the ether in A becomes cooler, and ultimately the temperature of the air in immediate contact with A sinks to that point at which its vapour is more than sufficient to saturate it, and it is, accordingly, deposited on the outside as a ring of dew corresponding to the surface of the ether. The temperature of this point is noted by means of the thermometer in the inside. The addition of ether to

the bulb B is then discontinued, the temperature of A rises, and the temperature at which the dew disappears is noted. In order to render the deposition of dew more perceptible, the bulb A is made of black glass.

These two points having been determined, their mean is taken as that of the dew-point. The temperature of the air at the time of the experiment is indicated by the thermometer on the stem. The pressure f, corresponding to the temperature of the dew-point, is then found in the table of pressures (358). This pressure is exactly that of the vapour present in the air at the time of the experiment. The pressure F of vapour saturated at the temperature of the atmosphere is found by means of the same table; the quotient obtained by dividing f by F represents the hygrometric state of the air (392). For instance, the temperature of the air being 15°, suppose the dew-point is 5°. From the table the corresponding pressures are $f = 6{\cdot}534$ millimetres, and $F = 12{\cdot}699$ millimetres, which gives 0·514 for the ratio of f to F, or the hygrometric state.

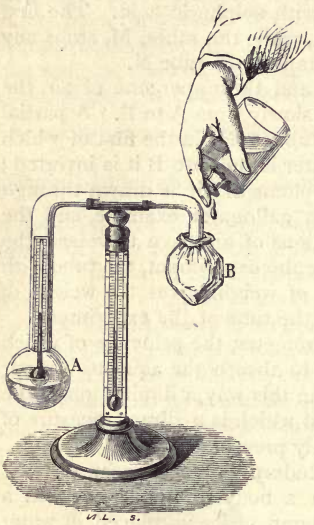


Fig. 367.

There are many sources of error in Daniell's hygrometer. The principal are: 1st, that as the evaporation in the bulb A only cools the liquid on the surface, the thermometer dipping on it does not exactly give the dew-point; 2nd, that the observer standing near the instrument modifies the hygrometric state of the surrounding air, as well as its temperature; the cold ether vapour also flowing from the upper bulb may cause inaccuracy.

397. **Regnault's hygrometer.**—Regnault's hygrometer is free from the sources of error incidental to the use of Daniell's. It consists of two very thin polished silver thimbles 1·75 inch in height, and 0·75 inch in diameter (fig. 368). In these are fixed two glass tubes, D and E, in each of which is a thermometer. A bent tube, A, open at both ends, passes through the cork of the tube D, and reaches nearly to the bottom of the thimble. There is a tubulure on the side of D, fitting in a brass tube which forms a support for the apparatus. The end of this tube is connected with an aspirator G. The tube E is not connected with the aspirator; its thermometer simply indicates the temperature of the atmosphere.

The tube D is then half filled with ether, and the stopcock of the aspirator opened. The water contained in it runs out, and just as much air enters through the tube A, bubbling through the ether, and causing it to evaporate. This evaporation produces a diminution of temperature, so that dew is deposited on the silver just as on the bulb in Daniell's hygrometer; the thermometer T is then instantly to be read, and the stream from the aspirator stopped. The dew will soon disappear again, and the thermometer T is

again to be read ; the mean of the two readings is taken ; the thermometer *t* gives the corresponding temperature of the air, and hence there are all the elements necessary for calculating the hygrometric state.

As all the ether in this instrument is at the same temperature in consequence of the agitation, and the temperatures may be read off at a distance by means of a telescope, the sources of error in Daniell's hygrometer are avoided.

A much simpler form of the apparatus may be constructed out of a common test-tube containing a depth of 1½ inch of ether. The tube is provided with a loosely fitting cork in which are a delicate thermometer and a narrow bent tube dipping in the ether. On blowing into the ether through a caoutchouc tube of considerable length, a diminution of temperature is caused, and dew is ultimately deposited on the glass ; after a little practice the whole process can be conducted almost as well as with Regnault's more complete instrument. The temperature of the air is indicated by a detached thermometer.

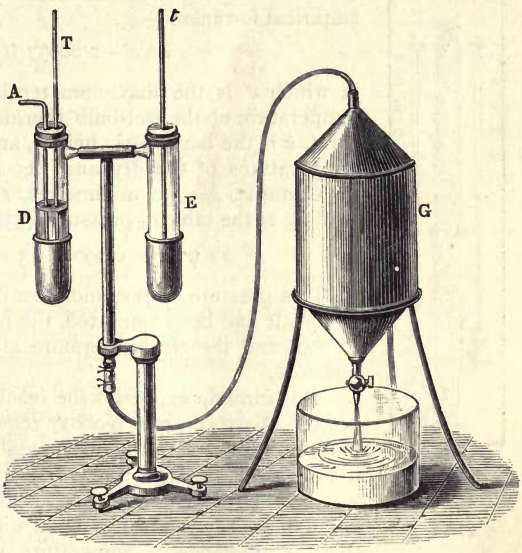


Fig. 368.

397*a*. **Dines' hygrometer.**—Dines constructed a hygrometer which is also one of condensation, but which dispenses with the use of such volatile liquids as ether. The principle of this instrument is to have a thin flat metal box, through which a small stream of cooled water is allowed to flow for a few seconds. The dew is deposited on the top of the box, which is of thin dark polished metal. By alternately stopping the flow and allowing it to continue, the disappearance and formation of the dew may be very accurately observed, and the corresponding temperatures read off by a delicate thermometer placed inside.

398. **Psychrometer. Wet-bulb hygrometer.**—A moist body evaporates in the air more rapidly in proportion as the air is drier, and the temperature of the body sinks in consequence of this evaporation. The *psychrometer*, or *wet-bulb hygrometer*, is based on this principle, the application of which to this purpose was first suggested by Leslie. The form usually adopted in this country is due to Mason. It consists of two delicate thermometers placed on a wooden stand (fig. 369). One of the bulbs is covered with muslin, and is kept continually moist by being connected with a reservoir of water

by means of a string. Unless the air is saturated with moisture the wet-bulb thermometer always indicates a lower temperature than the other, and the difference between the indications of the two thermometers is greater in proportion as the air can take up more moisture. The tension e of the aqueous vapour in the atmosphere may be calculated from the indications of the two thermometers by means of the following empirical formula :—

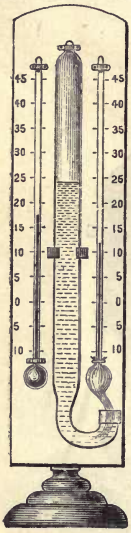


Fig. 369.

$$e = e' - 0{\cdot}00077\ (t - t')h,$$

in which e' is the maximum tension corresponding to the temperature of the wet-bulb thermometer, 0·00077 is a constant, h is the barometric height, and t and t' the respective temperatures of the dry and wet bulb thermometers. If, for example, $h = 750$ millimetres, $t = 15°$ C., $t' = 10°$ C. ; according to the table of pressures (358), $e' = 9{\cdot}165$, and we have

$$e = 9{\cdot}165 - 0{\cdot}00077 \times 5 \times 750 = 6{\cdot}342.$$

This pressure corresponds to a dew-point of about 4·5° C. If the air had been saturated, the pressure would have been 12·699, and the air is therefore about half saturated with moisture.

This formula expresses the result with tolerable accuracy, but the above constant 0·00077 requires to be controlled for different positions of the instrument ; in small closed rooms it is 0·00128, in large rooms it is 0·00100, and in the open air without wind it is 0·00090 : the number 0·00077 is its value in a large room with open windows. Regnault found that the difference in temperature of the two bulbs depends on the rapidity of the current of air ; he also found that at a low temperature, and in very moist air, the results obtained with the psychrometer differed from those yielded by his hygrometer. It is probable that the indications of the psychrometer are only true for mean and high temperatures, and when the atmosphere is not too moist.

A formula frequently used in this country is that given by Dr. Apjohn. It is

$$F = f - \frac{d}{88} \times \frac{h}{30}, \text{ or } F = f - \frac{d}{96} \times \frac{h}{30}$$

in which d is the difference of the wet and dry bulb thermometers in *Fahrenheit* degrees ; h the barometric height in *inches* ; f the pressure of vapour for the temperature of the *wet bulb*, and F the pressure of vapour at the dew-point, from which the dew-point may, if necessary, be found from the tables. The constant coefficient 88, for the specific heats of air and aqueous vapour, is to be used when the reading of the wet bulb is above 32° F., and 96 when it is below.

According to Glaisher the temperature of the dew-point may be obtained by multiplying the difference between the temperatures of the wet and dry bulb by a constant depending on the temperature of the air at the time of observation, and subtracting the product thus obtained from this last-named temperature.

The following table gives the numbers, which are known as *Glaisher's factors.*

Dry bulb Temperature F.°	Factor	Dry bulb Temperature F.°	Factor
Below 24°	8·5	34 to 35	2 8
24 to 25	6·9	35—40	2·5
25—26	6·5	40—45	2·2
26—27	6·1	45—50	2·1
27—28	5·6	50—55	2·0
28—29	5·1	55—60	1·9
29—30	4·6	60—65	1·8
30—31	4·1	65—70	1·8
31—32	3·7	70—75	1·7
32—33	3·3	75—80	1·7
33—34	3·0	80—85	1·6

399. **Absorption hygrometers.**—These hygrometers are based on the property which organic substances have of elongating when moist, and of again contracting as they become dry. The most common form is the *hair* or *Saussure's hygrometer.*

It consists of a brass frame (fig. 370), on which is fixed a hair, *c*, fastened at the top in a clamp, *a*, provided with a screw, *d*. This clamp is moved by a screw, *b*. The lower part of the hair passes round a pulley, *o*, and supports a small weight, *p*. On the pulley there is a needle, which moves along a graduated scale. When the hair becomes shorter the needle rises, when it becomes longer the weight *p* makes it sink.

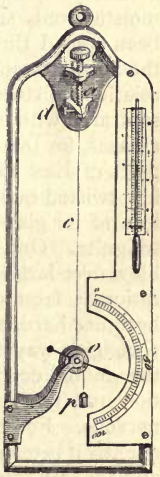


Fig. 370.

The scale is graduated by calling that point zero at which the needle would stand if the air were completely dry, and 100 the point at which it stands in air completely saturated with moisture. The distance between these points is divided into 100 equal degrees.

Regnault devoted much study in order to render the hair hygrometer scientifically useful, but without much success. The utmost that can be claimed for it is that it can be used as a *hygroscope*; that is, an instrument which shows approximately whether the air is more or less moist, without giving any indication as to the quantity of moisture present. To this class of hygroscopes belong the chimney ornaments, one of the most common forms of which is that of a small male and female figure, so arranged in reference to a little house with two doors, that when it is moist the man goes out and the woman goes in, and *vice versâ* when it is fine. They are founded on the property which twisted strings or pieces of catgut possess of untwisting when moist, and of twisting when dry. As these hygroscopes only change slowly, their indications are always behindhand with the state of the weather; nor are they, moreover, very exact.

A strip of drawing-paper, coated on one side with gelatine and varnished on the other, readily absorbs moisture, so that the strip curves outwards on the gelatine side, like the compensating strips in (320), when heated. If such a strip be coiled as a spiral, then, according to the greater or less quantity of moisture it absorbs, this twists and untwists like a Breguet's thermometer (309), and thus serves as a sensitive hygroscope.

400. **Moisture of the atmosphere.**—The absolute moisture varies with the temperature in the course both of the year and of the day. In summer there is a maximum at eight in the morning and evening, and a minimum at 3 P.M. and 3 A.M., because the ascending current of air carries the moisture upwards. The *absolute* moisture is greatest in the tropics, where it represents a pressure of 25 mm., while in our latitudes it does not exceed 10 mm. The relative moisture, on the other hand, is on the average greater in high than in low latitudes; it is at the minimum in the hottest and at its maximum in the coolest part of the day. It varies also in different regions. It is greater in the centre of continents than it is on the sea or the sea-coast. Thus in summer the relative moisture at Greenwich is 77, at Venice 64, Lugano 58, and Uralsk 42 per cent. That the dryness increases with the distance from the sea is shown by the clearer skies of continental regions. In Platowskya in Siberia the air, at a temperature of 24°, was found to contain a quantity of moisture only sufficient to saturate it at $-3°$; the air might therefore have been cooled through 27° without any deposit of moisture. On the ground the absolute moisture is greatest, and diminishes rapidly as we ascend; the relative moisture however increases, so that at a certain height the air is saturated with moisture. From this zone upwards the relative moisture decreases, for the aqueous vapour is confined to the lower regions. In some parts of East Africa the springs of powder-flasks exposed to the damp snap like twisted quills; on the contrary, paper becomes soft and sloppy by the loss of its glaze; and gunpowder, if not kept hermetically sealed, refuses to ignite. On the other hand in North America, where the south-west winds blow over large tracts of land, the relative moisture is less and the evaporation is far more rapid than in Europe; clothes dry quickly, bread soon becomes hard, newly built houses can be at once inhabited, European pianos soon give way there, while American ones are very durable. As regards the animal economy, liquids evaporate more rapidly, by which the circulation and the assimilation are accelerated, and the whole character is more nervous. For evaporation is quicker the drier the air, and the more frequently it is renewed; it is, moreover, more rapid the higher the temperature, and the less the pressure. This is not in disaccord with the statement that the quantity of vapour which saturates a given space is the same however this be filled with air; a certain space takes up the same weight of vapour whether it is vacuous, or filled with rarefied or dense air; the saturation with vapour takes place the more rapidly the lower the pressure of the air.

401. **Problem on hygrometry.**—To calculate the weight P of a volume of moist air V, the hygrometric state of which is E, the temperature t, and the pressure H, the density of the vapour being $\frac{5}{8}$ that of air.

From the second law of the mixture of gases and vapours, it will be seen that the moist air is nothing more than a mixture of V cubic inches of dry air at $t°$, under the pressure H minus that of the vapour, and of V cubic

inches of vapour at $t°$ and the pressure given by the hygrometric state; these two values must, therefore, be found separately.

The formula $f = F \times E$ (392) gives the pressure f of the vapour in the air, for E has been determined, and F is found from the tables. The pressure f being known, if f' is the pressure of the air, $f + f' = H$, from which

$$f' = H - f = H - FE$$

The question consequently resolves itself into calculating the weight of V cubic inches of dry air at $t°$, and the pressure $H - FE$, and then that of V cubic inches of aqueous vapour also at $t°$, but under the pressure FE.

Now V cubic inches of dry air under the given conditions weigh $\frac{0{\cdot}31\ V\ (H - FE)}{(1 + at)\ 760}$, and we readily see from problem iii. art. 384, that V cubic inches of vapour at $t°$, and the pressure FE, weigh $\frac{5}{8} \times \frac{0{\cdot}31\ VFE}{(1 + at)\ 760}$. Adding these two weights, and reducing, we get

$$P = \frac{0{\cdot}31\ V\ (H - \frac{3}{8}\ FE)}{(1 + at)\ 760}.$$

If the air were saturated we should have $E = 1$, and the formula would thus be changed into that already found for the mixture of gases and saturated vapours (384).

This formula contains, besides the weight P, many variable quantities, V, E, H, and t, and consequently, by taking successively each of these quantities as unknown, as many different problems might be proposed.

402. **Correction for the loss of weight experienced by bodies weighed in the air.**—It has been seen in speaking of the balance that the weight which it indicates is only an apparent weight, and is generally less than the real weight. The latter may be deduced from the former when it is remembered that every body weighed in the air loses a weight equal to that of the displaced air (195). This problem is, however, very complicated, for not only does the weight of the displaced air vary with the temperature, the pressure, and the hygrometric state, but the volume of the body to be weighed, and that of the weights, vary also with the temperature; so that a double correction has to be made; one relative to the *weights*, the other to the body weighed.

Correction relative to the weights.—In order to make this correction let P be their weight in air, and Π their weight *in vacuo*; further, let V be the volume of these weights at 0°, D the density of the substance of which they are made, and K its coefficient of linear expansion.

The volume V becomes $V\ (1 + 3Kt)$ at $t°$; hence this is the volume of air displaced by the *weights*. If μ be the weight of a cubic inch of air at t, and the pressure H at the time of weighing, we have

$$P = \Pi - \mu V\ (1 + 3Kt).$$

From the formula $P = VD$ (125) V may be replaced by $\frac{\Pi}{D}$, and the formula becomes

$$X = \Pi \left[1 - \frac{\mu(1 + 3Kt)}{D} \right] \quad . \quad . \quad . \quad . \quad . \quad (1)$$

which gives the value, in air, of a *weight* Π, when μ is replaced by its value But since μ is the weight of a cubic inch of air more or less moist, at the temperature t and the pressure H, its value may be calculated by means of the formula in the foregoing paragraph.

Correction relative to the body weighed.—Let p be the apparent weight of the body to be weighed, π its real weight *in vacuo*, d its density, k its coefficient of expansion, and t its temperature; by the same reasoning as above we have

$$p = \pi\left[1 - \frac{\mu(1 + 3kt)}{d}\right] \quad . \quad . \quad . \quad . \quad . \quad (2)$$

By using the method of double weighing, and of a counterpoise whose apparent weight is p', the real weight π', the density d', and the coefficient k', and assuming that the pressure does not change, which is usually the case, we have again

$$p' = \pi'\left[1 - \frac{\mu(1 + 3kt)}{d'}\right] \quad . \quad . \quad . \quad . \quad (3)$$

If a and b are the two arms of the beam, we have in the first weighing $ap = pb$; and in the second $aP = bp$, whence $p = P$. Replacing P and p by their values deduced from the above equations, we have

$$\pi\left[1 - \frac{\mu(1 + 3kt)}{d}\right] = \Pi\left[1 - \frac{\mu(1 + 3Kt)}{D}\right]$$

whence

$$\pi = \Pi \frac{1 - \frac{\mu(1 + 3Kt)}{D}}{1 - \frac{\mu(1 + 3Kt)}{d}}$$

which solves the problem.

CHAPTER VII

CONDUCTIVITY OF SOLIDS, LIQUIDS, AND GASES

403. **Transmission of heat.**—When we stand at a little distance from a fire or other source of heat we experience the sensation of warmth. The heat is not transmitted by the intervening air; it passes through it without raising its temperature, for if we place a screen before the fire the sensation ceases to be felt. The heat from the sun reaches us in the same manner. The heat, which, as in this case, is transmitted to a body from the source of heat without affecting the temperature of the intervening medium, is said to be *radiated*.

That heat can be transmitted through a medium without raising its temperature is proved by a remarkable experiment made by Prevost in 1811. Water from a spring was allowed to fall in a thin sheet; on one side of this was held a red-hot iron ball, and on the other a delicate thermometer. The temperature of the latter was observed to rise steadily, a result which could not have been due to any heating effect of the water itself, as this was cold, and was being continually renewed. It could only have been due to heat which traversed the water without raising its temperature. A similar experiment has been made by a hollow glass lens through which cold water flowed in a constant stream. The sun's rays concentrated by this arrangement ignited a piece of wood placed in the focus.

Heat is transmitted in another way. When the end of a metal bar is heated, a certain increase of temperature is presently observed along the bar. Where the heat is transmitted in the mass of the body itself, as in this case, it is said to be *conducted*. We shall first consider the transmission of heat by conduction.

404. **Conductivity of solids.**—Bodies conduct heat with different degrees of facility. *Good conductors* are those which readily transmit heat, such as are the metals; while *bad conductors*, to which class belong the resins, glass, wood, and more especially liquids and gases, offer a greater or less resistance to the transmission of heat.

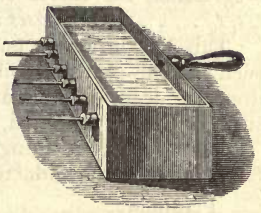


Fig. 371.

In order to compare roughly the conducting power or *conductivity* of different solids, Ingenhaus constructed the apparatus which bears his name and which is represented in fig. 371. It is a metal trough, in which, by means of tubulures and corks, are fixed rods of the same dimensions, but of different materials; for instance, iron, copper, wood, glass. These rods extend to a slight distance in the trough, and the parts

outside are coated with wax which melts at 61°. The box being filled with boiling water, it is observed that the wax melts to a certain distance on the metal rods, while on the others there is no trace of fusion. The conducting power is evidently greater in proportion as the wax has fused to a greater distance. The experiment is sometimes modified by attaching glass balls or marbles to the ends of the rods by means of wax. As the wax melts, the balls drop off, and this in the order of their respective conductivities. The quickness with which melting takes place is, however, only a measure of the conducting power, in case the metals have the same or nearly the same specific heat.

Despretz compared the conducting powers of solids by forming them into bars (fig. 372), in which small cavities are made at short intervals: these cavities contain mercury, and a delicate thermometer is placed in each of them. Such a bar, AB, is exposed at one end to a constant source of heat,

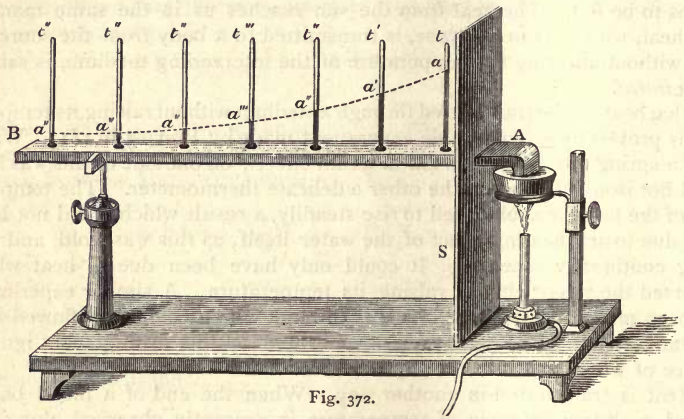


Fig. 372.

such as that of a bath of paraffin or of fusible metal heated by a Bunsen's burner; the thermometers gradually rise until they indicate fixed temperatures, which are less according as the thermometers are farther from the source of heat. By this method Despretz verified the following law:—*If the distances,* a, a_i a_{ii} . . . . a_{vi} *from the source of heat increase in arithmetical progression, the excess of temperature over that of the surrounding air,* t, t_i t_{ii} . . . . t_{vi}, *decreases in geometrical progression.*

This law, however, only prevails in the case of very good conductors, such as gold, platinum, silver, and copper; it is only approximately true for iron, zinc, lead, and tin, and does not apply at all to non-metallic bodies, such as marble, porcelain, &c.

Taking the conducting power of gold at 1000, Despretz constructed the following table of conductivities:—

Platinum	981	Tin	304
Silver	973	Lead	179
Copper	897	Marble	23
Iron	374	Porcelain . . .	12
Zinc	363	Brick earth . . .	21

By making cavities in the bars, as in Despretz's method, their form is altered, and the continuity partially destroyed. Wiedemann and Franz avoided this source of error by measuring the temperature of the bars in different places by applying to them the junction of a thermo-electric couple (412). The metal bars were made as regular as possible, one of the ends was heated to 100°, the rest of the bar being surrounded by air at a constant temperature. The thermo-electric couple was of small dimensions, in order not to abstract too much heat.

By this method Wiedemann and Franz obtained results which differ considerably from those of Despretz. Representing the conductivity of silver by 1000, they found the following numbers for the other metals :—

Silver	1000	Iron	119
Copper	736	Steel	116
Gold	532	Lead	85
Brass	231	Platinum	84
Zinc	190	Rose's alloy	28
Tin	145	Bismuth	18

These experimenters found that the conducting power of the pure metals for heat and electricity is the same.

Organic substances conduct heat badly. De la Rive and De Candolle showed that woods conduct better in the direction of their fibres than in a transverse direction, and this difference is greater with the soft than with the hard woods ; they remarked upon the influence which this feeble conducting power, in a transverse direction, exerts in preserving a tree from sudden changes of temperature, enabling it to resist alike a sudden abstraction of heat from within, and a sudden accession of heat from without. Tyndall also showed that this tendency is aided by the low conducting power of the bark, which in all cases is less than that of the wood. Cotton, wool, straw, bran, &c., are all bad conductors. The conductivity of snow is about $\frac{1}{7}$ that of moist clay.

Rocks and the earth are the worse conductors, the less dense and homogeneous is the mass. Hence the length of time required for the sun's heat to penetrate into the earth. The mean highest temperature of the air near the ground in Central Europe is in the month of July, but at a depth of 25 to 28 feet in the earth it is in the month of December.

405. **Coefficient of conductivity.**—The numbers given in the foregoing article only express the *relative* conducting powers of the respective substances. Numerous experiments have been made to determine the *quantity* of heat, W, which passes, for instance, through a plate the two sides of which are kept at a constant difference of temperature. This will clearly be proportional to the area of the plate A and to the time t. It is further proportional to the excess of the temperature of the one face θ_1 over that of the other θ—that is, to $\theta_1 - \theta$; and as the flow of heat is different in different substances, it will be proportional to a constant k.

On the other hand it will be inversely proportional to the thickness of the plate d. These results are expressed by the formula

$$W = \frac{k\,(\theta_1 - \theta)\,At}{d} \text{ from which } k = \frac{Wd}{(\theta_1 - \theta)\,At}.$$

On the CGS system of units, the *coefficient of thermal* or *calorimetrical conductivity*, k, is the quantity of heat which passes in a second of time, between the two opposite faces of a cube of the substance one centimetre in thickness, and which are kept at a constant difference of one degree. The mean values, as found by Neumann, are as follows : copper, 1·108 ; zinc, 0·307 ; iron, 0·163 ; argentan, 0·109 ; ice, 0·0057.

Thus if the two opposite faces of a cube of iron one centimetre in thickness—that is to say, a cubic centimetre of iron—are kept at a constant difference of 1° C., the quantity of heat which passes in each second of time will be sufficient to raise 0·163 gramme of water through 1° C. From this, which is often called the *calorimetrical measure of conductivity*, we must distinguish the *thermometric measure of conductivity* ; that is to say, the number of degrees through which the cube in question would be heated when the above quantity of heat passes through it under the given conditions. This is obtained from the constants given, by dividing them by the reduced value of the cube c, or the specific heat of unit volume ; that is, by the product of its specific heat into its specific gravity.

406. **Senarmont's experiment.**—It is only in homogeneous bodies that heat is conducted with equal facility in all directions. If an aperture be made in a piece of ordinary glass covered with a thin layer of wax, and a platinum wire ignited by a voltaic current be held through the aperture, the wax will be melted round the hole in a circular form. Senarmont made, on this principle, a series of experiments on the conductivity of heat in crystals. A plate cut from a crystal of the regular system was covered with wax, and a heated metallic point was held against it. The part melted had a circular form ; but when plates of crystals belonging to other systems were investigated in a similar manner, it was found that the form of the *isothermal line* or line of equal temperature—that is, the boundary of the melted part—varied with the different systems and with the position of the axes. In plates of uniaxial crystals cut parallel to the principal axis it was an ellipse (fig. 373), the major axis of which was in the direction of the principal axis. In plates cut perpendicular to the principal axis it was a circle. In biaxial crystals, for which selenite is well adapted, the line was always an ellipse. The isothermal surface agrees in general character with the wave surface of the extraordinary ray (641).

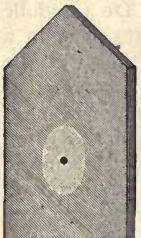


Fig. 373.

Instead of wax the plate may be coated with the double iodide of mercury and copper ; this substance is of a brick-red colour, which when heated changes into a purplish black.

Röntgen makes the experiment very simply by breathing on the plate, and then holding a hot steel point against it. When a space free from moisture has been found about the point, the whole plate is dusted with lycopodium, which shows the outline of the figure with great sharpness.

Pfaff found the conductivity of rock crystal 50·3 in the direction of the principal axis, and 39·1 in a direction at right angles thereto.

407. **Conductivity of liquids.**—The conductivity of liquids is very small, as is seen from the following experiment :—A delicate thermoscope, B, con-

sisting of two glass bulbs, joined by a tube *m*, in which there is a small index of coloured liquid, is placed in a large cylindrical glass vessel, D (fig. 374). This vessel is filled with water at the ordinary temperature, and a tin vessel, A, containing oil at a temperature of two or three hundred degrees, is dipped in it. The bulb near the vessel A is only very slightly heated, and the index *m* moves through a very small distance. Other liquids give the same result. That liquids conduct very badly is also demonstrated by a simpler experiment. A long test-tube is half filled with water, and some ice so placed in it that it cannot rise to the surface. By inclining the tube and heating the surface of the liquid by means of a spirit lamp, the liquid at the top may be made to boil, while the ice at the bottom remains unmelted.

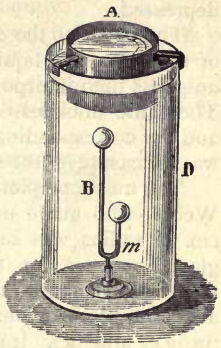


Fig. 374.

Despretz made a series of experiments with an apparatus analogous to that here described, but he kept the liquid in the vessel, A, at a constant temperature, and arranged a series of thermometers one below the other in the vessel D. In this manner he found that the conductivity of heat in liquid obeys the same laws as in solids, but is much more feeble. For example, the conductivity of water is $\frac{1}{95}$ that of copper.

Guthrie examined the conductivity of liquids in the following manner:— Two hollow brass cones are placed near each other so that the top of one

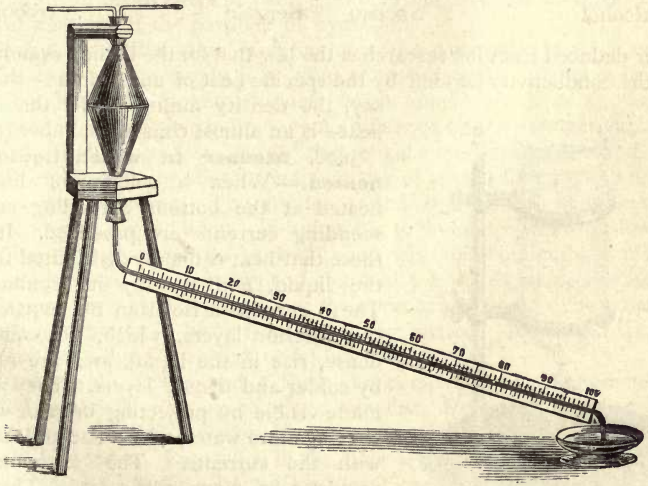


Fig. 375.

points upwards, that of the other downwards (fig. 375). The distance of the bases, which are of platinum, can be regulated by a micrometer screw. The liquid to be examined is introduced between the bases by means of a pipette. The lower cone is fitted with a glass tube which dips in a coloured liquid,

and thus constitutes an air thermometer. The base of the upper cone is kept at a constant temperature by means of a current of hot water; it thus warms the liquid, and the base of the lower cone, in consequence of which the air in the interior is expanded and the column of liquid in the stem depressed.

The bases of the cones were first brought in contact and the depression of the column of liquid was observed. A layer of liquid of a given thickness was then interposed and the depression observed after a certain time. The same thicknesses of other liquids were then successively introduced, and the corresponding depressions noted. The difference of the depressions was a measure for the resistance which the liquid offered to the passage of heat.

The most complete researches on the conductivity of liquids are those of Weber, who made use of the following method. A copper disc about 8 cm. in radius was separated from another similar one by three pieces of glass, about 0·2 cm. thick. The space thus formed between the two was filled with the liquid to be examined, and the system placed horizontally on a smooth block of ice. The lower plate rapidly assumed the temperature of the ice, and heat travelled through the liquid from the upper plate, the changes in temperature of which were noted by a thermo-electrical arrangement (412). He thus observed the following values for k (405):—

Water . . .	0·00124	Carbon bisulphide .	0·00042
Solution of $CuSO_4$.	0·00118	Ether . . .	0·00040
Solution of NaCl .	0·00115	Olive oil . . .	0·00039
Glycerine . . .	0·00067	Chloroform . .	0·00037
Alcohol . . .	0·00049	Benzole . . .	0·00032

Weber deduced from his researches the law that for the liquids examined by him, the conductivity divided by the specific heat of unit volume—that is to say, the density multiplied by the specific heat—is an almost constant number (405).

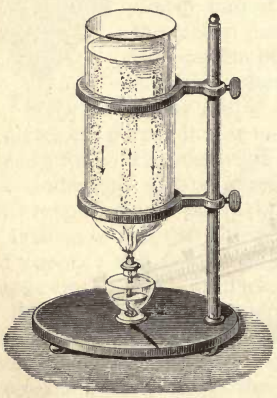


Fig. 376.

408. **Manner in which liquids are heated.**—When a column of liquid is heated at the bottom, ascending and descending currents are produced. It is by these that heat is mainly distributed through the liquid, and not by its conductivity. These currents arise from the expansion of the inferior layers, which, becoming less dense, rise in the liquid, and are replaced by colder and denser layers. They may be made visible by projecting bran or wooden shavings into water, which rise and descend with the currents. The experiment is arranged as shown in fig. 376. The mode in which heat is thus propagated in liquids and in gases is said to be by *convection*.

409. **Conductivity of gases.**—It has been a disputed question whether gases have a true conductivity, that is to say, a conduction from layer to layer as with metals; but certainly when they are restrained in their

motion their conductivity is very small. All substances, for instance, between whose particles air remains stationary, offer great resistance to the propagation of heat. This is well seen in straw, eider-down, and furs. The propagation of heat in a gaseous mass is effected by means of the ascending and descending currents formed in it, as is the case with liquids.

The following experiment, a modification of one originally devised by Sir W. Grove, is considered to prove that gases have a certain conductivity.

A glass tube, fig. 377, with two lateral tubes *d* and *e* opening into it at one end, is closed in the middle by a cork, *b*, through which a stout copper

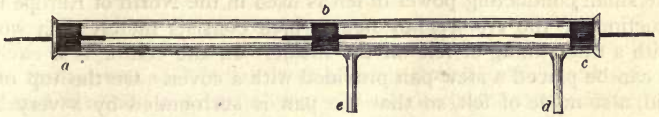


Fig. 377.

wire passes. This is connected by thin platinum wires with similar stout copper wires also passing through the corks *a* and *c*. When the current of a Grove's battery is passed through the wires, both platinums are equally incandescent. If, now, one half of the tube is filled with hydrogen by connecting one of the small tubes with a supply of that gas, and the current is again passed, the wire in the hydrogen is scarcely luminous, while that in air is still brightly incandescent.

This greater chilling of the wire in hydrogen than in air was considered by Magnus to be an effect of conduction; while Tyndall ascribes it to the greater mobility of the particles of hydrogen.

Stefan found the value of *k* for air to be 0·0000558 in CGS units, so that its conductivity is only $\frac{1}{1985}$ that of copper, and $\frac{1}{2921}$ that of iron. He also found that hydrogen conducts seven times as well as air, and that difference of density seems to have no influence on the conductivity.

Maxwell deduced from purely theoretical considerations, based on the kinetic theory of gases, that the conductivity of air must be $\frac{1}{3500}$ that of iron.

410. **Applications.**—The greater or less conductivity of bodies meets with numerous applications. If a liquid is to be kept warm for a long time, it is placed in a vessel and packed round with non-conducting substances, such as shavings, straw, or bruised charcoal. For this purpose water-pipes and pumps are wrapped in straw at the approach of frost. The same means are used to hinder a body from becoming heated. Ice is transported in summer by packing it in bran or folding it in flannel.

Double walls constructed of thick planks having between them any finely divided materials, such as shavings, sawdust, dry leaves, &c., retain heat extremely well; and are likewise advantageous in hot countries, for they prevent its access. Pure silica in the state of rock crystal is a better conductor than lead, but in a state of powder it conducts very badly. If a layer of asbestos is placed on the hand, a red-hot iron ball can be held without inconvenience. Red-hot cannon-balls can be wheeled to the gun's mouth in wooden barrows partially filled with sand. Lava has been known to flow over a layer of ashes underneath which was a bed of ice, and the non-conducting power of the ashes has prevented the ice from melting.

The clothes which we wear are not warm in themselves; they only hinder the body from losing heat, in consequence of their spongy texture and the air they enclose. The warmth of bed-covers and of counterpanes is explained in a similar manner. Double windows are frequently used in cold climates to keep a room warm—they do this by the non-conducting layer of air interposed between them. During the night the windows are opened, while during the day they are kept closed. It is for the same reason that two shirts are warmer than one of the same material but of double the thickness. Hence, too, the warmth of furs, eider-down, &c.

The small conducting power of felt is used in the North of Europe in the construction of the *Norwegian stove*, which consists merely of a wooden box with a thick lining of felt on the inside. In the centre is a cavity in which can be placed a stew-pan provided with a cover. On the top of this is a lid, also made of felt, so that the pan is surrounded by a very badly conducting envelope. Meat, with water and suitable additions, is placed in the pan, and the contents are then raised to boiling point. The whole is then enclosed in the box and left to itsel ; the cooking will go on without fire, and after the lapse of several hours it will be quite finished. The cooling down is very slow, owing to the bad conducting power of the lining; at the end of three hours the temperature is usually not found to have sunk more than from 10° to 15°.

That water boils more rapidly in a metallic vessel than in one of porcelain of the same thickness; that a burning piece of wood can be held close to the burning part with the naked hand, while a piece of iron heated at one end can only be held at a great distance, are easily explained by reference to their various conductivities.

The sensation of heat or cold which we feel when in contact with certain bodies is materially influenced by their conductivity. If their temperature is lower than ours, they appear colder than they really are, because from their conductivity heat passes away from us. If, on the contrary, their temperature is higher than that of our body, they appear warmer from the heat which they give up at different parts of their mass. Hence it is clear why carpets, for example, are warmer than wooden floors, and why the latter again are warmer than stone floors.

The closer the contact of the hand with a substance, the greater is the difference of temperature felt. With smooth surfaces there are more points of contact than with rough ones. A hot glass rod feels hotter than a piece of rusted iron of the same temperature, although the latter is a better conductor. The closer the substance is pressed, the more intimate the contact; an ignited piece of charcoal can be lifted by the fingers, if it is not closely pressed.

CHAPTER VIII

RADIATION OF HEAT

411. **Radiant heat.**—It has been already stated (403) that heat can be transmitted from one body to another without altering the temperature of the intervening medium. If we stand in front of a fire we experience a sensation of warmth which is not due to the temperature of the air, for if a screen be interposed the sensation immediately disappears, which would not be the case if the surrounding air had a high temperature. Hence bodies can send out rays which excite heat, and which penetrate through the air without heating it, as rays of light through transparent bodies. Heat thus propagated is said to be *radiated*; and we shall use the terms *ray of heat*, or *thermal*, or *calorific ray*, in a similar sense to that in which we use the term *ray of light*, or *luminous ray*.

We shall find that the property of radiating heat is not confined to luminous bodies, such as a fire or a red-hot ball, but that bodies of all temperatures radiate heat. It will be convenient to make a distinction between *luminous* and *obscure* rays of heat.

412. **Detection and measurement of radiant heat.**—In demonstrating the phenomena of radiant heat, very delicate thermometers are required, and the thermo-electrical multiplier of Melloni is used for this purpose with great advantage; for it not only indicates minute differences of temperature, but it also measures them with accuracy.

This instrument cannot be properly understood without a knowledge of the principles of thermo-electricity, for which Book X. must be consulted. It may, however, be stated here that when two different metals A and B are soldered together at one end (figs. 378, 379), the free ends being joined by a wire when the soldering C is heated, a current of electricity circulates through the system; if, on the contrary, the soldering be cooled, a current is also produced, but it circulates in exactly the opposite direction. This is called a *thermo-electric couple* or *pair*. If a number of such pairs be alternately soldered together, as represented in fig. 379, the strength of the current produced by heating the ends is increased; or, what amounts to the same thing, a smaller degree of heat will produce the same effect. Such an

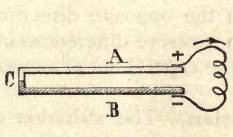


Fig. 378.

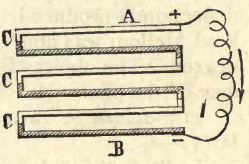


Fig. 379.

arrangement of a number of thermo-electric pairs is called a *thermo-electric battery* or *pile.*

Melloni's thermomultiplier consists of a thermo-electric pile connected with a delicate galvanometer. The thermo-electric pile is constructed of a number of minute bars of bismuth and antimony soldered together alternately, though kept insulated from each other, and contained in a rectangular box P (fig. 380). The terminal bars are connected with two binding screws *m* and *n*, which in turn are connected with the galvanometer G by means of the wires *a* and *b*.

The galvanometer consists of a quantity of fine insulated copper wire coiled round a frame, in the centre of which a delicate magnetic needle is suspended by means of a silk thread. When an electric current is passed through this coil, the needle is deflected through an angle which depends on the strength of the current. The angle is measured on a dial by an index connected with the needle.

It may then be sufficient to state that the thermo-electric pile being connected with the galvanometer by means of the wires *a* and *b*, an excess of

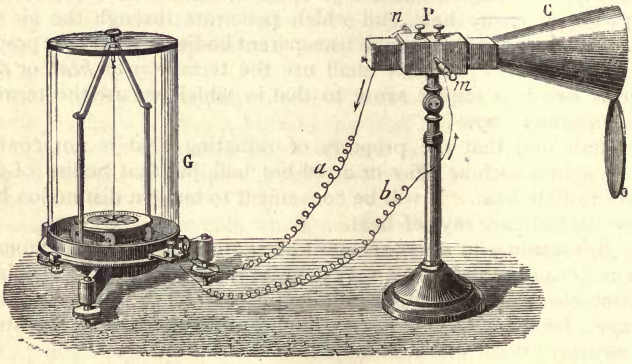


Fig. 380.

temperature at one end of the pile causes the needle to be deflected through an angle which depends on the extent of this excess; and similarly if the temperature is depressed below that of the other end, a corresponding deflection is produced in the opposite direction. By arrangements of this kind Melloni was able to measure differences of temperature of $\frac{1}{500}$th of a degree. The object of the cone C is to concentrate the thermal rays on the face of the pile.

413. **Laws of radiation.**—The radiation of heat is governed by three laws:—

I. *Radiation takes place in all directions round a body.* If a thermometer be placed in different positions round a heated body, it indicates everywhere a rise in temperature.

II. *In a homogeneous medium radiation takes place in a right line.* For if a screen be placed in a right line which joins the source of heat and the thermometer, the latter is not affected.

But in passing obliquely from one medium into another, as from air into glass, calorific like luminous rays become deviated, an effect known as *refraction.* The laws of this phenomenon are the same for heat as for light, and they will be more fully discussed under the latter subject.

III. *Radiant heat is propagated, in vacuo as well as in air.* This is demonstrated by the following experiment :—

In the bottom of a glass flask a thermometer is fixed in such a manner that its bulb occupies the centre of the flask (fig. 381). The neck of the flask is carefully narrowed by means of the blowpipe, and then the apparatus having been suitably attached to an air-pump, a vacuum is produced in the interior. This having been done, the tube is sealed at the narrow part. On immersing this apparatus in hot water, or on bringing near it some hot charcoal, the thermometer is at once seen to rise. This could only rise from radiation through the vacuum in the interior, for glass is so bad a conductor that the heat could not travel with this rapidity through the sides of the flask and the stem of the thermometer.

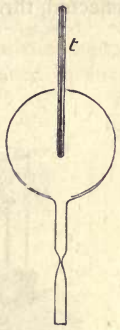


Fig. 381.

414. **Causes which modify the intensity of radiant heat.**—By the *intensity of radiant heat* is understood the quantity of heat received on the unit of surface. Three causes are found to modify this intensity : the temperature of the source of heat, its distance, and the obliquity of the calorific rays in reference to the surface which emits them. The laws which regulate these modifications may be thus stated :—

I. *The intensity of radiant heat is proportional to the temperature of the source.*

II. *The intensity is inversely as the square of the distance.*

III. *The intensity is less, the greater the obliquity of the rays with respect to the radiating surface.*

The first law is demonstrated by placing a metal box containing water at 10°, 20°, or 30° successively at equal distances from the bulb of a differential thermometer. The temperatures indicated by the latter are then found to be in the same ratio as those of the box : for instance, if the temperature of that corresponding to the box at 10° be 2°, those of others will be 4° and 6° respectively.

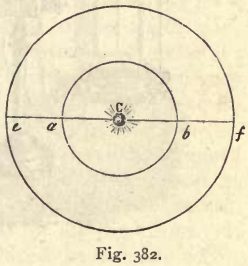


Fig. 382.

The truth of the second law follows from the geometrical principle that the surface of a sphere increases as the square of its radius. Suppose a hollow sphere *ab* (fig. 382) of any given radius, and a source of heat, C, in its centre; each unit of surface in the interior receives a certain quantity of heat. Now a sphere, *ef*, of double the radius will present a surface four times as great ; its internal surface contains, therefore, four times as many units of surface, and as the quantity of heat emitted is the same, each unit must receive one-fourth the quantity.

To demonstrate the same law experimentally, a narrow tin-plate box is

taken (fig. 383), filled with hot water, and coated on one side with lampblack. The thermopile with its conical reflector is placed so that its face is at a certain definite distance, *co*, say 9 inches, from this box, and the cover having been lowered, the needle of the galvanometer is observed to be deflected, through 80° for example.

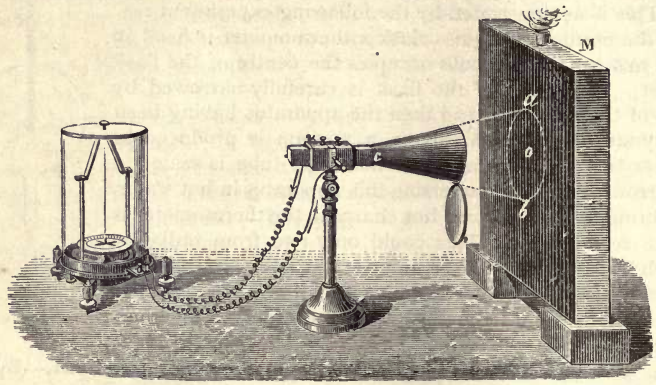


Fig. 383.

If now the pile is removed to a distance, CO (fig. 384), double that of *co*, the deflection of the galvanometer remains the same, which shows that the pile receives the same amount of heat; the same is the case if the pile is removed to three or four times the distance. This result, though

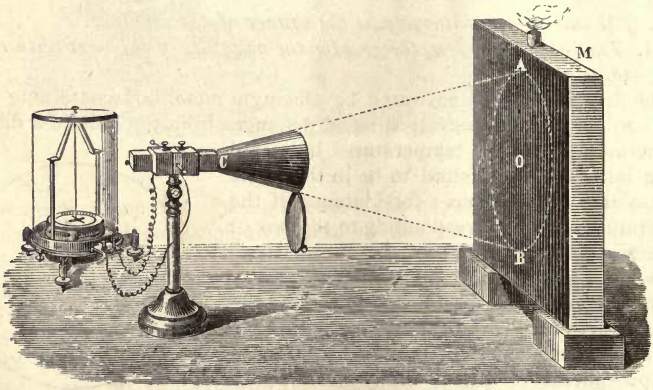


Fig. 384.

apparently in opposition to the second law, really confirms it. For at first the pile only receives heat from the circular portion *ab* of the side of the box, while, in the second case, the circular portion AB radiates towards it. But, as the two cones ACB and *acb* are similar, and the height of ACB is double that of *acb*, the diameter AB is double that of *ab*, and therefore the

area AB is four times as great as that of *ab*, for the areas of circles are proportional to the squares of the radii. But since the radiating surface increases as the square of the distance, while the galvanometer remains stationary, the heat received by the battery must be inversely as this same square.

The third law is demonstrated by means of the following experiment, which is a modification of one originally devised by Leslie (fig. 385):—P represents the thermomultiplier which is connected with its galvanometer, and A a metal cube full of hot water. The cube being first placed in such

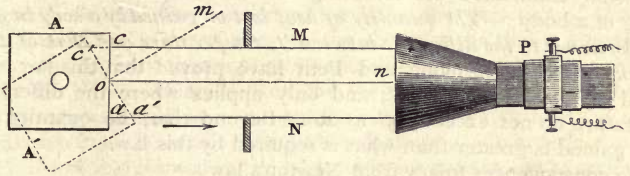


Fig. 385.

a position, A, that its front face, *ac*, is vertical, the deflection of the galvanometer is noted. Supposing it amounts to 45°, this represents the radiation from *ac*. If this now be turned in the direction represented by A′, the galvanometer is still found to mark 45°.

The second surface is larger than the first, and it therefore sends more rays to the mirror. But as the action on the thermometer is no greater than in the first case, it follows that in the second case, where the rays are oblique, the intensity is less than in the first case, where they are perpendicular.

In order to express this in a formula, let i be the intensity of the rays emitted perpendicularly to the surface, and i' that of the oblique rays. These intensities are necessarily inversely as the surfaces *ac* and *a′c′*, for the effect is the same in both cases, and therefore $i' \times$ surface $a'c' = i \times$ surface ac; hence $i' = i \dfrac{\text{surf. } ac}{\text{surf. } a'c'} = i' \dfrac{ac}{a'c'} = i \cos. aoa'$; which signifies that *the intensity of oblique rays is proportional to the cosine of the angle which these rays form with the normal to the surface*; for this angle is equal to the angle *aoa′*. This law is known as the *law of the cosine*; it is, however, not general; Desains and De la Provostaye have shown that it is only true within very narrow limits; that is, only with bodies which, like lampblack, are entirely destitute of reflecting power (423).

415. **Mobile equilibrium. Theory of exchanges.**—Prevost of Geneva suggested the following hypothesis in reference to radiant heat, known as Prevost's *theory of exchanges*, which is now universally admitted. All bodies, whatever their temperatures, constantly radiate heat in all directions. If we imagine two bodies at different temperatures placed near each other, the one at a higher temperature will experience a loss of heat, its temperature will sink, because the rays it emits are of greater intensity than those it receives; the colder body, on the contrary, will rise in temperature, because it receives rays of greater intensity than those which it emits. Ultimately the temperature of both bodies becomes the same, but heat is still exchanged

between them, only each receives as much as it emits, and the temperature remains constant. This state is called the *mobile equilibrium of temperature.*

416. **Newton's law of cooling.**—A body placed in a vacuum is only cooled or heated by radiation. In the atmosphere it becomes cooled or heated by its contact with the air, according as the latter is colder or hotter than the radiating body. In both cases the velocity of cooling or of heating —that is, *the quantity of heat lost or gained in a second*—is greater according as the difference of temperature is greater.

Newton enunciated the following law in reference to the cooling or heating of a body :—*The quantity of heat lost or gained by a body in a second is proportional to the difference between its temperature and that of the surrounding medium.* Dulong and Petit have proved that this law is not so general as Newton supposed, and only applies where the differences of temperature do not exceed 15° to 20°. Beyond that, the quantity of heat lost or gained is greater than what is required by this law.

Two consequences follow from Newton's law :—

I. When a body is exposed to a constant source of heat, its temperature does not increase indefinitely, for the quantity which it receives in the same time is always the same ; while that which it loses increases with the excess of its temperature over that of the surrounding medium. Consequently a point is reached at which the quantity of heat emitted is equal to that absorbed, and the temperature then remains stationary.

II. Newton's law, as applied to the differential thermometer, shows that its indications are proportional to the quantities of heat which it receives. If one of the bulbs of a differential thermometer receives rays of heat from a constant source, the instrument exhibits, first, increasing temperature, but afterwards becomes stationary. In this case, the quantity of heat which it receives is equal to that which it emits. But the latter is proportional to the excess of the temperature of the bulb above that of the surrounding atmosphere—that is, to the number of degrees indicated by the thermometer ; consequently, the temperature indicated by the differential thermometer is proportional to the quantity of heat it receives.

REFLECTION OF HEAT

417. **Laws of reflection.**—When thermal rays fall upon a body they are, speaking generally, divided into two portions, one of which penetrates the body while the other rebounds as if repelled from the surface like an elastic ball. This is said to be *reflected.*

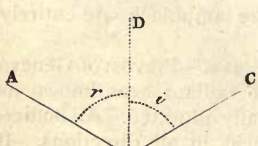


Fig. 386.

If *mn* be a plane reflecting surface (fig. 386), CB an *incident ray*, DB a line perpendicular to the surface called the *normal*, and BA the *reflected ray*, the angle CBD is called the *angle of incidence*, and DBA the *angle of reflection.* The reflection of heat, like that of light, is governed by the two following laws :—

I. *The angle of reflection is equal to the angle of incidence.*

II. *Both the incident and the reflected ray are in the same plane with the perpendicular to the reflecting surface.*

418. **Experimental demonstration of the laws of reflection of heat.**—This may be effected by means of Melloni's thermopile and also by the conjugate mirrors (420). Fig. 387 represents the arrangement adopted in the former case. MN is a horizontal bar, about a metre in length, graduated in millimetres, on which slide various parts, which can be clamped by means of screws. The source of heat, S, is a platinum spiral, kept at a white heat in a spirit lamp. A screen K, when raised, cuts off the radiation from the source; a second screen, F, with an aperture in the centre, cuts off all rays except a pencil which falls upon the mirror *m*. At the other end is an upright rod, I, with a graduated dial, the zero of which is in the direction of MN, and therefore parallel to the pencil S*m*. In the centre of the dial is an aperture, in which turns an axis that supports a metallic mirror *m*. About this axis turns an index, R, on which is fixed the thermopile, P, in connection with the galvanometer G; H is a screen, the object of which is to cut off any direct radiation from the source of heat towards the pile. In order not to mask the pile, it is not represented in the position it occupies in the experiment.

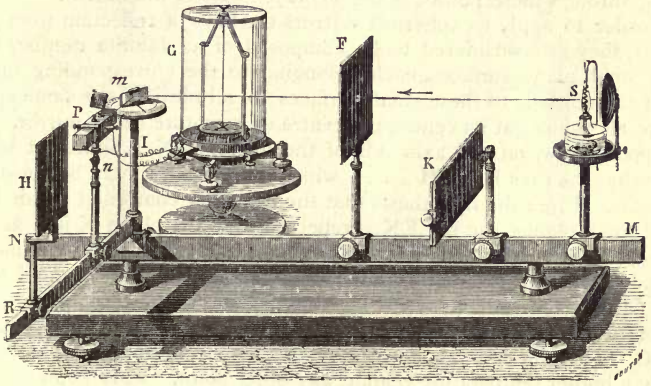


Fig. 387.

By lowering the screen K, a pencil of parallel rays, passing through the aperture F, falls upon the mirror *m*, and is there reflected. If the index R is not in the direction of the reflected pencil, this latter does not fall on the pile, and the needle of the galvanometer remains stationary: but by slowly turning the index R, a position is found at which the galvanometer attains its greatest deviation, which is the case when the pile receives the reflected pencil perpendicularly to its surface. Reading off then on the dial the position of a small needle perpendicular to the mirror, it is observed that this bisects the angle formed by the incident and the reflected pencil, which demonstrates the first law.

The second law is also proved by the same experiment, for the various pieces of the apparatus are arranged so that the incident and reflected rays are in the same horizontal plane, and therefore at right angles to the reflecting surface, which is vertical.

419. **Reflection from concave mirrors.**—*Concave mirrors* or *reflectors* are polished spherical or parabolic surfaces of metal or of glass, which are used to concentrate luminous or calorific rays in the same point.

We shall only consider the case of spherical mirrors. Fig. 389 represents two of these mirrors ; fig. 388 gives a medial section, which is called the *principal section.* The centre C of the sphere to which the mirror belongs is called the *centre of curvature*; the point A, the middle of the reflector, is the *centre of the figure* ; the straight line AB passing through these points, is the *principal axis* of the mirror.

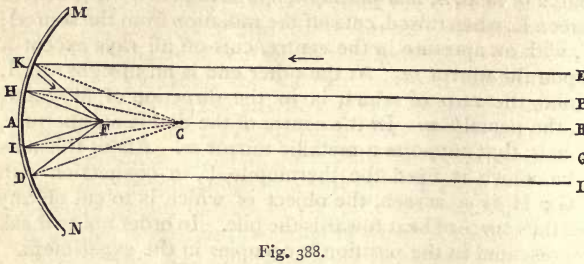


Fig. 388.

In order to apply to spherical mirrors the laws of reflection from plane surfaces, they are considered to be composed of an infinite number of infinitely small plane surfaces, each belonging to the corresponding tangent plane ; the normals to these small surfaces are all radii of the same sphere, and therefore meet at its centre, the centre of curvature of the mirror.

Suppose now, on the axis AB of the mirror MN, a source of heat so distant that the rays EK, PH . . . . which start from it may be considered as parallel. From the hypothesis that the mirror is composed of an infinitude of small planes, the ray EK is reflected from the plane K just as from a plane mirror ; that is to say, CK being the normal to this plane, the reflected ray takes a direction such that the angle CKF is equal to the angle CKE. The other rays, PH, GI . . . . are reflected in the same manner, and all converge approximately towards the same point F, on the line AC. There is then a concentration of the rays in this point, and consequently a higher temperature than at any other point. This point is called the *focus*, and the distance from the focus to the mirror at A is the *focal distance.*

In the above figure the heat is propagated along the lines EKF, LDF, in the direction of the arrows ; but, conversely, if the heated body be placed at F, the heat is propagated along the lines FKE, FDL, so that the rays emitted from the focus are nearly parallel after reflection.

420. **Verification of the laws of reflection.**—The following experiment, which was made for the first time by Pictet and Saussure, and which is known as the *experiment of the conjugate mirrors*, demonstrates not only the existence of the foci, but also the laws o reflection. Two reflectors, M and N (fig. 389), are arranged at a distance of 4 to 5 yards, and so that their axes coincide. In the focus of one of them, A, is placed a small wire basket containing a red-hot iron ball. In the focus of the other is placed B, an easily inflammable body, such as gun-cotton or phosphorus. The rays emitted from the focus A are first reflected from the mirror M, in a direction parallel to the axis (419), and impinging on the other mirror, N, are reflected

so that they coincide in the focus B. That this is so, is proved by the fact that the gun-cotton at this point takes fire, which is not the case if it is above or below it.

The experiment also serves to show that light and heat are reflected in the same manner. For this purpose a lighted candle is placed in the focus of A, and a ground-glass screen in the focus of B, when a luminous focus is seen on it exactly in the spot where the gun-cotton ignites. Hence the luminous and the calorific foci are produced at the same point, and the reflection takes place in both cases according to the same laws ; for it will be afterwards shown that for light, the angle of reflection is equal to the angle of incidence, and that both the incident and the reflected rays are in the same plane perpendicular to the plane reflecting surface.

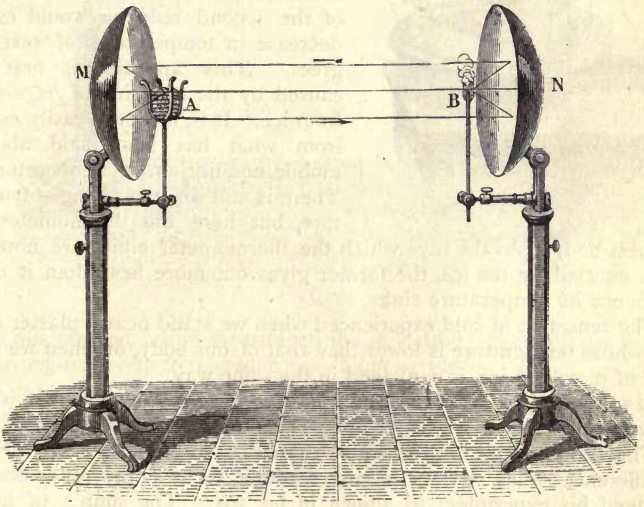


Fig 389.

In consequence of the high temperature produced in the foci of concave mirrors, they have been called *burning mirrors*. It is stated that Archimedes burnt the Roman vessels before Syracuse by means of such mirrors. Buffon constructed burning mirrors of such power as to prove that the feat attributed to Archimedes was not impossible. The mirrors were made of a number of silver plane mirrors about 8 inches long by 5 broad. They could be turned independently of each other in such a manner that the rays reflected from each coincided in the same point. With 128 mirrors and a hot summer's sun Buffon ignited a plank of tarred wood at a distance of 70 yards.

421. **Reflection in a vacuum.**—Heat is reflected in a vacuum as well as in air, as is seen from the following experiment (fig. 390), due to Sir Humphry Davy. Two small concave reflectors were placed opposite each other under the receiver of an air-pump. In the focus of one was placed a delicate

thermometer, and in the focus of the other a platinum wire made incandescent by means of a galvanic current. The thermometer was immediately seen to rise several degrees, which could only be due to reflected heat, for the thermometer did not show any increase of temperature if it were not exactly in the focus of the second reflector.

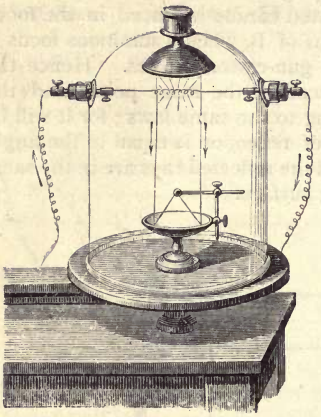


Fig. 390.

422. **Apparent reflection of cold.**—If two mirrors are arranged as represented in fig. 389, and a piece of ice is placed in one of the foci instead of the red-hot ball, the surrounding temperature being greater than zero, a differential thermometer placed in the focus of the second reflector would exhibit a decrease in temperature of several degrees. This appears at first to be caused by the emission of *frigorific* rays from ice. It is, however, easily explained from what has been said about the mobile equilibrium of temperature (415). There is still an interchange of temperature, but here the thermometer is the warmest body. As the rays which the thermometer emits are hotter than those emitted by the ice, the former gives out more heat than it receives, and hence its temperature sinks.

The sensation of cold experienced when we stand near a plaster or stone wall whose temperature is lower than that of our body, or when we stand in front of a wall of ice, is explained in the same way.

423. **Reflecting power.**—The *reflecting power* of a substance is its property of throwing off a greater or less proportion of incident heat.

This power varies in different substances. In order to study this power in different bodies without having recourse to as many reflectors, Leslie arranged his experiment as shown in fig. 391. The source of heat is a cubical canister, M, now known as *Leslie's cube*, filled with hot water. A plate, *a*, of the substance to be experimented upon is placed on the axis of a reflecting mirror between the focus and the mirror. In this manner the rays emitted by the source are first reflected from the mirror and impinge on the plate *a*, where they are again reflected and converge to the focus between the plate and the mirror, at which point a differential thermometer is placed. The reflector and the thermometer are always in the same position, and the water of the cube is always kept at 100°, but it is found that the temperature indicated by the thermometer varies with the nature of the plate. This method gives a means of determining, not the absolute reflecting power of a body, but its power relatively to that of some body taken as a standard of comparison. For from what has been said on the application of Newton's law to the differential thermometer (416), the temperatures which this instrument indicates are proportional to the quantities of heat which it receives. Hence, if in the above experiment a plate of glass causes the temperature to rise 1° and a plate of lead 6°, it follows that the quantity of heat reflected by the

latter is six times as great as that reflected by the former. For the heat emitted by the source remains the same, the concave reflector receives the same portion, and the difference can only arise from the reflecting power of the plate *a*.

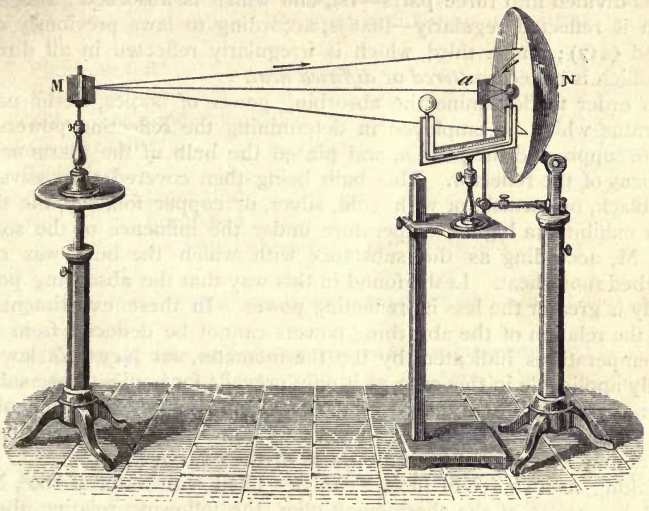


Fig. 391.

By this method Leslie determined the reflecting powers of the following substances, relatively to that of brass, taken as 100 :—

Polished brass	100	Indian ink	13
Silver	90	Glass	10
Steel	70	Oiled glass	5
Lead	60	Lampblack	0

The numbers only represent the relative reflecting power as compared with that of brass. Their *absolute power* is the *relation of the quantity of heat reflected to the quantity of heat received.* Desains and De la Provostaye, who examined the absolute reflecting power of certain metals, obtained the following results by means of Melloni's thermomultiplier (412), the heat being reflected at an angle of 50° :—

Silver plate	0·97	Steel	0·82
Gold	0·95	Zinc	0·81
Brass	0·93	Iron	0·77
Platinum	0·83	Cast iron	0·74

424. **Absorbing power.**—The *absorbing power* of a body is its property of allowing a greater or less quantity of the heat which falls upon it to pass into its mass. Its absolute value is the ratio of the quantity of heat absorbed to the quantity of heat received.

The absorbing power of a body is always inversely as its reflecting power : a body which is a good absorbent is a bad reflector, and *vice versâ*.

It was formerly supposed that the two powers were exactly complementary, that the sum of the reflected and absorbed heat was equal to the total quantity of incident heat. This is not the case ; it is always less : the incident heat is divided into three parts—1st, one which is absorbed ; 2nd, another which is reflected regularly—that is, according to laws previously demonstrated (417) ; and a third, which is irregularly reflected in all directions, and which is called *scattered* or *diffused heat.*

In order to determine the absorbing power of bodies, Leslie used the apparatus which he employed in determining the reflecting powers (423). But he suppressed the plate *a*, and placed the bulb of the thermometer in the focus of the reflector. This bulb being then covered successively with lampblack, or varnish, or with gold, silver, or copper foil, &c., the thermometer exhibited a higher temperature under the influence of the source of heat, M, according as the substance with which the bulb was covered absorbed more heat. Leslie found in this way that the absorbing power of a body is greater the less its reflecting power. In these experiments, however, the relation of the absorbing powers cannot be deduced from that of the temperatures indicated by the thermometer, for Newton's law is not exactly applicable in this case, as it only prevails for bodies whose substance does not vary, and here the covering of the bulb varied with each observation. But we shall presently show (426) how the comparative absorbing powers may be deduced from the ratios of the emissive powers.

Taking, as a source of heat, a canister filled with water at 100°, Melloni found, by means of the thermomultiplier, the following relative absorbing powers :—

Lampblack	100	Indian ink	85
White lead	100	Shellac	72
Isinglass	91	Metals	13

425. **Radiating power.**—The *radiating* or *emissive power* of a body is its capability of emitting, at the same temperature, and with the same extent of surface, greater or less quantities of heat.

The apparatus represented in fig. 391 was also used by Leslie in determining the radiating power of bodies. For this purpose the bulb of the thermometer was placed in the focus of the reflector, and the faces of the canister M were formed of different metals, or covered with different substances, such as lampblack, paper, &c. The cube being filled with hot water, at 100°, and all other conditions remaining the same, Leslie turned each face of the cube successively towards the reflectors, and noted the temperature each time. That face which was coated with lampblack caused the greatest elevation of temperature, and the metal faces the least. Applying Newton's law, and representing the heat emitted by lampblack as 100, Leslie formed the following table of radiating powers :—

Lampblack	100	Tarnished lead	45
White lead	100	Mercury	20
Paper	98	Polished lead	19
Ordinary white glass	90	Polished iron	15
Isinglass	80	Tin, gold, silver, copper, &c.	12

It will be seen that, in this table, the order of the bodies is exactly the reverse of that in the tables of reflecting powers.

The radiating powers of several substances were determined by Desains and De la Provostaye, who used the thermomultiplier. They found, in this manner, the following numbers compared with lampblack as 100 :—

Platinum foil	10·80
Burnished platinum	9·50
Silver deposited chemically	5·36
Copper foil	4·90
Gold leaf	4·28
Pure silver laminated	3·00
„ burnished	2·50
„ deposited chemically and burnished	2·25

It appears, therefore, that the radiating power found by Leslie for the metals is too large.

426. **Identity of the absorbing and radiating powers.**—The absorbing power of a body cannot be accurately deduced from its reflecting power, because the two are not exactly complementary. But the absorbing power would be determined if it could be shown that in the same body it is equal to the radiating power. This conclusion has been drawn by Dulong and Petit from the following experiments :—In a large glass globe, blackened on the inside, was placed a thermometer at a certain temperature, 15° for example ; the globe was kept at zero by surrounding it with ice, and having been exhausted by means of a tubulure connected with an air-pump, the time was noted which elapsed while the thermometer fell through 5°. The experiment was then made in the contrary direction ; that is, the sides of the globe were heated to 15°, while the thermometer was cooled to zero ; the time was then observed which the thermometer occupied in rising through 5°. It was found that this time was exactly the same as that which the thermometer had taken in sinking through 5°, and it was thence concluded that the radiating power is equal to the absorbing power for the same body, and for the same difference between its temperature and the temperature of the surrounding medium, because the quantities of heat emitted or absorbed in the same time are equal.

This point may also be demonstrated by means of the following apparatus devised by Ritchie. Fig. 392 represents what is virtually a differential thermometer, the two glass bulbs of which are replaced by two cylindrical reservoirs B and C, of metal, and full of air. Between them is a third and larger one A, which can be filled with hot water by means of a tubulure. The ends of B and of A, which face the right, are coated with lampblack ; those of C and of A, which face the left, are either painted white or are coated with silver foil. Thus one of the two faces opposite each other is black, and the other white ; hence when the cylinder A is filled with hot water, its white face radiates towards the black face of B, and its black face towards the white face of C. In these circumstances the liquid in the stem does not move, indicating that the two reservoirs are at the same temperature. On the one hand, the greater emissive power of the black

face of A is compensated by the smaller absorptive power of the white face of C ; while, on the other hand, the feebler radiating power of the white face of A is compensated by the greater absorbing power of the black face of B.

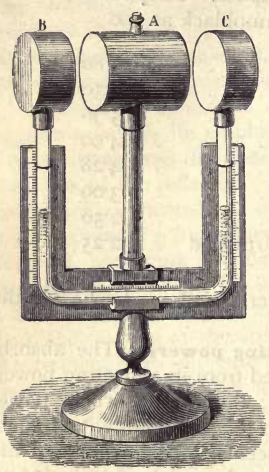


Fig. 392.

The experiment may be varied by replacing the two white faces by discs, of paper, glass, porcelain, &c.

427. **Causes which modify the reflecting, absorbing, and radiating powers.**—As the radiating and absorbing powers are equal, any cause which affects the one affects the other also. And as the reflecting power varies in an inverse manner, whatever increases it diminishes the radiating and absorbing powers, and *vice versâ*.

It has been already stated that these different powers vary with different bodies, and that metals have the greatest reflecting power, and lampblack the least. In the same body these powers are modified by the degree of polish, the density, the thickness of the radiating substance, the obliquity of the incident or emitted rays, and, lastly, by the nature of the source of heat.

It has been usually assumed that the reflecting power increases with the polish of the surface, and that the other powers diminish therewith. But Melloni showed that by scratching a polished metallic surface its reflecting power was sometimes diminished and sometimes increased. This phenomenon he attributed to the greater or less density of the reflecting surface. If the plate had been originally hammered, its homogeneity would be destroyed by this process, the molecules would be closer together on the surface than in the interior, and the reflecting power would be increased. But if the surface is scratched, the interior and less dense mass becomes exposed, and the reflecting power diminished. On the contrary, in a plate which has not been hammered, and which is homogeneous, the reflecting power is increased when the plate is scratched, because the density at the surface is increased by the scratches.

Melloni found that when the faces of a cube filled with water at a constant temperature were varnished, the emissive power increased with the number of layers up to 16 layers, while above that point it remained constant, whatever the number. The thickness of the 16 layers was calculated to be 0·04 mm. With reference to metals, gold leaves of 0·008, 0·004, and 0·002 of a millimetre in thickness, having been successively applied on the sides of a cube of glass, the diminution of radiant heat was the same in each case. It appears, therefore, that, beyond certain limits, the thickness of the radiating layer of metal is without influence.

The absorbing power is greatest when the rays are at right angles, and it diminishes in proportion as the incident rays deviate from the normal. This is one of the reasons why the sun is hotter in summer than in winter, because, in the former case, the sun's rays are less oblique.

The radiating power of gaseous bodies in a state of combustion is very weak, as is seen by bringing the bulb of a thermometer near a hydrogen flame, the temperature of which is very high. But if a platinum spiral be placed in this flame, it assumes the temperature of the flame, and radiates a great amount of heat, as is shown by the thermometer. For a similar reason the flames of oil and of gas lamps radiate more than a hydrogen flame in consequence of the excess of carbon which they contain, and which, not being entirely burned, becomes incandescent in the flame.

428. **Melloni's researches on radiant heat.**—For our knowledge of the phenomena of the reflection, emission, and absorption of heat which have up to now been described, science is indebted mainly to Leslie. But since his time the discovery of other and far more delicate modes of detecting and measuring heat has not only extended and corrected our previous knowledge, but has led to the discovery of other phenomena of radiant heat, which, without such improved means, must have remained unknown.

Fig. 393.

Fig. 394.

Fig. 395.

Fig. 396.

This advance in science is due to an Italian philosopher, Melloni, who first applied the thermo-electric pile, invented by Nobili, to the measurement of very small differences of temperature ; a method of which a preliminary account has already been given (412).

In his experiments Melloni used five sources of heat—1st, a Locatelli's lamp—one, that is, without a glass chimney, but provided with a reflector (fig. 393) ; 2nd, an Argand lamp, that is, one with a chimney and a double draught ; 3rd, a platinum spiral, kept red hot by a spirit lamp (fig. 394) 4th, a blackened copper plate, kept at a temperature of about 400° by a spirit lamp (fig. 395) ; 5th, a copper tube, blackened on the outside and filled with water at 100° (fig. 396).

429. **Dynamical theory of heat.**—Before describing the results arrived at by Melloni and others, it will be convenient to explain here the view now generally taken as to the mode in which heat is propagated. For additional information the chapter on the Mechanical Theory of Heat and the book on Light should be read. According to what has already been stated (292), a hot body is nothing more than one whose particles are in a state of vibration. The higher the temperature of the body, the more rapid are these vibrations, and a diminution in temperature is but a diminished rapidity of vibration of the particles. The propagation of heat through a bar is due to a gradual communication of this vibratory motion from the heated part to the rest of the bar. A good conductor is one which readily takes up and transmits the

vibratory motion from particle to particle, while a bad conductor is one which takes up and transmits the motion with difficulty. But even through the best conductors the propagation of this motion is comparatively slow. How then are we to explain the instantaneous perception of heat experienced when a screen is removed from a fire, or when a cloud drifts from the face of the sun? In this case the heat passes from one body to another without affecting the temperature of the medium which transmits it. In order to explain these phenomena, it is imagined that all space, the interplanetary spaces as well as the interstices in the hardest crystal or the heaviest metal—in short, matter of any kind—is permeated by a medium having the properties of a fluid of infinite tenuity, called *ether* (499). The particles of a heated body, being in a state of intensely rapid vibration, communicate their motion to the ether around them, throwing it into a system of waves which travel through space and pass from one body to another with the velocity of light. When the undulations of the ether reach a given body, the motion is again delivered up to the particles of that body, which in turn begin to vibrate; that is, the body becomes heated. This process of motion through the hypothetical ether is termed radiation, and what is called a 'ray of heat' is merely one series of waves moving in a certain direction.

It will facilitate the understanding of this to consider the analogous mode in which sound is produced and propagated. A sounding body is one whose entire mass is in a state of vibration (222); the more rapid the rate of vibration, the more acute the sound; the slower the rate of vibration, the deeper the sound. This vibratory motion is communicated to the surrounding air, by means of which the vibrations reach the auditory nerve, and there produce the sensation of sound. If a metal ball be heated, say, to the temperature of boiling water, we can ascertain that it radiates heat, although we cannot see any luminosity; and if its temperature be gradually raised, we see it becomes successively of a dull red, bright red, and dazzling white. At each particular temperature the heated body emits waves of a definite length; in other words, its particles vibrate in a certain period. As its temperature rises it sends out other and more rapid vibrations, which coexist, however, with all those which it had previously emitted. Thus the motion at each successive temperature is compounded of all preceding ones.

It has been seen that vibrations of the air below and above a certain rate do not affect the auditory nerve (244); it can only take up and transmit to the brain vibrations of a certain periodicity. So too with the vibrations which produce light. The optic nerve is insensible to a large number of wave-lengths. It can apprehend only those waves that form the visible spectrum. If the rate of undulation be slower than the red or faster than the violet, though intense motion may pass through the humours of the eye and fall upon the retina, yet we shall be utterly unconscious of the fact, for the optic nerve cannot take up and respond to the rate of vibrations which exist beyond the visible spectrum in both directions. Hence, these are termed *invisible* or *obscure* rays. A vast quantity of these obscure rays is emitted by flames which, though intensely hot, are yet almost non-luminous, such as the oxy-hydrogen flame, or that of a Bunsen's burner; for the vibrations which these emit, though capable in part of penetrating the media of the eye, are incapable of exciting in the optic nerve the sensation of light.

430. **Thermal analysis of solar light.**—When a beam of sunlight (fig. 397), admitted through an aperture in a dark room, is concentrated on a prism of rock salt by means of a lens of the same material, and then, after emerging from the prism, is received on a screen, it will be found to present a band of colours in the following order: red, orange, yellow, green, blue, and violet. This is called the *spectrum* (564).

If now a narrow and delicate thermopile be placed successively on the space occupied by each of the colours, it will be scarcely affected on the violet, but in passing over the other colours it will indicate a gradual rise of temperature, which is greatest at the red. Painters, thus guided by a correct but unconscious feeling, always speak of blue and green colours as cold, and of red and orange as warm tones. If the pile be now moved in the same direction beyond the limits of the luminous spectrum, the temperature will gradually rise up to CP, at which it attains its maximum. From this point the pile indicates a decrease of temperature until it reaches a point, O, where it ceases to be affected. This point is about as distant from R as the latter is from V; that is, there is a region in which thermal effects are pro-

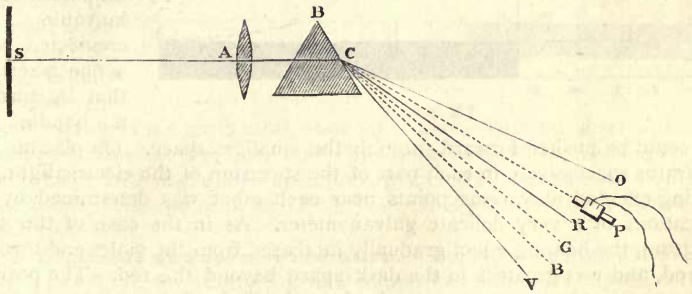


Fig. 397.

duced extending as far beyond the red end of the spectrum in one direction as the entire length of the visible spectrum in the other. In accordance with what we have stated, the sun's light consists of rays of different rates of vibration; by their passage through the prism they are unequally broken or refracted; those of greatest wave-length or slowest vibrating period are least bent aside, or are said to be the least refrangible, while those with shorter wave-lengths are the most refrangible.

These non-luminous rays outside the red are called the extra or ultra-red rays, or sometimes the *Herschelian* rays, from Sir W. Herschel, who first discovered their existence.

If, in the above case, prisms of other materials than rock salt be used, the position of the maximum heat will be found to vary with the nature of the prism, a fact first noticed by Seebeck. Thus with a prism of water it is in the yellow, with one of crown glass in the middle of the red, and so on. These changes are due to the circumstance that prisms of different materials absorb rays of different refrangibility to unequal extents. But rock salt practically allows heat of all kinds to pass with equal facility, and thus gives a normal spectrum.

431. **Tyndall's researches.**—Tyndall investigated the spectrum produced by the electric light, by the following mode of experimenting :—The electric light was produced between charcoal points by a Grove's battery of fifty cells. The beam, rendered parallel by a double rock-salt lens, was caused to pass through a narrow slit, and then through a second lens of rock salt; the slices of white light thus obtained being decomposed by a prism of the same material. To investigate the thermal conditions of the spectrum a *linear* thermo-electric pile was used; that is, one consisting of a number of elements arranged in a line, and in front of which was a slit that could be narrowed to any extent. The instrument was mounted on a movable bar connected with a fine screw, so that by turning a handle the pile could be pushed forward through the smallest space. On placing this apparatus successively in each part of the spectrum of the electric light, the heating effected at various points near each other was determined by the indications of a very delicate galvanometer. As in the case of the solar spectrum, the heating effect gradually increased from the violet end towards the red, and was greatest in the dark space beyond the red. The position of the greatest heat was about as far from the limit of the visible red as the latter was from the green, and the total extent of the invisible spectrum was found to be twice that of the visible.

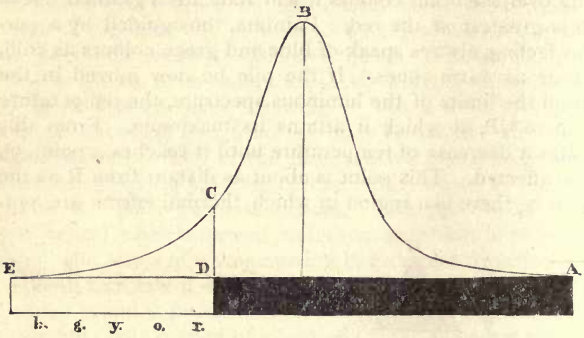


Fig. 398.

The increase of temperature in the dark space is very considerable. If thermal intensities are represented by perpendicular lines of proportionate length, erected at those parts of the spectrum to which they correspond, on passing beyond the red end these lines increase rapidly and greatly in length, reach a maximum, and then fall somewhat more suddenly. If these lines are connected, they form a curve (fig. 398), which beyond the red represents a peak, quite dwarfing that of the visible spectrum. In fig. 399, the dark parts at the end represent the obscure radiation. The curve is based, in the manner above stated, on the results obtained by Tyndall with the electric light. The upper curve in fig. 399 represents the spectrum of sunlight with a rock-salt prism, while the lower curve represents the results obtained with a flint-glass prism, which is thus seen to absorb some of the ultra-red radiation.

By interposing various substances, more especially water, in certain thicknesses, in the path of the electric light, the ultra-red radiation was greatly diminished. Now aqueous vapour, like water, absorbs the obscure rays. And probably the reason why the obscure part of the spectrum of

sunlight is not so intense as in the case of the electric light is that the obscure rays have been already partially absorbed by the aqueous vapour of the atmosphere. If a solar spectrum could be produced outside the atmosphere, it would probably give a spectrum more like that of the electric light, which is unaffected by the atmospheric absorption.

This has been confirmed in other ways. Melloni observed that the position of the maximum in the solar spectrum differs on different days; which is probably due to the varying absorption of the atmosphere, in consequence of its varying hygrometric state. Secchi, in Rome, found the same shifting of the maximum to occur in the different seasons of the year; for in winter, when there is least moisture in the atmosphere, the maximum is farther from the red than in summer, when the aqueous vapour in the air is most abundant. An important observation on the luminous rays has also been made by Cooke, in America, who found that the faint black lines in the solar spectrum attributed to the absorption of light by our atmosphere (see book on Optics) are chiefly caused by the presence of aqueous vapour.

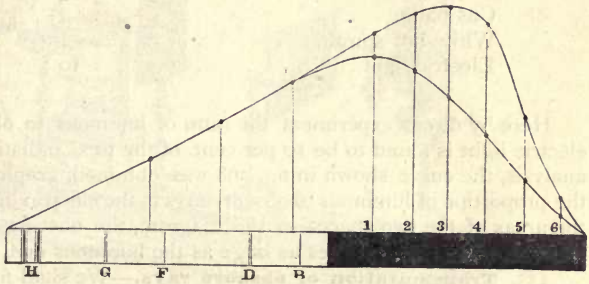


Fig. 399.

432. **Luminous and obscure radiation.**—The radiation from a luminous object, a gas flame, for example, is of a composite character; a portion consists of what we term light, but a far greater part consists of heat rays, which are insensible to our eyes, being unable to affect the optic nerve. When this mixed radiation falls upon the blackened face of a thermo-electric pile, the whole of it is taken to be absorbed, the light by this act being converted into heat, and affecting the instrument proportionally with the purely calorific rays. The total radiation of a luminous source, expressed in units of heat or force, can thus be measured. By introducing into the path of the rays a body capable of stopping either the luminous or the obscure radiation, we can ascertain by the comparative action on the pile the relative quantities of heat and light radiated from the source. Melloni sought to do this by passing a luminous beam through a layer of water containing alum in solution; a liquid which he found in previous experiments absorbed all the radiation from bodies heated under incandescence. Comparing the transmission through this liquid—which allowed the luminous but not the obscure part of the beam to pass—with the transmission through a plate of rock salt—which affected neither the luminous nor the obscure radiation, but gave the loss due to reflection—Melloni found that 90 per cent. of the radiation from an oil flame and 99 per cent. of the radiation from an alcohol flame consist of invisible calorific rays. Tyndall employed a solution of iodine in bisulphide of carbon, which he found to be impervious to

the most intense light, but very pervious to radiant heat; only a slight absorption being effected by the bisulphide. By comparing the transmission through the transparent bisulphide, and the transmission through the same liquid rendered opaque by iodine, the value of the luminous radiation from various sources was found to be as follows :—

Source	Luminous	Obscure
Red-hot spiral	0	100
Hydrogen flame	0	100
Oil flame	3	97
Gas flame	4	96
White-hot spiral	4·6	95·4
Electric light	10	90

Here by direct experiment the ratio of luminous to obscure rays in the electric light is found to be 10 per cent. of the total radiation. By prismatic analysis, the curve shown in fig. 398 was obtained, graphically representing the proportion of luminous to obscure rays in the electric light; by calculating the areas of the two spaces in the diagram, the obscure portion, DCBA, is found to be nearly 16 times as large as the luminous one, DCE.

433. **Transmutation of obscure rays.**—We shall find, in speaking of the luminous spectrum, that beyond the violet there are rays which are invisible to the eye, but which are distinguished by their chemical action, and are spoken of as the *actinic* or chemical rays; they are also known as the *Ritteric* rays, from the philosopher who first discovered their existence.

As we shall afterwards see, in the book on Optics, Stokes has succeeded in converting these rays into rays of lower refrangibility, which then become visible; so Tyndall has effected the corresponding but inverse change, and has increased the refrangibility of the Herschelian or extra red rays, and thus rendered them visible. The charcoal points of the electric light were placed in front of a concave silvered glass mirror in such a manner that the rays from the points after reflection were concentrated to a focus about 6 inches distant. On the path of the beam was interposed a cell full of a solution of iodine in bisulphide of carbon, which (432) has the power of completely stopping all luminous radiation, but gives free passage to the non-luminous rays. On now placing in the focus of the beam, thus sifted, a piece of platinum, it was raised to incandescence by the impact of perfectly invisible rays. In like manner a piece of charcoal *in vacuo* was heated to redness.

By a proper arrangement of the charcoal points a metal may be raised to whiteness, and the light now emitted by the metal yields on prismatic analysis a brilliant luminous spectrum, which is thus entirely derived from the invisible rays beyond the red. This transmutation of non-luminous into luminous heat, Tyndall calls *calorescence*.

When the eye was cautiously placed in the focus, guarded by a small hole pierced in a metal screen, so that the converged rays should only enter the pupil and not affect the surrounding part of the eye, no impression of light was produced, and there was scarcely any sensation of heat. A considerable portion was absorbed by the humours of the eye, but yet a powerful beam undoubtedly reached the retina; for, as Tyndall showed by a

separate experiment, about 18 per cent. of the obscure radiation from the electric light passed through the humours of an ox's eye.

434. **Transmission of thermal rays.**—Melloni was the first who examined extensively and accurately the absorption of heat by solids and liquids. The apparatus he employed is represented in fig. 400, where AB is the thermo-electric pile ; *a* is a support for the source of heat, in this case a Locatelli's lamp ; F and E are screens, and C is a support for the body experimented on ; while *m* is the support for the pile, and D the galvanometer

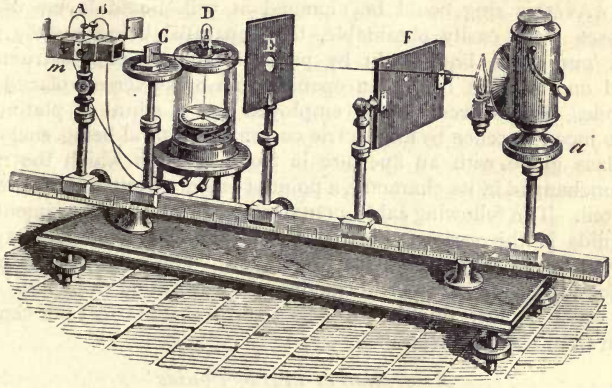


Fig. 400.

To express the power which bodies have of transmitting heat, Melloni used the term *diathermancy* : diathermancy bears the same relation to radiant heat that transparency does to light ; and in like manner the power of stopping radiant heat is called *athermancy*, which thus corresponds to opacity for light. In experimenting on the diathermancy of liquids, Melloni used glass troughs with parallel sides, the thickness of the liquid layer being 0·36 in. The radiant heat of an Argand lamp with a glass chimney was first allowed to fall directly on the face of the pile, and the deflection produced in the galvanometer taken as the total radiation ; the substance under examination was then interposed, and the deflection noted. This corresponded to the quantity of heat transmitted by the substance. If t indicate this latter number, and t' the total radiation, then

$$t' : t :: 100 : x,$$

which is the percentage of rays transmitted. Thus calling the total radiation 100, Melloni found that

Bisulphide of carbon transmitted	. . . . .	63
Olive oil	„	30
Ether	„	21
Sulphuric acid	„	17
Alcohol	„	15
Solution of alum or sugar	„	12
Distilled water	„	11

In experimenting with solids they were cut into plates 0·1 inch in thickness, and it was found that of every 100 rays there was transmitted by

Rock salt	92	Selenite . . .	20
Smoky quartz . . .	67	Alum	12
Transparent carbonate of lead	52	Sulphate of copper .	0

The transmission of heat through liquids has been re-examined by Tyndall, who used a cell consisting of parallel plates of rock salt separated by a ring of brass with an aperture on the top through which the liquid could be poured. As this ring could be changed at will, liquid layers of various thicknesses were easily obtainable, the apparatus being merely screwed together and made liquid-tight by paper washers. The instrument was mounted on a support before an opening in a brass screen placed in front of the pile. The source of heat employed was a spiral of platinum wire raised to incandescence by an electric current, the spiral being enclosed in a small glass globe with an aperture in front, through which the radiation passed unchanged in its character, a point of essential importance overlooked by Melloni. The following table contains the results of experiments made with liquids in the various thicknesses indicated, the numbers expressing the *absorption* per cent. of the total radiation. The *transmission* per cent. can be found in each case by subtracting the absorption from 100. Thus a layer of water 0·2 inch thick absorbs 80·7 and transmits 19·3 per cent. of the radiation from a red-hot spiral.

Absorption of heat by liquids

Liquid	Thickness of liquids in parts of an inch				
	0·02	0·04	0·07	0·14	0·27
Bisulphide of carbon .	5·5	8·4	12·5	15·2	17·3
Chloroform . . .	16·6	25·0	35·0	40·0	44·8
Iodide of methyl . .	36·1	46·5	53·2	65·2	68·6
Benzole	43·4	55·7	62·5	71·5	73·6
Amylene	58·3	65·2	73·6	77·7	82·3
Ether	63·3	73·5	76·1	78·6	85·2
Alcohol	67·3	78·6	83·6	85·3	89·1
Water	80·7	86·1	88·8	91·0	91·0

It appears from these tables that there is no connection between diathermancy and transparency. The liquids, except olive oil, are all colourless and transparent, and yet vary as much as 75 per cent. in the amount of heat transmitted. Among solids, smoky quartz, which is nearly opaque to light, transmits heat very well; while alum, which is perfectly transparent, cuts off 88 per cent. of heat rays. As there are different degrees of transparency, so there are different degrees of diathermancy; and the one cannot be predicated from the other.

By studying the transmission of heat from different parts of the spectrum separately, the connection between light and heat becomes manifest. With this view Masson and Jamin received the spectrum of the solar light

given by a prism of rock salt on a movable screen provided with an aperture, so that by raising or lowering the screen the action of any given part of the spectrum on different plates could be investigated. They thus found—

That glass, rock crystal, ice, and generally substances transparent for light, are also diathermanous for all kinds of *luminous* heat ;

That a coloured glass, red, for instance, which only transmits the red rays of the spectrum, and extinguishes the others, also extinguishes every kind of luminous heat, excepting that of the red rays ;

That glass and rock crystal, which are diathermanous for luminous heat, also transmit the obscure heat near the red—that is, the most refrangible—but extinguish the extreme obscure rays, or those which are the least deflected by the prism. Alum extinguishes a still greater proportion of the obscure spectrum, and ice stops it altogether.

Knoblauch has shown that very thin layers of gold, silver, and platinum, which are known to transmit luminous rays of a definite colour, also allow rays of heat to pass ; so that these substances are diathermanous, though in a small degree. This is also the case with thin sheets of ebonite.

435. **Influence of the nature of the heat.**—The diathermanous power differs greatly with the heat from different sources, as is seen from the following table, in which the numbers express what proportion of every 100 rays from the different sources of heat incident on the plates is transmitted :—

	Locatelli's lamp	Incandescent platinum wire	Copper at 400°	Copper at 100°
Rock salt . . .	92	92	92	92
Fluor spar. . .	78	69	42	33
Plate glass . .	39	24	6	0
Black glass . .	26	55	12	0
Selenite . . .	14	5	0	0
Alum . . .	9	2	0	0
Ice	6	0·5	0	0

These different sources of heat correspond to light from different sources. Rock salt is here stated to transmit all kinds of heat with equal facility, and to be the only substance which does so. It is analogous to white glass, which is transparent for light from all sources. Fluor spar transmits 78 per cent. of the rays from a lamp, but only 33 of those from a blackened surface at 100°. A piece of plate glass only one-tenth of an inch thick, and perfectly transparent to light, is opaque to all the radiation from a source of 100°, transmits only 6 per cent. of the heat from a source at 400°, and but 39 of the radiation from the lamp. Black glass, on the contrary, though it cuts off all heat from a source at 100°, allows 12 per cent. of the heat at 400° to pass, and is equally transparent to the heat from the spiral, but on account of its blackness is more opaque to the heat from the lamp. As we have already seen, every luminous ray is a heat ray ; now as several of the substances in this table are pervious to all the luminous rays, and yet, as in the case of ice, transmit about 6 per cent. of luminous heat, we have an apparent

anomaly ; which, however, is only a confirmation of the remarkably small ratio which the luminous rays of a lamp bear to the obscure.

From these experiments Melloni concluded that as the temperature of the source rose, more heat was transmitted. This has been confirmed by some experiments of Tyndall. The platinum lamp (434) was used as the source, the temperature of which could be varied from a dark to a brilliant white heat, by a gradual augmentation of the strength of the electric current which heated the platinum spiral. Instead of liquids, vapours were examined in a manner to be described subsequently ; the measurements are given in the following table :—

Absorption of heat by vapours

Name of vapour	Source, platinum spiral			
	Barely visible	Bright red	White hot	Near fusion
Bisulphide of carbon .	6·5	4·7	2·9	2·5
Chloroform . . .	9·1	6·3	5·6	3·9
Iodide of methyl . .	12·5	9·6	7·8	—
Benzole	26·4	20·6	16·5	—
Ether	43·4	31·4	25·9	23·7
Formic ether . . .	45·2	31·9	25·1	21·3
Acetic ether . . .	49·6	34·6	27·2	—

The percentage of rays absorbed is here seen to diminish in each case as the temperature of the source rises. Mere elevation of temperature does not, however, invariably produce a high penetrative power in the rays emitted ; the rays from sources of far higher temperature than any of the foregoing are more largely absorbed by certain substances than are the rays emitted from any one of the sources as yet mentioned. Thus, the radiation from a hydrogen flame was completely intercepted by a layer of water only 0·27 of an inch thick, the same layer transmitting 9 per cent. of the radiation from the red-hot spiral, a source of much lower temperature. The explanation of this is, that those rays which heated water emits (and water, the product of combustion, is the main radiant in a hydrogen flame) are the very ones which this substance most largely absorbs. This statement, which will become clearer after considering the analogous phenomena in the case of light, was exemplified by the powerful absorption of the heat from a carbonic oxide flame by carbonic acid gas. It will be seen presently (438) that of the rays from a heated plate of copper, olefiant gas absorbs 10 times the quantity intercepted by carbonic acid, whilst of the rays from a carbonic oxide flame Tyndall found carbonic acid absorbed twice as much as olefiant gas. A tenth of an atmosphere of carbonic acid, inclosed in a tube 4 feet long, absorbs 60 per cent. of the radiation from a carbonic oxide flame. Radiant heat of this character can thus be used as a delicate test for the presence of carbonic acid, the amount of which may even be accurately measured by the same means. Prof. Barrett made in this way a *physical* analysis of the human breath. In one experiment, the carbonic acid con-

tained in breath physically analysed was found to be 4·65 per cent., whilst the same breath chemically analysed gave 4·66 per cent.

436. **Influence of the thickness and nature of screens.**—It will be seen from the table (435) that of every 100 rays rock salt transmits 92. The other 8 may either have been absorbed or reflected from the surface of the plate. According to Melloni, the latter is the case; for if—instead of on one plate—heat be allowed to fall on two or more plates whose total thickness does not exceed that of the one, the quantity of heat arrested will be proportional to the number of reflecting surfaces. He therefore concluded that rock salt was quite diathermanous.

The experiments of later observers show that this conclusion is not strictly correct; rock salt does absorb a very small proportion of obscure rays.

The quantity of heat transmitted through rock salt is practically the same whether the plate be 1, 2, or 4 millimetres thick. But with other bodies absorption increases with the thickness, although by no means in direct proportion. This is seen to be the case in the table of absorption by liquids at different thicknesses. The following table tells what proportion of 1,000 rays from a Locatelli's lamp pass through a glass plate of the given thickness:—

Thickness in millimetres	0·5	1	2	3	4	5	6	7	8
Rays transmitted . .	775	733	682	653	634	620	609	600	592

The absorption takes place in the first layers; the rays which have passed these possess the property of passing through other layers in a higher degree, so that beyond the first layers the heat transmitted approaches a certain constant value. If a thin glass plate be placed behind another glass plate a centimetre thick, the former diminishes the transmission by little more than the reflection from its surface. But if a plate of alum were placed behind the glass plate, the result would be different, for the latter is opaque for much of the heat transmitted by glass.

Heat, therefore, which has traversed a glass plate traverses another plate of the same material with very slight loss, but is very greatly diminished by a plate of alum. Of 100 rays which had passed through green glass or tourmaline, only 5 and 7 were respectively transmitted by the same plate of alum. A plate of blackened rock salt only transmits obscure rays, while alum extinguishes them. Consequently, when these two substances are superposed, a system impervious to light and heat is obtained.

These phenomena find their exact analogies in the case of light. The different sources of heat correspond to flames of different colours, and the screens of various materials to glasses of different colours. A red flame looked at through a red glass appears quite bright, but through a green glass it appears dim or is scarcely visible. So in like manner heat which has traversed a red glass passes through another red glass with little diminution, but it is almost completely stopped by a green glass. Rock salt at 150° emits only one kind of heat; it is monothermal, just as sodium vapour is monochromatic.

Different luminous rays being distinguished by their *colours*, Melloni gave the name of *thermocrose* or heat coloration to these different obscure

calorific rays. The invisible portion of the spectrum is accordingly mapped out into a series of spaces, each possessing its own peculiar feature corresponding to the coloured spaces which are seen in that portion of the spectrum visible to our eyes.

Besides thickness and colour, the polish of a substance influences the transmission. Glass plates of the same size and thickness transmit more heat as their surface is more polished. Bodies which transmit heat of any kind very readily are not heated. Thus a window pane is not much heated by the strongest sun's heat; but a glass screen held before a common fire stops most of the heat, and is itself heated thereby. The reason of this is that by far the greater part of the heat from a fire is obscure, and glass is opaque to this kind of heat.

437. **Diffusion of heat.**—When a ray of light falls upon an unpolished surface in a definite direction, it is decomposed into a variety of rays which are reflected from the surface in all directions. This irregular reflection is called *diffusion*, and it is in virtue of it that bodies are visible when light falls upon them. A further peculiarity is, that all solar rays are not equally diffused from the surface of bodies. Certain bodies diffuse certain rays and absorb others, and accordingly appear coloured. The red colour of a geranium is caused by its absorbing all the rays, excepting the red, which are irregularly reflected. Just as is the case with transmitted light in transparent bodies, so with diffused light in opaque ones; for if a red body is illuminated by red light it appears of a bright red colour, but if green light fall upon it it is almost black. We shall now see that here again analogous phenomena prevail with heat.

Various substances diffuse the several thermal rays to a different extent; each possesses a peculiar thermocrose. Melloni placed a number of strips of brass foil between the source of heat and the thermo-pile. They were coated on the side opposite to the pile with lampblack, and on the other side with the substances to be investigated. Representing the quantity of heat absorbed by the lampblack by 100, the absorption of the other bodies was as follows:—

	Incandescent platinum	Copper at 400°	Copper at 100°
Lampblack	100	100	100
White lead	56	89	100
Isinglass	54	64	91
Indian ink	95	87	85
Shellac	47	70	72
Polished metal . . .	13·5	13	13

Hence white lead absorbs far less of the heat radiated from incandescent platinum than lampblack, but it absorbs the obscure rays from copper at 100° as completely as lampblack. Indian ink is the reverse of this; it absorbs obscure rays less completely than luminous rays. Lampblack absorbs the heat from all sources in equal quantities, and very nearly completely. In consequence of this property all thermoscopes which are used for investigating radiant heat are covered with lampblack, as it is the best-

known absorbent of heat. The behaviour of metals is the reverse of that of lampblack. They reflect the heat of different sources in the same degree. They are to heat what *white* bodies are to light.

As coloured light is altered by diffusion from several bodies, so Knoblauch has shown that the different kinds of heat are altered by reflection from different surfaces. The heat of an Argand lamp diffused from white paper passes more easily through calcspar than when it has been diffused from black paper.

The rays of heat, like the rays of light, are susceptible of polarisation and double refraction. These properties will be better understood after treating of light.

438. **Relation of gases and vapours to radiant heat.**—This subject has been investigated by Tyndall ; the apparatus he used is represented in the adjacent figure, the arrangement being looked upon from above.

A (fig. 401) is a cylinder about 4 feet in length and $2\frac{1}{2}$ inches in diameter, placed horizontally, the ends of which can be closed with rock-salt plates ; by means of a lateral tube at *r* it can be connected with an air-pump and

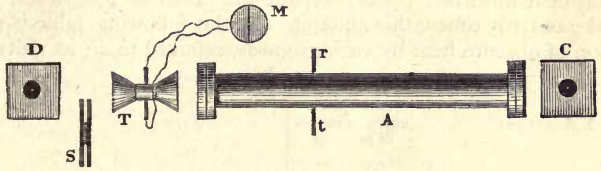


Fig. 401.

exhausted ; while at *t* is another tube which serves for the introduction of gases and vapours. T is a sensitive thermo-pile connected with an extremely delicate galvanometer, M.

The deflections of this galvanometer were proportional to the degrees of heat up to about 30° ; beyond this point the proportionality no longer held good, and accordingly, for the higher degrees, a table was empirically constructed, in which the value of the higher deflections was expressed in units ; the unit being the amount of heat necessary to move the needle through one of the lower degrees.

C was a source of heat, which usually was either a Leslie's cube filled with boiling water, or else a sheet of blackened copper heated by gas. Now, when the source of heat was permitted to radiate through the exhausted tube, the needle made a great deflection ; and in this position a very considerable degree of absorption would have been needed to produce an alteration of 1° of the galvanometer. And if to lessen this deflection a lower source of heat had been used, the fraction absorbed would be correspondingly less, and might well have been insensible. Hence Tyndall adopted the following device, by which he was enabled to use a powerful flux of heat, and at the same time to discover small variations in the quantity falling on the pile.

The source of heat at C was allowed to radiate through the tube at the end of which the pile was placed ; a deflection was produced of, say, 70° ; a second source of heat, D, was then placed near the other face of the pile, the amount of heat falling on the pile from this *compensating* cube being

regulated by means of a movable screen S. When both faces of the pile are warmed, two currents are produced, which are in opposite directions, and tend, therefore, to neutralise each other: when the heat on both faces is precisely equal, the neutralisation is perfect, and no current at all is produced, however high may be the temperature on both sides. In the arrangement just described, by means of the screen S, the radiation from the compensating cube was caused to neutralise exactly the radiation from the source C; the needle consequently was brought down from 70° to zero, and remained there so long as both sources were equal. If now a gas or vapour be admitted into the exhausted tube, any power of absorption it may possess will be indicated by the destruction of this equilibrium, and preponderance of the radiation from the compensating cube, by an amount corresponding to the heat cut off by the gas. Examined in this way, air, hydrogen, and nitrogen, when dried by passing through sulphuric acid, were found to exert an almost inappreciable effect; their presence as regards radiant heat being but little different from a vacuum. But with olefiant and other complex gases the case was entirely different. Representing by the number 1 the quantity of radiant heat absorbed by air, olefiant gas absorbs 970 times, and ammoniacal gas 1,195 times, this amount. In the following table is given the absorption of obscure heat by various gases, referred to air as unity:—

Name of gas	Absorption under 30 inches of pressure	Name of gas	Absorption under 30 inches of pressure
Air	1	Carbonic acid	90
Oxygen	1	Nitrous oxide	335
Nitrogen	1	Marsh gas	403
Hydrogen	1	Sulphurous acid	710
Chlorine	39	Olefiant	970
Hydrochloric acid	62	Ammonia	1195

If, instead of comparing the gases at a common pressure of one atmosphere, they are compared at a common pressure of an inch, their differences in absorption are still more strikingly seen. Thus, assuming the absorption by 1 inch of dry air to be 1, the absorption by 1 inch of olefiant gas is 7,950, and by the same amount of sulphurous acid 8,800.

439. **Influence of pressure and thickness on the absorption of heat by gases.**—The absorption of heat by gases varies with the pressure; this variation is best seen in the case of those gases which have considerable absorptive power. Taking the total absorption by atmospheric air under ordinary pressure at unity, the numbers of olefiant gas under a pressure of 1, 3, 5, 7, and 10 inches of mercury are respectively 90, 142, 168, 182, and 193. Thus one-thirtieth of an atmosphere of olefiant gas exerts 90 times the absorption of an entire atmosphere of air. And the absorption, it is seen, increases with the density, though not in a direct ratio. Tyndall showed, however, by special experiments, that for very low pressures the absorption does increase with the density. Employing as a unit volume of the gas a quantity which measured only $\frac{1}{50}$ of a cubic inch, and admitting successive measures of olefiant gas into the experimental tube, it was found that

up to 15 measures the absorption was directly proportionate to the density in each case.

In these experiments the length of the experimental tube remains the same whilst the pressure of the gas within it was caused to vary; in subsequent experiments the pressure of the gas was kept constant, whilst the length of the tube was, by suitable means, varied from 0·01 of an inch up to 50 inches. The source was a heated plate of copper; of the total radiation from this nearly 2 per cent. was absorbed by a film of olefiant gas ·01 of an inch thick, upwards of 9 per cent. by a layer of the same gas 0·1 of an inch thick, 33 per cent. by a layer 2 inches thick, 68 per cent. by a column 20 inches long, and 77 per cent. by a column rather more than 4 feet long.

440. **Absorptive power of vapours.**—Tyndall showed that the absorptive power of olefiant gas is exceeded by that of several vapours. The liquid from which the vapour was to be produced was inclosed in a small flask, which could be attached with a stop-cock to the exhausted experimental tube. The absorption was then determined after admitting the vapour into the tube in quantities measured by the pressure of the barometer gauge attached to the air-pump.

The following table shows the absorption of vapours under pressures varying from 0·1 to 0·1 inch of mercury:—

Name of vapours	Absorption under pressure in inches of mercury		
	0·1	0·5	1·0
Bisulphide of carbon . .	15	47	62
Benzole	66	182	267
Chloroform	85	182	236
Ether	300	710	870
Alcohol	325	622	—
Acetic ether	590	980	1195

These numbers refer to the absorption of a whole atmosphere of dry air as their unit, and it is thus seen that a quantity of bisulphide of carbon vapour, the feeblest absorbent yet examined, which only exerts a pressure of $\frac{1}{10}$ of an inch of mercury, or the $\frac{1}{300}$ of an atmosphere, gave fifteen times the absorption of an entire atmosphere of air; and $\frac{1}{10}$ of an inch of acetic ether 590 times as much. Comparing air at a pressure of 0·1 with acetic ether of the same pressure, the absorption of the latter would be more than 17,500 times as great as that of the former.

Tyndall found that the odours from the essential oils exercised a marked influence on radiant heat. Perfectly dry air was allowed to pass through a tube containing dried paper impregnated with various essential oils, and then admitted into the experimental tube. Taking the absorption of dry air as unity, the following were the numbers respectively obtained for air scented with various oils:—Patchouli 31, otto of roses 37, lavender 60, thyme 68, rosemary 74, cassia 109, aniseed 372. Thus the perfume of a flower-bed absorbs a large percentage of the heat of low refrangibility emitted from it.

Ozone prepared by electrolysing water was also found to have a remark-

ably absorptive effect. The small quantity of ozone present in electrolytic oxygen was found in one experiment to exercise 136 times the absorption of the entire mass of the oxygen itself.

But the most important results are those which follow from his experiments on the behaviour of aqueous vapour to radiant heat. The experimental tube was filled with air, dried as perfectly as possible, and the absorption it exercised was found to be one unit. Exhausting the tube, and admitting the ordinary undried, but not specially moist, air from the laboratory, the absorption now rose to 72 units. The difference between dried and undried air can only be ascribed to the aqueous vapour the latter contains. Thus on a day of average humidity the absorptive effect due to the transparent aqueous vapour present in the atmosphere is 72 times as great as that of the air itself, though in quantity the latter is about 200 times greater than the former. Analogous results were obtained on different days, and with specimens of air taken from various localities. When air which had been specially purified and dried was allowed to pass through a tube filled with fragments of moistened glass and examined, it was found to exert an absorption 90 times that of pure air.

In some other experiments Tyndall suppressed the use of rock-salt plates in his experimental tube, and even the tube itself, and yet in every case the results were such as to show the great power which aqueous vapour possesses as an absorbent of radiant heat.

The absorptive action which the aqueous vapour in the atmosphere exerts on the sun's heat has been established by a series of actinometrical observations made by Soret at Geneva and on the summit of Mont Blanc; he found that the intensity of the solar heat on the top of Mont Blanc is $\frac{8}{9}$ of that at Geneva; in other words, that of the heat which is radiated at the height of Mont Blanc, about $\frac{1}{9}$ is absorbed in passing through a vertical layer of the atmosphere 14,436 feet in thickness. The same observer has found that with the sun at heights which are virtually equal there is the smallest transmission of heat on those days on which the pressure of aqueous vapour is greatest; that is, when there is most moisture in the atmosphere.

441. **Radiating power of gases.**—Tyndall also examined the *radiating* power of gases. A red-hot copper ball was placed so that the current of heated air which rose from it acted on one face of a thermo-pile; this action was compensated by a cube of hot water placed in front of the opposite face. On then allowing a current of dry olefiant gas from a gasholder to stream through a ring burner over the heated ball and thus supplant the ascending current of hot air, it was found that the gas radiated energetically. By comparing in this manner the action of many gases it was discovered that, as is the case with solids, those gases which are the best absorbers are also those which radiate most freely.

442. **Dynamic radiation and absorption.**—A gas when permitted to enter an exhausted tube is heated in consequence of the collision of its particles against the sides of the vessel; it thus becomes a source of heat, which is perfectly capable of being measured. Tyndall calls this *dynamic heating*. In like manner, when a tube full of gas or vapour is rapidly exhausted, a chilling takes place owing to the loss of heat in the production of motion; this he calls *dynamic chilling or absorption.*

He could thus determine the radiation or absorption of a gas without any source of heat external to the gas itself. An experimental tube was taken, one end of which was closed with a polished metal plate, and the other with a plate of rock salt ; in front of the latter was the face of the pile. The needle being at zero, and the tube exhausted, a gas was allowed quickly to enter until the tube was full, the effect on the galvanometer being noted. This being only a transitory effect, the needle soon returned to zero ; the tube was then rapidly pumped out, by which a sudden chilling was produced and the needle exhibited a deflection in the opposite direction. Comparing in this way the dynamic heating and chilling of various gases, those gases which are the best absorbers were also found to be the best radiators.

Polished metallic surfaces are, as we have seen (427), bad radiators, but radiate freely when covered with varnish. Tyndall made the curious experiment of varnishing a metallic surface by a film of gas. A Leslie's cube was placed with its polished metal side in front of the pile, and its effect neutralised by a second cube placed before the other face of the pile. On allowing a stream of olefiant or coal gas to flow from a gasholder over the metal face of the first cube, a copious radiation from that side was produced as long as the flow of gas continued. Acting on the principle indicated in the foregoing experiment, Tyndall determined the dynamic radiation and absorption of vapours. The experimental tube containing a vapour under a small known pressure, air was allowed to enter until the pressure inside the tube was the same as that of the atmosphere. In this way the entering air, by its impact against the tube, became heated ; and its particles mixing with those of the minute quantity of vapour present, each of them became, so to speak, coated with a layer of the vapour. The entering air was in this case a source of heat, just as in the above experiments the Leslie's cube was. Here, however, one gas varnished another ; the radiation and subsequently the absorption of various vapours could thus be determined.

It was found that vapours differed very materially in their power of radiating under these circumstances ; of those which were tried bisulphide of carbon vapour was the worst and boracic ether the best radiator. And in all cases those which were the best absorbents were also the best radiators.

443. **Relation of absorption to molecular state.**—After examining the absorption of heat by vapours, Tyndall tried the same substances in a liquid form. The conditions of the experiments were in both cases the same ; the source of heat was a spiral of platinum heated to redness by an electric current of known strength ; and plates of rock salt were invariably employed to contain both vapours and liquids. Finally, the absorption by the vapours was re-measured ; in this case introducing into the experimental tube, not, as before, equal quantities of vapour, but amounts proportional to the density of the liquid. When this last condition had been attained, it was found that the order of absorption by a series of liquids, and by the same series when turned into vapour, was precisely the same. Thus the substances tried stood in the following order as liquid and as vapour, beginning with the feeblest absorbent, and ending with the most powerful :—

Liquids	Vapours
Bisulphide of carbon	Bisulphide of carbon
Chloroform	Chloroform
Iodide of ethyl	Iodide of ethyl
Benzole	Benzole
Ether	Ether
Alcohol	Alcohol
Water	

A direct determination of aqueous vapour could not be made, on account of its small elastic force and the hygroscopic nature of the rock salt. But the undeviating regularity of the absorption by all the other substances in the list, both as liquid and vapour, establishes the fact, which is corroborated by the experiments already mentioned, that aqueous vapour is one of the most energetic absorbents of heat.

In this table it will be noticed that those substances which have the simplest chemical constitution stand first in the list, with one anomalous exception, namely, that of water. In the absorption of heat by gases, Tyndall found that the elementary gases were the feeblest absorbents, while the gases of most complex constitution were the most powerful absorbents. Thus it may be inferred that absorption is mainly dependent on chemical constitution; that is to say, that absorption and radiation are molecular acts independent of the physical condition of the body.

Tyndall discovered that the radiation of powders is similar to that of the solids from which they were derived, and therefore differs greatly *inter se*. The absorbent power of powders was also found to correspond with their radiative power—which, as we have shown, is the case with solids and gases, and, though as yet we have no experiments on the subject, is doubtless also true for liquids. The powders were attached to the tin surfaces of a Leslie's cube, in such a manner that radiation took place from the surface of the powder alone. The following table gives the radiation in units from some of the powders examined by Tyndall; the metal surface of the cube giving a deflection of 15 units :—

Radiation from powders

Rock salt	35·3	Sulphate of calcium	77·7
Biniodide of mercury	39·7	Red oxide of iron	78·4
Sulphur	40·6	Hydrated oxide of zinc	80·4
Carbonate of calcium	70·2	Sulphide of iron	81·7
Red oxide of lead	74·0	Lampblack	84·0

These substances are of various colours. Some are white, such as rock salt, carbonate and sulphate of calcium, and hydrated oxide of zinc; some are red, such as biniodide of mercury and oxide of lead; whilst others are black, as sulphide of iron and lampblack; we have besides other colours. The colours, therefore, have no influence on the radiating power: rock salt, for example, is the feeblest radiator, and hydrated oxide of zinc one of the most powerful radiators.

Nearly a century ago Franklin made experiments on coloured pieces of cloth, and found their absorption, indicated by their sinking into snow on

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which they were placed, to increase with the darkness of the colour. But all the cloths were equally powerful absorbents of obscure heat, and the effects noticed were only produced by their relative absorptions of light. In fact, the conclusions to be drawn from Franklin's experiments only hold good for luminous heat, especially sunlight such as he employed.

444. **Applications.**—The properties which bodies possess of absorbing, emitting, and reflecting heat meet with numerous applications in domestic economy and in the arts. Leslie stated in a general manner that white bodies reflect heat very well, and absorb very little, and the contrary is the case with black substances. As we have seen, this principle is not generally true, as Leslie supposed; for example, white lead has as great an absorbing power for non-luminous rays as lampblack (437). Leslie's principle applies to powerful absorbents like cloth, cotton, wool, and other organic substances when exposed to luminous heat. Accordingly, the most suitable coloured clothing for summer is just that which experience has taught us to use, namely, white, for it absorbs less of the sun's rays than black clothing, and hence feels cooler.

The polished fire-irons before a fire are cold, whilst the black fender is often unbearably hot. If, on the contrary, a liquid is to be kept hot as long as possible, it must be placed in a brightly polished metallic vessel, for then, the emissive power being less, the cooling is slower. Hence it is advantageous that the steam pipes, &c., of locomotives should be kept bright. In the Alps, the mountaineers accelerate the fusion of the snow by covering it with earth, which increases the absorbing power.

In our dwellings, the outsides of stoves and of hot-water apparatus ought to be black, and the insides of fireplaces ought to be lined with firebrick, in order to increase the radiating power towards the apartment.

It is in consequence of the great diathermancy of dry atmospheric air that the higher regions of the atmosphere are so cold, notwithstanding the great heat which traverses them; whilst the intense heat of the sun's direct rays on high mountains is probably due to the comparative absence of aqueous vapour at these elevations.

As nearly all the luminous rays of the sun pass through water, and the sun's radiation as we receive it on the surface of the earth consists of a large proportion of luminous rays, accidents have often arisen from the convergence of these luminous rays by bottles of water which act as lenses. In this way gunpowder could be fired by the heat of the sun's rays concentrated by a water lens; and the drops of water on leaves in greenhouses have, it is said, been found to act as lenses, and burn the leaves on which they rest.

Certain bodies can be used (436) to separate the heat and light radiated from the same source. Rock salt covered with lampblack, or still better with iodine, transmits heat, but completely stops light. On the other hand, alum, either as a plate or in solution, or a thin layer of water, is permeable to light, but stops all the heat from obscure sources. This property is made use of in apparatus which are illuminated by the sun's rays, in order to sift the rays of their heating power; and a vessel full of water or a solution of alum is used with the electric light when it is desirable to avoid too intense a heat.

In gardens, the use of shades to protect plants depends partly on the diathermancy of glass for heat from luminous rays and its athermancy for obscure rays. The heat which radiates from the sun is largely of the former quality, but by contact with the earth it is changed into obscure heat, which, as such, cannot retraverse the glass. This explains the manner in which greenhouses accumulate their warmth, and also the great heat experienced in summer in rooms having glass roofs, for the glass in both cases acts, as it were, as a valve which effectually entraps the solar rays. On the same principle plates of glass are frequently used as screens to protect us from the heat of a fire; the glass allows us to see the cheerful light of the fire, but intercepts the larger part of the heat radiated from the fire. Though the screens thus become warm by the heat they have absorbed, yet, as they radiate this heat in all directions towards the fire as well as towards us, we finally receive less heat when they are interposed.

445. **Attraction and repulsion arising from radiation.**—Crookes has discovered a highly remarkable class of phenomena which are due to the radiant action of heated and of luminous bodies. These phenomena are most conveniently illustrated by means of an instrument which he has devised and which is called the *radiometer*, the construction of which is as follows:—A glass tube (fig. 402), with a bulb blown on it, is fused at the bottom to a glass tube which at one end serves to rest the whole apparatus in a wooden support. In the other end is fused a fine steel point. On this rests a small vane or fly, consisting of four arms of aluminium wire fixed at one end to a small cap, while at the others are fixed small discs or lozenges of thin mica, coated on one side with lampblack. The weight of the fly is not more than two grains.

In order to keep the fly on the pivot a tube is fused in the upper part of the bulb which reaches down to and just surrounds the top of the cap, without, however, touching it; the other end of this tube is drawn out and connected with an arrangement for exhausting the air by the Sprengel pump (205) or by chemical means; when the desired degree of exhaustion has been attained this can be sealed. By keeping the apparatus during exhaustion in a hot air bath at a temperature of 300°, the gases occluded on the inner surface of the glass, and by the vanes, are got rid of.

If a source of light or of heat, a candle for instance, is brought near the fly, it is attracted, and the fly rotates slowly in a direction showing that the blackened side moves towards the light; this movement, indicating an attraction, depends on a certain state of rarefaction. If, however, the apparatus be connected with an arrangement which allows the pressure to be varied, this rotation gradually diminishes in rapidity as the air within is further rarefied, until a certain point is reached at which it ceases. If now the rarefaction is pushed further, the highly remarkable phenomenon is observed that repulsion succeeds to attraction, and that the fly now rotates in the direction away from the source of heat. In a double radiometer, in which two flys are pivoted independently one over the other, having their blackened sides opposite each other, the flys rotate in opposite directions on the approach of a lighted candle. When a cold body, such as a piece of ice, is brought near, instead of a hot one, exactly the opposite effects are

observed ; when the vessel contains air a pith ball suspended at one end of a light arm is repelled, the neutral point is observed, while at high degrees of rarefaction attraction ensues.

One of the most important facts brought to light by these experiments is, that what has hitherto been looked upon as a complete vacuum is not so in reality ; the most perfect vacuum obtainable still contains a certain residue of gas, as has been proved by the experiments of Crookes and others, among which that of Kundt may be mentioned. The latter placed on the vanes a light disc of mica, and at a little distance above it a similar disc was arranged so as to rotate freely, in a horizontal plane independently of the first. When the lower vane was made to rotate by bringing a light near, it was found that the upper disc was also put in rotation in the same direction, being dragged by the viscosity of the residual air. Accordingly the explanation of the phenomena of the radiometer must take into account the existence of this gaseous residue.

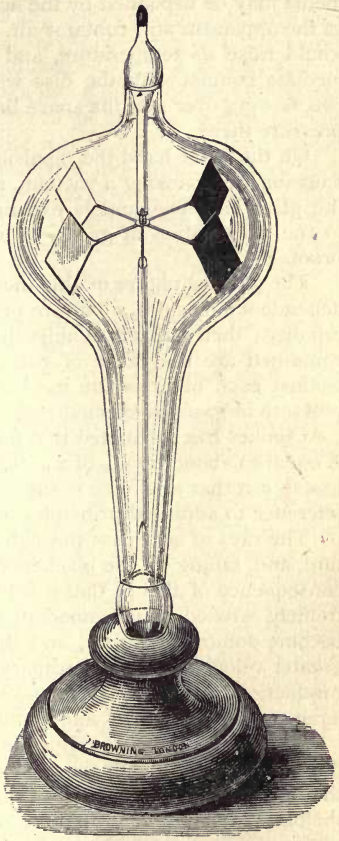


Fig. 402.

The nature of the gas seems to have no special influence on the phenomena ; whether the vacuum be one of hydrogen, of aqueous vapour, or of iodine vapour, seems immaterial ; though with hydrogen the exhaustion need not be pushed so far as with air. The repulsion takes place with all the rays of the spectrum, the intensity diminishing from the ultra red to the ultra violet. When the chemical rays act, the interposition of a plate of alum has no effect, while a solution of iodine in bisulphide of carbon diminishes the repulsion. The rate at which the vane rotates depends on the intensity of the source of light. With a strong light the rotation is so rapid that its rate cannot be determined. With two candles at the same distance the rotation is twice as rapid as with one. Two sources of light which, successively placed at the same distance, produce the same rate of rotation, are equal in intensity. If, when placed at different distances, they produce the same speed of rotation, their intensities are directly as the squares of these distances from the radiometer. On this is based the use of the instrument as a photometer (509) for comparing together various sources of artificial light. It may likewise be used for making comparative measurements of the intensity of

sunlight ; and the distribution of heat in the solar spectrum may be investigated by its means.

It is not difficult to understand that the attraction observed in the experiments may be explained by the action of convection currents (408), as long as the apparatus still contains air. For heat falling upon this blackened disc would raise its temperature, and the temperature of a layer of air in immediate contact with the disc would be raised too ; it would expand and rise, flowing over into the space behind the disc, and would thus increase the pressure there.

On the other hand the repulsion observed at the higher degrees of exhaustion, approaching a vacuum, is due to a reaction between the vane and the glass envelope, and is explained by reference to the modern views as to the constitution of gases, of which it is at once an illustration and a proof.

The general nature of this theory is that a gas is an assemblage of independent molecules, which are perfectly elastic, and which move with great rapidity ; their impacts against the sides of the vessel in which the gas is contained are the cause of the pressure. The impact of the molecules against each other is the mechanism by which the equal transmission of pressure in gases is effected (294).

Crookes has calculated that the mechanical effect of the force of repulsion is equal to about the $\frac{1}{100}$ of a milligramme on a square centimetre, and Stoney has shown that this force is sufficient to account for the effects observed, by reference to admitted principles of the mechanical theory of gases.

The rays of heat pass through the thin glass without raising its temperature, and, falling on the blackened side of the vane, are absorbed by it ; the consequence of this is, that it will become slightly hotter. The layer of extremely rarefied air in immediate contact with the blackened disc will also become somewhat hotter, and the molecules will fly from the disc with greater velocity. Under ordinary pressures or even at moderate degrees of rarefaction these more rapid motions would be equalised by their impacts against other molecules, and a uniformity of pressure—that is, of temperature —would be established. But the frequency of these intramolecular shocks diminishes rapidly with the increase of rarefaction ; and the consequence is, that a great number of molecules, after having been heated by contact with the blackened side of the palette, will strike against the cold glass. The effect of this will be to cool these molecules—that is, to diminish their velocity ; it will be chiefly molecules of this kind which fall on the back of the disc, and on the regions behind it. An excess of force equal and opposite to that on the glass acts against the front of the disc, and is sufficient to account for the phenomena exhibited by Crookes.

It follows from this explanation that, other things being equal, a fly will rotate more rapidly in a small than in a large bulb. This has been conclusively proved by Crookes, who constructed a double-bulb radiometer, the two bulbs being very different in size, and so connected that, by dexterous manipulation, the fly could be transferred from the pivot of the one to that of the other bulb.

The radiometer is well adapted for the lecture demonstration of many phenomena in heat. Thus the law of the inverse square (414) may be illus-

trated by counting the number of rotations when the instrument is placed at varying distances from the source of heat.

446. **Internal friction or viscosity of gases.**—In some recent experiments in connection with the radiometer, Crookes used an arrangement consisting of a long but light arm of straw suspended by a delicate glass fibre in a sort of T tube turned upside down ; in this way even a greater degree of delicacy was obtained than with the radiometer. Thus he was able to get a deflection by moonlight, which does not move the fly of the radiometer. He examined the internal friction or viscosity of the residual gas by causing the arm to oscillate, and then observing the rate at which the oscillations diminish under various pressures. He thus found that from ordinary pressures down to a pressure of 0·19 mm., or what may be called a Torricellian vacuum, the viscosity is practically constant, only diminishing from 0·126 to 0·112. It now begins to fall off, and at a pressure of 0·000076 mm. it has diminished to 0·01, or about $\frac{1}{12}$. Simultaneously with this decrease in viscosity the force of repulsion excited by a standard light on a blackened surface varies. It increases as the pressure diminishes until the exhaustion is about 0·05 mm., and attains its maximum at about 0·03 mm. It then sinks very rapidly until it is at 0·000076 mm., when it is less than $\frac{1}{10}$ of its maximum.

The viscosity varies in different gases ; it is considerably less in hydrogen than in air ; and hence with this gas it is not necessary to drive the exhaustion so far to produce a considerable degree of repulsion.

The researches of Crookes have opened the way to an entirely new field of experimental inquiry into the phenomena which occur in what is called the ultra-gaseous state of matter, or that in which the rarefaction of gases is pushed to its utmost limits. The state in which *molecular*, as distinguished from *molar*, actions come into play, has been aptly termed *Crookes's vacuum*. A further account of the researches requires too great an amount of detail for the purposes of this work ; and it must also be added that the explanations which have been given are still not beyond the range of controversy.

446*a*. **Relation of radiant heat to sound.**—This subject has of late engaged the attention of several physicists, among whom may be particularised Bell and Tainter, Tyndall, Preece, and Mercadier. A convenient way of showing the phenomena is by means of an apparatus constructed by Duboscq, the essential features of which are represented in fig. 404. It is an arrangement by which an intermittent beam of radiant heat may be made to act on various bodies, and consists of a disc D mounted on a horizontal axis, and which, by means of the multiplying wheels P and P′, may be rotated at any desired speed. In the disc is a series of holes, the numbers of which are in some multiple of the ratio 4 : 5 : 6 : 8. This apparatus constitutes in fact a sirene (242), and is very convenient for lecture purposes. If, while the disc is rotating with uniform speed, a current of air be successively directed against the rows of holes from the inside to the outside, we get the fundamental note, the third, the fifth, and the octave.

On the stand is a support on which the arrangement O may be fixed in any position by means of the screw *s* ; it consists of a screen and wide tube behind which is the source of radiant heat, not represented in the figure. To this support may be fitted a double convex lens, if the rays are to be

concentrated on one line of holes, or a cylindrical lens when a slice of thermal rays is to be used ; or the rays may be concentrated by a mirror, to get rid of the effects of absorption by glass. The support S is for holding various pieces of apparatus.

Tyndall found that when a flask like that represented in fig. 403, containing a small quantity of ether, was placed so that the intermittent beam arising from a lime-light could fall on it, and the top was connected with a flexible tube, a distinct musical note was heard when the ear-trumpet was held to the ear. Other liquids being tried it was found that those which his other experiments had revealed as the best absorbers of heat (440) gave the loudest sounds. The vapour was the operative cause, for when the beam was caused to strike against the liquid instead of against the vapour no sound was heard ; this was also the case when the rays fell on a rock-salt

Fig. 403.

Fig. 405.

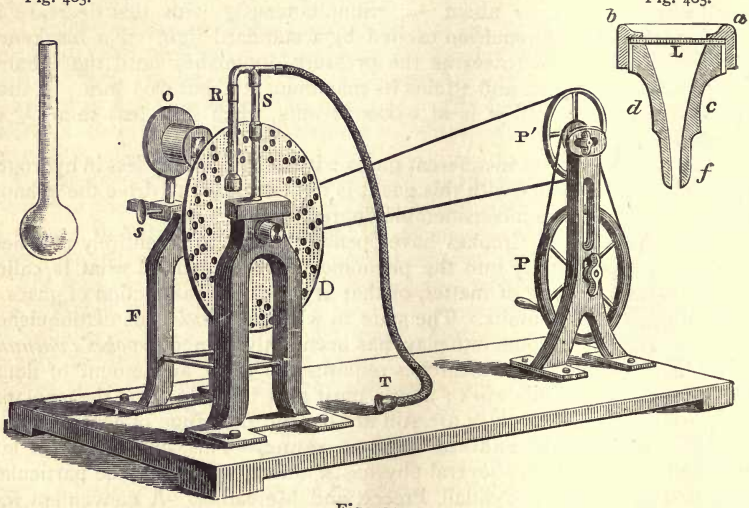


Fig. 404.

cell filled with the liquid. The pitch of the note depended on the velocity of rotation.

Dry air gave no sound, but air containing moisture did so ; and the more moisture was present the louder was the sound. Other gases gave sounds in the order of their absorption for heat ; and, indeed, all Tyndall's results in this direction are most strikingly confirmed.

The investigations of the other experimenters, Preece, Bell and Tainter, and Mercadier, were chiefly directed to the effects produced when the intermittent beam is allowed to fall on solid bodies. A sort of an ear-trumpet (fig. 405) was used by Mercadier, consisting of a movable piece *ab* fitting over *cd* so that plates L of various materials could be experimented upon. The other end *f* is fitted with a flexible tube and bell so that it could be applied to the ear.

When the intermittent beam is allowed to act on this plate it is set in

vibration and a sound is produced. This is not due, at any rate mainly, to transverse vibrations of the plate, for neither the pitch nor the quality of the note was altered when the thickness and nature of the plate were changed (282), nor was it altered when the plate was slit. The best effects were obtained when the diaphragm was of thin metal foil coated with lampblack on the side next the rays. Marked effects were also obtained when a transparent plate was used blackened on the side away from the rays. The effect is one of radiant heat, and is essentially due to alternate expansions and contractions of the layer of air in contact with the surfaces which absorb the radiant heat. The phenomenon may be very simply exhibited by blackening half the inside of a test-tube R, the open end of which is provided with a flexible tube which can be placed to the ear. When the rays fall on the blackened part a loud sound is heard, but very little when the bright side is exposed. The effect is also obtained when a blackened piece of foil is placed in the tube.

CHAPTER IX

CALORIMETRY

447. **Calorimetry. Thermal unit.**—The province of calorimetry is to measure the *quantity of heat* which a body parts with or absorbs, when its temperature sinks or rises through a certain number of degrees, or when it changes its condition.

Quantities of heat may be expressed by any of its directly measurable effects, but the most convenient is the alteration of temperature, and quantities of heat are usually defined by stating the extent to which they are capable of raising a known weight of a known substance, such as water. The unit chosen for comparison, and called the *thermal unit*, is not everywhere the same. In France it is the quantity of heat necessary to raise the temperature of *one* kilogramme of water through *one* degree Centigrade; this is called a *calorie*. In this book we shall adopt, as a thermal unit, *the quantity of heat necessary to raise one pound of water through one degree Centigrade:* 1 *calorie* = 2·2 thermal units, and one thermal unit = 0·45 *calorie*.

On the centimetre-gramme-second system of units the heat required to raise one gramme of water through one degree is taken as the unit. This is called the *gramme-degree* or *water-gramme-degree*.

448. **Specific heat.**—When equal weights of two different substances, at the same temperature, placed in similar vessels, are subjected for the same length of time to the heat of the same lamp, or are placed at the same distance in front of the same fire, it is found that their temperatures will vary considerably; thus mercury will be much hotter than water. But as, from the conditions of the experiment, they have each been receiving the same amount of heat, it is clear that the quantity of heat which is sufficient to raise the temperature of mercury through a certain number of degrees, will raise the temperature of the same quantity of water only through a less number of degrees; in other words, that it requires more heat to raise the temperature of water through one degree than it does to raise the temperature of mercury by the same extent. Conversely, if the same quantities of water and of mercury at 100° C. be allowed to cool down to the temperature of the air, the water will require a much longer time for the purpose than the mercury; hence, in cooling through the same number of degrees, water gives out more heat than does mercury.

It is readily seen that all bodies have not the same specific heat. If a pound of mercury at 100° is mixed with a pound of water at zero, the temperature of the mixture will be about 3° only; that is to say, that while the mercury has cooled through 97°, the temperature of the water has been raised only 3°. Consequently the same weight of water requires about 32 times as much heat as mercury does, to produce the same elevation of temperature.

If similar experiments are made with other substances, it will be found that the quantity of heat required to effect a certain change of temperature is different for almost every substance, and we speak of the *specific heat*, or *thermal* or *calorific capacity*, of a body as the quantity of heat which it absorbs when its temperature rises through a given range of temperature, from zero to 1° for example, compared with the quantity of heat which would be absorbed, in the same circumstances, by the same weight of water ; that is, water is taken as the standard for the comparison of specific heats. Thus, to say that the specific heat of lead is 0·0314, means that the quantity of heat which would raise the temperature of any given weight of lead through 1° C. would raise the temperature of the same weight of water through only 0·0314° C.

Temperature is the *vis viva* of the smallest particles of a body ; in bodies of the same temperature the atoms have the same *vis viva*, the smaller mass of the lighter atoms being compensated by their greater velocity. The heat absorbed by a body not only raises its temperature—that is, increases the *vis viva* of the progressive motion of the atoms—but in overcoming the attraction of the atoms it moves them further apart, and along with the expansion which this represents, some external pressure is overcome. In the conception of specific heat is included not merely that amount of heat which goes to raise the temperature, but also that necessary for the internal work of expansion, and that required for the external work. If these latter could be separated, we should get the true *heat of temperature*, that which is used solely in increasing the *vis viva* of the atoms. This is sometimes called the *true specific heat.*

Three methods have been employed for determining the specific heats of bodies : (i.) the method of the melting of ice, (ii.) the method of mixtures, and (iii.) that of cooling. In the latter, the specific heat of a body is determined by the time which it takes to cool through a certain temperature. Previously to describing these methods, it will be convenient to explain the expression for the quantity of heat absorbed or given out by a body of known weight and specific heat, when its temperature rises or falls through a certain number of degrees.

449. **Measure of the sensible heat absorbed by a body.**—Let m be the weight of a body in pounds, c its specific heat, and t its temperature. The quantity of heat necessary to raise a pound of water through one degree being taken as unity, m of these units would be required to raise m pounds of water through one degree, and to raise it through t degrees, t times as much, or mt. As this is the quantity of heat necessary to raise through t degrees m pounds of water, whose specific heat is unity, a body of the same weight, only of different specific heat, would require mtc. Consequently, when a body is heated through t degrees, the quantity of heat which it absorbs is *the product of its weight into the range of temperature into its specific heat.* This principle is the basis of all the formulæ for calculating specific heats.

If a body is heated or cooled from t to t' degrees, the heat absorbed or disengaged will be represented by the formula

$$m(t'-t)c, \text{ or } m(t-t')c.$$

450. **Method of the fusion of ice.**—This method of determining specific heats is based on the fact that to melt a pound of ice 80 thermal units are necessary, or more exactly 79·25. Black's calorimeter (fig. 406) consists of a block of ice in which a cavity is made, and which is provided with a cover of ice. The substance whose specific heat is to be determined is heated to a certain temperature, and is then placed in the cavity, which is covered. After some time the body becomes cooled to zero. It is then opened, and both the substance and the cavity wiped dry with a sponge which has been previously weighed. The increase of weight of this sponge obviously represents the ice which has been converted into water.

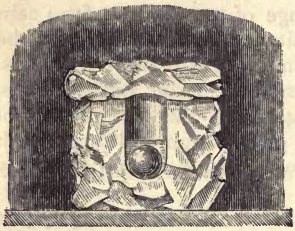


Fig. 406.

Now, since one pound of ice at 0° in melting to water at 0° absorbs 80 thermal units, P pounds absorbs 80 P units. On the other hand this quantity of heat is equal to the heat given out by the body in cooling from $t°$ to zero, which is mtc, for it may be taken for granted that in cooling from $t°$ to zero a body gives out as much heat as it absorbs in being heated from zero to $t°$. Consequently from

$$mtc = 80\ \mathrm{P} \text{ we have } c = \frac{80\mathrm{P}}{mt}.$$

It is difficult to obtain blocks of ice as large and pure as those used by Black in his experiments, and Lavoisier and Laplace replaced the block of ice by a more complicated apparatus which is called the *ice calorimeter*. Fig. 407 gives a perspective view of it, and fig. 408 represents a section. It consists of three concentric tin vessels; in the central one is placed the body M, whose specific heat is to be determined, while the other two are filled with pounded ice. The ice in the compartment A is melted by the heated body, while the ice in the compartment B cuts off the heating influence of the surrounding atmosphere. The two stopcocks E and D give issue to the water which arises from the melting of the ice.

Fig. 407.

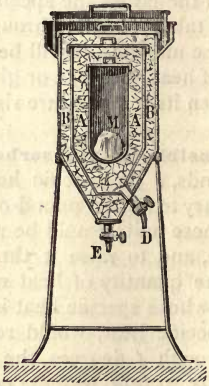


Fig. 408.

In order to find the specific heat of a body by this apparatus, its weight, m, is first determined; it is then raised to a given temperature, t, by keeping

it for some time in an oil or water bath, or in a current of steam. Having been quickly brought into the central compartment, the lids are replaced and covered with ice, as represented in the figure. The water which flows out by the stopcock D is collected. Its weight, P, is manifestly that of the melted ice. The calculation is then made as in the preceding case.

There are many objections to the use of this apparatus. From its size it requires some quantity of ice, and a body, M, of large mass; while the experiment lasts a considerable time. A certain weight of the melted water remains adhering to the ice, so that the water which flows out from D does not exactly represent the weight of the melted ice.

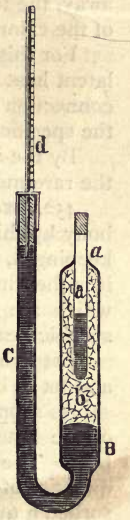


Fig. 409.

451. **Bunsen's ice calorimeter.**—On the very considerable diminution of volume which ice experiences on passing into water (347), Bunsen has based a calorimeter which is particularly suitable when only small quantities of a substance can be used in determinations. A small test-tube *a* (fig. 409) intended to receive the substance experimented upon is fused in the wider tube B. The part *ab* contains pure freshly boiled distilled water, and the prolongation of this tube BC, together with the capillary tube *d*, contains pure mercury. This tube *d* is firmly fixed to the end of the tube C; it is graduated, and the value of each division of the graduation is specially determined by calibration. When the apparatus is immersed in a freezing mixture, the water in the part *ab* freezes. Hence, if afterwards, while the apparatus is protected against the excess of heat from without, a weighed quantity of a substance at a given temperature is introduced into the tube, it imparts its heat to this in sinking to zero. In doing so it melts a certain quantity of ice, which is evidenced by a corresponding depression of the mercury in the tube *d*. Thus the weight of ice melted, and the weight and original temperature of the substance experimented upon, furnish all the data for calculating the specific heat.

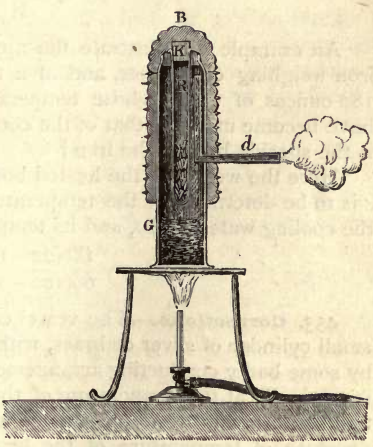


Fig. 410.

For heating the substance in this, and also in other calorimetrical experiments, the apparatus fig. 410 is well adapted. The cylindrical metal vessel G is narrower at the top, and a glass test-tube R is fitted into a cork which closes G. In this glass tube, which is also closed by a cork K, the substance is placed which is to be heated. The greater part of the vessel is covered by a thick mantle of felt, B. The water in the vessel is boiled, the steam emerging at *d*, until the

substance has acquired the temperature of boiling water, for which about twenty minutes is required. The mantle and the lamp having been taken away, the tube R is rapidly removed, and its contents tipped into the tube *a* of the calorimeter (fig. 407).

For this method of determining specific heat a new determination of the latent heat of ice was made, and it was found to be 80·025. It was also in connection with these experiments that Bunsen made his determination of the specific gravity of ice, which he found to be in the mean 0·91674.

By the above method Bunsen determined the specific heat of several of the rare metals for which a weight of only a few grains could be used.

452. **Method of mixtures.**—In determining the specific heat of a solid body by this method, it is weighed and raised to a known temperature, by keeping it, for instance, for some time in a closed place heated by steam; it is then immersed in a mass of cold water, the weight and temperature of which are known. From the temperature of the water after mixture the specific heat of the body is determined.

Let M be the weight of the body, T its temperature, c its specific heat; and let m be the weight of the cold water, and t its temperature.

As soon as the heated body is plunged into the water, the temperature of the latter rises until both are at the same temperature. Let this point be θ. The heated body has been cooled by $T-\theta$; it has, therefore, lost a quantity of heat, $M(T-\theta)c$. The cooling water has, on the contrary, absorbed a quantity of heat equal to $m(\theta-t)$, for the specific heat of water is unity. Now the quantity of heat given out by the body is manifestly equal to the quantity of heat absorbed by the water; that is, $M(T-\theta)c=m(\theta-t)$, from which

$$c=\frac{m(\theta-t)}{M(T-\theta)}.$$

An example will illustrate the application of this formula. A piece of iron weighing 60 ounces, and at a temperature of 100° C., is immersed in 180 ounces of water, whose temperature is 19° C. After the temperatures have become uniform, that of the cooling water is found to be 22° C. What is the specific heat of the iron?

Here the weight of the heated body, M, is 60, the temperature, T, is 100°, c is to be determined; the temperature of mixture, θ, is 22°, the weight of the cooling water is 180, and its temperature 12°. Therefore

$$c=\frac{180(22-19)}{60(100-22)}=\frac{9}{78}=0{\cdot}1153.$$

453. **Corrections.**—The vessel containing the cooling water is usually a small cylinder of silver or brass, with thin polished sides, and is supported by some badly conducting arrangement. It is obvious that this vessel, which is originally at the temperature of the cooling water, shares its increase of temperature, and in accurate experiments this must be allowed for. The decrease of temperature of the heated body is equal to the increase of temperature of the cooling water, and of the vessel in which it is contained. If the weight of this latter be m', and its specific heat c', its temperature, like that of the water, is t: consequently the previous equation becomes

$$Mc(T-\theta)=m(\theta-t)+m'c'(\theta-t);$$

from which, by obvious transformations,

$$c = \frac{(m + m'c')\,(\theta - t)}{M(T - \theta)}.$$

Generally speaking, the value $m'c'$ is put $= \mu$; that is to say, μ is the weight of water which would absorb the same quantity of heat as the vessel. This is said to be the *reduced value* in water of the vessel, or the *water-equivalent.* This expression accordingly becomes

$$c = \frac{(m + \mu)\,(\theta - t)}{M(T - \theta)}.$$

In accurate experiments it is necessary to allow also for the heat absorbed by the glass and mercury of the thermometer, by introducing into the equation their values reduced on the same principle.

In order to allow for the loss of heat due to radiation, a preliminary experiment is made with the body whose specific heat is to be determined, the only object of which is to ascertain approximately the increase of temperature of the cooling water. If this increase be 10°, for example, the temperature of the water is reduced by half this number—that is to say, 5°—below the temperature of the atmosphere, and the experiment is then carried out in the ordinary manner.

By this method of compensation, first introduced by Rumford, the water receives as much heat from the atmosphere, during the first part of the experiment, as it loses by radiation during the second part.

454. **Regnault's apparatus for determining specific heats.**—Fig. 411 represents one of the forms of apparatus used by Regnault in determining specific heats during the method of mixtures.

The principal part is a water bath, AA, of which fig. 412 represents a section. It consists of three concentric compartments; in the central one there is a small basket of brass wire, *c*, containing fragments of the substance whose specific heat is to be determined, in the middle of which is placed a thermometer, T. The second compartment is heated by a current of steam coming through the tube, *e*, from a boiler B, and passing into a worm, *a*, where it is condensed. The third compartment, *ii*, is an air chamber to hinder the loss of heat. The water bath, AA, rests on a chamber, K, with double sides, EE, forming a jacket, which is kept full of cold water, in order to exclude the heat from AA, and from the boiler, B. The central compartment of the water bath is closed by a damper, *r*, which can be opened at pleasure, so that the basket, *c*, can be lowered into the chamber, K.

On the left of the figure is represented a small and very thin brass vessel, D, suspended by silk threads on a small carriage, which can be moved out of, or into, the chamber, K. This vessel, which serves as a calorimeter, contains water, in which is immersed a thermometer, *t*. Another thermometer at the side, *t'*, gives the temperature of the air.

When the thermometer T shows that the temperature of the substance in the bath is stationary, the screen, *h*, is raised, and the vessel, D, moved to just below the central compartment of the water bath. The damper, *r*, is then withdrawn, and the basket, *c*, and its contents are lowered into the water in the vessel, D, the thermometer, T, remaining fixed in the cork. The

carriage and the vessel, D, are then moved out, and the water agitated until the thermometer, T, becomes stationary. The temperature which it indicates is θ. This temperature known, the rest of the calculation is made in the manner described in art. 449, care being taken to make all the necessary corrections.

In determining the specific heat of substances—phosphorus, for instance—which could not be heated without causing them to melt, or undergo some change which would interfere with the accuracy of the result, Regnault adopted an inverse process: he cooled them down to a temperature considerably below that of the water in the calorimeter, and then observed the diminution in the temperature of the latter, which resulted from immersing the cooled substance in it.

In determining the specific heat of substances, which, like potassium, would decompose water, some other liquid, such as turpentine or benzole,

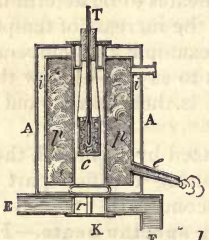


Fig. 412.

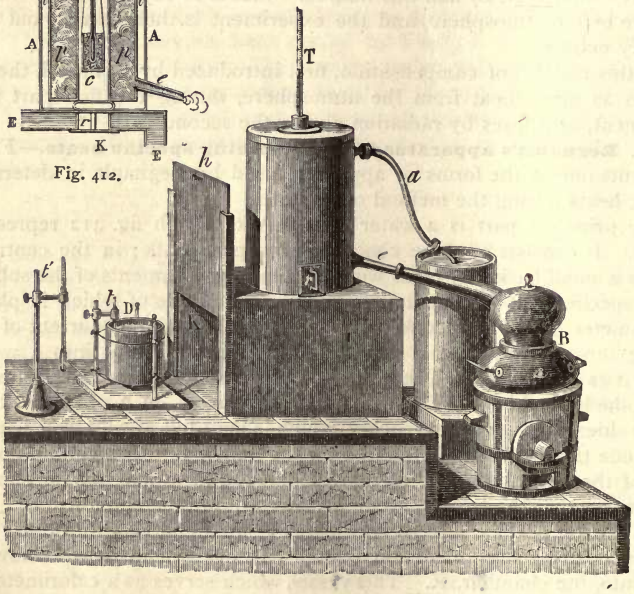


Fig. 411.

must be used. These liquids have the additional advantage of having a lower specific heat than water, which has the highest of any liquid, so that an error in determining the temperature of the cooling liquid has a less influence on the value of the specific heat. With this view use has been made of mercury, the specific heat of which is only one-thirtieth that of water.

455. **Method of cooling.**—Equal weights of different bodies whose specific heats are different will occupy different times in cooling through the same number of degrees. Dulong and Petit applied this principle in determining the specific heats of bodies in the following manner :—A small polished silver vessel is filled with the substance in a state of fine powder, and a thermometer placed in the powder, which is pressed down. This vessel is heated to a certain temperature, and is then introduced into a copper vessel, in which it fits hermetically. This copper vessel is exhausted, and maintained at the constant temperature of melting ice, and the time noted which the substance takes in falling through a given range of temperature, from 15° to 5° for example. The times which equal weights of different bodies require for cooling, through the same range of temperature, are directly as their specific heats.

Regnault has proved that with solids this method does not give trustworthy results ; it assumes, which is not quite the case, that the cooling in all parts is equal, and that all substances part with their heat to the silver case with equal facility. The method may, however, be employed with success in the determination of the specific heat of liquids.

In an investigation of the specific heats of various soils, Pfaundler found that a soil of low specific heat heats and cools rapidly, while earth of higher specific heat undergoes slow heating and slow cooling ; that moist earths rich in humus have a high specific heat, amounting in the case of turf to as much as 0·5 ; while dry soils free from humus, such as lime and sand, have a low specific heat, not more than about 0·2.

456. **Specific heat of liquids.**—The specific heat of liquids may be determined either by the method of cooling, by that of mixtures, or by that of the ice calorimeter. In the latter case they are contained in a small metal vessel, or a glass tube, which is placed in the central compartment (fig. 412), and the experiment then made in the usual manner.

A method devised by Pfaundler of determining the specific heat of liquids, which under certain circumstances is advantageous, depends on a property of the electrical current of heating any conductor through which it passes.

In two equal calorimeters containing the liquids to be tested, together with suitable thermometers and stirrers, two equal spirals of fine platinum wire are placed. These are connected with a voltaic battery by means of copper wires, and if the same current of electricity be simultaneously passed through each of them, which can be very accurately done, the heat produced in the wires will be equal, and the rise in temperature in the liquids will then be inversely as the specific heats. One of the liquids is usually water.

It will be seen from the table in the following article that water and oil of turpentine have a much greater specific heat than other substances, and more especially than the metals. It is because of its great specific heat that water requires a long time to heat or to cool, and that for the same weight and temperature it absorbs or gives out far more heat than other substances. This double property is applied in the hot-water apparatus, of which we shall afterwards speak, and it plays a most important part in the economy of nature.

457. **Specific heats of bodies.**—The list contained in the next article (458) gives the specific heats of a great number of elementary substances. We give here the specific heats of a few substances, mostly liquids :—

	Specific heat		Specific heat
Turpentine	0·426	Bisulphide of carbon	0·245
Alcohol	0·062	Thermometer glass	0·198
Ether	0·516	Steel	0·118
Glycerine	0·555	Brass	0·094

The specific heat of water is commonly taken at unity, which is not strictly correct. According to the most recent determinations the *mean* specific heat between 0° and t is expressed by the formula $1 + 0{\cdot}00015t$.

These numbers, as well as the numbers in (458), represent the mean specific heats between 0° and 100°. It was shown by Dulong and Petit that the specific heats increase with the temperature. Those of the metals, for instance, are greater between 100° and 200° than between 0° and 100°, and are still greater between 200° and 300° ; that is to say, a greater amount of heat is required to raise a body from 200° to 250° than from 100° to 150°, and still more than from 0° to 50°. For silver, the mean specific heat between 0° and 100° is 0·057, while between 0° and 200° it is 0·0611. The following table gives the specific heats at various temperatures :—

Copper	$0{\cdot}0910 + 0{\cdot}000046t$
Zinc	$0{\cdot}0865 + 0{\cdot}000088t$
Lead	$0{\cdot}0286 + 0{\cdot}000038t$
Platinum	$0{\cdot}0317 + 0{\cdot}0000062t$
Water	$1 + 0{\cdot}00030t$

The increase of specific heat with the temperature is greater as bodies are nearer their fusing point. Any action which increases the density and molecular aggregation of a body, diminishes its specific heat. Thus hard steel with the density 7·798 has the sp. heat 0·1175 ; while that of soft steel of density 7·861 is 0·1165. The specific heat of copper is diminished by its being hammered, but it regains its original value after the metal has been again heated.

The specific heat of a liquid increases with the temperature much more rapidly than that of a solid. Water is, however, an exception : its specific heat increases less rapidly than does that of solids.

The most remarkable examples of the influence of temperature on the specific heat are afforded by carbon, boron, and silicon. Weber has found that at 600° the specific heat of carbon is 7 times, and that of boron 2½ times, as great as their respective specific heats at −50°. The specific heat of diamond is 0·0635 at −50°, 0·1318 at 33°, 0·2218 at 140°, and 0·3026 at 247°. Although the specific heat increases thus rapidly between −50° and 250°, beyond that point the rate of increase is slower ; and beyond 600°, or at an incipient red heat, it seems to be pretty constant, or at any rate to exhibit no greater variations with the temperature than are afforded by other substances. Thus while at 600° the specific heat is 0·441, at 985° it is 0·459. Graphite also has at 22° the specific heat 0·168 ; this increases, but at a gradually diminishing rate, to 642°, where its specific heat is 0·445. Like

diamond, an incipient red heat seems to be a limiting temperature beyond which graphite exhibits only the ordinary variation with the temperature. Weber has also found that, in their thermal deportment, there are only two essentially different modifications of carbon—the transparent one (diamond), and the opaque ones (graphite, dense amorphous carbon, and porous amorphous carbon).

Crystallised boron is similar in its deportment to carbon ; its specific heat increases from 0·1915 at −40° to 0·2382 at 27°, and to 0·3663 at 233°. The rate of increase is very rapid up to 80° ; it increases beyond that temperature, but at a gradually diminished rate, and, no doubt, tends to an almost constant value of 0·5.

The specific heat of silicon also varies with the temperature ; between −40° and 200° it increases from 0·136 to 0·203 ; the rate of increase is less rapid with higher temperatures, being at 200° only $\frac{1}{14}$ what it is at 10°. At 200° it reaches its limiting value.

The specific heat of substances is greater in the liquid than in the solid state, as will be seen by the following table :—

	Solid	Liquid
Water	0·502	1·000
Sulphur	0·203	0·234
Bromine	0·084	0·110
Iodine	0·054	0·008
Mercury	0·031	0·033
Phosphorus	0·190	0·212
Tin	0·056	0·064
Lead	0·031	0·040

It also differs with the *allotropic* modification ; thus the specific heat of red phosphorus is 0·19, and that of white 0·17 ; of crystallised arsenic 0·083, and of amorphous 0·058 ; of crystallised selenium 0·084, and of amorphous 0·0953 ; of wood charcoal 0·241 ; of graphite 0·202 ; and of diamond 0·147.

Pouillet used the specific heat of platinum for measuring high degrees of heat. Supposing 200 ounces of platinum had been heated in a furnace, and had then been placed in 1,000 ounces of water, the temperature of which it had raised from 13° to 20°. From the formula we have $M = 200$, $m = 1000$; θ is 20, and t is 13. The specific heat of platinum is 0·033, and we have therefore, from the equation—

$$Mc(T - \theta) = m(\theta - t)$$

$$T = \frac{m(\theta - t) + Mc\theta}{Mc} = \frac{7000 + 132}{6{\cdot}6} = \frac{7132}{6{\cdot}6} = 1080°.$$

It is found, however, that the mean specific heat of platinum at temperatures up to about 1200° is 0·0377 ; if this value, therefore, be substituted for c in the equation, we have—

$$T = \frac{7150{\cdot}8}{7{\cdot}54} = 948° \text{ C.}$$

By this method, which requires great skill in the experimenter, Pouillet determined a series of high temperatures. He found, for example, the temperature of melting iron to be 1500° to 1600° C.

458. **Dulong and Petit's law.**—A knowledge of the specific heat of bodies became of great importance, in consequence of Dulong and Petit's discovery of the remarkable law, that the product of the specific heat of any solid element into its atomic weight is approximately a constant number, as will be seen from the following table :—

	Specific heat	Atomic weight	Atomic heat
Aluminium	0·2143	27·4	5·87
Antimony	0·0513	122	6·26
Arsenic	0·0822	75	6·17
Bismuth	0·0308	210	6·47
Bromine	0·0843	80	6·74
Cadmium	0·0567	112	6·35
Cobalt	0·1067	58·7	6·26
Copper	0·0939	63·5	5·99
Gold	0·0324	197	6·38
Iodine	0·0541	127	6·87
Iron	0·1138	56	6·37
Lead	0·0314	207	6·50
Magnesium	0·2475	24	5·94
Mercury	0·0332	200	6·64
Nickel	0·1092	58·7	6·41
Phosphorus	0·1740	31·0	5·39
Platinum	0·0324	197·5	6·40
Potassium	0·1655	39·1	6·47
Silver	0·0570	108·0	6·16
Sulphur	0·178	32	5·70
Tin	0·0555	118	6·55
Zinc	0·0950	65·2	6·23

It will be seen that the number is not a constant, but varies between 5·39 and 6·87. These variations may depend partly on the difficulty of getting the elements in a state of perfect purity, and partly on errors incidental to the determination of the specific heats, and of the atomic weights. Again, the specific heats of bodies vary with the state of aggregation of the bodies, and also with the temperatures at which they are determined; some, such as potassium, have been determined at temperatures very near their fusing points; others, like platinum, at temperatures much removed from them. A prominent cause, therefore, of the discrepancies is doubtless to be found in the fact that all the determinations have not been made under corresponding physical conditions.

The atomic weights of the elements represent the relative weights of equal number of atoms of these bodies, and the product, pc, of the specific heat, c, into the atomic weight, p, is the *atomic heat*, or the quantity of heat necessary to raise the temperature of the same number of atoms of different substances by one degree; and Dulong and Petit's law may be thus expressed : *the same quantity of heat is needed to heat an atom of all simple bodies to the same extent.*

The atomic heat of a body, when divided by its specific heat, gives the

atomic weight of a body. Regnault proposed to use this relation as a means of determining the atomic weight, and it certainly is of great service in deciding on the atomic weight of a body in cases where the chemical relations permit a choice between two or more numbers.

According to modern views, the heat imparted to a body is partly expended in external work, which in the case of a solid would be extremely small, being only that required for raising the pressure of the atmosphere through a distance representing the expansion; secondly, the internal work, or the heat used in overcoming the attraction of the atoms for each other, and forcing them apart; and thirdly, there is the *true specific heat*, or the heat applied in increasing the temperature—that is, in increasing the *vis viva* of the molecules (448). By far the most considerable of these is the latter; the amount of heat consumed in the two former operations is small, and the variations with different bodies must be inconsiderable. Until, however, the relation between the various factors is made out, absolute identity in the numbers for the atomic specific heat cannot be expected. Weber holds that even when due allowance has been made for these circumstances, the variations are too great to be accounted for, and he considers that they point for their explanation to an alteration in the constitution of the atom, and render probable a changing *valency* of the atom of carbon.

459. **Specific heat of compound bodies.**—In compound bodies the law also prevails: the product of the specific heat into the equivalent is an almost constant number, which varies, however, with different classes of bodies. Thus, for the class of oxides of the general formula RO, it is 11·30; for the sesquioxides R^2O^3 it is 27·15; for the sulphides RS, it is 18·88; and for the carbonates RCO^3, it is 21·54. The law, which is known as *Neumann's law*, may be expressed in the following general manner:—*With compounds of the same formula, and of a similar chemical constitution, the product of the atomic weight into the specific heat is a constant quantity.* This includes Dulong and Petit's law as a particular case.

Kopp propounded the following law, which is an extension of that of Neumann:—*The molecular heats of all solid bodies are equal to the sum of the molecular heats of the elements contained in them.* Dulong and Petit's law that all elements have the same atomic heat he does not consider universally valid. He assigns the number 6·4 to all elements excepting the following; with sulphur and phosphorus it is 5·4, fluorine 5·0, oxygen 4·0, silicon 3·8, boron 2·7, hydrogen 2·3, and carbon 1·8.

Even with this modification it is found that the calculated heats of compounds differ more from the observed ones than can be ascribed to errors in the determination of the specific heats. This is probably due to the fact that some of the heat is expended in internal work, and that it is this which brings about the discrepancies.

With mixtures of alcohol and water in certain proportions, the specific heat is greater than that of the water; thus, that of a mixture containing 20 per cent. of alcohol was found by Dupré and Page to be 1·0456. No general law can be laid down for solutions of acids or of salts in water; though the specific heat is most frequently less than that calculated from the constituents.

460. **Specific heat of gases.**—The specific heat of a gas may be referred either to that of water or to that of air. In the former case it represents the quantity of heat necessary to raise a given *weight* of the gas through one degree, as compared with the heat necessary to raise the same *weight* of water one degree. In the latter case it represents the quantity of heat necessary to raise a given *volume* of the gas through one degree, compared with the quantity necessary for the same *volume* of air treated in the same manner.

De la Roche and Berard determined the specific heats of gases in re-

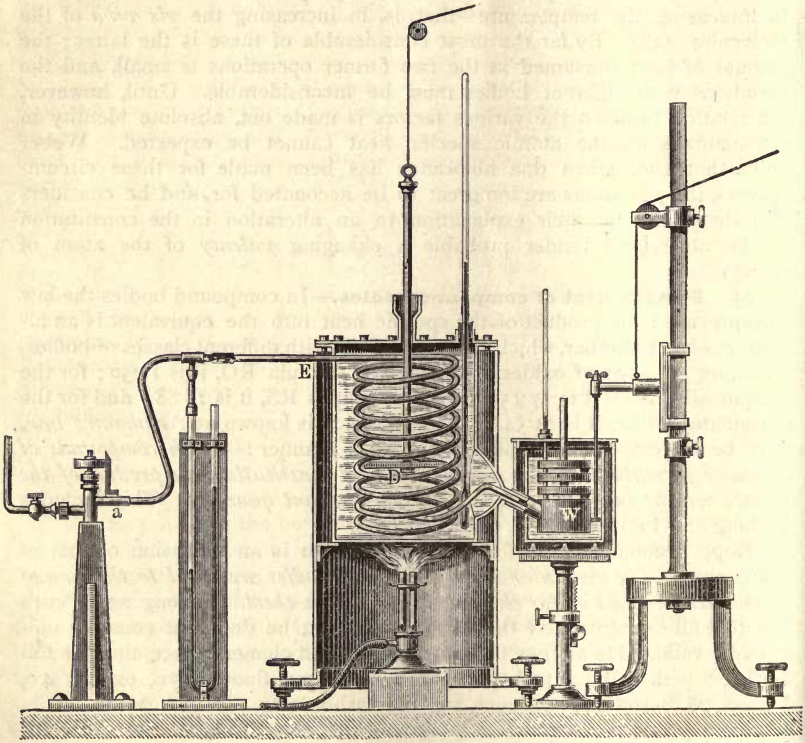


Fig. 413.

ference to water by causing known volumes of a given gas under constant pressure, and at a given temperature, to pass through a spiral glass tube placed in water. From the increase in temperature of this water, and from the other data, the specific heat was determined by a calculation analogous to that given under the method of mixtures. They also determined the specific heats of different gases relatively to that of air, by comparing the quantities of heat which equal volumes of a given gas, and of air at the same pressure and temperature, imparted to equal weights of water. Subsequently

to these researches, De la Rive and Marcet applied the method of cooling to the same determination ; and more recently Regnault made a series of investigations on the calorific capacities of gases and vapours, in which he adopted, but with material improvements, the method of De la Roche and Berard. Fig. 413 represents this apparatus. The gas issues from a gas-holder, not shown in the figure, through the tube, *a*, and enters the heating space, E, which is filled with oil ; the gas passes here through a serpentine tube, and thus acquires the temperature, which is kept uniform by a stirrer, D. From this it passes into the calorimeter, W, which consists of a metal box divided into a series of compartments, and forming in effect a serpentine, so that the gas has to traverse a long path before emerging into the air. The temperature of this bath is indicated by a delicate thermometer, and is kept uniform by moving the stirrer. The pressure of the gas is noted on a manometer. Regnault thus obtained the following results for the specific heats of the various gases and vapours, compared first with the specific heat of an equal weight of water taken as unity ; secondly, with that of an equal volume of air, referred, as before, to its own weight of water taken as unity :—

		Specific heats	
		Equal weights	Equal volumes
	Air	0·2374	0·2374
Simple gases	Oxygen	0·2174	0·2405
	Nitrogen	0·2438	0·2370
	Hydrogen	3·4090	0·2359
	Chlorine	0·1210	0·2962
Compound gases	Binoxide of nitrogen	0·2315	0·2406
	Carbonic oxide	0·2450	0·2370
	Carbonic acid	0·2163	0·3307
	Hydrochloric acid	0·1845	0·2333
	Ammonia	0·5083	0·2966
	Olefiant gas	0·4040	0·4106
Vapours	Water	0·4805	0·2984
	Ether	0·4810	1·2296
	Alcohol	0·4534	0·7171
	Turpentine	0·5061	2·3776
	Bisulphide of carbon	0·1570	0·4140
	Benzole	0·3754	1·0114

In making these determinations the gases were under a constant pressure, but variable volume ; that is, the gas as it was heated could expand, and this is called the *specific heat under constant pressure.* But if the gas when being heated is kept at a constant volume, its pressure or elastic force then necessarily increasing, it has a different capacity for heat ; this latter is spoken of as the *specific heat under constant volume.* That this latter is less than the former is evident from the following considerations :—

Suppose a given quantity of gas to have had its temperature raised t° while the pressure remained constant, this increase of temperature will have been accompanied by a certain increase in volume. Supposing now that the gas is so compressed as to restore it to its original volume, the result of this compression will be to raise its temperature again to a certain extent,

say $t'°$. The gas will now be in the same condition as if it had been heated and had not been allowed to expand. Hence, the same quantity of heat which is required to raise the temperature of a given weight of gas, $t°$, while the pressure remains constant and the volume alters, will raise the temperature $t + t'$ degrees if it is kept at a constant volume but variable pressure. The specific heat, therefore, of a gas at constant pressure, c, is greater than the specific heat under constant volume, $c_,$, and they are to each other as $t + t' : t$, that is $\frac{c}{c_,} = \frac{t + t'}{t}$.

It is not possible to determine by direct means the specific heat of gases under constant volume with much approach to accuracy; and it has been determined by some indirect method, of which the most accurate is based on the theory of the propagation of sound (229). A critical comparison of the most accurate recent determinations gives the number 1·405 for the value of $\frac{c}{c_,}$, which is usually designated by the symbol k.

461. **Latent heat of fusion.**—Black was the first to observe that during the passage of a body from the solid to the liquid state, a quantity of heat disappears, so far as thermometric effects are concerned, and which is accordingly said to become latent.

In one experiment he suspended in the room at a temperature 8·5° two thin glass flasks, one containing water at 0°, and the other the same weight of ice at 0°. At the end of half an hour the temperature of the water had risen 4°, that of the ice being unchanged, and it was 10½ hours before the ice had melted and attained the same temperature. Now the temperature of the room remained constant, and it must be concluded that both vessels received the same amount of heat in the same time. Hence 21 times as much heat was required to melt the ice and raise it to 4° as was sufficient to raise the same weight of water through 4°. So that the total quantity of heat imparted to the ice was $21 \times 4 = 84$; and as only 4 of this was used in raising the temperature, the remainder, 80, was used in simply melting the ice.

He also determined the latent heat by immersing 119 parts of ice at 0° in 135 parts of water at 87·7° C. He thus obtained 254 parts of water at 11·6° C. Taking into account the heat received by the vessel in which the liquid was placed, he obtained the number 79·44 as the latent heat of liquefaction of ice.

We may thus say,

Water at 0° = Ice at 0° + latent heat of liquefaction.

The method which Black adopted is essentially that which is now used for the determination of latent heats of liquids; it consists in placing the substance under examination at a known temperature in the water (or other liquid) of a calorimeter, the temperature of which is sufficient to melt the substance if it is solid, or to solidify it if liquid; and when uniformity of temperature is established in the calorimeter, this temperature is determined. Thus, to take a simple case, suppose it is required to determine the latent heat of the liquidity of ice. Let M be a certain weight of ice at zero, and m a weight of water at $t°$ sufficient to melt the ice. The ice is immersed in

the water, and as soon as it has melted, the final temperature $\theta°$ is noted. The water, in cooling from $t°$ to $\theta°$, has parted with a quantity of heat, $m\ (t-\theta)$. If x be the latent heat of the ice, it absorbs, in liquefying, a quantity of heat Mx; but, besides this, the water which it forms has risen to the temperature $\theta°$, and to do so has required a quantity of heat, represented by $M\theta°$. We thus get the equation

$$Mx + M\theta = m(t-\theta),$$

from which the value x is deduced.

By this method Desains and De la Provostaye found that the latent heat of the liquefaction of ice is 79·25 : that is, a pound of ice, in liquefying, absorbs the quantity of heat which would be necessary to raise 79·25 pounds of water 1°, or, what is the same thing, one pound of water from zero to 79·25°. Bunsen's most recent determination gives 80·025 (451).

This method is thus essentially that of the method of mixtures ; the same apparatus may be used, and the same precautions are required, in the two cases. In determining the latent heat of liquefaction of most solids, the different specific heats of the substance in the solid and in the liquid state require to be taken into account. In such a case, let m be the weight of the water in the calorimeter (the water equivalents of the calorimeter and thermometer, supposed to be included) ; M the weight of the substance worked with ; t the original and θ the final temperature of the calorimeter ; T the original temperature of the substance ; $\mathfrak{T}$ its melting (or freezing) point ; C the specific heat of the substance in the solid state between the temperature $\mathfrak{T}$ and θ ; c its specific heat in the liquid state between the temperatures T and $\mathfrak{T}$; and let L be the latent heat sought.

If the experiment be made on a melted substance which gives out heat to the calorimeter and is thereby solidified (it is taken for granted that a body gives out as much heat in solidifying as it absorbs in liquefying), it is plain that the quantity of heat absorbed by the calorimeter, $m(\theta-t)$, is made up of three parts : first, the heat lost by the substance in cooling from its original temperature T to the solidifying point $\mathfrak{T}$; secondly, the heat given out in solidification, ML ; and, thirdly, the heat it loses in sinking from its solidifying point $\mathfrak{T}$ to the temperature of the water of the calorimeter. That is :

$$m(\theta-t) = M\left[(T-\mathfrak{T})c + L + (\mathfrak{T}-\theta)C\right]$$

whence,

$$L = \frac{m(\theta-t)}{M} - (T-\mathfrak{T})c - (\mathfrak{T}-\theta)C.$$

The following numbers have been obtained for the latent heats of fusion :—

Water . . .	80·03	Cadmium . . .	13·66
Nitrate of sodium . .	62·97	Bismuth . . .	12·64
„ „ potassium .	47·37	Sulphur . . .	9·37
Zinc	28·13	Lead	5·37
Platinum . . .	27·18	Phosphorus . . .	5·03
Silver	21·07	D'Arcet's alloy . .	4·50
Tin	14·25	Mercury . . .	2·83

These numbers represent the number of degrees through which a pound of water would be raised by a pound of the body in question in passing from the liquid to the solid state ; or, what is the same thing, the number of pounds of water that would be raised 1° C. by one of the bodies in solidifying.

In order to melt a body the molecules must be driven so far apart that their attraction for each other is overcome ; this requires work which is done by the *vis viva* of the molecules ; for this purpose it must reach a certain amount, that is to say, the *vis viva* of the molecules, or their temperature, must increase until the pressure thereby caused is equal to the molecular attraction. This is the melting point, which in the case of crystallised bodies is fixed and definite.

But even when this temperature is attained, the work has still to be done, and in doing this work, *vis viva* is consumed ; this is the latent heat of fusion; and while the work is being done there is no rise of temperature. When however, the liquid is all melted, then the further application of heat simply varies the temperature.

The potential energy which bodies thus acquire in the act of melting is strictly comparable to the expenditure of work in the process of raising a weight. When the liquid solidifies, it reproduces the heat which had been expended in liquefying the solid : just as when a stone falls it produces by its impact against the ground the heat, the equivalent of which in work had been expended in raising it, and a similar explanation applies to the latent heat of gasification.

462. **Determination of the latent heat of vapour.**—Liquids, as we have seen, in passing into the state of vapour, absorb a very considerable quantity of heat, which is termed *latent heat of vaporisation.* In determining the heat absorbed in vapours, it is assumed that a vapour in liquefying gives out as much heat as it had absorbed in becoming converted into vapour.

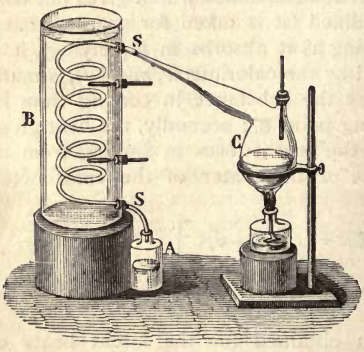


Fig. 414.

The method employed is essentially the same as that for determining the specific heat of gases. Fig. 414 represents the apparatus used by Despretz. The vapour is produced in a retort, C, where its temperature is indicated by a thermometer. It passes into a worm, SS, immersed in cold water, where it condenses, imparting its latent heat to the condensing water in the vessel B. The condensed vapour is collected in a vessel, A, and its weight represents the quantity of vapour which has passed through the worm. The thermometers in B give the change of temperature.

Let M be the weight of the condensed vapour, T its temperature on entering the worm, which is that of its boiling point, and x the latent heat of vaporisation. Similarly, let m be the weight of the condensing water (com-

prising the weight of the vessel B and of the worm SS *reduced* in water), let $t°$ be the temperature of the water at the beginning, and $\theta°$ its temperature at the end of the experiment.

It is to be observed that, at the commencement of the experiment, the condensed vapour passes out at the temperature $t°$, while at the conclusion its temperature is $\theta°$; we may, however, assume that its mean temperature during the experiment is $\frac{(t+\theta)}{2}$. The vapour M after condensation has therefore parted with a quantity of heat $M\left(T-\frac{t+\theta}{2}\right)c$, while the heat disengaged in liquefaction is represented by Mx. The quantity of heat absorbed by the cold water, the worm, and the vessel is $m(\theta-t)$. Therefore,

$$Mx+M\left(T-\frac{t+\theta}{2}\right)c=m(\theta-t),$$

from which x is obtained. Despretz found that the latent heat of aqueous vapour at 100° is 540; that is, a pound of water at 100° absorbs in vaporising as much heat as would raise 540 pounds of water through 1°. Regnault found the number 537, and Favre and Silbermann 538·8.

As in the case of the latent heat of water we may say,

Steam at 100° = water at 100° + latent heat of gasification.

Berthelot uses the very convenient apparatus represented in fig. 415 for determining latent heats of vaporisation. The liquid in the flask D is heated by the ring burner B, and the vapour which forms passes through the tube *ab* into the serpentine S, where it condenses and collects in the bulb R; this and the serpentine are contained in the calorimeter C, the top of which is closed by a wooden cover, HH, and a layer of felt, NN′; they cut off any heat from the flask D and from the burner B. As the serpentine SR can be detached from *ab*, it is easy to determine the weight of the distillate; from this, and from the rise in temperature of the water in the calorimeter, the latent heat can be readily calculated.

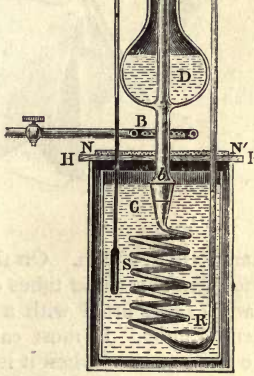


Fig. 415.

In the conversion of a body from the liquid into the gaseous state, as in the analogous process of fusion (461), one part of the heat is used in increasing the temperature and another in internal work. For vaporisation, the greater portion is consumed in the internal work of overcoming the reciprocal attraction of the particles of liquid, and in removing them to the far greater distances apart in which they exist in the gaseous state. In addition to this there is the external work—namely, that required to overcome the external pressure, usually that of the atmosphere: and as the increase of volume in vaporisation is considerable, this pressure has to be raised through a

greater space. Vaporisation may take place without having external work to perform, as when it is effected in vacuo ; but whether the evaporation takes place under a high or under a low pressure, on the surface of a liquid or in the interior, there is always a great consumption of heat in internal work.

463. **Favre and Silbermann's Calorimeter.**—The apparatus (fig. 416) furnishes a very delicate means of determining the calorific capacity of liquids, latent heats of evaporation, and the heat disengaged in chemical actions.

The principal part is a spherical iron reservoir, A, full of mercury, of which it holds about 50 pounds, and represents, therefore, a volume of more

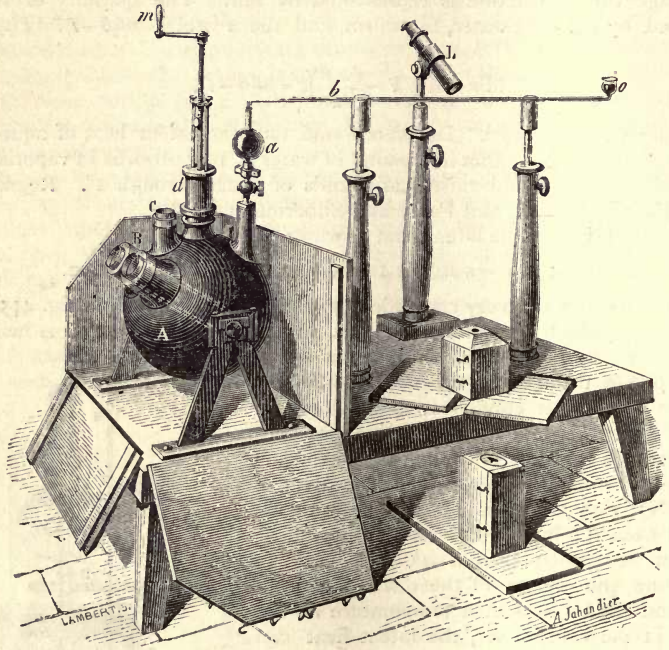


Fig. 416.

than half a gallon. On the left there are two tubulures, B, in which are fitted two sheet-iron tubes or *muffles*, projecting into the interior of the bulb. Each can be fitted with a glass tube for containing the substance experimented upon. In most cases one muffle and one glass tube are enough ; the two are used when it is desired to compare the quantities of heat produced in two different operations. In a third vertical tubulure, C, there is also a muffle, which can be used for determining calorific capacities by Regnault's method (454), in which case it is placed beneath the *r* of fig. 412.

The tubulure *d* contains a steel piston ; a rod turned by a handle, *m*, and which is provided with a screw thread, transmits a vertical motion to the piston ; but, by a peculiar mechanism, gives it no rotatory motion. In

the last tubulure is a glass bulb, *a*, in which is a long capillary glass tube, *bo*, divided into parts of equal capacity.

It will be seen from this description that the mercury calorimeter is essentially a thermometer with a very large bulb and a capillary stem : it is therefore extremely delicate. It differs, however, from a thermometer in the fact that the divisions do not indicate the temperature of the mercury in the bulb, but the number of thermal units imparted to it by the substances placed in the muffle.

This graduation is effected as follows :—By working the piston the mercury can be made to stop at any point of the tube, *bo*, at which it is desired the graduation should commence. Having then placed in the iron tube a small quantity of mercury, which is not afterwards changed, a thin glass tube, *e*, is inserted, which is kept fixed against the buoyancy of the mercury by a small wedge not represented in the figure. The tube being thus adjusted, the point of a bulb tube (see fig. 417) is introduced, containing water which is raised to the boiling point ; turning the position of the pipette, then, as represented in *n'*, a quantity of the liquid flows into the test-tube.

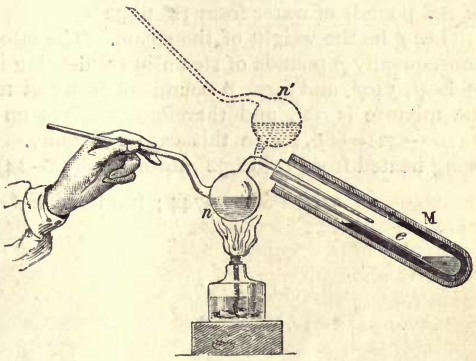


Fig. 417.

The heat which is thus imparted to the mercury makes it expand ; the column of mercury in *bo* is lengthened by a number of divisions, which we shall call *n*. If the water poured into the test-glass be weighed, and if its temperature be taken when the column *bo* is stationary, the product of the weight of the water into the number of degrees through which it has fallen indicates the number of thermal units which the water gives up to the entire apparatus (447). Dividing, by *n*, this number of thermal units, the quotient gives the number, *a*, of thermal units corresponding to a single division of the tube *bo*.

In determining the specific heat of liquids, a given weight, M, of the liquid in question is raised to the temperature T, and is poured into the tube C. Calling the specific heat of the liquid *c*, its final temperature θ, and *n* the number of divisions by which the mercurial column *bo* has advanced, we have

$$\mathrm{M}c(\mathrm{T}-\theta)=na, \text{ from which } c=\frac{na}{\mathrm{M}(\mathrm{T}-\theta)}.$$

The boards represented round the apparatus are hinged so as to form a box, which is lined with eider-down or wadding, to prevent any loss of heat. It is closed at the top by a board, which is provided with a suitable case, also lined, which fits over the tubulures *d* and *a*. A small magnifying glass,

which slides along the latter, enables the divisions on the scale to be read off.

464. **Examples.**—I. What weight of ice at zero must be mixed with 9 pounds of water at 20° in order to cool it to 5°?

Let M be the weight of ice necessary; in passing from the state of ice to that of water at zero, it will absorb 80M thermal units; and in order to raise it from zero to 5°, 5M thermal units will be needed. Hence the total heat which it absorbs is $80M + 5M = 85M$. On the other hand, the heat given up by the water in cooling from 20° to 5° is $9 \times (20 - 5) = 135$. Consequently,

$$85M = 135 \text{ ; from which } M = 1{\cdot}588 \text{ pounds.}$$

II. What weight of steam at 100° is necessary to raise the temperature of 208 pounds of water from 14° to 32°?

Let p be the weight of the steam. The latent heat of steam is 540°, and consequently p pounds of steam in condensing into water give up a quantity of heat, $540p$, and form p pounds of water at 100°. But the temperature of the mixture is 32°, and therefore p gives up a further quantity of heat $p(100 - 32) = 68p$, for in this case c is unity. The 208 pounds of water in being heated from 14° to 32° absorb $208(32 - 14) = 3744$ units. Therefore

$$540p + 68p = 3744 \text{ ; from which } p = 6{\cdot}581 \text{ pounds.}$$

CHAPTER X

STEAM ENGINES

465. **Steam engines.**—*Steam engines* are machines by which heat energy, obtained by the combustion of some fuel, is turned into mechanical work, aqueous vapour being used as a working fluid for effecting the transformation. In all but a few very exceptional cases the mechanical means used for the transformation of the one form of energy into the other are as follows :—the heat of combustion is, as far as possible, imparted to water in a closed vessel called a *boiler*; the water is thereby converted into steam, occupying an enormously greater volume, and this steam is allowed to pass from the boiler as fast as it is formed, and to act alternately on the two sides of a movable piston working backwards and forwards in a cylinder. As soon as the piston has been pushed to either end of the cylinder by the incoming steam acting on one side of it, the communication between that side and the boiler is shut off, and another communication opened either to a condenser or to the atmosphere. In either case the steam rushes out of the cylinder and the pressure against the piston falls, so that it can be pushed back by fresh steam from the boiler acting on its opposite side. If the purpose of the engine is merely to work pumps, or any other apparatus requiring only a reciprocating motion, a rod from the piston can be connected directly, or through a lever, to the pump to be worked. If, however, as in a majority of cases, the engine has to drive something having a rotary motion, a simple mechanism is used to change the reciprocating motion of the piston into the rotation of a crank. In this change itself there is no loss of work or energy (471), the work of the steam on the piston being exactly equal to the work done at the rotating crank-pin, minus only the lost work spent in overcoming the friction of the joints of the mechanism.

We shall first consider the boiler, or apparatus for generating steam, and then the engine itself.

466. **Steam boiler.**—Figs. 418 and 419 show one of the forms of boiler most commonly used in this country for supplying steam to stationary engines. This type of boiler is called *Cornish*, having been first used for the pumping engines in Cornwall. Fig. 418 shows a longitudinal section of the boiler and the brick flues in which it is set, and fig. 419 shows on the left a half-front view of the boiler and on the right a half-cross section. The boiler consists of an outer cylindrical shell A of wrought iron or steel plates riveted together, and a smaller internal flue or furnace B. The latter is open at both ends, and is crossed by a series of vertical tubes C, called *Galloway* tubes, which allow the water to circulate from the lower to the upper part of the boiler. The fire is placed on a *grate* D in the front part of the flue and ending

in a firebrick *bridge* over which the gases have to pass. These hot gases find their way past the tubes to the back of the boiler and then are compelled to diverge sideways and return by the side flues K to nearly the front of the shell where the flues are diverted downwards, as shown in fig. 419, and thence they return by the lower flue L to the chimney M. By thus

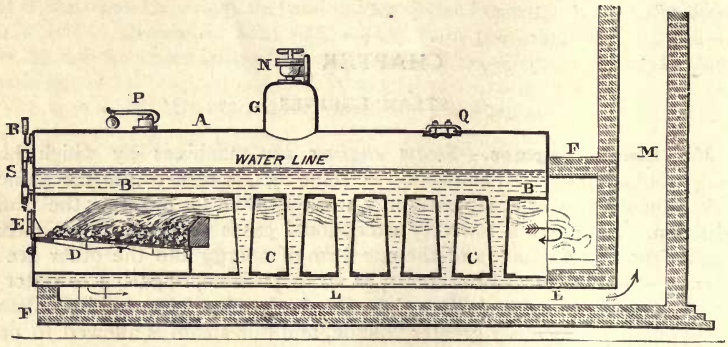


Fig. 418.

encircling the boiler with flues it is endeavoured to get all the heat possible from the gases before they are allowed to pass away up the chimney. The principal *fittings* or *mountings* of the boiler are indicated in the figures and are as follows: G is a *dome* on which stands the *stop-valve* N through which the steam is carried to the engine. The object of the dome is to take the steam from a point as far away from the water line as possible, so as to dry it. P is a *safety valve*, held down on its seat by the action of a weighted lever, and so adjusted that as soon as the pressure of steam reaches its intended maximum and tends to rise beyond it, the valve is lifted and the steam rushes away into the air. Q is a *manhole door* by which access is had to the interior of the boiler, when it is empty and out of use, for cleaning and repair. R is a *pressure gauge* or indicator, standing in front of the shell, showing, by a hand working in front of a dial plate, the 'boiler pressure' or amount which the pressure of steam inside the boiler exceeds that of the atmosphere surrounding it. S is a *water gauge*, a glass tube connected at top and bottom to the boiler, its upper end to the steam space, and the lower end to the water space. The water stands in the glass tube at the same level as in the boiler, and the fireman can see at a glance whether it is at the right height. This matter is of great importance, because an accidental fall of water-level is a frequent cause of boiler explosions. If, for instance, the water fell so low as to leave the top of the furnace B uncovered, the plates would get red-hot and soften so much as to collapse

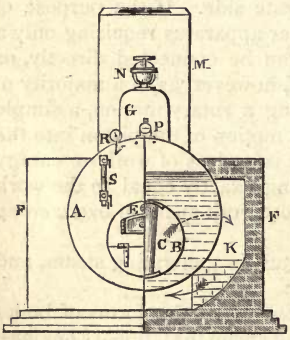


Fig. 419.

under the action of the steam pressure, with consequences that might be most serious.

In *marine boilers*, when it is of the greatest importance to get as much heating surface as possible into a small space, and similarly in the locomotive boiler to be presently described, the hot gases after leaving the furnace are made to pass through a number of small tubes instead of one large one as in fig. 418. Such boilers are called *multitubular boilers*.

Of late years the shells of large boilers have frequently been made of 'mild steel,' produced by the Bessemer or Siemens-Martin processes, rather than of wrought iron. In locomotive boilers, where the combustion is very rapid and intense, the fire-boxes are frequently made of copper, a much better conductor of heat than either iron or steel.

467. **Cornish engine.**—Fig. 420 shows the oldest of all the types of engines still in use, the *Cornish pumping engine*, which is worth examina-

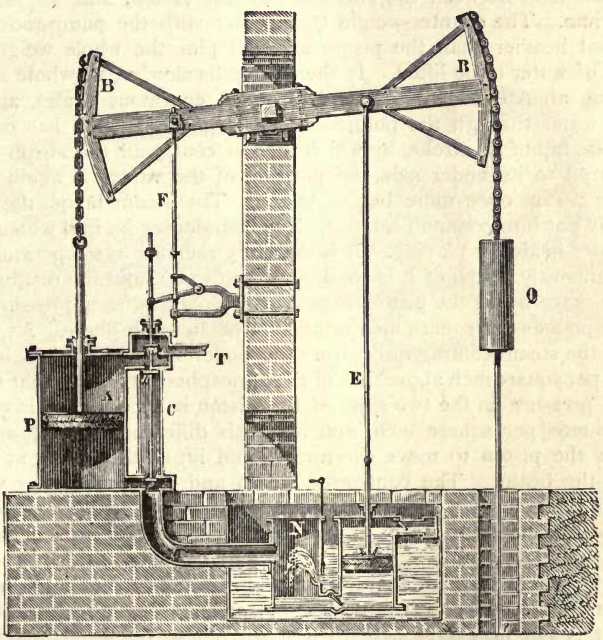


Fig. 420.

tion both for its historical interest and on account of the special way in which it works. (In the figure all details except those absolutely necessary to illustrate the action of the engine are omitted.) The engine has a vertical cylinder A (often of very great size, and with as much as 10 or 11 ft. stroke), in which works a piston P, whose rod is connected by a chain to a sector on the end of a beam B. Beside the cylinder is a chamber C containing the valves for admitting and discharging steam, whose mode of working will be presently described. At the further end of the beam a second sector is

connected with the pump-rod, at the upper end of which is placed a heavy counterweight Q. Below the cylinder a pipe M leads to a chamber N called the *condenser*, into which a jet of water from the tank in which it stands continually plays. The condenser in its turn is connected with a pump called an air-pump, worked from the beam by the rod E, and fitted with suction and discharge valves, and valves in its piston in the usual way.

We can follow the working of the engine easily by supposing the piston to start at the top of its stroke. The valves are then in the position shown, *m* open, *n* and *o* closed. Steam passes from the boiler through the pipe T to the top of the piston, and forces it down against the small pressure of the steam below it, this steam escaping into the condenser through the valve *o* and the pipe M. The pump-rods or *pit work*, and the weight Q, are thus lifted to the top of their stroke. When the piston arrives at the bottom of its stroke the valves *m* and *o* are shut and *n* is opened. This allows free communication between the two sides of the piston, and so puts it into equilibrium. The counter-weight Q, together with the pump-rods, is made somewhat heavier than the piston and rod plus the whole weight of the column of water to be lifted. It therefore falls slowly (the whole affair thus becoming an Attwood's machine (77) on an enormous scale), and forces up the water through the pumps. As soon as the piston has once more got to the top of its stroke, by which time of course all the steam has been transferred to its under side, the position of the valves is again reversed, and the piston once more begins to fall. The steam below the piston is suddenly put into communication with the condenser N, into which a jet of cold water is always playing. It is therefore reduced in temperature almost instantaneously, much of it is condensed into water, and the rest, which still fills the space below the piston, is necessarily reduced to a pressure of only about 3 pounds per square inch or about $\frac{1}{5}$ of an atmosphere. As the pressure of the steam coming direct from the boiler in such engines is often 50 pounds per square inch above that of the atmosphere, it follows that the difference of pressure on the two sides of the piston in such a case, is $50+15-3=62$ pounds per square inch, and it is this difference of pressure which compels the piston to move downwards and lift all the weight at the other end of the beam. The condensed steam and the condensing water fall together at the bottom of the condenser, and are continually removed (along with the uncondensed steam and any air that may be present) by the *air pump*, which is a simple lift pump with a valve in its piston (216).

In all modern Cornish engines the beams are of iron and the sector and chains are replaced by an arrangement of iron links forming a *parallel motion* which it is not necessary here to describe. The simple arrangement for working the valves, shown in outline in the figure, is also replaced by a much more complicated apparatus in which, by means of *cataracts*, any required length of pause can be made between the strokes of the engine, a matter which is sometimes of importance in heavy pumping work. It will be noticed that by the peculiar single-acting method of working adopted in the Cornish engine, the velocity of the down stroke (also called the *steam stroke*, or the *indoor stroke*) depends—other things being equal—upon the steam pressure, but the velocity of the up stroke (*equilibrium* or *outdoor stroke*) depends solely on the overplus weight put on the outer end of the

beam. In this way a slow and quiet upward motion can be given to the water, no matter how quickly the steam may move the piston.

468. **Ordinary horizontal engine.**—The engines now most largely used in factories for driving machinery differ altogether in their action from the Cornish engine. In them the cylinder is generally horizontal, and the crank is driven through a connecting rod only, without the intervention of any beam. Such an engine is shown in fig. 421. Here A is the steam cylinder, B the valve chest, or chamber in which works the valve whose mode of action is described in the next article. D is the main shaft, on the inner end of which is the crank driven by the connecting rod E. C is an *eccentric* (fig. 423), which works the valve by the rod N. F is a *governor* controlling the admission of steam to the cylinder by the valve H. M is the *bedplate* or frame of the engine, and L the flywheel.

A few words are necessary about the *governor*. This apparatus, an invention of James Watt's, consists of two weighted arms hinged at the top,

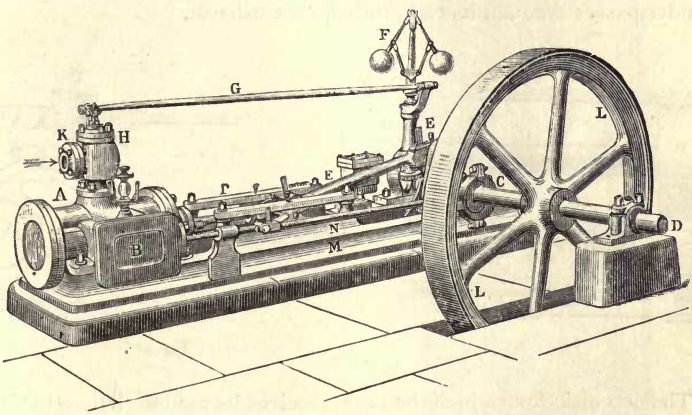


Fig. 421.

which fly outward when the speed of rotation is increased and drop together when it is reduced. The outward or inward motion of the arms is caused by a simple arrangement to turn the spindle G and so to close or open the valve H, which admits steam through K to the cylinder. In this way the engine automatically controls its own speed (471).

469. **Distribution of steam. Slide valves.**—Figs. 422 and 423 show details as to the working of the valve and the distribution of the steam in the engine just described. The former is a longitudinal section of the cylinder shown in fig. 421. A is the cylinder itself, B the piston, C the piston-rod, D the stuffing-box through which the piston passes steam-tight. It will be seen that a *port* or passage L communicates between each end of the cylinder and the surface on which the valve works, or *valve face*. On this face, and between the two steam-ports, comes a third port M, communicating directly with the atmosphere or with a condenser as the case may be. The valve G is shaped in section something like an irregular D, and is often

called a 'D' valve in consequence. It is moved continuously backwards and forwards upon the valve face by the valve rod H working in the stuffing-box K. When in the position shown in the figure the steam enters by F, and passes into the left-hand end of the cylinder (past the edge of the valve) and pushes the piston from left to right. The steam at present in the cylinder (as shown by the arrows) passes out at L, and through the under part of the valve G to the exhaust port M. As the piston moves on, the valve at first moves in the same direction, opening the port a little wider, then gradually moves back again and closes the admission port altogether. The point at which this occurs is called the point of *cut off*. No more steam is allowed to enter the cylinder for that stroke, the piston being pushed forward by the pressure of the elastic steam expanding behind it. By the time the piston has got to the end of its stroke, the position of the valve is just reversed from that in which it is shown, and steam passes into the cylinder through the right-hand port, driving the piston from right to left, while the steam which has already done duty in the left-hand end of the cylinder passes away, in its turn, through the exhaust.

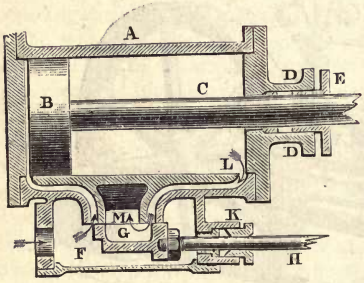


Fig. 422.

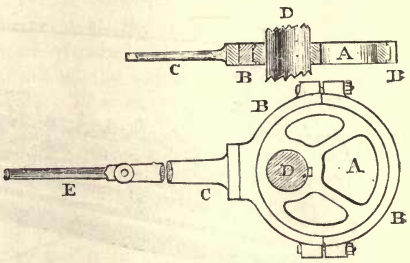


Fig. 423.

The eccentric from which the valve receives its motion (lettered C in fig. 421) is shown in detail in fig. 423. Here D is the crank-shaft and A a disc (solid or ribbed) fixed eccentrically on it so as to revolve with it. Encircling this disc (which is the *eccentric*) is a *strap* or ring B (made in two pieces for the sake of getting on and off), rigidly connected with a rod C, which is coupled by a pin to the valve-rod E. In each revolution of the eccentric the valve-rod is moved backwards and forwards through a space equal to twice the eccentricity of the eccentric, or distance between the centres of D and of A. The eccentric is thus equivalent exactly to a crank having a radius equal to its eccentricity. It is used instead of a crank because it does not require any gap to be left in the shaft, as a crank would do, but allows it to be carried continuously on.

In locomotive or marine engines two eccentrics are commonly used, one so placed as to give the valve the right motion when the shaft rotates in one direction, and one rightly placed for the other. By apparatus called *reversing gear* either one or the other can be caused to move the valve, so that the engine can be made, at pleasure, to turn the shaft in one or the other direction.

470. **Locomotives.**—*Locomotive engines*, or simply *locomotives*, are steam engines which, mounted on a carriage, propel themselves by transmitting their motion to wheels. The whole machine, fig. 424, boiler and engine, is fixed to a wrought-iron *frame*, which, therefore, is made strong

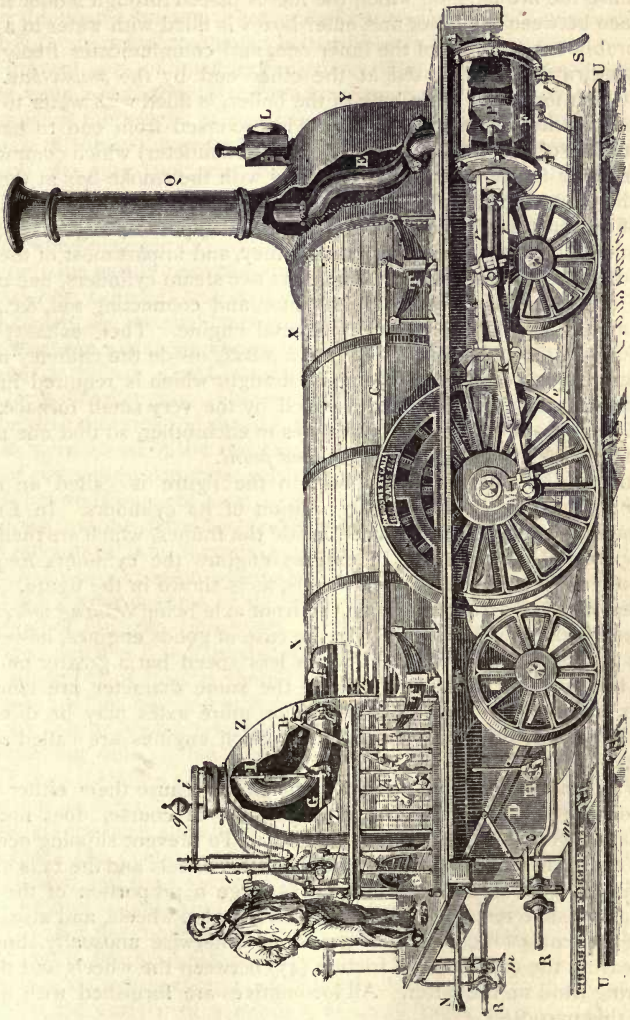


Fig. 424.

enough to carry the whole weight, and which in turn transmits that weight to the *axle-boxes* (or bearings in which the *axles* turn), by means of springs, and thence through the wheels to the rails. The *boiler* is of a special type, adopted in order to get the greatest possible heating surface in a very limited

space. It consists of three parts—the *fire-box*, *barrel*, and *smoke-box*. The fire-box, in the left of the engraving, is generally a more or less rectangular box, with a flat top, placed inside a second box of somewhat similar shape, but with a semi-cylindrical, or, as in the figure, domed top. In the inner fire-box are the fire-bars, on which the fuel is placed through a door in front. The space between the inner and outer boxes is filled with water to a height considerably over the top of the inner one, and communicates freely with a long cylindrical *barrel*, closed at the other end by the *smoke-box*. This barrel, which forms the main bulk of the boiler, is filled with water to within nine or ten inches of its upper side. It is traversed from end to end by a great number of small tubes (about $1\frac{1}{2}$ inch in diameter) which communicate with the inner fire-box at the one end, and with the smoke-box at the other. They, therefore, are entirely immersed in the water from end to end. The gases of combustion, formed in the inner fire-box, pass through these tubes to the smoke-box, and thence up the chimney, and impart most of their heat to the water as they pass along. There are two steam cylinders, one on each side of the frame, each one with its piston and connecting rod, &c., being simply an ordinary high-pressure horizontal engine. Their exhaust steam is discharged through a *blast pipe* into a *nozzle* inside the chimney near its base, and this serves to excite the fierce draught which is required in order that the necessary heat may be developed by the very small furnace. The two cylinders work cranks at right angles to each other, so that one may be in full action when the other is at its *dead point*.

A locomotive such as that shown in the figure is called an *outside cylinder* engine, on account of the position of its cylinders. In England many engines have cylinders placed inside the frames, which are then called *inside cylinder* locomotives. In express engines the cylinders frequently drive only one very large pair of wheels, as is shown in the figure. These are called *driving wheels*, those on the front axle being *leading wheels*, and on the rear axle *trailing wheels*. In the case of goods engines, however (as well as in many other instances), when less speed but a greater pull is required, two or more pairs of wheels of the same diameter are connected together by *coupling rods*, so that two or more axles may be directly or indirectly actually driven by the engine. Such engines are called *coupled engines*.

The action of the engine upon the wheels may cause them either to slip round on the rails (in which case the engine, of course, does not move onwards) or to roll on them in the usual way. To prevent slipping occurring it is necessary to make the friction between the wheels and the rails as great as possible. This is done by making as large a proportion of the whole weight as possible rest on the driving or the coupled wheels, and also—when bad weather causes the rails to be greasy or otherwise unusually slippery—by increasing the coefficient of friction (47) between the wheels and the rails by pouring sand on the latter. All locomotives are furnished with a sand-box for this purpose.

The steam pressure in locomotives is greater than that commonly used in any other engines, being often 120 to 130 lbs. per square inch above the atmosphere. In marine engines 70 to 80 lbs. is often used, in stationary engines seldom quite so much.

The following is an explanation of the reference letters in fig. 424 :—A, the main steam-pipe, conveying steam to the cylinder F, in which works a piston P, driving the crank M through the connecting rod K, *rr* are the piston-rod guides, V the stuffing-box. The exhaust steam is discharged through the pipe E. (It will be remembered that the cylinder and all this gear are duplicated on the other side of the engine.) DZ is the outer fire-box and X the barrel of the boiler, both covered with felt and wood or sheet iron to prevent loss of heat by radiation. The small tubes are seen at *a*, Y is the smoke-box, and Q the chimney or funnel. TT are the springs which transmit the weight of the frame to the axle-boxes. Of the smaller details, GI is the arrangement for closing or opening the steam-admission valve, B*b*C the reversing gear, RR feed-water pipes, N coupling rod for attaching tender and rest of train, *ei* safety valves, *g* whistle, *m* steps, *n* water gauge, *t* cocks for blowing water out of cylinders, H cock for blowing out boiler when necessary.

It is perhaps hardly necessary to explain that the breaking away of part of the fire-box, cylinder, &c., is done in the drawing only for the sake of showing clearly the internal construction.

471. **Various kinds of steam engine.**—Three types of steam engine have been described : the Cornish engine, the ordinary horizontal engine, and the locomotive engine. Others ought to be mentioned, although they cannot be here described in detail. *Compound engines* are those in which the steam is first used in the ordinary way in one cylinder and then transferred—of course at a comparatively low pressure—to another cylinder and used in it before being sent away to the condenser. This type is practically universal for marine purposes, and is very common for stationary engines. Its main advantage is a thermodynamic one. In an ordinary engine the cylinder walls are exposed alternately to the hot steam from the boiler and the cool vapour passing to the condenser. The latter so reduces the temperature of the iron, that when the first rush of fresh steam comes into the cylinder, much of it is immediately condensed on the cool metal, and an enormous quantity of heat is thereby lost. By passing the steam through an intermediate, or *low-pressure*, cylinder on its way to the condenser, the sides of the first or *high-pressure* cylinder are never exposed to condenser temperature, but only to that of the steam as it passes to the low-pressure cylinder ; they therefore are not so much cooled, and the loss of steam by condensation on them is very much reduced. There is no *mechanical* gain, as has sometimes been stated, in the use of two cylinders instead of one.

Sometimes the cylinder of an engine is inclosed in a second, slightly larger, cylinder, and fresh steam at boiler pressure admitted to the annular space so formed outside the working cylinder. The object of this is to reduce still further the condensation in the cylinder just alluded to. Such an engine is said to be *steam-jacketed.*

A *surface-condensing* engine is one in which the steam is condensed by contact with the surface of a number of small tubes through which cold water is kept continually circulating without being itself actually mixed with the condensing water. By this arrangement the condensed steam is kept by itself, and being distilled water it can be used very advantageously to feed the boiler again. Compound marine engines are almost invariably surface-

condensing. In this case the air pump only takes away the condensed steam, a separate pump, called a *circulating* pump, being used to force the condensing water through the tubes.

Engines without any condenser, like that shown in fig. 423, in which the steam is exhausted directly into the atmosphere after it has done its work, are often called *high-pressure* engines, but high pressures (of 80 to 90 pounds per square inch) are now frequently used in condensing engines, so that the name may be somewhat misleading.

In such an engine as is shown in fig. 423 we have seen that the governor keeps the speed constant, by closing or opening an exterior valve through which the steam passes on its way to the main valve. An artificial resistance is in this way opposed to the passage of the steam, by increasing which the pressure can be reduced, and therefore the work done by the steam, so that the engine will not run too fast if the resistance to its motion be diminished (as by the disconnecting of some of the machines it is driving, &c.). The actual weight of steam passing into the cylinder at each stroke remains unchanged, but the amount of *useful* work the steam can do is diminished artificially by giving it some *useless* work to do in addition, in forcing its way through a constricted passage. This is now known to be a wasteful way of controlling speed. In modern engines, therefore, the governor is frequently made to act by regulating the quantity of steam admitted by each stroke, and thus making the consumption of steam as nearly as possible proportional to the work done. Engines so arranged, of which the *Corliss* engine is one of the best-known examples, are said to be fitted with *automatic cut-off gear.*

There is a popular misconception, that somehow or other work is lost in an engine of the ordinary type between the piston and the crank, the latter receiving less work than is done on the former in consequence of the nature of the mechanism connecting them. It is probably unnecessary to point out here the fallacy of this notion, but it has received sufficient acceptance to lead to the invention of a host of *rotary* engines, in which it is endeavoured to obtain the desired rotary motion in a somewhat more direct fashion. Reuleaux has shown that in almost every case the mechanisms used in the rotary engines are the same as those of ordinary engines, although disguised in form, so that the idea of mechanical advantage is doubly a mistake, while in almost every case the rotary engines possess such grave mechanical defects that none of them have practically come into use.

472. **Work of an engine. Horse-power.**—The unit of work by which the performance of an engine is measured is in this country always the foot-pound. The number of foot-pounds of work done by the engine in any given time is equal to the average effective pressure upon its piston during that time, multiplied by the total distance through which the piston has moved under that pressure. By *average effective pressure* is meant the average value of the difference between the pressures on its two sides. Taking the time as one minute, this quantity of work in foot-pounds is equal to :—

Area of piston × *mean intensity of pressure on piston* × *length of stroke* × *number of strokes per minute.*

The stroke must be taken in feet. If the area is in square feet, the

pressure must be in pounds per square foot; if the area is in square inches, the pressure must be in pounds per square inch. If the strokes are *double* strokes, each corresponding, that is, to one whole revolution of the shaft, the length of stroke must be multiplied by 2. To find, for example, the work done in one minute by an engine with cylinder 16 inches diameter and 24 inches stroke, making 50 (double) strokes per minute with a mean pressure of 52 pounds per square inch, we have

$$(8^2 \times 3{\cdot}1416) \times 52 \times \left(\frac{24 \times 2}{12}\right) \times 50 = 2{,}091{,}000 \text{ ft.-lbs.}$$

The rate at which an engine does work is often measured in *horse-power* of 33,000 ft.-lbs. per minute, an arbitrary unit supposed to represent the maximum rate at which work could actually be done by a horse. In the case supposed the horse-power would be $\frac{2{,}091{,}000}{33{,}000} = 63{\cdot}4$.

On the Continent the unit of work is a kilogrammetre, which is very closely equal to $7\frac{1}{4}$ ft.-lbs. The horse-power used abroad, of 75 kilogrammetres per second, is nearly 2 per cent. smaller than that in use in this country.

473. **Indicator. Brake.**—By the expression *work done by an engine* we may mean either of two things, viz.—the *total* work done by the engine, or what is called its *useful*, or *effective*, work. The total work is the actual work done by the steam on the piston and obtained by calculation, as described in the last paragraph. The useful work is what remains of this total after deduction has been made of the work necessary to drive the engine itself against its own frictional resistances. The total work of an engine is measured by means of an apparatus called an *indicator*, invented by Watt, of which fig. 425 shows one of the most recent forms (Richard's) omitting a number of constructional details. The steam-engine indicator consists of a small cylinder A, half a square inch in area, in which works a piston B, the under side of which can be put into full communication with the cylinder of the engine by opening the cock C. Between the top side of the piston and the under side of the cylinder-cover is a spiral spring. The motion of the piston-rod is transferred to a parallel motion DD, and so causes a point E to move in a straight line up and down, its stroke being about four times as great as that of the small piston. The indicator is fixed on to the cylinder of the steam-engine near one end, so that when the cock C is opened, there is the same pressure of steam on the indicator piston as on the engine piston. This pressure forces up the piston, and the amount of compression of the spring so caused is proportionate to the pressure causing it. The upward motion of E, therefore, is proportional to the steam pressure. In front of E is a vertical drum F, on which a strip of paper can be fixed, and this drum is caused to rotate about its axis by attaching the cord G to any suitable part of the engine. The paper thus moves horizontally under the pencil, with a motion proportional to the stroke of the engine, while the pencil moves up and down on the paper with a motion proportional to the steam pressure on the piston. The two motions occurring simultaneously, the pencil traces on the paper a curve whose horizontal and vertical ordinates are proportional to the two quantities just named, and

whose area is therefore proportional to the product of these quantities, or, which is the same thing, to the work done by the piston as defined in the last paragraph. The curve is called an indicator card, or *indicator diagram*, and while its whole *area* shows the whole work done by the steam, its *form* shows the engineer what is happening within the cylinder at each point of the stroke, which he may often require to know.

Figs. 426 and 427 show two forms of indicator diagram. The curves themselves, as drawn by the indicators, are lettered ABCD. Beside them a scale of pressure in atmospheres is placed. In fig. 426 the steam is expanded about seven times, and the back pressure is about $\frac{1}{3}$ of an atmosphere, the pressure during admission being five atmospheres. The engine is a condensing one, and the diagram is fairly good. Fig. 427 is for a non-condensing engine, the back pressure being above that of the atmosphere.

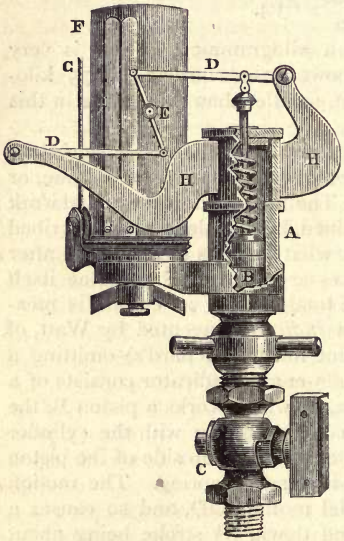


Fig. 425.

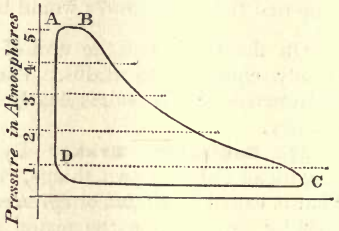


Fig. 426.

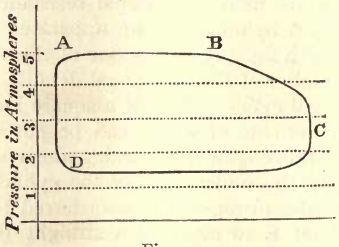


Fig. 427.

The steam is cut off (at B) only at about $\frac{3}{4}$ of the stroke, so that it is not working economically, and from the roundness of its corners the diagram would be considered a poor one.

The *useful* work of an engine is measured by an entirely different piece of apparatus, called a *dynamometer*. This is used in many forms, but fig. 428 shows the principle upon which the majority act. The apparatus shown in the figure is known as a *Prony's friction brake*. A is the shaft, the usual work transmitted by which we require to find. Upon the shaft is a fixed pulley B, embraced by two blocks B_1 and B_2, which can be tightened up by the screws at C_1 and C_2. To the lower block is fixed a lever D, from which hangs a weight, and which has at its extremity a small pointer working against a short scale F. If such an apparatus be set in motion by turning the shaft A, one of two things must happen ; either the pulley must

slip round in the blocks, or it must so grip them as to carry both them and the lever D round its own axis. The moment of resistance to the former is rF, if r be the radius of the pulley and F the frictional resistance at its

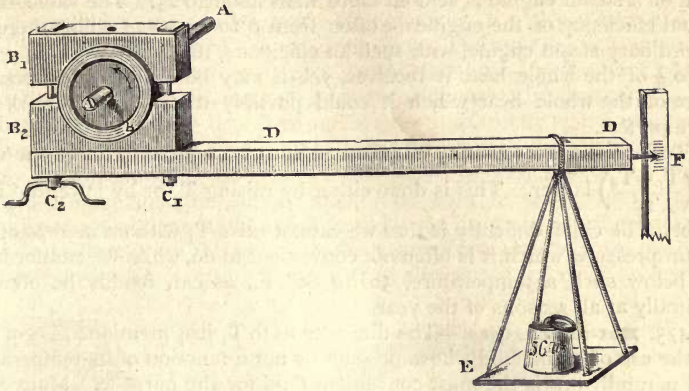


Fig. 428.

periphery; that of the latter is RW, where R is the radius of the weight and W the weight itself. In practice the screw C_2 is loosened just sufficiently to keep the weight just lifted from the ground, while the pulley is always turning round in the blocks, so that, therefore,

$$rF = RW.$$

The work done at the brake per minute is equal to the frictional resistance multiplied by the distance through which it is overcome in the same time, or, if n be the number of revolutions per minute,

$$= 2\pi rFn = 2\pi RWn.$$

It is therefore just the same as if a resistance = W were continually being overcome at the periphery of a wheel of radius R, making n turns per minute. As the values of all the quantities in the expression $2\pi RWn$ are very readily determined, it will be seen that this brake affords a very simple way of measuring the net work transmitted through the shaft of an engine.

The ratio $\frac{\text{useful work}}{\text{total work}}$, or $\frac{\text{work shown by brake}}{\text{work shown by indicator}}$, is called the *efficiency* of the engine as a machine, or its *mechanical efficiency*. It is often as much as 0·85, and sometimes even higher than 0·9 or 90 per cent., being generally greatest in large engines.

474. **Efficiency of heat engines.**—There is another ratio of efficiency connected with the steam-engine, namely the ratio

$$\frac{\text{Total work done by engine}}{\text{Total heat expended}}$$

which is called the *efficiency of the engine as a heat engine* or its *thermodynamic efficiency*. If T_1 and T_2 be respectively the *absolute* temperatures (496) of the steam and the feed water in any engine, then it can be shown

that such an engine, if working quite perfectly, could transform no more than $\left(\frac{T_1 - T_2}{T_1}\right)$ of the heat which it receives into work. This fraction in the case of a steam engine is seldom more than about 0·25. The value of the actual efficiency of the engine is often from 0·10 to 0·14; while, therefore, an ordinary steam engine, with such an efficiency, turns into work only from $\frac{1}{10}$ to $\frac{1}{6}$ of the whole heat it receives, yet it may be turning into work $\frac{1}{2}$ or more of the whole heat which it could possibly transform into work if it were perfect.

To increase the economy of steam engines we require to make the value of $\left(\frac{T_1 - T_2}{T_1}\right)$ larger. This is done either by raising T_1 or by lowering T_2, or both. The chief difficulty is that we cannot raise T_1 without increasing the steam pressure, which it is often not convenient to do, while we cannot lower T_2 below such a temperature, 50° to 60° F., as can readily be obtained naturally at all seasons of the year.

475. **Hot-air engines.**—The difficulty as to T_1 just mentioned is got over by the use of some fluid whose pressure is not a function of its temperature, and naturally *air* is the most convenient fluid for the purpose. Many 'hot-air' engines have been designed, and some have found a considerable measure of success commercially, as Rider's, Hock's, and Lehmann's. In all cases the engines consist essentially of one (or two) chambers placed so that one end can be heated by a furnace and the other cooled by a refrigerator. The air is compelled to move from the cold space to the hot and back again continually. When hot it is allowed to expand and push forward a piston, when cold it is compressed by pushing back the piston again to its original position. The difference between these two quantities of work is the whole work done by the engine. By making T_1 a very high temperature, the theoretical efficiency $\left(\frac{T_1 - T_2}{T_1}\right)$ of an air engine may be made much higher than that of a steam engine. But it is so much more difficult to attain the theoretical efficiency in the air than in the steam engine, that its actual efficiency is generally much lower than that of a steam engine. There are constructive difficulties connected with the hot-air chambers, and with the regulation of the speed, and these, as well as with the large bulk of most air engines in proportion to their power, have stood greatly in the way of their development. No doubt, however, much more improvement would have taken place in these engines had not gas engines come into prominence of late years and proved much more convenient.

476. **Gas engines.**—Gas engines, like steam engines and air engines, are heat engines, but in them the working fluid is ordinary coal gas mixed with air, in the proportion of about 1 to 11 by volume. The principle of action is very simple :—The explosive mixture after being drawn into the cylinder is set light to, the heat generated by the very rapid combustion which we call an explosion causes the mixed gases to expand and drive forward the piston. The great difficulty for many years was that the explosion was so rapid that the comparatively slow-going piston could not keep up with it, and the greater part of the energy of the explosion was lost by radiation and conduction. In the more modern gas engines, however (Otto's and Clerk's

and others), this difficulty is got over by compressing the charge before igniting it, a treatment which is found to decrease very much the rapidity of the explosion and so greatly increase the actual efficiency of the engine. Fig. 429 shows the principal parts of an Otto 'Silent' gas engine, as now made. A is the cylinder, open at front and single-acting, in which works a deep piston F, driving a crank in the usual manner. The cylinder is surrounded by a water jacket, to prevent it from getting too hot. At the back of the cylinder is a slide valve B, worked by a cam, not shown in drawing, on the lay shaft G. The valve B is kept up against its face by spiral springs E. D is a chamber in which a small jet of gas for igniting the mixture is continually burning. C_1 is the cock for admission of gas, and C_2 an india-rubber bag to equalise the gas pressure. The working of the engine is as follows :—the piston moves from left to right and draws into the cylinder the explosive mixture. On the return stroke it compresses the mixture to about 3 atmospheres. The igniting flame is then allowed to come for an instant

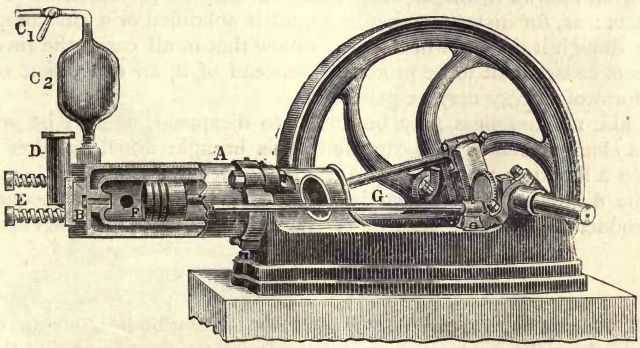


Fig. 429.

into contact with the compressed mixture, which burns very rapidly (or explodes slowly, whichever expression be preferred) and pushes the piston forward again, the pressure rising to 10 or 12 atmospheres. On the next return stroke the burnt gases are pushed out through the opening shown in the drawing, and the process begins again once more. There are many ingenious arrangements about this type of engine which our space will not allow us to mention in detail. It must suffice to say that the engine has proved distinctly economical, and has such very great conveniences as may fairly account for the rapid way in which its use (and that of other gas engines) has extended.

In conclusion, it is as well to point out that, as long as they work between the same temperatures, there is no difference between steam, air, and gas engines as to theoretical economy. The last two gain by the possibility of using higher limits of temperature than can be employed in a steam engine, but, so far, have lost by constructive and mechanical difficulties which prevent their theoretical efficiency from being attained.

CHAPTER XI

SOURCES OF HEAT AND COLD

477. **Different sources of heat.**—The following different sources of heat may be distinguished: i. the *mechanical sources*, comprising friction, percussion, and pressure; ii. the *physical sources*—that is, solar radiation, terrestrial heat, molecular actions, change of condition, and electricity; iii. the *chemical sources*, or molecular combinations, and more especially combustion.

In what follows it will be seen that heat may be produced by reversing its effects; as, for instance, when a liquid is solidified or a gas compressed (479); though it does not necessarily follow that in all cases the reversal of its effects causes heat to be produced—instead of it, an equivalent of some other form of energy may be generated.

In like manner heat may be forced to disappear, or cold be produced when a change such as heat can produce is brought about by other means, as when a liquid is vaporised or a solid liquefied by solution; though here also the disappearance of heat is not always a necessary consequence of the production, by other means, of changes such as might be effected by heat.

MECHANICAL SOURCES

478. **Heat due to friction.**—The friction of two bodies, one against the other, produces heat, which is greater the greater the pressure and the more rapid the motion. For example, the axles of carriage wheels, by their friction against the boxes, often become so strongly heated as to take fire. By rubbing together two pieces of ice in a vacuum below zero, Sir H. Davy partially melted them. In boring a brass cannon Rumford found that the heat developed in the course of 2½ hours was sufficient to raise 26½ pounds of water from zero to 100°, which represents 2,650 thermal units (447). Mayer raised water from 12° to 13° by shaking it. At the Paris Exhibition, in 1855, Beaumont and Mayer exhibited an apparatus, which consisted of a wooden cone covered with hemp, and moving with a velocity of 400 revolutions in a minute, in a hollow copper cone, which was fixed and immersed in the water of an hermetically-closed boiler. The surfaces were kept covered with oil. By means of this apparatus 88 gallons of water were raised from 10 to 130 degrees in the course of a few hours.

In the case of flint and steel, the friction of the flint against the steel raises the temperature of the metallic particles, which fly off, heated to such an extent that they take fire in the air.

The luminosity of aerolites is considered to be due to their friction against the air, and to their condensation of the air in front of them (479), their velocity attaining as much as 150 miles in a second.

Tyndall devised an experiment by which the great heat developed by friction is illustrated in a striking manner. A brass tube (fig. 430), about 7 inches in length and $\frac{3}{4}$ of an inch in diameter, is fixed on a small wheel. By means of a cord passing round a much larger wheel, this tube can be rotated with any desired velocity. The tube is three parts full of water, and is closed by a cork. In making the experiment, the tube is pressed between a wooden clamp, while the wheel is rotated with some rapidity. The water rapidly becomes heated by the friction, and its temperature soon exceeding the boiling-point, the cork is projected to a height of several yards by the elastic force of the steam.

479. **Heat due to pressure and percussion.**—If a body be so compressed that its density is increased, its temperature rises according as the volume diminishes. Joule verified this in the case of water and of oil, which were exposed to pressures of 15 to 25 atmospheres. In the case of water at 1·2° C., increase of pressure caused lowering of temperature—a result which agrees with the fact that water contracts by heat at this temperature.

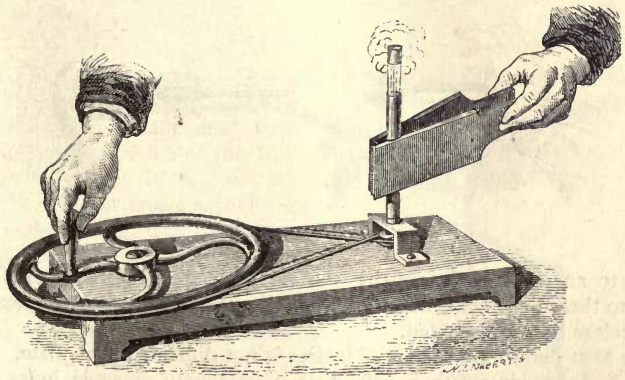


Fig. 430.

Similarly, when weights are laid on metal pillars, heat is evolved, and absorbed when they are removed. So in like manner the stretching of a metal wire is attended with a diminution of temperature.

The production of heat by the compression of gases is easily shown by means of the *pneumatic syringe* (fig. 431). This consists of a glass tube with thick sides, closed hermetically by a leather piston. At the bottom of this there is a cavity in which a small piece of cotton, moistened with ether or bisulphide of carbon, is placed. The tube being full of air, the piston is suddenly plunged downwards ; the air thus compressed disengages so much heat as to ignite the cotton, which is seen to burn when the piston is rapidly withdrawn. The ignition of the cotton in this experiment indicates a temperature of at least 300°.

The rise of temperature produced by the compression in the above experiment is sufficient to effect the combination, and therefore the detonation, of a mixture of hydrogen and oxygen.

A curious application of the principle of the pneumatic syringe is met

with in the American *powder ram* for pile-driving. On the pile to be driven is fixed a powder mortar, above which is suspended at a suitable distance an iron rammer, shaped like a gigantic stopper, which just fits in the mortar. Gunpowder is placed in the mortar, and when the rammer is detached it falls into the mortar, compresses the air, producing so much heat that the powder is exploded. The force of the gases projects the rammer into its original position, where it is caught by a suitable arrangement; at the same time the reaction of the mortar on the pile drives this in with far greater force than the fall of the rammer. After adding a fresh charge of powder, the rammer is again allowed to fall, again produces heat, explosion, and so forth, so that the driving is effected in a surprisingly short time.

Percussion is also a source of heat. In firing shot at an iron target, a sheet of flame is frequently seen at the moment of impact; and Sir J. Whitworth has used iron shells which are exploded by the concussion on striking an iron target. A small piece of iron hammered on the anvil becomes very hot. The heat is not simply due to an approximation of the molecules—

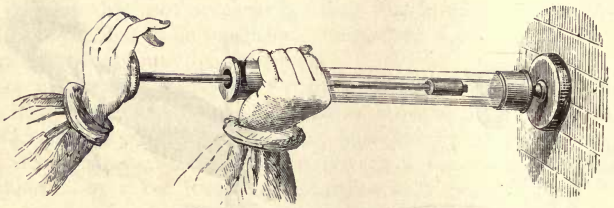


Fig. 431

that is, to an increase in density—but arises from a vibratory motion imparted to them; for lead, which does not increase in density by hammering, nevertheless becomes heated.

The heat due to the impact of bodies is not difficult to calculate. Whenever a body moving with a velocity v is suddenly arrested in its motion, its *vis viva* is converted into heat. This holds equally whatever be the cause to which the motion is due: whether it be that acquired by a stone falling from a height, by a bullet fired from a gun, or the rotation of a copper disc by means of a turning-table. The *vis viva* of any moving body is expressed by $\frac{mv^2}{2}$ or in foot-pounds by $\frac{pv^2}{2g}$, where p is the weight in pounds, v the velocity in feet per second, and g is about 32 (29); and if the whole of this be converted into heat, its equivalent in thermal units will be $\frac{pv^2}{2g \times 1390}$. Suppose, for instance, a lead ball weighing a pound be fired from a gun, and strike against a target, what amount of heat will it produce? We may assume that its velocity will be about 1,600 feet per second; then its *vis viva* will be $\frac{1 \times 1600^2}{2 \times 32} = 40{,}000$ foot-pounds. Some of this will have been consumed in producing the vibrations which represent the sound of the shock, some of it also in its change of shape; but neglecting these two, as being small, and assuming that the heat is equally divided between the ball

and the target, then, since 40,000 foot-pounds is the equivalent of 28·7 thermal units, the share of the ball will be 14·3 thermal units; and if, for simplicity's sake, we assume that its initial temperature is zero, then, taking its specific heat at 0·0314, we shall have

$$1 \times 0{\cdot}0314 \times t = 14{\cdot}3 \text{ or } t = 457°,$$

which is a temperature considerably above that of the melting point of lead (338).

By allowing a lead ball to fall from various heights on an iron plate, both experience an increase of temperature which may be measured by the thermopile; and from these increases it may be easily shown that the heat is directly proportional to the height of fall, and therefore to the square of the velocity.

By similar methods Mayer has calculated that if the motion of the earth were suddenly arrested the temperature produced would be sufficient to melt and even volatilise it; while, if it fell into the sun, as much heat would be produced as results from the combustion of 5,000 spheres of carbon the size of our globe.

PHYSICAL SOURCES

480. **Solar radiation.**—The most intense of all sources of heat is the sun. Different attempts have been made to determine the quantity of heat which it emits. Pouillet made the first accurate measurements of the heat of the sun by means of an instrument called the *pyrheliometer*. The form represented in fig. 432 consists of a flat cylindrical metal box 3 inches in diameter and ½ an inch deep, containing a known weight of water. To it is fitted a metal tube which contains the stem of a delicate thermometer, the bulb of which dips in the liquid of the box, being fitted by means of a cork. The tube works in two collars, so that by means of a milled head it can be turned, and with it the vessel, and the liquid thus be uniformly mixed. The face of the vessel is coated with lampblack, and is so adjusted that the sun's rays fall perpendicularly upon it. This can be ascertained by observing when the shadow exactly covers the lower disc which is fitted to the same axis.

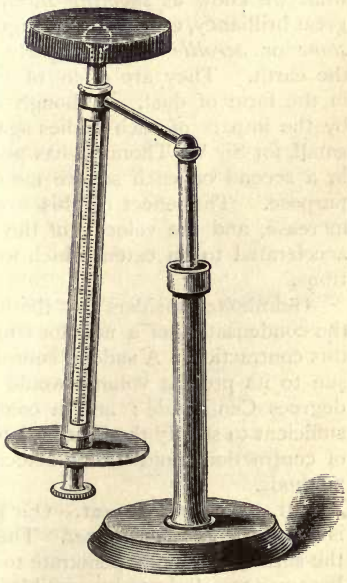


Fig. 432.

The instrument was exposed for five minutes at a time to the sun's rays; knowing the weight of the water, its rise in temperature could be easily calculated (449). Corrections were necessary for the heat reflected by the lampblack, and also for the heat absorbed by the air.

Pouillet calculated from the results of experiments with this apparatus that if the total quantity of heat which the earth receives from the sun in the course of a year were employed to melt ice, it would be capable of melting a layer of ice all round the earth of 35 yards in thickness. Another statement is that the heat emitted by the sun is equal to that produced by the combustion of 1,500 pounds of coal in an hour on each square foot of its surface. But from the surface which the earth exposes to the sun's radiation, and from the distance which separates the earth from the sun, the quantity of heat which the earth receives can only be $\frac{1}{2,381,000,000}$ of the heat emitted by the sun. Violle estimated the thickness of ice melted by the sun's heat at the equator, apart from absorption by the atmosphere, at 55 metres in thickness; and, deducting this absorption, at 37 metres.

Faraday calculated that the average amount of heat radiated in a day on each acre of ground in the latitude of London is equal to that which would be produced by the combustion of sixty sacks of coal.

The heat of the sun cannot be due to combustion, for even if the sun consisted of hydrogen, which of all substances gives the most heat in combining with oxygen, it can be calculated that the heat thus produced would not last more than 3,000 years. Another supposition is that originally put forth by Mayer, according to which the heat which the sun loses by radiation is replaced by the fall of aerolites against its surface. One class of these is what we know as *shooting stars*, which often appear in the heavens with great brilliancy, especially on August 14 and November 15; the term *meteoric stone* or *aerolite* being properly restricted to the bodies which fall on the earth. They are often of considerable size, and are even met with in the form of dust. Although some of the sun's heat may be restored by the impact of such bodies against the sun, the amount must be very small, for Sir W. Thomson has proved that a fall of 0·3 gramme of matter in a second on each square metre of surface would be necessary for this purpose. The effect of this would be that the mass of the sun would increase, and the velocity of the earth's rotation about the sun would be accelerated to an extent which would be detected by astronomical observations.

Helmholtz considers that the heat of the sun was produced originally by the condensation of a nebulous mass, and is kept up by a continuance of this contraction. A sudden contraction of the primitive nebular mass of the sun to its present volume would produce a temperature of 28 millions of degrees Centigrade; and a contraction of $\frac{1}{10000}$ of its mass would be sufficient to supply the heat radiated by the sun in 2,000 years. This amount of contraction could not be detected even by the most refined astronomical methods.

481. **Terrestrial heat.**—Our globe possesses a heat peculiar to it, which is called the *terrestrial heat.* The variations of temperature which occur at the surface gradually penetrate to a certain depth, at which their influence becomes too slight to be sensible. It is hence concluded that the solar heat does not penetrate below a certain internal layer, which is called the *layer of constant annual temperature*; its depth below the earth's external surface varies, of course, in different parts of the globe; at Paris it is about 30 yards, and the temperature is constant at 11·8° C.

Below the layer of constant temperature, the temperature is observed to increase, on the average, 1° C. for every 90 feet. The most rapid increase is at Irkutsk in Siberia, where it is 1° for 20 feet, and the slowest in the mines at Mansfield, where it is about 1° C. for 330 feet. This increase has been verified in mines and artesian wells. According to this at a depth of 3,000 yards, the temperature of a corresponding layer would be 100°, and at a depth of 20 to 30 miles there would be a temperature sufficient to melt all substances which exist on the surface. Hot springs and volcanoes confirm the existence of this central heat.

Various hypotheses have been proposed to account for the existence of this central heat. The one usually admitted by physicists is that the earth was originally in a liquid state in consequence of the high temperature, and that by radiation the surface has gradually solidified, so as to form a solid crust. The thickness of this crust is not believed to be more than 40 to 50 miles, and the interior is probably still in a liquid state. The cooling must be very slow, in consequence of the imperfect conductivity of the crust. For the same reason the central heat does not appear to raise the temperature of the surface more than $\frac{1}{30}$ of a degree.

Fourier calculated that the heat given off by the earth in 100 years would be sufficient to melt a layer of ice 3 metres in thickness, which therefore is only $\frac{1}{1000}$ of that received by the sun in the same time.

482. **Heat produced by absorption and imbibition.**—Molecular phenomena, such as imbibition, absorption, capillary actions, are usually accompanied by disengagement of heat. Pouillet found that whenever a liquid is poured on a finely-divided solid, an increase of temperature is produced which varies with the nature of the substances. With inorganic substances, such as metal, the oxides, the earths, the increase is $\frac{4}{10}$ of a degree ; but with organic substances, such as sponge, flour, starch, roots, dried membranes, the increase varies from 1 to 10 degrees.

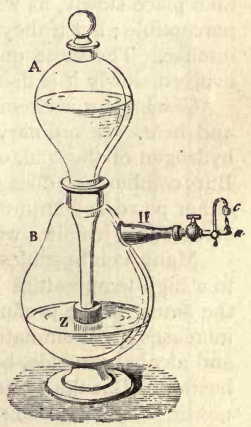


Fig. 433.

The absorption of gases by solid bodies presents the same phenomena. Döbereiner found that when platinum, in the fine state of division known as platinum black, is placed in oxygen, it absorbs many hundred times its volume, and that the gas is then in such a state of density, and the temperature so high, as to give rise to intense combustions, Spongy platinum produces the same effect. A jet of hydrogen directed on it takes fire.

The apparatus known as *Döbereiner's Lamp* depends on this property of finely-divided platinum. It consists of two glass vessels (fig. 433). The first, A, fits in the lower vessel by means of a tubulure which closes it hermetically. At the end of the tubulure is a lump of zinc, Z, immersed in dilute sulphuric acid. By the chemical action of the zinc on the dilute acid hydrogen gas is generated, which, finding no issue, forces the liquid out of the vessel B into the vessel A, so that the zinc is not in contact with the liquid. The stopper of the

upper vessel is raised to give exit to the air in proportion as the water rises. On a copper tube, H, fixed in the side of the vessel B, there is a small cone, *a*, perforated by an orifice ; above this there is some spongy platinum in the capsule, *c*. As soon now as the cock, which closes the tube, H, is opened, the hydrogen escapes, and, coming in contact with the spongy platinum, is ignited.

The condensation of vapours by solids often produces an appreciable increase of temperature. This is particularly the case with humus, which, to the benefit of plants, is warmer in moist air than the air itself.

Favre has found that when a gas is absorbed by charcoal the amount of heat produced by the absorption of a given weight of sulphurous acid, or of protoxide of nitrogen, greatly exceeds that which is disengaged in the liquefaction of the same weight of gas ; for carbonic acid, the heat produced by absorption exceeds even the heat which would be disengaged by the solidification of the gas. The heat produced by the absorption of these gases cannot, therefore, be explained by assuming that the gas is liquefied, or even solidified in the pores of the charcoal. It is probable that it is in part due to that produced by the liquefaction of the gas, and in part to the heat due to the imbibition in the charcoal of the liquid so produced.

CHEMICAL SOURCES

483. **Chemical combination. Combustion.**—*Chemical combinations* are usually accompanied by a rise of temperature. When these combinations take place slowly, as when iron oxidises in the air, the heat produced is imperceptible ; but if they take place rapidly, the disengagement of heat is very intense. The same quantity of heat is produced in both cases, but when evolved slowly it is dissipated as fast as formed.

Combustion is chemical combination attended with the evolution of light and heat. In ordinary combustion in lamps, fires, candles, the carbon and hydrogen of the coal, or of the oil, &c., combine with the oxygen of the air. But combustion does not necessarily involve the presence of oxygen. If either powdered antimony or a fragment of phosphorus be placed in a vessel of chlorine, it unites with chlorine, producing thereby heat and flame.

Many combustibles burn with flame. A *flame* is a gas or vapour raised to a high temperature by combustion. Its illuminating power varies with the nature of the product formed. The presence of a solid body in the flame increases the illuminating power. The flames of hydrogen, carbonic oxide, and alcohol are pale, because they only contain gaseous products of combustion. But the flames of candles, lamps, coal gas, have a high illuminating power. They owe this to the fact that the high temperature produced decomposes certain of the gases, with the production of carbon, which, not being perfectly burnt, becomes incandescent in the flame. Coal gas, when burnt in an arrangement by which it obtains an adequate supply of air, such as a Bunsen's burner, is almost entirely devoid of luminosity. A non-luminous flame may be made luminous by placing in it platinum wire or asbestos. The temperature of a flame does not depend on its illuminating power. A hydrogen flame, which is the palest of all flames, gives the greatest heat.

Chemical decomposition, in which the attraction of heterogeneous molecules for each other is overcome, and they are moved further apart, is an operation requiring an expenditure of work or an equivalent consumption of heat; and conversely, in chemical combination, motion is transformed into heat. When bodies attract each other chemically their molecules move towards each other with gradually increasing velocity, and when impact has taken place the progressive motion of the molecules ceases, and is converted into a rotating, vibrating, or progressive motion of the molecules of the new body.

The heat produced by chemical combination of two elements may be compared to that due to the impact of bodies against each other. Thus the action of the atoms of oxygen, which in virtue of their progressive motion, and of chemical attraction, rush against ignited carbon, has been likened by Tyndall to the action of meteorites which fall into the sun.

484. **Heat disengaged during chemical action.**—Many physicists, more especially Lavoisier, Rumford, Dulong, Despretz, Hess, Favre and Silbermann, Berthelot, Thomsen, and Andrews, have investigated the quantity of heat disengaged by various bodies in chemical actions.

Lavoisier used in his experiments the ice calorimeter already described. Rumford used a calorimeter known by his name, which consists of a rectangular copper canister filled with water. In this canister there is a worm which passes through the bottom of the box, and terminates below in an inverted funnel. Under this funnel is burnt the substance experimented upon. The products of combustion, in passing through the worm, heat the water of the canister, and from the increase of its temperature the quantity of heat evolved is calculated. Despretz and Dulong successively modified Rumford's calorimeter by allowing the combustion to take place, not outside the canister, but in a chamber placed in the liquid itself; the oxygen necessary for the combustion entered by a tube in the lower part of the chamber, and the products of combustion escaped by another tube placed at the upper part and twisted in a serpentine form in the mass of the liquid to be heated. Favre and Silbermann improved this calorimeter very greatly (463), not only by avoiding or taking account of all possible sources of error, but by arranging it for the determination of the heat evolved in such chemical actions as take place between gases and vapours. The gases enter by tubes BB′ and CC′, fig. 434, into a metal chamber A, where the reaction takes

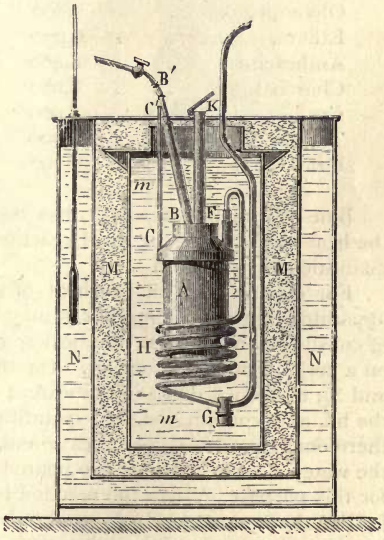


Fig. 434.

place, the course of which can be watched through a glass plate which closes a wider tube FK. The gaseous products before passing into the air traverse a long serpentine tube H, at the lower end of which is a small box G which receives the liquids arising from the condensation of the vapours. The cylinder A and the serpentine are contained in a known mass of water contained in a calorimeter, and from the rise in temperature of this water the heat developed can be calculated. To avoid any loss of heat this is placed within a metal case, MM, containing swan's down. The whole is contained in a vessel of water NN in which is a thermometer, to eliminate the influence of changes in the temperature of the air.

The experiments of Favre and Silbermann are the most trustworthy, as having been executed with the greatest care. They agree very closely with those of Dulong. Taking as thermal unit the heat necessary to raise the temperature of a pound of water through *one* degree Centigrade, the following table gives the thermal units in round numbers disengaged by a pound of each of the substances while burning in oxygen :—

Hydrogen	34,462	Diamond	7,770
Marsh gas	13,063	Absolute alcohol	7,180
Olefiant gas	11,858	Coke	7,000
Oil of turpentine	10,852	Phosphorus	5,750
Olive oil	9,860	Wood, dried at 120°	3,616
Ether	9,030	Bisulphide of carbon	3,401
Anthracite	8,460	Wood, ordinary	2,756
Charcoal	8,080	Carbonic oxide	2,400
Coal	8,000	Sulphur	2,220
Tallow	8,000	Iron	1,181
Graphite	7,797	Zinc	1,300

Bunsen's calorimeter (451) has been used with advantage for studying the heat produced in chemical reactions, for cases in which only very small quantities are available.

For experiments on the heat of neutralisation of acids and bases the apparatus represented in fig. 435 may be used. W is a large vessel of water of constant temperature ; the beaker glass, B, which is the calorimeter, rests on a cork in the outer one, A. On the wooden lid, H, are two weights, S_1 and S_2, to keep A down in the water ; *c* and *d* are tubes placed in holes in the lid, and contain weighed quantities of the two liquids ; *b* is a delicate thermometer. After the tubes *c* and *d* have acquired the temperature of the water t, their contents are poured into B through an aperture in the lid for this purpose. When the reaction is complete, the temperature indicated by the thermometer, which reaches to the middle of B, rises to t, so that, knowing the weight of the substances, and the rise $t_1 - t$, the quantity of heat produced in the reaction is easily determined.

All chemical actions, whether of combination or of decomposition, are attended by a disturbance of the thermal equilibrium ; and the quantity of heat disengaged is a measure of the physical and chemical work.

In most cases the act of chemical combination is attended by a rise of temperature, and the quantity of heat is a measure of the energy developed

in the reaction. Thus in the formation of one *molecule* of water there are liberated 68,924 thermal units, which may be written thus,

$$H_2 + O = H_2O + 68{,}924.$$

Those reactions which take place with disengagement of heat are said to be *exothermic*; there are, however, cases where bodies do not directly combine without the intervention of extraneous heat—for instance, iodine and hydrogen to form hydriodic acid; the equation for this is

$$I + H + 6{,}000 = IH.$$

Such reactions are called *endothermic*.

Those bodies are most stable in the formation of which most heat is developed; thus the oxides of iron and zinc, in the formation of which 1,181 and 1,300 units are respectively developed, are much more stable than oxide of silver, in the formation of which only 27 units are developed. The heat of decomposition is the reciprocal of that of combination; those bodies which develop most heat in their formation require conversely an equivalent quantity to decompose them; decompositions which require an expenditure of heat to produce them are called *endothermic*. Those compounds, on the contrary, which absorb heat in their formation, develop an equivalent quantity in being decomposed, and the reactions are *exothermic*; they often take place with explosive violence, as in the case of the chlorides and iodide of nitrogen. An exothermic reaction gives rise to an endothermic compound; and, conversely, an endothermic reaction forms an exothermic compound.

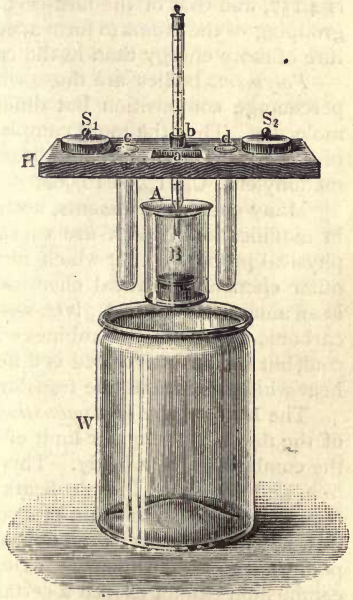


Fig. 435.

If there be any system of bodies which act on each other without the supply of extraneous energy, then that body, or set of bodies, results in the formation of which most heat is produced. This is called the *principle of greatest chemical action*.

The heat developed in any chemical reaction depends on the relation between the initial and the final products, and is independent of the nature and succession of the intermediate stages. It is equal to the sum of the quantities of heat produced in each stage, regard being had to the negative quantities produced in such processes as solution and gasification.

Thus a unit weight of carbon in burning to carbonic acid produces 8,080 units. If the same weight of carbon burns so as to form carbonic oxide it forms 2,473; and the combustion of the carbonic oxide resulting from this reaction yields 5,607, making together 8,080.

Potassium combines directly with chlorine to form potassium chloride, the heat of formation of which is 15,000, and is equal to that produced by the same weight of salt, whether this be formed by the direct union of hydrochloric acid and potash, or whether it be produced by the action of potassium on aqueous solution of hydrochloric acid.

The heat of combustion of a compound is not equal to the sum of that of each of its constituents. The heat of combustion of bisulphide of carbon is 3,401, while that calculated from its constituents is 3,145; the compound accordingly possesses more energy than its constituents, and its formation is due to an endothermic reaction.

Metameric bodies are those which contain the same number of elements but in different groupings; thus acetic acid and methylic formate have each the composition $C_2H_4O_2$; but the heat of combustion of the latter is 4,157, and that of the former 3,505; from this it is to be inferred that the grouping of the atoms to form acetic acid has been attended with the expenditure of more energy than in the case of methylic formate.

Polymeric bodies are those which have the same elements and the same percentage composition but differ in the number of atoms which form a molecule. Thus the more complex the molecule the smaller is the quantity of heat. That of amylene, for instance, C_5H_{10}, is 11,401, and that of metamylene, $C_{20}H_{40}$, is 10,908.

Many chemical elements, such as carbon, sulphur, and phosphorus, exist in modifications which are essentially different from each other in their physical properties, but which form when they enter into combination with other elements identical chemical products. Such bodies are said to exist in an allotropic form. A given weight of carbon produces the same weight of carbonic acid when it combines with oxygen, whether it be diamond or charcoal, but the heat produced is different, and this difference corresponds to the heat which represents the transformation from one modification into another.

The *temperature of combustion*, or, in the case of gases, the temperature of the flame, is the upper limit of the temperature which can be attained by the combustion of a body. This can be deduced from the heat of combustion, and from the specific heats of the bodies produced. The theoretical temperature of combustion of hydrogen in oxygen is calculated at 6,715°; this, however, is never even approximately reached, for at much lower temperatures aqueous vapour is *dissociated* (389) into its constituents, and the combustion cannot exceed a certain limit.

485. **Animal heat.**—In all the organs of the human body, as well as those of all animals, processes of oxidation are continually going on. Oxygen passes through the lungs into the blood, and so into all parts of the body. In like manner the oxidisible bodies, which are principally hydrocarbons, pass by the process of digestion into the blood, and likewise into all parts of the body, while the products of oxidation, carbonic acid and water, are eliminated by the skin, the lungs, &c. Oxidation in the muscle produces motions of the molecules, which are changed into contraction of the muscular fibres; all other oxidations produce heat directly. When the body is at rest, all its functions, even involuntary motions, are transformed into heat. When the body is at work, the more vigorous oxidations of the working parts are transferred to the others. Moreover, a great part of the muscular work is

changed into heat, by friction of the muscle and of the sinews in their sheaths, and of the bones in their sockets. Hence the heat produced by the body when at work is greater than when at rest. The blood distributes heat uniformly through the body, which in the normal condition has a temperature of 37° C. = 98·6 F. The blood of mammalia has the same temperature, that of birds is somewhat higher. In fever the temperature rises to 42° to 43°, and in cholera, or when near death, sinks as low as 35°.

The function of producing work in the animal organism was formerly considered as separate from that of the production of heat. The latter was held to be specially due to the oxidation of the hydrocarbons of the fat, while the work was ascribed to the chemical activity of the nitrogenous matter. This view has now been generally abandoned; for it has been found that during work there is no increase in the secretion of urea, which is the result of the oxidation of nitrogenous matter; moreover, the organism while at rest produces less carbonic acid, and requires less oxygen than when it is at work; and the muscle itself, both in the living organism and also when removed from it and artificially stimulated, requires more oxygen in a state of activity than when at rest. For these reasons the production of work is ascribed to the oxidation of the organic matter generally.

The process of vegetation in the living plant is not in general connected with any oxidation. On the contrary, under the influence of the sun's rays, the green parts of plants decompose the carbonic acid of the atmosphere into free oxygen gas and into carbon, which, uniting with the elements of water, form cellulose, starch, sugar, and so forth. In order to effect this, an expenditure of heat is required which is stored up in the plant, and which reappears during the combustion of the wood, or of the coal arising from its decomposition.

At the time of blossoming a process of oxidation goes on, which, as in the case of the blossoming of the *Victoria regia*, is attended with an appreciable rise of temperature.

HEATING

486. **Different kinds of heating.**—*Heating* is the art of utilising for domestic and industrial purposes the sources of heat which nature offers to us. Our principal source of artificial heat is the combustion of coal, coke, turf, wood, and charcoal.

487. **Fireplaces.**—Fireplaces are open hearths built against a wall under a chimney, through which the products of combustion escape.

However much they may be improved, fireplaces will always remain the most imperfect and costly mode of heating, for they only render available 13 per cent. of the total heat yielded by coal or coke, and 6 per cent. of that by wood. This enormous loss of temperature arises from the fact that the current of air necessary for combustion always carries with it a large quantity of the heat produced, which is dissipated in the atmosphere. Hence Franklin said 'fireplaces should be adopted in cases where the smallest quantity of heat was to be obtained from a given quantity of fuel.' Notwithstanding their want of economy, however, they will always be preferred as the healthiest and pleasantest mode of heating, on account of the cheerful light which they emit, and the ventilation which they ensure.

488. **Draught of fireplaces.**—The *draught* of a fire is the upward current in the chimney caused by the ascent of the products of combustion; when the current is rapid and continuous, the chimney is said to *draw* well.

The draught is caused by the difference between the temperature of the inside and that on the outside of the chimney; for, in consequence of this difference, the gaseous bodies which fill the chimney are lighter than the air of the room, and consequently equilibrium is impossible. The weight of the column of gas CD, fig. 436, in the chimney being less than that of the external column of air AB of the same height, there is a pressure from the outside to the inside which causes the products of combustion to ascend the more rapidly in proportion as the difference in weight of the two gaseous masses is greater.

The velocity of the draught of a chimney may be determined theoretically by the formula

$$v = \sqrt{2ga(t' - t)h},$$

in which g is the acceleration of gravity, a the coefficient of the expansion of air, h the height of the chimney, t' the mean temperature of the air inside the chimney, and t the temperature of the surrounding air.

The currents caused by the difference in temperature of two communicating gaseous masses may be demonstrated by placing a candle near the top and near the bottom of the partially-opened door of a warm room. At the top, the flame will be turned from the room towards the outside, while the contrary effect will be produced when the candle is placed on the ground. The two effects are caused by the current of heated air which issues by the top of the door, while the cold air which replaces it enters at the bottom.

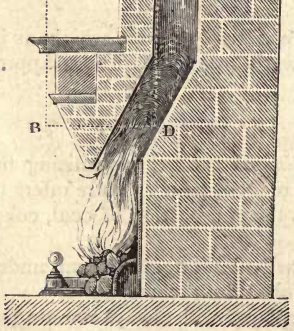


Fig. 436.

In order to have a good draught, a chimney ought to satisfy the following conditions:—

i. The section of the chimney ought not to be larger than is necessary to allow an exit for the products of combustion; otherwise ascending and descending currents are produced in the chimney, which cause it to smoke. It is advantageous to place on the top of the chimney a conical pot narrower than the chimney, so that the smoke may escape with sufficient velocity to resist the action of the wind.

ii. The chimney ought to be sufficiently high; for, as the draught is caused by the excess of the external over the internal pressure, this excess is greater in proportion as the column of heated air is longer.

iii. The external air ought to pass into the chamber with sufficient rapidity to supply the wants of the fire. In an hermetically-closed room combustibles would not burn, or descending currents would be formed which

would drive the smoke into the room. Usually air enters in sufficient quantity by the crevices of the doors and windows.

iv. Two chimneys should not communicate, for if one draws better than the other, a descending current of air is produced in the latter, which carries smoke with it.

For the strong fires required by steam-boilers and the like, very high chimneys are needed ; of course the increase in height would lose its effect if the hot column above became cooled down. Hence chimneys are often made with hollow walls—that is, of separate concentric layers of masonry or brickwork—the space between them containing air.

489. **Stoves.**—*Stoves* are apparatus for heating with a detached fire, placed in a room to be heated, so that heat radiates in all directions round the stove. At the lower part is the draught-hole by which the air necessary for combustion enters. The products of combustion escape by means of iron chimney-pipes. This mode of heating is one of the most economical, but it is by no means so healthy as that by open fire-places, for the ventilation is very bad, more especially where the stoves are fed from the outside of the room. These stoves also emit a bad smell, arising in part from the decomposition of organic substances which are always present in the air by their contact with the heated sides of the chimney-pipes ; or possibly, as Deville and Troost's researches seem to show, from the diffusion of gases through the heated sides of the stove.

The heating is very rapid with blackened metal stoves, but they also cool very rapidly. Stoves constructed of polished earthenware, which are common on the Continent, heat more slowly, but more pleasantly, and they retain the heat longer.

490. **Heating by steam.** Steam, in condensing, gives up its latent heat of vaporisation, and this property has been used in heating baths, workshops, public buildings, hothouses, &c. For this purpose steam is generated in boilers similar to those used for steam-engines, and is then made to circulate in pipes placed in the room to be heated. The steam condenses, and in doing so imparts to the pipes its latent heat, which becomes free, and thus heats the surrounding air.

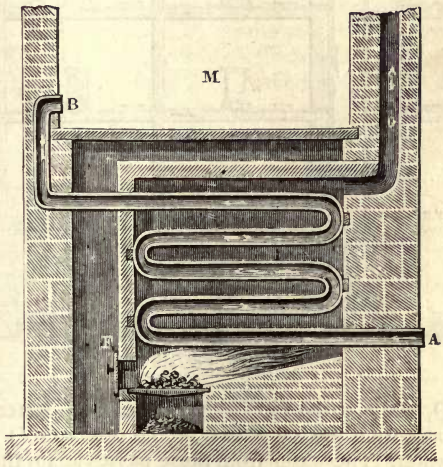


Fig. 437.

491. **Heating by hot air.**—Heating by hot air consists in heating the air in the lower part of a building, from whence it rises to the higher parts

in virtue of its lessened density. The apparatus is arranged as represented in fig. 437.

A series of tubes, AB, only one of which is shown in the figure, is placed in a furnace F, in the cellar. The air passes into the tubes through the lower end, A, where it becomes heated, and, rising in the direction of the arrows, reaches the room M by a higher aperture, B. The various rooms to be heated are provided with one or more of these apertures, which are placed as low in the room as possible. The conduit O is an ordinary chimney. These apparatus are more economical than open fire-places, but they are less healthy, unless special provision is made for ventilation.

492. **Heating by hot water.**—This consists of a continuous circulation of water, which, having been heated in a boiler, rises through a series of tubes, and then, after becoming cool, passes into the boiler again by a similar series.

Fig. 438 represents an apparatus for heating a building of several stories. The heating apparatus, which is in the basement, consists of a bell-shaped boiler, *o o*, with an internal flue, F. A long pipe, M, fits in the upper part of the boiler, and also in the reservoir Q, placed in the upper part of the building to be heated. At the top of this reservoir there is a safety valve, *s*, by which the pressure of the vapour in the interior can be regulated.

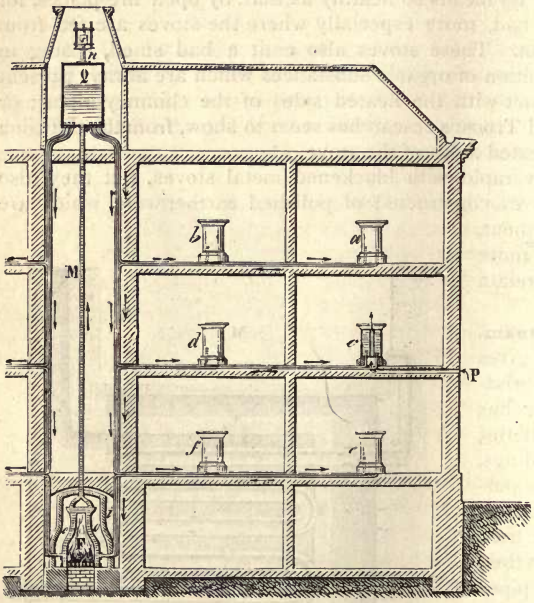


Fig. 438.

The boiler, the pipe M, and a portion of the reservoir Q, being filled with water, as it becomes heated in the boiler an ascending current of hot water rises to the reservoir Q, while at the same time descending currents of colder and denser water pass from the lower part of the reservoir Q into receivers *b*, *d*, *f*, filled with water. The water from these passes again through pipes into other receivers, *a*, *c*, *e*, and ultimately reaches the lower part of the boiler.

During this circulation the hot water heats the pipes and the receivers, which thus become true water-stoves. The number and the dimensions of these parts are determined from the fact that a cubic foot of water in falling through a temperature of one degree can theoretically impart the same in-

crease of temperature 3,200 cubic feet of air (460). In the interior of the receivers, *a*, *b*, *c*, *d*, *e*, *f*, there are cast-iron tubes which communicate with the outside by pipes, P, placed underneath the flooring. The air becomes heated in these tubes, and issues at the upper part of the receiver.

The principal advantage of this mode of heating is that of giving a temperature which is constant for a long time, for the mass of water only cools slowly. It is much used in hot-houses, baths, artificial incubation, drying rooms, and generally wherever a uniform temperature is desired.

SOURCES OF COLD

493. **Various sources of cold.**—Besides the cold caused by the passage of a body from a solid to the liquid state, of which we have already spoken, cold is produced by the expansion of gases, by radiation in general, and more especially by radiation at night.

494. **Cold produced by the expansion of gases. Ice machines.**—We have seen that when a gas is compressed the temperature rises (479). The reverse of this is also the case: when a gas is rarefied, a reduction of temperature ensues, because a quantity of sensible heat disappears when the gas becomes increased to a larger volume. This may be shown by placing a delicate Breguet's thermometer under the receiver of an air-pump, and exhausting; at each stroke of the piston the needle moves in the direction of zero, and regains its original position when air is admitted.

The production of cold when a gas is expanded has been extensively applied in machines for artificial refrigeration on a large scale. By Windhausen's ice machine, from 15,000 to 150,000 feet of air can be cooled in an hour, through 40 to 100 degrees in temperature, by means of a steam-engine of from 6 to 20 horse-power. The essential parts of this machine are represented in fig. 439. The piston B in the cylinder A is worked to the right by

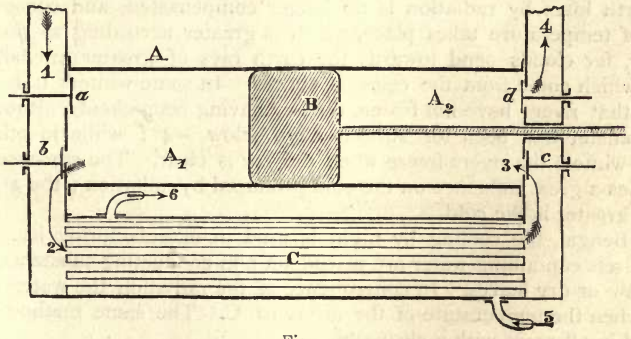


Fig. 439.

a steam-engine and to the left by a steam-engine and by the compressed air. As it moves towards the right the valve *a* opens, and air under the ordinary atmospheric pressure enters the space A_1. When this is full the piston moves towards the left, the air in A is compressed to about 2 atmospheres, the

valve *a* is closed, the valve *b* opens, and air passes in the direction of the arrows into the cooler, C. By its compression it has become strongly heated, and the necessary cooling is effected by means of pipes through which cold water circulates, entering at 5 and emerging at 6. The air, thus compressed and cooled, passes out through the valve *c*, which is automatically worked by the machine, into the space A_2, where, in conjunction with the steam-engine, it moves the piston to the left, and compresses the air in A_1; for at a certain position of the piston the valve *c* is closed, the compressed air in the cylinder A_2 expands, and thereby is cooled far below the freezing point. As the piston moves again to the right, the valve *d* is opened by the working of the machine, and the cooled air emerges through the tube 4 to its destination. If it passes into an ordinary room, by condensing the moisture it fills the air with snowflakes. Machines of this kind are extensively employed in the arts; in breweries, oil refineries, in the artificial production of ice, and in cooling rooms on board ship for the transport of dead meat, &c., which has become an industry of the greatest importance.

In the Linde machine the material used is ammoniacal gas, which is liquefied by compression and surface condensation. This liquid ammonia being allowed to evaporate takes the heat for this change of state from the surrounding bodies, which are thereby cooled. The ammonia vapour thus formed is again liquefied, and flowing back to the refrigerator is once more evaporated, so that a small quantity of ammonia is always passing through the same cycle of operations.

A machine of this kind worked by a steam-engine of half a horse-power can cool in an hour 3,400 cubic yards of air from 10° to 5° C., or 1,400 cubic yards from 6° to −4° C.; or it will produce 1 cwt. of ice in the same time. The larger machines are relatively more advantageous.

495. **Cold produced by radiation at night.**—During the day the ground receives from the sun more heat than radiates into space, and the temperature rises. The reverse is the case during night. The heat which the earth loses by radiation is no longer compensated, and consequently a fall of temperature takes place, which is greater according as the sky is clearer, for clouds send towards the earth rays of greater intensity than those which come from the celestial spaces. In some winters it has been found that rivers have not frozen, the sky having been cloudy, although the thermometer had been for several days below −4°; while in other less severe winters the rivers freeze when the sky is clear. The emissive power exercises a great influence on the cold produced by radiation; the greater it is, the greater is the cold.

In Bengal, the cooling by night is used in manufacturing ice. Large flat vessels containing water are placed on non-conducting substances, such as straw or dry leaves. In consequence of the radiation the water freezes, even when the temperature of the air is 10° C. The same method can be applied in all cases with a clear sky.

The Peruvians, in order to preserve the shoots of young plants from freezing, light great fires in their neighbourhood, the smoke of which, producing an artificial cloud, hinders the cooling produced by radiation.

496. **Absolute zero of temperature.**—As a gas is increased $\frac{1}{273}$ of its volume for each degree Centigrade, it follows that at a temperature of 273°

C. the volume of any gas measured at zero is doubled. In like manner, if the temperature of a given volume at zero were lowered through $-273°$, the contraction would be equal to the volume: that is, the volume would not exist. At this temperature the motion of the molecules of the gas would completely cease, and the pressure thereby occasioned. In all probability, before reaching this temperature, gases would undergo some change.

This point on the Centigrade scale is called the *absolute zero of temperature*; the temperatures reckoned from this point are called *absolute temperatures*. They are clearly obtained by adding 273 to the temperature on the Centigrade scale. Thus $-35°$ C. is 238° on the absolute scale of temperature, and $+15°$ C. is 288°.

CHAPTER XII

MECHANICAL EQUIVALENT OF HEAT

497. **Mechanical equivalent of heat.**—If the various instances of the production of heat by motion be examined, it will be found that in all cases mechanical force is consumed. Thus in rubbing two bodies against each other, motion is apparently destroyed by friction ; it is not, however, lost, but appears in the form of a motion of the particles of the body ; the motion of the mass is transformed into a motion of the molecules.

Again, if a body be allowed to fall from a height, it strikes against the ground with a certain velocity. According to older views, its motion is destroyed, *vis viva* is lost. This, however, is not the case ; the *vis viva* of the body appears as *vis viva* of its molecules.

In the case, too, of chemical action, the most productive artificial source of heat, it is not difficult to conceive that there is, in the act of combining, an impact of the dissimilar molecules against each other, an effect analogous to the production of heat by the impact of masses of matter against each other (483).

In like manner, heat may be made to produce motion, as in the case of the steam-engine, and the propulsion of shot from a gun.

Traces of a view that there is a connection between heat and motion are to be met with in the older writers, Bacon for example ; and Locke says, 'Heat is a very brisk agitation of the insensible parts of the object, which produces in us that sensation from whence we denominate the object hot ; so that what in our sensation is heat, in the object is nothing but motion.' Rumford, in explaining his great experiment of the production of heat by friction, was unable to assign any other cause for the heat produced than motion ; and Davy, in the explanation of his experiment of melting ice by friction *in vacuo*, expressed similar views. Carnot, in a work on the steam engine, published in 1824, also indicated a connection between heat and work.

The views, however, which had been stated by isolated writers had little or no influence on the progress of scientific investigation, and it is in the year 1842 that the modern theories may be said to have had their origin. In that year Dr. Mayer, a physician in Heilbronn, formally stated that there exists a connection between heat and work ; and he it was who first introduced into science the expression '*mechanical equivalent of heat*.' Mayer also gave a method by which this equivalent could be calculated ; the particular results, however, are of no value, as the method, though correct in principle, is founded on incorrect data.

In the same year, too, Colding of Copenhagen published experiments on

the production of heat by friction, from which he concluded that the evolution of heat was proportional to the mechanical energy expended.

About the same time as Mayer, but quite independently of him, Joule commenced a series of experimental investigations on the relation between heat and work. These first drew the attention of scientific men to the subject, and were admitted as a proof that the transformation of heat into mechanical energy, or of mechanical energy into heat, always takes place in a definite numerical ratio.

Subsequently to Mayer and Joule, several physicists, by their theoretical and experimental investigations, have contributed to establish the mechanical theory of heat : namely, in this country, Sir W. Thomson and Rankine ; in Germany, Helmholtz, Clausius, and Holtzmann ; and in France, Clapeyron and Regnault. The following are some of the most important and satisfactory of Joule's experiments.

A copper vessel, B (fig. 440), was provided with a brass paddle-wheel (indicated by the dotted lines), which could be made to rotate about a

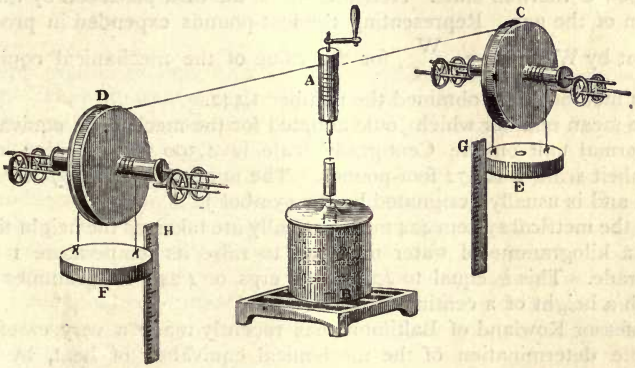


Fig. 440.

vertical axis. Two weights, E and F, were attached to cords which passed over the pulleys C and D, and were connected with the axis A. These weights in falling cause the wheel to rotate. The height of the fall, which in Joule's experiments was about 63 feet, was indicated on the scales G and H.

The roller A was so constructed that by detaching a pin the weights could be raised without moving the wheel. The vessel B was filled with water and placed on a stand, and the weights allowed to sink. When they had reached the ground, the roller was detached from the axis, and the weights again raised, the same operations being repeated twenty times. The heat produced was measured by ordinary calorimetric methods (447).

The work expended is measured by the product of the weight into the height through which it falls, or ph, less the work lost by the friction of the various parts of the apparatus. This is diminished as far as possible by the use of friction wheels (77), and its amount is determined by connecting C and D without causing them to pass over A, and then determining the weight necessary to communicate to them a uniform motion.

In this way it has been found that a thermal unit—that is, the quantity of heat by which a pound of water is raised through 1° C.—is generated by the expenditure of the same amount of work as would be required to raise 1,392 pounds through 1 foot, or 1 pound through 1,392 feet. This is expressed by saying that the mechanical equivalent of the thermal unit is 1,392 foot-pounds.

The friction of an iron paddle-wheel in mercury gave 1,397 foot-pounds, and that of the friction of two iron plates gave 1,395 foot-pounds, as the mechanical equivalent of one thermal unit.

In another series of experiments, the air in a receiver was compressed by means of a force-pump, both being immersed in a known weight of water at a known temperature. After 300 strokes of the piston the heat, C, was measured which the water had gained. This heat was due to the compression of the air and to the friction of the piston. To eliminate the latter influence, the experiment was made under the same conditions, but leaving the receiver open. The air was not compressed, and 300 strokes of the piston developed C′ thermal units. Hence $C - C'$ is the heat produced by the compression of the gas. Representing the foot-pounds expended in producing this heat by W, we have $\frac{W}{C - C'}$ for the value of the mechanical equivalent. By this method Joule obtained the number 1,442.

The mean number which Joule adopted for the mechanical equivalent of one thermal unit on the Centigrade scale is 1,390 foot-pounds; on the Fahrenheit scale it is 772 foot-pounds. The number is called *Joule's equivalent*, and is usually designated by the symbol J.

On the metrical system 424 metres usually are taken as the height through which a kilogramme of water must fall to raise its temperature 1 degree Centigrade. This is equal to 42,400,000 ergs or $4{\cdot}24 \times 10^7$ grammes raised through a height of a centimetre.

Professor Rowland of Baltimore has recently made a very careful and complete determination of the mechanical equivalent of heat, by Joule's method, in which he has examined and allowed for all possible sources of error. His results give 426·9 kilogramme-metres as the mean value of this constant for the latitude of Baltimore.

Hirn made the following determination of the mechanical equivalent by means of the heat produced by the compression of lead. A large block of sandstone, CD (fig. 441), is suspended vertically by cords; its weight is P. E is a piece of lead, fashioned so that its temperature may be determined by the introduction of a thermometer. The weight of this is Π, and its specific heat c. AB is a cylinder of cast iron, whose weight is p. If this be raised to A′B′, a height of h, and allowed to fall again, it compresses the lead, E, against the anvil, CD. It remains to measure on the one hand the work lost, and on the other the heat gained.

The hammer AB being raised to a height h, the work of its fall is ph; but as, by its elasticity, it rises again to a height h_1, the work is $p(h - h_1)$. The anvil CD, on the other hand, has been raised through a height H to C′D′ and has required in so doing PH units of work. The work, W, definitely absorbed by the lead is $p(h - h_1) - PH$. On the other hand, the lead has been heated by θ, it has gained $\Pi c\theta$ thermal units, c being the

specific heat of lead, and the mechanical equivalent J is equal to the quotient $\frac{W}{\Pi c\theta}$. A series of six experiments gave 1,394 for the mechanical equivalent as thus obtained.

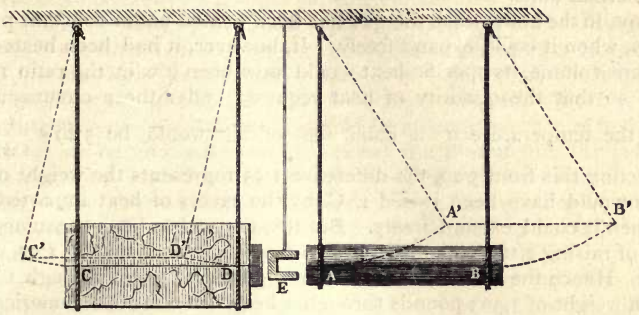


Fig. 441.

Recent experiments of Cantoni and Gerosa in this direction are the simplest. They allowed mercury to fall from a funnel through a narrow tube into a vessel below, when its temperature was measured. In this way the number 1,390 was obtained.

Experiments in the opposite direction have also been made, in which the work produced by one thermal unit was determined. This was done on a large scale by Hirn by means of a steam-engine of one hundred horse-power. He determined the quantity of heat contained in the steam before its action, and then the amount contained in the water after its condensation. This was less, for some had been expended in work; and this work as measured by the dynamometer was equivalent to that which had disappeared, the number 1390·7 being thus obtained.

The following is the method which Mayer first employed in calculating the mechanical equivalent of heat. It is taken, with slight modifications, from Professor Tyndall's work on *Heat*, who, while strictly following Mayer's reasoning, has corrected his data.

Let us suppose that a rectangular vessel with a section of a square foot contains at 0° a cubic foot of air under the ordinary atmospheric pressure; and let us suppose that it is inclosed by a piston without weight.

Suppose now that the cubic foot of air is heated until its volume is doubled; from the coefficient of expansion of air we know that this is the case at 273°C. The gas in doubling its volume will have raised the piston through a foot in height; it will have lifted the atmospheric pressure through this distance. But the atmospheric pressure on a square foot is in round numbers $15 \times 144 = 2{,}160$ pounds. Hence a cubic foot of air in doubling its volume has lifted a weight of 2,160 pounds through a height of a foot.

Now, a cubic foot of air at zero weighs 1·29 ounce, and the specific heat of air under constant pressure—that is, when it can expand freely—as compared with that of an equal weight of water, is 0·24; so that the quantity of heat which will raise 1·29 ounce of air through 273° will only raise $0{\cdot}24 \times 1{\cdot}29$

= 031 oz. of water through the same temperature ; but 0·31 oz. of water raised through 273° is equal to 5·29 pounds of water raised through 1° C.

That is, the quantity of heat which will double the volume of a cubic foot of air, and in so doing will lift 2,160 pounds through a height of a foot, is 5·29 thermal units.

Now, in the above case the gas has been heated under constant pressure, that is, when it could expand freely. If, however, it had been heated under constant volume, its specific heat would have been less in the ratio 1 : 1·414 (460), so that the quantity of heat required under these circumstances to raise the temperature o a cubic foot of air would be $5{\cdot}29 \times \frac{1}{1{\cdot}41} = 3{\cdot}74$. Deducting this from 5·29, the difference 1·55 represents the weight of water which would have been raised 1° C. by the excess of heat imparted to the air when it could expand freely. But this excess has been consumed in the work of raising 2,160 pounds through a foot. Dividing this by 1·55, we have 1,393. Hence the heat which will raise a pound of water through 1° C. will raise a weight of 1,393 pounds through a height of a foot ; a numerical value of the mechanical equivalent of heat agreeing as closely as can be expected with that which Joule adopted as the most certain of his experimental results.

The law of the relation of heat to mechanical energy may be thus stated :— *Heat and mechanical energy are mutually convertible ; and heat requires for its production, and produces by its disappearance, mechanical energy in the ratio of* 1,390 *foot-pounds for every thermal unit.*

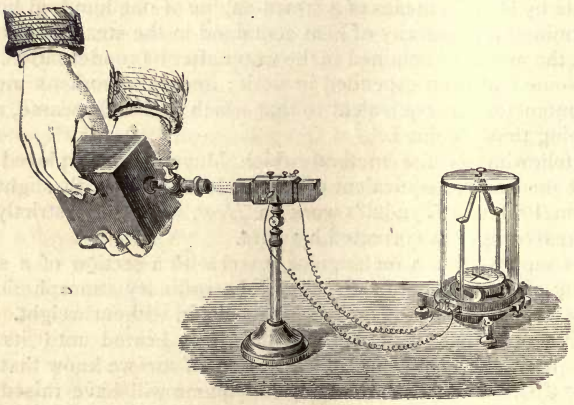


Fig. 442.

A variety of experiments may in like manner be adduced to show that whenever heat disappears work is produced. For example, if in a reservoir immersed in water the air be compressed to the extent of 10 atmospheres : supposing that now, when the compressed air has acquired the temperature of the water, it be allowed to act upon a piston loaded by a weight, the weight is raised. At the same time the water becomes cooler, showing that

a certain quantity of heat had disappeared in producing the mechanical work of raising the weight. This may also be illustrated by the following experiment (fig. 442), due to Professor Tyndall :—

A strong metal box is taken, provided with a stopcock, on which can be screwed a small condensing pump. Having compressed the air by its means since it becomes heated by this process, the box is allowed to stand for some time, until it has acquired the temperature of the surrounding medium. On opening the stopcock the air rushes out : it is expelled by the expansive force of the internal air ; in short, the air drives itself out. Work is therefore performed by the air, and there should be a disappearance of heat ; and if the jet of air be allowed to strike against the thermopile, the galvanometer is deflected, and the direction of its deflection indicates a cooling (fig. 442). The same effect is observed when, on opening a bottle of soda water, the carbonic gas which escapes is allowed to impinge on the thermopile.

If, on the contrary, the experiment is made with an ordinary pair of bellows, and the current of air is allowed to strike against the pile, the deflection of the galvanometer is in the opposite direction, indicating an increase of temperature (fig. 443). In this case the hand of the experimenter performs the work, which is converted into heat.

Joule placed in a calorimeter two equal copper reservoirs, which could be connected by a tube. One of these contained air at 22 atmospheres, the other was exhausted. When they were connected, they came into equilibrium under a pressure of 11 atmospheres ; but as the gas in expanding had done no work, there was no alteration in temperature. When, however, the second reservoir was full of water, the air in entering was obliged to

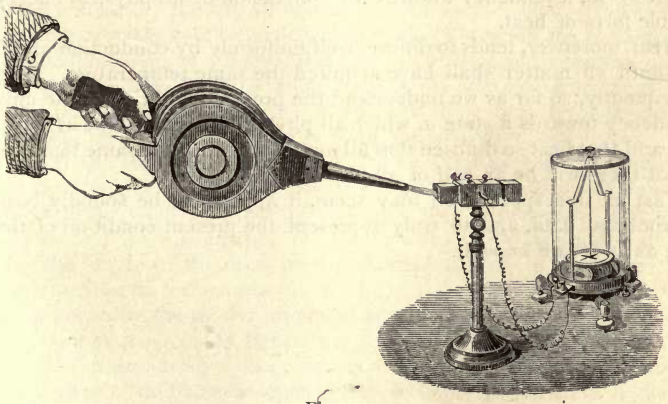


Fig. 443.

expel it and thus perform work, and the temperature sank, owing to an absorption of heat.

For further information the student of this subject is referred to the following works :—Tyndall on *Heat as a Mode of Motion*, Maxwell on *Heat*, Wormell's *Thermodynamics* (Longmans), and Tait on *Thermodynamics*

(Edmondston & Douglas). A condensed, though complete and systematic, account of the dynamical theory of heat is met with in Professor Foster's articles on 'Heat,' in *Watts' Dictionary of Chemistry.*

498. **Dissipation of energy.**—Rankine has the following interesting observations on a remarkable consequence of the mutual convertibility which has been shown to exist between heat and other forms of energy:—Sir W. Thomson has pointed out the fact that there exists, at least in the present state of the known world, a predominating tendency to the conversion of all other forms of physical energy into heat, and to the uniform diffusion of heat throughout all matter. The form in which we generally find energy originally collected is that of a store of chemical power consisting of uncombined elements. The combination of these elements produces energy in the form known by the name of electrical currents, part only of which can be employed in analysing chemical compounds, and thus reconverted into a store of chemical power; the remainder is necessarily converted into heat; a part only of this heat can be employed in analysing compounds or in reproducing electric currents. If the remainder of the heat be employed in expanding an elastic substance, it may be converted entirely into visible motion, or into a store of visible mechanical power (by raising weights, for example), provided the elastic substance is enabled to expand until its temperature falls to the point which corresponds to the absolute privation of heat; but unless this condition is fulfilled a certain proportion only of the heat, depending on the range of temperature through which the elastic body works, can be converted, the rest remaining in the state of heat. On the other hand, all visible motion is of necessity ultimately converted into heat by the agency of friction. There is, then, in the present state of the known world, a tendency towards the conversion of all physical energy into the sole form of heat.

Heat, moreover, tends to diffuse itself uniformly by conduction and radiation, until all matter shall have acquired the same temperature. There is, consequently, so far as we understand the present condition of the universe, a tendency towards a state in which all physical energy will be in the state of heat, and that heat so diffused that all matter will be at the same temperature; so that there will be an end of all physical phenomena.

Vast as this speculation may seem, it appears to be soundly based on experimental data, and to truly represent the present condition of the universe as far as we know it.

BOOK VII

ON LIGHT

CHAPTER I

TRANSMISSION, VELOCITY, AND INTENSITY OF LIGHT

499. **Theories of light.**—*Light* is the agent which, by its action on the retina, excites in us the sensation of vision. That part of physics which deals with the properties of light is known as *optics*.

In order to explain the origin of light, various hypotheses have been made, the most important of which are the *emission* or *corpuscular* theory, and the *undulatory* theory.

On the *emission* theory it is assumed that luminous bodies emit, in all directions, an imponderable substance, which consists of molecules of an extreme degree of tenuity: these are propagated in right lines with an almost infinite velocity. Penetrating into the eye they act on the retina, and determine the sensation which constitutes vision.

On the undulatory theory, all bodies, as well as the celestial spaces, are filled by an extremely subtle elastic medium, which is called the *luminiferous ether*. The luminosity of a body is due to an infinitely rapid vibratory motion of its molecules, which, when communicated to the ether, is propagated in all directions in the form of spherical waves, and this vibratory motion, being thus transmitted to the retina, calls forth the sensation of vision. The vibrations of the ether take place not in the direction of the wave, but in a plane at right angles to it. The latter are called the *transversal* vibrations. An idea of these may be formed by shaking a rope at one end. The vibrations, or to and fro movements, of the particles of the rope, are at right angles to the length of the rope, but the onward motion of the wave's form is in the direction of the length.

On the emission theory the propagation of light is effected by a motion or *translation* of particles of light thrown out from the luminous body, as a bullet is discharged from a gun; on the undulatory theory there is no progressive motion of the particles themselves, but only of the state of disturbance which was communicated by the luminous body; it is a motion of *oscillation*, and, like the propagation of waves in water, takes place by a series of vibrations.

The luminiferous ether penetrates all bodies, but on account of its extreme tenuity it is uninfluenced by gravitation; it occupies space, and although it presents no appreciable resistance to the motion of the denser

bodies, it is possible that it hinders the motion of the smaller comets. It has been found, for example, that Encke's comet, whose period of revolution is about $3\frac{1}{3}$ years, has its period diminished by about 0·11 of a day at each successive rotation, and this diminution is ascribed by some to the resistance of the ether.

Graetz has calculated that the density of ether is 9×10^{-16} that of water. From a formula of Sir W. Thomson it is calculated to be greater than 10^{-18}; it may accordingly be admitted to be 10^{-17}. While the air over a square metre weighs 10,000 kilogrammes, the ether in it, taking the height of the atmosphere at 30 miles, would only weigh 0·0022 milligramme. Thomson concludes that if the density of the air followed Boyle's law, and the temperature were constant, at a height equal to that of the earth's radius it would only be 10^{-350} that of water. The ether is therefore far more dense than air so rarefied.

The fundamental principles of the undulatory theory were enunciated by Huyghens, and subsequently by Euler. The emission theory, principally owing to Newton's powerful support, was for long the prevalent scientific creed. The undulatory theory was adopted and advocated by Young, who showed how a large number of optical phenomena, particularly those of diffraction, were to be explained by that theory. Subsequently, too, though independently of Young, Fresnel showed that the phenomena of diffraction, and also those of polarisation, are explicable on the same theory, which, since his time, has been generally accepted.

The undulatory theory not only explains the phenomena of light, but it reveals an intimate connection between these phenomena and those of heat (429); it shows, also, how completely analogous the phenomena of light are to those of sound, regard being had to the differences of the media in which these two classes of phenomena take place.

500. **Luminous, transparent, translucent, and opaque bodies.**—*Luminous* bodies are those which emit light, such as the sun, and ignited bodies. *Transparent* or *diaphanous* bodies are those which readily transmit light, and through which objects can be distinguished: water, gases, polished glass are of this kind. *Translucent* bodies transmit light, but objects cannot be distinguished through them: ground glass, oiled paper, &c., belong to this class. *Opaque* bodies do not transmit light; for example, wood, metals, &c. No bodies are quite opaque; they are all more or less translucent when cut in sufficiently thin leaves.

Foucault showed that when the object-glass of a telescope is thinly silvered, the layer is so transparent that the sun can be viewed through it without danger to the eyes, since the metallic surface reflects the greater part of the heat and light.

501. **Luminous ray and pencil.**—A *luminous ray* is the direction of the line in which light is propagated; a *luminous pencil* is a collection of rays from the same source; it is said to be *parallel* when it is composed of parallel rays, *divergent* when the rays separate from each other, and *convergent* when they tend towards the same point. Every luminous body emits divergent rectilinear rays from all its points, and in all directions.

502. **Propagation of light in a homogeneous medium.**—A *medium* is any space or substance which light can traverse, such as a vacuum, air, water,

glass, &c. A medium is said to be *homogeneous* when its chemical composition and density are the same in all parts.

In every homogeneous medium light is propagated in a right line. For, if an opaque body is placed in the right line which joins the eye and the luminous body, the light is intercepted. The light which passes into a dark room by a small aperture is visible from the light falling on the particles of dust suspended in the atmosphere.

Light changes its direction on meeting an object which it cannot penetrate, or when it passes from one medium to another. These phenomena will be described under the heads *reflection* and *refraction.*

503. **Shadow, penumbra.**—When light falls upon an opaque body it cannot penetrate into the space immediately behind it, and this space is called the *shadow.*

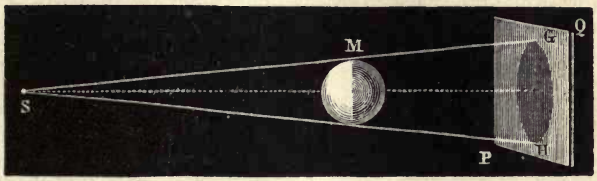


Fig. 444.

In determining the extent and the shape of a shadow projected by a body, two cases are to be distinguished; that in which the source of light is a single point, and that in which it is a body of any given extent.

In the first case, let S (fig. 444) be the luminous point, and M a spherical body, which causes the shadow. If an infinitely long straight line, SG, move

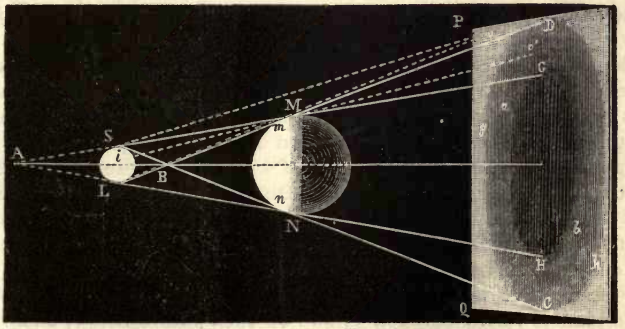


Fig. 445.

round the sphere M tangentially, always passing through the point S, this line will produce a conical surface, which, beyond the sphere, separates that portion of space which is in shadow from that which is illuminated. In

the present case, on placing a screen, PQ, behind the opaque body the limit of the shadow HG will be sharply defined. This is not, however, usually the case, for luminous bodies have always a certain magnitude, and are not merely luminous points.

Suppose that the luminous and illuminated bodies are two spheres, SL and MN (fig. 445). If an infinite straight line, AG, moves tangentially to both spheres, always cutting the line of the centre in the point A, it will produce a conical surface with this point for a summit, and which traces behind the sphere MN a perfectly dark space MGHN. If a second right line, LD, which cuts the line of centre in B, moves tangentially to the two spheres, so as to produce a new conical surface, BDC, it will be seen that all the space outside this surface is illuminated, but that the part between the two conical surfaces is neither quite dark nor quite light. So that if a screen, PQ, is placed behind the opaque body, the portion *cGdH* of the screen is quite in the shadow, while the space *ab* receives light from certain parts of the luminous body, and not from others. It is brighter than the true shadow, and not so bright as the rest of the screen, and it is accordingly called the *penumbra*.

Shadows such as these are *geometrical shadows; physical shadows*, or those which are really seen, are by no means so sharply defined. A certain quantity of light passes into the shadow, even when the source of light is a mere point, and conversely the shadow influences the illuminated part. This phenomenon, which will be afterwards described, is known by the name of *diffraction* (646).

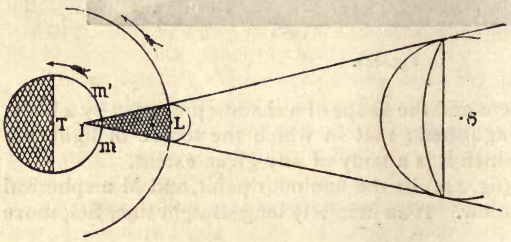


Fig. 446.

The explanation of the phenomena of *eclipses* follows directly from the theory of shadows.

When the opaque disc of the moon comes, according to the conditions, between the sun and the earth, the shadow cast by the moon causes a more or less complete solar eclipse on those parts of the earth which it meets.

Let S be the sun, T the earth, and L the moon placed in a position favourable for an eclipse (fig. 446). If we can suppose the three bodies represented with their *relative* magnitudes and distances we need only repeat the graphical construction of fig. 446 to determine the dimensions of the cone of the shadow, and of the penumbra of the moon. The length LI of the cone of the shadow varies between 57 and 59 terrestrial radii, according to the relative positions of the earth and its satellite; the distance of the two planets varies between 55 and 62 such radii; hence, under favourable conditions the cone of the shadow may reach the earth, and in those points of the earth thus touched, *m*, there is a total eclipse of the sun. As this area has relatively a small extent, an eclipse which is visible by the inhabitants of this area is not so by those in the neighbourhood. After the lapse of a time which never exceeds 3 min. 13 sec. the cone will have left

the place *m* and will pass to *m'*, which is not necessarily on the same parallel of latitude. It will thus sweep over the surface of the earth, in virtue of the special motion of the two heavenly bodies, along a line which astronomers can determine beforehand. On all points along this line (fig. 447) there will successively be a total eclipse ; for adjacent ones, which are within the cone of the penumbra, the eclipse will be *partial*.

If the cone of the shadow does not reach the earth, there will nowhere be a total eclipse ; but on a point *m'* (fig. 448) there will be no light from the

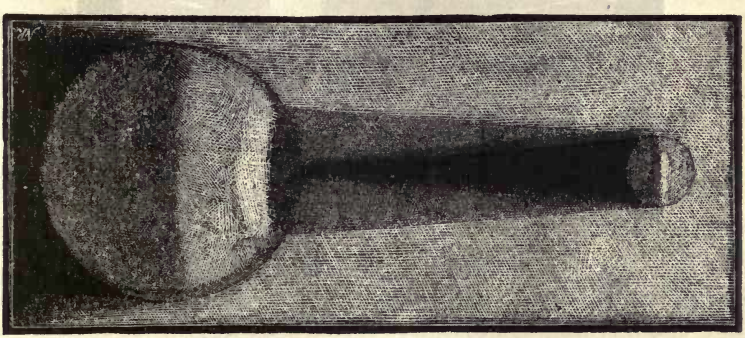


Fig. 447.

central part of the sun ; this will then appear like a black circle surrounded by a bright ring (fig. 449), and forms what is called an *annular eclipse*.

Total or partial eclipses of the moon are produced by the total or partial immersion of the moon in the cone of the shadow cast by the earth ; for an observer on the moon they would constitute total or partial eclipses of the sun ; *total* at those parts of the moon in the shadow, *partial* at those in the penumbra.

The *transits* of Venus or of Mercury over the sun are phenomena of the same kind as eclipses, being produced by the projection on the earth of the

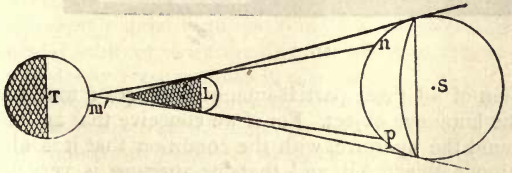


Fig. 448.

Fig. 449.

penumbral cones of shadow of those planets. The eclipses of the satellites of certain planets such as Jupiter are identical with the eclipses of the moon.

504. **Images produced by small apertures.**—When luminous rays, which pass into a dark chamber through a *small* aperture, are received upon a screen, they form images of external objects. These images are inverted, their shape is always that of the external objects, and is independent of the shape of the aperture.

The inversion of the images arises from the fact that the luminous rays proceeding from external objects, and penetrating into the chamber, cross one another in passing the aperture, as shown in fig. 450. Continuing in a straight line, the rays from the higher parts meet the screen at the lower parts; and conversely, those which come from the lower parts meet the higher parts of the screen. Hence the inversion of the image. In the

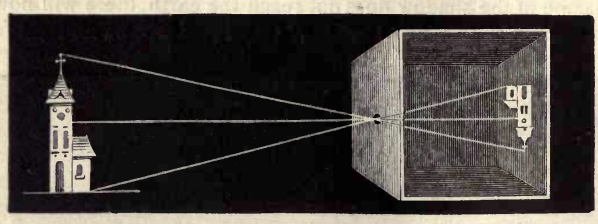


Fig. 450.

article Camera Obscura (602) it will be seen that the brightness and precision of these images are increased by means of lenses.

In order to show that the shape of the image is independent of that of the aperture, when the latter is sufficiently small and the screen at an adequate distance, imagine a triangular aperture, O (fig. 451), made in the door of a dark chamber, and let *ab* be a screen on which is received the image of a flame, AB. A divergent pencil from each point of the flame passes through the aperture, and forms on the screen a triangular image resembling the

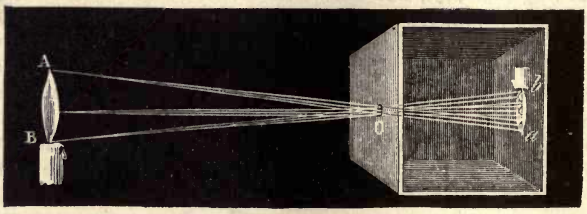


Fig. 451.

aperture. But the union of all these partial images produces a total image of the same form as the luminous object. For if we conceive that an infinite straight line moves round the aperture, with the condition that it is always tangential to the luminous object AB, and that the aperture is very small, the straight line describes two cones, the apex of which is the aperture, while one of the bases is the luminous object and the other the luminous object on the screen—that is, the image. Hence, if the screen is perpendicular to the right line joining the centre of the aperture and the centre of the luminous body, the image is similar to the body; but if the screen is oblique, the image is elongated in the direction of its obliquity. This is what is seen in the shadow produced by foliage; the luminous rays passing through the leaves produce images of the sun, which are either round or

elliptical, according as the ground is perpendicular or oblique to the solar rays; and this is the case whatever be the shape of the aperture through which the light passes.

505. **Velocity of light.**—Light moves with such a velocity that at the surface of the earth there is, to ordinary observation, no appreciable interval between the occurrence of any luminous phenomenon and its perception by the eye. And, accordingly, this velocity was first determined by means of astronomical observations. Römer, a Danish astronomer, in 1675, first deduced the velocity of light from an observation of the eclipses of Jupiter's first satellite.

Jupiter is a planet, round which four satellites revolve, as the moon does round the earth. This first satellite, E (fig. 452), suffers *occultation*—that is, passes into Jupiter's shadow—at equal intervals of time, which are 42h. 28m. 36s. While the earth moves in that part of its orbit, *ab*, nearest Jupiter its distance from that planet does not materially alter, and the intervals between two successive occultations of the satellite are approximately the same; but, in proportion as the earth moves away in its revolution round the sun, S, the interval between two occultations increases, and when, at the end of six months, the earth has passed from the position T to the position T′, a *total* retardation of 16m. 36s. is observed between the time at which the phenomenon is seen and that at which it is calculated to take place. But when the earth was in the position T, the sun's light reflected from the satellite E had to traverse the distance ET, while in the second position the light had to traverse the distance ET′. This distance exceeds the first by the quantity TT′, for, from the great distance of the satellite E, the rays ET and ET′ may be considered parallel. Consequently, light requires 16m. 36s. to travel the diameter TT′ of the terrestrial orbit, or twice the distance of the earth from the sun, which gives for its velocity 190,000 miles in a second.

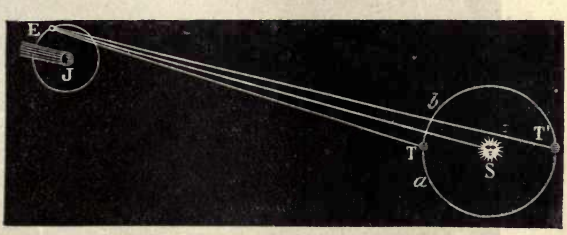


Fig. 452.

The stars nearest the earth are separated from it by at least 206,265 times the distance of the sun. Consequently, the light which they send requires more than three years to reach us. Those stars, which are only visible by means of the telescope, are possibly at such a distance that thousands of years would be required for their light to reach our planetary system. They might have been extinguished for ages without our knowing it.

506. **Foucault's apparatus for determining the velocity of light.**—Notwithstanding the prodigious velocity of light, Foucault succeeded in determining it experimentally by the aid of an ingenious apparatus, based on the use of the rotating mirror, which was adopted by Wheatstone in measuring the velocity of electricity.

In the description of this apparatus, a knowledge of the principal pro-

perties of mirrors and of lenses is presupposed. Fig. 453 represents the chief parts of Foucault's arrangement. The window shutter, K, of a dark chamber is perforated by a rectangular slit, behind which the platinum wire *o* is stretched vertically. A beam of sunlight reflected from the outside upon a mirror enters the dark room by the slit, meets the platinum wire, and then traverses an achromatic lens, L, with a long focus placed at a distance from the platinum wire less than double the principal focal distance. The image of the platinum wire, more or less magnified, would thus be formed on the axis of the lens; but the pencil of light, having traversed the lens, impinges on a plane mirror, *m*, rotating with great velocity; it is reflected from this, and forms in space an image of the platinum wire, which is displaced with an angular velocity double that of the mirror (520). This image is reflected by a concave mirror, M, whose centre of curvature coincides with the axis of rotation of the mirror *m*, and with its centre of figure. The pencil reflected from the mirror M returns upon itself, is again reflected from the mirror *m*, traverses the lens a second time, and

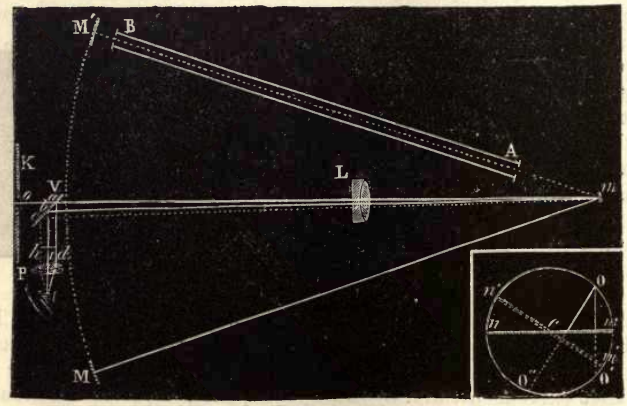


Fig. 453.

Fig. 454.

forms an image of the platinum wire, which appears on the wire itself so long as the mirror *m* turns slowly.

In order to see this image without hiding the pencil of light which enters by the aperture in K, a plane parallel mirror of unsilvered glass, V, is placed between the lens and the wire, and is inclined so that the reflected rays fall upon a powerful eyepiece, P.

The apparatus being arranged, if the mirror *m* is at rest, the pencil after meeting M is reflected to *m*, and from thence returns along its former path, till it meets the glass plate V in *a*, and being partially reflected, forms at *d*—the distance *ad* being equal to *ao*—an image of the wire, which the eye is enabled to observe by means of the eyepiece, P. If the mirror, instead of being fixed, is moving slowly round—its axis being at right angles to the plane of the paper—there will be no sensible change in the position of the mirror *m* during the brief interval elapsing while light travels from *m* to M

and back again, but the image will alternately disappear and reappear. If now the velocity of M is increased to upwards of 30 turns per second, the interval between the disappearance and reappearance is so short that the impression on the eye is persistent, and the image appears perfectly steady.

Lastly, if the mirror turns with sufficient velocity, there is no appreciable change in its position during the time which the light takes in making the double journey from m to M, and from M to m; the return ray, after its reflection from the mirror m, takes the direction mb, and forms its image at i; that is, the image has undergone a total deviation di. Speaking precisely, there is a deviation as soon as the mirror turns, even slowly; but it is only appreciable when it has acquired a certain magnitude, which is the case when the velocity of rotation is sufficiently rapid, or the distance Mm sufficiently great, for the deviation necessarily increases with the time which the light takes in returning on its own path.

In Foucault's experiment the distance Mm was only 13½ feet; when the mirror rotated with a velocity of 600 to 800 turns in a second, deviations of $\frac{2}{10}$ to $\frac{3}{10}$ of a millimetre were obtained.

Taking M$m = l$, L$m = l'$, oL $= r$, and representing by n the number of turns in a second, by δ the absolute deviation di, and by V the velocity of light, Foucault arrived at the formula

$$V = \frac{8\pi l^2 n r}{\delta(l + l')},$$

from which the velocity of light is calculated at 185,157 miles in a second; this number, which is less than that ordinarily assumed, agrees remarkably well with the value deduced from the new determinations of the value of the solar parallax.

The mechanism by which the mirror was turned consisted of a small steam turbine, bearing a sort of resemblance to the sirene, and, like that instrument, giving a higher sound as the rotation is more rapid: in fact, it is by the pitch of the note that the velocity of the rotation is determined.

In this apparatus liquids can be experimented upon. For that purpose a tube, AB, 10 feet long, and filled with distilled water, is placed between the turning mirror m, and a concave mirror M′, identical with the mirror M. The luminous rays reflected by the rotating mirror, in the direction mM′, traverse the column of water AB twice before returning to V. But the return ray then becomes reflected at c, and forms its image at h: the deviation is consequently greater for rays which have traversed water than for those which have passed through air alone; hence the velocity of light is less in water than in air.

This is the most important part of these experiments. For it had been shown theoretically that on the undulatory theory the velocity of light must be less in the more highly refracting medium (638), while the opposite is a necessary consequence of the emission theory. Hence Foucault's result may be regarded as a crucial test of the validity of the undulatory theory.

507. **Experiments of Fizeau.**—In 1849 Fizeau measured directly the velocity of light, by ascertaining the time it took to travel from Suresnes to Montmartre and back again. The apparatus employed was a toothed wheel,

capable of being turned more or less quickly, and with a velocity that could be exactly ascertained. The teeth were made of precisely the same width as the intervals between them. The apparatus being placed at Suresnes, a pencil of parallel rays was transmitted through an interval between two teeth to a mirror placed at Montmartre. The pencil, directed by a properly arranged system of tubes and lenses, returned to the wheel. As long as the apparatus was at rest the pencil returned exactly through the same interval as that through which it first set out. But when the wheel was turned sufficiently fast, a tooth was made to take the place of an interval, and the ray was intercepted. By causing the wheel to turn more rapidly, it re-appeared when the interval between the next two teeth had taken the place of the former tooth at the instant of the return of the pencil.

The distance between the two stations was 28,334 feet. By means of the data furnished by this distance, by the dimensions of the wheel, its velocity of rotation, &c., Fizeau found the velocity of light to be 196,000 miles per second—a result agreeing with that given by astronomical observation as closely as can be expected in a determination of this kind.

Cornu recently investigated the velocity of light by Fizeau's method, but with improvements so that the probable error did not exceed $\frac{1}{400}$ of the total amount; the two stations, which were 6·4 miles apart, were a pavilion of the École Polytechnique and a room in the barracks of Mont Valérien. By means of electromagnetic arrangements the rotation of the toothed disc, and the times of obscuration and illumination, were registered on a blackened cylinder, on the principle of the method described in (245). Cornu thus obtained the number 185,420 miles—a result closely agreeing with that of Foucault, and which is supported by calculations based on the results of astronomical observations of the transit of Venus in 1874.

Newcomb improved Foucault's method by using a slightly concave mirror instead of a plane one, by which the image of the slit was brighter; it was observed by a telescope through a distance of 4,000 metres. The rotations also were reversed, by which the angle between the two positions of the telescope was observed with greater accuracy; he thus obtained the number 186,364 miles, while Michelson repeated a former determination and found 186,354, a difference of only about 10 miles.

508. **Laws of the intensity of light.**—The *intensity* of illumination is the quantity of light received on the unit of surface; it is subject to the following laws:—

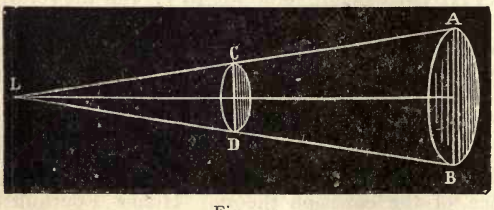


Fig. 455.

I. *The intensity of illumination on a given surface is inversely as the square of its distance from the source of light.*

II. *The intensity of illumination which is received obliquely is proportional to the cosine of the angle which the luminous rays make with the normal to the illuminated surface.*

In order to demonstrate the first law, let there be two circular screens,

CD and AB (fig. 455), one placed at a certain distance from a source of light, L, and the other at double this distance, and let s and S be the areas of the two screens. If a be the total quantity of light which is emitted by the source in the direction of the cone ALB, the intensity of the light on the screen CD—that is, the quantity which falls on the unit of surface—is $\frac{a}{s}$, and the intensity on the screen AB is $\frac{a}{S}$.

Now as the triangles ALB and CLD are similar, the diameter of AB is double that of CD ; and as the surfaces of circles are as the squares of their diameters, the surface S is four times s, consequently the intensity $\frac{a}{S}$ is one-fourth that of $\frac{a}{s}$.

The same law may also be demonstrated by an experiment with the apparatus represented in fig. 457. It is made by comparing the shadows of an opaque rod cast upon a glass plate, in one case by the light of a single candle, and in another by that of a lamp equalling four candles, placed at double the distance of the first. In both cases the shadows have the same intensity.

Fig. 455 shows that it is owing to the divergence of the luminous rays emitted from the same source that the intensity of light is inversely as the square of the distance. The illumination of a surface placed in a beam of parallel luminous rays is the same at all distances in a vacuum ; in air and in other transparent media the intensity of light decreases, in consequence of absorption, more rapidly than the square of the distance.

The second law of intensity corresponds to the law which we have found to prevail for heat : it may be theoretically deduced as follows :—Let DA, EB (456) be a pencil of parallel rays falling obliquely on a surface, AB, and let *om* be the normal to this surface. If S is the section of the pencil, a the total quantity of light which falls on the surface AB, and I that which falls on the unit of surface—that is, the intensity of illumination—we have $I = \frac{a}{AB}$. But

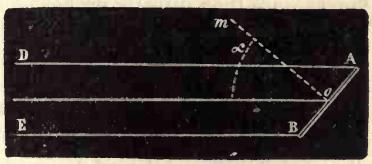


Fig. 456.

as S is only the projection of AB on a plane perpendicular to the pencil, we know from trigonometry that $S = AB \cos a$, from which $AB = \frac{S}{\cos a}$. This value substituted in the above equation gives $I = \frac{a}{S} \cos a$; a formula which demonstrates the law of the cosine, for as a and S are constant quantities, I is proportional to $\cos a$.

The law of the cosine applies also to rays emitted obliquely by a luminous surface ; that is, the rays are less intense in proportion as they are more inclined to the surface which emits them. In this respect they correspond to the third law of the intensity of radiant heat.

509. **Photometers.**—A *photometer* is an apparatus for measuring the relative intensities of different sources of light.

Rumford's photometer.—This consists of a ground-glass screen, in front of which is fixed an opaque rod (fig. 457) ; the lights to be compared—for instance, a lamp and a candle—are placed at a certain distance in such a manner that each projects on the screen a shadow of the rod. The shadows thus projected are at first of unequal intensity, but by altering the position of the lamp, it may be so placed that the intensity of the two shadows is the same. Then, since the shadow thrown by the lamp is illuminated by the candle, and that thrown by the candle is illuminated by the lamp, the illumination of the screen due to each light is the same. The intensities of the

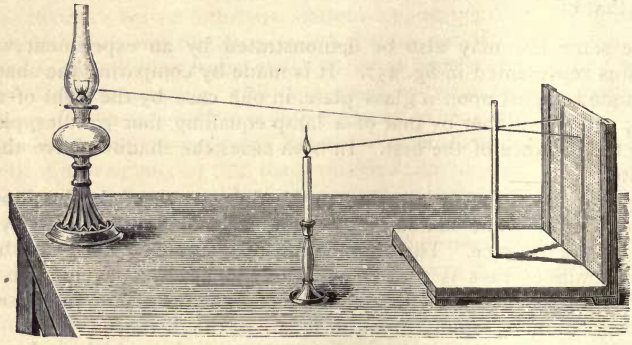


Fig. 457.

two lights—that is, the illuminations which they would give at equal distances—are then directly proportional to the squares of their distances from the shadows ; that is to say, if the lamp is three times the distance of the candle, its illuminating power is nine times as great.

For if i and i' are the intensities of the lamp and the candle at the unit of distance, and d and d' their distances from the shadows, it follows, from the first law of the intensity of light, that the intensity of the lamp at the distance d is $\frac{i}{d^2}$ and that of the candle $\frac{i'}{d'^2}$ at the distance d'. On the screen these two intensities are equal ; hence $\frac{i}{d^2} = \frac{i'}{d'^2}$ or $\frac{i}{i'} = \frac{d^2}{d'^2}$, which was to be proved.

Bunsen's photometer.—When a grease-spot is made on a piece of bibulous paper, the part appears translucent. If the paper be illuminated by a light placed in front, the spot appears darker than the surrounding space ; if, on the contrary, it be illuminated from behind, the spot appears light on a dark ground. If the greased part and the rest appear unchanged, the intensity of illumination on both sides is the same. Bunsen's photometer depends on an application of this principle. Its essential features are represented in fig. 458. A circular spot is made on a paper screen by means of a solution of spermaceti in naphtha : on one side of this is placed a light of a certain intensity, which serves as a standard ; in London it is a sperm

candle of six to the pound, and burning 120 grains in an hour. The light to be tested, a petroleum lamp or a gas burner consuming a certain volume of gas in a given time, is then moved in a right line to such a distance on the other side of the screen that there is no difference in brightness between the greased part and the rest of the screen. By measuring the distances of the lights from the screen by means of the scale, their relative illuminating powers are respectively as the squares of their distances from the screen.

The difficulty of getting more carefully constructed candles to give a light sufficiently uniform for standard purposes has led Harcourt to adopt as unit the light formed by burning a mixture of 7 volumes pentane gas and

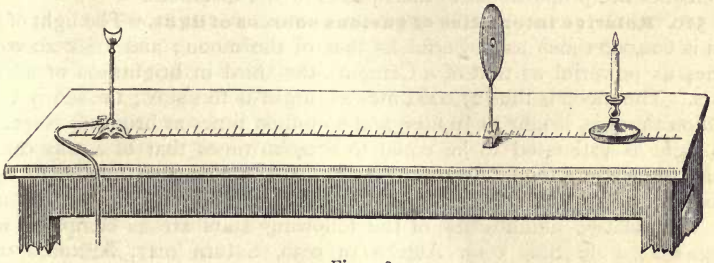


Fig. 458.

20 volumes of air, at the rate of half a cubic foot in an hour, in a specially constructed burner so as to produce a flame of a definite height. This has been found to answer well in practice. By this kind of determination the degree of accuracy which can be attained is not so great as in many physical determinations, more especially when the lights to be compared are of different colours ; one, for instance, being yellow, and the other of a bluish tint. It gives, however, results which are sufficiently accurate for practical purposes, and is almost universally employed for determining the illuminating power of coal gas and of other artificial lights.

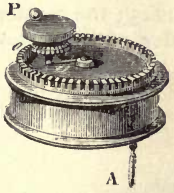


Fig. 459.

Fig. 460.

In Germany the *Hefner Alteneck* lamp is much used as standard ; the combustible is amylic acetate or artificial pear oil, and a flame of constant height is produced in a burner of special construction.

The absolute unit of light adopted by the International Congress of Electricians is that emitted by a square centimetre of melted platinum at the moment of its solidification. It is equal to about fifteen standard candles.

Wheatstone's photometer.—The principal part of this instrument is a steel bead P (fig. 459), fixed on the edge of a disc, which rotates on a pinion, *o*, working in a larger toothed wheel. The wheel fits in a cylindrical brass box which is held in one hand, while the other works a handle, A, which turns a central axis, the motion of which is transmitted by a spoke, *a*, to the pinion *o*. In this way the latter turns on itself, and at the same time revolves round the circumference of the box ; the bead shares the

double motion, and consequently describes a curve in the form of a rose (fig. 460).

Now, let M and N be the two lights whose intensities are to be compared; the photometer is placed between them and rapidly rotated. The brilliant points produced by the reflection of the light on the two opposite sides of the bead give rise to two luminous bands, arranged as represented in fig. 460. If one of them is more brilliant than the other—that which proceeds from the light M, for instance—the instrument is brought nearer the other light until the two bands exhibit the same brightness. The distance of the photometer from each of the two lights being then measured, their intensities are proportional to the squares of the distances.

510. **Relative intensities of various sources of light.**—The light of the sun is 600,000 times as powerful as that of the moon; and 16,000,000,000 times as powerful as that of *a* Centauri, the third in brightness of all the stars. The moon is thus 27,000 times as bright as this star; the sun is 5,500 million times as bright as Jupiter, and 80 billion times as bright as Neptune. Its light is estimated to be equal to 670,000 times that of a wax candle at a distance of 1 foot. According to Fizeau and Foucault the electric light produced by 50 Bunsen's cells is about $\frac{1}{4}$ as strong as sunlight.

The relative luminosities of the following stars are as compared with Vega = 1; Pole Star 0·13, Aldebaran 0·30, Saturn 0·47, Arcturus 0·79, Mars 2·93, Sirius 4·291, Jupiter 8·24, Venus 38·9.

A difference in the strength of light or shadow is perceived when the duller light is $\frac{98}{99}$ of the brightness of the other, and both are near together, especially when the shadow is moved about.

Our requirements as regards illumination are constantly on the increase; thus for public receptions of a State character in recent times in Paris, a number of lamps was used corresponding to over 13 candles per square yard, which is 6 times as much as was used on the occasion of the marriage of the Dauphin in 1745; the former, however, is still far removed from perfect illumination, that of daylight, which is estimated at about 180 candles per square yard.

CHAPTER II

REFLECTION OF LIGHT. MIRRORS

511. **Laws of the reflection of light.**—When a ray of light meets a polished surface, it is reflected according to the two following laws, which, as we have seen, also hold for heat.

I. *The angle of reflection is equal to the angle of incidence.*

II. *The incident and the reflected ray are both in the same plane, which is perpendicular to the reflecting surface.*

The words are here used in the same sense as in article 417, and need no further explanation.

First proof.—The two laws may be demonstrated by the apparatus represented in fig. 461. It consists of a graduated circle in a vertical plane. Two brass slides move round the circumference; on one of them there is a piece of ground glass, P, and on the other an opaque screen, N, in the centre of which is a small aperture. Fixed to the latter slide there is also a mirror, M, which can be more or less inclined, but always remains in a plane perpendicular to the plane of the graduated circle. Lastly, there is a small polished metallic mirror, *m*, placed horizontally in the centre of the circle.

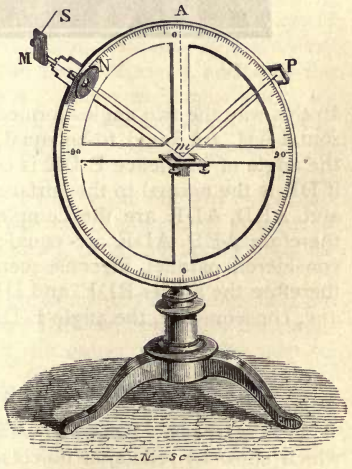


Fig. 461.

In making the experiment, a pencil of solar or any suitable artificial light, S, is caused to fall on the mirror M, which is so inclined that the reflected light passes through the aperture in N, and falls on the centre of the mirror, *m*. The luminous pencil then experiences a second reflection in a direction *m*P, which is ascertained by moving P until an image of the aperture is found in its centre. The number of degrees comprised in the arc AN is then read off, and likewise that in AP; these being equal, it follows that the angle of reflection A*m*P is equal to the angle of incidence A*m*M.

The second law follows from the arrangement of the apparatus, the plane of the rays M*m* and *m*P being parallel to the plane of the graduated circle, and, consequently, perpendicular to the mirror *m*.

Second proof.—The law of the reflection of light may also be demonstrated by the following experiment, which is susceptible of greater accuracy than that just described :—In the centre of a graduated circle, M (fig. 462), placed in a vertical position, there is a small telescope movable in a plane parallel to the limb ; at a suitable distance there is a vessel D full of mercury, which forms a perfectly horizontal plane mirror. Some particular star of the first or second magnitude is viewed through the telescope in the direction AE, and the telescope is then inclined so as to receive the ray AD coming from the star after being reflected from the brilliant surface of the mercury.

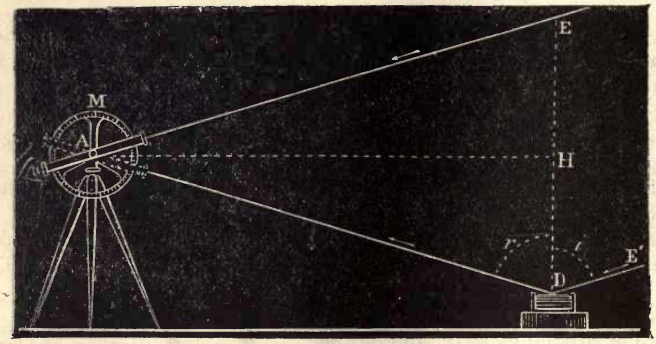


Fig. 462.

In this way the two angles formed by the rays EA and DA, with the horizontal AH, are found to be equal, from which it may easily be shown that the angle of incidence E′DE is equal to the angle of reflection EDA. For if DE is the normal to the surface of the mercury, it is perpendicular to AH, and AED, ADE are the complements of the equal angles EAH, DAH ; therefore AED, ADE are equal ; but the two rays AE and DE′ may be considered parallel, in consequence of the great distance of the star, and therefore the angles EDE′ and DEA are equal, for they are alternate angles, and, consequently, the angle E′DE is equal to the angle EDA.

REFLECTION OF LIGHT FROM PLANE SURFACES

512. **Mirrors. Images.**—*Mirrors* are bodies with polished surfaces which show by reflection objects presented to them. The place at which objects appear is their *image.* According to their shape, mirrors are divided into *plane*, *concave*, *convex*, *spherical*, *parabolic*, *conical*, &c.

513. **Formation of images by plane mirrors.**—The determination of the position and size of images resolves itself into investigating the images of a series of points. And first, the case of a single point, A, placed in front of a plane mirror, MN (fig. 463), will be considered. Any ray, AB, incident from this point on the mirror is reflected in the direction BO, making the angle of reflection DBO equal to the angle of incidence DBA.

If, now, a perpendicular, AN, be let fall from the point A on the mirror,

and if the ray OB be prolonged below the mirror until it meets this perpendicular in the point *a*, two triangles are formed, ABN and BN*a*, which are equal, for they have the side BN common to both, and the angles ANB, ABN, equal to the angles *a*NB, *a*BN ; for the angles ANB and *a*NB are right angles, and the angles ABN and *a*BN are each equal to the angle OBM. From the equality of these triangles, it follows that *a*N is equal to AN ; that is, that any ray, AB, takes such a direction after being reflected, that its prolongation below the mirror cuts the perpendicular A*a* in the point *a*, which is at the same distance from the mirror as the point A. This applies also to the case of any other ray from the point A ; AC, for example.

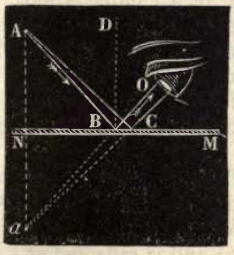


Fig. 463.

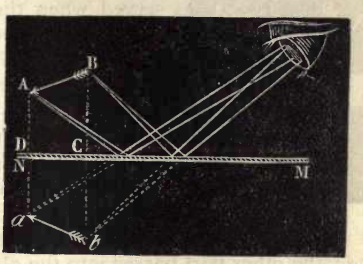


Fig. 464.

From this the important consequence follows, that all rays from the point A, reflected from the mirror, *follow, after reflection, the same direction as if they had all proceeded from the point a.* The eye is deceived, and sees the point A at *a*, as if it were really situated at *a*. Hence in plane mirrors *the image of any point is formed behind the mirror at a distance equal to that of the given point, and on the perpendicular let fall from this point on the mirror.*

It is manifest that the image of any object will be obtained by constructing, according to this rule, the image of each of its points, or, at least, of those which are sufficient to determine its form. Fig. 464 shows how the image *ab* of any object, AB, is formed.

It follows from this construction that in plane mirrors *the image is of the same size as the object* ; for if the trapezium ABCD be applied to the trapezium DC*ab*, they are seen to coincide, and the object AB agrees with its image.

A further consequence from the above construction is, that in plane mirrors the image is symmetrical in reference to the object, and not inverted.

514. **Virtual and real images.**—There are two cases relative to the direction of rays reflected by mirrors according as the rays after reflection are convergent or divergent. In the first case the reflected rays do not meet, but if they are supposed to be produced on the other side of the mirror, their prolongations coincide in the same point, as shown in figs. 463 and 464. The eye is then affected just as if the rays proceeded from this point, and it sees an image. But the image has no real existence, the luminous rays do not come from the other side of the mirror : this appearance is called the *virtual image.* The images of real objects produced by plane mirrors are of this kind.

In the second case, where the reflected rays converge, of which we shall soon have an example in concave mirrors, the rays coincide at a point in front of the mirror, and on the same side as the object. They form there an image called the *real image*, for it can be received on a screen. The distinction may be expressed by saying that *real images are those formed by the reflected rays themselves, and virtual images those formed by their prolongations.*

515. **Multiple images formed by glass mirrors.**—Metal mirrors which have but one reflecting surface give only one image; glass mirrors give rise to several images, which are readily observed when the image of a candle is looked at obliquely in a looking-glass. A very feeble image is first seen, and then a very distinct one; behind this there are several others, whose intensities gradually decrease until they disappear.

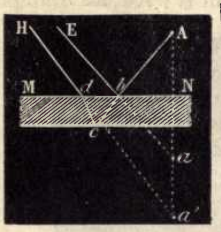


Fig. 465.

This phenomenon arises from the looking-glass having two reflecting surfaces. When the rays from the point A meet the surface, fig. 465, a part is reflected and forms an image, *a*, of the point A, on the prolongation of the ray *b*E, reflected by this surface; the other part passes into the glass (536), and is reflected at *c* from the layer of metal which covers the hinder surface of the glass, and reaching the eye in the direction *d*H gives the image *a'*. This image is distant from the first by double the thickness of the glass. It is more distinct, because metal reflects better than glass.

In regard to other images it will be remarked that whenever light is transmitted from one medium to another—for instance, from glass to air—(536), only some of the rays get through; the remainder are reflected at the surface which bounds the two media. Consequently when the pencil *cd*, reflected from *c*, attempts to leave the glass at *d*, most of the rays composing it pass into the air, but some are reflected at *d*, and continue within the glass. These are again reflected by the metallic surface, and form a third image of A; after this reflection they come to MN, when many emerge and render the third image visible; but some are again reflected within the glass, and in a similar manner give rise to a fourth, fifth, &c., image, thereby completing the series above described. It is manifest from the above explanation that each image must be much feebler than the one preceding it, and consequently only a small number are visible—ordinarily not more than eight or ten in all.

This multiplicity of images is objectionable in observations, and, accordingly, metal mirrors are to be preferred in optical instruments.

516. **Multiple images from two plane mirrors.**—When an object is placed between two plane mirrors, which form an angle with each other, either right or acute, images of the object are formed, the number of which increases with the inclination of the mirrors. If they are at right angles to each other, three images are seen, arranged as represented in fig. 466. The rays OC and OD from the point O, after a single reflection, give the one an image O′, and the other an image O″, while the ray OA, which has undergone two reflections at A and B, gives the third image O‴. When the

angle of the mirrors is 60°, five images are produced, and seven if it is 45°. The number of images continues to increase in proportion as the angle diminishes, and when it is zero—that is, when the mirrors are parallel—the number of images is theoretically infinite. In general, if two mirrors are inclined to each other, the number of images they produce is equal to the number of times the angle between them is contained in 360.

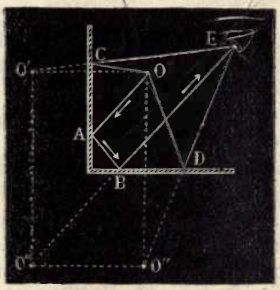


Fig. 466.

The *kaleidoscope*, invented by Sir D. Brewster, depends on this property of inclined mirrors. It consists of a tube, in which are three mirrors inclined at 60°; one end of the tube is closed by a piece of ground glass, and the other by a cap provided with an aperture. Small irregular pieces of coloured glass are placed at one end between the ground glass and another glass disc, and on looking through the aperture, the other end being held towards the light, the objects and their images are seen arranged in beautiful symmetrical forms; by turning the tube, an almost endless variety of these shapes is obtained.

517. **Multiple images in two plane parallel mirrors.**—In this case the number of images of an object placed between them is theoretically infinite. Physically the number is limited, for as the incident light is never totally reflected, some of it being always absorbed, the images gradually become fainter, and are ultimately quite extinguished.

Fig. 467 shows how the pencil L*a* reflected once from M gives at I the image of the object L at a distance $mI = mL$; then the pencil L*b* reflected once from the mirror M, and once from N, furnishes the image I′ at a distance $nI' = nI$; in like manner the pencil L*c*, after two reflections on M, and one on N, forms the image I″ at a distance $mI'' = mI'$, and so on for an infinite series. The images *i*, *i*′, *i*″ are formed in the same manner by rays of light which, emitted by the object L, fall first on the mirror N.

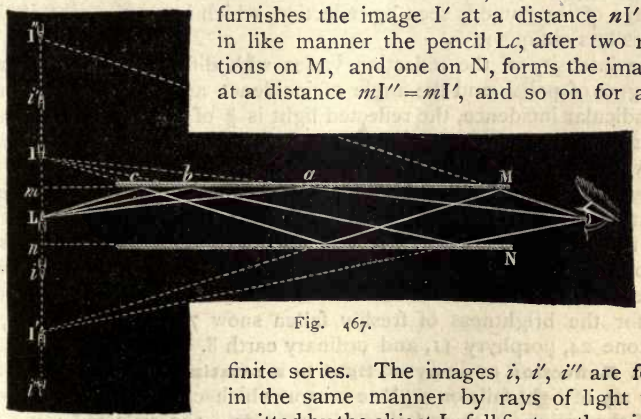


Fig. 467.

518. **Irregular reflection. Diffused light.**—The reflection from the surfaces of polished bodies, the laws of which have been just stated, is called the *regular* or *specular reflection*; but the quantity thus reflected is less

than that of the incident light. The light incident on an opaque body separates, in fact, into three parts: one is reflected *regularly*; another *irregularly*—that is, in all directions; while a third is extinguished, or *absorbed* by the reflecting body. If light falls on a transparent body, a considerable portion is transmitted with regularity.

The irregularly reflected light is called *scattered light*: it is that which makes bodies visible (502). The light which is reflected regularly does not give us the image of the reflecting surface, but that of the body from which the light proceeds. If, for example, a beam of sunlight be incident on a well-polished mirror in a dark room, the more perfectly the light is reflected the less visible is the mirror in the different parts of the room. The eye does not perceive the image of the mirror, but that of the sun. If the reflecting power of the mirror be diminished by sprinkling on it a light powder, the sun's image becomes feebler, and the mirror is visible from all parts of the room. Perfectly smooth, polished reflecting surfaces, if such there were, would be invisible. The beam of light itself is only seen in the room owing to irregular reflections from the particles of dust, and the like, which are floating in the air. Tyndall has shown that when this floating matter in the air in an inclosed space is completely removed, the beam of sunlight or the electric light is quite invisible. The atmosphere diffuses the light which falls on it from the sun in all directions, so that it is light in places which do not receive the direct rays of the sun. Thus, the upper layers of the air diffuse the light which they receive before sunrise and after sunset, and accordingly give rise to the phenomena of *twilight*.

519. **Intensity of reflected light.**—The intensity of the light reflected is always less than that of the incident light, for some of the original vibrations are converted into vibrations of the reflecting surfaces. The intensity increases with the obliquity of the incident ray. For instance, if a sheet of white paper be placed before a candle, and be looked at very obliquely, an image of the flame is seen by reflection, which is not the case if the eye receives less oblique rays.

The intensity of the reflection varies with different bodies, even when the degree of polish and the angle of incidence are the same. Thus with perpendicular incidence, the reflected light is $\frac{2}{3}$ of the incident in the case of that reflected from a metal mirror, $\frac{3}{4}$ from mercury, $\frac{1}{25}$ from glass, and $\frac{1}{50}$ from water. It also varies with the nature of the medium which the ray is traversing before and after reflection. Polished glass immersed in water loses a great part of its reflecting power.

In the case of scattered reflection the actual lustre or brightness of a luminous surface is only a fraction of the light which falls upon it, and depends on the nature of the surface. If we call the incident light 100, we have for the brightness of freshly fallen snow 78, white paper 70, white sandstone 24, porphyry 11, and ordinary earth 8.

520. **Reflection of a ray of light in a rotating mirror.**—When a horizontal ray of light falls on a plane mirror which can rotate about a vertical axis, if the mirror is turned through an angle α, the reflected ray is turned through double the angle.

Let *nm* (fig. 469) be the first position of the mirror, *n'm'* its position after it has been turned through the angle α; and let OD be the fixed incident

ray. If from the centre of rotation C, with any radius we describe the circumference O*mn*, and from the point O, where it cuts the incident ray, chords OO′ and OO″ are drawn perpendicular respectively to *mn* and *m′n′*; the points O′ and O″ are the images of the point O in the two positions of the mirror, and the angles CO′D and CO″D′ are each equal to COD. The lines O′D and O″D′ thus making equal angles with O′C and O″C, the angle between the two former lines is equal to that between the two latter; that is, it will be equal to O′CO″, and will be measured by the arc O′O″. The rotations of the reflected ray and of the mirror are thus measured by the two arcs O′O″ and *mm′* respectively.

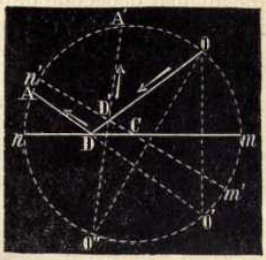


Fig. 468.

Now, the two angles O′OO″ and *m*C*m′* are equal, for they have their sides perpendicular each to each; but the angle O′OO″, which is an angle at the circumference, is measured by half the arc O′O″, and the angle *m*C*m′* by the whole arc *mm′*; hence O′O″ is the double of *mm′*, which shows that when the mirror has turned through an angle *a*, the reflected ray has turned through 2*a*.

521. **Hadley's reflecting sextant.**—The principal features of this instrument, which is used to measure the angular distance of any two distant objects, are represented in fig. 469. It consists of a metal sector, the arc, *cd*, of which is graduated. About the centre of the sector, an index arm, *ab*, turns; this is provided with a vernier and a micrometer screw, by which the index may be accurately adjusted and also clamped. A mirror at *a* is fixed perpendicularly to the arm *ab*, and therefore moves with it. A telescope *de* is permanently fixed to the arm *ac*, and opposite to it is a second mirror *m*, also permanently fixed; the lower half of this is silvered, and the axis of the telescope just traverses the boundary of the silvered and unsilvered part of the mirror.

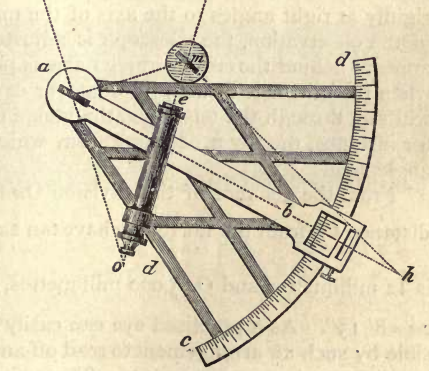


Fig. 469.

In making an observation the sextant is held so that its plane may pass through both the objects whose angular distance is to be measured. The index arm is at the zero of the graduation, which indicates the parallelism of the two mirrors. One of the objects is then viewed in the direction *om*, through the telescope, and the unsilvered part of the mirror *m*. The index arm is then moved until the eye sees simultaneously with this the image of another object *g*, which

reaches the eye after successive reflections from the mirror *a*, and from the silvered part of the mirror *m*; that is, by the path *gamedo*. The angle *mha* which the two mirrors now form is measured by the graduation of the sector *cd*, and is half the angle *gom*. For when the two mirrors were parallel, the angular deflection of the ray *ga*, after two reflections, would be zero, and its deflection is now the angle *gom*; whence, by the last article, the mirror *a* must have turned through half that angle, the mirror *m* having been fixed in position throughout.

522. **Measurement of small angles by reflection from a mirror.**—An important application of the laws of reflection in measuring small angles of deflection in magnetic and other observations was first made by Gauss. The principle of this method will be understood from fig. 470, in which AO represents a telescope, underneath which, and at right angles to its axis, is fixed a graduated scale *ss*; the centre of which, the zero, corresponds to the axis of the telescope.

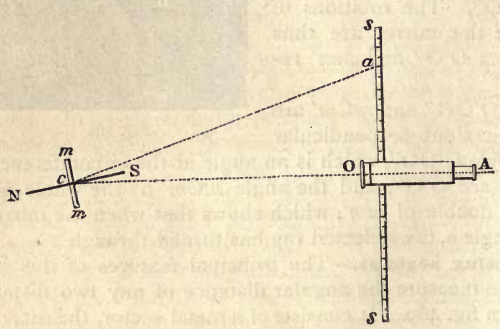


Fig. 470.

Let NS be the object whose angular deflection is to be measured, a magnet for instance, and let *mm* represent a small perfectly plane mirror fixed rigidly at right angles to the axis of the magnet. If now, at the beginning of the observation, the telescope is adjusted so that the image of the zero appears behind the cross wires, its axis is perpendicular to the mirror. Now when the mirror is turned, by whatever cause, through an angle α, the eye will see, through the telescope, the image of another division of the scale, a for instance, the ray proceeding from which makes with the line cOA the angle 2α.

From the distance of this division Oa from the zero of the scale and the distance Oc from the mirror we have $\tan 2\alpha = \frac{Oa}{Oc}$. Thus, for instance, if Oa is 12 millimetres and Oc 5,000 millimetres, then $\tan 2\alpha = \frac{12}{5{,}000}$, from which $2\alpha = 8' 15''$. As a practised eye can easily read $\frac{1}{10}$ of a millimetre, it is possible by such an arrangement to read off an angular deflection of two seconds.

523. **Mance's heliograph.**—The reflection of light from mirrors has been applied by Sir H. Mance in signalling at great distances by means of the sun's light.

The apparatus consists essentially of a mirror about 4 inches in diameter mounted on a tripod, and provided with suitable adjustments, so that the sun's light can be received upon it and reflected to a distant station. An observer then can see through a telescope the reflection of the sun's rays as a spot of light. The mirror has an adjustment by which it can be made to

follow the sun in its apparent motion. There is also a lever key by which the signaller can deflect the mirror through a very small angle either to the right or left, and thus the observer at the distant station sees corresponding flashes to the right or left. Under the subject of Telegraphy it will be seen how these alternate motions can be used to form an alphabet.

The heliograph proved of essential service in the campaigns in Africa and Afghanistan. Instead of any special form of apparatus, an ordinary shaving mirror or handglass is frequently used; and the proper inclination having been given so as to send the sun's rays to the distant station, which is very easily effected, the signals are produced by obscuring the mirror by sliding a piece of paper over it for varying lengths of time. In this way longer or shorter flashes of light are produced, which, properly combined, form the alphabet.

Of course this mode of signalling can only be used where the sun's light is available, but it has the advantage of being cheap, simple, and portable. Signals have been sent at the rate of 12 words a minute, through distances, in very fine weather, of 40 miles.

REFLECTION OF LIGHT FROM CURVED SURFACES

524. **Spherical mirrors.**—It has been already stated (512) that there are several kinds of curved mirrors; those most frequently employed are spherical and parabolic mirrors.

Spherical mirrors are those whose curvature is that of a sphere; their surface may be supposed to be formed by the revolution of an arc MN (fig. 471) about the radius CA, which unites the middle of the arc to the centre of the circle of which it is a part. According as the reflection takes place from the internal or from the external face of the mirror, it is said to be *concave* or *convex*. C, the centre of the hollow sphere of which the mirror forms part, is called the *centre of curvature*, or *geometrical centre*: the point A is the centre of the figure. The infinite right line AL, which passes through A and C, is the *principal axis* of the mirror; any right line which simply passes through the centre C, and not through the point A, is a *secondary* axis. The angle MCN, formed by joining the centre and extremities of the mirror, is the *aperture*. A *principal* or *meridional section* is the section made by a plane through its principal axis. In speaking of mirrors those lines alone will be considered which lie in the same principal section.

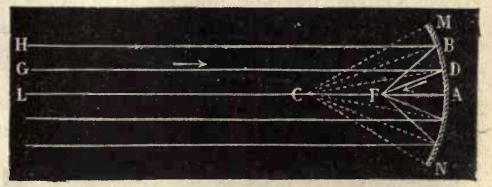


Fig. 471.

The theory of the reflection of light from curved mirrors is easily deduced from the laws of reflection from plane mirrors, by considering the surface of the former as made up of an infinitude of extremely small plane surfaces, which are its *elements*. The normal to the curved surface at a given point is the perpendicular to the corresponding element, or, what is the same thing, to its correspondent tangent plane. It is shown in geometry that in spheres

all the normals pass through the centre of curvature, so that the normal may readily be drawn to any point of a spherical mirror.

525. **Focus of a spherical concave mirror.**—In a curved mirror the *focus* is a point in which the reflected rays meet or tend to meet, if produced either backwards or forwards; there may either be a *real focus* or a *virtual focus*.

Real focus.—We shall first consider the case in which the rays of light are parallel to the principal axis, which presupposes that the luminous body is at an infinite distance. Let GD (fig. 471) be such a ray.

From the hypothesis that curved mirrors are composed of a number of infinitely small plane elements, this ray would be reflected from the element corresponding to the point D, according to the laws of the reflection from plane mirrors (513); that is, that CD being the normal at the point of incidence D, the angle of reflection CDF is equal to the angle of incidence GDC, and is in the same plane. It follows from this that the point F, where the reflected ray cuts the principal axis, divides the radius of curvature AC very nearly into two equal parts. For in the triangle DFC the angle DCF is equal to the angle CDG, for they are alternate and opposite angles; likewise the angle CDF is equal to the angle CDG, from the laws of reflection; therefore the angle FDC is equal to the angle FCD, and the sides FC and FD are equal as being opposite to equal angles. Now the smaller the arc AD, the more nearly does DF equal AF; and when the arc is only a small number of degrees, the right lines AF and FC may be taken as approximately equal, and the point F may be taken as the middle of AC. So long as the aperture of the mirror does not exceed 8 to 10 degrees, any other ray HB will, after reflection, pass very nearly through the point F. Hence, for practical purposes, we may say that when a pencil of rays parallel to the axis falls on a concave mirror the rays intersect after reflection in the same point, which is at an equal distance from the centre of curvature, and from the mirror. This point is called the *principal focus* of the mirror, and the distance AF is the *principal focal distance*.

All rays parallel to the axis meet in the point F; and, conversely, if a luminous point be placed at F, the rays emitted by this point will after reflection take the directions DG, BH, parallel to the principal axis; for in this case the angles of incidence and reflection have changed places; but these angles always remain equal.

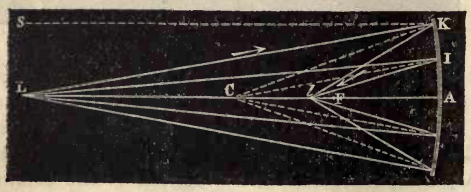


Fig. 472.

The case is now to be considered in which the rays are emitted from a luminous point, L (fig. 472), placed on the principal axis, but at such a distance that they are not parallel, but divergent. The angle LKC, which the incident ray LK forms with the normal KC, is smaller than the angle SKC, which the ray SK, parallel to the axis, forms with the same normal; and, consequently, the angle of reflection corresponding to the ray LK must be smaller than the angle CKF, corresponding to the ray SK. And therefore the ray

LK will meet the axis after reflection in the point *l*, between the centre C and the principal focus F. So long as the aperture of the mirror does not exceed a small number of degrees, all the rays from the point L will intersect after reflection in the point *l*. This point is called the *conjugate focus;* for there is this connection between the points L and *l*, that if the luminous point were transferred to *l*, its conjugate focus would be at L, *l*K being the incident and KL the reflected ray.

On considering the figure 472 it will be seen that when the point L is brought near to or removed from the centre C, its conjugate focus approaches or recedes in a corresponding manner, for the angles of incidence and reflection increase or decrease together.

If the point L coincides with the centre C, the angle of incidence is null, and as the angle of reflection must be the same, the ray is reflected on itself and the focus coincides with the luminous point. When the luminous point is between the centre C and the principal focus, the conjugate focus in turn is on the other side of the centre, and is further from the centre according as the luminous point is nearer the principal focus. If the luminous point coincides with the principal focus, the reflected rays, being parallel to the axis, will not meet, and there is, consequently, no focus.

Virtual focus.—There is, lastly, the case in which the point is placed at L, between the principal focus and the mirror (fig. 473). Any ray LM,

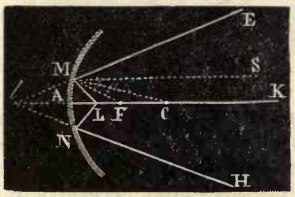


Fig. 473.

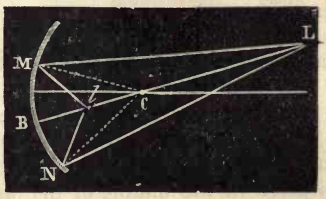


Fig. 474.

emitted from the point L, makes with the normal CM an angle of incidence LMC, greater than FMC; the angle of reflection must be greater than CMS, and therefore the reflected ray ME diverges from the axis AK. This is also the case with all rays from the point L, and hence these rays do not intersect, and, consequently, form no conjugate focus; but if they are conceived to be prolonged on the other side of the mirror, their prolongations will intersect in the same point, *l*, on the axis, and the eye experiences the same impression as if the rays were directly emitted from the point *l*. Hence a *virtual focus* is formed quite analogous to those formed by plane mirrors (514).

In all these cases it is seen that the position of the principal focus is constant, while that of the conjugate foci and of the virtual foci varies. *The principal and the conjugate foci are always on the same side of the mirror as the luminous point*, while *the virtual focus is always on the other side of the mirror.*

Hitherto the luminous point has always been supposed to be placed on the principal axis itself, and then the focus is formed on this axis. In the case in which the luminous point is situate on a secondary axis, LB (fig. 474), by applying to this axis the same reasoning as in the preceding case, it will

be seen that the focus of the point L is formed at a point *l* on the secondary axis, and that, according to the distance of the point L, the focus may be either principal, conjugate, or virtual.

526. **Foci of convex mirrors.**—In convex mirrors there are only virtual foci. Let SI, TK . . . (fig. 475) be rays parallel to the principal axis of a

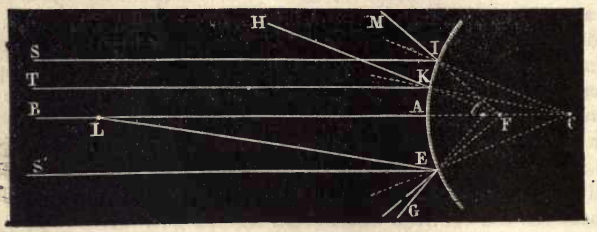


Fig. 475.

convex mirror. These rays, after reflection, take the diverging directions IM, KH, which, when continued, meet in a point F, which is the *principal virtual focus* of the mirror. By means of the triangle CKF, it may be shown, in the same manner as with concave mirrors, that the point F is approximately the centre of the radius of curvature, CA.

If the incident luminous rays, instead of being parallel to the axis, proceed from a point L, situated on the axis at a finite distance, it is at once seen that a virtual focus will be formed at a point *l*, between the principal focus F and the mirror.

527. **Determination of the principal focus of a mirror.**—In the applications of concave and convex mirrors it is often necessary to know the radius of curvature. This is tantamount to finding the principal focus; for being situated at the middle of the radius, it is simply necessary to double the focal distance.

To find this focus with a concave mirror, it is exposed to the sun's rays, so that its principal axis is parallel to them, and then with a small screen of ground glass the point is sought at which the image is formed with the greatest intensity; this is the principal focus. The radius of the mirror is double this distance.

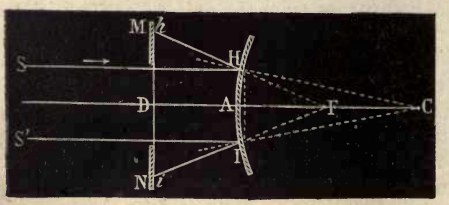


Fig. 476.

If the mirror is convex, it is covered with paper; but two small portions, H and I, are left exposed at equal distances from the centre of the figure A, and on the same principal section (fig. 476). A screen MN, in the centre of which is an opening larger than the distance HI, is placed before the mirror. If a pencil of the sun's rays, SH, S'I, parallel to the axis, fall on the mirror, the light is reflected at H and I, on the parts where the mirror is left exposed, and forms on the screen two brilliant images at *h* and *i*. By moving the screen MN nearer

to or farther from the mirror, a position is found at which the distance *hi* is double that of HI. The distance AD from the screen to the mirror then equals the principal focal distance. For the arc HAI does not sensibly differ from its chord ; and because the triangles FHI and F*hi* are similar, $\frac{HI}{hi} = \frac{FA}{FD}$, but HI is half of *hi*, and therefore also FA is the half of FD, and therefore AD is equal to AF. Further, FA is the principal focal distance ; for the rays SH and S′I are parallel to the axis ; consequently also twice the distance AD equals the radius of curvature of the mirror.

528. **Formation of images in concave mirrors.**—Hitherto it has been supposed that the luminous or illuminated object placed in front of the mirror was simply a point ; but if this object has a certain magnitude, we can conceive a secondary axis drawn through each of its points, and thus a series of real or virtual foci could be determined the collection of which composes the image of the object. By the aid of the constructions which have served for determining the foci, we shall investigate

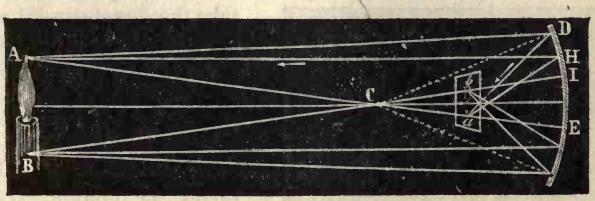


Fig. 477.

the position and magnitude of these images in concave and in convex mirrors.

Real image.—We shall first take the case in which the mirror is concave, and the object AB (fig. 477) is on the other side of the centre. To obtain the image or the focus of any point A, a secondary axis, AE, is drawn from this point, and then drawing from the point A an incident ray AD, the normal to this point, CD, is taken, and the angle of reflection CD*a* is made equal to the angle of incidence ADC. The point *a*, where the reflected ray cuts the secondary axis AE, is the conjugate focus of the point A, because every other ray drawn from this point passes through *a*. Similarly if a secondary axis, BI, be drawn from the point B, the rays from this point meet after reflection in *b*, and form the conjugate focus of B. And as the images of all the points of the object are formed between *a* and *b*, *ab* is the complete image of AB. From what has been said about foci (525), it follows that *this image is real, inverted, smaller than the object, and placed between the centre of curvature and the principal focus.* This image may be seen in two ways : by placing the eye in the continuation of

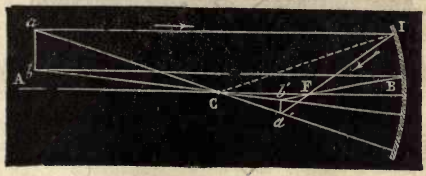


Fig. 478.

the reflected rays, and then it is an aërial image which is seen ; or the rays are collected on a screen, on which the image appears to be depicted.

If the luminous or illuminated object is placed at *ab*, between the principal focus and the centre, its image is formed at AB. It is then a real but inverted image ; it is larger than the object, *and the larger as the object, ab, is nearer the focus.*

If the object is placed in the principal focus itself, no image is produced ; for then the rays emitted from each point form, after reflection, as many pencils respectively parallel to the secondary axis, which is drawn through the point from which they are emitted (524), and hence neither foci nor images are formed.

When all points of the object AB are above the principal axis (fig. 478), it is readily seen, by repeating the preceding construction, that the image of the object is formed at *ab*.

Virtual image.—The case remains in which the object is placed between the principal focus and the mirror. Let AB be this object (fig. 479) ; the incident rays after reflection take the directions DI and KH, and their prolongations form a virtual image, *a*, of the point A, on the secondary axis. Similarly, an image of B is formed at *b* ; consequently the eye sees at *ab* the image of AB. *This image is virtual, erect, and larger than the object.*

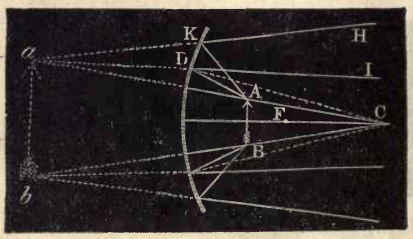


Fig. 479.

From what has been stated, it is seen that, according to the distance of the object, concave mirrors produce two kinds of images, or none at all ; a person notices this by placing himself in front of a concave mirror. At a certain distance he sees an image of himself inverted and smaller ; this is the real image ; at a less distance the image becomes confused, and disappears when he is at the focus ; still nearer the image appears erect, but larger—it is then a virtual image.

529. **Formation of images in convex mirrors.**—Let AB (fig. 480) be an object placed in front of a mirror at any given distance. AC and BC are secondary axes, and it follows, from what has been already stated, that all the rays from A are divergent after reflection, and that their prolongations pass through a point *a*, which is the virtual image of the point A. Similarly the rays from B form a virtual image of it in the point *b*. The eye which receives the divergent rays DE, KH . . . sees in *ab* an image of AB. Hence, whatever the position of an object in front of a convex mirror, *the image is always virtual, erect, and smaller than the object.*

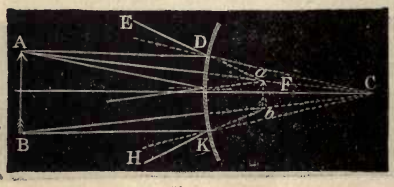


Fig. 480.

530. **Formulæ for spherical mirrors.**—The relation between the position of an object and that of its image in spherical mirrors may be expressed by a very simple formula. In the case of concave mirrors, let R be its radius of curvature, p the distance LA of the object L (fig. 481), and p' the distance lA of the image from the mirror. In the triangle LMl, the perpendicular MC divides the angle LMl into two equal parts, and from geometry it follows that the two segments LC, Cl are to each other as the two sides containing the angle ; that is,

$$\frac{C\,l}{CL} = \frac{l\,M}{LM} : \text{therefore } C\,l \times LM = CL \times l\,M.$$

If the arc AM does not exceed 5 or 6 degrees, the lines ML and Ml are approximately equal to AL and Al; that is, to p and p'.

Further, $C\,l = CA - A\,l = R - p'$,

and also $CL = AL - AC = p - R$.

The value substituted in the preceding equations gives

$$(R - p')\,p = (p - R)\,p'.$$

Fig. 481.

From which transposing and reducing we have

$$Rp + Rp' = 2pp' \quad . \quad . \quad . \quad . \quad (1)$$

If the terms of this equation be all divided by $pp'R$, we obtain

$$\frac{1}{p} + \frac{1}{p'} = \frac{2}{R} \quad . \quad . \quad . \quad . \quad (2)$$

which is the usual form of the equation.

From the equation (1) we get

$$p' = \frac{pR}{2p - R} \quad . \quad . \quad . \quad . \quad (3)$$

which gives the distance of the image from the mirror, in terms of the distance of the object, and of the radius of curvature.

531. **Discussion of the formulæ for mirrors.**—We shall now investigate the different values of p', according to the values of p in the formula (3).

i. Let the object be placed at an infinite distance on the axis, in which case the incident rays are parallel. To obtain the value of p', both terms of the fraction (3) must be divided by p, which gives

$$p' = \frac{R}{2 - \frac{R}{p}} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

as p is infinite, $\frac{R}{p}$ is zero, and we have $p' = \frac{R}{2}$; that is, the image is formed in the principal focus, as ought to be the case, for the incident rays are parallel to the axis.

ii. If the object approaches the mirror, p decreases, and as the denominator of the formula (4) diminishes, the value of p' increases ; consequently the image approaches the centre at the same time as the object, but it is always between the principal focus and the centre, for so long as

p is $>$ R, we have $\frac{R}{2-\frac{R}{p}} > \frac{R}{2}$ and $<$ R.

iii. When the object coincides with the centre, $p = R$, and, consequently, $p' = R$; that is, the image coincides with the object.

iv. When the luminous object is between the centre and the principal focus, $p < R$, and hence from the formula (4), $p' > R$; that is, the image is formed on the other side of the centre. When the object is in the focus, $p = \frac{R}{2}$, which gives $p' = \frac{R}{0} = \infty$; that is, the image is at an infinite distance, for the reflected rays are parallel to the axis.

v. Lastly, if the object is between the principal focus and the mirror, we get $p < \frac{R}{2}$; p' is then negative, because the denominator of the formula (4) is negative. Therefore, the distance p' of the mirror from the image must be calculated on the axis in a direction opposite to p. The image is then virtual, and is on the other side of the mirror.

Making p' negative in the formula (2), it becomes $\frac{1}{p} - \frac{1}{p'} = \frac{2}{R}$; in this form it comprehends all cases of virtual images in concave mirrors.

In the case of convex mirrors the image is always virtual (526); p' and R are of the same sign, since the image and the centre are on the same side of the mirror, while the object being on the opposite side, p is of the contrary sign; hence in the formula (2) we get

$$\frac{1}{p'} - \frac{1}{p} = \frac{2}{R} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

as the formula for convex mirrors. It may also be found directly by the same geometrical considerations as those which have led to the formula (2) for concave mirrors.

It must be observed that the preceding formulæ are not rigorously true, inasmuch as they depend upon the assumption that the lines LM and lM (fig. 481) are equal to LA and Al: although this is not true, the error diminishes without limit with the angle MCA; and when this angle does not exceed a few degrees, the error is so small that it may, in practice, be neglected.

532. **Calculation of the magnitude of images.**—By means of the above formulæ the magnitude of an image may be calculated when the distance of the object, its magnitude, and the radius of the mirror are given. For if BD be the object (fig. 482), bd its image, and if the distance A and the radius AC be known, Ao can be calculated by means of formula (3) of article 530. Ao known, oC can be calculated. But as the triangles BCD and dCb are similar, their bases and heights are in the proportion bd : BC = Co : CK, or

Fig. 482.

Length of the image : length of the object
= distance from image to centre : distance from the object to the centre.

The brightness of an image formed by a concave mirror is nearly proportional to its surface, and to the coefficient of reflection ; and is inversely as the square of the focal distance.

533. **Spherical aberration. Caustics.**—In the foregoing explanation of the formation of foci and images of spherical mirrors, it has already been observed that the reflected rays only pass through a single point when the aperture of the mirror does not exceed 8 or 10 degrees (525). With a larger aperture the rays reflected near the edges meet the axis nearer the mirror than those that are reflected at a small distance from the neighbourhood of the centre of the mirror. Hence arises a want of sharpness in these images, which is called *spherical aberration by reflection*, to distinguish it from the *spherical aberration by refraction*, which occurs in the case of lenses.

Every reflected ray cuts the one next to it (fig. 483), and their points of intersection form in space a curved surface which is called the *caustic by reflection*. The curve FM represents one of the branches of a section of this surface made by the plane of the paper. When the light of a candle is reflected from the inside of a tea-cup or a glass tumbler, a section of the caustic surface can be seen by partly filling the cup or tumbler with milk.

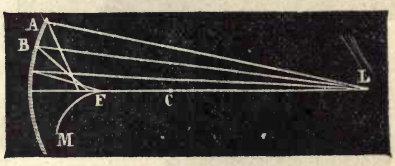


Fig. 483.

534. **Applications of mirrors. Heliostat.**—The applications of plane mirrors in domestic economy are well known. Mirrors are also frequently used in physical apparatus for sending light in a certain direction. We have already seen an application of this in the heliograph (523). The light of the sun can only be sent in a constant direction by making the mirror movable. It must have a motion which compensates for the continual change in the direction of the sun's rays produced by the apparent diurnal motion of the sun. This result is obtained by means of a clockwork motion, to which the mirror is fixed, and which causes it to follow the course of the sun. Such an apparatus is called a *heliostat*. The reflection of light is also used to measure the angles of crystals by means of the instruments known as *reflecting goniometers*.

Concave spherical mirrors are also often used. They are applied for magnifying *mirrors*, as in the older forms of shaving mirrors. They have been employed for burning mirrors, and are still used in telescopes. They also serve as reflectors, for conveying light to great distances, by placing a luminous object in their principal focus. For this purpose, however, parabolic mirrors are preferable.

The images of objects seen in concave or convex mirrors appear smaller or larger, but otherwise similar geometrically, except in the case where some parts of a body are nearer the mirror than others. The distortion of features observed on looking into a spherical garden mirror is more marked the nearer we are to the glass. Objects seen in *cylindrical* or

conical mirrors appear ludicrously distorted. From the laws of reflection the shape of such a distorted figure can be geometrically constructed. In like manner distorted images of objects can be constructed which, seen in such mirrors, appear in their normal proportions. They are called *anamorphoses.*

535. **Parabolic mirrors.**—*Parabolic mirrors* are concave mirrors whose surface is generated by the revolution of the arc of a parabola, AM, about its axis AX (fig. 484).

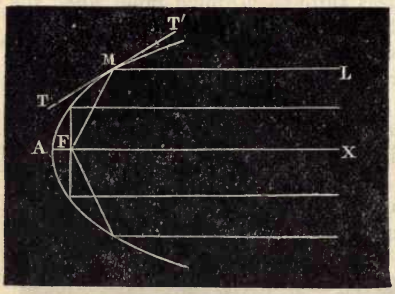


Fig. 484.

It has been already stated that in spherical mirrors the rays parallel to the axis converge only approximately to the principal focus ; and reciprocally, when a source of light is placed in the principal focus of these mirrors, the reflected rays are not exactly parallel to the axis. Parabolic mirrors are free from this defect ; they are more difficult to construct, but are better for reflectors. It is a property of a parabola that the right line FM, drawn from the focus F to any point M of the curve, and the line ML, parallel to the axis AF, make equal angles with the tangent TT′ at this point. Hence all rays parallel to the axis after reflection meet in the focus of the mirror F ; and conversely, when a source of light is placed in the focus, the rays incident on the mirror are reflected exactly parallel to the axis. The light thus reflected tends to maintain its intensity even at a great distance, for it has been seen (508) that it is the divergence of the luminous rays which principally weakens the intensity of light.

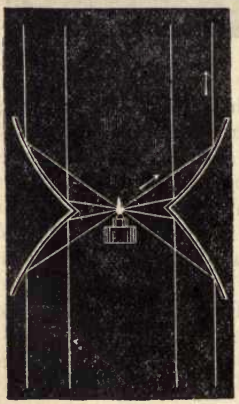


Fig. 485.

From this property parabolic mirrors are used in carriage lamps, and in the lamps placed in front of and behind railway trains. These reflectors were formerly used for lighthouses, but have been replaced by lenticular glasses.

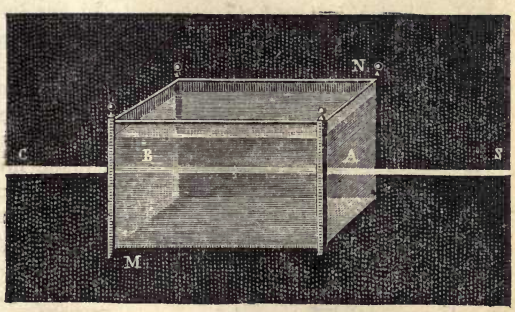
When two equal parabolic mirrors are cut by a plane perpendicular to the axis passing through the focus, and are then united at their intersections as shown in fig. 485, so that their foci coincide, a system of reflectors is obtained with which a single lamp illuminates in two directions at once. This arrangement is used in lighting staircases and passages.

CHAPTER III

SINGLE REFRACTION. LENSES

536. **Phenomenon of refraction.**—*Refraction* is the deflection or bending which the rays of light experience in passing *obliquely* from one medium to another: for instance, from air into water (fig. 487). If the incident ray is perpendicular to the surface separating the two media, it is not bent, but continues its course in a right line (fig. 486).

Fig. 486.



The *incident ray* being represented by SO (fig. 488), the *refracted ray* is the direction OH which light takes in the second medium; and of the angles SOA and HOB, which these rays form with the line AB, at right angles to the surface which separates the two media, the first is the *angle of incidence*, and the other the *angle of refraction*. According as the refracted ray approaches or deviates from the normal, the second medium is said to be more or less *refringent* or *refracting* than the first.

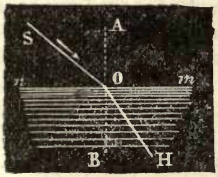


Fig. 488.

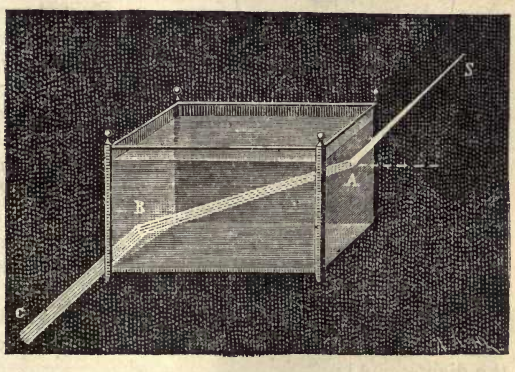


Fig. 487.

All the light which falls on a refracting surface does not completely pass into it; one part is reflected and scattered (518), while another penetrates into the medium.

Mathematical analysis shows that the direction of refraction depends on the relative velocity of light in the two media. On the undulatory theory

the more highly refracting medium is that in which the velocity of propagation is least.

In uncrystallised media, such as air, liquids, ordinary glass, the luminous ray is singly refracted; but in certain crystallised bodies, such as Iceland spar, selenite, &c., the incident ray gives rise to two refracted rays. The latter phenomenon is called *double refraction*, and will be discussed in another part of the book. We shall here deal exclusively with *single refraction.*

537. **Laws of single refraction.**—When a luminous ray is refracted in passing from one medium into another of a different refractive power, the following laws prevail :—

I. *Whatever the obliquity of the incident ray, the ratio which the sine of the incident angle bears to the sine of the angle of refraction is constant for the same two media, but varies with different media.*

II. *The incident and the refracted ray are in the same plane, which is perpendicular to the surface separating the two media.*

These have been known as *Descartes's law*; they are, however, really due to Willibrod Snell, who discovered them in 1620; they are demonstrated by the same apparatus as that used for the laws of reflection (511). The plane mirror in the centre of the graduated circle is replaced by a semi-cylindrical glass vessel, filled with water to such a height that its level is exactly the height of the centre (fig. 489). If the mirror, M, be then so inclined that a reflected ray, MO, is directed towards the centre, it is refracted on passing into the water, but it passes out without refraction, because its direction is then at right angles to the curved sides of the vessel. In order to observe the course of the refracted ray, it is received on a screen, P, which is moved until the image of the aperture in the screen N is formed at its centre. In all positions of the screens N and P, the sines of the angles of incidence and refraction are measured by means of two graduated rules, movable so as to be always horizontal, and hence perpendicular to the diameter AD.

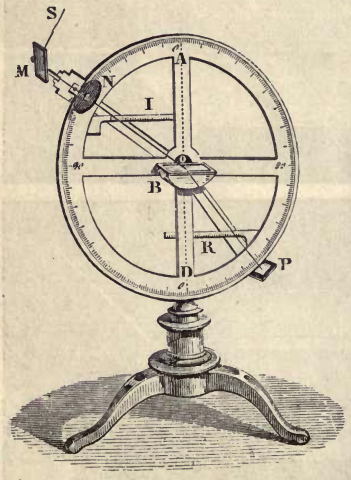


Fig. 489.

On reading off the length of the sines of the angles MOA and DOP in the scales I and R, the numbers are found to vary with the position of the screens, but their ratio is constant; that is, if the sine of incidence becomes twice or three times as large, the sine of refraction increases in the same ratio, which demonstrates the first law. The second law follows from the arrangement of the apparatus, for the plane of the graduated limb is perpendicular to the surface of the liquid in the semi-cylindrical vessel.

538. **Index of refraction.**—The ratio between the sines of the incident and refracted angle is called *index of refraction*, or *refractive index.* It

varies with the media ; for example, from air to water it is $\frac{4}{3}$, and from air to glass it is $\frac{3}{2}$.

If the media are considered in an inverse order—that is, if light passes from water to air, or from glass to air—it follows the same course, but in a contrary direction, PO becoming the incident and OM the refracted ray. Consequently the index of refraction is reversed ; from water to air it is then $\frac{3}{4}$, and from glass to air $\frac{2}{3}$.

539. **Effects produced by refraction.**—In consequence of refraction, bodies immersed in a medium more highly refracting than air appear nearer the surface of this medium, but they appear to be more distant if immersed in a less refracting medium. Let L (fig. 490) be an object immersed in a mass of water. In passing thence into air, the rays LA, LB . . . diverge from the normal to the point of incidence, and take the direction AC, BD . . . , the prolongations of which intersect approximately in the point L′, placed on the perpendicular L′K. The eye receiving these rays sees the object L at L′. The greater the obliquity of the rays LA, LB . . . the higher the object appears.

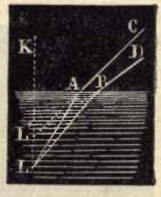


Fig. 490.

It is for the same reason that a stick plunged obliquely into water appears bent (fig. 491), the immersed part appearing raised.

An experimental illustration of the effect of refraction is the following :—A coin is placed in an empty porcelain basin, and the position of the eye is so adjusted that it is just not visible. If now, the position of the eye remaining unaltered, water be poured into the basin, the coin becomes visible. A consideration of fig. 490 will suggest the explanation of this phenomenon.

Owing to an effect of refraction, stars are visible to us even when they are below the horizon. For as the layers of the atmosphere are denser in proportion as they are nearer the earth, and as the refractive power of a gas increases with its density (550), it follows that

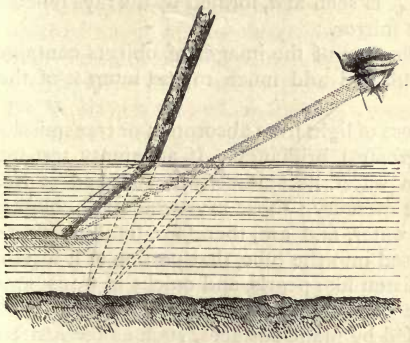


Fig. 491.

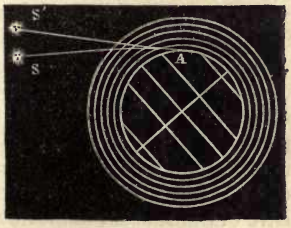


Fig. 492.

on entering the atmosphere the luminous rays become bent, as seen in fig. 492, describing a curve before reaching the eye, so that we can see the star at S′ along the tangent of this curve instead of at S. In our climate the atmospheric refraction does not raise the stars when on the horizon more than half a degree.

The effect of refraction is that objects at a distance appear higher than

they are in reality ; our horizon is thereby widened. When individual layers of air refract more strongly than usual, objects may thereby become visible which are usually below the horizon. Thus, from Hastings, the coast of France, which is at a distance of 56 miles, is not unfrequently seen.

540. **Total reflection. Critical angle.**—When a ray of light passes from one medium into another which is less refracting, as from water into air, it has been seen that the angle of incidence is less than the angle of refraction. Hence, when light is propagated in a mass of water from S to O (fig. 493), there is always a value of the angle of incidence SOB, such that the angle of refraction AOR is a right angle, in which case the refracted ray emerges parallel to the surface of the water.

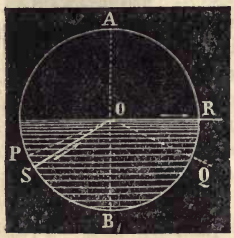


Fig. 493.

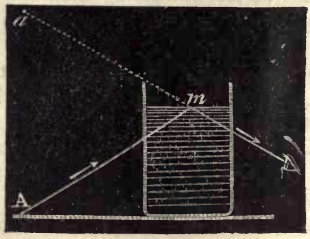


Fig. 494.

This angle, SOB, is called the *critical angle*, since for any greater angle, POB, the incident ray cannot emerge, but undergoes an internal reflection, which is called *total reflection* because the incident light is entirely reflected. From water to air the critical angle is 48° 35′ : from glass to air, 41° 48′.

The occurrence of this internal reflection may be observed by the following experiment :—An object, A, is placed before a glass vessel filled with water (fig. 494) ; the surface of the liquid is then looked at as shown in the figure, and an image of the object A is seen at *a*, formed by the rays reflected at *m*, in the ordinary manner of a mirror.

Similar effects of the total reflection of the images of objects contained in aquaria are frequently observed, and add much to the interest of their appearance.

In total reflection there is no loss of light from absorption or transmission, and accordingly it produces the greatest brilliancy. If an empty test-tube be placed in a slanting position in water, its surface, when looked at from above, shines as brilliantly as pure mercury ; those rays which fall obliquely on the side cannot pass into the water, and are, therefore, totally reflected upwards. If a little water be passed into the tube, that portion of it loses its lustre. Bubbles, again, in water glisten like pearls, and cracks in transparent bodies like strips of silver, for the oblique rays are totally reflected. The lustre of transparent bodies bounded by plane surfaces, such as the lustre of chandeliers, arises mainly from total reflection. This lustre is the more frequent and the more brilliant the smaller the limiting angle ; the lustre of diamond, therefore, is the most brilliant.

541. **Mirage.**—The *mirage* is an optical illusion by which inverted images of distant objects are seen as if below the ground or in the atmosphere. This phenomenon is of most frequent occurrence in hot climates, and more especially on the sandy plains of Egypt. The ground there has often the aspect of

a tranquil lake, on which are reflected trees and the surrounding villages. Monge, who accompanied Napoleon's expedition to Egypt, was the first to give an explanation of the phenomenon.

It is a phenomenon of refraction, which results from the unequal density of the different layers of the air when they are expanded by contact with the heated soil. The least dense layers are then the lowest, and the pencil of light from an elevated object, A (fig. 495), traverses layers which are gradually less refracting; for, as will be shown presently (550), the refracting power of a gas diminishes with lessened density. The angle of incidence accordingly increases from one layer to the other, and ultimately reaches the critical angle, beyond which internal reflection succeeds to refraction (540). The pencil then rises, as seen in the figure, and undergoes a series of successive refractions, but in the direction contrary to the first, for it now passes through layers which are gradually more refracting. The pencil then reaches the eye with the same direction as if it had proceeded from a point below the ground, and hence it gives an inverted image of the object, just as if it had been reflected at the point O, from the surface of a tranquil lake.

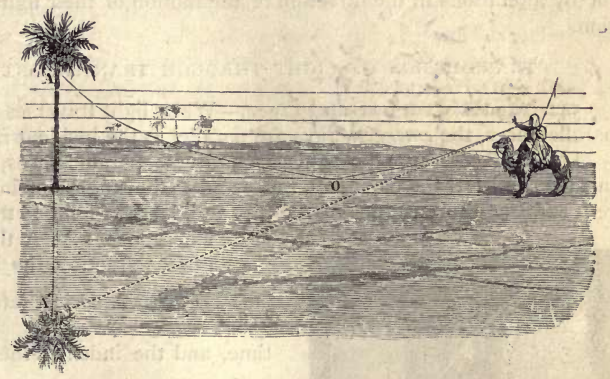


Fig 495.

The effect of the mirage may be illustrated artificially, though feebly, as Dr. Wollaston showed, by looking along the side of a red-hot poker at a word or object ten or twelve feet distant. At a distance less than three-eighths of an inch from the line of the poker, an inverted image was seen, and within and without that an erect image. A better arrangement than a red-hot poker is a flat sheet-iron box, about 3 feet in length by 5 to 7 inches in height and breadth (fig. 496); it is filled with red-hot charcoal and held at *a* about the level of the eye. Looking over the lid of the box in the direction *pm* a *direct*, and in the direction *pm'* an *inverted*, image of a distant point, *m*, is seen. The same phenomenon is observed by looking along the sides.

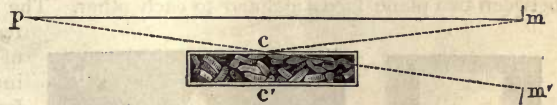


Fig. 496.

Mariners sometimes see inverted images in the air of ships and distant objects which are still under the horizon; this is due to the same cause as the mirage, but is in a contrary direction. The lower layers of the air being

in contact with the water are cold and dense. The rays of an object, a ship for instance, bent in an upward direction are more and more bent away from the vertical as they are continually passing into gradually less dense layers, and ultimately fall so obliquely on an upper attenuated layer that they are totally reflected downwards, and can thus reach the eye of an observer on the sea or on the shore. Scoresby observed several such cases in the Polar seas.

The *twinkling* or *scintillation* of the fixed stars is also to be accounted for by alterations in the direction of the motion of their light due to refraction.

TRANSMISSION OF LIGHT THROUGH TRANSPARENT MEDIA

542. **Media with parallel faces.**—When light traverses a medium with parallel faces, the *emergent* rays are parallel to the incident rays.

Let MN (fig. 497) be a glass plate with parallel faces, let SA be the incident and DB the emergent ray, i and r the angles of incidence and of refraction at the entrance of the ray, and, lastly, i' and r' the same angles at its emergence. At A the light undergoes a first refraction, the index of which is $\frac{\sin i}{\sin r}$ (537). At D it is refracted a second time, and the index is then $\frac{\sin i'}{\sin r'}$. But we have seen that the index of refraction of glass to air is the reciprocal of its refraction from air to glass ; hence

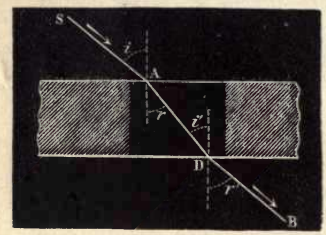


Fig. 497.

$$\frac{\sin i'}{\sin r'} = \frac{\sin r}{\sin i}$$

But as the two normals AG and DE are parallel, the angles r and i' are equal, as being alternate interior angles. As the numerators in the above equation are equal, the denominators must also be equal ; the angles r' and i are therefore equal, and hence DB is parallel to SA.

543. **Prism.**—In optics a *prism* is any transparent medium comprised between two plane faces inclined to each other. The intersection of these two faces is the *edge* of the prism, and their inclination is its refracting angle. Every section perpendicular to the edge is called a *principal section.*

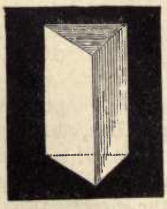


Fig. 498.

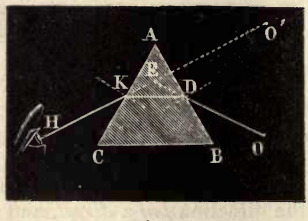


Fig. 499.

The prisms used for experiments are generally right triangular prisms of glass, as shown in fig. 498, and their principal section is a triangle (fig. 499). In this section the point A is called the *summit* of the prism, and the right line BC is called the *base* : these expressions have reference to the triangle ABC, and not to the prism.

544. **Path of rays in prism. Angle of deviation.**—When the laws of refraction are known, the path of the rays in a prism is readily determined. Let O be a luminous point (fig. 499) in the same plane as the principal section ABC of a prism, and let OD be an incident ray. This ray is refracted at D, and approaches the normal, because it passes into a more highly refracting medium. At K it experiences a second refraction, but it then deviates from the normal, for it passes into air, which is less refractive than glass. The light is thus refracted twice in the same direction, so that *the ray is deflected towards the base*, and consequently the eye which receives the emergent ray KH sees the object O at O′; that is, *objects seen through a prism appear deflected towards its summit.* The angle OEO′, which the incident and emergent rays form with each other, expresses the deviation of light caused by the prism, and is called *the angle of deviation.*

Besides this, objects seen through a prism appear in all the colours of the rainbow: this phenomenon, known as *dispersion*, will be afterwards described (564).

This angle increases with the refractive index of the material of the prism, and also with its refracting angle. It also varies with the angle under which

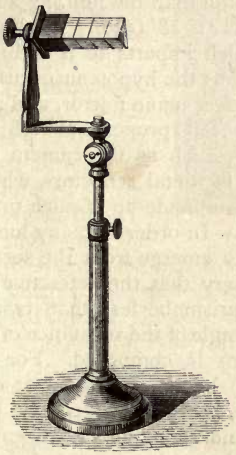


Fig. 500.

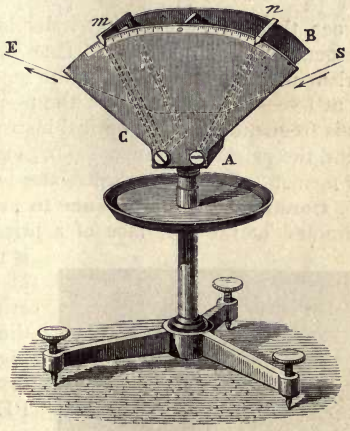


Fig. 501.

the luminous ray enters the prism. The angle of deviation increases up to a certain limit, which is determined by calculation, knowing the angle of incidence of the ray, and the refracting angle of the prism.

That the angle of deviation increases with the refractive index may be shown by means of the *polyprism.* This name is given to a prism formed of several prisms of the same angle connected at their ends (fig. 500). These prisms are made of substances unequally refringent, such as flint glass, rock crystal, or crown glass. If any object—a line, for instance—be looked at through the polyprism, its different parts are seen at unequal heights. The highest portion is that seen through the flint glass, the refractive index of

which is greatest; then the rock crystal; and so on in the order of the decreasing refractive indices.

The *prism with variable angle* (fig. 501) is used for showing that the angle of deviation increases with the refracting angle of the prism. It consists of two parallel brass plates, B and C, fixed on a support. Between these are two glass plates, moving on a hinge, with some friction against the plates, so as to close it. When water is poured into the vessel the angle may be varied at will. If a ray of light, S, be allowed to fall upon one of them, by inclining the other more the angle of the prism increases, and the deviation of the ray is seen to increase.

545. **Application of right-angled prisms in reflectors.**— Prisms whose principal section is an isosceles right-angled triangle afford an important application of total reflection (540). For let ABC (fig. 502) be the principal section of such a prism, O a luminous point, and OH a ray at right angles to the face BC. This ray enters the glass without being refracted, and makes with the face AB an angle equal to B—that is, to 45 degrees—and therefore greater than the limiting angle of glass, which is 41° 48′ (540). The ray OH undergoes, therefore, at H total reflection, which imparts to it a direction HI perpendicular to the second face AC. Thus the hypotenuse surface of this prism produces the effect of the most perfect plane mirror, and an eye placed at I sees O′, the image of the point O. This property of right-angle prisms is frequently used in optical instruments such as the camera lucida (603) and the prismatic compass (697) instead of metal reflectors, which so readily tarnish. The newer lighthouse lenses are made up of such prisms.

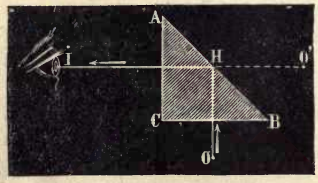


Fig. 502.

546. **Conditions of emergence in prisms.**—In order that any luminous ray refracted at the first face of a prism may emerge from the second, it is necessary that the refractive angle of the prism be less than twice the critical angle of the substance of which the prism is composed. For if LI (fig. 503) be the ray incident on the first face, IE the refracted ray, PI and PE the normals, the ray IE can only emerge from the second face when the incident angle IEP is less than the critical angle (540). But as the incident angle LIN increases, the angle EIP also increases, while IEP diminishes. Hence, according as the direction of the ray LI tends to become parallel with the face AB, does this ray tend to emerge at the second face.

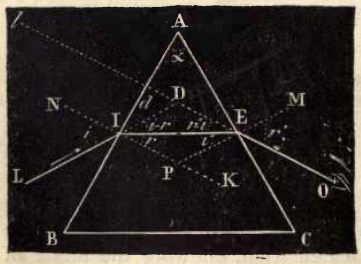


Fig. 503.

Let LI be now parallel to AB, the angle r is then equal to the critical angle l of the prism, because it has its maximum value. Further, the angle EPK, the exterior angle of the triangle IPE, is equal to $r + i'$; but the angles EPK and A are equal, because their sides are perpendicular, and

therefore $A = r + i'$; therefore also $A = l + i'$, for in this case $r = l$. Hence, if $A = 2l$ or is $> 2l$, we shall have $i' = l$ or $> l$, and therefore the ray would not emerge at the second face, but would be parallel to AC or would undergo internal reflection, and emerge at a third face, BC. This would be much more the case with rays whose incident angle is less than BIN, because we have already seen that i' would continually increase. Thus in the case in which the refracting angle of a prism is equal to $2l$ or is greater, no luminous ray could pass through the faces of the refracting angle.

As the critical angle of glass is 41° 48′, twice this angle is less than 90°, and, accordingly, objects cannot be seen through a glass prism whose refracting angle is a right angle. As the critical angle of water is 48° 35′, light could pass through a hollow rectangular prism formed of three glass plates and filled with water.

If we suppose A to be greater than l and less than $2l$, then of rays incident at I, some within the angle NIB will emerge from AC, others will not emerge, nor will any emerge that are incident within the angle NIA. If we suppose A to have any magnitude less than l, all rays incident at I within the angle NIB will emerge from AC, as also will some of those incident within the angle NIA.

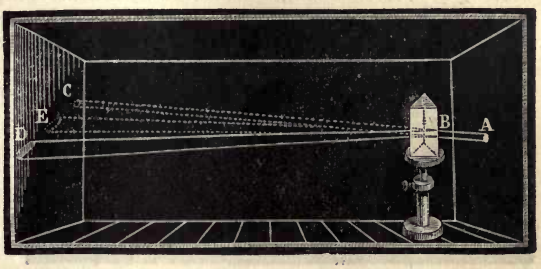


Fig. 504.

547. **Minimum deviation.**—When a pencil of sunlight passes through an aperture A, in the side of a dark chamber (fig. 504), the pencil is projected in a straight line AC, on a distant screen. But if a vertical prism be interposed between the aperture and the screen, the pencil is deviated towards the base of the prism, and the image is projected at D, at some distance from the point C. If the prism be turned so that the incident angle decreases, the disc of light approaches the point C up to a certain position, E, from which it reverts to its original position even when the prism is rotated in the same direction. Hence there is a deviation, EBC, less than any other. It may be demonstrated mathematically that this *minimum deviation* takes place when the angles of incidence and of emergence are equal.

The angle of minimum deviation may be calculated when the incident angle and the refracting angle of the prism are known. For when the deviation is a minimum, then since the angle of emergence r' is equal to the incident angle i (fig. 503), r must equal i'. But it has been shown above (546) that $A = r + i'$; consequently

$$A = 2r. \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

If the minimum angle of deviation LDl be called d, this angle being exterior to the triangle DIE, we readily obtain the equation

$$d = i - r + r' - i' = 2i - 2r,$$

whence $$d = 2i - A \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

which gives the angle d, when i and A are known.

From the formulæ (1) and (2) a third may be obtained, which serves to calculate the index of refraction of a prism when its refracting angle and the minimum of deviation are known. The index of refraction, n, is the ratio of the sines of the angles of incidence and refraction; hence $n = \frac{\sin i}{\sin r}$; replacing i and r from their values in the above equations (1) and (2) we get

$$n = \frac{\sin\left(\frac{A+d}{2}\right)}{\sin\frac{A}{2}} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

548. **Measurement of the refractive index of solids.**—By means of the preceding formula (3) the refractive index of a solid may be calculated when the angles A and d are known.

In order to determine the angle A, the substance is cut in the form of a triangle prism, and the angle measured by means of a goniometer (534).

The angle d is measured in the following manner:—A ray, LI, emitted from a distant object (fig. 505), is received on the prism, which is turned in order to obtain the minimum deviation EDL′. By means of a telescope with a graduated circle the angle EDL′ is read off, which the refracted ray DE makes with the ray DL′, coming directly from the object; now this is the angle of minimum deviation, assuming that the object is so distant that the two rays LI and L′D are approximately parallel. These values then only need to be substituted in the equation (3) to give the value of n.

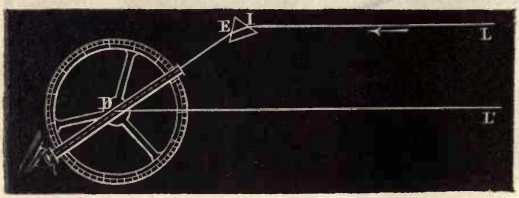


Fig. 505.

549. **Measurement of the refractive index of liquids.**—Biot applied Newton's method to determining the refractive index of liquids. For this purpose a cylindrical cavity O, of about 0·75 in diameter, is perforated in a glass prism, PQ (fig. 506), from the incident face to the face of emergence. This cavity is closed by two plates of thin glass which are cemented on the sides of this prism. Liquids are introduced through a small stoppered aperture, B. The refracting angle and the minimum deviation of the liquid prism in the cavity O having been determined, their values are introduced into the formula (3), which gives the index.

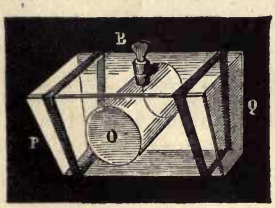


Fig. 506.

550. **Measurement of the refractive index of gases.**—A method for this purpose, founded on that of Newton, was devised by Biot and Arago.

The apparatus which they used consists of a glass tube (fig. 507), bevelled at its two ends, and closed by glass plates, which are at an angle of 143°. This tube is connected with a bell-jar, H, in which there is a siphon barometer, and with a stopcock by means of which the apparatus can be exhausted, and different gases introduced. After having exhausted the tube AB, a ray of light, SA, is transmitted, which is bent away from the normal through an angle $r-i$ at the first incidence, and towards it through an angle $i'-r'$ at the second. These two deviations being added, the total deviation d is $r-i+i'-r'$. In the case of a minimum deviation, $i=r'$ and $r=i'$, whence $d=\mathrm{A}-2i$, since $r+i=\mathrm{A}$ (547). The index from vacuum to air, which is evidently $\frac{\sin r}{\sin i}$, has therefore the value

$$\frac{\sin\frac{\mathrm{A}}{2}}{\sin\left(\frac{\mathrm{A}-d}{2}\right)} \quad . \quad . \quad . \quad (4)$$

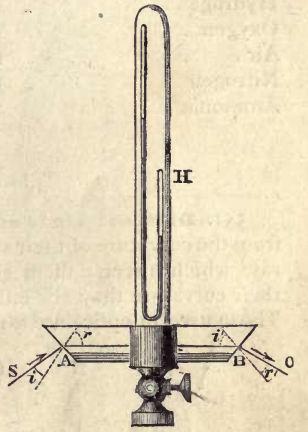


Fig. 507.

Hence, in order to deduce the refractive index n from vacuum into air, which is the *absolute index* or *principal index*, it is merely necessary to know the refracting angle A, and the angle of minimum deviation d. To obtain the absolute index of any other gas, after having produced a vacuum, this gas is introduced; the angles A and d having been measured, the above formula gives the index of refraction from gas to air. Dividing the index of refraction from vacuum to air by the index of refraction from the gas to air, we obtain the index of refraction from vacuum to the gas; that is, its absolute index.

The square of the refractive index of a substance, less unity, that is n^2-1, measures what is called the *refractive action*. On the undulatory theory n^2 is the density of the ether in the medium, when 1 is the density of the ether in a vacuum. The refractive action is therefore a measure of the excess of the density of the ether in the refracting medium. The refractive action divided by the density or $\frac{n^2-1}{d}$ is called the *absolute refractive power*.

Table of refractive indices

Diamond	2·470 to 2·750	Iceland spar, ordinary ray	1·654
Rutile	2·616	Plate glass, St. Gobin	1·587
Phosphorus	2·224	Rock crystal	1·548
Sulphur	2·215	Rock salt	1·545
Arsenic bromide	1·781	Turpentine	1·471
Ruby	1·779	Alcohol	1·363
Flint glass	1·702	Albumen	1·360
Bisulphide of carbon	1·678	Ether	1·358

Crystalline lens	1·384	Vitreous lens	1·339
Iceland spar, extraordinary ray	1·483	Aqueous „	1·357
Crown glass	1·608	Water	1·336
Oil of cassia	1·600	Ice	1·310

Refractive indices of gases

Vacuum	1·000000	Carbonic acid	1·000449
Hydrogen	1·000138	Hydrochloric acid	1·000449
Oxygen	1·000272	Nitrous oxide	1·000503
Air	1·000294	Sulphurous acid	1·000665
Nitrogen	1·000300	Olefiant gas	1·000678
Ammonia	1·000385	Chlorine	1·000772

LENSES. THEIR EFFECTS

551. **Different kinds of lenses.**—*Lenses* are transparent media which, from the curvature of their surfaces, have the property of causing the luminous rays which traverse them either to converge or to diverge. According to their curvature they are either *spherical*, *cylindrical*, *elliptical*, or *parabolic*. Those used in optics are always exclusively spherical. They are commonly

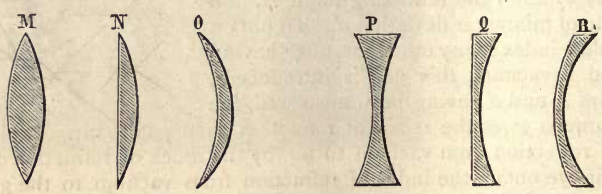


Fig. 508.

made either of *crown glass*, which is free from lead, or of *flint glass*, which contains lead, and is more refractive than crown glass.

The combination of spherical surfaces, either with each other or with plane surfaces, gives rise to six kinds of lenses, sections of which are represented in fig. 508; four are formed by two spherical surfaces, and two by a plane and a spherical surface.

M is a *double convex*, N is a *plano-convex*, O is a *converging concavo-convex*, P is a *double concave*, Q is a *plano-concave*, and R is a *diverging concavo-convex*. The lenses O and R are also called *meniscus* lenses, from their resemblance to the crescent-shaped moon.

The first three, which are thicker at the centre than at the borders, are *converging*; the others, which are thinner in the centre, are *diverging*. In the first group the double convex lens only need be considered, and in the second the double concave, as the properties of each of these lenses apply to all those of the same group.

In lenses whose two surfaces are spherical, the centres for these surfaces are called *centres of curvature*, and the right line which passes through

these two centres is the *principal axis*. In a plano-concave or plano-convex lens the principal axis is the perpendicular let fall from the centre of curvature of the spherical face on the plane face.

In order to compare the path of a luminous ray in a lens with that in a prism, the same hypothesis is made as for curved mirrors (525); that is, the surfaces of these lenses are supposed to be formed of an infinity of small plane surfaces or elements (fig. 509): the *normal* at any point is then the perpendicular to the plane of the corresponding element. It is a geometrical principle that all the normals to the same spherical surface pass through its centre. On the above hypothesis we can always conceive two plane surfaces at the points of incidence and emergence, which are inclined to each other, and thus produce the effect of a prism. Pursuing this comparison, the three lenses M, N, and O may be compared to a succession of prisms having their summits outwards, and the lenses P, Q, and R to a series having their summits inwards: from this we see that the first ought to condense the rays, and the latter to disperse them, for we have already seen that when a luminous ray traverses a prism it is deflected towards the base (544).

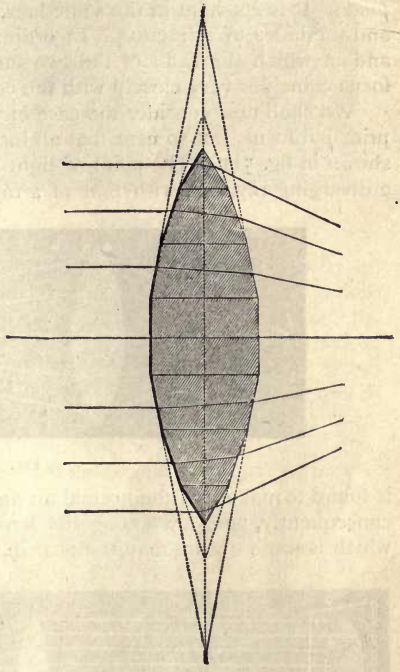


Fig. 509.

552. **Foci in double convex lenses.**—The focus of a lens is the point where the refracted rays, or their prolongations, meet. Double convex lenses have both real and virtual foci, like concave mirrors.

Real foci.—We shall first consider the case in which the luminous rays which fall on the lens are parallel to its principal axis, as shown in fig. 510. In this case, any incident ray, LB, in approaching the normal of the point of incidence B, and in diverging from it at the point of emergence D, is twice refracted towards the axis, which it cuts at F. As all rays parallel to the axis are refracted in the same manner,

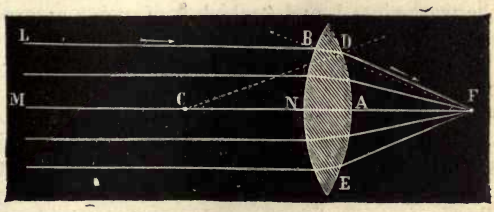


Fig. 510.

it can be shown by calculation that they all pass very nearly through the point F, so long as the arc DE does not exceed 10° to 12°. This point is called the *principal focus*, and the distance FA is the *principal focal distance*. It is constant in the same lens, but varies with the radii of curvature and the index of refraction. In ordinary lenses, which are of crown glass, and in which the radii of the two surfaces are nearly equal, the principal focus coincides very closely with the centre of curvature.

We shall now consider the case in which the point of light is outside the principal focus, but so near that all incident rays form a divergent pencil, as shown in fig. 511. The point of light being at L, by comparing the path of a diverging ray, LB, with that of a ray, SB, parallel to the axis, the former

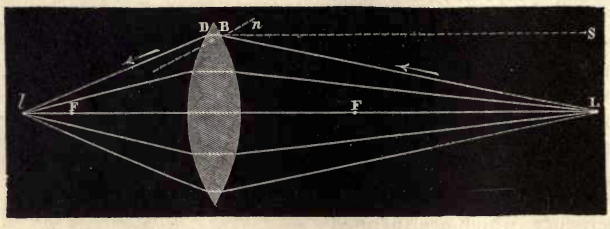


Fig. 511.

is found to make with the normal an angle, LB*n*, greater than the angle SB*n*; consequently, after traversing the lens, the ray cuts the axis at a point, *l*, which is more distant than the principal focus F. As all rays from the point L intersect approximately in the same point *l*, this latter is the *conjugate focus* of the point L; this term has the same meaning here as in the case of mirrors, and expresses the relation existing between the two points L and *l*, which is of such a nature that, if the luminous point is moved to *l*, the focus passes to L.

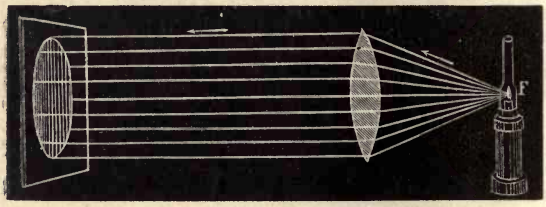


Fig. 512.

According as the point of light comes nearer the lens, the convergence of the emergent rays decreases, and the focus *l* becomes more distant; when the point L coincides with the principal focus, the emergent rays on the other side are parallel to the axis, and there is no focus, or, what is the same thing, it is infinitely distant. As the refracted rays are parallel in this case, the intensity of light only decreases slowly, and a simple lamp can illuminate great distances. It is merely necessary to place it in the focus of a double convex lens, as shown in fig. 513.

Virtual foci.—A double convex lens has a virtual focus when the luminous object is placed between the lens and the principal focus, as shown in fig. 513. In this case the incident rays make with the normal greater angles than those

made with the rays FI from the principal focus; hence, when the former rays emerge, they move farther from the axis than the latter, and form a diverging pencil, HK, GM. These rays cannot produce a real focus, but their prolongations intersect in some point, *l*, on the axis, and this point is the virtual focus of the point L (514).

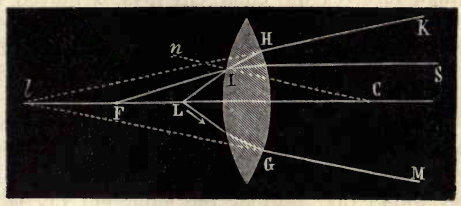


Fig. 513.

553. **Foci in double concave lenses.**—In double concave lenses there are only virtual foci, whatever the distance of the object. Let SS′ be any pencil of rays parallel to the axis (fig. 514); any ray SI is refracted at the point of incidence I, and approaches the normal CI. At the point of emergence it is also refracted, but diverges from the normal GC′, so that it is twice refracted in a direction which moves it from the axis CC′. As the same thing takes place for every other ray, S′KMN, it follows that the rays, after traversing the lens, form a diverging pencil, GHMN. Hence there is no real focus, but the prolongations of these rays cut one another in a point F, which is the principal virtual focus.

In the case in which the rays proceed from a point, L (fig. 515), on the

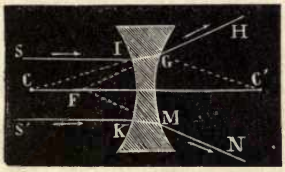


Fig. 514.

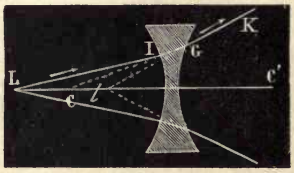


Fig. 515.

axis, it is found by the same construction that a virtual focus is formed at *l* which is between the principal focus and the lens.

554. **Experimental determination of the principal focus of lenses.**—To determine the principal focus of a convex lens, it may be exposed to the sun's rays so that they are parallel to its axis. The emergent pencil being received on a ground glass screen, the point to which the rays converge is readily seen; it is the principal focus.

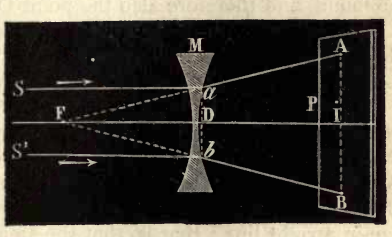


Fig. 516.

Or an image of an object is formed on a screen, their respective distances from which are then measured, and from these distances the focus is calculated from the dioptric formula (561).

With a double concave lens, the face *ab* (fig. 516) is covered with an opaque substance, such as lampblack, two small apertures *a* and *b* being left

in the same principal section, and at an equal distance from the axis; a pencil of sunlight is then received on the other face, and the screen P, which receives the emergent rays, is moved nearer to or farther from the lens, until A and B, the spots of light from the small apertures *a* and *b*, are distant from each other by twice *ab*. The distance DI is then equal to the focal distance FD, because the triangles F*ab* and FAB are similar. Another method of determining the focus of a concave lens is given in article 560.

555. **Optical centre, secondary axis.**—In or near every lens there is a point called the *optical centre*, which is situate on the axis, and which has the property that any luminous ray passing through it experiences no angular deviation; that is, that the emergent ray is parallel to the incident ray. The existence of this point may be demonstrated in the following manner:—Let two parallel radii of curvature, CA and C′A′ (fig. 517), be drawn to the

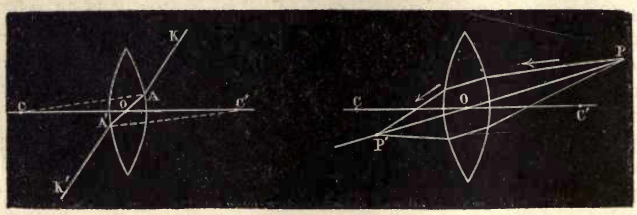


Fig. 517.

Fig. 518.

two surfaces of a double convex lens. Since the two plane elements of the lens A and A′ are parallel, as being perpendicular to two parallel right lines, it will be granted that the refracted ray AA′ is propagated in a medium with parallel faces. Hence a ray KA, which reaches A at such an inclination that after refraction it takes the direction AA′, will emerge parallel to its first direction (542); the point O, at which the right line cuts the axis, is therefore the optical centre. The position of this point may be determined for the case in which the curvature of the two faces is the same, which is the usual condition, by observing that the triangles COA and C′OA′ are equal, and therefore that OC = OC′, which gives the point O. If the curvatures are unequal, the triangles COA and COA′ are similar, and either CO or C′O may be found, and therefore also the point O.

In double concave or concavo-convex lenses the optical centre may be determined by the same construction. In lenses with a plane face this point is at the intersection of the axis by the curved face.

Every right line PP′ (fig. 518), which passes through the optical centre without passing through the centres of curvature, is a *secondary axis*. From this property of the optical centre, every secondary axis represents a luminous rectilinear ray passing through this point: for, from the slight thickness of the lenses, it may be assumed that rays passing through the optical centre are in a right line; that is, that the small deviation may be neglected which rays experience in traversing a medium with parallel faces (fig. 517).

So long as the secondary axes only make a small angle with the principal axis, all that has hitherto been said about the principal axis is applicable to them; that is, that rays emitted from a point P (fig. 518) on the secondary

axis PP′ nearly converge to a certain point of the axis P′, and according as the distance from the point P to the lens is greater or less than the principal focal distance, the focus thus formed will be conjugate or virtual. This principle is the basis of what follows as to the formation of images.

556. **Formation of images by double convex lenses.**—In lenses, as well as in mirrors, the image of an object is the collection of the foci of its several points ; hence the images furnished by lenses are real or virtual in the same case as the foci, and their construction resolves itself into determining the position of a series of points, as was the case with mirrors (528).

i. *Real image.*—Let AB (fig. 519) be placed beyond the principal focus. If a secondary axis, A*a*, be drawn from the outside point A, any ray AC, from this point, will be twice refracted at C and D, and both times in the same direction approaching the secondary axis, which it cuts at *a*. From what has been said in the last paragraph, the other rays from the point A will intersect in the point *a*, which is accordingly the conjugate focus of the point A. If the secondary axis be drawn from the point B, it will be seen, in like manner, that the rays from this point intersect in the point *b* ; and as the points between A and B have their foci between *a* and *b*, a *real* but *inverted* image of AB will be formed at *ab*. To see this image, it may be received on a white screen, on which it will be depicted, or the eye may be placed in the path of the rays emerging from it.

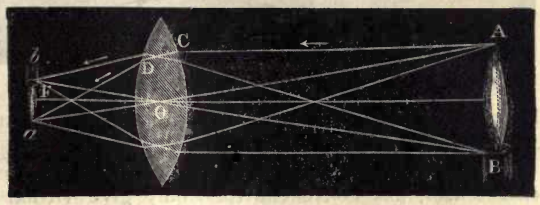


Fig. 519.

Conversely, if *ab* were the luminous or illuminated object, its image would be formed at AB. Two consequences important for the theory of optical instruments follow from this : that, 1st, *if an object, even a very large one, is at a sufficient distance from a double convex lens, the real and inverted image which is obtained of it is very small—it is near the principal focus, but somewhat farther from the lens than this is* ; 2nd, *if a very small object be placed near the principal focus, but a little in front of it, the image which is formed is at a great distance—it is much larger, and that in proportion as the object is near the principal focus.* In all cases the object and the image are in the same proportion as their distances from the lens.

These two principles are experimentally confirmed by receiving on a screen the image of a lighted candle, placed successively at various distances from a double convex lens.

ii. *Virtual image.*—There is another case in which the object AB (fig. 520) is placed between the lens and its principal focus. If a secondary axis O*a* be drawn from the point A, every ray AC, after having been twice refracted, diverges from this axis on emerging, since the point A is at a less distance than the principal focal distance (552). This ray, continued in an opposite direction, will cut the axis O*a* in the point *a*, which is the virtual focus of the

point A. Tracing the secondary axis of the point B, it will be found, in the same manner, that the virtual focus of this point is formed at *b*. There is, therefore, an image of AB at *ab*. *This is a virtual image; it is erect, and larger than the object.*

The magnifying power is greater in proportion as the lens is more convex, and the object nearer the principal focus. We shall presently show how the magnifying power may be calculated by means of the formulæ relating to lenses (561). Double convex lenses, used in this manner as magnifying glasses, are called *simple microscopes*.

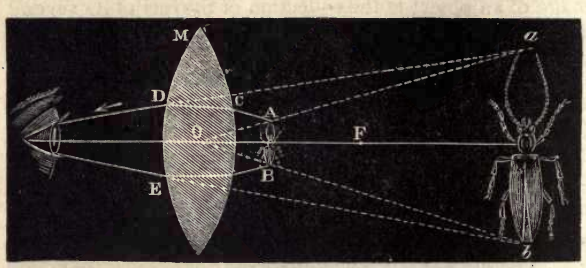


Fig. 520.

557. **Formation of images in double concave lenses.**—Double concave lenses, like convex mirrors, only give virtual images, whatever the distance of the object.

Let AB (fig. 521) be an object placed in front of such a lens. If the secondary axis AO be drawn from the point A, all rays, AC, AI, from this point are twice refracted in the same direction, diverging from the axis AO; so that the eye, receiving the emergent rays DE and GH, supposes them to proceed from the point where their prolongations cut the secondary axis AO in the point *a*. In like manner, drawing a secondary axis from the point B, the rays from this point form a pencil of divergent rays, the directions of which, prolonged, intersect in *b*. Hence the eye sees at *ab* a virtual image of AB, *which is always erect, and smaller than the object.*

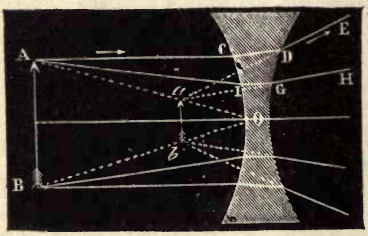


Fig. 521.

558. **Spherical aberration. Caustics.**—In speaking about foci, and about the images formed by different kinds of spherical lenses, it has been hitherto assumed that the rays emitted from a single point intersect also after refraction in a single point. This is virtually the case with a lens whose *aperture*—that is, the angle obtained by joining the edges to the principal focus—does not exceed 10° or 12°.

Where, however, the aperture is larger, the rays which traverse the lens near the edge are refracted to a point F nearer the lens than the point G, which is the focus of the rays which pass near the axis. The phenomenon thus produced is named *spherical aberration by refraction*; it is analogous to the spherical aberration produced by reflection (533). The luminous sur-

faces formed by the intersection of the refracted rays are termed *caustics by refraction.*

Spherical aberration is prejudicial to the sharpness and definition of an image. If a ground glass screen be placed exactly in the focus of a lens, the image of an object will be sharply defined in the centre, but indistinct at the edges; and, *vice versâ*, if the image is sharp at the edges, it will be indistinct in the centre. This defect is very objectionable, more especially in lenses used for photography. It is partially obviated by placing in front of the lenses diaphragms provided with a central aperture, called *stops*, which admit the rays passing near the centre, but cut off those which pass near the edges. The image thereby becomes sharper and more distinct, though the illumination is less.

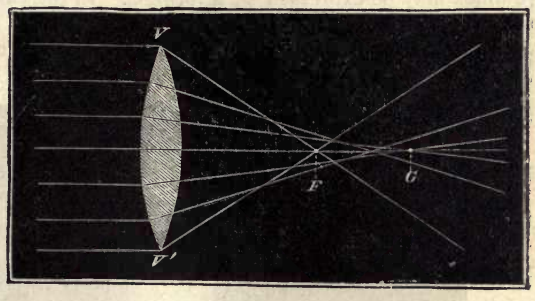


Fig. 522.

If a screen be held between the light and an ordinary double convex lens which quite covers the lens, but has two concentric series of holes, two images are obtained, and may be received on a sheet of paper. By closing one or the other series of holes by a flat paper ring, it can be easily ascertained which image arises from the central, and which from the marginal rays. When the paper is at a small distance the marginal rays produce the image in a point, and the central ones in a ring; the former are converged to a point, and the latter not. At a somewhat greater distance the marginal rays produce a ring, and the central ones a point. It is thus shown that the focus of the marginal rays is nearer the lens than that of the central rays.

Mathematical investigation shows that convex lenses whose radii of curvature stand in the ratio expressed by the formula

$$\frac{r}{r_1} = \frac{4 - 2n^2 + n}{2n^2 + n}$$

are most free from spherical aberration, and are called *lenses of best form:* in this formula r is the radius of curvature of the face turned to the parallel rays, and r_1 that of the other face, while n is the refractive index. Thus, with a glass whose refractive index is $\frac{3}{2}$, $r_1 = 6r$. Spherical aberration is also destroyed by substituting for a lens of short focus two lenses of double focal length, which are placed at a little distance apart. Greater length of focus has the result that for the same diameter the aperture and also the aberration are less; and as it is not necessary to stop a great part of the lens there is a gain in luminosity, which is not purchased by indistinctness of the images, while the combination of the two lenses has the same focus.

as that of the single lens (560). Lenses which are free from spherical aberration are called *aplanatic.*

559. **Formulæ relating to lenses.**—In all thin lenses the relations between the distances of the image and object, the radii of curvature, and the refractive index may be expressed by a formula. In the case of a double convex lens, let P be a luminous point situate on the axis (fig. 523), let PI be an incident ray, IE its direction within the lens, EP′ the emergent ray, so that P′ is the conjugate focus of P. Further, let C′I and CE be the normals to the

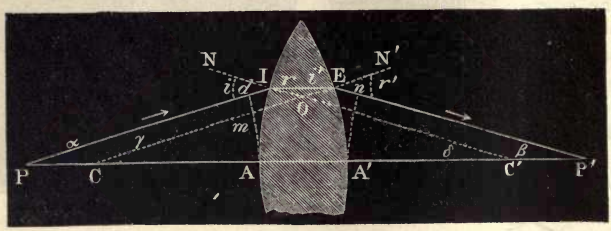


Fig. 523.

points of incidence and emergence, and IPA be put equal to a, EP′A′ = β, ECA′ = γ, IC′A = δ, NIP = i, EIO = r, IEO = i', N′EP′ = r'.

Because the angle i is the exterior angle of the triangle PIC′, and the angle r' the exterior angle of the triangle CEP′, therefore $i = a + \delta$, and $r' = \gamma + \beta$, whence

$$i + r' = a + \beta + \gamma + \delta \quad . \quad . \quad . \quad . \quad . \quad (1)$$

But at the point I, sin $i = n$ sin r, and at the point E, sin $r' = n$ sin i (538), n being the refractive index of the lens. Now if the arc AI is only a small number of degrees, these sines may be considered as proportional to the angles i, r, i' and r'; whence, in the above formula, we may replace the sines by their angles, which gives $i = nr$ and $r' = ni'$, from which $i + r' = n\ (r + i')$. Further, because the two triangles IOE and COC′ have a common equal angle O, therefore $r + i' = \gamma + \delta$, from which $i + r' = n\ (\gamma + \delta)$. Introducing this value into the equation (1) we obtain

$$n\ (\gamma + \delta) = a + \beta + \gamma + \delta, \text{ from which } (n - 1)\ (\gamma + \delta) = a + \beta \quad . \quad (2)$$

Let CA′ be denoted by R, C′A by R′, PA by p, and P′A′ by p'. Then with centre P and radius PA describe the arc Ad, and with centre P′ and radius P′A′ describe the arc A′n. Now when an angle at the centre of a circle subtends a certain arc of the circumference, the quotient of the arc divided by the radius measures the angle; consequently

$$a = \frac{Aa}{AP} \text{ or } \frac{Ad}{p}, \beta = \frac{A'n}{p'}, \gamma = \frac{A'E}{R}, \text{ and } \delta = \frac{AI}{R'}.$$

Therefore by substitution in (2), $(n-1)\left(\frac{A'E}{R} + \frac{AI}{R'}\right) = \frac{Ad}{p} + \frac{A'n}{p'}$.

Now since the thickness of the lens is very small, the angles are also small, and Ad, AI, A′E, A′n differ but little from coincident straight lines, and are therefore virtually equal. Hence the above equation becomes

$$(n-1)\left(\frac{I}{R} + \frac{I}{R'}\right) = \frac{I}{p} + \frac{I}{p'} \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

This is the formula for double convex lenses; if p be $=\infty$—that is, if the rays are parallel—we have

$$(n-1)\left(\frac{1}{R}+\frac{1}{R'}\right)=\frac{1}{p'},$$

p' being the principal focal distance. Calling this f, we get

$$(n-1)\left(\frac{1}{R}+\frac{1}{R'}\right)=\frac{1}{f} \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

from which the value of f is easily deduced. Considered in reference to equation (4), the equation (3) assumes the form

$$\frac{1}{p}+\frac{1}{p'}=\frac{1}{f} \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

which is that in which it is usually employed. When the image is virtual, p' changes its sign, and formula (5) takes the form

$$\frac{1}{p}-\frac{1}{p'}=\frac{1}{f} \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

In double concave lenses p' and f retain the same sign, but that of p changes; equation (5) then becomes

$$-\frac{1}{p}-\frac{1}{p'}=-\frac{1}{f} \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Equation (7) may be obtained by the same reasoning as the other.

560. **Combination of lenses.**—If parallel rays fall on a convex lens A, which has the focal distance f, and then on a similar lens B with the focal distance f', at a distance d from A, the distance from the lens B at which the image is formed is F, then

$$\frac{1}{F}=\frac{1}{f'}+\frac{1}{f-d}$$

If the lenses are close together, so that $d=0$, then

$$\frac{1}{F}=\frac{1}{f}+\frac{1}{f'}.$$

If the lenses have the same curvature, that is $f=f'$, then $\frac{1}{F}=\frac{2}{f}$; that is to say, the focal distance of the combination is half that of a single lens.

If the second lens is a dispersing one of the focal distance f', then $\frac{1}{F}=\frac{1}{f-d}-\frac{1}{f'}$; and if the lenses are close together, then $\frac{1}{F}=\frac{1}{f}-\frac{1}{f'}$.

This formula can be used to determine the focal distance of a concave lens, by combining it with a convex lens of longer focus, and then determining the focal distance of the combination.

561. **Relative magnitudes of image and object. Determination of focus.**—From the similarity of the triangles AOB, aOb (fig. 520), we get for the relative magnitudes of image and object the proportion $\frac{AB}{ab}=\frac{p}{p'}$; whence $\frac{I}{O}=\frac{p'}{p}$, where AB = O is the magnitude of the object, and $ab=I$

that of the image; while p and p' are their respective distances from the lens. Replacing p' by its value from the equation $\frac{1}{p}+\frac{1}{p'}=\frac{1}{f}$ where the image is real, or from the equation $\frac{1}{p}-\frac{1}{p'}=\frac{1}{f}$ where it is virtual, we shall obtain the different values of the ratio $\frac{I}{O}$ for various positions of the object.

In the first case we have $$\frac{I}{O}=\frac{f}{p-f}.$$

Thus if
$$p>2f \quad I>O$$
$$p=2f \quad I=O$$
$$p<2f \quad I>O$$

In the second case when the image is virtual we shall have

$$\frac{I}{O}=\frac{f}{f-p}, \text{ so that in all cases } I<O.$$

By using the above formula we may easily deduce the focal length of a convex lens where direct sunlight is not available. For if it be placed in front of a scale, and if a screen be placed on the other side, then by altering the relative positions of the lens and the screen, a position may be found by trial, such that an image of the object is formed on the screen of exactly the same size. Dividing now by 4 the total distance between the object and the screen, we get the focal distance of the lens.

Another method is to place on one side of the lens, and a little beyond its principal focus, a brightly illuminated scale, which is best of glass, on which a strong light falls; on the other side a screen is placed at such a distance as to produce a greatly magnified distinct image of the scale. Then if l and L are the lengths of the scale and its image respectively, and d the distance of the screen from the lens,

$$f=d\frac{f}{l+L}$$

562. **Determination of the refractive index of a liquid.**—By measurements of focal distance the refractive index of a liquid (549) may be ascertained in cases in which only small quantities of liquid are available. One face of a double convex lens of known focal distance f, and known curvature r, is pressed against a drop of the liquid in question on a plane of plate glass (fig. 524). The liquid forms thereby a plano-concave lens whose radius of curvature is r. The focal distance F of the whole system is then determined experimentally; this gives the focal length of the liquid lens f' from the formula

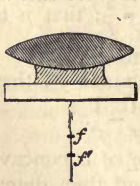


Fig. 524.

$$\frac{I}{F}=\frac{I}{f}-\frac{I}{f'},$$

while from the formula $\frac{I}{f'}=(n-1)\frac{I}{r}$ we get the value of n.

563. **Laryngoscope.**—As an application of lenses may be adduced the *laryngoscope*, which is an instrument invented to facilitate the investigation

of the larynx and other cavities of the mouth. It consists of a plano-convex lens L, and a concave reflector M, both fixed to a ring which can be adjusted to any convenient lamp (fig. 525). The flame of a lamp is in the principal focus of the lens, and at the same time is at the centre of curvature of the reflector. Hence the divergent pencil proceeding from the lamp to

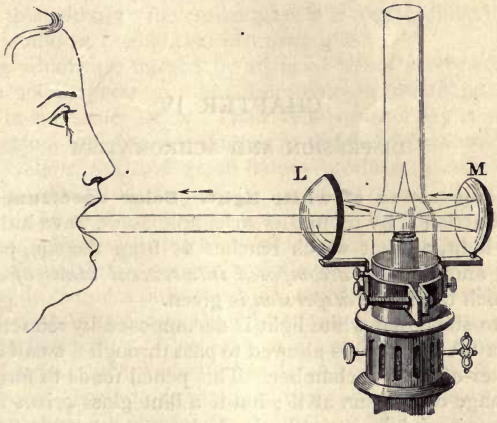


Fig. 525.

the lens is changed after emerging into a parallel pencil. Moreover, the pencil from the lamp, impinging upon the mirror, is reflected to the focus of the lens, and traverses the lens, forming a second parallel pencil which is superposed on the first. This being directed into the mouth of a patient, its condition may be readily observed.

CHAPTER IV

DISPERSION AND ACHROMATISM

564. **Decomposition of white light. Solar spectrum.**—The phenomenon of refraction is by no means so simple as we have hitherto assumed. When *white* light, or that which reaches us from the sun, passes from one medium into another, *it is decomposed into several kinds of light*, a phenomenon to which the name *dispersion* is given.

In order to show that white light is decomposed by refraction, a pencil of the sun's rays SA (fig. 526) is allowed to pass through a small aperture in the window shutter of a dark chamber. This pencil tends to form a round and colourless image of the sun at K; but if a flint glass prism arranged horizontally be interposed in its path, the beam, on emerging from the prism, becomes refracted towards its base, and produces on a distant screen a vertical band rounded at the ends, coloured in all the tints of the rainbow, which is called the *solar spectrum* (see Plate I.). In this spectrum there is, in reality, an infinity of different tints, which imperceptibly merge into each other, but it is customary to distinguish seven principal colours. These are *violet*, *indigo*, *blue*, *green*, *yellow*, *orange*, *red*; they are arranged in this order in the spectrum, the violet being the most refrangible, and the red the least so. They do not all occupy an equal extent in the spectrum, violet having the greatest extent, and orange the least.

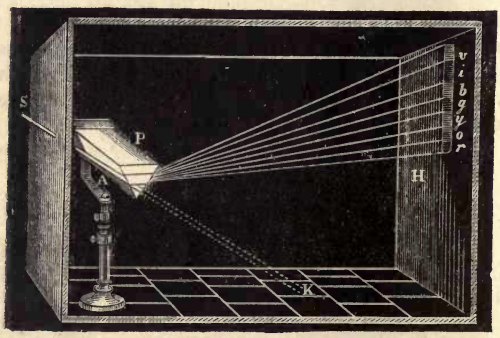


Fig. 526.

With transparent prisms of different substances, or with hollow glass prisms filled with various liquids, spectra are obtained formed of the same colours, and in the same order; but when the deviation produced is the same, the length of the spectrum varies with the substance of which the prism is made. The angle of separation of two selected rays (say in the red and the violet) produced by a prism is called the *dispersion*, and the ratio of

this angle to the mean deviation of the two rays is called the *dispersive power*. This ratio is constant for the same substance so long as the refracting angle of the prism is small. For the deviation of the two rays is proportional to the refracting angle; their difference and their mean vary in the same manner, and therefore the ratio of their difference to their mean is constant. For flint glass this is 0·043; for crown glass it is 0·0246, since the dispersive power of flint is almost double that of crown glass.

The spectra which are formed by artificial lights rarely contain all the colours of the solar spectrum; but their colours are found in the solar spectrum, and in the same order. Their relative intensity is also modified. The shade of colour which predominates in the flame predominates also in the spectrum; yellow, red, and green flames produce spectra in which the dominant tint is yellow, red, or green.

565. **Production of a pure solar spectrum.**—In the above experiment, when the light is admitted through a wide slit, the spectrum formed is built up of a series of overlapping spectra, and the colours are confused and indistinct. In order to obtain a pure spectrum, the slit, in the shutter of the dark room through which light enters, should be from 15 to 25 mm. in height and from 1 to 2 mm. in breadth. The sun's rays are directed upon the slit by a mirror, or still better by a heliostat (534). An achromatic double convex lens is placed at a distance from the slit of double its own focal length, which should be about a metre, and a screen is placed at the same distance from the lens. An image of the slit of exactly the same size is thus formed on the screen (561). If now there is placed near the lens, between it and the screen, a prism with an angle of about 60°, and with its refracting edge parallel to the slit, a very beautiful, sharp, and pure spectrum is formed on the screen. The prism should be free from striæ, and should be placed so that it produces the minimum deviation.

566. **The colours of the spectrum are simple, and unequally refrangible.**—If one of the colours of the spectrum be isolated by intercepting the others by means of a screen E, as shown in fig. 527, and if the light thus isolated be allowed to pass through a second prism, B, a refraction will be observed, but the light remains unchanged; that is, the image received on the screen H is violet if the violet pencil has been allowed to pass, blue if the blue pencil, and so on. Hence the colours of the spectrum are *simple*; that is, they cannot be further decomposed by the prism.

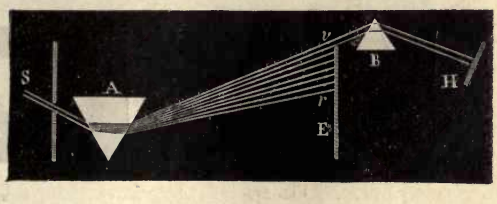


Fig. 527

Moreover, the colours of the spectrum are unequally refrangible; that is, they possess different refractive indices. The elongated shape of the spectrum would be sufficient to prove the unequal refrangibility of the simple colours, for it is clear that the violet, which is most deflected towards the base of the prism, is also most refrangible; and that red, which is least deflected, is least refrangible. But the unequal refrangibility of simple colours

may be shown by numerous experiments, of which the two following may be adduced :—

i. Two narrow strips of coloured paper, one red and the other violet, are fastened close to each other on a sheet of black paper. On looking at them through a prism, they are seen to be unequally displaced, the red band to a less extent than the violet ; hence the red rays are less refrangible than the violet.

ii. The same conclusion may be drawn from Newton's experiment with crossed prisms. On a prism A (fig. 528), in a horizontal position, a pencil

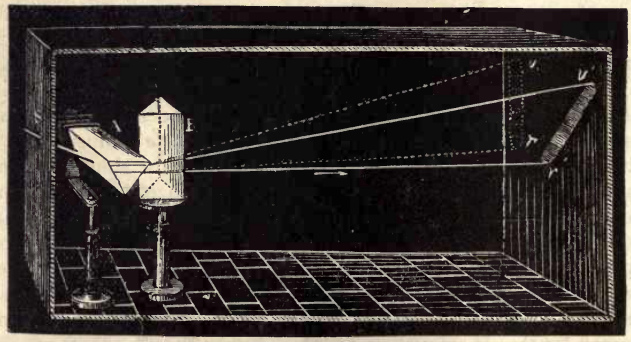


Fig. 528.

of white light, S, is received, which, if it had merely traversed the prism A, would form the spectrum *rv*, on a distant screen. But if a second prism, B, be placed in a vertical position behind the first, in such a manner that the refracted pencil passes through it, the spectrum *rv* becomes deflected towards the base of the vertical prism ; but, instead of being deflected in a direction parallel to itself, as would be the case if the colours of the spectrum were

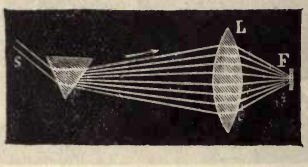


Fig. 529.

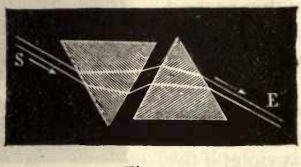


Fig. 530.

equally refracted, it is obliquely refracted in the direction *r'v'*, proving that from red to violet the colours are more and more refrangible.

These different experiments show that the refractive index differs in different colours ; even rays which are to perception undistinguishable have not the same refractive index. In the red band, for instance, the rays at the extremity of the spectrum are less refracted than those which are nearer the orange zone. In determining indices of refraction (538), it is usual to take, as the index of any particular substance, the refrangibility of the yellow ray in a prism formed of that substance.

567. **Recomposition of white light.**—Not merely can white light be resolved into lights of various colours, but by combining the different pencils separated by the prism white light can be reproduced. This may be effected in various ways.

i. If the spectrum produced by one prism be allowed to fall upon a second prism of the same material and the same refracting angle as the first, but inverted, as shown in fig. 530, the latter reunites the different colours of the spectrum, and it is seen that the emergent pencil E, which is parallel to the pencil S, is colourless.

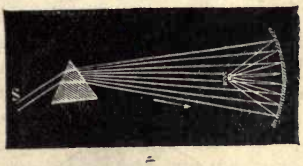


Fig. 531.

ii. If the spectrum falls upon a double convex lens (fig. 529), a white image of the sun will be formed on a white screen placed in the focus of the lens ; a glass globe filled with water produces the same effect as the lens.

iii. When the spectrum falls upon a concave mirror, a white image is formed on a screen of ground glass placed in its focus (fig. 531).

iv. Light may be recomposed by means of a pretty experiment, which consists in receiving the seven colours of the spectrum on seven small glass mirrors with plane faces, and which can be so inclined in all positions that the reflected light may be transmitted in any given direction (fig. 532).

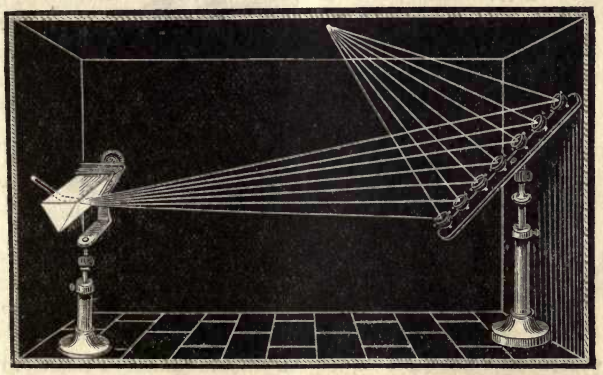


Fig. 532.

When these mirrors are suitably arranged, the seven reflected pencils may be caused to fall on the ceiling, in such a manner as to form seven distinct images—red, orange, yellow, &c. When the mirrors are moved so that the separate images become superposed, a single image is obtained, which is white.

v. By means of *Newton's disc* (fig. 533) it may be shown that the seven colours of the spectrum form white. This is a cardboard disc of about a foot in diameter ; the centre and the edges are covered with black paper, while in the space between there are pasted strips of paper of the colours of the spectrum. They proceed from the centre to the circumference, and their

relative dimensions and tints are such as to represent five spectra (fig. 534). When this disc is rapidly rotated, the effect is the same as if the retina received simultaneously the impression of the seven colours.

vi. If by a mechanical arrangement a prism, on which the sun's light falls, is made to oscillate rapidly, so that the spectrum also oscillates, the middle of the spectrum appears white.

These latter phenomena depend on the physiological fact that sensation always lasts a little longer than the impression from which it results (625). If a new impression is allowed to act, before the sensation arising from the former one has ceased, a sensation is obtained consisting of two impressions. And by choosing the time short enough, three, four, or more impressions may be mixed with each other. With a rapid rotation the disc (fig. 533)

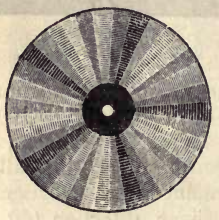


Fig. 534.

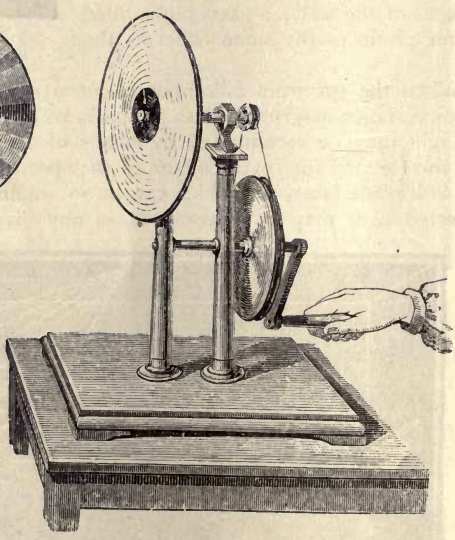


Fig. 533.

is nearly white. It is not quite so, for the colours cannot be exactly arranged in the same proportions as those in which they exist in the spectrum, and moreover *pigment* colours are not pure (571).

568. **Newton's theory of the composition of light.**—Newton was the first to decompose white light by the prism, and to recompose it. From the various experiments which we have described, he concluded that white light was not homogeneous, but formed of seven lights unequally refrangible, which he called *simple* or *primitive* lights. Owing to the difference in refrangibility they become separated in traversing the prism.

The designation of the various colours of the spectrum is to a very great extent arbitrary ; for, in strict accuracy, the spectrum is made up of an infinite number of simple colours, which pass into one another by imperceptible graduations of colour and refrangibility.

569. **Colour of bodies.**—The natural colour of bodies results from the fact that one portion of the coloured rays contained in white light is absorbed at the surface of the body. If the unabsorbed portion traverses the body, it is coloured and transparent; if, on the contrary, it is reflected, it is coloured and opaque. In both cases the colour results from the constituents which have not been absorbed. Those which reflect or transmit all colours in the proportion in which they exist in the spectrum are white; those which reflect or transmit none are black. Between these two limits there are infinite tints according to the greater or less extent to which bodies reflect or transmit some colours and absorb others. Thus a body appears yellow because it absorbs all colours with the exception of yellow. In like manner, a solution of ammoniacal sulphate of copper absorbs preferably the red and yellow rays, transmits the blue rays almost completely, the green and violet less so; hence the light seen through it is blue.

Accordingly bodies have no colour of their own; the colour of the body changes with the nature of the incident light. Thus, if a white body in a dark room be successively illuminated by each of the colours of the spectrum, it has no special colour, but appears red, orange, green, &c., according to the position in which it is placed. If homogeneous light falls upon a body, it appears brighter in the colour of this light if it does not absorb this colour; but black if it does absorb it. In the light of a lamp fed by spirit in which some common salt is dissolved, everything white and yellow seems bright, while other colours, such as vermilion, ultramarine, and malachite, are black. This is well seen in the case of a stick of red sealing-wax viewed in such a light. In the light of lamps and of candles, which from the want of blue rays appear yellow, yellow and white appear the same, and blue seems like green. In bright twilight or in moonshine the light of gas has a reddish tint.

570. **Mixed colours. Complementary colours.**—By mixed colours we understand the impression of colour which results from the coincident action of two or more colours on the same position of the retina. This new impression is single; it cannot be resolved into its components; in this respect it differs from a complex sound, in which the ear, by practice, can learn to distinguish the constituents. Mixed colours may be produced by *Lambert's method*, which consists in looking in an oblique direction through a vertical glass plate P (fig. 535) at a coloured wafer *b*, while, at the same time, a wafer of another colour *g* sends its light by reflection towards the observer's eye; if *g* is placed in a proper position, which is easily found by trial, its image exactly coincides with that of *b*. The method of the colour disc (567) affords another means of producing mixed colours.

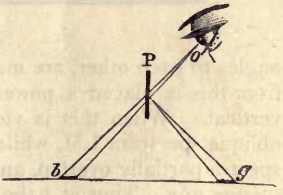


Fig. 535.

A very convenient way of investigating the phenomena of mixed colours is that of *Maxwell's colour-discs*. These consist of discs of cardboard with an aperture in the centre, by which they can be fastened on the spindle of the turning-table (fig. 536). Each disc is painted with a separate colour, and, having a radial slit, they may be slid over each other so as to overlap to any

desired extent (figs. 537 and 538); and thus, when in this way two such discs are rotated, we get the effect due to this mixture of these two colours. It is clear also that the effect of three colours may be investigated in the same way.

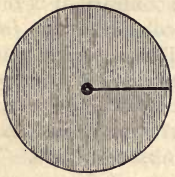


Fig. 536.

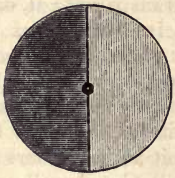


Fig. 537.

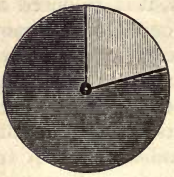


Fig. 538.

If in any of the methods by which the impression of mixed spectral colours is produced, one or more colours be suppressed, the residue corresponds to one of the tints of the spectrum; and the mixture of the colours taken away produces the impression of another spectral colour. Thus, if in fig. 529 the red rays are cut off from the lens L, the light on the focus is no longer white, but greenish blue. In like manner, if the violet, indigo, and blue of the colour disc be suppressed, the rest seems yellow, while the mixture of that which has been taken out is a bluish violet. Hence white can always be compounded of *two* tints; and two tints which together give white are called *complementary colours*. Thus of spectral tints *red* and *greenish yellow* are complementary, so are *orange* and *Prussian blue; yellow* and *indigo blue; greenish yellow* and *violet.*

The method by which Helmholtz investigated the mixture of spectral colours is as follows:—Two very narrow slits, A and B (fig. 539), at right

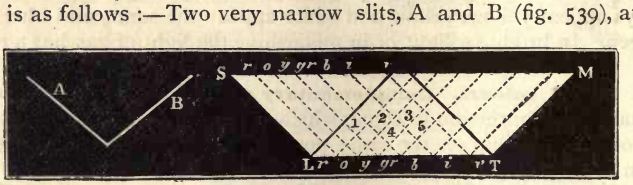


Fig. 539.

angles to each other, are made in the shutter of a dark room; at a distance from this is placed a powerfully dispersing prism with its refracting edge vertical. When this is viewed through a telescope, the slit B gives the oblique spectrum LM, while the slit A gives the spectrum ST. These two spectra partially overlap, and when this is the case *two homogeneous spectral colours* mix. Thus at 1 the red of one spectrum coincides with the green of the other; at 3, indigo and yellow coincide; and so forth.

When the experiment is made with suitable precautions, the colours obtained by mixing the spectral colours are given in the table on the next page, where the fundamental spectra to be mixed are given in the first horizontal and vertical column, and the resultant colours where these cross.

Prismatic spectrum colours may also be investigated by the method of Von Bezold, which consists in producing two images of colours by double refraction, and making one cover the other.

The mixture of mixed colours gives rise to no new colours. Only the

same colours are obtained as a mixture of the primitive spectral colours would yield, except that they are less *saturated*, as it is called ; that is, more mixed with white.

571. **Spectral colours and pigment colours.**—A distinction must be made between *spectral colours* and *pigment colours*. Thus a mixture of pigment yellow and pigment blue produces green, and not white, as is the case when the blue and yellow of the spectrum are mixed. The reason of this is that in the mixture of pigments we have a case of subtraction of colours, and not of addition. For the pigment blue in the mixture absorbs almost entirely the yellow and red light ; and the pigment yellow absorbs the blue and violet light, so that only the green remains.

In the above series are two spectral colours very remote in the spectrum, which have nearly the same complementary tints ; these are red, the complementary colour to which is greenish blue ; and violet, whose complementary colour is greenish yellow. Now when two pairs of complementary colours are mixed together they must produce white, just as if only two complementary colours were mixed. But a mixture of greenish blue and of greenish yellow is green. Hence it follows that from a mixture of red, green, and violet, white must be formed. This may easily be ascertained to be the case by means of a colour disc on which are these three colours in suitable proportions.

	Violet	Blue	Green	Yellow	Red
Red	Purple	Rose	Dull yellow	Orange	Red
Yellow	Rose	**White**	Yellowish green	Yellow	
Green	Pale blue	Bluish green	Green		
Blue	Indigo	Blue			
Violet	Violet				

From the above facts it follows that from a mixture of red, green, and violet all possible colours may be constructed, and hence these three spectral colours are called the *fundamental colours*. It must be remarked that the tints resulting from the mixture of these three have never the saturation of the individual spectral colours.

We have to discriminate three points in regard to *colour*. In the first place, the *tint*, or colour proper, by which we mean that special property which is due to a definite refrangibility of the rays producing it ; secondly, the *saturation*, which depends on the greater or less admixture of white light with the colours of the spectrum, these being colours which are fully satu-

rated; and thirdly, there is the *intensity*, which depends on the amplitude of vibration.

572. **Homogeneous light.**—The light emitted from luminous bodies is seldom or never quite pure; on being examined by the prism it will be found to contain more than one colour. In optical researches it is frequently of great importance to procure *homogeneous* or *monochromatic* light. Common salt, or, still better, bromide of sodium, in the flame of a Bunsen's lamp gives a yellow of great purity. For red light, ordinary light is transmitted through glass coloured with suboxide of copper, which absorbs nearly all the rays excepting the red. A very pure blue is obtained by transmitting ordinary light through a glass trough containing an ammoniacal solution of sulphate of copper, and a nearly pure red by transmitting it through a solution of sulphocyanide of iron.

573. **Properties of the spectrum.**—Besides its luminous properties, the spectrum is found to produce calorific and chemical effects.

Luminous properties. It appears from the experiments of Fraunhofer and of Herschel, that the light in the yellow part of the spectrum has the greatest intensity, and that in the violet the least.

Heating effects. It was long known that the various parts of the spectrum differed in their calorific effects. Leslie found that a thermometer placed in different parts of the spectrum indicated a higher temperature as it moved from violet towards red. Herschel fixed the maximum intensity of the heating effects just outside the red; Berard in the red itself. Seebeck showed that those different effects depend on the nature of a prism; with a prism of water the greatest calorific effect is produced in the yellow; with one of alcohol it is in the orange-yellow; and with a prism of crown glass it is in the middle of the red.

Melloni, by using prisms and lenses of rock salt, and by availing himself of the extreme delicacy of the thermo-electric apparatus, first made a complete investigation of the calorific properties of the thermal spectrum. This result led, as we have seen, to the confirmation and extension of Seebeck's observations.

Chemical properties. In numerous phenomena, light exerts a chemical action. For instance, chloride of silver blackens under the influence of light; transparent phosphorus becomes opaque; vegetable colouring matters fade; hydrogen and chlorine gases, when mixed, combine slowly in diffused light, and with explosive violence when exposed to direct sunlight. The chemical action differs in different parts of the spectrum. Scheele found that when chloride of silver was placed in the violet, the action was more energetic than in any other part. Wollaston observed that the action extended beyond the violet, and concluded that, besides the visible rays, there are some invisible and more highly refrangible rays. These are the chemical or *actinic* rays.

The most remarkable chemical action which light exerts is in the growth of plant life. The vast masses of carbon and hydrogen accumulated in the vegetable world owe their origin to the carbonic acid and aqueous vapour present in the atmosphere. The light which is absorbed by the green parts of plants acts as a reducing agent. The reduction does not extend to the complete isolation of carbon and hydrogen, and the individual stages of the

process are unknown to us; but the general result is, undoubtedly, that under the influence of the sun's rays the chemical attraction which holds together the carbon and oxygen is overcome; the carbon, which is set free, assimilates at that moment the elements of water, forming cellulose or woody fibre, while the oxygen returns to the atmosphere in the form of gas. The equivalent of the sunlight which has been absorbed is to be sought in the chemical energy of the separated constituents. When we burn petroleum or coal, we reproduce, in some sense, the light which the sun has expended in former ages in the production of a primeval vegetable growth.

The researches of Bunsen and Roscoe show that whenever chemical action is induced by light, an absorption of light takes place, preferably of the more refrangible parts of the spectrum. Thus, when chlorine and hydrogen unite, under the action of light, to form hydrochloric acid, light is absorbed, and the quantity of chemically active rays consumed is directly proportional to the amount of chemical action.

There is a curious difference in the action of the different spectral rays. Moser placed an engraving on an iodised silver plate, and exposed it to the light until an action had commenced, and then placed it under a violet glass in the sunlight. After a few minutes a picture was seen with great distinctness, while when placed under a red or yellow glass it required a very long time, and was very indistinct. When, however, the iodised silver plate was first exposed in a camera obscura to blue light for two minutes, and was then brought under a red or yellow glass, an image quickly appeared, but not when placed under a green glass. It appears as if there are vibrations of a certain velocity which could commence an action, and that there are others which are devoid of the property of commencing, but can continue and complete an action when once set up. Becquerel, who discovered these properties in luminous rays, called the former *exciting rays* and the latter *continuing* or *phosphorogenic rays*. The phosphorogenic rays, for instance, have the property of rendering certain objects self-luminous in the dark after they have been exposed for some time to the light. Becquerel found that the phosphorogenic spectrum extended from indigo to beyond the violet.

574. **Dark lines of the spectrum.**—The colours of the solar spectrum are not continuous. For several grades of refrangibility rays are wanting, and, in consequence, throughout the whole extent of the spectrum there are a great number of very narrow dark lines. To observe them, a pencil of solar rays is admitted into a darkened room, through a narrow slit. At a distance of three or four yards we look at this slit through a prism of flint glass, which must be very free from flaws, taking care to hold its edge parallel to the slit. We then observe a great number of very delicate dark lines parallel to the edge of the prism, and at very unequal intervals.

The existence of the dark lines was first observed by Wollaston in 1802; but Fraunhofer, a celebrated optician of Munich, first studied and gave a detailed description of them. Fraunhofer mapped the lines, and indicated the most marked of them by the letters A, *a*, B, C, D, E, *b*, F, G, H; they are therefore generally known as *Fraunhofer's lines*.

The dark line A (see fig. ii. of Plate I.) is at the middle and B halfway between this and the end of the red ray; C at the boundary of the red and

orange ray; D is in the yellow ray; E, in the green; F, in the blue; G, in the indigo; H, in the violet. There are certain other noticeable dark lines, such as *a* in the red and *b* in the green. In the case of sunlight the positions of the dark lines are fixed and definite; on this account they are used for obtaining an exact determination of the refractive index (538) of each colour; for example, the refractive index of the blue ray is, strictly speaking, that of the dark line F. In the spectra of artificial lights, and of the stars, the relative positions of the dark lines are changed. In the electric light the dark lines are replaced by brilliant lines. In coloured flames—that is to say, flames in which certain chemical substances undergo evaporation—the dark lines are replaced by very brilliant lines of light, which differ for different substances. Lastly, some of the dark lines are constant in position and distinctness, such as Fraunhofer's lines; but some of the lines only appear as the sun nears the horizon, and others are strengthened. They are also influenced by the state of the atmosphere. The fixed lines are due to the sun; the variable lines have been proved by Janssen and Secchi to be due to the aqueous vapour in the air, and are called atmospheric or telluric lines.

Fraunhofer counted in the spectrum more than 600 dark lines, more or less distinct, distributed irregularly from the extreme red to the extreme violet ray. Brewster counted 2,000. By causing the refracted rays to pass successively through several analysing prisms (576), not merely has the existence of 3,000 dark lines been ascertained, but several which had been supposed to be single have been shown to be compound.

575. **Applications of Fraunhofer's lines.**—Subsequently to Fraunhofer, several physicists studied the dark lines of the spectrum. In 1822 Sir J. Herschel remarked that by volatilising substances in a flame a very delicate means is afforded of detecting certain ingredients by the colours they impart to certain of the dark lines of the spectrum; and Fox Talbot in 1834 suggested optical analysis as probably the most delicate means of detecting minute portions of a substance. To Kirchhoff and Bunsen, however, is really due the merit of basing a method of analysis on the observation of the lines of the spectrum. They ascertained that the salts of the same metal, when introduced into a flame, always produced lines identical in colour and position, but that lines different in colour, position, or number were produced by different metals; and finally, that an exceedingly small quantity of a metal suffices to disclose its existence. Hence has arisen a new and powerful method of analysis, known by the name of *spectrum analysis*.

576. **Spectroscope.**—The name of spectroscope has been given to the apparatus employed by Kirchhoff and Bunsen for the study of the spectrum. One of the forms of this apparatus is represented in fig. 540. It is composed of three telescopes mounted on a common foot, and whose axes converge towards a prism, P, of flint glass. The telescope A is the only one which can turn round the prism. It is fixed in any required position by a clamping screw *n*. The screw-head *m* is used to *focus* the eyepiece. The screw-head *n* serves to change the inclination of the axis.

To explain the use of the telescopes B and C we must refer to fig. 541, which shows the passage of the light through the apparatus. The rays

emitted by the flame G fall on the lens *a*, and are caused to converge to a point *b*, which is the principal focus of a second lens *c*. Consequently the pencil, on leaving the telescope B, is formed of parallel rays (552). This pencil

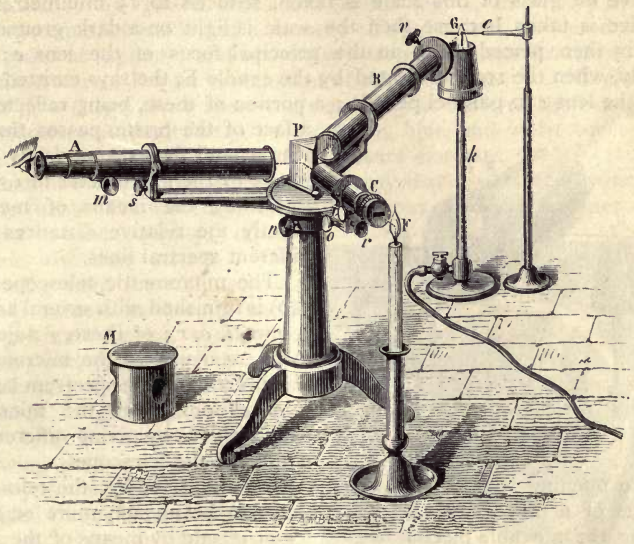


Fig. 540.

enters the prism P. On leaving the prism the light is decomposed, and in this state falls on the lens *x*. By this lens *x* a real and reversed image of the spectrum is formed at *i*. This image is seen by the observer through a

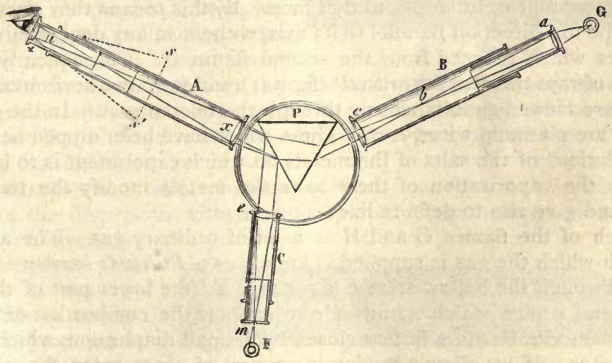


Fig. 541.

magnifying glass, which forms at *ss'* a virtual image of the spectrum magnified about eight times.

The telescope C serves to measure the relative distances of the lines

of the spectrum. For this purpose a micrometer is placed at *m*, divided into 25 equal parts. A micrometer is formed thus :—A scale of 250 millimetres is divided with great exactness into 25 equal parts. A photographic negative on glass of this scale is taken, reduced to 15 millimetres. The negative is taken because then the scale is light on a dark ground. The scale is then placed at *m* in the principal focus of the lens *e*; consequently, when the scale is lighted by the candle F, the rays emitted from it leave the lens *e* in parallel pencils ; a portion of these, being reflected from a face of the prism, passes through a lens *x*, and forms a perfectly distinct image of the micrometer at *i*, thereby furnishing the means of measuring exactly the relative distances of the different spectral lines.

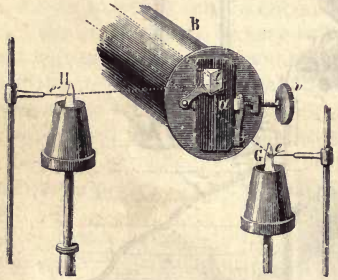


Fig. 542.

The micrometric telescope C (fig. 540) is furnished with several adjusting screws, *i*, *o*, *r*; of these, *i* adjusts the focus ; *o* displaces the micrometer in the direction of the spectrum laterally ; *r* raises or lowers the micrometer, which it does by giving different inclinations to the telescope.

The opening whereby the light of the flame G enters the telescope B consists of a narrow vertical slit, which can be opened more or less by causing the movable piece *a* to advance or recede by means of the screw *v* (fig. 542). When, for purposes of comparison, the spectra of two flames are to be examined simultaneously, a small prism, whose refracting angle is 60°, is placed over the upper part of the slit. Rays from one of the flames, H, fall at right angles on one face of the prism ; they then experience total reflection on a second face, and leave the prism by the third face, and in a direction at right angles to that face. By this means they pass into the telescope in a direction parallel to its axis, without in any degree mixing with the rays which proceed from the second flame, G. Consequently the two pencils of rays traverse the prism P (fig. 541), and form two horizontal spectra, which are viewed simultaneously through the telescope A. In the flames G and H are platinum wires, *e*, *e'*. These wires have been dipped beforehand into solutions of the salts of the metals on which experiment is to be made ; and by the vaporisation of these salts the metals modify the transmitted light, and give rise to definite lines.

Each of the flames G and H is a jet of ordinary gas. The apparatus through which the gas is supplied is known as a *Bunsen's burner*. The gas comes through the hollow stem *k* (fig. 540). At the lower part of this there is a lateral orifice which admits air to support the combustion of the gas. This orifice can be more or less closed by a small diaphragm, which acts as a regulator. If we allow a moderate amount of air to enter, the gas burns with a luminous flame, and the lines are obscured. But if a strong and steady current of air enters, the carbon is rapidly oxidised, the flame loses its brightness, and burns with a pale blue light, but with an intense heat. In this state it no longer yields a spectrum. If, however, a metallic salt is in-

troduced either in a solid state or in a state of solution, the spectrum of the metal makes its appearance, and in a fit state for observation.

There are three chief types of spectra: the *continuous* spectra, or those furnished by ignited solids and liquids (fig. 1, Plate I.); the *band* or *line* spectrum, consisting of a number of bright lines, and produced by ignited gases or vapours; and *absorption* spectra, or those furnished by the sun or fixed stars. For an explanation of these see art. 579. Bodies at a red heat give only a short spectrum, extending at most to the orange; as the temperature gradually rises, yellow, green, blue, and violet successively appear, while the intensity of the lower colours increases.

Instead of the prism very pure spectra may also be obtained by means of a grating (647). For more detailed investigations of the spectral lines a *train of prisms* is used. Fig. 543 represents one with nine prisms. The light issuing from the collimeter A passes in succession through each of the prisms. As the successive deviations add themselves the dispersion is very much increased, and a spectrum of great extent is obtained. It is, however, feebly luminous, owing partly to its extension, and partly to the loss of light which is observed through the telescope B, which it undergoes in traversing all these refracting surfaces. In the case of ten prisms the loss of light has been found to amount to ninety-nine per cent.

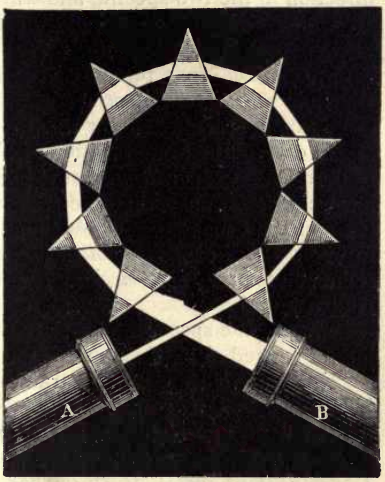


Fig. 543.

Christie has used with advantage a *semi-prism* obtained by cutting an isosceles prism by a plane at right angles to the base. These semi-prisms have the advantage that they produce as much dispersion as with several prisms without any appreciable loss in the sharpness of the images; and without that absorption of light which in the case of a number of prisms is so very considerable.

577. **Direct vision spectroscope.**—Prisms may be combined so as to get rid of the dispersion without entirely destroying the refraction (584); they may, conversely, be combined so that the light is not refracted, but is decomposed and produces a spectrum. Combinations of prisms of this kind are used in what are called *direct vision spectroscopes*. Fig. 544 represents the section of such an instrument in about $\frac{2}{3}$ the natural size. A system of two flint and three crown-glass prisms is placed in a tube which moves in

Fig. 544.

a second one ; at the end of this is an aperture *o*, and inside it a slit the width of which can by a special arrangement be regulated by simply turning a ring *r*. A small achromatic lens is introduced at *aa*, the focus of which is at the slit, so that the rays pass parallel through the train of prisms, and the spectrum is viewed at *e*.

The *reversion spectroscope* contains two equal systems of direct vision prisms arranged close to each other, but *reversed*, so that two spectra are obtained with the colours in opposite order. By suitable micrometric movement of a split lens, the position of the two spectra may be moved apart or nearer each other. Hence it is possible to bring any two identical lines so that they are in the same vertical line. If now the position of these lines in the spectrum is altered, the displacement will take place in the opposite direction in the two spectra, and will therefore be twice as distinct.

578. **Experiments with the spectroscope.**—The coloured plate at the beginning of the book shows certain spectra observed by means of the spectroscope. No. 1 represents the continuous spectrum.

No. 2 shows the spectrum of sodium. The spectrum contains neither red, orange, green, blue, nor violet. It is marked by a very brilliant yellow ray in exactly the same position as Fraunhofer's dark line D. Of all metals, sodium is that which possesses the greatest spectral sensibility. In fact, it has been ascertained that one two-hundred-millionth of a grain of sodium is enough to make the yellow line visible. Consequently it is very difficult to avoid the appearance of this line. A very little dust in the apartment is enough to produce it—a circumstance which shows how abundantly sodium is distributed.

No. 3 is the spectrum of *lithium*. It is characterised by a well-marked line in the red called Liα, and by the feebler orange line Liβ.

Nos. 4 and 5 show the spectra of *cæsium* and *rubidium*, metals discovered by Bunsen and Kirchhoff by means of spectrum analysis. The former is distinguished by two blue lines, Csα and Csβ; the latter by two very brilliant dark red lines, Rbγ and Rbδ, and by two less intense violet lines, Rbα and Bbβ. A third metal, *thallium*, has been discovered by the same method by Mr. Crookes in England, and independently by M. Lamy in France. Thallium is characterised by a single green line. Subsequently to this Richter and Reich discovered a new metal associated with zinc, and which they call *indium* from a couple of characteristic lines which it forms in the indigo ; and quite recently Boisbaudran discovered a new metal which he calls *gallium* existing in zinc in very minute quantities.

The extreme delicacy of the spectrum reactions, and the ease with which they are produced, constitute them a most valuable help in the qualitative analysis of the alkalies and alkaline earths. It is sufficient to place a small portion of the substance under examination on platinum wire as represented in fig. 542, and compare the spectrum thus obtained either directly with that of another substance or with the charts in which the positions of the lines produced by the various metals are laid down.

With other metals the production of their spectra is more difficult, especially in the case of some of their compounds. The heat of a Bunsen's burner is insufficient to vaporise the metals, and a more intense temperature must be used. This is effected by taking electric sparks between

wires consisting of the metal whose spectrum is required, and the electric sparks are most conveniently obtained by means of Ruhmkorff's coil. In order to investigate solutions of salts the apparatus shown in fig. 545 is used. A platinum wire is fixed in the bottom, and over one end a small conical glass tube D is placed ; only so much solution is poured in, that by capillary action it just rises to the top of D. Another glass tube fused in a platinum wire is fixed in the cork C, and its free end can be placed at any distance from D. Thus all the metals may be brought within the sphere of spectrum observation.

Fig. 545.

The power of the apparatus has great influence on the nature of the spectrum ; while an apparatus with one prism only gives in a sodium flame the well-known yellow line, an apparatus with more prisms resolves it into two or three lines.

It has been observed that the character of the spectrum changes with the temperature ; thus chloride of lithium in the flame of a Bunsen's burner gives a single intense peach-coloured line ; in a hotter flame, as that of hydrogen, it gives an additional orange line ; while in the oxyhydrogen jet or the voltaic arc a broad brilliant blue band comes out in addition. The sodium spectrum produced by a Bunsen's burner consists of a single yellow line ; if, by the addition of oxygen, the heat be gradually increased, more bright lines appear ; and with the aid of the oxyhydrogen flame the spectrum is continuous. Sometimes also, in addition to the appearance of new lines, an increase in temperature resolves those bands which exist into a number of fine lines, which in some cases are more and in some less refrangible than the bands from which they are formed. It may be supposed that the glowing vapour formed at the low temperature consists of the oxide of some difficultly reducible metal, whereas at the enormously high temperature of the spark these compounds are decomposed, and the true bright lines of the metal are formed.

The delicacy of the reaction increases very considerably with the temperature. With the exception of the alkalies, it is from 40 to 400 times greater at the temperature of the electric spark than at that of Bunsen's burner.

The spectra of the permanent gases are best obtained by taking the electric spark of a Ruhmkorff's coil, or Holtz's machine, through glass tubes of a special construction, consisting of two wide ones connected by a capillary tube (fig. 546), which in the wider parts are provided with electrodes of platinum or aluminium ; they are filled with the gas in question in a state of great attenuation, and are usually known as *Geissler's tubes* ; if the spark be passed through hydrogen, the light emitted is bright red, and its spectrum consists of one red, two blue lines, No. 7, the first two of which appear to coincide with Fraunhofer's lines C and F, and the third with a line between F and G. No. 6 represents the spectrum of oxygen. No. 8 is the spectrum of nitrogen. The light of this gas in a Geissler's tube is purple, and the spectrum very complicated.

If the electric discharge takes place through a compound gas or vapour, the spectra are those of the elementary constituents of the gas. It seems as

if, at very intense temperatures, chemical combination were impossible, and oxygen and hydrogen, chlorine and the metals, could coexist in a separate form, as though mechanically mixed with each other.

The nature of the spectra of the elementary gases is very materially influenced by alterations of temperature and pressure. Wüllner made a series of very accurate observations on the gases oxygen, hydrogen, and nitrogen. He not only used gases in closed tubes, which by various electrical means he raised to different temperatures ; but in one and the same series of experiments, in which a small inductorium was used, he employed pressures varying from 100 millimetres to a fraction of a millimetre ; while in another series in which a larger apparatus was used, he extended the pressure to 2,000 millimetres. At the lowest pressure of less than one millimetre, the spectrum of hydrogen was found to be green, and consisting of six splendid groups of lines, which at a higher pressure than 1 millimetre changed to continuous bands ; at 2 to 3 millimetres the spectrum consisted of the often-mentioned three lines, which did not disappear under a higher pressure, but gradually became less brilliant as the continuous spectrum increased in extent and lustre. From this point the light, and therefore the spectrum, became feebler. Using the larger apparatus, the band spectrum appeared only under a higher pressure ; at the highest pressure of 2,000 millimetres it gave place to the continous spectrum, since the bright lines continually extended and ultimately merged into each other.

Fig 546.

579. **Explanation of the dark lines of the solar spectrum.**—It has been already seen that incandescent sodium vapour gives a bright yellow line corresponding to the dark line D of the solar spectrum. Kirchhoff found that, when the brilliant light produced by incandescent lime passes through a flame coloured by sodium in the usual manner, a spectrum is produced in which is a dark line coinciding with the dark line D of the solar spectrum ; what would have been a bright yellow line becomes a dark line when formed on the background of the limelight. By allowing in a similar manner the limelight to traverse vapours of potassium, barium, strontium, &c., the bright lines which they would have formed were found to be converted into dark lines : such spectra are called *absorption spectra.*

It appears, then, that the vapour of sodium has the power of absorbing rays of the same refrangibility as that which it emits. And the same is true of the vapours of potassium, barium, strontium, &c. This absorptive power is by no means an isolated phenomenon. These substances share it, for example, with the vapour of nitrous acid, which Brewster found to possess the following property :—when a tube filled with this vapour is placed in the path of the light either of the sun or of a gas flame, and the light is subsequently decomposed by a prism, a spectrum is produced which is full of dark lines (No. 9, Plate I.) ; and Miller showed that iodine and bromine vapour produced analogous effects.

Hence the origin of the above phenomenon is, doubtless, the absorption

by the sodium vapour of rays of the same kind—that is, having the same refrangibility—as those which it has itself the power of emitting. Other rays it allows to pass unchanged, but these it either totally or in great part suppresses. Thus the particular lines in the spectrum to which these rays would converge are illuminated only by the feebly luminous sodium flame, and accordingly appear dark by contrast with the other portions of the spectrum which receive light from the powerful flame behind.

By replacing one of the flames G and H (fig. 542) by a pencil of sunlight reflected from a heliostat, Kirchhoff ascertained by direct comparison that the bright lines which characterise iron correspond to dark lines in the solar spectrum. He also found the same to be the case with sodium, magnesium, calcium, nickel, and some other metals.

This reversal of the sodium light may be produced even without a prism by an apparatus devised by Bunsen, and shown in fig. 547. It consists of a Woolf's bottle in which a small quantity of zinc, dilute sulphuric acid, and common salt are placed so that hydrogen is slowly liberated, charged with particles of sodium chloride, or, better, bromide. Through the india-rubber tube L ordinary coal gas is admitted, and issues through the tubes R and R′. On each of these tubes is a metal burner. The gas burns at the top A with a broad flat flame, C; the burner B is cylindrical, and over it is placed a conical mantle closed at the top with wire gauze. In this way a small yellow flame is produced. On looking through this against the wide flame, the former appears dark, as if smoky on a light background. The light of the posterior and far brighter flame is absorbed by the front and cooler one, and replaced by light of lesser intensity, which thus appears dark by contrast.

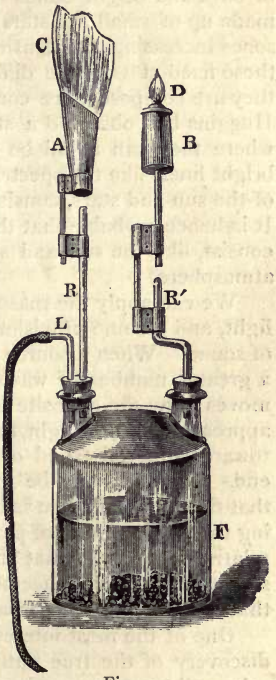


Fig. 547.

From such observations we may draw important conclusions with respect to the constitution of the sun. Since the solar spectrum has dark lines where sodium, iron, &c., give bright ones (No. 11, Plate I.), it is probable that around the solid, or more probably liquid, body of the sun which throws out the light, there exists a vaporous envelope which, like the sodium flame in the experiment described above, absorbs certain rays; namely, those which the envelope itself emits. Hence those parts of the spectrum which, but for this absorption, would have been illuminated by these particular rays, appear feebly luminous in comparison with the other parts, since they are illuminated only by the light emitted by the envelope, and not by the solar nucleus; and we are at the same time led to conclude that in this vapour there exist the metals sodium, iron, &c.

Huggins and Miller applied spectrum analysis to the investigation of the

heavenly bodies. The spectra of the moon and planets, whose light is reflected from the sun, give the same lines as those of the sun. Uranus proves an exception to this, and is probably still in a self-luminous condition. The spectra of the fixed stars contain, however, dark lines differing from the solar lines, and from one another. Four distinct types of spectra were distinguished by Secchi. The first embraces the white stars, and includes the well-known Sirius and α Lyræ. Their spectra (No. 12, Plate I.) usually contain a number of very fine lines, and always contain four broad dark lines which coincide with the bright lines of hydrogen. Out of 346 stars, 164 were found to belong to this group. The second group embraces those having spectra intersected by numerous fine lines like those of our sun. About 140 stars, among them Pollux, Capella, ϕ Aquilæ, belong to this group. The third group embraces the red and orange stars, such as α Orionis, β Pegasi ; the spectra of these (Nos. 13, 14, Plate I.) are divided into eight or ten parallel columnar clusters of dark and bright bands increasing in intensity to the red. Group four is made up of small red stars with spectra, and is constructed of three bright zones increasing in intensity towards the violet. It would thus appear that these fixed stars, while differing from one another in the matter of which they are composed, are constructed on the same general plan as our sun. Huggins has observed a striking difference in the spectra of the nebulæ ; where they can at all be observed they are found to consist generally of bright lines, like the spectra of the ignited gases, instead of, like the spectra of the sun and stars, consisting of a bright ground intersected by dark lines. It is hence probable that the nebulæ are masses of glowing gas, and do not consist, like the sun and stars, of a photosphere surrounded by a gaseous atmosphere.

We can apply the reasoning of Doppler's principle (233) to the case of light, and assume provisionally that the motion of light is analogous to that of sound. When a source of light is approaching the earth, the eye receives a greater number of waves in a given time, the waves are shorter ; as it moves away the opposite is the case, the waves are longer. Hence, on the approach of yellow light, for instance, the bright band D will seem displaced towards the violet end of the spectrum, and in receding, towards the red end. This will also be the case with the corresponding dark line, proving that the whole medium is moved at the same time. Accordingly, by observing the displacement of particular lines, conclusions may be drawn as to the relative motions of what are called the fixed stars. Thus, from careful observation of the displacement of the F line in Sirius, Huggins has inferred that it is moving away from the earth with a velocity of 42 miles per second.

One of the most interesting triumphs of spectrum analysis has been the discovery of the true nature of the *protuberances*, which appear during a solar eclipse as mountains or cloud-shaped luminous objects varying in size, and surrounding the moon's disc.

During the eclipse of 1868 it had been ascertained by Janssen that protuberances emitted certain bright lines coinciding with those of hydrogen. They have, however, been fully understood only since Lockyer and Janssen have discovered a method of investigating them at any time. The principle of this method is as follows :—When a line of light admitted through a slit is decomposed by a prism, the length of the spectrum may be increased by

passing it through two or more prisms ; as the quantity of light is the same, it is clear that the intensity of the spectrum will be diminished. This is the case with the ordinary sources of light, such as the sun ; if the light be homogeneous, it will be merely deviated, and not reduced in intensity, by dispersion. And if the source of light emit light of both kinds, the image of the slit of light of a definite refrangibility, which the mixture may contain, will stand out, by its superior intensity, on the weaker ground of the continuous spectrum. This is the case with the spectrum of the protuberances. Viewed through an ordinary spectroscope, the light they emit is overshadowed by that of the sun ; but by using prisms of great dispersive power the sun's light becomes weakened, and the spectrum of the protuberances may be observed. Lockyer's researches leave no doubt that they are ignited masses of gas, principally of hydrogen. By altering the position of the slit a series of sections of the prominences is obtained, by collating which the form of the prominence may be inferred. They are thus found to enclose the sun usually to a depth of about 5,000 miles, but sometimes in enormous local accumulations, which reach the height of 70,000 miles. Lockyer has not merely examined these phenomena right on the edge of the sun, but he has been able to observe them on the disc itself. He has shown that some of these protuberances are the results of sudden outbursts or storms, which move with the enormous velocity of 120 miles in a second ; and, by reasoning as above, the direction of this motion has been determined.

For a fuller account of this branch of physics, which is incompatible with the limits of this work, the reader is referred to Sir H. Roscoe's 'Lectures on Spectrum Analysis,' and to the same writer's articles, and those of Schuster, in Watts's 'Dictionary of Chemistry,' or to Schellen's 'Spectrum Analysis,' translated by Lassell, or to Lockyer 'On the Spectroscope.'

580. **Uses of the spectroscope.**—When a liquid placed in a glass tube or in a suitable glass cell is interposed between a source of light and the slit of the spectroscope, the spectrum observed on looking through the telescope will in many cases be found to be traversed by dark bands. No. 10, Plate I., represents the appearance of the spectrum when a solution of *chlorophyl*, the green colouring matter of plants, is thus interposed. In the red, the yellow, and the violet parts, dark bands are formed, and the blue gives way to a reddish shimmer. If, instead of chlorophyl, arterial blood greatly diluted be used, the red of the spectrum appears brighter, but green and violet are nearly extinguished. As these bands thus differ in different liquids as regards position, breadth, and intensity, in many cases they afford the most suitable means of identifying bodies. Sorby and Browning have devised a combination of the microscope and spectroscope called the *microspectroscope*, which renders it possible to examine even very minute traces of substances.

This application of the spectroscope has been very useful in investigating substances which have special importance in physiology and pathology ; thus in examining normal and diseased blood, and in ascertaining the rate at which certain substances pass into the various fluids of the system. The characteristic absorption bands with certain liquids, such as wine, beer, &c., present in their normal state, compared with those yielded by adulterated substances, furnish a delicate and certain means of detecting the latter.

Thus the adulteration of claret with the juice of elderberries is detected by the appearance of faint bands near line D, which are not seen with pure red wine. The colouring matter of malt and hops is quite distinct from that of many other substances with which it is alleged to be adulterated. An alkaline solution of blood, to which ammonium sulphide is added, gives two very powerful absorption bands between D and E, and between E and *b*; this is the most valuable test for toxicological cases. Blood charged with carbonic oxide is unchanged on the addition of ammonium sulphide, and thus the poisoning by carbonic oxide can be detected. So, too, the appearance of the characteristic bands of gall in blood, and of albumen in urine, are indications of jaundice and of Bright's disease respectively.

Suppose the slit of the spectroscope be divided into two halves, S_1 and S_2 (fig. 548), the aperture of each of which can be varied to any measured extent by means of micrometric screws. If then a layer of a substance of known thickness be placed in front of the slit S_1, for instance, and the spectrum of a particular portion be observed, there will be a difference between the luminosity of the two parts of the spectrum; but by regulating the width of the slit they may be made the same. The luminosities will then be inversely as the width of the slit. Thus, if the widths of each were originally 1, and the uncovered slit had to be narrowed to 0·4, the intensity of the light transmitted through the screen would only be 0·4 of the incident light. Vierordt has based on this a method of quantitative spectrum analysis; thus if the absorption produced by a definite thickness of known strength be known, the relative concentration of any other solution of the same substance for the same thickness may be determined.

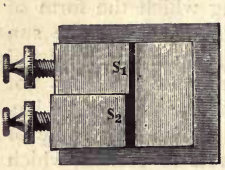


Fig. 548.

581. **Abnormal dispersion.**—A remarkable exception to the ordinary law of dispersion was discovered by Christiansen, and subsequently confirmed and extended by Soret and Kundt—that the solutions of certain substances, such as indigo and potassium permanganate, give spectra in which the *order* of the colours is not the same as in the prismatic spectrum. Thus, when a hollow glass prism is filled with an alcoholic solution of fuchsine, the order of the colours in the spectrum which it yields is as follows. Violet is *least* refracted, then red, and then yellow, which is *most* refracted. If we imagine that the central green of an ordinary spectrum is removed, and then the position of the rest is interchanged, we get an idea of the abnormal spectrum of fuchsine. This will be seen from fig. 549, in which I represents the position of Fraunhofer's lines in the abnormal dispersion of fuchsine, while II represents the position in the normal spectrum. Kundt examined a great number of substances in this direction, mostly the colours derived from aniline, and found that the abnormal dispersion is exhibited by all substances with *surface colour*. These bodies have the peculiarity that when viewed in diffused light they exhibit a

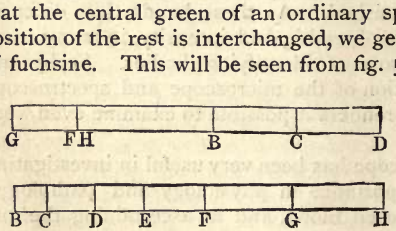


Fig. 549.

different colour from that which they transmit. Thus a thin flake of fuchsine appears green in diffused, but red in transmitted light.

The substances in solution are examined by placing them in hollow glass prisms ; if the solutions are weak, the abnormal dispersion of the substance is concealed by that of the solvent, while stronger solutions absorb so much light as to be almost opaque, and prisms of very small refracting angle have to be used. Soret gets rid of this difficulty by immersing the prism containing the solution in glass vessels with parallel sides filled with the solvent. The dispersion due to the solvent is thereby eliminated, and only that of the substance comes into play. Cyanine gives a well-marked abnormal spectrum, the order of the colours being the following : green, light blue, dark blue, a dark space, red, and traces of orange, the green being the colour which is least diffused.

The same explanation cannot be given of this as of the ordinary colour of bodies (569), but must be ascribed to the fact that the bodies in question totally reflect light of certain wave-lengths (637) at almost all incidences, and that these colours are reflected on the surface. Hence it follows that the colour of these bodies in diffused light must be almost complementary to the transmitted light—a prevision which experiment confirms.

582. **Fluorescence.**—Stokes made the remarkable discovery that under certain circumstances the rays of light are capable of undergoing a change of refrangibility. The discovery originated in the study of a phenomenon observed by Brewster and by Herschel, that some varieties of fluorspar, and also the solutions of certain substances, when looked at by transmitted light appear colourless, but when viewed in reflected light present a bluish appearance. Stokes has found that this property, which he calls *fluorescence* from having been observed in fluorspar, is characteristic of a large number of bodies.

If by means of a lens of long focus, preferably of quartz, a line of the sun's rays be focussed on a solution of sulphate of quinine contained in a glass trough, a beautiful cerulean blue cone of light (fig. 550) is formed, which is much the brightest on the surface and the intensity of which rapidly diminishes as it penetrates the liquid.

It thus appears that fluorescence is due to an absorption of certain rays ; rays of light which have passed through a sufficient thickness of a fluorescent substance lose thereby the power of exciting fluorescence when they are passed through a second layer of the same substance ; thus a test tube containing a fluorescent liquid is brightly luminous when exposed to the sun's rays, but loses this lustre at once when it is dipped in a trough of the same liquid, on the front of which the sun's rays fall. This also results from a comparison of the absorption spectrum of a fluorescent substance with the appearance presented by this substance when the spectrum falls on it. Where the fluorescence begins there also begins the absorption, and to a maximum of absorption corresponds a maximum of fluorescence.

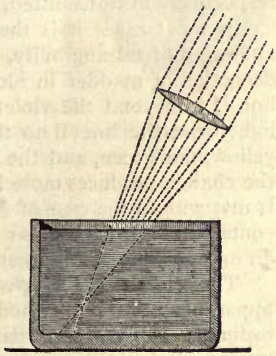


Fig. 550.

The phenomenon is seen when a solution of sulphate of quinine, contained in a trough with parallel sides, is placed in different positions in the solar spectrum. No change is observed in the upper part of the spectrum, but from about the middle of the lines G and H (coloured Plate) to some distance beyond the extreme range of the violet, rays of a beautiful sky-blue colour are seen to proceed. These invisible ultra-violet rays also become visible when the spectrum is allowed to fall on paper impregnated with a solution of *æsculine* (a substance extracted from horse-chestnut), an alcoholic solution of stramonium, or a plate of *canary glass* (which is coloured by means of uranium). If light be allowed to fall on paper impregnated with platinomanganide of barium, a beautiful green fluorescence is observed.

If a few drops of a strong solution of fluoresceine in soda fall into a large beaker of water on the front of which the sun's rays fall, beautiful fluorescent clouds are first produced, and on shaking the liquid the whole vessel fluoresces with a bright green light.

This change arises from a diminution in the refrangibility of those rays outside the violet, which are ordinarily too refrangible to affect the eye.

Glass appears to absorb many of these more refrangible rays, which is not the case nearly to the same extent with quartz. When a prism and trough formed of plates of quartz are used, and the spectrum is received on a sheet of paper on which a wash of solution of sulphate of quinine has been made, two juxtaposed spectra can be obtained. That which is on the part coated with sulphate of quinine extends beyond the line H to an extent equal to that of the visible spectrum. In the spectrum, thus made visible, dark lines may be seen analogous to those in the ordinary spectrum.

The phenomena may be observed without the use of a prism. When an aperture in a dark room is closed by means of a piece of blue glass, and the light is allowed to fall upon a piece of canary glass, it instantly appears self-luminous from the emission of the altered rays. If a test tube be half filled with a solution of sulphate of quinine, and on it be poured a freshly prepared solution of chlorophyl in ether, the two layers appear colourless and green respectively in transmitted, and sky-blue and blood-red in reflected light.

In most cases it is the violet and ultra-violet rays which undergo an alteration of refrangibility, but the phenomenon is not confined to them. A decoction of madder in alum gives yellow and violet light from about the line D to beyond the violet; an alcoholic solution of chlorophyl gives red light from the line B to the limit of the spectrum. In these cases the yellow, the green, and the blue rays experience increase of refrangibility; the change produces more highly refrangible rays. An exception to this rule is met with in the case of Magdala red. If on a solution of this substance contained in a rectangular glass vessel a solar spectrum be allowed to fall, an orange-yellow fluorescence is found even in the red part of the spectrum.

The electric light gives a very remarkable spectrum. With quartz apparatus Stokes obtained a spectrum six or eight times as long as the ordinary one. Several flames of no great illuminating power emit very peculiar light. Characters traced on paper with solution of stramonium, which are almost invisible in daylight, appear at once when illuminated by the flame of burning sulphur or of bisulphide of carbon. Robinson found that the light of the aurora is rich in rays of high refrangibility.

583. **Chromatic aberration.**—The various lenses hitherto described (551) possess the inconvenience that, when at a certain distance from the eye, they give images with coloured edges. This defect, which is most observable in condensing lenses, is due to the unequal refrangibility of the simple colours (564), and is called *chromatic aberration.*

For, since a lens may be compared to a series of prisms with infinitely small faces, and united at their bases (551), it not only refracts light, but also decomposes it like a prism. On account of this dispersion, therefore, lenses have really a distinct focus for each colour. In condensing lenses, for example, the red rays, which are the least refrangible, form their focus at a point R on the axis of the lens (fig. 551); while the violet rays, which are most refrangible, coincide in the nearer point V. The foci of the orange, yellow, green, blue, and indigo are between these points. The chromatic aberration is more perceptible in proportion as the lenses are more convex, and as the point at which the rays are incident is farther from the axis; for then the deviation, and therefore the dispersion, are increased.

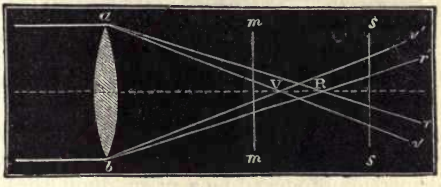


Fig. 551.

If a pencil of rays which has passed through a condensing lens be received on a screen placed at *mm* within the focal distance, a bright spot is seen with a red border; if it is placed at *ss*, the bright spot has a violet border.

The inequality in the refraction of the blue and red rays may be demonstrated by closing a small aperture, half with red and half with blue glass (fig. 552); on each half a black arrow is painted, and a lamp is placed behind it. By means of a lens of 60 cm. focus an image is formed on a screen at a distance of about 2 metres. If the screen be placed so that a sharp image is obtained of the black object on the blue ground, the outlines of the other are confused. To get a sharp image of the arrow on the red ground the screen must be moved farther away.

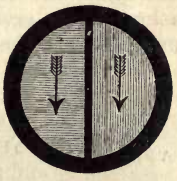


Fig. 552.

584. **Achromatism.**—By combining prisms which have different refracting angles (544), and are formed of substances of unequal dispersive powers (564), white light may be refracted without being dispersed. The same result is obtained by combining lenses of different substances, the curvatures of which are suitably combined. The images of objects viewed through such lenses do not appear coloured, and they are accordingly called *achromatic* lenses; *achromatism* being the term applied to the phenomenon of the refraction of light without decomposition.

By observing the phenomenon of the dispersion of colours in prisms of water, of oil of turpentine, and of crown glass, Newton was led to suppose that dispersion was proportional to refraction. He concluded that there could be no refraction without dispersion, and, therefore, that achromatism was impossible. Almost half a century elapsed before this was found to be incorrect. Hall, an English philosopher, in 1733, was the first to construct

achromatic lenses, but he did not publish his discovery. It is to Dollond, an optician in London, that we owe the greatest improvement which has been made in optical instruments. He showed in 1757 that by combining two lenses—one a double convex crown glass lens, the other a concavo-convex lens of flint glass (fig. 554)—a lens is obtained which is virtually achromatic.

To explain this result, let two prisms, BFC and CDF, be joined and turned in a contrary direction, as shown in fig. 553. Let us suppose in the first case that both prisms are of the same material, but that the refracting angle of the second, CDF, is less than the refracting angle of the first; the two prisms will produce the same effect as a single prism, BAF; that is to say, white light which traverses it will not only be refracted, but also decomposed. If, on the contrary, the first prism BCF were of crown glass, and the other CFD of flint glass, the dispersion might be destroyed without destroying the refraction. For, as flint glass is more dispersive than crown, and as the dispersion produced by a prism diminishes with its refracting angle (564), it follows that by suitably lessening the refracting angle of the flint glass prism CFD, as compared with the refracting angle of the crown glass prism BCF, the dispersive power of these prisms may be equalised; and as, from their position, the dispersion takes place in a contrary direction, it is neutralised; that is, the emergent rays EO are parallel, and therefore give white light. Nevertheless, the ratio of the angles BCF and CFD, which is suitable for the parallelism of the red rays and violet rays, is not so for the intermediate rays, and, consequently, only two of the rays of the spectrum can be exactly combined, and the achromatism is not quite perfect. To obtain perfect achromatism, several prisms would be necessary, of unequally dispersive materials, and the angles of which were suitably combined.

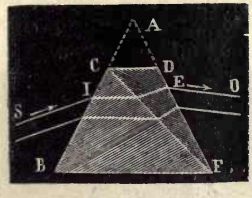


Fig. 553.

The refraction is not destroyed at the same time as the dispersion; that could only happen if the refracting power of a body varied in the same ratio as its dispersive power, which is not the case. Consequently, the emergent ray EO is not exactly parallel to the incident ray, and there is a refraction without appreciable decomposition.

Fig. 554.

Achromatic lenses are made of two lenses of unequal dispersive materials: one, A, of flint glass, is a diverging concavo-convex (fig. 554); the other, B, of crown glass, is double convex, and one of its faces may exactly coincide with the concave face of the first. As with prisms, several lenses would be necessary to obtain perfect achromatism; but for optical instruments two are sufficient, their curvatures being such as to combine not the extreme red and violet, but the blue and orange rays, while at the same time regard is had to the correction for spherical aberration.

CHAPTER V

OPTICAL INSTRUMENTS

585. **The different kinds of optical instruments.**—By the term *optical instrument* is meant any combination of lenses, or of lenses and mirrors. Optical instruments may be divided into three classes, according to the ends they are intended to answer, viz. :—i. *Microscopes*, which are designed to obtain a magnified image of any object whose real dimensions are too small to admit of its being seen distinctly by the naked eye. ii. *Telescopes*, by which very distant objects, whether celestial or terrestrial, may be observed. iii. *Instruments* designed to project on a screen a magnified or diminished image of any object which can thereby be either depicted or rendered visible to a crowd of spectators; such as the *camera lucida*, the *camera obscura*, *photographic apparatus*, the *magic lantern*, the *solar microscope*, the *photo-electric microscope*, &c. The two former classes yield virtual images; the last, with the exception of the *camera lucida*, yield real images.

MICROSCOPES

586. **The simple microscope.**—The *simple microscope*, or *magnifying glass*, is merely a convex lens of short focal length, by means of which we look at objects placed between the lens and its principal focus. Let AB (fig. 555) be the object to be observed, placed between the lens and its principal focus, F. Draw the secondary axes AO and BO, and also from A and B rays parallel to the axis of the lens FO. Now these rays, on passing out of the lens, tend to pass through the second principal focus F′; consequently they are divergent with reference to the secondary axes, and therefore, when produced, will cut those axes in A′ and B′ respectively. These points are the virtual foci of A and B respectively. The lens, therefore, produces at A′B′ an erect and magnified virtual image of the object AB.

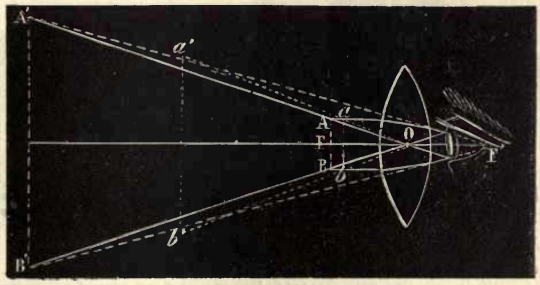


Fig. 555.

The position and magnitude of this image depend on the distance of the object from the focus. Thus, if AB is moved to *ab*, nearer the lens, the secondary axes will contain a greater angle, and the image will be formed at *a'b'*, and will be much smaller, and nearer the eye. On the other hand, if the object is moved farther from the lens, the angle between the secondary axes is diminished, and their intersection with the prolongation of the refracted rays taking place beyond A'B', the image is formed farther from the lens, and is larger.

In a simple microscope both chromatic aberration and spherical aberration increase with the degree of magnification. We have already seen that the former can be corrected by using achromatic lenses (584), and the latter by using stops, which allow the passage of such rays only as are nearly parallel to the axis, the spherical aberration of these rays being nearly inappreciable. Spherical aberration may be still further corrected by using two plano-convex lenses, instead of one very convergent lens. When this is done, the plane face of each lens is turned towards the object (fig. 556). Although each lens is less convex than the simple lens which together they replace, yet their joint magnifying power is as great, and with a less amount of spherical aberration, since the first lens diverts towards the axis the rays which fall on the second lens. This combination of lenses is known as *Wollaston's doublet.*

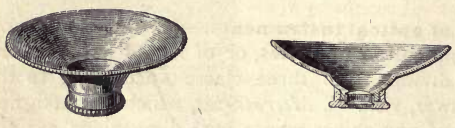


Fig. 556.

There are many forms of the simple microscope. One of the best is that represented in fig. 557. On a horizontal support E, which can be raised and lowered by a rack K and pinion D, there is a black *eyepiece m*, in the centre of which is fitted a small convex lens. Below this is the *stage b*, which is fixed, and on which the object is placed between glass plates. In order to illuminate the object powerfully, diffused light is reflected from a concave glass mirror, M, so that the reflected rays fall upon the object. In using this microscope the eye is placed very near the lens, which is lowered or raised until the position is found at which the object appears in its greatest distinctness.

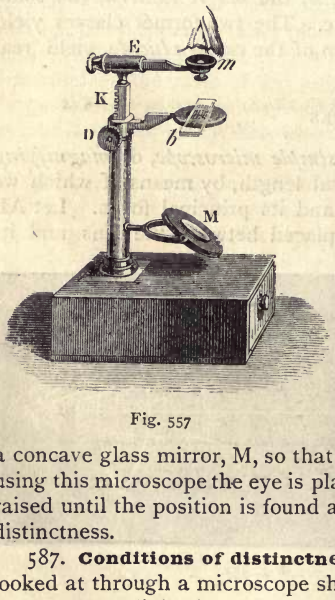


Fig. 557

587. **Conditions of distinctness of the images.**—In order that objects looked at through a microscope should be seen with distinctness, they must have a strong light thrown upon them, but this is by no means enough. It is necessary that the image be formed at a determinate distance from the eye. In fact, there is for each person a *distance of most distinct vision*—a distance, that is to say, at which an object must be placed from an observer's

eye in order to be seen with greatest distinctness. This distance is different for different observers, but ordinarily is between 10 and 12 inches. It is, therefore, at this distance from the eye that the image ought to be formed. Moreover, this is why each observer has to *focus* the instrument; that is, to adapt the microscope to his own distance of most distinct vision. This is effected by slightly varying the distance from the lens to the object, for we have seen above that a slight displacement of the object causes a great displacement of the image. With a common magnifying glass, such as is held in the hand, the adjustment is effected by merely moving it nearer to or farther from the object. In the microscope the adjustment is effected by means of a rack and pinion, which in the case of the instrument shown in fig. 556 moves the eyepiece, but moves the object in the case of the instrument depicted in fig. 557. What has been said about *focussing* the microscope applies equally to telescopes. In the latter instrument the eyepiece is generally adjusted with respect to the image formed in the focus of the object-glass.

In respect of the distinctness of the image the general rules for convex lenses apply.

In order to lessen dispersion, lenses have been constructed of diamond, of ruby, and of other precious stones, which for a small amount of dispersion have a great degree of refrangibility. A drop of water or of Canada balsam in a small hole in a thin piece of wood or of metal, acts as a microscope.

588. **Apparent magnitude of an object.**—The apparent magnitude or apparent diameter of a body is the angle it subtends at the eye of the

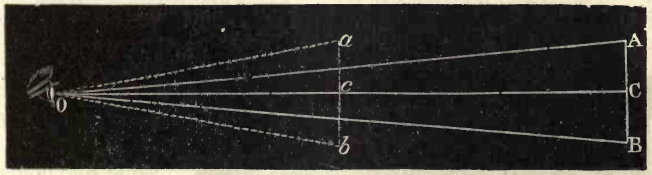


Fig. 558.

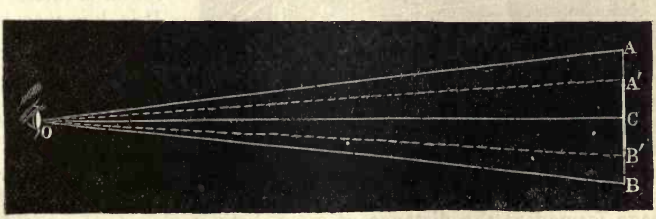


Fig. 559.

observer. Thus, if AB is the object, and O the observer's eye (figs. 558, 559) the apparent magnitude of the object is the angle AOB contained by two visual rays drawn from the centre of the pupil to the extremities of the object.

In the case of objects seen through optical instruments, the angles which they subtend are so small that the arcs which measure the angles do not differ sensibly from their tangents. The ratio of two such angles is

therefore the same as that of their tangents. Hence we deduce the two following principles :—

i. *When the same object is seen at unequal distances, the apparent diameter varies inversely as the distance from the observer's eye.*

ii. *In the case of two objects seen at the same distance, the ratio of the apparent diameters is the same as that of their absolute magnitudes.*

These principles may be proved as follows :—i. In fig. 558, let AB be the object in its first position, and *ab* the same object in its second position. For the sake of distinctness these are represented in such positions that the line OC passes at right angles through their middle points C and *c* respectively. It is, however, sufficient that *ab* and AB should be the bases of isosceles triangles having a common vertex at O. Now, by what has been said above, AB is virtually an arc of a circle described with centre O and radius OC ; likewise *ab* is virtually an arc of a circle whose centre is O and radius O*c*. Therefore,

$$\text{AOB} : a\text{O}b = \frac{\text{AB}}{\text{OC}} : \frac{ab}{\text{O}c} = \frac{1}{\text{OC}} : \frac{1}{\text{O}c}.$$

Therefore, AOB varies inversely as OC.

ii. Let AB and A′B′ be two objects placed at the same perpendicular distance, OC, from the eye, O, of the observer (fig. 559). Then they are virtually arcs of a circle whose centre is O and radius OC. Therefore,

$$\text{AOB} : \text{A}'\text{OB}' = \frac{\text{AB}}{\text{OC}} : \frac{\text{A}'\text{B}'}{\text{OC}} = \text{AB} : \text{A}'\text{B}',$$

a proportion which expresses the second principle.

589. **Measure of magnification.**—In the simple microscope the measure of the magnification produced is the ratio of the apparent diameter of the image to that of the object, both being at the distance of most distinct vision. The same rule holds good for other microscopes. It is, however, important to obtain an expression for the magnification depending on data that are of easier determination.

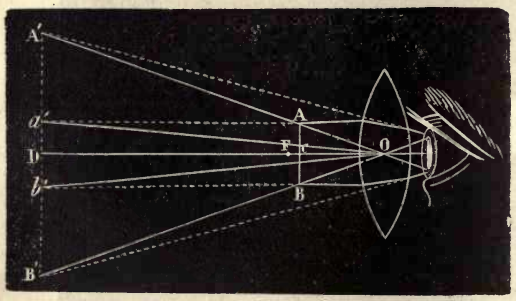


Fig. 560.

In fig. 560 let AB be the object, and A′B′ its image formed at the distance of most distinct vision. Let *a′b′* be the projection of AB on A′B′. Then, since the eye is very near the glass, the magnification equals $\frac{\text{A}'\text{OB}'}{a'\text{O}b'}$, or $\frac{\text{A}'\text{B}'}{a'b'}$; that is, $\frac{\text{A}'\text{B}'}{\text{AB}}$. But since the triangles A′OB′ and AOB are similar, A′B′ : AB = DO : CO. Now DO is the distance of most distinct vision, and CO is very nearly equal to FO, the focal length of the lens. Therefore, the magnification equals the ratio of the distance of most distinct vision to the focal length of the lens. Hence we con-

clude that the magnification is greater, 1st, as the focal length of the lens is smaller—in other words, as the lens is more convergent; 2ndly, as the observer's distance of most distinct vision is greater.

A simpler and more general definition of the measure of magnification may be stated thus :—Let α be the angular magnitude of the object as seen by the naked eye, β the angular magnitude of the image, whether real or virtual, actually present to the eye, then the magnification is $\beta \div \alpha$. This rule applies to telescopes.

By changing the lens the magnification may be increased, but only within certain limits if we wish to obtain a distinct image. By means of a simple microscope distinct magnification may be obtained up to 120 diameters.

The magnification we have here considered is *linear* magnification. *Superficial* magnification equals the square of the *linear* magnification; for instance, the former will be 1,600 when the latter is 40.

590. **Principle of the compound microscope.**—The compound microscope in its simplest form consists of two condensing lenses: one, with a short focus, is called the *object-glass*, or *objective*, because it is turned towards the object; the other is less condensing, and is called the *eyepiece*, or *power*, because it is close to the observer's eye.

Fig. 561 represents the path of the luminous rays and the formation of the image in the simplest form of a compound microscope. An object AB

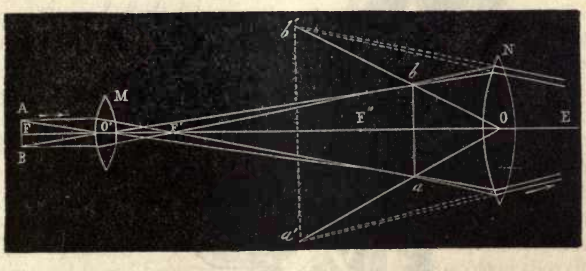


Fig. 561.

being placed very near the principal focus F of the object-glass M, but a little farther from the glass, a real image *ab*, inverted and somewhat magnified, is formed on the other side of the object-glass (556). Now the distance of the two lenses M and N is regulated so that the position of the image *ab* is between the eyepiece N and its focus F″. From this it follows that for the eye at E, looking at the image through the eyepiece, this glass produces the same effect as a simple microscope, and instead of this image *ab*, another image, *a′b′*, is seen, which is virtual, and still more magnified. This second image, although erect as regards the first, is inverted in reference to the object. It may thus be said that the compound microscope is in effect a simple microscope applied not to the object but to its image already magnified by the first lens.

591. **Compound microscope.**—The principle of the compound microscope has been already (590) explained; the principal accessories to the instrument remain to be described.

Fig 562 represents a perspective view, and fig. 563 a section of a compound microscope. The body of the microscope consists of a series of brass tubes, DD′, H, and I ; in H is fitted the eyepiece O, and in the lower part of DD′, the object-glass *o*. The tube I moves with gentle friction in the tube DD′, which in turn can also be moved in a larger tube fixed in the ring E. This latter is fixed to a piece BB′, which, by means of a very fine screw worked by the milled head T, can be moved up and down an inner rod, *c*, not represented in the figure. The whole body of the microscope is raised

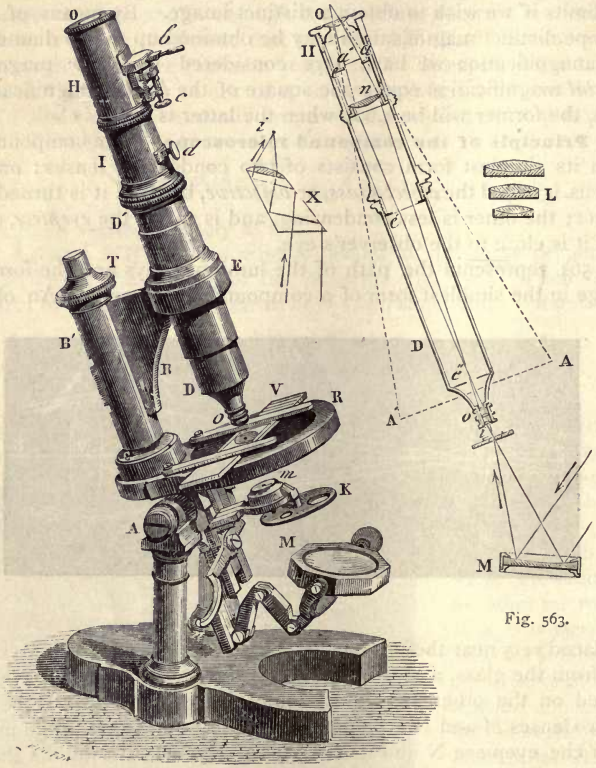


Fig. 563.

Fig. 562.

and lowered with the piece BB′, so that it can be placed near or far from the object to be examined. Moreover, the rod *c*, and all the other pieces of the apparatus, rest on a horizontal axis A, with which they turn under so much friction as to remain fixed in any position in which they may be placed.

The object to be observed is placed between two glass plates, V, on a *stage*, R. This is perforated in the centre, so that light can be reflected upon the object by a concave reflecting glass mirror, M. The mirror is mounted

on a jointed support so that it can be placed in any position whatever, in order to reflect to the object either the diffused light of the atmosphere, or that from a candle or lamp. Between the reflector and the stage is a *diaphragm* or *stop*, K, perforated by four holes of different sizes, any one of which can be placed over the perforation in the stage ; thus the light falling on the object may be regulated by raising, by a lever *n*, the diaphragm K, which is movable in a slide. Above the diaphragm is a piece, *m*, to which can be attached either a very small stop, so that only very little light can reach the object, or a condensing lens, which illuminates it strongly, or an oblique prism, represented at X. The rays from the reflector undergo two total reflections in this prism, and emerge by a lenticular face that concentrates them on the object, but in an oblique direction, which in some microscopic observations is an advantage. Objects are generally so transparent that they can be lighted from below ; but where, owing to their opacity, this is not possible, they are lighted from above by means of a condensing lens mounted on a jointed support, and so placed that they receive the diffused light of the atmosphere.

Fig. 563 shows the arrangement of the lenses and the path of the rays in the microscope. At *o* is the object-glass, consisting of three small condensing lenses, represented on a larger scale at L, on the right of the figure. The effect of these lenses being added to each other is that they act like a single very powerful condensing lens. The object being placed at *i*, a very little beyond the principal focus of the system, the emerging rays fall upon a fourth condensing lens, *n*, the use of which will be seen presently (592, 593). Having become more convergent, owing to their passage through the lens *n*, the rays form at *aa'* a real and amplified image of the object *i*. This image is between a fifth condensing lens, O, and the principal focus of this lens. Hence, on looking through this, it acts as a magnifier (556), and gives at AA' a virtual and highly magnified image of *aa'*, and therefore of the object. The two glasses *n* and O constitute the eyepiece, in the same manner as the three glasses *o* constitute the object-glass.

The first image, *aa'*, must not merely be formed between the glass O and its principal focus, but at such a distance from this glass that the second image, AA', is formed at the observer's distance of distinct vision. This result is obtained in moving, by the hand, the body DH of the microscope in the larger tube fixed to the ring E, until a tolerably distinct image is obtained ; then turning the milled head T in one direction or the other, the piece BB', and with it the whole microscope, are moved until the image AA' attains its greatest distinctness, which is the case when the image *aa'* is formed at the distance of distinct vision : a distance which can always be ultimately obtained, for as the object-glass approaches or recedes from the object, the image *aa'* recedes from or approaches the eyepiece, and at the same time the image AA'.

This operation is called the *focussing*. In the microscope, where the distance from the object-glass to the eyepiece is constant, it is effected by altering their distance from the object. In telescopes, where the objects are inaccessible, the focussing is effected by varying the distance of the eyepiece and the object-glass.

The microscope possesses numerous eyepieces and object-glasses, by

means of which a great variety of magnifying power is obtained. A lower magnifying power is also obtained by removing one or two of the lenses of the object-glass.

The above contains the essential features of the microscope ; it is made in a great variety of forms, which differ mainly in the construction of the stand, the arrangement of the lenses, and in the illumination. For descriptions of these the student is referred to special works on the microscope.

592. **Achromatism of the microscope. Campani's eyepiece.**—When a compound microscope consists of two single lenses, as in fig. 561, not only is the spherical aberration uncorrected, but also the chromatic aberration, the latter defect causing the images to be surrounded by fringes of the prismatic colours, these fringes being larger as the magnification is greater. With a view to correcting these aberrations the object-glass (see fig. 563) is composed of three achromatic lenses, and the eyepiece of two lenses, n and O ; for the first of these, n, would produce colour unless the magnifying power were low.

The effect of this eyepiece in correcting the colour may be explained as follows :—It will be borne in mind that with respect to red rays the focal length of a lens is *greater* than the focal length of the same lens with reference to the violet rays.

In fact, if in the equation (4) (559) we write $R' = \infty$, we obtain $f = \frac{R}{n-1}$, which gives the focal length of a plano-convex lens whose refractive index is n. Now, in flint glass, and for the red ray, $n-1$ equals 0·63, and for the violet ray $n-1$ equals 0·67.

Let *ab* be the object, O the object-glass, which is corrected for colour. Consequently, a pencil (fig. 564) of rays falling from *a* on O would converge

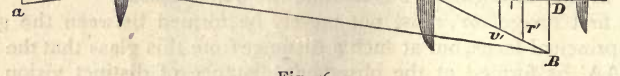


Fig. 564.

to the focus A without any separation of colours ; but falling on the *field-glass* C, the red rays would converge to r, the violet rays to v, and intermediate colours to intermediate points. In like manner the rays from b, after passing through the field-glass, would converge to r', or v', and intermediate points. So that on the whole there would be formed a succession of coloured images of *ab* ; viz. a red image at rr', a violet image at vv', and between them images of intermediate colours. Let d be the point of the object which is situated on the axis. The rays from d will converge to R, V, and intermediate points. Now suppose the *eye-glass* O′ to be placed in such a manner that R is the principal focus of O′ for the red rays, then V will be its principal focus for the violet rays. Consequently, the red rays, after emerging from O, will be parallel to the axis, as will the violet rays coming from V and so of any other colour. Accordingly, the colours of d, which are separated by C, are again combined by O′. The same is very

nearly true of r and v, and of r' and v'. Hence a combination of lenses C and O′ corrects the chromatic aberration that would be produced by the use of a single eye-glass. Moreover, by drawing the rays towards the axis, it diminishes the spherical aberration, and, as we shall see in the next article, enlarges the field of view.

In all eyepieces consisting of two lenses the lens to which the eye is applied is called the *eye-lens*; the one towards the object-glass is called the *field-lens*. The eyepiece above described was invented by Huyghens, who was not, however, aware of its property of achromatism. He designed it for use with the telescope. It was applied to the microscope by Campani. The relation between the focal length of the lenses is as follows :—The focal length of the field-glass is three times that of the eye-lens, and the distance between their centres is half the sum of the focal length. It easily follows from this that the image of the point d would, but for the interposition of the field-lens, be formed at D, which is so situated that CD is three times DO′; then the mean of the coloured images would be formed midway between C and O′.

593. **Field of view.**—By the field of view of an optical instrument is meant all those points which are visible through the eyepiece. The advan-

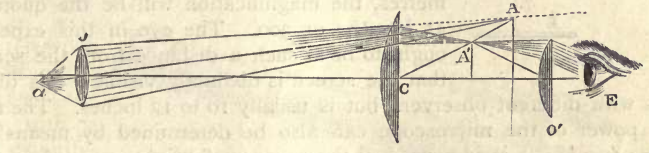


Fig. 565.

tage obtained by the use of an eyepiece in enlarging the field of view will be readily understood by an inspection of the accompanying figure. As before (fig. 565), O is the object-glass, C the field-lens, O′ the eye-lens, and E the eye placed on the axis of the instrument. Let a be a point of the object; if we suppose the field-lens removed, the pencil of rays from a would be brought to a focus at A, and none of them would fall on the eye-lens O′, nor pass into the eye E. Consequently, a is beyond the field of view. But when the field-glass C is interposed, the pencil of rays is brought to a focus at A′, and emerges from O′ into the eye. Consequently, a is now within the field of view. In this manner the substitution of an eyepiece for a single eye-lens enlarges the field of view.

594. **Magnifying power. Micrometer.**—The magnifying power of any optical instrument is the ratio of the magnitude of the image to the magnitude of the object. The magnifying power in a compound microscope is the product of the respective magnifying powers of the object-glass and of the eyepiece; that is, if the first of these magnifies 20 times, and the other 10, the total magnifying power is 200. The magnifying power depends on the greater or less convexity of the object-glass and of the eyepiece, as well as on the distance between these two glasses, together with the distance of the object from the object-glass. A magnifying power of 1,500 and even upwards has been obtained; but the image then loses in sharpness what it

gains in extent. To obtain precise and well-illuminated images, the magnifying power ought not to exceed 500 to 600 diameters, which gives a superficial enlargement 250,000 to 360,000 times that of the object.

The magnifying power is determined experimentally by means of the *glass micrometer*: this is a small glass plate, on which, by means of a diamond, a series of lines is drawn at a distance from each other of $\frac{1}{10}$ or $\frac{1}{100}$ of a millimetre. The micrometer is placed in front of the object-glass, and then, instead of viewing directly the rays emerging from the eyepiece O, they are received on a piece of glass A (fig. 566), inclined at an angle of 45°, and the eye is placed above so as to see the image of the micrometer lines, which is formed by reflection on a screen E, on which is a scale divided into millimetres. By counting the number of divisions of the scale corresponding to a certain number of lines of the image, the magnifying power may be deduced. Thus, if the image occupies a space of 45 millimetres on the scale and contains 15 lines of the micrometer, the distance between each of which shall be assumed at $\frac{1}{100}$ millimetre, the absolute magnitude of the object will be $\frac{15}{100}$ millimetre; and as the image occupies a space of 45 millimetres, the magnification will be the quotient of 45 by $\frac{15}{100}$, or 300. The eye in this experiment ought to be at such a distance from the screen E that the screen is distinctly visible: this distance varies with different observers, but is usually 10 to 12 inches. The magnifying power of the microscope can also be determined by means of the *camera lucida*; it is increased at the expense of brightness, definition, and field. Hence it is usual to have several eyepieces with each microscope so as to obtain greater definition of higher magnification.

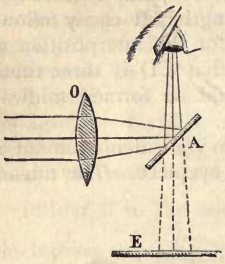


Fig. 566.

Nobert's lines are frequently used as test objects; these are lines ruled on glass in series; in the first group the lines are at a distance of $\frac{1}{1200}$ of an inch from the middle of one line to the middle of the next; in the finest the lines are at a distance of $\frac{1}{120000}$ of an inch. Other test objects are the scales of certain butterflies, and various kinds of diatoms.

When once the magnifying power is known, the absolute magnitude of objects placed under the microscope is easily deduced. For, as the magnifying power is the quotient of the size of the image by the size of the object, it follows that the size of the image divided by the magnifying power gives the size of the object: in this manner the diameters of all microscopic objects are determined.

TELESCOPES

595. **Astronomical telescope.**—The *astronomical telescope* is used for observing the heavenly bodies; like the microscope, it consists of a condensing eyepiece and object-glass. The object-glass, M (fig. 567), forms between the eyepiece, N, and its principal focus an inverted image of the heavenly body; and this eyepiece, which acts as a magnifying glass, then gives a virtual and highly magnified image, *a'b'*, of the image *ab*. The astronomical telescope appears, therefore, analogous to the microscope:

but the two instruments differ in this respect, that in the microscope, the object being very near the object-glass, the image is formed much beyond the principal focus, and is greatly magnified, so that both the object-glass and the eyepiece magnify ; while in the astronomical telescope, the heavenly

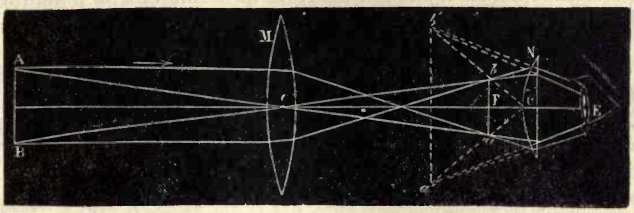


Fig. 567.

body being at a great distance, the incident rays are parallel, and the image formed in the principal focus of the object-glass is much smaller than the object. There is thus no magnification except by the eyepiece, and this ought, therefore, to be of very short focal length.

Fig. 568 shows an astronomical telescope mounted on its stand. Above it there is a small telescope which is called the *finder*. Telescopes with a large magnifying power are not convenient for finding a star, as they have but a small field of view : the position of the star is, accordingly, first sought by the finder, which has a much larger field of view—that is, takes in a far greater extent of the heavens ; it is then viewed by means of the telescope.

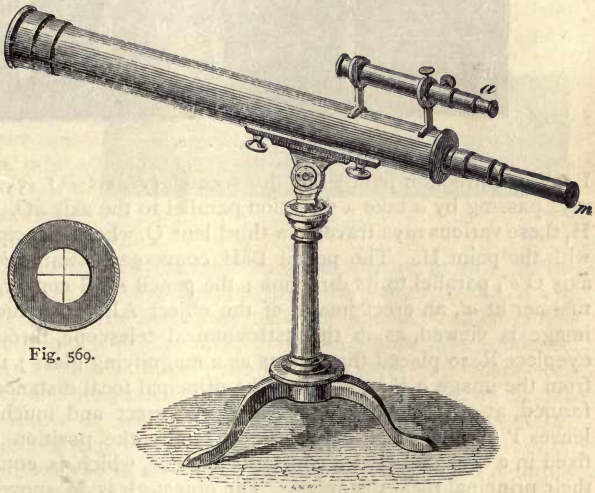


Fig. 568.

Fig. 569.

The magnification (589) equals $\frac{a'Ob'}{ACB}$ (fig. 567) ; that is, it equals $\frac{bOC}{bCO'}$ and therefore is approximately equal to $\frac{CF}{OF}$, F being the focus of the object-glass M, and being supposed very nearly to coincide with the focus of the eyepiece N ; it may, therefore, be concluded that the magnifying power is

greater in proportion as the object-glass is less convex, and the eyepiece more so.

When the telescope is used to make an accurate observation of the stars—for example, the zenith distance, or their passage over the meridian—a *cross wire* is added. This consists of two very fine metal wires or spider threads stretched across a circular aperture in a small metal plate (fig. 569). The wires ought to be placed in the position where the inverted image is produced by the object-glass, and the point where the wires cross ought to be on the optical axis of the telescope, which thus becomes the *line of sight* or *collimation.*

596. **Terrestrial telescope.**—The *terrestrial telescope* differs from the astronomical telescope in producing images in their right positions. This is effected by means of two condensing glasses, P and Q (fig. 570), placed between the object-glass M and the eyepiece R. The object being supposed to be at AB, at a greater distance than can be shown in the drawing, an inverted and much smaller image is formed at *ba* on the other side of the object-glass. But the second lens, P, is at such a distance that its principal focus coincides with the image *ab* ; from which it follows that the luminous rays which pass through *b*, for example, after traversing the lens

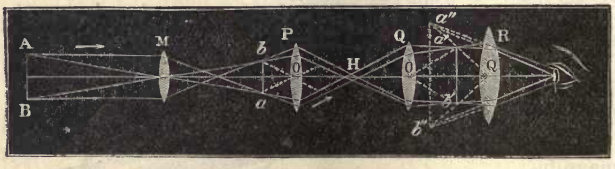


Fig. 570.

P, take a direction parallel to the secondary axis *b*O (552). Similarly, the rays passing by *a* take a direction parallel to the axis *a*O. After crossing in H, these various rays traverse a third lens Q, whose principal focus coincides with the point H. The pencil B*b*H converges towards *b'*, on a secondary axis O'*b'*, parallel to its direction ; the pencil A*a*H converging in the same manner at *a'*, an erect image of the object AB is produced at *a'b'*. This image is viewed, as in the astronomical telescope, through a condensing eyepiece R, so placed that it acts as a magnifying glass ; that is, its distance from the image *a'b'* is less than the principal focal distance ; hence there is formed, at *a''b''*, a virtual image of *a'b'*, erect and much magnified. The lenses P and Q, which only serve to rectify the position of the image, are fixed in a brass tube, at a constant distance, which is equal to the sum of their principal focal distances. The object-glass M moves in a tube, and can be moved to or from the lens P, so that the image *ab* is always formed in the focus of the lens, whatever be the distance of the object. The distance of the lens R may be varied so that the image *a''b''* may be formed at the distance of distinct vision.

This instrument may also be used as an astronomical telescope by using a different eyepiece : this must have a much greater magnifying power than in the former case.

In the terrestrial telescope the magnifying power is the same as in the

astronomical telescope, provided always that the correcting glasses, P and Q, have the same convexity.

In order to determine directly the magnifying power of a telescope, when this is not great, a divided scale at a distance, or the tiles of a house, may be viewed through the telescope with one eye and directly with the other. This with a little practice is not difficult. It is thus observed how many unmagnified divisions correspond to a single magnified one. Thus, if two seen through the telescope appear like seven, the magnifying power is $3\frac{1}{2}$. Reading ordinary printing from a distance is an excellent means of testing and comparing telescopes.

Fig. 571.

The excellence of a telescope depends also on the sharpness of the images. To test this, various circular and angular figures are painted in black on a white ground, as shown in fig. 571, in about $\frac{1}{10}$ the full size. When these are looked at through the telescope at a distance of 80 or 100 paces, they should appear sharply defined, perfectly black, without distortion, and without coloured edges. The *penetration* or *penetrating power* of a telescope by which stars are seen which are not visible to the naked eye depends mainly on the aperture of the object-glass. Even with the strongest magnification the fixed stars appear as luminous points without apparent diameter.

597. **Galileo's telescope.**—*Galileo's telescope* is the simplest of all telescopes, for it only consists of two lenses; namely, an object-glass, M, and a diverging or double concave eyepiece, R (fig. 572), and it gives at once an erect image. *Opera-glasses* are constructed on this principle.

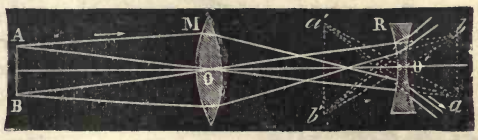


Fig. 572.

If the object be represented by the right line AB, a real but inverted and smaller image would be formed at *ba*; but in traversing the eyepiece R, the rays emitted from the points A and B are refracted and diverge from the secondary axis *b*O′ and *a*O′ which correspond to the points *b* and *a* of the image. Hence, these rays produced backward meet their axes in *a′* and *b′*; the eye which receives them sees accordingly an erect and magnified image in *a′b′*, which appears nearer because it is seen under an angle, *a′*O′*b′*, greater than the angle, AOB, under which the object is seen.

The magnifying power is equal to the ratio of the angle *a′*O′*b′* to the angle AOB, and is usually from 2 to 4.

The distance of the eyepiece R from the image *ab* is pretty nearly equal to the principal focal distance of this eyepiece; it follows, therefore, that the difference between the two lenses is the distance between their respective focal distances; hence Galileo's telescope is very short and portable. It has the advantage of showing objects in their right position; and, further, as it has only two lenses, it absorbs very little light: in consequence, however, of the divergence of the emergent rays, it has only a small field of view, and in using it the eye must be placed very near the eyepiece. The eye-

piece can be moved to or from the object-glass, so that the image *a'b'* is always formed at the distance of distinct vision.

The opera-glass is usually double, so as to produce an image in each eye, by which greater brightness is attained.

The time at which telescopes were invented is not known. Some attribute their invention to Roger Bacon in the thirteenth century; others to J. B. Porta at the end of the sixteenth; others, again, to a Dutchman, Jacques Metius, who, in 1609, accidentally found that by combining two glasses, one concave and the other convex, distant objects appeared nearer and much larger. Galileo's was the first telescope directed towards the heavens. By its means Galileo discovered the mountains of the moon, Jupiter's satellites, and the spots on the sun.

598. **Reflecting telescopes.**—The telescopes previously described are *refracting* or *dioptric* telescopes. It is, however, only in recent times that it has been possible to construct achromatic lenses of large size; before this a concave metallic mirror was used instead of the object-glass. Telescopes of this kind are called *reflecting* or *catoptric telescopes*. The principal forms are those devised by Gregory, Newton, Herschel, and Cassegrain.

599. **The Gregorian telescope.**—Fig. 573 is a representation of Gregory's telescope; it is mounted on a stand, about which it is movable, and can be inclined at any angle. This mode of mounting is optional; it may be equatorially mounted. Fig. 574 gives a longitudinal section. It consists of a long brass tube closed at one end by a concave metallic mirror, M, which is perforated in the centre by a round aperture through which rays reach the eye. There is a second concave metal mirror, N, near the end of the tube: it is somewhat larger than the central aperture in the large mirror, and its radius of curvature is much smaller than that of the large mirror. The axes of both mirrors coincide with the axis of the tube. As the centre of curvature of the large mirror is at O, and its focus at *ab*, rays such as SA emitted from a heavenly body are reflected from the mirror M, and form at *ab* an inverted and very small image of the heavenly body. The distance of the mirrors and their curvatures is so arranged that the position of this image is between the centre, *o*, and the focus, *f*, of the small mirror; hence the rays, after being reflected a second time from the mirror N, form at *a'b'* a magnified and inverted image of *ab*, and therefore in the true position of the heavenly body. This image is viewed through an eye-piece, P, which may either be simple or compound, its object being to magnify it again, so that it is seen at *a''b''*.

Fig. 573.

As the objects viewed are not always at the same distance, the focus of the large mirror, and therefore that of the small one, vary in position.

And as the distance of distinct vision is not the same with all eyes, the image $a''b''$ ought to be formed at different distances. The required adjust-

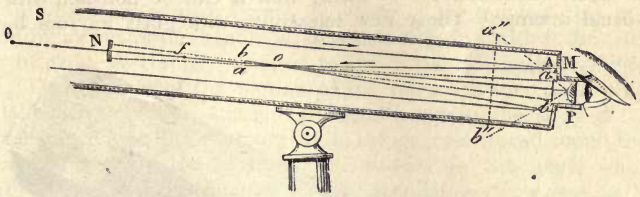


Fig. 574.

ments may be obtained by bringing the small mirror nearer to or farther from the larger one ; this is effected by means of a milled head, A (fig. 573), which turns a rod, and this by a screw moves a piece to which the mirror is fixed.

600. **The Newtonian telescope.**—This instrument does not differ much from that of Gregory ; the large mirror is not perforated, and there is a small plane mirror inclined at an angle of 45° towards an eyepiece placed in the side of the telescope.

The difficulty of constructing metallic mirrors caused telescopes of Gregorian and Newtonian construction to fall into disuse. Of late, however, the process of silvering glass mirrors has been carried to a high state of perfection, and Foucault applied these mirrors to Newtonian telescopes

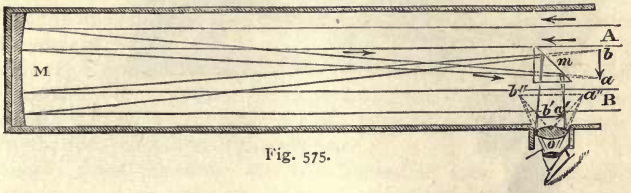


Fig. 575.

with great success. His first mirror was only four inches in diameter, but he successively constructed mirrors of 8, 12, and 13 inches, and at the time of his death had completed one of 32 inches in diameter.

Fig. 576 represents a Newtonian telescope mounted on an equatorial stand, and fig. 575 gives a longitudinal section of it. This section shows how the luminous rays reflected from the parabolic mirror M meet a small rectangular prism, *m*, which replaces the inclined plane mirror used in the old form of Newtonian telescope. After undergoing a total reflection from *m*, the rays form at $a'b'$ a very small image of the heavenly body. This image is viewed through an eyepiece with four lenses placed on the side of the telescope, and magnifying from 50 to 800 times according to the size of the silvered mirror.

In reflectors the mirror acts as object-glass, but there is, of course, no chromatic aberration. The spherical aberration is corrected by the form given to the reflector, which is paraboloid, but slightly modified by trial to suit the eyepiece fitted to the telescope.

The mirror when once polished is immersed in a silvering liquid, which consists essentially of ammoniacal solution of nitrate of silver, to which some reducing agent is added. When a polished glass surface is immersed in this solution, silver is deposited on the surface in the form of a brilliant metallic layer, which adheres so firmly that it can be polished with rouge in the usual manner. These new telescopes with glass mirrors have the

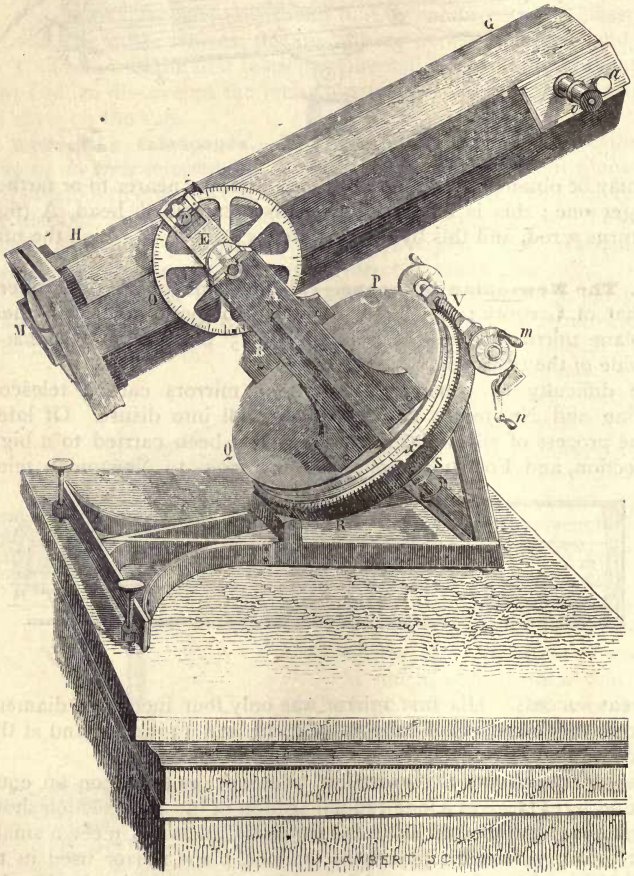


Fig. 576.

advantage over the old ones that they give purer images, they weigh less and are much shorter, their focal distance being only about six times the diameter of the mirror.

These details known, the whole apparatus remains to be described. The body of the telescope (fig. 576) consists of an octagonal wooden tube. The end G is open ; the mirror is at the other end. At a certain distance from this

end two axles are fixed, which rest on bearings supported by two wooden uprights, A and B. These are themselves fixed to a table, PQ, which turns on a fixed plate, RS, placed exactly parallel to the equator. On the circumference of the turning-table there is a brass circle divided into 360 degrees; and beneath it, but also fixed to the turning-table, there is a circular toothed wheel, in which an endless screw, V, works. By moving this in either direction by means of the handle *m*, the table PQ, and with it the telescope, can be turned. A vernier, *x*, fixed to the plate RS, gives fractions of a degree. On the axis of the motion of the telescope there is a graduated circle, O, which serves to measure the *declination* of the star—that is, its angular distance from the equator; while the degrees traced round the table RS serve to measure the *right ascension*—that is, the angle which the declination circle of the star makes with the declination circle passing through the first point of Aries.

In order to fix the telescope in declination, a brass plate, E, is fixed to the upright; it is provided with a clamp, in which the limb O works, and which can be screwed tight by means of a screw with a milled head *r*. On the side of the apparatus is the eyepiece *o*, which is mounted on a sliding copper plate, on which there is also the small prism *m*, represented in section in fig. 574. To bring the image to the right place, this plate may be moved by means of a rack and a milled head *a*. The handle *n* serves to *clamp* or *unclamp* the screw V. The drawing was one taken from a telescope the mirror of which is only 6½ inches in diameter, and which gives a magnifying power of 150 to 200.

601. **The Herschelian telescope.**—Sir W. Herschel's telescope, which until recently was the most celebrated instrument of modern times, was constructed on a method differing from those described. The mirror was so inclined that the image of the star was formed at *ab* on the side of the telescope near the eyepiece *o*; hence it is termed the *front-view* telescope. As the rays in this telescope only undergo a single reflection, the loss of light is less than in either of the preceding cases, and the image is therefore brighter. The magnifying power is the quotient of the principal focal distance of the mirror by the focal distance of the eyepiece.

Herschel's great telescope was constructed in 1789; it was 40 feet in length, the great mirror was 50 inches in diameter. The quantity of light obtained by this instrument was so great as to enable its inventor to use magnifying powers far higher than anything which had hitherto been attempted.

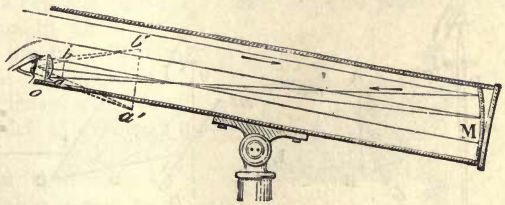


Fig. 577.

Herschel's telescope has been exceeded by one constructed by the late Earl of Rosse. This magnificent instrument has a focal distance of 53 feet, the diameter of the speculum being six feet. It is at present used as a Newtonian telescope, but it can also be arranged as a front-view telescope.

INSTRUMENTS FOR FORMING PICTURES OF OBJECTS

602. **Camera obscura.**—The *camera obscura* (dark chamber) is, as its name implies, a closed space impervious to light. The principle of this apparatus is illustrated by fig. 578. The rays proceeding from an external object AB, and entering by the aperture O, form on the opposite an image of the object *ba* in its natural colours, but of reduced dimensions, and in an inverted position.

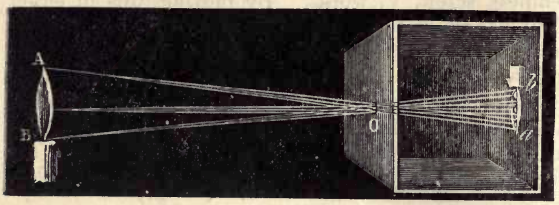


Fig 578.

Porta, a Neapolitan physician, the inventor of this instrument, found that by fixing a double convex lens in the aperture, and receiving the image on a white screen, it was much brighter and more definite.

603. **Camera lucida.**—The *camera lucida* is a small instrument depending on internal reflection, and serves for taking an outline of any object. It was invented by Wollaston in 1804. It consists of a small four-sided glass prism, of which fig. 579 gives a section perpendicular to the edges. A is a right angle, and C an angle of 135°; the other angles, B and D, are 67½°. The prism rests on a stand, on which it can be raised or lowered, and turned more or less about an axis parallel to the prismatic edges. When the face AB is turned towards the object, the rays from the object fall nearly perpendicular on this face, pass into the prism without any appreciable refraction, and are totally reflected from BC; for as the line *ab* is perpendicular to BC, and *n*L to AB, the angle *an*L will equal the angle B; that is, it will contain 67½°, and this being greater than the critical angle of glass (540), the ray L*n* will undergo total reflection. The rays are again totally reflected from *o*, and emerge near the summit, D, in a direction almost perpendicular to the face DA, so that the eye which receives the rays sees at L′ an image of the object L. If the outlines of the image are traced with a pencil, a very correct design is obtained; but unfortunately there is a great difficulty in seeing both the image and the point of the pencil, for the rays from the object give an image which is farther from the eye than the pencil. This is corrected by placing between the eye and prism a lens, I, which gives to the rays from the pencil and those from the object the same divergence. In this case, however, it is necessary to place the eye very near the edge of the prism, so that the aperture of the

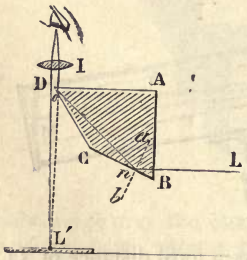


Fig. 579

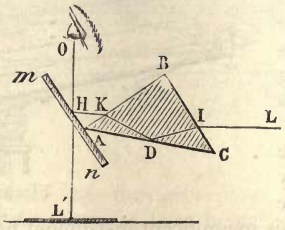


Fig. 580.

pupil is divided into two parts, one of which sees the image and the other the pencil.

Amici's camera lucida, represented in fig. 579, is preferable to that of Wollaston, inasmuch as it allows the eye to change its position to a considerable extent without ceasing to see the image and the pencil at the same time. It consists of a rectangular glass prism ABC, having one of its perpendicular faces turned towards the object to be depicted, while the other is at right angles to an inclined plate of glass, *mn*. The rays LI, proceeding from the object, and entering the prism, are totally reflected from its base at D, and emerge in the direction KH. They are then partially reflected from the glass plate *mn* at H, and form a vertical image of the object L, which is seen by the eye in the direction OL'. The eye at the same time sees through the glass the point of the pencil applied to the paper, and thus the outline of the picture may be traced with great exactness.

604. **Magic lantern.**—This is an apparatus by which a magnified image of small objects may be projected on a white screen in a dark room. A typical form is the *sciopticon*, fig. 581. The box C, the side of which is shown removed, is constructed of sheet iron ; *e* is the flame of a lamp V, with two long flat wicks, fed by petroleum from the reservoir B. The box is air-tight, and the chimney F producing a good draught, the air is compelled to pass through the wicks, by which smoke and smell are avoided, and a flame of high illuminating power is produced.

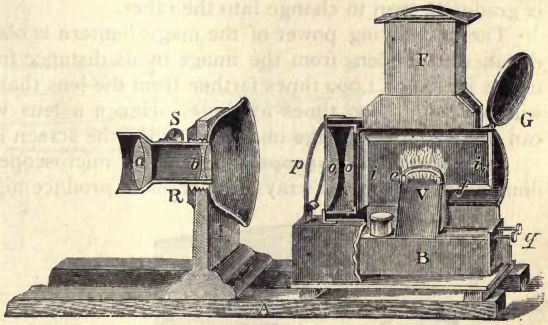


Fig. 581.

The ends of the box are closed by glass plates *i* and *i*,. G is a hinged door, and on its inside is a concave mirror ; *o* and *o*, are two plano-convex lenses ; *p* a spring clamp, in which is placed the transparent picture. The sliding piece supports the lens tube, in which are two achromatic lenses *a* and *b*, the fine adjustment of which is effected by the screw S.

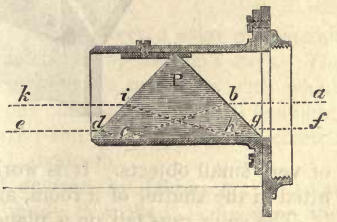


Fig. 582.

The rays from the flame *e*, reinforced by the reflection from G, falling upon the lenses *o*, *o*,, are made parallel, or, at all events, very slightly divergent ; these lenses are accordingly called the *condensing lenses*. Passing through the object which is depicted on the slide placed in *p*, they are concentrated to an image which is received on a

screen. The image is inverted, and hence, if objects are to be seen in their erect position, they must be drawn inverted. But ordinary drawings are easily adjusted by fixing an equilateral rectangular prism, P (fig. 582), in front of the lens tube, so that the hypotenuse surface is horizontal. The parallel rays falling on the prism are inverted in consequence of refraction at the sides and total reflection from the hypotenuse surface, so that an upright position is obtained instead of a reverse one. The dotted lines *abcde* and *fghik* give the path of two rays.

The apparatus can be used for projecting on a screen not only flat images, but also simple physical experiments, such as the expansion of a liquid in a thermometer, the divergence of the gold leaves of an electroscope, and so forth.

Dissolving views are obtained by arranging two magic lanterns, which are quite alike, with different pictures, in such a manner that both pictures are produced on exactly the same part of a screen. The object-glasses of both lanterns are closed by shades, which are so arranged that according as one is raised the other is lowered, and *vice versâ*. In this way one picture is gradually seen to change into the other.

The magnifying power of the magic lantern is obtained by dividing the distance of the lens from the image by its distance from the object. If the image is 100 or 1,000 times farther from the lens than the object, the image will be 100 or 1,000 times as large. Hence a lens with a very short focus can produce a very large image, provided the screen is sufficiently large.

605. **Solar microscope.**—The solar microscope is a magic lantern illuminated by the sun's rays ; it serves to produce highly magnified images

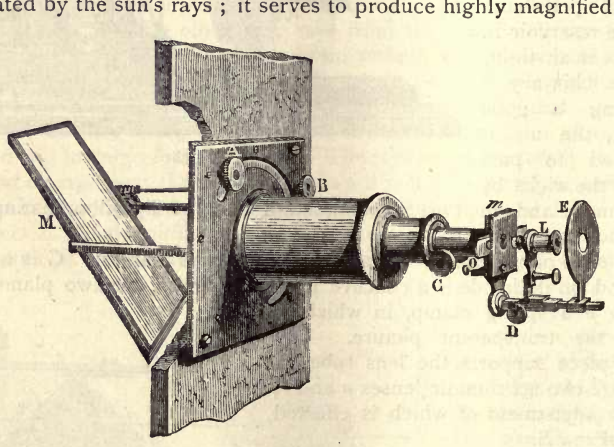


Fig. 583.

of very small objects. It is worked in a dark room : fig. 583 represents it fitted in the shutter of a room, and fig. 584 gives the internal details.

The sun's rays fall on a plane mirror, M, placed outside the room, and are reflected towards a condensing lens, *l*, and thence to a second lens, *o* (fig. 584), by which they are concentrated at its focus. The object to be magnified is at this point ; it is placed between two glass plates, which, by

means of a spring, *n*, are kept in a firm position between two metal plates, *m*. The object thus strongly illuminated is very near the focus of a system of three condensing lenses, *x*, which forms upon a screen at a suitable distance an inverted and greatly magnified image, *ab*. The distance of the lenses *o* and *x* from the object is regulated by means of screws, C and D.

As the direction of the sun's light is continually varying, the position of the mirror outside the shutter must also be changed, so that the reflection is always in the direction of the axis of the microscope. The most exact apparatus for this purpose is the heliostat (534); but as this instrument is very expensive, the object is usually attained by inclining the mirror to a greater or less extent by means of an endless screw B, and at the same time turning the mirror itself round the lens *l* by a knob A, which moves in a fixed slide.

The solar microscope labours under the objection of concentrating great heat on the object, which soon alters it. This is partially obviated by inter-

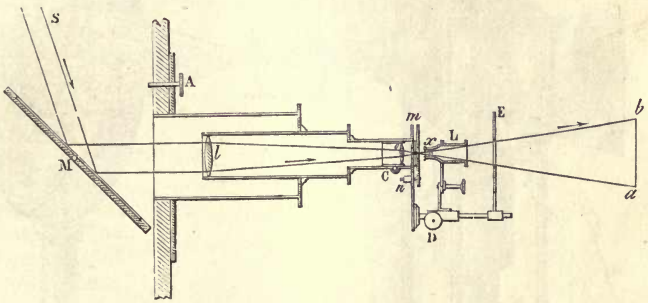


Fig. 584.

posing a layer of a saturated solution of alum, which, being a powerfully athermanous substance (434), cuts off a considerable portion of the heat.

The magnifying power of the solar microscope may be deduced experimentally by substituting for the object a glass plate marked with lines at a distance of $\frac{1}{10}$ or $\frac{1}{100}$ of a millimetre. Knowing the distance of these lines on the image, the magnifying power may be calculated. The same method is used with the electric light. According to the magnifying power which it is desired to obtain, the objective *x* is formed of one, two, or three lenses, which are all achromatic.

606. **Photo-electric microscope.**—This is in effect a solar microscope which is illuminated by the electric light instead of by the sun's rays. The electric light, by its intensity, its steadiness, and the readiness with which it can be produced at any time of the day, has completely replaced sun light. The microscope alone will be described here: the production of the electric light will be considered under the head of Galvanism.

Fig. 585 represents the arrangement devised by Duboscq. A solar microscope, ABD, identical with that already described, is fixed on the outside of a brass box. In the interior are two charcoal points which do not quite touch, the space between them being exactly on the axis of the lenses. The electricity of one end of a powerful battery reaches the charcoal

a by means of a copper wire K; while the electricity from the opposite end of the battery reaches *c* by a second copper wire H.

During the passage of the electricity a luminous arc is formed between the two ends of the carbons, which gives a most brilliant light, and powerfully illuminates the microscope. This is effected by placing at D in the inside of the tube a condensing lens, whose principal focus corresponds to the space between the two charcoals. In this manner the luminous rays which enter the tubes D and B are parallel to their axis, and the same

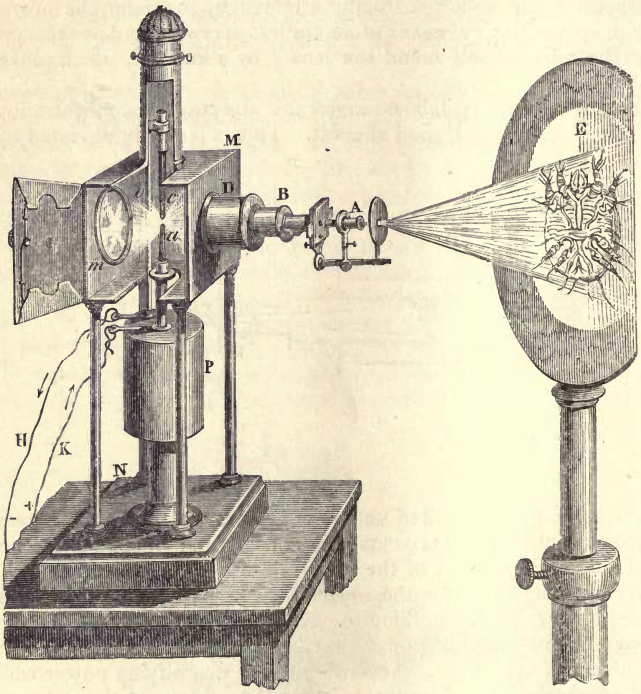


Fig. 585.

effects are produced as with the ordinary solar microscope; a magnified image of the object placed between two plates of glass is produced on the screen.

In continuing the experiment the two carbons become consumed, and to an unequal extent, *a* more quickly than *c*. Hence, their distance increasing, the light becomes weaker, and is ultimately extinguished. In speaking afterwards of the electric light, the working of the apparatus P, which keeps these charcoals at a constant distance, and thus ensures a constant light, will be explained.

The part of the apparatus MN may be considered as a universal *photogenic apparatus*. The microscope can be replaced by the headpieces of the phantasmagoria, the polyorama, the megascope, by polarising apparatus, &c.,

and in this manner is admirably adapted for exhibiting optical phenomena to a large auditory. Instead of the electric light, we may use with this apparatus the *oxyhydrogen* or *Drummond's* light, which is obtained by heating a cylinder of lime in the flame produced by the combustion of a mixture of hydrogen or of coal gas with oxygen gas.

607. **Lighthouse lenses.**—Lenses of large dimensions are very difficult of construction; they further produce a considerable spherical aberration, and their thickness causes the loss of much light. In order to avoid these inconveniences, *echelon* lenses have been constructed. They consist of a plano-convex lens, C (figs. 586 and 587), surrounded by a series of annular and concentric segments, A, B, each of which has a plane face on the same side as the plane face of the central lens, while the faces on the other side have such a curvature that the foci of the different segments coincide in the same point. These rings form, together with the central lens, a single lens, a section of which is represented in fig. 587. The drawing was made from a lens of about 2 feet in diameter, the segments of which are formed of a single piece of glass; but, with larger lenses, each segment is likewise formed of several pieces.

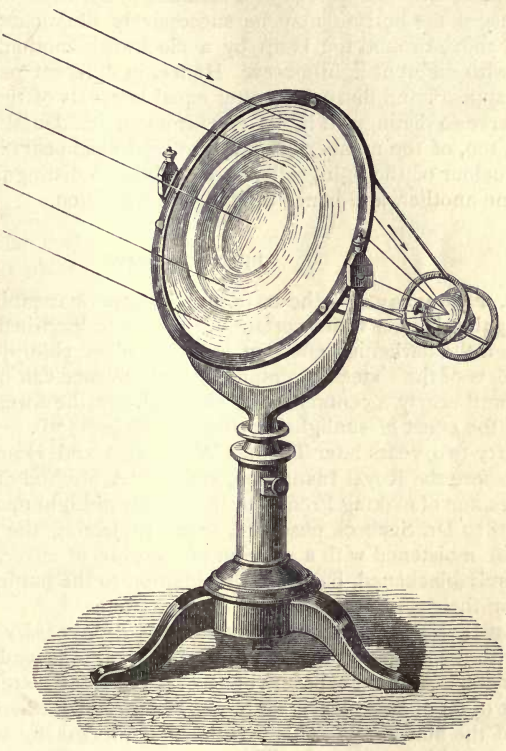


Fig. 586.

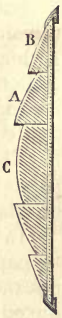


Fig. 587.

Behind the lens there is a support fixed by three rods, on which a body can be placed and submitted to the sun's rays. As the centre of the support coincides with the focus of the lens, the substances placed there are melted and volatilised by the high temperature produced. Gold, platinum, and quartz are melted. The experiment proves that heat is refracted in the same way as light; for the position of the calorific focus is identical with that of the luminous focus.

Formerly parabolic mirrors were used in sending the light of beacons and lighthouses to great distances, but they have been supplanted by the use of lenses of the above construction. Oil is used in a lamp of peculiar construction, which gives as much light as twenty moderators, but of late years this has been often superseded by the electric light. The light is placed in the principal focus of the lens, so that the emergent rays form a parallel beam (fig. 512), which loses intensity only by absorption in the atmosphere, and can be seen at a distance of above 40 miles. In order that all points of the horizon may be successively illuminated, the lens is continually moved round the lamp by a clockwork motion, the rate of which varies with different lighthouses. Hence, in different parts the light alternately appears and disappears after equal intervals of time. These alternations serve to distinguish lighthouses from an accidental fire or a star. By means, too, of the number of times the light disappears in a given time, and by the colour of the light, sailors are enabled to distinguish the lighthouses from one another, and hence to know their position.

PHOTOGRAPHY

608. **Photography** is the art of producing permanent images of objects by utilising the changes which certain substances undergo in the presence of light. Although the darkening effect of light on silver chloride was known to the alchemists of the sixteenth century, no real advance can be said to have been made until nearly a century later, when Scheele, the Swedish chemist, investigated the effect of sunlight on silver chloride (1770).

Thirty-two years later Thomas Wedgwood and Humphry Davy read a paper before the Royal Institution, entitled 'A Method of copying Paintings on Glass, and of making Profiles by the agency of Light upon Nitrate of Silver.'

In 1810 Dr. Seebeck observed, when projecting the solar spectrum on to paper moistened with a solution of chloride of silver, that the silver was not merely blackened, but an approximation to the natural colours was produced in their relative position in the spectrum.

In 1814 Nièpce patented a process of *héliographie* by coating a metal plate with a solution of bitumen dissolved in oil of lavender, and exposing in the camera. After an exposure of several hours, the plate was developed by a mixture of oil of lavender in white petroleum, which dissolved the unaffected parts of the film away. This was the first process by which photographic images could be preserved. This process was tedious and inefficient, and quite useless for portraying living objects, and it was not until 1839 that a really practical method was discovered. In that year, Daguerre in Paris and Fox Talbot in England published their respective processes, and laid the foundation of modern photography.

In Daguerre's process, the *Daguerrotype*, the picture is produced on a piece of highly burnished electro-plated copper. This is rendered sensitive by exposing it to the action of iodine vapour, which forms a thin layer of iodide of silver on the surface, and further sensitised by treatment with bromine and calcium hydrate, by which a bromiodide of silver is formed. The plate is then exposed in a camera, such as is depicted in figs. 588 and 589. The brass tube A contains an achromatic condensing lens, which can

be moved by means of a rackwork motion, by the milled head D. At the opposite end of the box is a ground plate, E, which slides in a grove, B, in which the case containing the plate also fits. The camera being placed in position before the object, the sliding part of the box is adjusted until the image is produced on the glass with the utmost sharpness; this is the case when the glass slide is exactly in focus. The final adjustment is made by means of the milled head D.

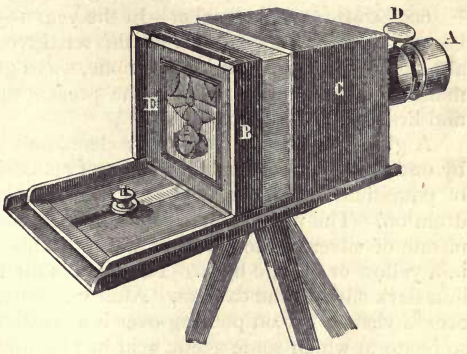


Fig. 588.

The glass slide is then replaced by the case containing the sensitive plate: the slide which protects it is raised, and the plate exposed for a given time, which varies with the amount and nature of light reflected from the object, the size of the aperture of the lens, and other causes. The plate is now developed by submitting its surface to the action of mercury vapour. If correctly exposed, a brilliant image will appear. This development is due to the fact that mercury vapour will attack all the bromiodide of silver, which has been reduced by the light to a sub-bromiodide, while the unreduced bromiodide is unaffected by the mercury. In order to 'fix' the plate, this latter salt must be dissolved off, or it would

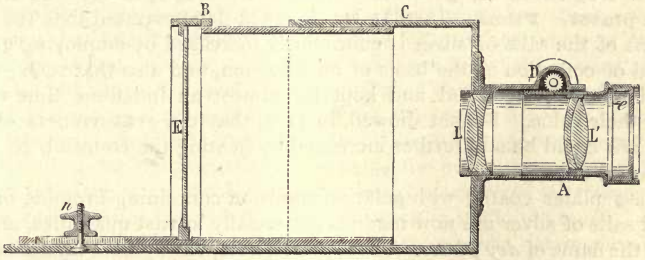


Fig. 589.

darken when exposed to light. The plate is therefore placed in a solution of hyposulphite of soda, which dissolves out all the bromiodide of silver, leaving the image as a white amalgamated deposit of mercury and silver. The picture, after washing and drying, is varnished.

Contemporaneously with Daguerre, Fox Talbot introduced his *calotype* process, a method possessing many advantages over that of Daguerre, and which is employed occasionally even at the present day. In the latter process the images are produced directly on metal plates, the lights and shades being in their natural position (positive pictures); but in Talbot's process the images are produced on paper, and the lights and shadows re-

versed—*i.e.* the shadows appear transparent, and the lights dark (negative pictures). In order to obtain positive copies, Talbot made his paper negatives transparent by melting a thin layer of wax over them with a hot iron; they could then be used for printing like glass negatives are now.

609. **Collodion pictures.**—In the year 1850 Scott Archer and Fry made use of collodion as a vehicle for the sensitive salt. This beautiful process, although a slow and cumbersome one, was a great advance over the previous methods, and is unsurpassed at the present day by any process for delicacy and brilliancy of image.

A glass plate made chemically clean, and polished with a soft rag, is held by one of its corners while a stream of collodion holding iodide and bromide of potassium in solution is poured over the plate and the excess allowed to drain off. The plate is then immersed in a bath containing 30 grains of nitrate of silver to the ounce of water. This operation must be performed in a yellow or orange light. The plate is then removed, drained, and placed in a dark slide in the camera. After exposure, no change on the plate whatever is visible, but on pouring over it a solution of pyrogallic acid, or ferrous sulphate to which some acetic acid has been added as a restrainer, the image slowly appears. When all the details are visible, the plate is fixed by immersion in a bath of hyposulphite of soda or cyanide of potassium.

In 1864 Bolton and Sayce made a further advance. Instead of coating the plate with iodised collodion and then immersing in the silver bath, they discovered that this latter operation, which was open to many objections could be entirely done away with by previously impregnating the collodion with the silver salts, and then letting it flow over the plate. By increasing the amount of silver nitrate and bromide, and diminishing the iodide, the rapidity was materially increased.

610. **Negatives on gelatine emulsions. Dry plates. Orthochromatic plates. Films.**—In 1871 Dr. Maddox demonstrated that the sensitiveness of the salts of silver is enormously increased by employing gelatine instead of collodion as the basis of an emulsion, and also that such gelatine emulsions could be dried and kept for almost an indefinite time without losing their value. Bennet showed, in 1878, that the sensitiveness of these emulsions could be still further increased by heating the emulsion to 32° C. for several days.

Glass plates coated with gelatine emulsion containing bromide or other haloid salts of silver are now made commercially in vast quantities, and sold under the name of *dry plates*. These plates, if kept away from injurious fumes and all traces of light and moisture, will keep good for months, or even years, and are ready for use at any moment. These advantages, together with their rapidity, have in a great measure led to their universal adoption, to the exclusion of the collodion or 'wet' plates.

In 1878 Abney showed that gelatine bromide plates could be made sensitive to the red and infra-red rays of the spectrum.

In 1881 Dr. Eder found that a bromiodide emulsion could be made sensitive to a much greater range of the spectrum by adding a minute proportion of eosin, or other red aniline dye. These *orthochromatic* plates, as they are called, are not merely sensitive to the violet and blue rays, but are highly sensitive to rays about the D line of the spectrum, and thus yellow objects,

instead of appearing black, or blue objects appearing white, as in photographic prints from ordinary plates, appear in their true visible relation of brightness to one another.

In 1889 Eastman introduced transparent celluloid as a support for the gelatine film in the place of glass. This has been made sufficiently thin to allow of strips many yards in length being rolled on a reel. By means of a box of peculiar construction, termed a *rollholder*, successive portions of the film can be unwound and exposed in the camera like an ordinary glass plate.

Lastly, in 1890, Lippmann of Paris obtained permanent images of the colours of the spectrum on gelatinochloride plates.

611. **Photographic printing.**—When once a negative is obtained by any of these processes it may be used for printing an indefinite number of positive pictures. For this purpose paper is coated with a layer of egg-albumen containing a certain proportion of chloride of ammonium, and is then left to dry. The paper is then made sensitive by floating it on a bath of solution of nitrate of silver. Chloride of silver is formed by the double decomposition of the two salts, and this again acting on the albumen forms an obscure compound containing chloride and albuminate of silver. The negative is placed on a sheet of paper in the copying frame, and exposed to the action of light for a certain time. The chloride of silver becomes acted upon—the light parts of the negative being most affected, and the dark parts least so. A copy is thus obtained, on which the lights of the negative are replaced by shades, and inversely. The picture is then immersed in a bath of chloride of gold, which gives it its tone, and preserves it from fading. In order to fix the picture it is now immersed in a solution of hyposulphite of sodium, which dissolves the unaltered chloride of silver. The print must now be washed in a stream of water for several hours in order to get rid of all traces of the hyposulphite, which if left in would ruin the picture.

Of late years permanent and very beautiful prints have been obtained by utilising the chemical change produced by light on a mixture of bichromate of potass and gelatine. On this reaction are based the various carbon processes, and those for mechanical printing. Very beautiful prints, with an effect resembling that of steel engravings, are produced by what is known as the *platinum process*. This consists in exposing paper charged with ferric oxalate to the light, and then developing the prints thus produced by a platinum salt; the ferric salt by exposure to light is reduced to ferric oxalate, which in turn reduces the platinum salt to black metallic platinum. Prints similar in appearance but more varied in tone can also be obtained on paper specially coated with a solution of silver bromide, which are developed in the same way as an ordinary dry-plate. In this case the paper may be either exposed in a dark slide in the camera, and the negative placed in a good light and copied by the lens, or the paper may be placed in direct contact with the negative, and exposed for a few seconds to artificial light while in the printing frame; the result after development being in either case the same. This is known as the 'bromide' process, and is much in favour for printing enlargements of negatives. The great advantage of these last-mentioned processes over the ordinary silver prints lies in the fact that they do not fade with time.

CHAPTER VI

THE EYE CONSIDERED AS AN OPTICAL INSTRUMENT

612. **Structure of the human eye.**—The eye is placed in a bony cavity called the orbit; it is maintained in its position by the muscles which serve to move it, by the optic nerve, the conjunctiva, and the eyelids.

Fig. 590 represents a transverse section of the eye from back to front. The general shape is that of a sphere, or, more strictly speaking, it consists of the segments of two spheres of unequal size, of which the anterior is much the smaller and constitutes the *cornea*, while the posterior, forming the chief envelope of the eyeball, receives the name of the *sclerotic*. The eye is composed of the following parts: the *cornea*, the *sclerotic*, the *iris*, the *pupil*, the *aqueous humour*, the *crystalline*, the *vitreous body*, the *hyaloid membrane*, the *choroid*, the *retina*, and the *optic nerve*.

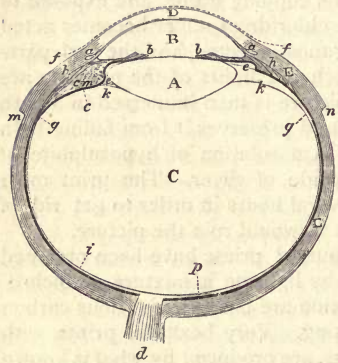


Fig. 590.

Cornea.—The cornea, D, is a transparent circular tunic forming the anterior segment of the eye. It is nothing more than a continuation of the sclerotic forwards, and is formed by the fibres of the latter becoming more systematically arranged and rendered quite transparent. Its front surface is lined throughout by the *conjunctiva*. This is a soft membrane which not only covers the cornea, but, passing in a loose fold to the circumference of the orbit, is reflected over the under surface of the lids, thus completely closing-in the cavity of the eyeball, and yet being so loose that the eye can roll freely in its socket. The two surfaces of the cornea are so nearly parallel that optically they may be considered as a single surface.

Sclerotic.—This (fig. 590, E) is a strong tough tunic enveloping the whole of the eye behind the cornea. At its back part it is reflected over the optic nerve, forming a sheath for it as far as the apex of the orbit. The chief functions of the sclerotic are to maintain the shape of the organ, and to protect it from injury and pressure.

Iris.—The iris, *bb*, is an annular, opaque diaphragm, placed between the cornea and the crystalline lens. It constitutes the coloured part of the eye, and is perforated by an aperture called the *pupil*, which in man is circular. In some animals, especially those belonging to the genus *Felis*, it is narrow and elongated in a vertical direction; in the ruminants it is elongated in a

transverse direction. It contains a large number of muscular fibres, which are disposed partly as a narrow ring close to the pupil, called the *sphincter iridis*, and partly in the form of fibres radiating from the circumference to the sphincter, called the *dilator iridis*. Thus, as the one or the other set of fibres are stimulated, the pupil is able to contract or dilate. The diameter of the pupil is constantly varying, the variation ranging from $\frac{1}{3}$ to $\frac{1}{20}$ of an inch, but these limits may be exceeded. In total darkness the pupil is enlarged to its utmost limits, but it contracts instantly in a bright light. The movements of the iris are involuntary.

It appears from this description that the iris is a screen with a variable aperture, whose function is to regulate the quantity of light which penetrates into the eye; for the size of the pupil diminishes as the intensity of light increases. The iris serves also to correct the spherical aberration, as it prevents the marginal rays from passing through the edges of the crystalline lens. It thus plays the same part with reference to the eye that a stop does in optical instruments (558).

Aqueous humour.—Between the posterior part of the cornea and the front of the crystalline there is a transparent liquid called the aqueous humour. The space, B, occupied by this humour is called the *anterior* in contradistinction to the *posterior chamber*, a space which the older anatomists imagined to exist between the iris and the capsule of the lens. But inasmuch as later observations have shown that the iris lies in contact with the lens in its capsule throughout the greater part of its length, this space has no practical existence.

Crystalline lens.—This (fig. 590, A) is a double convex transparent body placed immediately behind the iris, which it supports, though not attached to it. The lens is enclosed in a transparent membrane, called its *capsule*. The structure of the lens can be best seen by boiling it in water, which converts it into a hard opaque mass. A succession of concentric laminæ, like the coats of an onion, may be stripped off, leaving a hard central spherical nucleus of the same material. These laminæ increase in density and refracting power from the circumference to the centre. They consist entirely of long ribbon-shaped fibres, which overlap one another concentrically, and are united together by a kind of cement. Optically, the lens may be considered as a system made up of a biconvex lens of high refracting power and short focal length, enclosed by two meniscus lenses of lower refracting power. Opticians have constructed achromatic lenses on the same lines for photographic purposes by cementing two meniscus crown lenses to an intermediate biconvex flint lens.

To the anterior surface of the capsule, near its margin, is fixed a firm transparent membrane, and known as the *suspensory ligament* or *zonule of Zinn*, *ee*, which is attached behind to the front of the hyaloid membrane, and indirectly to the ciliary muscle, *m*. This ligament exerts traction, all round, on the front surface of the lens, and renders it less convex than it would otherwise be, and its relaxation plays an important part in the adaptation of the eye for sight at different distances.

Vitreous body. Hyaloid membrane.—The vitreous body, or vitreous humour, is a transparent gelatinous mass resembling the white of an egg, which occupies all the part of the ball of the eye, C, behind the lens. The

vitreous humour is surrounded by the *hyaloid membrane*, *kk*, which lines the posterior face of the crystalline capsule, and also the inner face of another membrane called the retina.

Retina. Optic nerve.—The retina, *i*, is the name given to a layer of specially modified cells which receives the impression of light. It is really nothing more than the terminal fibres of the optic nerve, altered in such a way as to be sensitive to the waves of light. Each optic nerve which conveys to the mind the impression produced by light arises from three centres in the brain, and the fibres, becoming collected into a thick cord, pass forward along the base of the brain. Here each cord after forming a junction with its fellow of the opposite side again separates, and, passing through a hole at the back of each socket, reaches and enters the eyeball, inside which it expands into a cup-shaped network of nerves called the *retina*. The nerve fibres themselves are not sensitive to light, but are only stimulated by it indirectly through the intervention of certain specially adapted cells. The fibres of the optic nerve, when they spread out to form the inner layer of the retina, after running a shorter or longer distance turn abruptly outward, and each fibre becomes connected with a larger ganglion cell, which again is connected by other processes with smaller cells; and each group of these finally ends in either a peculiarly shaped cylinder called a *rod*, or a thicker flask-shaped structure called a *cone*. All are ranged perpendicular to the surface of the retina, closely packed together, so as to form a regular mosaic layer when viewed from the outside. In the retina is a remarkable spot, *p*, which is situated in the axis of vision a little to the outside of the place where the optic nerve enters the eyeball. From its colour it is called the *macula lutea* or *yellow spot*. The retina is here somewhat thick, but in the middle of the yellow spot is found a depression, the *fovea centralis*, where the retina is reduced to those elements alone which are absolutely necessary for exact vision. This *fovea*, or pit of the retina, is of great importance for vision, since it is the spot where the most exact discrimination of distance is made. Only those parts of the retinal image which fall on the yellow spot are sharp, all the rest are more inaccurate the nearer they fall to the limits of the retina. The field of view of the eye is like a drawing, the centre of which is done with great accuracy and delicacy, while the surrounding part is only roughly sketched. Where the optic nerve enters there are no rods or cones; this part of the retina, therefore, is insensitive to light, and is called the *punctum cæcum* or *blind spot*. When examined in the living subject by means of the ophthalmoscope it appears as a slightly oval pinkish disc crossed by numerous blood-vessels.

The only property of the retina and optic nerve is that of receiving and transmitting to the brain the impression of objects. These organs have been cut and pricked without causing any pain to the animals submitted to these experiments; but there is reason to believe that irritation of the optic nerve causes the sensation of a flash of light.

Choroid.—The choroid, *g*, shown by a thick black line in the figure, is a membrane between the retina and the sclerotic. It is highly vascular, and supplies the nourishment necessary for the chemical and physiological processes concerned in vision. On its inner surface, and in close contact with the ends of the rods and cones, is a layer of densely black pigment cells,

which secrete a peculiar yellowish purple pigment called the *visual purple*, and which is rapidly bleached by light. It is evidently connected with the act of vision, but its precise use is uncertain.

The choroid forms a series of convoluted folds in front, called *ciliary processes*, which penetrate between the iris and the lens capsule, forming round the latter a disc resembling a radiated flower. The ciliary processes secrete the colourless fluid necessary for the nourishment of the transparent parts of the eye, which, being transparent for visual purposes, cannot be nourished by means of blood-vessels.

613. **Refractive indices of the transparent media of the eye.**—The refractive indices from air into the transparent parts of the eye were determined by Brewster, and have since been carefully examined by Von Helmholtz. Their results are contained in the following table, compared with water as a standard :—

	Brewster	Helmholtz
Water	1·3358	1·3358
Aqueous humour	1·3366	1·3365
Vitreous humour	1·3394	1·3365
Cornea	—	1·3365
External coating of the lens	1·3767	1·3930
Centre of the lens	1·3990	1·4541
Mean refraction of the lens	1·3839	1·4371

From this it will be noticed that, according to the latter, the refractive index of all the media excepting the lens is the same.

614. **Curvatures and dimensions of various parts of the human eye.**—According to the latest tables of Von Helmholtz (1888), these are :—

	mm
Radius of curvature of the cornea	7·83
„ „ anterior surface of the crystalline	10·00
„ „ posterior surface „ „	6·00
Distance from apex of the cornea to the anterior surface of the lens	3·60
„ „ „ posterior „ „	7·20
Thickness of the crystalline lens	3·60

615. **Path of rays in the eye.**—From what has been said as to the structure of the eye, it may be compared to a camera obscura (602), of which

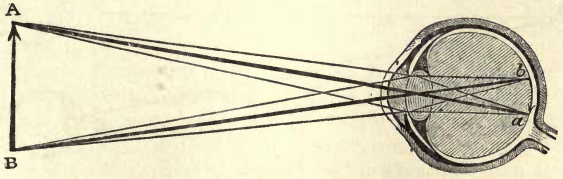


Fig. 591.

the pupil is the aperture, the crystalline is the condensing lens, and the retina is the screen on which the image is formed. Hence, the effect is the same as when the image of an object placed in front of a double convex lens is formed at its conjugate focus. Let AB (fig. 591) be an object placed

before the eye, and let us consider the rays emitted from any point of the object A. Of all these rays, those which are directed towards the pupil are the only ones which penetrate the eye, and are operative in producing vision. These rays, on passing into the aqueous humour, experience a first refraction which brings them near the secondary axis A*a* drawn through the optic centre of the crystalline; they then traverse the crystalline, which again refracts them like a double convex lens, and, having experienced a final refraction by the vitreous humour, they meet in a point *a*, and form the image of the point A. The rays issuing from the point B form in like manner an image of it at the point *b*, so that a very small, real, and inverted image is formed exactly on the retina, provided the eye is in its normal condition.

616. **Inversion of images.**—In order to show that the images formed on the retina are really inverted, the eye of an albino or any animal with pink eyes may be taken; this has the advantage that, as the choroid is destitute of pigment, light can traverse it without loss. This is then deprived at its posterior part of the cellular tissue surrounding it, and fixed in a hole in the shutter of a dark room; by means of a lens it may be seen that the inverted images of external objects are depicted on the retina.

The inversion of images in the eye has greatly occupied both physicists and physiologists, and many theories have been proposed to explain how it is that we do not see inverted images of objects. The chief difficulty seems to have arisen from the conception of the mind or brain as something behind the eye, looking into it, and seeing the image upon the retina; whereas really this image simply causes a stimulation of the optic nerve, which produces some molecular change in some part of the brain; and it is only of this change, and not of the image as such, that we have any consciousness. The mind has thus no direct cognisance of the image upon the retina, nor of the relative positions of its parts, and, sight being supplemented by touch in innumerable cases, it learns from the first to associate the sensations brought about by the stimulation of the retina (although due to an inverted image) with the correct position of the object as taught by touch.

617. **Optic axis, optic angle, visual angle.**—The *principal optic axis* of an eye is the axis of its figure; that is to say, the straight line in reference

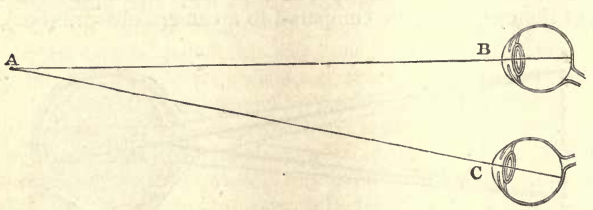


Fig. 592.

to which it is symmetrical. In a well-shaped eye it is the straight line passing through the centre of the pupil and of the crystalline. The lines A*a*, B*b* (fig. 592) are secondary axes. The eye sees objects most distinctly in the direction of the principal optic axis, since rays of light following this direction impinge upon the yellow spot where vision is most acute.

The *optic angle* is the angle BAC (fig. 592), formed between the principal optic axis of the two eyes when they are directed towards the same point. This angle is smaller in proportion as the objects are more distant.

The *visual angle* is the angle AOB (fig. 593), under which an object is seen; that is to say, the angle formed by the secondary axes drawn from the optic centre of the crystalline to the opposite extremities of the object. For the

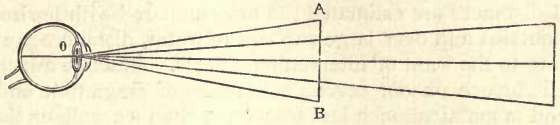


Fig. 593.

same distance, this angle increases with the magnitude of the object, and for the same object it decreases as the distance increases, as is the case when the object passes from AB to A′B′. It follows, therefore, that objects appear smaller in proportion as they are more distant; for as the secondary axes, AO, BO, cross in the centre of the crystalline, the size of the image projected on the retina depends on the size of the visual angle AOB.

618. **Estimation of the distance and size of objects.**—The estimation of distance and of size depends on numerous circumstances; these are—the visual angle, the optic angle, the comparison with objects whose size is familiar to us; to these must be added the effect of what is called *aërial perspective;* that is, a more or less vaporous medium which enshrouds the distant objects, and thereby diminishes not only the sharpness of the outlines, but also softens the contrast between light and shade, which close at hand are marked.

When the size of an object is known, as the figure of a man, the height of a tree or of a house, the distance is estimated by the magnitude of the visual angle under which it is seen. If its size is unknown, it is judged relatively to that of objects which surround it.

A colonnade, an avenue of trees, the gas-lights on the side of a road, appear to diminish in size in proportion as their distance increases, because the visual angle decreases; but the habit of seeing the columns, trees, &c., in their proper height, leads our judgment to rectify the impression produced by vision. Similarly, although distant mountains are seen under a very small angle, and occupy but a small space in the field of view, our familiarity with the effects of aërial perspective enables us to form a correct idea of their real magnitude.

As regards the estimation of near objects, the muscular sense of accommodation (620) and convergence (621) play a very important part. Thus it is well known that people who lose the sight of one eye experience great difficulty in estimating the distance of objects near at hand. This anyone can prove for himself by covering up one of his eyes, and then attempting to thread a needle. The explanation of this difficulty is very simple. When one eye is destroyed or covered up, then owing to the absence of binocular vision there is no optic angle, and, therefore, convergence ceases to be called into play. Now the two chief guides for estimating the distance of near

objects are the stereoscopic relief of objects due to binocular vision, and the muscular sense of convergence. When only one eye is used both these factors are wanting. Nevertheless, it is only by long custom that we can establish a relation between our distance from the objects and the corresponding motion of the eyes. It is a curious fact that persons born blind, and whose sight has been restored by the operation for cataract, imagine at first that all objects are at the same distance away.

Vertical distances are estimated too low compared with horizontal ones; on high mountains and over large surfaces of water, distances are estimated too low owing to the want of intervening objects. Practice and experience have great influence on our correct estimation of magnitude and distance. As we ascend mountains much less frequently than we walk on the level, we err more easily in estimating a height than in judging a horizontal distance. A room filled with furniture appears larger than an empty room of the same size.

We cannot recognise the true form of an object if, with moderate illumination, the visual angle is less than half a minute. A white square, a metre in the side, appears at a distance of about 5 miles under this angle as a bright spot which can scarcely be distinguished from a circle of the same size.

A very bright object, however, such as an incandescent platinum wire, is seen in a dark ground under an angle of 2 seconds. So, too, a small dark object is seen against a bright ground; thus a hair held against the sky can be seen at a distance of 1 or 2 metres.

618*a*. **Scheiner's experiment.**—If we look at a small object placed either within or beyond the point on which the eye is focussed, through a number of minute openings in a diaphragm, arranged so close together that they fall within the circumference of the pupil, the object appears multiple, each object furnishing a separate retinal image. This forms what is known as *Scheiner's experiment.* This may be made as follows: By means of a sewing needle, two small holes are pricked in a piece of cardboard, not more than $\frac{1}{16}$ of an inch apart, *i.e.* less than the diameter of the pupil. The card is held before one eye with the holes horizontally in front of the pupil, and with the other hand a needle is held at ordinary reading distance in the line of vision. If the eye be fixed on the needle itself, it appears single and clearly defined; as soon, however, as we look at a more distant object, the needle appears double, and at the same time blurred. If we block out the right-hand hole, the left-hand image disappears, and *vice versâ.*

If we now fix the eye on an object nearer than the needle, the latter again appears double and blurred, and blocking either hole causes the image on the same side to vanish. The explanation of these phenomena may be simplified by the following diagram.

Let AB (fig. 594) be the two holes in the card CC, O a luminous point in the needle, OA, OB the pencils of rays passing through the apertures in the card. Let H, E, M represent the position of a hypermetropic, normal, and myopic eye respectively. When the normal eye E is accommodated for O, the rays OA, OB meet at the point E, and the needle appears sharply defined and single. If the eye is fixed on a point beyond, or, what amounts to the same thing, if the eye be hypermetropic, the retina may be considered to lie no longer at E, but in front at H, and the rays OAp not only do not

meet in a focus at p, but do not meet the rays OBp'; hence the luminous spot O will be seen at two points, and the points themselves being out of focus will appear blurred. Moreover, the rays passing through the right-hand hole A will cut the retina at p, and will appear to the mind on the reverse side, *i.e.* on the left; therefore blocking the right-hand hole A causes a disappearance of the left-hand image, and *vice versâ*.

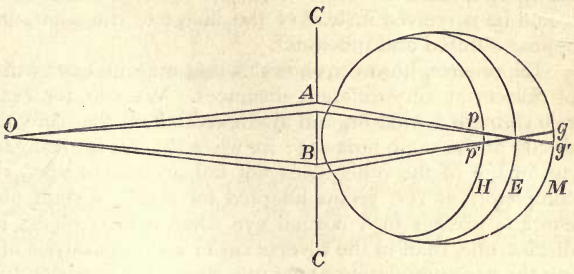


Fig. 594.

For similar reasons, if the eye be accommodated for a point nearer the eye than O, or, what amounts to the same thing, if the eye be myopic, the retina may be considered to lie behind E at M, and the image will again be seen doubled and blurred; only in this case blocking out the right-hand hole A will cause the right-hand image to disappear. Stampfer constructed an optometer based on this principle. He employed a tube containing two diaphragms, one furnished with two slits 1^{mm} apart, the other with a single slit covered with ground glass. The diaphragm is moved to or from the eye until the slit is seen single. This distance from the eye is the measure of distinct vision.

619. **Distance of distinct vision.**—The *distance of distinct vision*, as already stated (587), is the distance at which objects must be placed so as to be seen with the greatest distinctness. It varies in different individuals, and in the same individual it is often different in the two eyes. For small objects, such as print, it is from 10 to 12 inches in normal cases.

Persons who see objects distinctly only at a very short distance away are called *myopic*, or *short-sighted*, and those who require a convex glass to see objects distinctly at a long distance are *hypermetropic*, or *long-sighted* (629).

Sharpness of sight may be compared by reference to that of a normal eye taken as a unit. Such a standard eye, according to Snellen, recognises quadrangular letters when they are seen under an angle of 6′; if, for instance, such letters are 1^{cm} high at a distance of 6 metres. The sharpness of vision of one who recognises these letters at a distance of 6 metres is then said to be $\frac{6}{6}$, and in like manner, if the letters can only be distinctly seen when they subtend an angle of 9′, 12′, or 18′, the sharpness of vision would be indicated by the equation $V = \frac{6}{9}, \frac{6}{12}$, and $\frac{6}{18}$ respectively.

620. **Accommodation.**—By this term is meant the changes which occur in the eye to fit it for seeing distinctly objects at different distances from it.

If the eye be supposed fixed and its parts immovable, it is evident that there could only be one surface whose image would fall exactly upon the retina; the distance of this surface from the eye being dependent on the refractive indices of the media and the curvatures of the refractive surfaces

of the eye. The image of any point nearer the eye than this distinctly seen surface would fall behind the retina ; the image of any more distant point would be formed in front of it ; in each case the section of a luminous cone would be perceived instead of the image of the point, and the latter would appear diffused and indistinct.

Experience, however, shows us that a normal eye can see distinct images of objects at very different distances. We can, for example, see a distant tree through a window, and also a scratch on the pane, though not both distinctly at the same moment ; for when the eye is arranged to see one clearly, the image of the other does not fall accurately upon the retina. An eye completely at rest seems adapted for seeing distant objects ; the sense of effort is greater in a normal eye when a near object is looked at, after a distant one, than in the reverse case ; and in paralysis of the nerves governing the accommodating apparatus, the eye is persistently adapted for distant sight. There must, therefore, be some mechanism in the eye by which it can be voluntarily altered, so that the more divergent rays proceeding from near objects shall come to a focus upon the retina. There are several conceivable methods by which this might be effected ; it is actually brought about by a drawing forwards of the crystalline lens and a greater convexity of its front surface.

This is shown by the following experiment :—If a candle be placed on one side of the eye of a person looking at a distant object, and his eye be observed from the other side, three distinct images of the flame will be seen : the first, virtual and erect, is reflected from the anterior surface of the cornea ; the next, erect and less bright, is reflected from the anterior surface of the lens ; the third, inverted and brilliant, is formed on the posterior surface of the lens. If now the person look at a near object, no change is observed in the first and third images, but the second image becomes smaller and approaches the first ; which shows that the anterior surface of the crystalline lens becomes more convex and approaches the cornea. In place of the candle, Von Helmholtz throws light through two holes in the screen upon the eye, and observes the distance on the eye between the two shining points, instead of the size of the flame of the candle.

This change in the lens is effected chiefly by means of a circular muscle (ciliary muscle), the contraction of which relaxes the suspensory ligament, and so allows the front surface of the lens to assume more or less of that greater convexity which it would normally exhibit were it not for the drag exercised upon it by the ligament. Certain other less important changes occur, tending to make the lens more convex and to push it forwards, which cannot, however, be explained without entering into minute anatomical details. When the eye is accommodated for near vision, the pupil contracts and so partially remedies the greater spherical aberration.

The *range of accommodation*, called by Donders $\frac{1}{A}$, is measured by first of all determining the greatest distance, R, at which a person can see distinctly. In this case the ciliary muscle is in a state of rest, that is, the accommodation is relaxed to the utmost. The shortest distance, P, is observed at which the person can see distinctly. In this case the ciliary muscle is contracted to its utmost power, that is, the accommodation has arrived at its maximum. The total accommodation power an eye can bring into play

is therefore represented by the difference between the refraction of the eye when at rest, and when it is at the maximum effort of accommodation; then

$$\frac{I}{A} = \frac{I}{P} - \frac{I}{R}.$$

Here each of the three terms represent a lens expressed by the inverse of its focal distance.

By the dioptric method (630) this formula is simplified, and becomes

$$a = p - r,$$

where a is the number of diopters represented by the range of accommodation; p = the number represented by the eye when accommodated to its maximum, and r = the number of diopters when relaxed to see the *punctum remotum.*

621. **Binocular vision.**—A single eye sees most distinctly any point situated on its optical axis, and less distinctly other points also, towards which it is not directly looking, but which are still within its circle of vision.

It is able to judge of the *direction* of any such point, but unable by itself to estimate its *distance.* Of the distance of an *object* it may, indeed, learn to judge by such criteria as loss of colour, indistinctness of outline, decrease in magnitude, &c.; but if the object is near, the single eye is not infallible, even with these aids.

When the two eyes are directed upon a single point, we then gain the power of judging of its distance as compared with that of any other point, and this we seem to gain by the sense of greater or less effort required in causing the optical axes to converge upon the one point or upon the other. Now a solid object may be regarded as composed of points which are at different distances from the eye. Hence, in looking at such an object, the axes of the two eyes are rapidly and insensibly varying their angle of convergence, and we as rapidly are gaining experience of the difference in distance of the various points of which the object is composed, or, in other words, an assurance of its solidity. Such kind of assurance is necessarily unattainable in monocular vision.

622. **The principle of the stereoscope.**—Let any solid object, such as a small box, be supposed to be held at some short distance in front of the two eyes. On whatever point of it they are fixed, they will see that point the most distinctly, and other points more or less clearly. But it is evident that, as the two eyes see from different points of view, there will be formed in the right eye a picture of the object different from that formed in the left; and it is by the apparent union of these two dissimilar pictures that we see the object in relief. If, therefore, we delineate the object, first as seen by the right

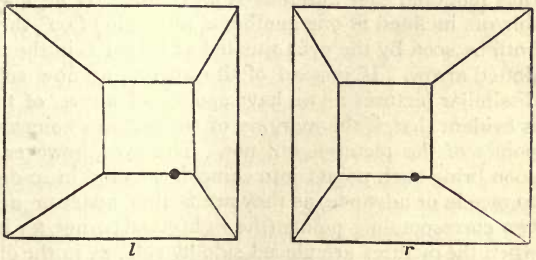


Fig. 595.

eye, and then as seen by the left, and afterwards present these dissimilar pictures again to the eyes, taking care to present to each eye that picture which was drawn from its own point of view, there would seem to be no reason why we should not see a representation of the object, as we saw the object itself, in relief. Experiment confirms the supposition. If the object held before the eyes were a truncated pyramid, *r* and *l*, fig. 595, would represent its principal lines, as seen by the right and left eyes respectively. If a card is held between the figures, and they are steadily looked at, *r* by the right eye, and *l* simultaneously by the left, for a few seconds, there will be seen a single picture having the unmistakable appearance of relief. Even without a card interposed, the eye, by a little practice, may soon be taught so to combine the two as to form this solid picture. Three pictures will in that case be seen, the central one being solid, and the two outside ones plane. Fig. 596 will explain this. Let *r* and *l* be any two corresponding points, say the points marked by a large dot in the figures drawn above; R and L the positions of the right and left eyes; then the right eye sees the point *r* in the direction R*o*, and the left eye the point *l* in the direction L*o*, and accordingly each by itself judging only by the direction, they together see these two points as one, and imagine it to be situated at *o*. But the right eye, though looking in the direction R*r*, also receives an image of *l* on another part of the retina, and the left eye in the same way an image of *r*, and thus three images are seen. A card, however, placed in the position marked by the dotted line will, of course, cut off the two side pictures. To assist the eye in combining such pairs of dissimilar pictures, both mirrors and lenses have been made use of, and the instruments in which either of these are adapted to this end are called *stereoscopes*.

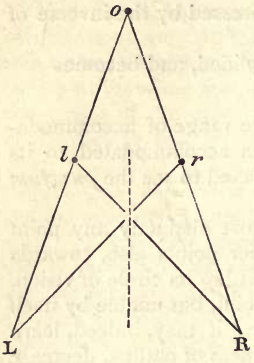


Fig. 596.

623. **The reflecting stereoscope.**—In the reflecting stereoscope plane mirrors are used to change the apparent position of the pictures, so that they are both seen in the same direction, and their combination by the eye is thus rendered easy and almost inevitable. If *ab*, *ab* (fig. 597) are two plane mirrors inclined to one another at an angle of 90°, the two arrows, *x*, *y*, would both be seen by the eyes situated at R and L in the position marked by the dotted arrow. If, instead of the arrows, we now substitute such a pair of dissimilar pictures as we have spoken of above, of the same solid object, it is evident that, if the margins of the pictures coincide, other corresponding points of the pictures will not. The eyes, however, almost without effort, soon bring such points into coincidence, and in so doing make them appear to recede or advance, as they are farther apart or nearer together than any two corresponding points (the right-hand corner, for instance) of the margins when the pictures are placed side by side, as in the diagram, fig. 597. It will be plain, also, on considering the position for the arrows in fig. 597, that to adapt such figures as those in fig. 596 for use in a reflecting stereoscope one of them must be reversed, or drawn as it would be seen through the paper if held up to the light.

624. **The refracting stereoscope.**—Since the rays passing through a convex lens are bent always towards the thicker part of the lens, any segment of such a lens may be readily adapted to change the apparent position of any object seen through it. Thus, if (fig. 598) two segments be cut from a double convex lens, and placed with their edges together, the arrows, x, y,

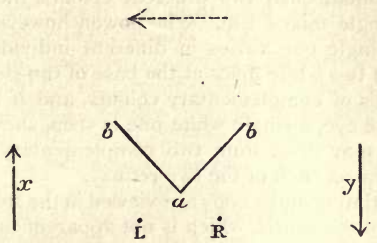


Fig. 597.

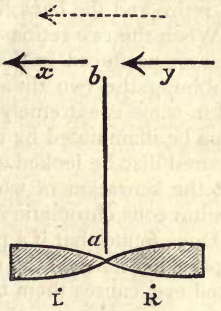


Fig. 598.

would both be seen in the position of the dotted arrow by the eyes at R and L.

If we substitute for the arrows two dissimilar pictures of the same solid object, or the same landscape, we shall then, if a diaphragm, *ab*, be placed between the lenses to prevent the pictures being seen crosswise by the eyes, see but one picture, and that apparently in the centre, and magnified. As before, if the margins are brought by the power of the lenses to coincide, other corresponding points will not be coincident until combined by an almost insensible effort of the eyes. Any pair of corresponding points which are farther apart than any other pair will then be seen farther back in the picture, just as any point in the background of a landscape would be found (if we came to compare two pictures of the landscape, one drawn by the right eye, and the other by the left) to be represented by two points farther apart from one another than two others which represented a point in the foreground.

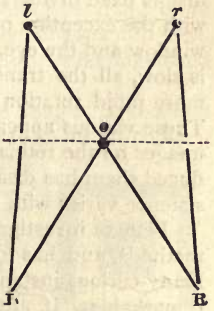


Fig. 599.

It will be instructive to notice that there is also a second point on *this side* of the paper, at which, if a person look steadily, the diagrams in fig. 599 will combine, and form quite a different stereoscopic picture. Instead of a solid pyramid, a hollow pyramidal box will then be seen. The point may easily be found by experiment. Here again two external images will also be seen. If we wish to shut these out, and see only their central stereoscopic combination, we must use a diaphragm of paper held parallel to the plane of the picture with a square hole in it. This paper screen must be so adjusted that it may conceal the right-hand figure from the left eye, and the left-hand figure from the right eye, while the central stereoscopic picture may be seen through the hole. It will be plain from

the diagram (fig. 599) that *o* is the point to which the eyes must be directed, and at which they will imagine the point to be situated, which is formed by the combination of the two points *r* and *l*. The dotted line shows the position of the screen. A stereoscope with or without lenses may easily be constructed, which will thus give us, with the ordinary stereoscopic slides, a reversed picture ; for instance, if the subject be a landscape, the foreground will retire and the background come forward.

When the two retinas view simultaneously two different colours, the impression produced is that of a single mixed tint. The power, however, of combining the two tints into a single one varies in different individuals, and in some is extremely weak. If two white discs at the base of the stereoscope be illuminated by two pencils of complementary colours, and if each coloured disc be looked at with one eye, a single white one is seen, showing that the sensation of white light may arise from two complementary and simultaneous chromatic impressions on each of the two retinas.

Dove found that if a piece of printing and a copy are viewed in the stereoscope, a difference in the distance of the words, which is not apparent to the naked eye, causes them to stand out from the plane of the paper.

625. **Persistence of impressions on the retina.**—When an ignited piece of charcoal is rapidly rotated, we cannot distinguish it ; the appearance of a circle of fire is produced ; similarly, rain, in falling drops, appears in the air like a series of liquid threads. In a rapidly rotating toothed wheel the individual teeth cannot be seen. But if, during darkness, the wheel be suddenly illuminated, as by the electric spark, the individual parts may be clearly made out. The following experiment is a further illustration of this property :—A series of equal sectors are traced on a disc of glass, and they are alternately blackened ; in the centre there is a pivot, on which a second disc is fixed of the same dimensions as the first, but completely blackened with the exception of a single sector ; then, placing the apparatus between a window and the eye, the second disc is made to rotate. If the movement is slow, all the transparent sectors are seen, but only one at a time ; by a more rapid rotation we see simultaneously two, three, or a greater number. These various appearances are due to the fact that the impression of these images on the retina remains for some time after the object which has produced them has disappeared or become displaced. The duration of the persistence varies with the sensitiveness of the retina and the intensity of light.

Plateau investigated the duration of the impression by numerous similar methods, and has found that it is, on the average, half a second. Among many curious instances of these phenomena, the following is one of the most remarkable. If, after having looked at a brightly illuminated window, the eyes are suddenly closed, the image remains for a few instants—that is, a sashwork is seen consisting of luminous panes surrounded by dark frames ; after a few seconds the colours become interchanged, the same framework is seen, but the frames are now bright, and the glasses are perfectly black ; this new appearance may again revert to its original appearance.

The impression of colours remains as well as that of the form of objects ; for if circles divided into sectors are painted in different colours, they become confounded, and give the sensation of the colour which would result from their mixture. Yellow and red give orange ; blue and red violet ; the seven

colours of the spectrum give white, as shown in Newton's disc (fig. 533). This is a convenient method of studying the tints produced by mixed colours.

A great number of pieces of apparatus are founded on the persistence of sensation on the retina; such are the *thaumatrope*, the *phenakistoscope*, *Faraday's wheel*, the *kaleidophone*, and the *zoetrope*.

The *zoetrope*, or *wheel of life*, is very convenient for representing a number of optical, acoustical, and other vibratory motions. It consists of an open cylinder which can be rotated about its vertical axis. At the top are a number of vertical slits. If now the various positions of a vibrating pendulum, for instance, are drawn on a narrow strip of paper, the length of which is equal to the circumference, and this is placed inside the cylinder, when the wheel is rapidly rotated, on looking through the slits the pendulum seems as if it were steadily vibrating.

626. **Accidental images.**—When a coloured object placed upon a black ground is steadily looked at for some time, the eye is soon tired, and the intensity of the colour is enfeebled; if now the eyes are directed towards a white sheet, or to the ceiling, an image will be seen of the same shape as the object, but of the complementary colour (570); that is, such a one as united to that of the object would form white. For a green object the image will be red; if the object is yellow, the image will be violet.

Accidental colours are of greater duration in proportion as the object has been more brilliantly illuminated, and has been longer looked at. When a lighted candle has been looked at for some time, and the eyes are turned towards a dark part of the room, the appearance of the flame remains, but it gradually changes colour; it is first yellow, then it passes through orange to red, from red through violet to greenish blue, which is gradually feebler until it disappears. If the eye which has been looking at the light be turned towards a white wall, the colours follow almost the opposite direction: there is first a dark picture on a white ground, which gradually changes into blue, is then successively green and yellow, and ultimately cannot be distinguished from a white ground.

The reason of this phenomenon is, doubtless, to be sought in the fact that the action of light on the retina is not of equal duration for all colours, and that the decrease in the intensity of the subsequent action does not follow the same law for all colours. According to Külp, the durations of the after-image with moderate illumination are for white yellow, red, and blue, 0·1, 0·09, 0·08, and 0·066 of a second respectively.

Fig. 600.

627. **Irradiation.**—This is a phenomenon in virtue of which white objects, or those of a very bright colour, when seen on a dark ground, appear larger than they really are. Thus a white square upon a black ground seems larger than an exactly equal black square upon a white ground (fig. 600). Irradiation arises from the fact that the impression produced on the retina extends beyond the outline of the image. It bears the same relation to the space occupied by the image, that the duration of the impression does to the time during which the image is seen.

The effect of irradiation is very perceptible in the apparent magnitude of stars, which may thus appear much larger than they really are; also in the appearance of the moon when two or three days old, the brightly illuminated crescent seeming to extend beyond the darker portion of the disc, and hold it in its grasp.

Plateau found that irradiation differs very much in different people, and even in the same person on different days. He also found that irradiation increases with the lustre of the object, and the length of time during which it is viewed. It manifests itself at all distances; diverging lenses increase and condensing lenses diminish it.

Accidental haloes are the colours which, instead of succeeding the impression of an object like accidental colours, appear round the object itself when it is looked at fixedly. The impression of the halo is the opposite to that of the object: if the object is bright the halo is dark, and *vice versâ*. These appearances are best produced in the following manner:—A white surface, such as a sheet of paper, is illuminated by coloured light, and a narrow opaque body held so as to cut off some of the coloured rays. In this manner a narrow shadow is obtained which is illuminated by the surrounding white daylight, and appears complementary to the coloured ground. If red glass is used, the shadow appears green, and blue when a yellow glass is used.

The *contrast of colours* is a reciprocal action exerted between two adjacent colours, and in virtue of which to each one is added the complementary colour of the other. Chevreul found that when red and yellow colours are adjacent, red acquires a violet and yellow an orange tint. If the experiment is made with red and blue, the former acquires a yellow, and the latter a green tint; with yellow and blue, yellow passes to orange, and blue towards indigo; if a narrow strip of grey paper be laid on a sheet of light green paper, it appears reddish, if laid on blue paper it seems yellow, and so on for a vast number of combinations; in all cases the colour is complementary to the colour of the base. The importance of this phenomenon in its application to the manufacture of coloured cloths, carpets, curtains, and other fabrics may be readily conceived.

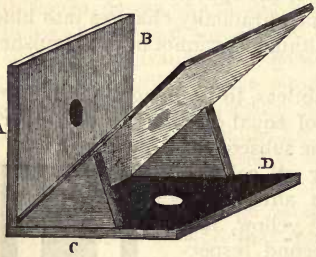


Fig. 601.

The contrast may be conveniently examined by means of the apparatus shown in fig. 601 in about $\frac{1}{3}$ scale. It consists of a thin vertical board, AB, painted white, and the base, DC, painted black, on which are painted circles about $\frac{3}{8}$ of an inch in diameter, black and white respectively. A sheet of coloured glass is inclined at an angle of 45°; if now the eye be so held that the image of the white circle on DC reflected from the under surface of the glass plate is looked at in front of the circle on AB, the image appears of a colour complementary to that of the glass. Thus with a green plate a red spot is seen on a green ground.

628. **The eye is not achromatic.**—It had long been supposed that the human eye was perfectly achromatic; but this is clearly impossible, as all

the refractions are made the same way, viz. towards the axis; moreover, the experiments of Wollaston, of Young, of Fraunhofer, and of Müller have shown that it is not true in any absolute sense.

Fraunhofer showed that in a telescope with two lenses, a very fine wire placed inside the instrument in the focus of the object-glass is seen distinctly through the eyepiece, when the telescope is illuminated with red light; but it is invisible by violet light even when the eyepiece is in the same position. In order to see the wire again, the distance of the lenses must be diminished to a far greater extent than would correspond to the degree of refrangibility of violet light in glass. In this case, therefore, the effect must be due to a chromatic aberration in the eye.

Müller, on looking at a white disc on a dark ground, found that the image is sharp when the eye is accommodated to the distance of the disc—that is, when the image forms on the retina; but he found that, if the image is formed in front of or behind the retina, the disc appears surrounded by a very narrow blue edge. If a finger be held up in front of one eye (the other being closed) in such a manner as to allow the light to enter only one half of the pupil, and, of course, obliquely, and the eye be then directed to any well-defined line of light, such as a slit in the shutter of a darkened room, or a strip of white paper on a black ground, this line of light will appear as a complete spectrum.

Müller concluded from these experiments that the eye is sensibly achromatic as long as the image is received at the focal distance, or when it is accommodated to the distance of the object. The cause of this apparent achromatism cannot be exactly stated. It has generally been attributed to the tenuity of the luminous beams which pass through the pupillary aperture, and because these unequally refrangible rays, meeting the surfaces of the media of the eye almost at the normal incidence, are very little refracted, from which it follows that the chromatic aberration is imperceptible (584).

Spherical aberration, as we have already seen, is corrected by the iris (612). The iris is, in point of fact, a diaphragm, which stops the marginal rays and only allows those to pass which are near the axis.

629. **Short sight and long sight. Emmetropia. Ametropia. Myopia and hypermetropia. Astigmatism. Presbyopia.**—The most usual deviations of the refraction of the eye are *myopia*, *hypermetropia*, *presbyopia*, and *astigmatism*. *Myopia*, or short sight, is the inability to see objects clearly defined beyond a variable but always limited distance. The usual cause of myopia is an abnormal increase in length of the eyeball along the axis of vision, so that the retina lies behind the focus of the dioptric systems of the eye for parallel rays, thereby rendering objects on the retina indistinct. It may be remedied by means of diverging concave glasses (630), which, in making the rays deviate from their common axis, throw the focus farther back, and cause the image to be formed on the retina.

An eye which, when the accommodation is completely relaxed, can see distant objects distinctly—in other words, one in which parallel rays are sharply focussed on the retina—is said to be *emmetropic*. All deviations from the normal refraction are included in the term *ametropia*.

The habitual contemplation of small objects, sedentary occupations, a stooping position while studying, in fact anything which tends to congest

the eyes, and cause an unequal strain on the muscles of convergence, may produce short sight. It is common in the case of young people, and, when once acquired, tends to become hereditary; hence the percentage of myopes is continually on the increase in countries where education is becoming more advanced.

Hypermetropia, or long sight, is due to causes contrary to those which produce short sight. The eye is abnormally short along the axis of vision, so that the retina lies in front of the dioptric system of the eye for parallel rays, thereby rendering objects on the retina indistinct unless the rays be rendered more convergent by exerting the muscles of accommodation. Hence the ciliary muscle can never be relaxed without the image becoming blurred, even when looking at distant objects. When regarding near objects, however, the accommodation has to be brought into play, not from a position of rest, but from the state of contraction of the ciliary muscle, which was necessary to see distant objects clearly. Owing to this increased strain of accommodation, the eye becomes easily fatigued when regarding near objects, which thus become blurred. Hypermetropia is corrected by means of converging (convex) lenses. These glasses converge the rays before their entrance into the eye, and, therefore, if the converging power is properly chosen, the image will be formed exactly on the retina.

On applying a few drops of solution of atropine to the eye, the amount of hypermetropia will be found to be considerably increased. This additional increase is termed *latent* hypermetropia, in contradistinction to the amount of hypermetropia discovered before the atropine was applied, or *manifest* hypermetropia.

Presbyopia.—As we grow older, the range of accommodation—in other words, the power of focussing near objects—decreases. Now there comes a time with everyone who is not myopic when an object cannot be distinctly seen nearer than eight inches (the distance arbitrarily chosen by Donders). This occurs in a normal or *emmetropic* eye at forty years of age. Hence presbyopia, as it is called, may be defined as the contraction of the visual range due to physiological weakening of the accommodating mechanism. It is clear, according to the standard of Donders, that it can never occur in very short-sighted persons, as their near point is always much less than eight inches to begin with, whereas in hypermetropes, on the other hand, it may become evident at a much earlier age. Presbyopia is corrected by suitable convex glasses, which, by converging the rays, bring the point of near vision to eight inches. Donders found that the failure of accommodation needed to be compensated by a convex lens of one diopter for every five years of life after forty.

Astigmatism.—We have hitherto considered the dioptric surfaces as portions of true spheres. Should, however, one of the surfaces have a curve of shorter radius in one of its meridians, all the rays from a luminous point cannot be focussed on the same plane, but will possess two linear foci, one anterior corresponding to the curvature of shorter radius, and the other behind corresponding to the curve of greater radius. This defect is called *astigmatism*; it is usually most marked in the cornea, and sometimes causes serious impairment of vision. It may be corrected by applying a lens ground on a cylindrical surface in which one of the axes only is a plane; a curve of

such a radius being chosen as will enable the two linear foci to unite on the same plane.

630. **Eye-glasses. Spectacles.**—The glasses commonly used by short- or long-sighted persons are known under the general name of *eye-glasses* or *spectacles*. Generally speaking, numbers are engraved on the trial glasses which express their focal length in *inches* or *diopters*. This latter term is applied to the standard focal length of all spectacle glasses adopted by the Ophthalmological Congress at Heidelberg, and now the only standard officially recognised throughout Europe and America. This standard is the refractive power of a lens having a focal length of a metre (39·37 inches), and is represented by the letter D. Here the refractive power is the inverse of the focal distance, i.e. $D = \frac{1}{F}$ and $F = \frac{1}{D}$. Hence to find the number of diopters which represent the focal length in inches, we must divide 39·37 by that focal distance, and, conversely, to find the number of inches corresponding to a given number of diopters, we have only to divide 39·37 by this latter.

In choosing spectacles it is necessary in practice to adapt them to the special requirements of a patient rather than to correct the absolute error of refraction of the eye. The following rules, therefore, only represent the theoretical values necessary to correct the error of refraction (ametropia) present for seeing near or distant objects.

In myopia the defect is measured by the distance of the farthest point of distinct vision from the eye (*punctum remotum*) F, (fig. 602). Thus, if the punctum remotum (p.r.) be situated at 50 cm., the myopia equals $\frac{1}{50}$ cm. = 2 diopters, and we correct it by a concave lens having a focus of 50 cm., i.e. −2D.

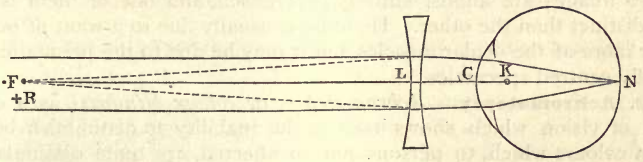


Fig. 602.

For if a concave lens be placed in front of the eye, parallel rays proceeding from infinity will enter the eye as if they came from the focus of the lens—that is, from the punctum remotum. But all rays from the p.r. meet on the retina, and the effect of the −2D lens is to cause rays proceeding from infinity to diverge after passing through the lens as if they came from the p.r.—in other words, distant objects will appear sharply defined on the retina. A concave pair of spectacles will, therefore, be the proper correction to order. In high myopia the distance between the spectacles and the eye must be taken into account. Let the distance of the p.r. from the eye be 100 mm. = $\frac{1}{10}$ metres, or 10D, and let the spectacle glass be placed 13 mm. in front of the eye—the correcting-glass will need to have a focal length of 100 − 13 = 87 mm., i.e. $\frac{1}{87}$, or 11·5D. A concave lens of 11·5D will therefore be necessary to correct a myopia of 10D.

In hypermetropia the whole case is reversed. The retina being situated in front of the focus, it would be necessary in order that the image should fall on the retina, that the parallel incident rays coming from infinity should be made to converge by means of a convex lens, for convergent rays do not exist in nature. The punctum remotum in this case is therefore negative, i.e. it is situated behind the retina, F (fig. 603). The defect is, therefore, measured by a convex lens whose focus coincides with the punctum remotum. Parallel rays proceeding from infinity will then enter the eye as if they came along

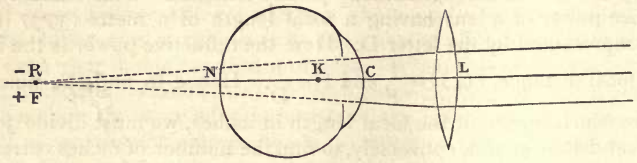


Fig. 603.

the path of rays which converge to the p.r.—in other words, distant objects will appear sharply defined on the retina.

Suppose the p.r. be situated 125 mm. behind the eye, the hypermetropia $= \frac{1000}{125}$, i.e. 8D, and a lens of that refractive power will correct the error. But as the spectacles are placed about 13 mm. in front of the eye, the correcting lens will need to have a focal length of $125 + 13 = 138$ mm., or about 7D. In lower degrees of hypermetropia the distance of the lens from the eye may be disregarded.

631. **Diplopia.**—*Diplopia* is an affection of the eye which causes objects to be seen double; that is, two images are seen instead of one. Usually the two images are almost entirely superposed, and one of them is much more distinct than the other. Diplopia is usually due to a want of power in one or more of the ocular muscles, but it may be due to the prismatic action of badly centred spectacles.

632. **Achromatopsy.**—*Achromatopsy*, or *colour blindness*, is a curious defect of vision which shows itself in the inability to distinguish between certain colours which, to persons not so affected, are quite dissimilar. In other respects their vision may be quite normal. Four forms of colour blindness have been described, viz.: *red blindness*, *green blindness*, *violet blindness*, and *total colour blindness*. By far the commonest form of this defect is that of *red blindness*. Dalton suffered from this defect, and from the fact that he very carefully described it the defect has been called *Daltonism*. Persons so affected are unable to distinguish between certain shades of red and green, being blind for two particular groups of hues which are complementary. Green or bluish green and red colours to them vary only in shade, but not in colour. Thus red cherries on a green tree are distinguishable only by their form and shade. If such a person be examined with the spectroscope, the red end is found to be more or less shortened, and the whole spectrum appears to consist of two distinct colours only, yellow and blue. In many cases the two colours are separated by a neutral band of greyish colour.

Blue yellow blindness, or *violet blindness*, as it is called by some, is an

exceedingly rare phenomenon. It is characterised by the inability to distinguish blue and its shades from yellow, but red and green, with all their shades, are clearly defined. In this form the spectrum is shortened at the violet end. *Green blindness* has also been described, but as in colour blindness the complementary colours are always defective, it is difficult to distinguish such cases from red blindness, and what appears to be green blindness by one test may be considered to be red blindness by another.

Total colour blindness.—Lastly, cases have been occasionally met with in which all sensation of colour is absent, the spectrum appearing to such persons as a greyish band of varying brightness. In such cases the vision will be somewhat impaired as well.

Colour blindness occurs chiefly in the male sex. It is usually congenital, and is found to affect from 3 to 4 per cent. of the community. When acquired it is almost always due to disease. Owing to the danger which may arise from the faulty observation of coloured signals on railways and at sea, numerous methods have been proposed for the qualitative and quantitative observation of the colour sense.

The best test for ordinary use is to give the patient a standard skein of wool of a particular tint, green, rose, or red, and to require him to match it, with others which appear to him of the same tint, among a large bundle of skeins of many colours. Coloured glasses which can be rotated in front of a lantern to imitate railway signals are also largely used for this purpose.

633. **Ophthalmoscope.**—This instrument, as its name indicates, is designed for the examination of the eye, and was invented in 1851 by Von Helmholtz. It consists :—1. Of a concave spherical reflector of glass or metal, M (figs. 604, 605), in the middle of which is a small hole about a sixth of an inch in diameter. The focal length of the reflector is from 8 to 10 inches. 2. Of a converging lens, *o*, which is held in front of the eye of the patient.

To make use of the ophthalmoscope, the patient is placed in a dark room,

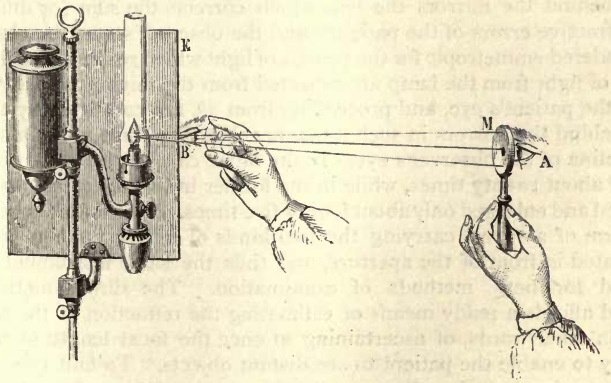


Fig. 604.

and a lamp put beside him, E. The screen serves to shade the light from his head, and keep it in darkness. The observer, A, holding in one hand

the reflector, employs it to concentrate the light of the lamp near the eye, B, of the patient, and with his other hand holds the achromatic lens, *o*, in front of the eye. By this arrangement the back of the eye is lighted up, and its structure can be clearly discerned.

Fig. 605 shows how the image of the back of the eye is produced, which the observer, A, sees on looking through the hole in the reflector. Let *ab* be the part of the retina on which the light is concentrated, pencils of rays proceeding from *ab* would form an inverted and aërial image of *ab* at *a′b′*. These pencils, however, on leaving the eye, pass through the lens *o*, and

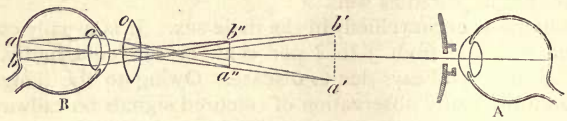


Fig. 605.

thus the image *a″b″* is in fact formed, inverted, but distinct, and in a position fit for vision.

Modern ophthalmoscopes are now usually provided with either one or more discs of metal carrying a complete series of convex and concave lenses, or with a similar series of lenses forming a chain of discs fitted in the body of the instrument. These lenses are so arranged that the observer by rotating a small wheel can bring a lens of any focal length he pleases behind the aperture of the mirror. This mirror is usually of a much shorter focal length than in the instrument previously described, and is tilted at an angle so that its plane is not parallel to the lenses behind the mirrors. By this means, the ophthalmoscope can be held almost touching the patient's eye, while the light can still be reflected into the patient's eye from the mirror. In this form of ophthalmoscope the lens *o* is dispensed with, and by placing behind the mirrors the lens which corrects the sum (or difference) of the refractive errors of the patient's and the observer's eye, the observer's eye is rendered emmetropic for the pencils of light which reach it. In this case the rays of light from the lamp are reflected from the mirrors directly on the back of the patient's eye, and proceeding from *ab*, are converted by the lens placed behind the mirrors in such a manner that they form a distinct image on the retina of the observer's eye. In the latter case the image is erect and enlarged about twenty times, while in the former indirect method the image is inverted and enlarged only about four or five times. By a simple contrivance in the form of a swivel carrying the two kinds of mirrors, either can be at once rotated in front of the aperture, and thus the same instrument can be employed for both methods of examination. The direct method just described affords a ready means of estimating the refraction of the patient's eye, or, in other words, of ascertaining at once the focal length of the lens necessary to enable the patient to see distant objects. To find this, all that is necessary is that the observer should previously ascertain the exact number of diopters necessary to correct his eye for distant vision, and to accustom himself to relax his accommodation to the full when using the instrument. On the patient's part this relaxation occurs insensibly. The

eye of both persons being adjusted for distant objects, the observer now looks through the aperture of the mirror, holding the instrument as close to the patient's eye as possible. Should both eyes be emmetropic (normal), the rays of light which are practically parallel would be focussed on the retinæ of both the eyes, and no correcting lens would be needed. Should the observer's eye be at fault, the lens which will correct it for parallel rays will enable him to see the details of the patient's retina. Should both eyes want correcting, then the number of diopters which are found necessary to add to or subtract from the number which correct the observer's eye will indicate the error in the patient's eye. By thus correcting for the vessels in the retina which run in every direction, both the axis and the amount of astigmatism present may be readily ascertained.

CHAPTER VII

SOURCES OF LIGHT. PHOSPHORESCENCE

634. **Various sources of light.**—The various sources of light are the sun, the stars, heat, chemical combination, phosphorescence, electricity, and meteoric phenomena. The last two sources will be treated under the articles Electricity and Meteorology.

The origin of the light emitted by the sun and by the stars is unknown ; the sun is the chief source. The ignited envelope by which the sun is surrounded is gaseous, because the light of the sun, like that emitted from all gaseous bodies, gives no trace of polarisation in the polarising telescope, Chap. viii.

Terrestrial bodies become sources of light when they are raised to a sufficiently high temperature; according to Draper all bodies begin to glow with a red heat at 525°; the light is brighter as the temperature is higher, and at 1,170° it is a white heat.

The luminous effects witnessed in many chemical combinations are due to the high temperatures produced. Ordinary luminous flames are essentially gases containing solids heated to incandescence.

635. **Phosphorescence.**—Certain bodies have the property of becoming luminous in the dark without any considerable rise of temperature. This phenomenon, which is well seen in phosphorus, is for this reason known as *phosphorescence*. Here it is undoubtedly due to a slow oxidation, for it ceases in spaces where no oxygen is present. Phosphorescence is also exhibited under certain conditions by decaying animal and vegetable matter. This is also due to slow oxidation.

Phosphorescence is observed in living animals, of which the best known case is that of the *glowworm* ; here it is very intense, and the brightness seems to depend on the will. Its light consists of a continuous spectrum from C to near *h*, and is particularly rich in blue and green rays. In tropical climates the sea is often covered with a bright phosphorescent light due to myriads of small luminous infusoria (*Noctiluca miliaris*).

Langley showed that in ordinary oil or gas flames more than 99 per cent. of the total energy is expended as heat, and is useless as regards light; and in sources at a high temperature, such as the arc and incandescent electric lights, the proportion although less is still considerable.

The light of the *Pyrophorus noctilucus*, which is found abundantly in Cuba, when examined by the spectroscope and bolometer (960) is found to be devoid of red and also obscure rays, that is, to contain only luminous rays. It thus represents an economical form of light. As the more refrangible rays are those which affect the eye, it is desirable to obtain in our artificial

lights as great a proportion of the energy in this form as possible, and it is in this direction that improvement in artificial illumination is to be looked for. At present, to use the striking comparison of Professor Lodge, we are in the position of an organist who, in order to produce a few very high notes, has to turn on the whole of the register.

Phosphorescence by rise of temperature. This is best seen in certain species of diamonds, and particularly in *chlorophane*, a variety of fluorspar, which, when heated to 300° or 400°, suddenly becomes luminous, emitting a greenish-blue light which lasts for several days.

Hagenbach found that the spectrum of phosphorescent fluorspar consisted of only nine bands: four blue, two green, two yellow, and one orange. As the relative intensities of these bands are continually changing, it is easy to understand the different colours presented by different specimens of this mineral.

Phosphorescence by mechanical effects, such as friction, percussion, cleavage, &c.; for example, when two crystals of quartz are rubbed against each other in darkness, when a lump of sugar is broken, or when a plate of mica is cleft. To this category belongs also the disengagement of light when arsenious acid crystallises.

Phosphorescence by electricity, like that which results from the friction of mercury against the glass in a barometric tube.

636. **Phosphorescence by insolation.**—A large number of substances, after having been exposed to the direct action of sunlight, or even of the diffused light of the atmosphere, emit in darkness a phosphorescence the colour and intensity of which depend on the nature and physical condition of these substances.

This was first observed in 1604 in Bolognese phosphorus (sulphide of barium), but it also exists in a great number of substances. The sulphides of calcium and strontium are those which present it in the highest degree. They must be prepared in the dry way and at high temperatures. When well prepared, after being exposed to the light, they are luminous for several hours in darkness. But as this phosphorescence takes place in a vacuum as well as in a gaseous medium, it cannot be attributed to a chemical action, but rather to a temporary modification which the body undergoes from the action of light. A phosphorescent sulphide of calcium is prepared for industrial purposes, and is known as Balmain's luminous paint.

After the substances above named, the best phosphorescents are the following, in the order in which they are placed: a large number of diamonds (especially yellow ones), and most specimens of fluorspar; then arragonite, calcareous concretions, chalk, apatite, heavy spar, dried nitrate of calcium, and dried chloride of calcium, cyanide of calcium, a large number of strontium or barium compounds, magnesium and its carbonate, &c. Besides these a large number of organic substances also become phosphorescent by insolation; for instance, dry paper, silk, sugar, amber, the teeth, &c.

The different spectral rays are not equally well fitted to render substances phosphorescent. The maximum effect takes place in the violet rays, or even a little beyond; while the light emitted by phosphorescent bodies generally corresponds to rays of a smaller refrangibility, that is, of greater wave-length, than those of the light received by them and giving rise to the action.

The tint which phosphorescent bodies assume is very variable, and even in the same body it changes with the manner in which it is prepared. In strontium compounds green and blue tints predominate; and orange, yellow, and green tints in the sulphides of barium.

The colour also alters with the temperature during insolation. Thus with sulphide of strontium the light is dark violet at −20° C., bright blue at +40°, bluish-green at 70°, greenish-yellow at 100°, and reddish-yellow of feeble luminosity at 200° C.

The duration of phosphorescence varies also in different bodies. In the sulphides of calcium and strontium, phosphorescence lasts as long as thirty hours; with other substances it does not exceed a few seconds, or even a fraction of a second.

When a phosphorescent body has been heated the light emitted is brighter, but the greater the emission of light the shorter is the duration of the phosphorescence. Heat, therefore, produces a more rapid irradiation of the light.

Phosphoroscope.—In experimenting with bodies whose phosphorescence lasts a few minutes or even a few seconds, it is simply necessary to expose them to solar or diffused light for a short time, and then place them in darkness: their luminosity is very apparent, especially if care has been taken to close the eyes previously for a few moments. But in the case of bodies whose phosphorescence lasts only a very short time, this method is inadequate. Becquerel invented an ingenious apparatus, the *phosphoroscope*, by which bodies can be viewed immediately after being exposed to light: the interval which separates the insolation and observation can be made as small as possible, and measured with great precision.

This apparatus consists of a closed cylindrical box, AB (fig. 606), of blackened metal; on the ends are two apertures opposite each other which have the form of a circular sector. One only of these, *o*, is seen in the figure. The box is fixed, but it is traversed in the centre by a movable axis, to which are fixed two circular screens, MM and PP, of blackened metal (fig. 607). Each of these screens is perforated by four apertures of the same shape as those in the box; but while the latter correspond to each other, the apertures of the screens alternate, so that the open parts of the one correspond to the closed parts of the other. The two screens, as already mentioned, are placed in the box, and fixed to the axis, which by means of a train of wheels, worked by a handle, can be made to turn with any velocity.

In order to investigate the phosphorescence of any body by means of this instrument, the body is placed on a stirrup interposed between the two rotating screens. The light cannot pass at the same time through the opposite apertures of the sides A and B, because one of the closed parts of the screen MM, or of the screen PP, is always between them. So that when a body, *a*, is illuminated by light from the other side of the apparatus, it could not be seen by an observer looking at the aperture, *o*, for then it would be masked by the screen PP. Accordingly, when an observer saw the body *a*, it would not be illuminated, as the light would be intercepted by the closed parts of a screen MM. The body *a* would alternately appear and disappear; it would disappear during the time of its being illuminated, and appear when it was no longer so. The time which elapses between the appearance and disappearance depends on the velocity of rotation of the screens. Suppose,

for instance, that they made 150 turns in a second; as one revolution of the screens is effected in $\frac{1}{150}$ of a second, there would be four appearances and four disappearances during that time. Hence the length of time elapsing between the time of illumination and of observation would be $\frac{1}{8}$ of $\frac{1}{150}$ of a second or 0·0008 of a second.

Observations with the phosphoroscope are made in a dark chamber, the observer being on that side on which is the wheelwork. A ray of solar or

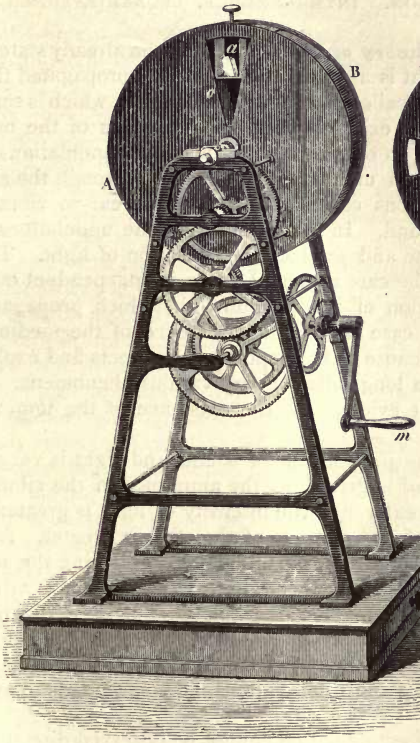


Fig. 606.

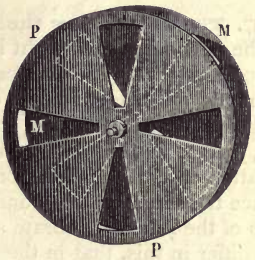


Fig. 607.

electric light is allowed to fall upon the substance *a*, and, the screens being made to rotate more or less rapidly, the body *a* appears luminous by transparence in a continuous manner, when the interval between insolation and observation is less than the duration of the phosphorescence of the body. By experiments of this kind, Becquerel found that substances which usually are not phosphorescent become so in the phosphoroscope; such, for instance, is Iceland spar. Uranium compounds present the most brilliant appearance in this apparatus; they emit a very bright luminosity when the observer can see them 0·03 or 0·04 of a second after insolation. But a large number of bodies produce no effect in the phosphoroscope; for instance, quartz, sulphur, phosphorus, metals, and liquids.

CHAPTER VIII.

DOUBLE REFRACTION. INTERFERENCE. POLARISATION.

637. **The undulatory theory of light.**—It has been already stated (499) that the phenomenon of light is ascribed to undulations propagated through an exceedingly rare medium called the luminiferous ether, which is supposed to pervade all space, and to exist between the molecules of the ordinary forms of matter. It is held, in short, that light is due to undulations of the ether, just as sound is due to undulations propagated through the air. In the latter case the undulations cause the drum of the ear to vibrate and produce the sensation of sound. In the former case, the undulations cause points of the retina to vibrate and produce the sensation of light. The two cases differ in this, that in the case of sound there is independent evidence of the existence and vibration of the medium (air) which propagates the undulation; whereas in the case of light the existence of the medium and its vibrations is *assumed*, because that supposition connects and explains in the most complete manner a long series of very various phenomena. There is, however, no independent evidence of the existence of the luminiferous ether.

The analogy between the phenomena of sound and light is very close; thus, the intensity of a sound is greater as the amplitude of the vibration of each particle of the air is greater, and the intensity of light is greater as the amplitude of the vibration of each particle of the ether is greater. Again, a sound is more acute as the length of each undulation producing the sound is less, or, what comes to the same thing, according as the number of vibrations per second is greater. In like manner, the colour of light is different according to the length of the undulation producing the light: a red light is due to a comparatively long undulation, and corresponds to a deep sound, while a violet light is due to a short undulation, and corresponds to an acute sound.

Although the length of the undulation cannot be observed directly, yet it can be inferred from certain phenomena with great exactness. The following table gives the lengths, in inches and millimetres, of the undulations corresponding to the light at the principal dark lines of the spectrum :—

Dark line	Length of Undulation in inches	Length of Undulation in millimetres
B	0·0000271	0·0006874
C	0·0000258	0·0006562
D_1	0·0000232	0·0005897
E	0·0000207	0·0005271
F	0·0000191	0·0004862
G	0·0000169	0·0004311
H_1	0·0000159	0·0003969

It will be remarked that the limits are very narrow within which the lengths of the undulations of the ether must be comprised, if they are to be capable of producing the sensation of light. In this respect light is in marked contrast to sound. For the limits are very wide within which the lengths of the undulations of the air may be comprised when they produce the sensation of sound (244).

The undulatory theory readily explains the colours of different bodies. According to that theory, certain bodies have the property of exciting undulations of different lengths, and thus producing light of given colours. White light or daylight results from the coexistence of undulations of all possible lengths.

The colour of a body is due to the power it has of extinguishing certain vibrations, and of reflecting others ; and the body appears of the colour produced by the coexistence of the reflected vibrations. A body appears white when it reflects all different vibrations in the proportion in which they are present in the spectrum ; it appears black when it reflects light in such small quantities as not to affect the eye. A red body is one which has the property of reflecting in predominant strength those vibrations which produce the sensation of red. This is seen in the fact that, when a piece of red paper is held against the daylight, and the reflected light is caught on a white wall, this also appears red. A piece of red paper in the red part of the spectrum appears of a brighter red, and a piece of blue paper held in the blue part appears a brighter blue ; while a red paper placed in the violet or blue part appears almost black. In the last case the red paper can only reflect red rays, while it extinguishes the blue rays, and as the blue of the spectrum is almost free from red, so little is reflected that the paper appears black.

The undulatory theory likewise explains the colours of transparent bodies. Thus, a vibrating motion on reaching a body sets it in vibration. So also the vibrations of the luminiferous ether are communicated to the ether in a body, and, setting it in motion, produce light of different colours. When this motion is transmitted through any body, it is said to be *transparent* or *translucent*, according to the different degrees of strength with which this transmission is effected. In the opposite case it is said to be *opaque*.

When light falls upon a transparent body, the body appears colourless if all the vibrations are transmitted in the proportion in which they exist in the spectrum. But if some of the vibrations are checked or extinguished, the emergent light will be of the colour produced by the coexistence of the unchecked vibrations. Thus, when a piece of blue glass is held before the eye, the vibrations producing red and yellow are extinguished, and the colour is due to the emergent vibrations which produce blue light.

The undulatory theory also accounts for the reflection and refraction of light, as well as other phenomena which are yet to be described. The explanation of the refraction of light is of so much importance that we shall devote to it the following article.

638. **Physical explanation of single refraction.**—The explanation of this phenomenon by means of the undulatory theory of light presupposes that of the mode of propagation of a plane wave. Now, if a disturbance originated at any *point* of the ether, it would be propagated as a spherical

wave in all directions round that point with a uniform velocity. If, instead of a single point, we consider the front of a plane wave, it is evident that disturbances originate simultaneously at all points of the front, and that spherical waves proceed from each *point* with the same uniform velocity. Consequently, all these spheres will at any subsequent instant be touched by a plane parallel to the original plane. The disturbances propagated from the points in the first position of the wave will mutually destroy each other, except in the tangent plane; consequently the wave advances as a plane wave, its successive positions being the successive positions of the tangent plane. If the wave moves in any medium with a velocity v, it will describe a space vt in a time t, in a direction at right angles to the wave-front.

In any given moment let mn (fig. 608) be the position of the wave-front of a ray of light, which, moving through any medium, meets the plane surface AB of any denser refracting medium. In the same moment in which the wave-front reaches n, m becomes the centre of a spherical wave system which moves in the second medium; and, as the elasticity of the second medium is different from that of the first, the velocity of propagation of the wave in the two media will be different. While the plane wave moves from n to K, the corresponding wave starting from m reaches the surface of a sphere the radius of which is less than nK, if the second medium is denser than the first. The incident wave in like manner reaches m' and n' simultaneously, and while n' moves to K, m' moves to o', the surface of a sphere the radius of which, $m'o'$, is to mo as n'K is to nK. All the elementary waves proceeding from points intermediate to n and K which arise from the same incident wave, touch one and the same plane K$o'o$, and the refracted ray proceeds in the new medium perpendicular to this tangent plane.

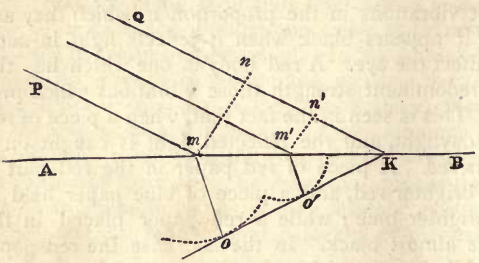


Fig. 608.

Now nK and mo are proportional to the velocities of light in the two media respectively: let mK be taken as unit of length, then

$$n\text{K} = \sin nm\text{K} \text{ and } mo = \sin m\text{K}o.$$

Now nmK is the angle of incidence of the ray, and mKo is the angle of refraction, and nK and mo are proportional to the velocities of light in the two media respectively; hence we see that these velocities are to each other in the same ratio as the sines of the angles of incidence and refraction; a conclusion which agrees with the results of direct observation (506), and forms a beautiful confirmation of the truth of the undulatory theory.

DOUBLE REFRACTION

639. **Double refraction.**—It has been already stated (536) that a large number of crystals possess the property of double refraction, in virtue of

which a single incident ray in passing through any one of them is divided into two, or undergoes *bifurcation*, whence it follows that, when an object is seen through one of these crystals, it appears double. The fact of the existence of double refraction in Iceland spar was first stated by Bartholin in 1669, but the law of double refraction was first enunciated exactly by Huyghens, in his treatise on light, written in 1678 and published in 1690.

Crystals which possess this peculiarity are said to be *double-refracting*. It is found to a greater or less extent in all crystals which do not belong to the cubical system. Bodies which crystallise in this system, and those which, like glass, are destitute of crystallisation, have no double refraction. The property can, however, be imparted to them when they are unequally compressed, or when they are cooled quickly after having been heated, in which state glass is said to be *unannealed*. Of all substances, that which possesses it most remarkably is Iceland spar or crystallised carbonate of calcium. In many substances, the power of double refraction can hardly be proved to exist directly by the bifurcation of an incident ray; but its existence is shown indirectly by their being able to depolarise light (665).

Fresnel explained double refraction by assuming that the ether in double-refracting bodies is not equally elastic in all directions; from which it follows that the vibrations, in certain directions at right angles to each other, are transmitted with unequal velocities; these directions being dependent on the constitution of the crystal. This hypothesis is confirmed by the property which glass acquires of becoming double-refracting by pressure and when unannealed.

640. **Uniaxial crystals.**—In all double-refracting crystals there is *one* direction, and in some a second direction, possessing the following property:—When a point is looked at through the crystal in this particular direction, it does *not* appear double. The lines fixing these directions are called *optic axes*; and sometimes, though not very properly, axes of double refraction. A crystal is called *uniaxial* when it has *one* optic axis; that is to say, when there is one direction within the crystal along which a ray of light can proceed without bifurcation. When a crystal has *two* such axes, it is called a *biaxial* crystal.

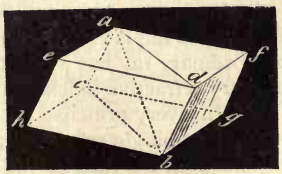


Fig. 609.

The uniaxial crystals most frequently used in optical instruments are Iceland spar, quartz, and tourmaline. Iceland spar crystallises in rhombohedra, whose faces form with each other angles of 105° 5′ or 74° 55′. It has eight solid angles (see fig. 609). Of these, two, situated at the extremities of one of the diagonals, are severally contained by three obtuse angles. A line drawn within one of these two angles in such a manner as to be equally inclined to the three edges containing the angle is called the *axis of the crystal*. If all the edges of the crystal were equal, the axis of the crystal would coincide with the diagonal, *ab*.

Brewster showed that in all uniaxial crystals the optic axis coincides with the axis of crystallisation.

The principal plane with reference to a point of any face of a crystal, whether natural or artificial, is a plane drawn through that point at right

angles to the face and parallel to the optic axis. If in fig. 609 we suppose the edges of the rhombohedron to be equal, the diagonal plane *abcd* contains the optic axis (*ab*), and is at right angles to the faces *aedf* and *chbg*; consequently it is parallel to the principal plane at any point of either of those two faces. For this reason *abcd* is often called the principal plane with respect to those faces.

641. **Ordinary and extraordinary ray.**—One of the two rays into which an incident ray is divided on entering a uniaxial crystal is called the *ordinary*, and the other the *extraordinary* ray. The ordinary ray follows the laws of single refraction; that is, with respect to that ray the sine of the angle of incidence bears a constant ratio to the sine of the angle of refraction, and the plane of incidence coincides with the plane of refraction. Except in particular positions, the extraordinary ray follows neither of these laws. The images corresponding to the ordinary and extraordinary rays are called the ordinary and extraordinary images respectively.

If a transparent specimen of Iceland spar be placed over a dot of ink, on a sheet of white paper, two images will be seen. One of them, the ordinary image, will seem slightly nearer to the eye than the other, the extraordinary image. Suppose the spectator to view the dot in a direction at right angles to the paper, then, if the crystal, with the face still on the paper, be turned round, the *ordinary* image will continue fixed, and the *extraordinary* image will describe a circle round it, the line joining them being always in the direction of the shorter diagonal of the face of the crystal, supposing its edges to be of equal length. In this case it is found that the angle between the ordinary and extraordinary ray is 6° 12′.

642. **The laws of double refraction in a uniaxial crystal.**—These phenomena are found to obey the following laws :—

i. Whatever be the plane of incidence, the ordinary ray always obeys the two general laws of single refraction (537). The refractive index for the ordinary ray is called the ordinary refractive index.

ii. In every section perpendicular to the optic axis the extraordinary ray also follows the laws of single refraction. Consequently, in this plane, the extraordinary ray has a constant refractive index, which is called the extraordinary refractive index.

iii. In every principal section the extraordinary ray follows the second law only of single refraction; that is, the planes of incidence and refraction coincide, but the ratio of the sines of the angles of incidence and refraction is not constant.

iv. The velocities of light along the rays are unequal. It can be shown that the difference between the squares of the reciprocals of the velocities along the ordinary and extraordinary rays is proportional to the square of the sine of the angle between the latter ray and the axis of the crystal.

There is an important difference between the velocity of the *ray* and the velocity of the corresponding *plane wave*. If the velocities of the plane waves corresponding to the ordinary and extraordinary rays are considered, the difference between the squares of these velocities is proportional to the square of the sine of the angle between the axis of the crystal, and the normal to that plane wave which corresponds to the extraordinary ray. The normal and the ray do not generally coincide.

Huyghens gave a very simple geometrical construction, by means of which the directions of the refracted rays can be determined when the directions of the incident ray and of the axis are known relatively to the face of the crystal. This construction was not generally accepted by physicists until Wollaston, and subsequently Malus, showed its truth by numerous exact measurements.

643. **Positive and negative uniaxial crystal.**—The term extraordinary refractive index has been defined in the last article. For the same crystal, its magnitude always differs from that of the *ordinary* refractive index; for example, in Iceland spar the ordinary refractive index is 1·654, while the extraordinary refractive index is 1·483. In this case the ordinary index exceeds the extraordinary index. When this is so, the crystal is said to be negative. On the other hand, when the extraordinary index exceeds the ordinary index, the crystal is said to be positive. The following list gives the names of some of the principal uniaxial crystals :—

Negative Uniaxial Crystals

Iceland spar	Ruby	Pyromorphite
Tourmaline	Emerald	Ferrocyanide of potassium
Sapphire	Apatite	Nitrate of sodium

Positive Uniaxial Crystals

Zircon	Apophyllite	Titanite
Quartz	Ice	Boracite

644. **Double refraction in biaxial crystals.**—A large number of crystals, including all those belonging to the *trimetric*, the *monoclinic*, and the *triclinic* systems, possess two *optic axes*; in other words, in each of these crystals there are two directions along which a ray of light passes without bifurcation. A line bisecting the acute angle between the optic axes is called the medial line; one that bisects the obtuse angle is called the supplementary line. It has been found that the medial and supplementary lines and a third line at right angles to both are closely related to the fundamental form of the crystal to which the optic axes belong. The acute angle between the optic axes is different in different crystals. The following table gives the magnitude of this angle in the case of certain crystals :—

Nitre	5° 20′	Mica	45° 0′
Strontianite	6 56	Sugar	50 0
Arragonite	18 18	Selenite	60 0
Anhydrite	28 7	Epidote	84 19
Heavy spar	37 42	Sulphate of iron	90 0

When a ray of light enters a biaxial crystal, and passes in any direction not coinciding with an optic axis, it bifurcates; in this case, however, neither ray conforms to the laws of single refraction, but both are extraordinary rays. To this general statement the following exception must be made :—In a section of a crystal at right angles to the medial line one ray follows the laws of ordinary refraction, and in a section at right angles to the supplementary line the other ray follows the laws of ordinary refraction.

INTERFERENCE AND DIFFRACTION

645. **Interference of light.**—The name *interference* is given to the reciprocal action which two rays of light exert upon each other when they are emitted from two neighbouring sources and meet each other under a very small angle. This action may be observed by means of the following experiment:—In the shutter of a dark room two very small apertures of the same diameter are made close to each other. The apertures are closed by pieces of coloured glass—red, for example—by which two pencils of homogeneous light are introduced. These two pencils form two divergent luminous cones, which meet at a certain distance; they are received on a white screen a little beyond the place at which they meet, and in the segment common to the two discs which form upon this screen some very well-defined alternations of red and black bands are seen. If one of the two apertures be closed, the fringes disappear, and are replaced by an almost uniform red tint. From the fact that the dark fringes disappear when one of the beams is intercepted, it is concluded that they arise from the interference of the two pencils which cross obliquely.

This experiment was first made by Grimaldi, but was modified by Young. Grimaldi had drawn from it the conclusion that light added to light

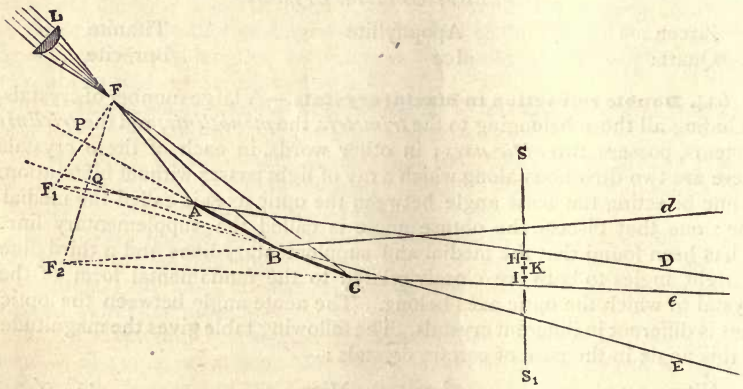


Fig. 610.

produced darkness. The full importance of this principle remained for a long time unrecognised, until these inquiries were resumed by Young and by Fresnel, of whom the latter, by a modification of Grimaldi's experiment, rendered it an *experimentum crucis* of the truth of the undulatory hypothesis.

In Grimaldi's experiment diffraction (646) takes place, for the luminous rays pass by the edge of the aperture. In the following experiment, which is due to Fresnel, the two pencils interfere without the possibility of diffraction.

Two plane mirrors, AB and BC (fig. 610), of metal, are arranged close to each other, so as to form a very obtuse angle, ABC, which must be very little

less than 180°. A pencil of monochromatic light—red, for instance—which passes into the dark chamber, is brought to a focus, F, by means of a lens, L. On diverging from F the rays fall partly on AB, and partly on BC. If BA is produced to P and FPF_1 is drawn at right angles to AP, and if PF_1 is made equal to PF, then the rays which fall on AB will, after reflection proceed as if they diverged from F_1. If a similar construction is made for the rays falling on BC, they will proceed after reflection as if they diverged from F_2. A little consideration will show that F_1 and F_2 are very near each other. Suppose the reflected rays to fall on a screen SS_1 placed nearly at right angles to their directions. Every point of the screen which receives light from both pencils is illuminated by both rays, viz. one from F_1, the other from F_2 : thus the point H is illuminated by two rays, as also are K and I. Now the combined action of these two pencils is to form a series of parallel bands alternately light and dark on the screen at right angles to the plane of the paper (fig. 611). They are distributed symmetrically in reference to one of them, *cc*, which is more brilliant than the others, and which is called the central fringe. This is the fundamental phenomenon of interference ; and that it results from the *joint action of the two pencils* is plain, for if the light which falls upon either of the mirrors is cut off, the dark bands altogether disappear.

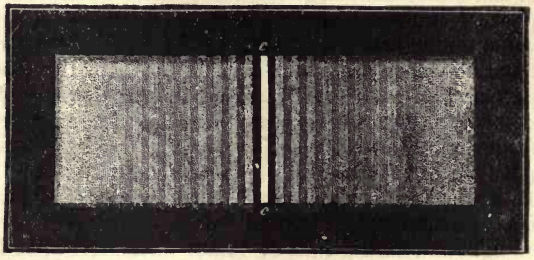


Fig. 611.

The experiment may also be made by means of *Ohm's prism*, which is a prism in which the refracting angle is very nearly 180°.

A very simple arrangement of a mirror for experiments on the interference of light is thus constructed. Four small wax pellets, *a*, *b*, *c*, *d*, are placed on a small block of wood, and on this two strips of plate glass, which accurately fit along the line *bc*. A larger glass plate is then laid on this, and the finger moved with gentle pressure along this line. In this way the two mirrors form a very slight angle with each other, and give in sunlight the coloured fringes.

Fig. 612.

This remarkable experiment is explained in the most satisfactory manner by the undulatory theory of light. The explanation exactly resembles that already given of the formation of nodes and loops by the combined action of two aërial waves (262) ; the only difference being that in that case the vibrating particles were supposed to be particles of air, whereas, in the present case, the vibrating particles are supposed to be those of the luminiferous ether. Consider any point K on the screen, and first let us suppose the distance of K from F_1 and F_2 to be equal. Then the undulations which reach K will always be in the same *phase*, and the particle of ether at K will vibrate as if the light came from one source : the

amplitude of the vibration, however, will be increased in exactly the same manner as happens at a loop or ventral point; consequently, at K the intensity of the light will be increased. And the same will be true for all parts on the screen, such that the difference between their distances from the two images equals the length of *one*, *two*, *three*, &c., undulations. If, on the other hand, the distances of K from F_1 and F_2 differ by the length of half an undulation, then the two waves would reach K in exactly opposite phases. Consequently, whatever velocity would be communicated at any instant to a particle of ether by the one undulation, an exactly equal and opposite velocity would be communicated by the other undulation, and the particle would be *permanently* at rest, or there would be darkness at that point; this result being produced in a manner precisely resembling the formation of a *nodal* point already explained. The same will be true for all positions of K, such that the difference between its distances from F_1 and F_2 is equal to three halves, or five halves, or seven halves, &c., of an undulation. Accordingly, there will be on the screen a succession of alternations of light and dark points, or rather lines—for what is true of points in the plane of the paper (fig. 611) will be equally true of other points on the screen, which is supposed to be at right angles to the plane of the paper. Between the light and dark lines the intensity of the light will vary, increasing gradually from darkness to its greatest intensity, and then decreasing to the second dark line, and so on.

If instead of red light any other coloured light were used—for example, violet light—an exactly similar phenomenon would be produced, but the distance from one dark line to another would be different. If white light were used, each separate colour would tend to produce a different set of dark lines. Now these sets being superimposed on each other, and not coinciding, the dark lines due to one colour are illuminated by other colours, and instead of dark lines a succession of coloured bands is produced. The number of coloured bands produced by white light is much smaller than the number of dark lines produced by a homogeneous light; since at a small distance from the middle band the various colours are completely blended, and a uniform white light produced.

646. **Diffraction and fringes.**—Diffraction is a modification which light undergoes when it passes the edge of a body, or when it traverses a small aperture—a modification in virtue of which the luminous rays appear to become bent, and to penetrate into the shadow.

This phenomenon may be observed in the following manner:—A beam of sunlight is allowed to pass through a very small aperture in the shutter of

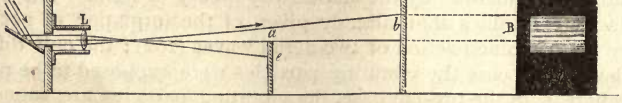


Fig. 613.

a dark room, where it is received on a condensing lens, L (fig. 613), with a short focal length. A red glass is placed in the aperture so as to allow only red light to pass. An opaque screen, *e*, with a sharp edge *a*—a razor, for

instance—is placed behind the lens beyond its focus, and intercepts one portion of the luminous cone, while the other is projected on the screen *b*, of which B represents a front view. The following phenomena are now seen :—Within the geometrical shadow, the limit of which is represented by the line *ab*, a faint light is seen, which gradually fades in proportion as it is farther from the limits of the shadow. In the upper part of the screen—which, being above the line *ab*, might be expected to be uniformly illuminated—a series of alternate dark and light bands or fringes is seen parallel to the line of shadow, which gradually become more indistinct and ultimately disappear. The limits between the light and dark fringes are not quite sharp lines : there are parts of maximum and minimum intensity which gradually fade off into each other.

All the colours of the spectrum give rise to the same phenomenon, but the fringes are broader in proportion as the light is less refrangible. Thus with red light they are broader than with green, and with green than with violet. Hence, with white light, which is composed of different colours, the dark spaces of one tint overlap the light spaces of another, and thus a series of prismatic colours will be produced.

If, instead of placing the edge of an opaque body between the light and the screen, a very narrow body be interposed, such as a hair or a fine metal wire, the phenomena will be different. Outside the space corresponding to the geometrical shadow there is a series of fringes, as in the former case. But within the shadow also there is a series of alternate light and dark bands. They are called interior fringes, and are much narrower and more numerous than the external fringes.

When a small opaque circular disc is interposed, white light being used, its shadow on the screen shows in the middle a bright spot surrounded by a series of coloured concentric rings ; the bright spot is of various colours according to the relative positions of the disc and screen. The haloes sometimes seen round the sun and moon belong to this class of phenomena. They are due, as Fraunhofer showed, to the diffraction of light by small globules of fog in the atmosphere. Fraunhofer even gave a method of estimating the mean diameter of these globules from the dimensions of the haloes.

647. **Gratings.**—Phenomena of diffraction of another class are produced by allowing the pencil of light from the luminous point to traverse an aperture in the form of a narrow slit in an opaque screen. The diffracted light may be received on a sheet of white paper, but the images are much better seen through a small telescope placed behind the aperture. If the aperture is very small, the telescope may be dispensed with, and the figure may be viewed by placing the aperture before the eye. If now monochromatic light—red, for instance (572)—be allowed to fall through such a narrow slit, a bright band of red light is seen, and right and left of it a series of similar bands gradually diminishing in brightness and separated by dark bands.

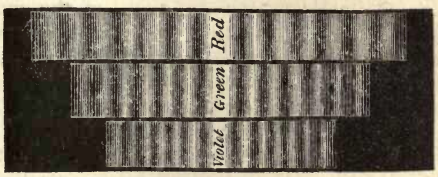


Fig. 614.

The breadth of these bands differs with the nature of the light, being narrower and nearer together in violet than in green, and these again narrower and nearer than in red, as shown in fig. 614. If ordinary white light be used, then the colours are not exactly superposed, but a series of equidistant spectra is formed on each side of the bright line, with their violet side turned inwards.

In order to explain this, let us refer to fig. 615, which represents the formation of the first dark band. When light is incident on the slit, AB, the particles of ether there, which we will represent by the dotted lines, will be set in vibration, and each point will become the centre of a new series of oscillations. Consider now the undulations which constitute a ray proceeding at right angles to the plane of the slit: all such undulations will form a band of light on the screen MN. Those which are not parallel but proceed at equal inclinations, and meet at the point *r*, will be in the same phase and will reinforce each other, and the line of maximum brightness will be at *r*. Consider, however, a pencil of rays which proceeds from the slit in an oblique direction and which meets the screen, or the retina, in the point *s*, and let us suppose that the difference between the lengths of the paths of the undulations proceeding from the edges *b* and *a*—that is, *bs* and *as*—is equal to the length of an undulation. Make $sc = sb$ and join *bc*; then *ac* is the length of the undulation.

Let us suppose that the whole set of undulations which proceeds from the slit *ab* is divided at *d* into two equal groups of undulations. Then a little consideration will show that at any part of the path there will be a difference of phase of half an undulation between the ray from the margin *a*, and that from the centre *d*; and to each of the undulations constituting the group on the left there will be a corresponding one among the groups on the right, which just differs from it by half an undulation; the general effect will be that the group on the left will be half an undulation behind the group on the right, and both arriving at the screen in opposite phases neutralise each other and produce darkness.

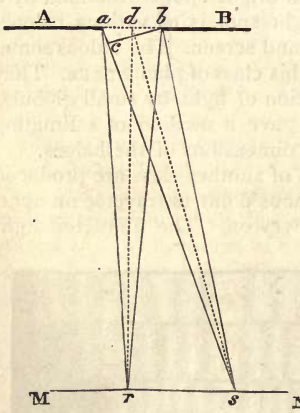


Fig. 615.

When the difference between the paths of the marginal undulations is equal to half a wave-length, a partial destruction of light takes place; the luminous intensity corresponding to this obliquity is a little less than half that of the undiffracted light. If the marginal distance is one and a half undulation, we can, as before, conceive the whole pencil divided into three parts, of which two will neutralise each other, and the third only will be effective. There will be a luminous band, but one of less intensity. In like manner where the marginal undulations differ by two whole wave-lengths, they will again extinguish each other, and a dark band will be the result. Thus there will be formed a series of alternate dark and bright

bands of rapidly diminishing intensity. In general, when the difference of path of the rays proceeding from the margin of the slit amounts to n wave-lengths, n being any whole number, we have a dark band, and when it amounts to $n + \frac{1}{2}$ wave-lengths, a bright band.

The phenomena of diffraction produced when other than straight lines are used are often of great beauty. They have been more particularly examined by Schwerdt, and the whole of the phenomena are in exact accordance with the undulatory theory, though the explanation is in many cases somewhat intricate. The theory renders it possible to predict the appearance which any particular aperture will produce, just as astronomy enables us to foretell the motions of the heavenly bodies. Some of the simpler forms—such as straight lines, triangles, squares—may be cut out of tinfoil pasted on glass, and apertures of any form may be produced with great accuracy by taking on glass a collodion photograph of a sheet of paper on which the required shapes are drawn in black.

Looking through any of these apertures at a luminous point, we see it surrounded with coloured spectra of very various forms, and of great beauty. The beautiful colours seen on looking through a bird's feather at a distant source of light, and the colours of striated surfaces, such as mother-of-pearl, are due to a similar cause. A beautiful phenomenon of the same kind is the aureole observed on looking at a candle flame through lycopodium powder strewn on glass. Two crossed gratings give a splendid picture, in which a bright point is surrounded in all directions by spectra.

648. **Diffraction spectra.**—The most important of these figures are the *gratings proper*, which may be produced by arranging a series of fine wires parallel to each other, or by careful ruling on a piece of smoked glass, or by photographic reduction. Nobert has made such gratings by ruling lines on glass with a diamond, in which there are no less than 12,000 lines in an inch in breadth. The late Dr. Stone constructed such gratings for reflection, by ruling lines on plates of nickel; this metal has the advantage of hardness, non-liability to tarnish, and great reflecting power.

If a grating be used instead of a single slit, as above described, the phenomena are in general the same, though of greater brilliancy. With homogeneous light and such a grating, there is seen, on each side of the central bright line, a series of sharply defined narrow bands and lines of light, gradually increasing in breadth and diminishing in intensity as their distance from the central line increases. If white light be used the white band is seen in the centre, and on each side of it a sharply defined isolated spectrum with the violet edges inwards. Next to this, and separated by a dark interval, is on each side a somewhat broader but similar spectrum, and then follow others which become fainter and broader and overlap each other. The brightness and sharpness of these spectra depend on the closeness of the lines, and on the opacity of the intermediate space. In those which are ruled by diamond on glass, the parts scratched represent the opaque parts.

For objective representation the image of a slit in a dark shutter, through which the sunlight enters, is focussed by means of a convex lens on a screen at a distance, and then a grating is placed in the path of the rays.

The spectra produced by means of a grating are known as *interference* or

diffraction spectra. Very accurate gratings can now be easily and cheaply prepared by means of photography, and their use for scientific purposes is extending.

There are many points of difference between these spectra and those produced by the prism, and for scientific work the former are preferable.

A diffraction spectrum is the purer the greater the number of lines in the grating, provided they are equidistant. The spectra are, however, not more than $\frac{1}{10}$ as bright as prismatic spectra ; and, to obtain the maximum brightness, the opaque intervals should be as opaque and the transparent ones as transparent as possible.

On the other hand, in diffraction spectra, the colours are uniformly distributed in their true order and extent according to the difference in their wave-lengths, and according therefore to a property which is inherent in the light itself; while in prismatic spectra the red rays are concentrated, and the violet ones dispersed. In diffraction spectra the centre is the brightest part.

Fig. 616 represents a grating spectrum, together with an equally long spectrum produced by a flint-glass prism; the upper one being that produced by the grating. It will be seen that D in the one spectrum is in almost exactly the same position as F in the other.

Diffraction spectra have, moreover, the advantage of giving a far larger number of dark lines, and of giving them in their exact relative positions. Thus, in a particular region in which Angström had mapped 118 lines

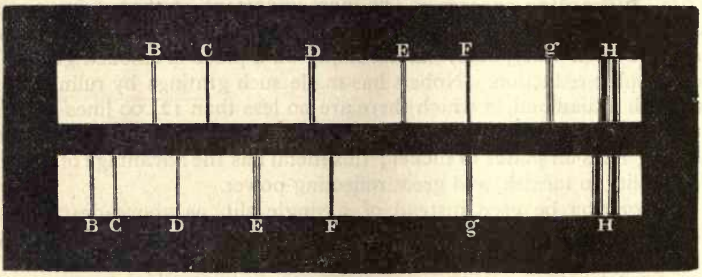


Fig. 616.

Draper, by means of a diffraction spectrum, was able to photograph at least 293. Diffraction spectra also extend farther in the direction of the ultra-violet, and give more dark lines in that region.

The most perfect gratings have quite recently been constructed by Professor Rowland, of Baltimore, by means of a machine specially planned and constructed for the purpose, and the chief feature in which is a practically perfect screw. Using this machine, he has been able to rule gratings with as many as 43,000 lines to the inch, nor does this represent the limit of the power of the machine. Gratings with 14,000 or 28,000 lines give, however, the best definition. Another great improvement is to rule the gratings on spherical instead of on flat surfaces ; in this way the spectrum can be formed without a telescope, which is a matter of great importance, as telescopes interfere with a great many experiments. The spectroscope is thus reduced

to its simplest form, so that an instrument of very high power may be constructed at a small cost.

By means of his gratings Professor Rowland has been able to resolve lines in the spectrum which had never hitherto been separated.

It has been proposed to use the fine quartz threads prepared by Mr. Boys (89) for making gratings.

649. **Determination of wave-length.**—The relative positions of these bright and dark lines furnish a means of calculating the wave-length or length of undulation of any particular colour. We must first of all know the distance *rs* of the first dark band from the bright one. The bands are not uniform in brightness or darkness, but there is in each case a position of maximum intensity, and it is from these that the distances are measured. If the bands are viewed through a telescope, the angle is observed through which the axis must be turned from the position in which the cross wire coincides with the centre of the bright band to that in which it coincides with the centre of the dark band. From this angle, which can be very accurately measured, the distance is easily calculated. When the diffraction bands are received on a screen, the distance may be directly measured, and most accurately by taking half the distance between the centres of the first pair of dark bands.

We have thus the similar triangles *abc*, and *rds*, in which $ac : bc = rs : rd$ (fig. 615). Now *bc* may be taken equal to *ab*, the width of the slit, which can be measured directly with great accuracy by means of a micrometric screw (11), and *rd* is the distance of the screen. Hence

$$ac = \frac{rs \times ab}{rd}.$$

Now *ac*, the difference between *as* and *sc*, is equal to the length of an undulation of this particular colour. In one experiment with red light the width of the slit *ab* was 0·015 in., the distance *rs* 0·15 in., and the distance of the screen 93 in., which gave $ac = \frac{0{\cdot}15 \times 0{\cdot}015}{93} = 0{\cdot}000024$ in. as the wave-length of red light. Using blue light the distance of *rs* was found to be 0·1, which gives 0·000016.

Knowing the length of the undulations, we can easily calculate their number in a second, *n*, from the formula $n = \frac{v}{l}$ (232), where *v* is the velocity of light. Taking this at 186,000 miles, we get for the red corresponding to the dark line B 434,420,000,000,000 as the number of oscillations in a second, and for the H in the violet 758,840,000,000,000 undulations.

If, instead of a single slit, gratings be used, we have the possibility of more accurate results, for the contrast is greater, and thus the distance is more easily determined. The width of the slit is easily calculated by counting the number of lines in a given space.

650. **Colours of thin plates. Newton's rings.**—All transparent bodies, solids, liquids, or gases, when in sufficiently fine laminæ, appear coloured with very bright tints, especially by reflection. Crystals which cleave easily, and can be obtained in very thin plates, such as mica and selenite, show this

phenomenon, which is also well seen in soap-bubbles and in the layers of air in cracks in glass and in crystals. Steel becoming covered with a thin layer of oxide exhibits the colour of thin plates, which change during heating as the oxide changes its thickness. A drop of oil spread rapidly over a large sheet of water exhibits all the colours of the spectrum in a constant order. A soap-bubble appears white at first, but, in proportion as it is blown out, brilliant iridescent colours appear, especially at the top, where it is thinnest. These colours are arranged in horizontal zones around the summit, which appears black when there is not thickness enough to reflect light, and the bubble then suddenly bursts.

Newton, who first studied the phenomena of the coloured rings in soap-bubbles, wishing to investigate the relation between the thickness of the thin plate, the colour of the rings, and their extent, produced them by means of a layer of air interposed between two glasses, one plane and the other convex, and with a very long focus (fig. 617). The two surfaces being cleaned and exposed to ordinary light in front of a window, so as to reflect light, there is seen at the point of contact a black spot surrounded by six or seven coloured rings, the tints of which become gradually less strong. If the glasses are viewed by transmitted

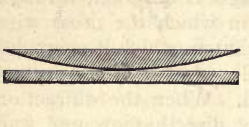


Fig. 617.

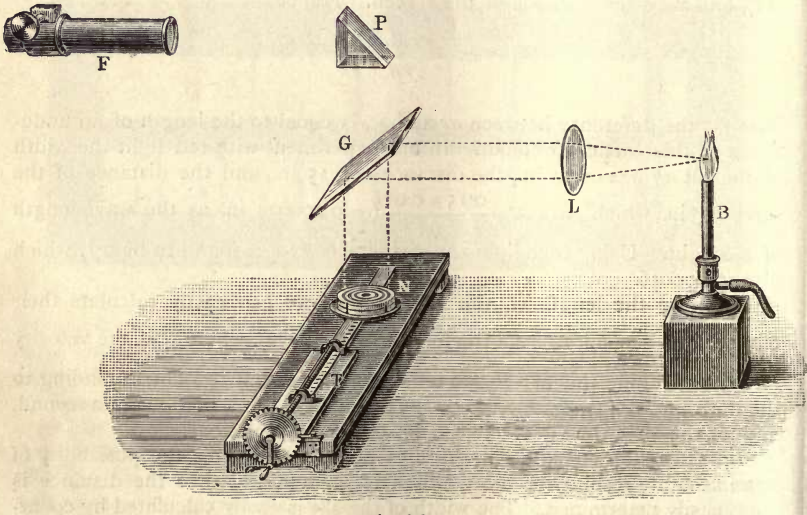


Fig. 618.

light, the centre of the rings is white, and each of the colours is exactly complementary of that of the rings by reflection. The lens and the glass plate are usually arranged in a brass mount, which by means of three screws allows the pressure to be regulated.

With homogeneous light, red for example, the rings are successively black and red; the diameters of corresponding rings are less as the colour is more refrangible; but with white light the rings are of the different colours of the spectrum, which arises from the fact that, as the rings of the different simple colours have different diameters, they are not exactly superposed, but are more or less separated.

For experiments on Newton's rings the apparatus represented in fig. 618 is suited. The glasses, N, for producing the rings are placed on the sliding part of a dividing machine. A pencil of monochromatic light from B, made parallel by the lens L, is thrown vertically downwards by the plate G. Above this is a totally reflecting prism, P, which sends the rays from N towards the telescope F.

The telescope is sighted so that the cross wire goes through the centre of the first ring; then by turning the screw the ten innermost bright and dark rings are successively brought in position; the respective diameters are then very easily determined by reading off on the scale the corresponding divisions.

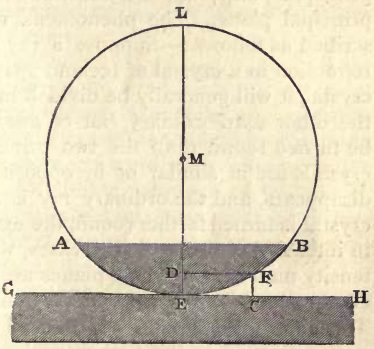


Fig. 619.

It is usual to speak of the successive rings as the first, second, third, &c. By the *first* ring is understood that of least diameter. The thickness of the layer of air which corresponds to any particular ring is obtained as follows. Let GH (fig. 619) be the plane glass surface on which is laid the convex glass lens AEB, the radius, R, of which is ME. Let DF be the radius, ρ, of any given ring; FC = DE, or e is the thickness of the layer of air corresponding to this ring. Now from a well-known geometrical principle, $\rho^2 = e \times (2R - e)$; but since e is infinitely small, its value in the bracket may be written $\rho^2 = e \times 2R$, so that knowing ρ and R it is easy to calculate e.

Newton found that the thicknesses corresponding to the successive *dark* rings are proportional to the numbers 0, 2, 4, 6 . . . . . , while for the *bright* rings the thicknesses were proportional to 1, 3, 5 . . . . . He found that for the first bright ring the thickness was $\frac{1}{178000}$ of an inch, when the light used was the brightest part of the spectrum; that is, the part on the confines of the orange and yellow rays.

If the focal length of the lens is from three to four yards, the rings can be seen with the naked eye; but if the length is less, the rings must be looked at with a lens.

651. **Explanation of Newton's rings.**—Newton's rings, and all phenomena of thin plates, are simple cases of interference.

In fig. 620, let MNOP represent a thin plate of a transparent body, on which a pencil of parallel rays of homogeneous light, *ab*, impinges; this will be partially reflected in the direction *bc*, and partially refracted towards *d*. But the refracted ray will undergo a second reflection at the surface OP;

the reflected ray will emerge at *e* in the same direction as the pencil of light reflected at the first surface ; and consequently the two pencils *bc* and *ef* will destroy or augment each other's effect according as they are in the same or different phases. We shall thus have an effect produced similar to that of fringes (646).

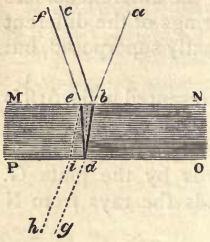


Fig. 620.

POLARISATION OF LIGHT

652. **Polarisation by double refraction.**—It has been already seen that when a ray of light passes through a crystal of Iceland spar (641), it becomes divided into two rays of *equal intensity* ; viz. the ordinary ray, and the extraordinary ray. These rays are found to possess other peculiarities, which are expressed by saying they are *polarised* ; namely, the ordinary ray in a principal plane, and the extraordinary ray in a plane at right angles to a principal plane. The phenomena which are thus designated may be described as follows :—Suppose a ray of light which has undergone *ordinary* refraction in a crystal of Iceland spar, to be allowed to pass through a second crystal, it will generally be divided into two rays ; namely, one ordinary, and the other extraordinary, but of *unequal intensities*. If the second crystal be turned round until the two principal planes coincide—that is, until the crystals are in similar or in opposite positions—then the extraordinary ray disappears, and the ordinary ray is at its greatest intensity ; if the second crystal is turned farther round, the extraordinary ray reappears, and increases in intensity as the angle increases, while the ordinary ray diminishes in intensity until the principal planes are at right angles to each other, when the extraordinary ray is at its greatest intensity and the ordinary ray vanishes. These are the phenomena produced when the ray which experienced ordinary refraction in the first crystal passes through the second. If that ray which has experienced extraordinary refraction in the first crystal is allowed to pass through the second crystal, the phenomena are similar to those above described ; but when the principal planes coincide, an extraordinary ray alone emerges from the second crystal, and when the planes are at right angles, an ordinary ray alone emerges.

These phenomena may also be thus described :—Let O and E denote the ordinary and extraordinary rays produced by the first crystal. When O enters the second crystal, it generally gives rise to two rays, an ordinary (O*o*), and an extraordinary (O*e*), of unequal intensities. When E enters the second crystal, it likewise gives rise to two rays, viz. an ordinary (E*o*) and an extraordinary (E*e*), of unequal intensities, the intensities varying with the angle between the principal planes of the crystals. When the principal planes coincide, only two rays, viz. O*o* and E*e*, emerge from the second crystal, and when the planes are at right angles, only two rays, viz. O*e* and E*o*, emerge from the second crystal. Since O gives rise to an ordinary ray when the principal planes are parallel, and E gives rise to an ordinary ray when they are at right angles, it is manifest that O is related to the principal plane in the same manner that E is related to a plane at right angles to a principal plane.

This phenomenon, which is produced by all double-refracting crystals, was first observed by Huyghens in Iceland spar, and in consequence of a suggestion of Newton's was afterwards called *polarisation*. It remained, however, an isolated fact until the discovery of polarisation by reflection recalled the attention of physicists to the subject. The latter discovery was made by Malus in 1808.

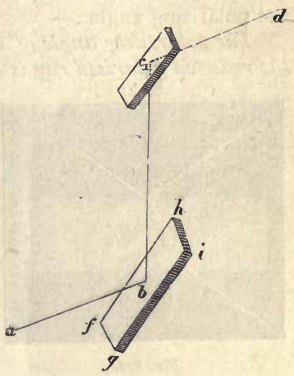


Fig. 621.

653. **Polarisation by reflection.**—When a ray of light, *ab* (fig. 621), falls on a polarised unsilvered glass surface, *fghi*, inclined to it at an angle of 35° 25′, it is reflected, and the reflected ray is polarised in the plane of reflection. If it were transmitted through a crystal of Iceland spar, it would pass through without bifurcation, and undergo an ordinary refraction; when the principal plane coincides with the plane of reflection, it would also be transmitted without bifurcation, but undergo extraordinary refraction, when the principal plane is at right angles to the plane of reflection; in other positions of the crystal it would give rise to an ordinary and an extraordinary ray of different intensities, according to the angle between the plane of reflection and the principal plane of the crystal. The peculiar property which the light has acquired by reflection at the surface *fghi* can also be exhibited as follows:—Let the polarised ray *bc* be received at *c*, on a second surface of unsilvered glass, at the same angle, viz. 35° 25′. If the surfaces are parallel, the ray is reflected; but if the second plate is caused to turn round *cb*, the intensity of the reflected ray continually diminishes, and when the glass surfaces are at right angles to each other, no light is reflected. By continuing to turn the upper mirror the intensity of the reflected ray gradually increases, and attains a maximum value when the surfaces are again parallel.

The above statement will serve to describe the phenomenon of polarisation by reflection so far as the principles are concerned; the apparatus best adapted for exhibiting the phenomenon will be described farther on.

654. **Angle of polarisation.**—The *polarising angle* of a substance is the angle which the incident ray must make with the perpendicular to a plane polished surface of that substance in order that the polarisation be complete. For glass this angle is 54° 35′, and if in the preceding experiment the lower mirror were inclined at any other angle than this, the light would not be completely polarised in any position; this would be shown by its being partially reflected from the upper surface in all positions. Such light is said to be *partially polarised*. The polarising angle for water is 52° 45′; for quartz, 57° 32′; for diamond, 68°; and it is 56° 30′ for obsidian, a kind of volcanic glass which is often used in these experiments.

Light which is reflected from the surface of water, from a slate roof, from a polished table, or from oil paintings, is all more or less polarised. The ordinary light of the atmosphere is frequently polarised, especially in the earlier and later periods of the day, when the solar rays fall obliquely on

the atmosphere. Almost all reflecting surfaces may be used as polarising mirrors. Metallic surfaces form, however, an important exception.

Brewster discovered the following remarkably simple law in reference to the polarising angle :—

The polarising angle of a substance is that angle of incidence for which the reflected polarised ray is at right angles to the refracted ray.

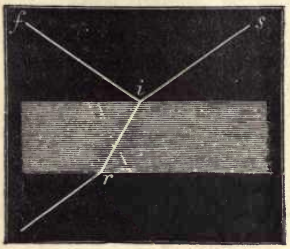


Fig. 622.

Thus, in fig. 622, if *si* is the incident, *ir* the refracted, and *if* the reflected ray, the polarisation is most complete when *fi* is at right angles to *ir*.

The *plane of polarisation* is the plane of reflection in which the light becomes polarised ; it coincides with the plane of incidence, and therefore contains the polarising angle.

A simple geometrical consideration will show that the above law may be thus expressed :—*The tangent of the angle of polarisation of a substance is equal to its refractive index.* As the refractive index differs with the different colours, it follows that the angle of polarisation cannot be the same for all colours. This explains why a ray of white light is never completely polarised.

655. **Polarisation by single refraction.**—When an unpolarised luminous ray falls upon a glass plate placed at the polarising angle, one part is reflected ; the other part becomes refracted in passing through the glass, and the transmitted light is now found to be partially polarised. If the light which has passed through one plate, and whose polarisation is very feeble, be transmitted through a second plate parallel to the first, the effects become more marked, and by ten or twelve plates are tolerably complete. A bundle of such plates, for which the best material is the glass used for covering microscopic objects, fitted in a tube at the polarising angle, is frequently used for examining or producing polarised light.

If a ray of light fall at any angle on a transparent medium, the same holds good with a slight modification. In fact, part of the light is reflected and part refracted, and both are found to be partially polarised, *equal quantities in each being polarised, and their planes of polarisation being at right angles to each other.* It is, of course, to be understood that the polarised portion of the reflected light is polarised in the plane of reflection, which is likewise the plane of refraction.

656. **Polarising instruments.**—Every instrument for investigating the properties of polarised light consists essentially of two parts—one for polarising the light, the other for ascertaining or exhibiting the fact of light having undergone polarisation. The former part is called the polariser, the latter the analyser. Thus in art. 652 the crystal producing the first refraction is the *polariser*, that producing the second refraction is the *analyser*. In art. 653 the mirror at which the first reflection takes place is the polariser, that at which the second reflection takes place is the analyser. Some of the most convenient means of producing polarised light will now be described, and it will be remarked that any instrument that can be used as a polariser

can also be used as an analyser. The experimenter has, therefore, considerable liberty of selection.

657. **Norremberg's apparatus.**—The most simple but complete instrument for polarising light is that invented by Norremberg. It may be used for repeating most of the experiments on polarised light.

It consists of two brass rods, *b* and *d* (fig. 623), which support an unsilvered mirror, *n*, of ordinary glass, movable about a horizontal axis. A small graduated circle indicates the angle of inclination of the mirror. Between the feet of the two columns there is a silvered glass, *p*, which is fixed and horizontal. At the upper end of the columns is a graduated plate, *i*, in which a circular disc, *o*, rotates. This disc, in which there is a square aperture, supports a mirror of black glass, *m*, which is inclined to the vertical at the polarising angle. An annular disc, *k*, can be fixed at different heights on the columns by means of a screw. A second ring, *a*, may be moved around the axis. It supports a black screen, in the centre of which there is a circular aperture.

When the mirror *n* makes with the vertical an angle of 35° 25′, which is the complement of the polarising angle for glass, the rays of light, S*n*, which meet the mirror at this angle become polarised, and are reflected in the direction *np* towards the mirror *p*, which sends them in the direction *pnr*. After having passed through the glass, *n*, the polarised ray falls upon the blackened glass *m* under an angle of 35° 25′, because the mirror makes exactly the same angle with the vertical. But if the disc, *o*, to which the mirror, *m*, is fixed, be turned horizontally, the intensity of the light reflected from the upper mirror gradually diminishes, and totally disappears when it has been moved through 90°. The position is that represented in the diagram: the plane of incidence on the upper mirror is then perpendicular to the plane of incidence, S*np*, on the mirror, *n*. When the upper mirror is again turned, the intensity of the light increases until it has passed through 180°, when it again reaches a maximum. The mirrors *m* and *n* are then parallel. The same phenomena are repeated as the mirror *m* continues to be turned in the same direction, until it again comes into its original

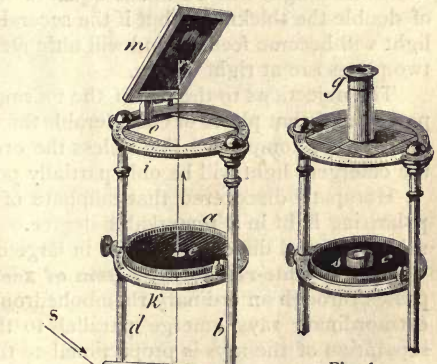


Fig. 624.

Fig. 623.

position ; the intensity of the reflected light being greatest when the mirrors are parallel, and being reduced to zero when they are at right angles. If the mirror *m* is at a greater or less angle than 35° 25′, a certain quantity of light is reflected in all positions of the plane of incidence.

658. **Tourmaline.**—The primary form of this crystal is a regular hexagonal prism. Tourmaline, as already stated, is a negative uniaxial crystal, and its optic axis coincides with the axis of the prism. For optical purposes a plate is cut from it parallel to the axis. When a ray of light passes through such a plate, an ordinary ray and an extraordinary ray are produced polarised in planes at right angles to each other ; viz. the former in a plane at right angles to the plate parallel to the axis, and the latter in a plane at right angles to the axis. The crystal possesses, however, the remarkable property of rapidly absorbing the ordinary ray ; consequently, when a plate of a certain thickness is used, the extraordinary ray alone emerges—in other words, a beam of common light emerges from the plate of tourmaline polarised in a plane at right angles to the axis of the crystal. If the light thus transmitted be viewed through another similar plate held in a parallel position, little change will be observed, excepting that the intensity of the transmitted light will be about equal to that which passes through a plate of double the thickness ; but if the second tourmaline be slowly turned, the light will become feebler, and will ultimately disappear when the axes of the two plates are at right angles.

The objections to the use of the tourmaline are that it is not very transparent, and that plates of considerable thickness must be used if the polarisation is to be complete. For unless the ordinary ray is completely absorbed the emergent light will be only partially polarised.

Herapath discovered that sulphate of iodoquinine has the property of polarising light in a remarkable degree. Unfortunately, it is a very fragile substance, and difficult to obtain in large crystals.

659. **Double-refracting prism of Iceland spar.**—When a ray of light passes through an ordinary rhombohedron of Iceland spar, the ordinary and extraordinary rays emerge parallel to the original ray, consequently the separation of the rays is proportional to the thickness of the prism. But if the crystal is cut so that its faces are inclined to each other, the deviations of the ordinary and extraordinary rays will be different, they will not emerge parallel, and their separation will be greater as their distance from the prism increases. The light, however, becomes decomposed in passing through the prism, and the rays will be coloured. It is therefore necessary to *achromatise* (584) the prism, which is done by combining it with a prism of glass with its refracting angle turned in the contrary direction (fig. 625). In order to obtain the greatest amount of divergence, the refracting edges of the prism should be cut parallel to the optic axis, and this is always done.

Fig. 625.

Let us suppose that a ray of polarised light passes along the axis of the cylinder (fig. 625), and let us suppose that the cylinder is caused to turn slowly about its axis ; then the resulting phenomena are exactly like those already described (643). Generally there will be an ordinary and extraordinary ray produced, whose relative intensities will vary as the tube is

turned. But in two opposite positions the ordinary ray alone will emerge, and in two others at right angles to the former the extraordinary ray will alone emerge. When the ordinary ray alone emerges, the principal plane of the crystal—that is, a plane at right angles to its face, and parallel to its refracting edge—coincides with the original plane of polarisation of the ray. Consequently, by means of the prism, it can be ascertained both that the ray is polarised, and likewise the plane in which it is polarised.

660. **Nicol's prism.**—The Nicol's prism is one of the most valuable means of polarising light, for it is perfectly colourless, it polarises light completely, and it transmits only one beam of polarised light, the other being entirely suppressed.

It is constructed from a rhombohedron of Iceland spar, about an inch in height and $\frac{1}{3}$ of an inch in breadth. This is bisected in the plane which passes through the obtuse angles as shown in fig. 628 ; that is, along the plane *acbd* (fig. 609). The two halves are then again joined in the same order by means of Canada balsam.

The principle of the Nicol's prism is this :—The refractive index of Canada balsam, 1·549, is less than the ordinary index of Iceland spar 1·654,

Fig. 626.

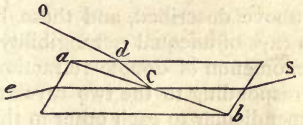


Fig. 627.

but greater than its extraordinary index 1·483. Hence when a luminous ray SC (fig. 627) enters the prism, the ordinary ray is totally reflected on the surface, *ab*, and takes the direction C*d*O, by which it is refracted out of the crystal, while the extraordinary ray, C*e*, emerges alone. Since the Nicol's prism allows only the extraordinary ray to pass, it may be used, like a tourmaline, as an analyser or as a polariser.

Foucault replaced the layer of Canada balsam by one of air, the two prisms being kept together by the mounting. The advantage of this is that the section *ab* (fig. 627) need not be so acute, so that the prism becomes shorter, and therefore cheaper.

Nicol's prism is the most important feature of most polarising apparatus. It is better than the polarising mirror on account of its more complete polarisation, and has the advantage over tourmaline of giving a colourless field of view.

661. **Physical theory of polarised light.**—The explanation of the dark bands produced by the interference of light is stated in art. 645 to resemble that of the formation of nodes and loops given in art. 276.

It might hence be supposed that the vibrations producing light are quite similar to those producing sound. But this is by no means the case. In fact, no assumption is made in art. 645 as to the *direction* in which the vibrating particles move, and accordingly the explanation is equally true whether the particles vibrate in the direction AB, BA, or at right angles to AB. As a matter of fact, the former is the case with the vibrations pro-

ducing sound, the latter with the vibrations producing light. In other words, the vibrations producing sound take place in the direction of propagation, the vibrations producing light are *transversal* to the direction of propagation.

This assumption as to the direction of the vibration of the particles of ether producing light is rendered necessary, and is justified, by the phenomena of polarisation.

When a ray of light is polarised, all the particles of ether in that ray vibrate in straight lines parallel to a certain direction in the front of the wave corresponding to the ray.

When a ray of light enters a double-refracting medium, such as Iceland spar, it becomes divided into two, as we have already seen. Now it can be shown to be in strict accordance with mechanical principles that, if a medium possesses unequal elasticity in different directions, a plane wave produced by transversal vibrations entering that medium will give rise to two plane waves moving with different velocities within the medium, and the vibrations of the particles in front of these waves will be in directions parallel respectively to two lines at right angles to each other. If, as is assumed in the undulatory theory of light, the ether exists in a double-refracting crystal in such a state of unequal elasticity, then the two plane waves will be formed as above described, and these, having different velocities, will give rise to two rays of unequal refrangibility (638). This is the physical account of the phenomenon of double refraction. It will be remarked that the vibrations corresponding to the two rays are transversal, rectilinear, and in directions perpendicular to each other in the rays respectively. Accordingly the same theory accounts for the fact that the two rays are both polarised, and in planes at right angles to each other.

It is a point still unsettled whether, when a ray of light is polarised with respect to a given plane, the vibrations take place in directions within or perpendicular to that plane. Fresnel was of the latter opinion. It is, however, convenient in some cases to regard the plane of polarisation as that plane in which the vibrations take place.

COLOURS PRODUCED BY THE INTERFERENCE OF POLARISED LIGHT

662. **Laws of the interference of polarised rays.**—After the discovery of polarisation, Fresnel and Arago tried whether polarised rays presented the same phenomena of interference as ordinary rays. They were thus led to the discovery of the following laws in reference to the interference of polarised light, and, at the same time, of the brilliant phenomena of coloration, which will be presently described :—

I. When two rays polarised in the same plane interfere with each other, they produce, by their interference, fringes of the very same kind as if they were common light.

II. When two rays of light are polarised at right angles to each other, they produce no coloured fringes in the same circumstances in which two rays of common light would produce them. When the rays are polarised in planes inclined to each other at any other angles, they produce fringes of intermediate brightness ; and if the angle is made to change, the fringes

gradually decrease in brightness from 0° to 90°, and are totally obliterated at the latter angle.

III. Two rays originally polarised in planes at right angles to each other may be subsequently brought into the same plane of polarisation without acquiring the power of forming fringes by their interference.

IV. Two rays polarised at right angles to each other, and afterwards brought into the same plane of polarisation, produce fringes by their interference like rays of common light, provided they originated in a pencil the whole of which was originally polarised in any one plane.

V. In the phenomena of interference produced by rays that have suffered double refraction, a difference of half an undulation must be allowed, as one of the pencils is retarded by that quantity, from some unknown cause.

663. **Effect produced by causing a pencil of polarised rays to traverse a double-refracting crystal.**—The following important experiment may be made most conveniently by Norremberg's apparatus (fig. 623). At *g* (fig. 624) there is a Nicol's prism. A plate of a double-refracting crystal cut parallel to its axis is placed on the disc at *e*. In the first place, however, suppose the plate of the crystal to be removed. Then, since the Nicol's prism allows only the extraordinary ray to pass when it is turned so that its principal plane coincides with the plane of reflection, no light will be transmitted (660). Place the plate of doubly refracting crystal, which is supposed to be of moderate thickness, in the path of the reflected ray at *e*. Light is now transmitted through the Nicol's prism. On turning the plate, the intensity of the transmitted light varies ; it reaches its maximum when the principal plane of the plate is inclined at an angle of 45° to the plane of reflection, and disappears when these planes either coincide with or are at right angles to each other. The light in this case is white. The interposed plate may be called the *depolarising plate*. The same or equivalent phenomena are produced when any other analyser is used. Thus, assume the double-refracting prism to be used, and suppose the depolarising plate to be removed. Then, generally, two rays are transmitted ; but if the principal plane of the analyser is turned in the plane of primitive polarisation, the ordinary ray only is transmitted, and when turned through 90°, the extraordinary ray only is transmitted. Let the analyser be turned into the former position, when, the depolarising plate being interposed, both ordinary and extraordinary rays are seen, and, when the depolarising plate is slowly turned round, the ordinary and extraordinary rays will be found to vary in intensity, the latter vanishing when the principal plane of the polarising plate either coincides with, or is at right angles to, the plane of primitive polarisation.

664. **Effect produced when the plate of crystal is very thin.**—In order to exhibit this, take a thin film of *selenite* or *mica* between the twentieth and sixtieth of an inch thick, and interpose it as described in the last article. If the thickness of the film is uniform, the light now transmitted through the analyser will be no longer white, but of a uniform tint ; the colour of the tint being different for different thicknesses—for instance, red, or green, or blue, or yellow, according to the thickness ; the intensity of the colour depending on the inclination of the principal plane of the film to the plane of reflection, being greatest when the angle of inclination is 45°. Let us

now suppose the crystalline film to be fixed in that position in which the light is brightest, and its colour to be *red*. Let the analyser (the Nicol's prism) be turned round, the colour will grow fainter, and when it has been turned through 45°, the colour disappears, and no light is transmitted; on turning it further, the complementary colour, *green*, makes its appearance, and increases in intensity until the analyser has been turned through 90°; after which the intensity diminishes until an angle of 135° is attained, when the light again vanishes, and, on increasing the angle, it changes again into red. Whatever be the colour proper to the plate, the same series of phenomena will be observed, the colour passing into its complementary when the analyser is turned. That the colours are really complementary is proved by using a double-refracting prism as analyser. In this case two rays are transmitted, each of which goes through the same changes of colour and intensity as the single ray described above; but whatever be the colour and intensity of the one ray in a given position, the other ray will have the same when the analyser has been turned through an angle of 90°. Consequently, these two rays give simultaneously the appearances which are successively presented in the above case by the same ray at an interval of 90°. If now the two rays are allowed to overlap, they produce white light; thereby proving their colours to be complementary.

Instead of using plates of different thicknesses to produce different tints, the same plate may be employed inclined at different angles to the polarised ray. This causes the ray to traverse the film obliquely, and, in fact, amounts to an alteration in its thickness.

With the same substance, but with plates of increasing thickness, the tints follow the laws of the colours of Newton's rings (650). The thickness of the depolarising plate must, however, be different from that of the layer of air in the case of Newton's rings to produce corresponding colours. Thus corresponding colours are produced by a plate of mica and a layer of air when the thickness of the former is about 400 times that of the latter. In the case of selenite the thickness is about 230 times, and in the case of Iceland spar about 13 times, that of the corresponding layer of air.

665. **Theory of the phenomena of depolarisation.**—The phenomena described in the last articles admit of complete explanation by the undulatory theory, but not without the aid of abstruse mathematical calculations. What follows will show the nature of the explanation. Let us suppose, for convenience, that in the case of a polarised ray the particles of ether vibrate in the plane of polarisation (661), and that the analyser is a double refracting prism, with its principal plane in the plane of primitive polarisation; then the vibrations, being wholly in that plane, have no resolved part in a plane at right angles to it, and, consequently, no extraordinary ray passes through the analyser; in other words, only an ordinary ray passes. Now take the depolarising plane cut parallel to the axis, and let it be interposed in such a manner that its principal plane makes any angle (θ) with the plane of primitive polarisation. The effect of this will be to cause the vibrations of the primitive ray to be resolved in the principal plane and at right angles to it, thereby giving rise to an ordinary ray (O) and an extraordinary ray (E), which, however, do not become separated on account of the thinness of the depolarising plate. They will not form a single plane polarised ray

on leaving the plate, since they are unequally retarded in passing through it, and consequently leave it in different phases. Since neither of the planes of polarisation of O and E coincides with the principal plane of the analyser, the vibrations composing them will again be resolved—viz. O gives rise to O*o* and O*e*, and E gives rise to E*o* and E*e*. But the vibrations composing O*o* and E*o*, being in the same phase, give rise to a single ordinary ray, I*o*, and in like manner O*e* and E*e* give rise to a single extraordinary ray, I*e*. Thus the interposition of the depolarising plate restores the extraordinary ray.

Suppose the angle θ to be either 0° or 90°. In either case the vibrations are transmitted through the depolarising plate without resolution, consequently they remain wholly in the plane of primitive polarisation, and on entering the analyser cannot give rise to an extraordinary ray.

If the Nicol's prism is used as an analyser, the ordinary ray is suppressed by mechanical means. Consequently only I*e* will pass through the prism, and that for all values of θ except 0° and 90°.

A little consideration will show that the joint intensities of all the rays existing at any stage of the above transformations must continue constant, but that the intensities of the individual rays will depend on the magnitude of θ; and when this circumstance is examined in detail, it explains the fact that I*e* increases in intensity as θ increases from 0° to 45°, and then decreases in intensity as θ increases from 45° to 90°.

In regard to the colour of the rays it is to be observed that the formulæ for the intensities of I*o* and I*e* contain a term depending on the length of the wave and the thickness of the plate. Consequently, when white light is used the relative intensities of its component colours are changed, and therefore I*o* and I*e* will each have a prevailing tint, which will be different for different thicknesses of the plate. The tints will, however, be complementary, since the joint intensities of I*o* and I*e* being the same as that of the original ray, they will, when superimposed, restore all the components of that ray in their original intensities, and therefore produce white light.

666. **Coloured rings produced by polarised light in traversing double refracting films.**—In the experiments with Norremberg's apparatus which have just been described (663), a pencil of parallel rays traverses the film of crystal perpendicularly to its faces, and as all parts of the film act in the same manner, there is everywhere the same tint. But when the incident rays traverse the plate under different obliquities, which comes to the same thing as if they traversed plates differing in thickness, coloured rings are formed similar to Newton's rings.

Fig. 628.

The best method of observing these new phenomena is by means of the *tourmaline pincette* or tongs (fig. 628). This is a small instrument consisting of two tourmalines, cut parallel to the axis, each of them being fitted in a copper disc. These two discs, which are perforated in the centre, and blackened, are mounted in two rings of silvered copper, which is coiled, as

shown in the figure, so as to form a spring, and press together the tourmalines. The tourmalines turn with the disc, and may be so arranged that their axes are either perpendicular or parallel.

The crystal to be experimented upon, being fixed in the centre of a cork disc, is placed between the two tourmalines, and the pincette is held before the eye so as to view diffused light. The tourmaline farthest from the eye acts as polariser and the other as analyser. If the crystal thus viewed is uniaxial, and cut perpendicularly to the axis, and a homogeneous light—red, for instance—is looked at, a series of alternately dark and red rings is seen. With another simple colour similar rings are obtained, but their diameter decreases with the refrangibility of the colour. On the other hand, the diameters of the rings diminish when the thickness of the plates increases, and beyond a certain thickness no more rings are produced. If, instead of illuminating the rings by homogeneous light, white light be used, then since the rings of the different colours produced have not the same diameter, they are partially superposed, and produce very brilliant variegated colours.

The position of the crystal has no influence on the rings, but this is not the case with the relative position of the two tourmalines. For instance, in experimenting on Iceland spar cut perpendicular to the axis, and from 1 to 20 millimetres in thickness, when the axes of the tourmalines are perpendicular, a beautiful series of rings is seen, brilliantly coloured, and traversed by a black cross, as shown in fig. 1, Plate II. If the axes of the tourmalines are parallel, the rings have tints complementary to those they had at first, and there is a white cross (fig. 2, Plate II.) instead of a black one.

In order to understand the formation of these rings when polarised light traverses double-refracting films, it must first be premised that these films are traversed by a converging conical pencil, whose summit is the eye of the observer. Hence it follows that the virtual thickness of the film which the rays traverse increases with their divergence; but for rays of the same obliquity this thickness is the same; hence there result different degrees of retardation of the ordinary with respect to the extraordinary ray at different points of the plate, and consequently different colours are produced at different distances from the axis, but the same colours will be produced at the same distance from the axis, and consequently the colours are arranged in circles round the axis. The arms of the black cross are parallel to the optic axis of each of the tourmalines, and are due to an absorption of the polarised light in these directions. When the tourmalines are parallel the vibrations are transmitted, and hence the white cross.

Analogous effects are produced with all uniaxial crystals; for instance, tourmaline, emerald, sapphire, beryl, mica, pyromorphite, and ferrocyanide of potassium.

667. **Rings in biaxial crystals.**—In biaxial crystals, coloured rings are also produced, but their form is more complicated. The coloured bands, instead of being circular and concentric, have the form of curves, with two centres, the centre of each system corresponding to an axis of the crystal. Figs. 4, 5, and 6, Plate II., represent the curves seen when a plate of either cerussite, topaz, or nitre, cut perpendicularly to the axis, is placed between the two tourmalines, the plane containing the axis of the crystal being in the

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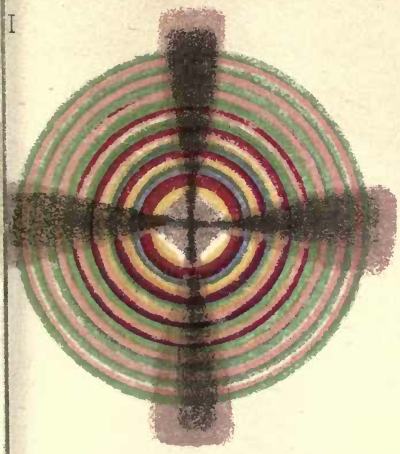

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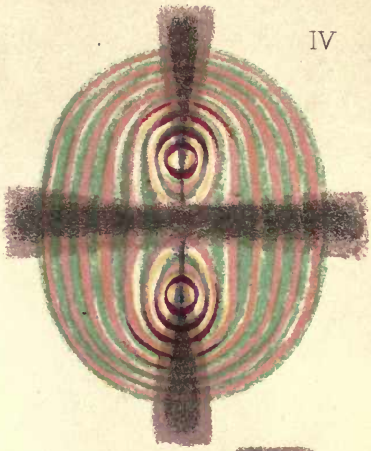

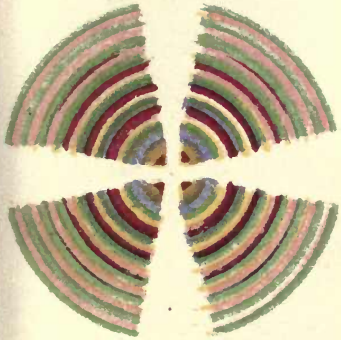

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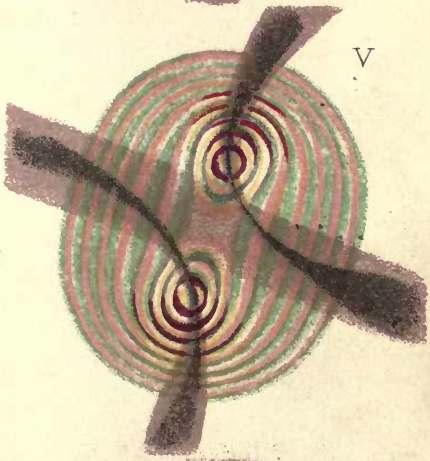

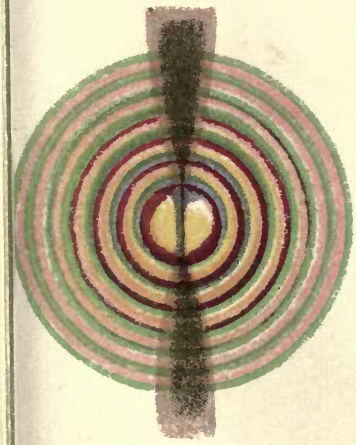

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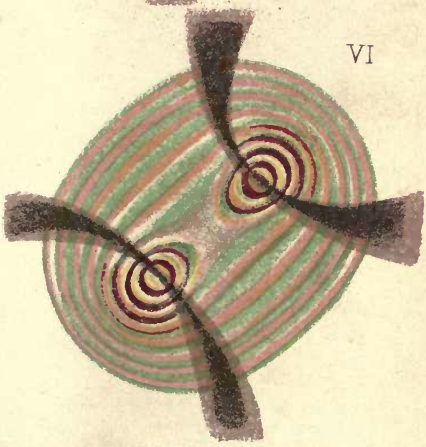


plane of primitive polarisation. When the axes of the two tourmalines are at right angles to each other, fig. 4, Plate II., is obtained. On turning the crystal without altering the tourmalines, fig. 5, Plate II., is seen, which changes into fig. 6, Plate II., when the crystal has been turned through 45°. If the axes of the tourmalines are parallel, the same coloured curves are obtained, but the colours are complementary, and the black cross changes into white. The angle of the optic axis in the case of nitre is only 5° 20′, and hence the whole system can be seen at once. But when the angle exceeds 20° to 25°, the two systems of curves cannot be simultaneously seen. There is then only one dark bar instead of the cross, and the bands are not oval, but circular. Fig. 3, Plate II., represents the phenomenon as seen with arragonite.

Sir John Herschel, who carefully measured the rings produced by biaxial crystals, referred them to the kind of curve known in geometry as the *lemniscate*, in strict accordance with the principles of the undulatory theory of light.

The observation of the system of rings which plates of crystals give in polarised light presents a means of distinguishing between optical uniaxial and optical biaxial crystals, even in cases in which no conclusion can be drawn as to the system in which a mineral crystallises from mere morphological reasons. In this way the optical investigation becomes a valuable aid in mineralogy; as, for example, in the case of mica, of which there are two mineralogical species, the uniaxial and the biaxial.

All the phenomena which have been described are only obtained by means of polarised light. Hence, a double-refracting film, with either a Nicol's prism or a tourmaline as analyser, may be used to distinguish between polarised and unpolarised light; that is as a polariscope.

668. **Colours produced by compressed or by unannealed glass.**— Ordinary glass is not endowed with the power of double refraction. It acquires this property, however, if by any cause its elasticity becomes more modified in one direction than in another. In order to effect this, it may be strongly compressed in a given direction, or it may be curved, or tempered; that is to say, cooled after having been heated. If the glass is then traversed by a beam of polarised light, effects of colour are obtained which are entirely analogous to those described in the case of doubly refracting crystals. They are, however, susceptible of far greater variety, according as the plates of glass have a circular, square, rectangular, or triangular shape, and according to the degree of tension of their particles.

When the polariser is a mirror of black glass, on which the light of the sky is incident, and the analyser is a Nicol's prism, through which the glass plates traversed by polarised light are viewed, figs. 629, 630, 632 represent the appearances presented successively, when a square plate of compressed glass is turned in its own plane; figs. 631 and 634 represent the appearances produced by a circular plate under the same circumstances; and fig. 633 that produced when one rectangular plate is superposed on another. This figure also varies when the system of plates is turned.

In consequence of being rapidly cooled, glass often acquires a strained condition. Hence, when the masses of glass, more especially the larger ones,

from which lenses are made, are examined by polarised light, the existence of strains may be revealed which would render it useless to go to the trouble

Fig. 629. Fig. 630. Fig. 631.

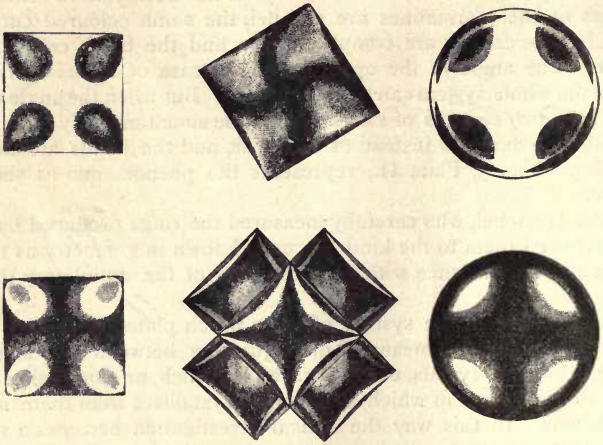


Fig. 632. Fig. 633. Fig. 634.

and expense of working such masses, as they would probably break in the operation.

ELLIPTICAL, CIRCULAR, AND ROTATORY POLARISATION

669. **Definition of elliptical and circular polarisation.**—In the cases hitherto considered, the particles of ether composing a polarised ray vibrate in parallel straight lines; to distinguish this case from those we are now to consider, such light is frequently called *plane polarised light.* It sometimes happens that the particles of ether describe *ellipses* about their positions of rest, the planes of the ellipses being perpendicular to the direction of the ray. If the axes of these ellipses are unequal and parallel, the ray is said to be *elliptically polarised.* In this case the particles which, when at rest, occupied a straight line, are, when in motion, arranged in a helix round the line of their original position as an axis, the helix changing from instant to instant. If the axes of the ellipses are equal, they become circles, and the light is said to be *circularly polarised.* If the minor axes become zero, the ellipses coincide with their major axes, and the light becomes *plane polarised.* Consequently, *plane* polarised light and *circularly* polarised light are particular cases of elliptically polarised light.

670. **Theory of the origin of elliptical and circular polarisation.**—Let us in the first place consider a simple pendulum (55) vibrating in any plane, the arc of vibration being small. Suppose that, when in its lowest position, it received a blow in a direction at right angles to the direction of its motion, such as would make it vibrate in an arc at right angles to its arc of primitive vibration, it follows from the law of the composition of velocities (52) that the joint effect will be to make it vibrate in an arc inclined

at a certain angle to the arc of primitive vibration, the magnitude of the angle depending on the magnitude of the blow. If the blow communicated a velocity equal to that with which the body is already moving, the angle would be 45°. Next suppose the blow to communicate an equal velocity, but to be struck when the body is at its highest point, this will cause the particle to describe a circle, and to move as a conical pendulum. If the blow is struck under any other circumstances, the particle will describe an ellipse. Now as the two blows would produce separately two simple vibrations in directions at right angles to each other, we may state the result arrived at as follows:—If two rectilinear vibrations are superinduced on the same particle in directions at right angles to each other, then: 1. If they are in the same or opposite phases, they make the point describe a rectilinear vibration in a direction inclined at a certain angle to either of the original vibrations. 2. But if their phases differ by 90° or a quarter of a vibration, the particle will describe a circle, provided the vibrations are equal. 3. Under other circumstances the particle will describe an ellipse.

To apply this to the case of polarised light. Suppose two rays of light polarised in perpendicular planes to coincide, each would separately cause the same particles to vibrate in perpendicular directions. Consequently—1. If the vibrations are in the same or opposite phases, the light resulting from the two rays is plane polarised. 2. If the rays are of equal intensity, and their phases differ by 90°, the resulting light is circularly polarised. 3. Under other circumstances the light is elliptically polarised.

As an example, if reference is made to arts. 665 and 666, it will be seen that the rays denoted by O and E are superimposed in the manner above described. Consequently, the light which leaves the depolarising plate is elliptically polarised. If, however, the principal plane of the depolarising plate is turned so as to make an angle of 45° with the plane of primitive polarisation, O and E have equal intensities; and if, further, the plate is made of a certain thickness, so that the phases of O and E may differ by 90°, or by a quarter of a vibration, the light which emerges from the plate is circularly polarised. This method may be employed to produce circularly polarised light.

Circular or elliptical polarisation may be either *right-handed* or *left-handed*, or what is sometimes called *dextrogyrate* and *lævogyrate*. If the observer looks along the ray in the direction of propagation, from polariser to analyser, then, if the particles move in the same direction as the hands of a watch with its face to the observer, the polarisation is right-handed.

671. **Fresnel's rhomb.**—This is a means of obtaining circularly polarised light. We have just seen (670) that, to obtain a ray of circularly polarised light, it is sufficient to decompose a ray of plane polarised light in such a manner as to produce two rays of light of equal intensity polarised in planes at right angles to each other, and differing in their paths by a quarter of an undulation. Fresnel effected this by means of a rhomb which has received his name. It is made of glass; its acute angle is 54°, and its obtuse 126°. If a ray (*a*, fig. 635) of plain polarised light falls perpendicularly on the face of AB, it will undergo two total internal reflections at an angle of about 54°, one at E, and the other at F, and will emerge perpendicularly.

If the plane ABCD be inclined at an angle of 45° to the plane of polarisation, the polarised ray will be divided into two coincident rays, with their planes of polarisation at right angles to each other, and it appears that one of them loses exactly a quarter of an undulation, so that on emerging from the rhomb the ray is circularly polarised. If the ray emerging as above from Fresnel's rhomb is examined, it will be found to differ from plane polarised light in this, that, when it passes through a double-refracting prism, the ordinary and extraordinary rays are of equal intensity in all positions of the prism. Moreover, it differs from ordinary light in this, that, if it pass through a second rhomb placed parallel to the first, a second quarter of an undulation will be lost, so that the parts of the original plane polarised ray will differ by half an undulation, and the emergen ray will be plane polarised ; moreover the plane of polarisation will be inclined at an angle of 45° to ABCD, but on the *other side* from the plane of primitive polarisation.

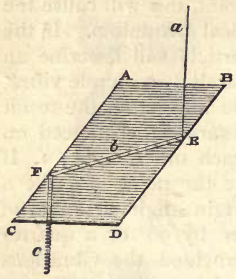


Fig. 635.

672. **Elliptical polarisation.**—In addition to the method already mentioned (671), elliptically polarised light is generally obtained whenever plane polarised light suffers reflection. Polarised light reflected from metals becomes elliptically polarised, the degree of ellipticity depending on the direction of the incident ray and of its plane of polarisation, as well as on the nature of the reflecting substance. When reflected from silver, the polarisation is almost circular, and when from galena almost plane. If elliptically polarised light be analysed by the Nicol's prism, it never vanishes, though at alternate positions it becomes fainter ; it is thus distinguished from plane and from circular polarised light. If analysed by Iceland spar, neither image disappears, but each undergoes changes in intensity.

Light can also be polarised elliptically in Fresnel's rhomb. If the angle between the planes of primitive polarisation and of incidence be any other than 45°, the emergent ray is elliptically polarised.

673. **Rotatory polarisation.**—Rock crystal or quartz possesses a remarkable property which was long regarded as peculiar to itself among all crystals, though it has been since found to be shared by tartaric acid and its salts, together with some other crystallised bodies. This property is called *rotatory polarisation*, and may be described as follows : Let a ray of homogeneous light be polarised, and let the analyser, say a Nicol's prism, be turned till the light does not pass through it. Take a thin section of a quartz crystal cut at right angles to its axis, and place it between the polariser and the analyser with its plane at right angles to the rays. The light will now pass through the analyser. The phenomenon is not the same as that previously described (663), for, if the rock crystal is turned round its axis, no effect is produced, and if the analyser is turned, the ray is found to be *plane polarised* in a plane inclined at a certain angle to the plane of primitive polarisation. If the light is red, and the plate 1 millimetre thick, this angle is about 17°. In some specimens of quartz the plane of polarisation is turned to the right hand, in others to the left hand. Specimens of the

former kind are said to be right-handed, those of the latter kind left-handed (670). This difference corresponds to a difference in crystallographic structure. The property possessed by rock crystal of turning the plane of polarisation through a certain angle was thoroughly investigated by Biot, who, amongst other results, arrived at this :—For a given colour, the angle, through which the plane of polarisation is turned, is proportional to the thickness of the quartz.

674. **Physical explanation of rotatory polarisation.**—The explanation of the phenomenon described in the last article is as follows : When a ray of polarised light passes along the axis of the quartz crystal, it is divided into two rays of *circularly* polarised light of equal intensity, which pass through the crystal with different velocities. In one the circular polarisation is right-handed, in the other left-handed (670). The existence of these rays was proved by Fresnel, who succeeded in separating them. On emerging from the crystal, they are compounded into a plane polarised ray ; but, since they move with unequal velocities within the crystal, they emerge in different phases, and consequently the plane of polarisation will not coincide with the plane of primitive polarisation. This can be readily shown by reasoning similar to that employed in art. 670. The same reasoning will also show that the plane of polarisation will be turned to the right or left, according as the right-handed or left-handed ray moves with the greater velocity. Moreover, the amount of the rotation will depend on the amount of the retardation of the ray whose velocity is least ; that is to say, it will depend on the thickness of the plate of quartz. In this manner the phenomena of rotary polarisation can be completely accounted for.

675. **Coloration produced by rotatory polarisation.**—The rotation is different with different colours ; its magnitude depends on the refrangibility, and is greatest with the most refrangible rays. In the case of red light a plate 1 millimetre in thickness will rotate the plane 17°, while, in the case of violet light, a plate of the same thickness will rotate it 44°. Hence with white light there will, in each position of the analysing Nicol's prism, be a greater or less quantity of each colour transmitted. In the case of a right-handed crystal, when the Nicol's prism is turned to the right, the colours will successively appear from the less refrangible to the more so—that is, in the order of the spectrum, from red to violet ; with a left-handed crystal in the reverse order. Obviously, in turning the Nicol's prism to the left, the reverse of these results will take place.

Fig. 636

When a quartz plate cut perpendicularly to the axis, and traversed by a ray of polarised light, is looked at through a doubly refracting prism, two brilliantly coloured images are seen, of which the tints are complementary : for their images are partially superposed, and in this position there is white light (fig. 636). When the prism is turned from left to right, the two images change colour and assume successively all the colours of the spectrum.

This will be understood from what has been said about the different rotation for different colours. Quartz rotates the plane of polarisation for red 17° for each millimetre, and for violet 44° ; hence from the great difference of these two angles, when the polarised light which has traversed the quartz

plate emerges, the various simple colours which it contains are polarised in different planes. Consequently, when the rays thus transmitted by the quartz pass through a double-refracting prism, they are each decomposed into two others polarised at right angles to each other : the various simple colours are not divided in the same proportion between the ordinary and extraordinary rays furnished by the prism ; the two images are, therefore, coloured ; but, since those which are wanting in one occur in the other, the colours of the images are perfectly complementary.

These phenomena of coloration may be well seen by means of Norremberg's apparatus (fig. 633). A quartz plate, *s*, cut at right angles to the axis and fixed in a cork disc, is placed on a screen *e* ; the mirror *n* being then so inclined that a ray of polarised light passes through the quartz, the latter is viewed through a double-refracting prism, *g* ; when this tube is turned, the complementary images furnished by the passage of polarised light through the quartz are seen.

676. **Rotatory power of liquids.**—Biot found that a great number of liquids and solutions possess the property of rotatory polarisation. He further observed that the deviation of the plane of polarisation can reveal differences in the composition of bodies where none is exhibited by chemical analysis. For instance, one of the two sugars obtained by the action of dilute acids on cane-sugar deflects the plane of polarisation to the right and the other to the left, although their chemical composition is the same.

The rotatory power of liquids is far less than that of quartz. In concentrated syrup of cane-sugar, which possesses the rotatory power in the highest degree, the power is $\frac{1}{36}$ that of quartz, so that it is necessary to operate upon columns of liquids of considerable length—8 inches, for example.

Fig. 637 represents an apparatus devised by Biot for measuring the rotatory power of liquids. On a metal groove, *g*, fixed to a support, *r*, is a brass tube, *d*, 20 centimetres long, tinned inside, in which is contained the liquid experimented upon. This tube is closed at each end by glass plates fastened by screw collars. At *m* is a mirror of black glass, inclined at the polarising angle to the axis of the tubes *bd* and *a*, so that the ray reflected by the mirror *m*, in the direction *bda*, is polarised. In the centre of the graduated circle *h*, inside the tube *a*, and at right angles to the axis *bda*, is a double-refracting achromatic prism, which can be turned about the axis of the apparatus by means of a button *n*. The latter is fixed to a limb *c*, on which is a vernier, to indicate the number of degrees turned through. Lastly, from the position of the mirror *m*, the plane of polarisation, *Sod*, of the reflected ray is vertical, and the zero of the graduation of the circle *h* is on this plane.

Before placing the tube *d* in the groove *g*, the extraordinary image furnished by the double-refracting prism disappears whenever the limb *c* corresponds to the zero of the graduation, because then the double-refracting prism is so turned that its principal section coincides with the plane of polarisation (659). This is the case also when the tube *d* is full of water or any other *inactive* liquid, like alcohol, ether, &c., which shows that the plane of polarisation has not been turned. But if the tube be filled with a solution of cane-sugar or any other *active* liquid, the extraordinary image reappears, and to extinguish it, the limb must be turned to a certain extent either to the right

or to the left of zero, according as the liquid is right-handed or left-handed, showing that the polarising plane has been turned by the same angle. With solution of cane-sugar the rotation takes place to the right ; and if with the same solution tubes of different lengths are taken, the rotation is found to increase proportionally to the length, in conformity with art. 673 ; further, with the same tube, but with solutions of various strengths, the rotation increases with the quantity of sugar dissolved, so that the quantitative analysis of a solution may be made by means of its angle of deviation.

In this experiment homogeneous light must be used ; for, as the various tints of the spectra have different rotatory powers, white light is decomposed in traversing an active liquid, and the extraordinary image does not disappear completely in any position of the double-refracting prism—it simply changes the tint. The transition tint (677) may, however, be observed. To avoid this inconvenience, a piece of red glass is placed in the tube between the eye and the double-refracting prism, which only allows red light to pass. The extraordinary image disappears in that case, whenever the principal section of the prism coincides with the plane of polarisation of the red ray.

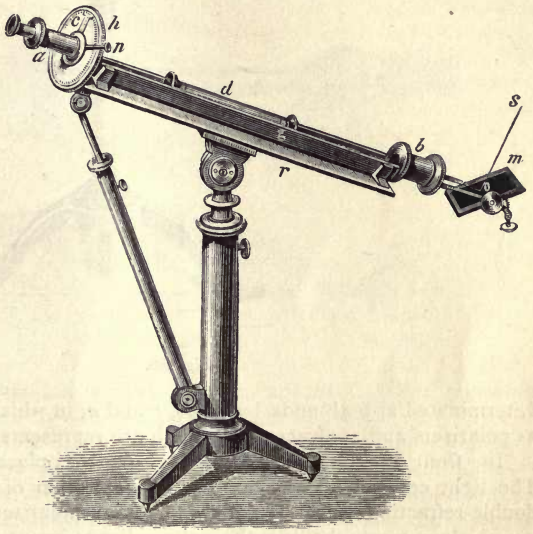


Fig. 637.

677. **Soleil's Saccharimeter.**—Soleil constructed an apparatus, based upon the rotatory power of liquids, for analysing saccharine substances, to which the name *saccharimeter* is applied. Fig. 638 represents the saccharimeter fixed horizontally, and fig. 639 gives a longitudinal section.

The principle of this instrument is not that of observing the amplitude of the rotation of the plane of polarisation, as in Biot's apparatus, but that of *compensation* ; that is to say, a second active substance is used acting in the opposite direction to that analysed, and whose thickness can be altered until the contrary actions of the two substances completely neutralise each other. Instead of measuring the deviation of the plane of polarisation, the thickness is measured which the plate of quartz must have in order to obtain perfect compensation.

The apparatus consists of three parts—a tube containing the liquid to be analysed, a polariser, and an analyser.

The tube *m*, containing the liquid, is made of copper, tinned on the inside, and closed at both ends by two glass plates. It rests on a support

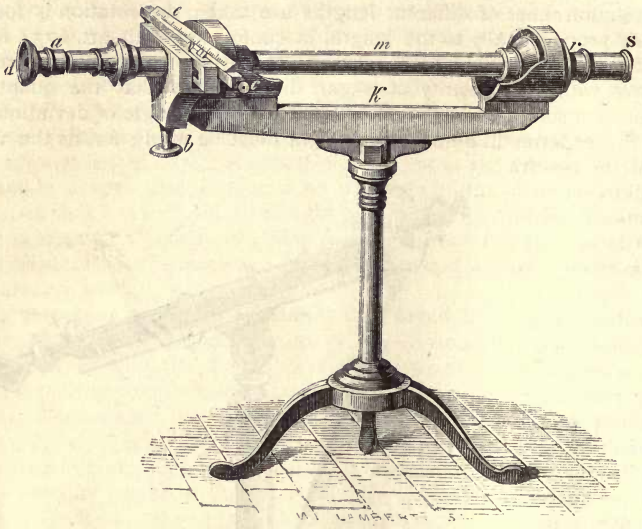


Fig. 638.

k, terminated at both ends by tubes, *r* and *a*, in which are the crystals used as polarisers and analysers, and which are represented in section (fig. 639).

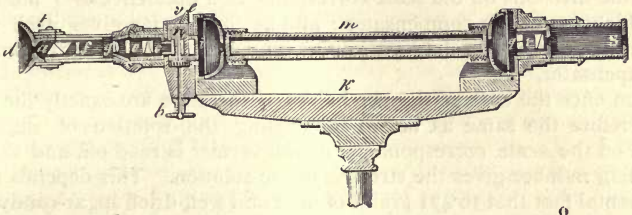
In front of the aperture S (fig. 639) is placed an ordinary lamp. The light emitted by this lamp in the direction of the axis first meets a double-refracting prism *r*, which serves as polariser (659). The ordinary image alone meets the eye, the extraordinary image being projected out of the field of vision in consequence of the amplitude of the angle which the ordinary makes with the extraordinary ray. The double-refracting prism is in such a position that the plane of polarisation is vertical, and passes through the axis of the apparatus.

Emerging from the double-refracting prism, the polarised ray meets a plate of quartz with double rotation; that is, this plate rotates the plane both to the right and to the left. This is effected by constructing the plate of two quartz plates of opposite rotation placed one on the other, as shown in fig. 640, so that the line of separation is vertical and in the same plane as the axis of the apparatus. These plates, cut perpendicularly to the axis, have a thickness of 3·65 millimetres, corresponding to a rotation of 90°, and give a rose-violet tint, called the *tint of passage*, or *transition tint*. As the quartz, whether right-handed or left-handed, turns always to the same extent for the same thickness, it follows that the two quartz plates *a* and *b* turn the plane of polarisation equally, one to the right and the other to the left. Hence, looked at through a double-refracting prism, they present exactly the same tint.

Having traversed the quartz, *q*, the polarised ray passes into the liquid in the tube *m*, and then meets a single plate of quartz, *i*, of any thickness,

the use of which will be seen presently. The compensator, *n*, which destroys the rotation of the column of liquid, *m*, consists of two quartz plates, with the same rotation either to the right or the left, but opposite to that of the plate *i*. These two quartz plates, a section of which is represented in fig. 640, are obtained by cutting obliquely a quartz plate with parallel sides so as to form two prisms of the same angle, N, N′, which is called a *biquartz*; superposing, then, these two prisms, as shown in the figure, a single plate is obtained with parallel faces, which can be varied at will. This is effected by fixing each prism to a slide, so as to move it either way without disturbing

Fig. 639



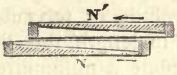


Fig. 640.

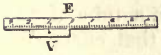


Fig. 641.

Fig. 642.

the parallelism. This motion is effected by means of a double rackwork and pinion motion turned by a milled head, *b* (figs. 638, 639).

When these plates move in the direction indicated by the arrows (fig. 640), it is clear that the sum of their thicknesses increases, and that it diminishes when the plates are moved in the contrary direction. A scale and a vernier follow the plates in their motion, and measure the thickness of the compensator. This scale, represented with its vernier in fig. 641, has two divisions with a common zero, one from left to right for right-handed liquids, and another from right to left for left-handed.

When the vernier is at zero, the sum of the thicknesses of the plates NN′ is exactly equal to that of the plate *i*, and as the rotation of the latter is opposed to that of the compensator, the effect is zero. But by moving the plates of the compensator in one or the other direction, either the compensator or the quartz, *i*, preponderates, and there is a rotation from left to right.

Behind the compensator is a double-refracting prism, *c* (fig. 642), serving as analyser to observe the polarised ray which has traversed the liquid and the various quartz plates. In order to understand more easily the object of the prism *c*, we will neglect for a moment the crystals and the lenses on the left of the drawing. If at first the zero of the vernier *v* coincides with that of the scale, and if the liquid in the tube is inactive, the actions of the compensator, and of the plate *i*, neutralise each other; and, the liquid having no action, the two halves of the plate *q*, seen through the prism *c*, give exactly the same tint as has been observed above. But if the tube filled with inactive liquid be replaced by one full of solution of sugar, the rotary power of this solution is added to that of one of the halves (*a* or *b*) of the plate *q* (viz.

that half which tends to turn the plane of polarisation in the same direction as the solution), and subtracted from that of the other. Hence the two halves of the plate *q* no longer show the same tint; the half *a*, for instance, is red, while the half *b* is blue. The prisms of the compensator are then moved by turning the milled head *b*, either to the right or to the left, until the difference of action of the compensator and of the plate *i* compensates the rotatory power of the solution, which takes place when the two halves of the plate *q*, with double rotation, revert to their original tint.

The direction of the deviation and the thickness of the compensator are measured by the relative displacement of the scale *e* and of the vernier *v*. Ten of the divisions on the scale correspond to a difference of 1 millimetre in the thickness of the compensator; and as the vernier gives itself tenths of these divisions, it therefore measures differences of $\frac{1}{100}$ in the thickness of the compensator.

When once the tints of the two halves of the plate are exactly the same, and therefore the same as before interposing the solution of sugar, the division on the scale corresponding to the vernier is read off, and the corresponding number gives the strength of the solution. This depends on the experimental fact that 16·471 grains of pure and well-dried sugar-candy being dissolved in water, and the solution diluted to the volume of 100 cubic centimetres, and observed in a tube of 20 centimetres in length, the deviation produced is the same as that effected by a quartz plate a millimetre thick. In making the analysis of raw sugar, a weight of 16·471 grains of sugar is taken, dissolved in water, and the solution made up to 100 cubic centimetres, with which a tube 20 centimetres in length is filled, and the number indicated by the vernier read off, when the primitive tint has been obtained. This number being 42, for example, it is concluded that the amount of crystallisable sugar in the solution is 42 per cent. of that which the solution of sugar-candy contained, and, therefore, 16·471 grains $\times \frac{42}{100}$, or 6·918 grains. This result is only valid when the sugar is not mixed with uncrystallisable sugar or some other left-handed substance. In that case the crystallisable sugar, which is right-handed, must be, by means of hydrochloric acid, converted into uncrystallisable sugar, which is left-handed; and a new determination is made, which, together with the first, gives the quantity of crystallisable sugar.

The arrangement of crystals and lenses, *o*, *g*, *f*, and *a*, placed behind the prism *c*, forms what Soleil calls the *producer of sensible tints*. For the most delicate tint—that by which a very feeble difference in the coloration of the two halves of the rotation plate can be distinguished—is not the same for all eyes; for most people it is of a violet-blue tint, like flax blossom; and it is important either to produce this tint, or some other equally sensible to the eye of the observer. This is effected by placing in front of the prism, *c*, at first a quartz plate, *o*, cut perpendicular to the axis, then a small Galileo's telescope consisting of a double convex glass, *g*, and a double concave glass, *f*, which can be approximated or removed from each other according to the distance of distinct vision of each observer. Lastly, there is a double-refracting prism, *c*, acting as polariser in reference to the quartz, and the prism *a* as analyser; and hence, when the latter is turned either right or left, the light which has traversed the prism *c*, and the plate *o*, changes its tint, and finally gives that which is the most delicate for the experimenter.

678. **Analysis of diabetic urine.**—In the disease *diabetes*, the urine contains a large quantity of fermentable sugar, called diabetic sugar, which in the natural condition of the urine turns the plane of polarisation to the right. To estimate the quantity of this sugar, the urine is first clarified by heating it with acetate of lead and filtering; the tube is filled with the clear liquid thus obtained; and the milled head *b* turned until, by means of the double-rotating plate, the same tint is obtained as before the interposition of the urine. Experiment has shown that 100 parts of the saccharimetric scale represent the displacement which the quartz compensators must have when there are 225·6 grains of sugar in a litre; hence each division of the scale represents 2·256 of sugar. Accordingly, to obtain the quantity of sugar in a given urine, the number indicated by the vernier, at the moment at which the primitive tint reappears, must be multiplied by 2·256.

679. **Polarisation of heat.**—The rays of heat, like those of light, may become polarised by reflection and by refraction. The experiments on this subject are difficult of execution; they were first made by Malus and Berard, in 1810; after the death of Malus they were continued by the latter philosopher.

In his experiments, the heat rays reflected from one mirror were received upon a second, just as in Norremberg's apparatus; from the second they fell upon a small metallic reflector, which concentrated them upon the bulb of a differential thermometer. Berard observed that heat was not reflected when the plane of reflection of the second mirror was at right angles to that of the first. As this phenomenon is the same as that presented by light under the same circumstances, Berard concluded that heat became polarised in being reflected.

The double refraction of heat may be shown by concentrating the sun's rays by means of a heliostat on a prism of Iceland spar, and investigating the resultant pencil by means of a thermopile, which must have a sharp narrow edge. In this case also there is an ordinary and an extraordinary ray, which follow the same laws as those of light. In the optic axis of the calcspar, heat is not doubly refracted. A Nicol's prism can be used for the polarisation of heat as well as for that of light: a polarised ray does not traverse the second Nicol if the plane of its principal section is perpendicular to the vibrations of the ray. The phenomena of the polarisation of heat may also be studied by means of plates of tourmaline and of mica. The angle of polarisation is virtually the same for heat as for light. In all these experiments the prisms must be very near each other.

The diffraction, and therefore the interference, of rays of heat has recently been established by the experiments of Knoblauch and others. And Forbes, who has repeated Fresnel's experiment with a rhombohedron of rock salt, has found that by two total internal reflections, heat is circularly polarised, just as is the case with light.

BOOK VIII

ON MAGNETISM

CHAPTER I

PROPERTIES OF MAGNETS

680. **Natural and artificial magnets.**—*Magnets* are substances which have the property of attracting iron, and the term *magnetism* is applied to the cause of this attraction and to the resulting phenomena.

This property exists in the highest degree in an ore of iron which is known in chemistry as the *magnetic oxide of iron.* Its composition is represented by the formula Fe_3O_4.

This mineral, which is also called *lodestone*, was first found at Magnesia, in Asia Minor, the name magnet being derived from this circumstance. The name lodestone, which is applied to this natural magnet, was given on account of its being used when suspended as a guiding or leading stone, from the Saxon *lædan*, to lead; so also the word *lodestar.* Lodestone is very abundant in nature; it is met with in the older geological formations, especially in Sweden and Norway, where it is worked as an iron ore, and furnishes the best quality of iron.

When a bar or needle of steel is rubbed with a magnet, it exhibits magnetic properties without the magnet losing anything of its own force. Such bars are called *artificial magnets*; they can be made more powerful than natural magnets, and, as they are also more convenient, they will be exclusively referred to in describing the phenomena of magnetism. The best modes of preparing them will be explained in a subsequent article.

681. **Poles and neutral lines.**—When a small piece of soft iron is suspended by a thread and a magnet is approached to it, the iron is attracted towards the magnet, and some force is required for its removal. The force of the attraction varies in different parts of the magnet; it is strongest at the two ends, and is totally wanting in the middle.

This variation may also be seen very clearly when a bar magnet is

placed in iron filings; these become arranged round the ends of the bar in feathery tufts, which decrease towards the middle of the bar, where there are none. That part of the surface of the bar where there is no visible magnetic force is called the *neutral line*; and the parts near the ends of the bar where the attraction is greatest are called the *poles*. Every magnet, whether natural or artificial, has two poles and a neutral line: sometimes, however, in magnetising bars and needles, poles are produced lying between the extreme points. Such magnets are abnormal, and these points are called *intermediate* or *consequent poles*. The shortest line joining the two poles is termed the *axis* of the magnet; in a horseshoe magnet the axis is in the direction of the keeper. The plane at right angles to the axis of a bar magnet and passing through the neutral line is sometimes called the *equator* of the magnet, and the *length* of a magnet, as far as magnetic actions are concerned, is the distance of the poles.

We shall presently see that a freely suspended magnet always sets with one pole pointing towards the north, and the other towards the south. The end pointing towards the north is called in this country the *north pole*, and the other end the *south pole*. The end of the magnetic needle pointing to the north is called the *marked end* of the needle. Sometimes also the end pointing to the north is called the *red* pole, and that to the south the *blue* pole; the corresponding terms red and blue magnetisms are occasionally used.

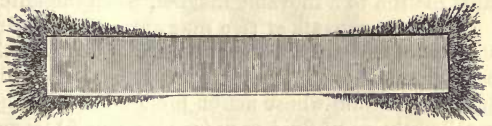


Fig. 643.

682. **Reciprocal action of two poles.**—The two poles of a magnet appear identical when they are brought in contact with iron filings (fig. 643), but this identity is only apparent, for when a small magnetic needle, *ab* (fig. 644), is suspended by a fine thread, and the north pole, A, of another needle is brought near its north pole, *a*, a repulsion takes place. If, on the contrary, A is brought near the south pole, *b*, of the movable needle, the latter is strongly attracted. Hence these two poles, *a* and *b*, are not identical, for one is repelled, and the other attracted, by the same pole of the magnet A. It may be shown in like manner that the two poles of the latter are also different, by successively presenting them to the same pole, *a*, of the movable needle. In one case there is repulsion, in the other attraction. Hence the following law may be enunciated :—

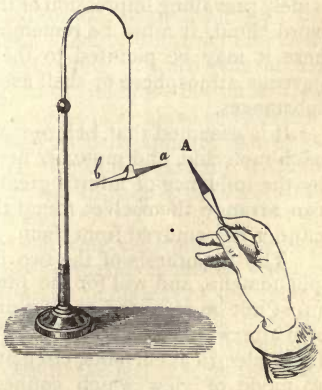


Fig. 644.

Poles of the same name repel, and poles of contrary name attract, one another.

The opposite actions of the north and south poles may be shown by the

following experiment:—A piece of iron, a key for example, is supported by a bar magnet. A second bar magnet of the same dimensions is then moved along the first, so that their poles are contrary (fig. 645). The key remains suspended so long as the two poles are at some distance, but when they are sufficiently near the key drops, just as if the bar which supported it had lost its magnetism. This, however, is not the case, for the key would be again supported if the first magnet were presented to it after the removal of the second bar.

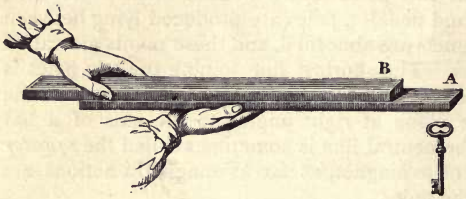


Fig. 645.

The attraction which a magnet exerts upon iron is reciprocal, which is indeed a general principle of all attractions. It is easily verified by presenting a mass of iron to a movable magnet, when the latter is attracted.

683. **Hypothesis of two magnetic fluids.**—In order to explain the phenomena of magnetism, the existence of two hypothetical *magnetic fluids* has been assumed, each of which acts repulsively on itself, but attracts the other fluid. The fluid whose action predominates at the north pole of the magnet is called the *north* fluid or *red* magnetism (681); and that at the south pole the *south* fluid, or *blue* magnetism. It is usual also to speak of north magnetism as *positive* and of south as *negative*, or + and − respectively. The term 'fluid' is apt to puzzle beginners, from its ambiguity. The idea of a liquid is ordinarily associated with the term 'a fluid'; hence the use of this term to explain the phenomena of magnetism and electricity has produced a widely prevailing impression of the material nature of these two forces. The word 'fluid,' it must be remembered, embraces gases as well as liquids, and here it may be pictured to the mind as representing an invisible, elastic, gaseous atmosphere or shell associated with the molecules of all magnetic substances.

It is assumed that, before magnetisation, these fluids are combined about each molecule, and mutually neutralise each other; they can be separated by the influence of a force greater than that of their mutual attraction, and can arrange themselves round the molecules to which they are attached, but cannot be removed from them.

The hypothesis of the two fluids is convenient in explaining magnetic phenomena, and will for the present be adhered to in what follows. But it must not be regarded as anything more than a provisional hypothesis, and it will afterwards be shown (879) that magnetic phenomena appear to result from electrical currents, circulating in the molecules of magnetic bodies; a mode of view which connects the theory of magnetism with that of electricity.

684. **Precise definition of poles.**—By aid of the preceding hypothesis we are enabled to obtain a clear idea of the distribution of the magnetism in a magnetised bar, and to account for the circumstance that there is no free magnetism in the middle of the bar, and that it is strongest at the poles. If AB (fig. 646) represent a magnet, then the alternate black and white spaces may be taken to represent the position of the magnetisms in a series

of particles after magnetisation: the black spaces, representing the south magnetism, all point in one direction, and the white ones the north in the opposite direction. The last half of the terminal molecule at one end would have north polarity, and at the other south polarity. Let N represent the north pole of a magnetic needle placed near the magnet AB; then the south magnetism *s* in the terminal molecule would tend to attract N, and the north magnetism *n* would tend to repel it; but as the molecule of south magnetism *s* is nearer N than the molecule of the north magnetism *n*, the attraction between *s* and N would be greater than the repulsion between *n* and N. Similarly the attraction between *s'* and N would be greater than the repulsion between *n'* and N, and so on with the following *s''* and *n''*, &c. And all these forces would give a resultant tending to attract N, whose point of application would have a certain fixed position, which would be the south pole of AB. In like manner it might be shown that the resultant of the forces acting at the other end of the bar would form a north pole, and would hence repel the north pole of the needle, but would attract its south pole.

Fig. 646.

A magnet is thus made up of a series of polarised particles, and that such a series really acts like an ordinary magnet may be shown by partly filling a glass tube with steel filings, and passing the pole of a strong magnet several times along the outside in one constant direction, taking care not to shake the tube. The individual filings will thus be magnetised, and the whole column of them presented to a magnetic needle will attract and repel its poles just like an ordinary bar magnet, exhibiting a north pole at one end, a south pole at the other, and no polarity in the middle; but on shaking the tube, or turning out the filings, and putting them in again so as to destroy the regularity, every trace of polarity will disappear. It appears hence that the polarity at each end of a magnet is caused by the fact that the resultant action on a magnetic body is strongest near the ends, and does not arise from any accumulation of magnetisms at the ends.

The same point may be illustrated by the following experiment, which is due to Sir W. Grove:—In a glass tube with flat glass ends is placed water in which is diffused magnetic oxide of iron. Round the outside of the tube is coiled some insulated wire. On looking at a light through the tube the liquid appears dark and muddy, but on passing a current of electricity through the wire it becomes clearer (879). This is due to the fact that by the magnetising action of the current, the particles, becoming magnetised, set with their longest dimension parallel to the axis of the tube, in which position they obstruct the passage of light to a less extent.

685. **Experiments with broken magnets.**—That the two magnetisms are present in all parts of the bar, and are not simply accumulated at the ends, is also evident from the following experiment:—A steel knitting-needle (fig. 647) is magnetised by rubbing it with one of the poles of a mag-

net, and then, the existence of the two poles AB and of the neutral line N having been ascertained by means of iron filings, it is broken in the middle. But now, on presenting successively the two halves to a magnet, each will be found to possess two opposite poles AB′ and A′B with a neutral line N, and to be in fact a perfect magnet. If these new magnets are broken in turn into two halves, each will be a complete magnet AB″ and A″B′ with its two poles and neutral line, and so on, as far as the division can be continued. It is, therefore, concluded by analogy that the smallest parts of a magnet, the ultimate molecules, contain the two magnetisms; that magnetism, in short, is a phenomenon the course of which resides in the elementary particle or molecule itself. Each molecule is a magnet. It follows also from this experiment that it is impossible to obtain an independent positive or negative mass of magnetism which is not associated with an equal mass of the opposite sign; in other words, that *unipolar* magnets have no existence.

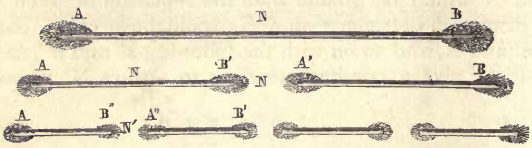


Fig. 647.

686. **Magnetic induction.**—When a magnetic substance is placed in contact with a magnet, the two magnetisms of the former become separated; and so long as the contact remains, it is a complete magnet, having its two poles and its neutral line. For instance, if a small cylinder of soft iron, *ab* (fig. 648), be placed in contact with one of the poles of a magnet, the cylinder can in turn support a second cylinder; this in turn a third, and so on, to as many as seven or eight, according to the power of the magnet. Each of these little cylinders is a magnet; if it be the north pole of the magnet to which the cylinders are attached, the part *a* will have south and *b* north magnetism; *b* will in like manner develop in the nearest end of the next cylinder south magnetism, and so on. But these cylinders, of course, are only magnets so long as the influence of a magnetised bar continues. For, if the first cylinder be removed from the magnet, the other cylinders immediately drop, and retain no trace of magnetism. The separation of the two magnetisms is only momentary, which proves that the magnets yield nothing to the iron. Hence we may have *temporary* magnets as well as *permanent* magnets; the former of iron and nickel, the latter of steel and cobalt (688).

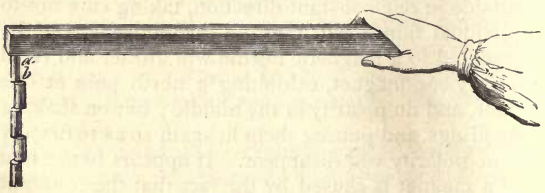


Fig. 648.

This action, in virtue of which a magnet can develop magnetisation in iron, is called *magnetic induction* or *influence*, and it can take place without actual contact between the magnet and the iron, as is seen in the following experiment:—A bar of soft iron is held with one end near a magnetic needle. If now the north pole of a magnet be approached to the iron without touch-

ing it, the needle will be attracted or repelled, according as its south or north pole is near the bar. For the north pole of the magnet will develop south magnetism in the end of the bar nearest it, and therefore north magnetism at the other end, which would thus attract the south but repel the north end of the needle. Obviously, if the other end of the magnet were brought near the iron, the opposite effects would be produced on the needle; or if the opposite pole of a second magnet of equal strength simultaneously be brought near the iron, the needle would be unaffected, as one magnet would undo the work of the other.

The formation of the tufts of iron filings which become attached to the poles of magnets (fig. 643) is due to induction. The parts in contact with the magnet are converted into magnets; these act inductively on the adjacent parts, these again on the following ones, and so on, producing a filamentary arrangement of the filings. The bush-like appearance of these filaments is due to the repulsive action which the free poles exert upon each other. Any piece of soft iron while being attracted by a magnet is for the time being converted into a magnet; hence is explained the paradoxical statement that 'magnets only attract magnets.'

687. **Coercive force.**—We have seen from the above experiments that soft iron becomes instantaneously magnetised under the influence of a magnet, but that this magnetism is not permanent, and ceases when the magnet is removed. Steel likewise becomes magnetised by contact with a magnet; but the operation is effected with difficulty, and in general the more so the more highly the steel is tempered. Placed in contact with a magnet, a steel bar acquires magnetic properties very slowly; and, to make the magnetism complete, the steel must be rubbed with one of the poles. But this magnetism, once evoked in steel, is permanent, and does not disappear when the inducing force is removed.

These different effects in soft iron and steel are ascribed to a kind of resistance analogous to friction which is often called *coercive force*, and which, in a magnetic substance, offers a hindrance to the separation of the two magnetisms, but which also prevents their recombination when once separated. In steel this coercive force is very great; in soft iron it is very small or almost absent. By oxidation, stretching, pressure, torsion, or hammering, &c., a certain amount of coercive force may be imparted to soft iron; and by heat the coercive force may be lessened, as will be afterwards seen.

688. **Difference between magnets and magnetic substances.**—*Magnetic substances* are substances which, like iron, steel, and nickel, are attracted by the magnet. They are assumed to possess the two magnetisms, but in a condition in which they neutralise each other as regards external action. Compounds containing iron are usually magnetic, and the more so in proportion as they contain a larger quantity of iron. Some, however, like iron pyrites, are not attracted by the magnet.

A magnetic substance is readily distinguished from a magnet. The former has no poles; if successively presented to the two ends of a magnetic needle, *ab* (fig. 644), it will attract both ends equally, while with one and the same end a magnet would attract the one end of the needle, but repel the other. Magnetic substances also have no action on each other; while mag-

nets attract or repel each other, according as unlike or like poles are presented. Attraction is no proof that a body is a magnet; repulsion is.

In the magnetic but unmagnetised condition the molecular magnets (685) are arranged quite irregularly, and their mutual action neutralises one another, so that there is no action on an external body. But if they are acted on by any magnetising power—a magne, tfor example—the effect is to give the molecular magnets a direction parallel to those of the magnet, and as soon as more molecular magnets set in one certain direction than in another, the magnet shows polarity; this polarity increases the more any one direction preponderates, and reaches a maximum when all the molecular magnets set in one direction.

Iron is not the only substance which possesses magnetic properties; nickel has considerable magnetic power, but far less than that of iron; cobalt is less magnetic than nickel: while to even a slighter extent chromium and manganese are magnetic. Further, we shall see that powerful magnets exert a peculiar influence on all substances.

CHAPTER II

TERRESTRIAL MAGNETISM. COMPASSES

689. **Directive action of the earth on magnets.**—When a magnetic needle is suspended by a thread, as represented in fig. 645, or is placed on a pivot on which it can move freely (fig. 649), it ultimately sets in a position which is more or less north and south. If removed from this position, it always returns to it after making a certain number of oscillations.

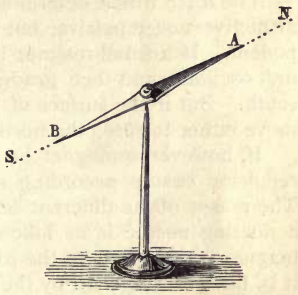


Fig. 649.

Analogous observations have been made in different parts of the globe, from which the earth has been compared to an immense magnet, whose poles are very near the terrestrial poles, and whose neutral line virtually coincides with the equator.

The polarity in the northern hemisphere is called the *northern* or *boreal* polarity, and that in the southern hemisphere the *southern* or *austral* polarity. In French works the end of the needle pointing north is called the *austral* or *southern* pole, and that pointing to the south the *boreal* or *northern* pole ; a designation based on this hypothesis of a terrestrial magnet, and on the law that unlike magnetisms attract each other. In practice it will be found more convenient to use the English names, and call that end of the magnet which points to the north the *north pole*, and that which points to the south the *south pole* ; the north pole of a magnet is a *north-seeking* pole, and a south pole a *south-seeking* pole. To avoid ambiguity, that end of the needle pointing north is in England sometimes spoken of as the *marked end* of the needle (681).

690. **Terrestrial magnetic couple.**—From what has been stated, it is clear that the magnetic action of the earth on a magnetised needle may be compared to a *couple* ; that is, to a system of two equal forces, parallel, but acting in contrary directions.

For let *ab* (fig. 650) be a movable magnetic needle making an angle with the magnetic meridian M′M (691). The earth's north pole acts attractively on the marked pole, *a*, and repulsively on the other pole, *b*, and two contrary forces are produced, *an* and *bn′*, which are equal and parallel : for the terrestrial pole is so distant, and the needle so small, as to justify the assumption that the two directions *an* and *bn′* are parallel, and that the two poles

are equidistant from the earth's north pole. But the earth's south pole acts similarly on the poles of the needle, and produces two other forces, *as* and *bs*, which are also equal and parallel ; but the two forces *an* and *as* may be reduced to a single resultant *a*N (33), and the forces *bn'* and *bs'* to a resultant *b*S ; the two forces *a*N and *b*S are equal, parallel, and act in opposite directions, and they constitute the *terrestrial magnetic couple* ; it is this couple which makes the needle set ultimately in the magnetic meridian, a position in which the two forces N and S are in equilibrium.

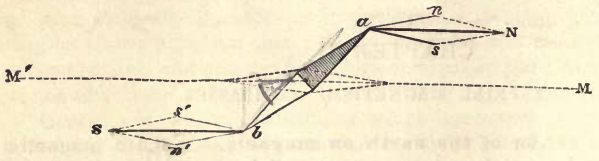


Fig. 650.

The force which determines the direction of the needle thus is neither attractive nor repulsive, but simply directive. It has no horizontal component. If a small magnet be placed on a cork floating in water, it will at first oscillate, and then gradually set in a line which is virtually north and south. But if the surface of the water be quite smooth, the needle will not move either towards the north or towards the south.

If, however, a magnet be approached to a floating needle, attraction or repulsion ensues, according as one or the other of the poles is presented. The reason of the different actions exerted by the earth and by a magnet on a floating needle is as follows :—When the north pole, for instance, of the magnet is presented to the south pole of the needle, the latter is attracted ; it is, however, repelled by the south pole of the magnet. Now the force of magnetic attraction or repulsion decreases with the distance ; and, as the distance between the south pole of the needle and the north pole of the magnet is less than the distance between the south pole of the needle and the south pole of the magnet, the attraction predominates over the repulsion, and the needle moves towards the magnet. But the earth's magnetic north pole is so distant from the floating needle that its length may be considered infinitely small in comparison, and one pole of the needle is just as strongly repelled as the other is attracted.

The action of the earth on a magnet has also no component which is directed vertically ; for if a steel bar be carefully equipoised and then magnetised there is not the least alteration in the weight.

691. **Magnetic elements. Declination.**—In order to obtain a full knowledge of the earth's magnetism at any place, three essentials are requisite ; these are—i. Declination ; ii. Inclination ; iii. Force or Intensity. These three are termed the *magnetic elements* of the place. We shall explain hem in the order in which they stand.

The *geographical meridian* of a place is the imaginary plane passing through this place and through the two terrestrial poles, and the *meridian* is the outline of this plane upon the surface of the globe. Similarly the *magnetic meridian* of a place is the vertical plane passing at this place through the two poles of a movable magnetic needle in equilibrium about its vertical axis.

In general the magnetic meridian does not coincide with the geographical meridian, and the angle which the magnetic makes with the geographical meridian—that is to say, the angle which the direction of the needle makes with the meridian—is called the *declination* or *variation of the magnetic needle.* The declination is said to be *east* or *west*, according as the north pole of the needle is to the east or west of the geographical meridian.

692. **Variations in declination.**—The declination of the magnetic needle, which varies in different places, is at present west in Europe and in Africa, but east in Asia and in the greater part of North and South America. It shows further considerable variations even in the same place. These variations are of two kinds ; some are regular, and are either secular, annual, or diurnal ; others, which are irregular, are called *magnetic storms* (694).

Secular variations.—In the same place the declination varies in the course of time, and the needle appears to make oscillations to the east and west of the meridian, the duration of which extends over centuries. The declination has been known at Paris since 1580, and the following table represents the variations which it has undergone :—

Year	Declination	Year	Declination
1580	11° 30′ E.	1850	20° 30′ W.
1663	0°	1855	19° 57′ W.
1700	8° 10′ W.	1860	19° 32′ W.
1780	19° 55′ W.	1865	18° 44′ W.
1785	22° 00′ W.	1875	17° 21′ W.
1805	22° 5′ W.	1880	16° 53′ W.
1814	22° 34′ W.	1883	16° 33′ W.
1825	22° 22′ W.	1888	15° 58′ W.
1830	22° 12′ W.	1891	15° 36′ W.
1835	22° 4′ W.		

This table shows that since 1580 the declination has varied at Paris as much as 34°, and that the greatest westerly declination was attained in 1814, since which time the needle has gradually tended towards the east.

At London the needle showed in 1580 an easterly declination of 11° 36′ ; in 1663 it was at zero ; from that time it gradually tended towards the west, and reached its maximum declination of 24° 41′ in 1818 ; since then it has steadily diminished ; it was 22° 30′ in 1850, 19° 32′ in 1873, 19° 24′ in 1874, 19° 16′ in 1875, 19° 10′ in 1876, 19° 3′ in 1877, 18° 52′ in 1878, 18° 40′ in 1881, 18° 15′ in 1883, and is now (1892) 17° 12′ W.

At Yarmouth and Dover the variation is about 40′ less than at London ; at Hull and Southampton about 20′ greater ; at Newcastle and Swansea about 1° 45′, and at Liverpool 2° 0′, at Edinburgh 3° 0′, and at Glasgow and Dublin about 3° 50′ greater than at London.

The following are the observations of the magnetic elements at Kew, extending over twenty-six years :—

Year	Declination	Inclination	Horizontal force
1865	20° 59′	68° 7′	3·829
1875	19° 41′	67° 48′	3·885
1880	18° 59′	67° 42′	3·899
1882	18° 45′	67° 41′	3·904
1884	18° 32′	67° 39′	3·916
1885	18° 26′	67° 38′	3·917
1886	18° 12′	67° 37′	3·923
1887	18° 7′	67° 37′	3·927
1888	18° 1′	67° 36′	3·930
1889	17° 57′	67° 34′	3·936
1890	17° 51′	67° 33′	3·941
1891	17° 42′	67° 31′	3·946

In certain parts of the earth the magnet coincides with the geographical meridian. These points are connected by an irregularly curved imaginary line, called a *line of no variation*, or *agonic line*. Such a line cuts the east of South America, and, passing east of the West Indies, enters North America near Philadelphia, and traverses Hudson's Bay; thence it passes through the North Pole, entering the Old World east of the White Sea, traverses the Caspian, cuts the east of Arabia, turns then towards Australia, and passes through the South Pole, to join itself again.

Isogonic lines are lines connecting those places on the earth's surface in which the declination is the same. The first of the kind was constructed in 1700 by Halley; since the elements of the earth's magnetism are continually changing, the course of such a line can only be determined for a definite time.

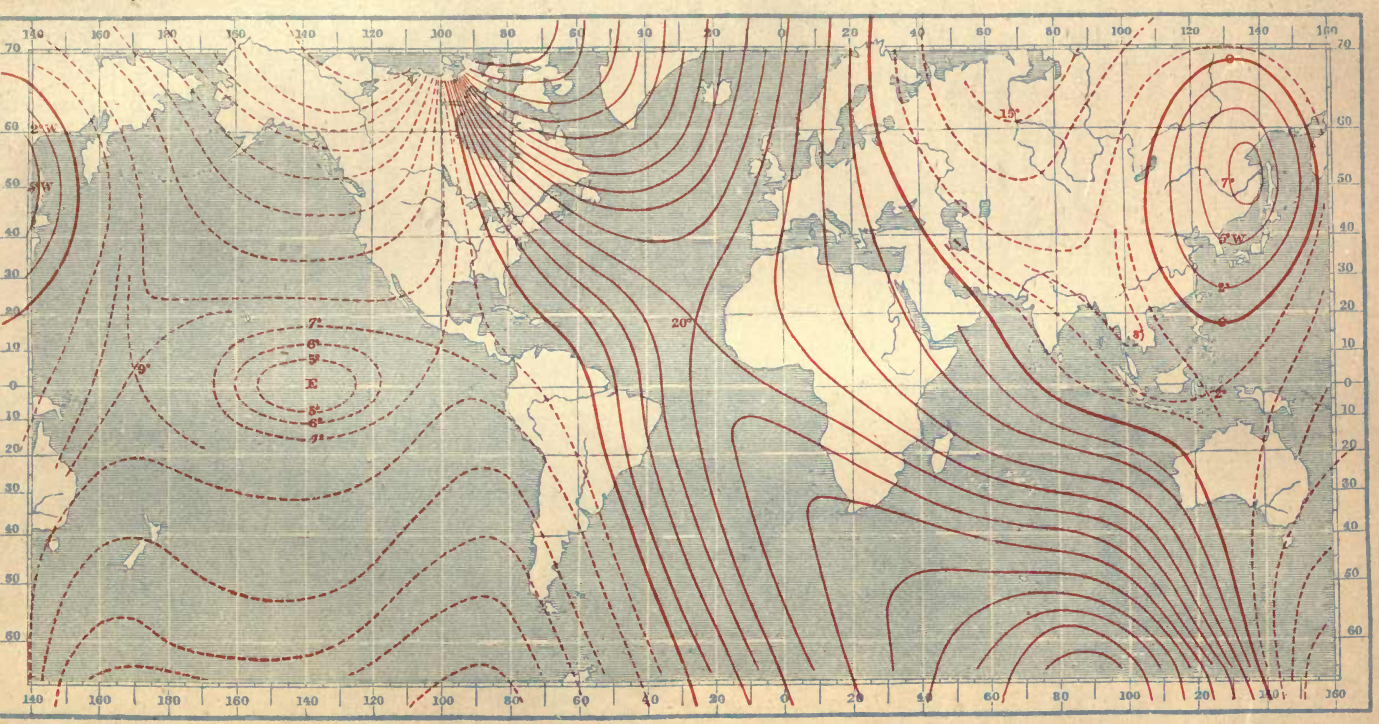
Maps on which such isogonic lines are depicted are called *declination* or *variation maps*; and a comparison of these in various years is well fitted to show the variation which this magnetic element undergoes. Plate III. represents a map on Mercator's projection giving these lines for the year 1882. It will be seen that the surface of the globe is divided by them into two regions: one, the smaller, in which the variation is westerly, as indicated by the continuous lines; the other, in which the variation is easterly, as indicated by the dotted lines. This chart is useful to the mariner as not only giving him the declination in any place, but also as showing him the places on the globe where it changes most rapidly. Of these the most remarkable are the coast of Newfoundland, the Gulf of St. Lawrence, the seaboard of North America, and the English Channel and its approaches.

693. **Annual variations.**—Cassini first discovered in 1780 that the declination is subject to small annual variations. At Paris and London it is greatest about the vernal equinox, diminishes from that time to the summer solstice, and increases again during the nine following months. It does not exceed from 15′ to 18′, and it varies somewhat at different epochs.

The *daily variations* were first discovered by Graham in 1722; they can only be observed by means of long needles or delicate indicators such as the reflection of a ray of light (522) and very sensitive instruments (702).

LINES OF EQUAL MAGNETIC VARIATION, 1882.

No. 1.



In this country the north pole moves every day from east to west from sunrise until one or two o'clock; it then tends towards the east, and at about ten o'clock regains its original position. During the night the needle is almost stationary. Thus the westerly declination is greatest during the warmest part of the day.

At Paris the mean amplitude of the diurnal variation from April to September is from 13′ to 15′, and for the other months from 8′ to 10′. On some days it amounts to 25′, and on others does not exceed 5′. The greatest variation is not always at the same time. The amplitude of the daily variations decreases from the poles towards the equator, where it is very feeble. Thus in the island of Rewak it never exceeds 3′ to 4′.

694. **Accidental variations and perturbations.**—The declination is accidentally disturbed in its daily variations by many causes, such as earthquakes, the *aurora borealis*, and volcanic eruptions. The effect of the aurora is felt at great distances. Auroras, which are only visible in the most northerly parts of Europe, act on the needle even in these latitudes, where accidental variations of 1° or 2° have been observed. In polar regions the needle frequently oscillates several degrees; its irregularity on the day before the aurora borealis is a presage of the occurrence of this phenomenon.

Another remarkable phenomenon is the simultaneous occurrence of magnetic perturbations in very distant countries. Thus Sabine mentions a magnetic disturbance which was felt simultaneously at Toronto, the Cape, Prague, and Van Diemen's Land. Such simultaneous perturbations have received the name of *magnetic storms* (702).

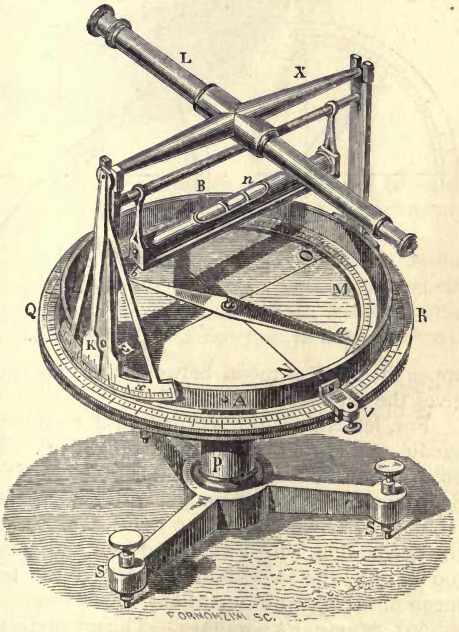


Fig. 651.

695. **Declination compass.**—The *declination compass* is an instrument by which the magnetic declination of any place may be determined when its astronomical meridian is known. The form represented in fig. 651 consists of a brass box, AB, in the bottom of which is a graduated circle, M. In the centre is a pivot on which oscillates a very light lozenge-shaped magnetic needle, *ab*. To the box are attached two uprights supporting a horizontal axis, X, on which is fixed an astronomical telescope, L, movable in a vertical plane. The box rests on a foot, P, about which it can turn in a horizontal plane, taking with it the telescope. A fixed

circle, QR, which is called the *azimuth circle*, measures the number of degrees through which the telescope has been turned, by means of a vernier, V, fixed to the box. The inclination of the telescope, in reference to the horizon, may be measured by another vernier, K, which moves with the axis of the telescope, and is read off on a fixed graduated arc, *x*.

The first thing in determining the declination is to adjust the compass horizontally by means of the screws SS and the level *n*. The astronomical meridian is then found, either by an observation of the sun at noon exactly, or by any of the ready methods known to astronomers. The box AB is then turned until the telescope is in the plane of the astronomical meridian. The angle made by the magnetic needle with the diameter N, which corresponds with the zero of the scale, and is exactly in the plane of the telescope, is then read off on the graduated limb, and this is east or west, according as the pole *a* of the needle stops at the east or west of the diameter N.

696. **Correction of errors.**—These indications of the compass are only correct when the magnetic axis of the needle—that is, the right line passing through the two poles—coincides with its axis of figure, or the line connecting its two ends. This is not usually the case, and a correction must therefore be made, which is done by the *method of reversion*. For this purpose the needle is not fixed in the cap, but merely rests on it, so that it can be removed and its position reversed; thus what was before the lower is now the upper face. The mean between the observations made in the two cases gives the true declination.

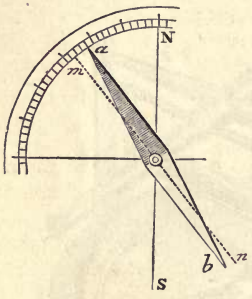


Fig. 652.

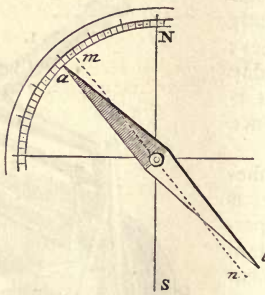


Fig. 653.

For, let NS be the astronomical meridian, *ab* the axis of figure of the needle, and *mn* its magnetic axis (fig. 652). The true declination is not the arc N*a*, but the arc N*m*, which is greater. If now the needle be turned, the line *mn* makes the same angle with the meridian NS; but the north end of the needle, which was on the right of *mn*, is now on the left (fig. 653), so that the declination, which was previously too small by a certain amount, is now too large by the same amount. Hence the true declination is given by the mean of these two observations.

697. **Mariner's compass.**—The magnetic action of the earth has received its most important application in the *mariner's compass*. This is a declination compass used in guiding the course of a ship. Fig. 654 represents a view of the whole, and fig. 655 a vertical section. It consists of a cylindrical case, BB′, which, to keep the compass in a horizontal position in spite of the rolling of the vessel, is supported on *gimbals*. These are two concentric rings, one of which, attached to the case itself, moves about the axis *xd* which

plays in the outer ring AB, and this moves in the supports PQ, about the axis *mn*, at right angles to the first.

In the bottom of the box is a pivot, on which is placed, by means of an agate cap, a magnetic bar, *ab*, which is the needle of the compass. On this is fixed a disc of mica, a little larger than the length of the needle, on which is traced a star or *rose*, with thirty-two branches, making the eight points or

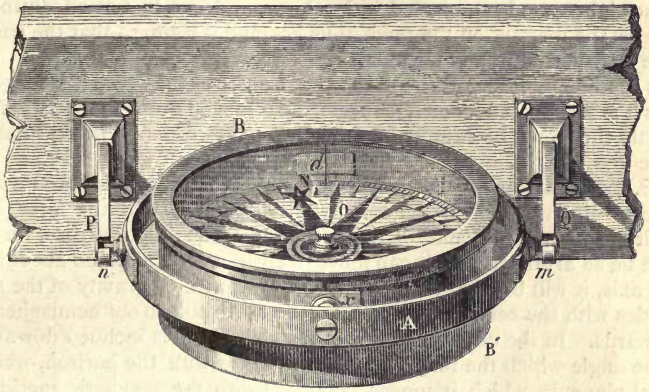


Fig. 654.

rhumbs of the wind, the demi-rhumbs, and the quarters. The branch ending in a small star, and called N, corresponds to the bar *ab*, which is underneath the disc.

The compass is placed near the stern of the vessel in the *binnacle*. Knowing the direction of the compass in which the ship is to be steered, the pilot has the rudder turned till the direction coincides with the sight-vane, passing through a line *d* marked on the inside of the box, and parallel with the keel of the vessel.

The *prismatic compass* is greatly used for surveying and more especially for military purposes; it differs from the mariner's compass mainly in its dimensions, and in the way in which observations are made. It consists of a shallow metal box about 2½ inches in diameter (fig. 656); the needle, which is fixed below the compass card, plays on a pivot much as in

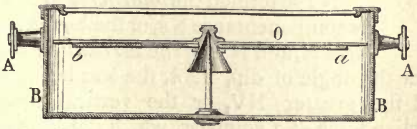


Fig. 655.

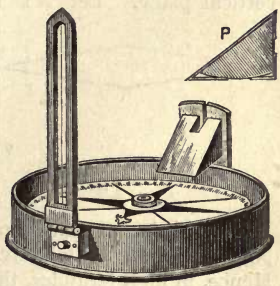


Fig. 656.

fig. 655. A is a metal frame across which is stretched a horse-hair, forming a sight-vane. Exactly opposite this is a right-angled prism P enclosed in a

metal case, with an eyehole and a slit as represented at the side of the figure (fig. 656).

In order to make an observation the compass is held horizontally, and so that the slit in the prism, the hair of the sight-vane, and the distant object are seen to be in the same line; looking through the eyehole, the angle which the needle makes is then noted; a similar observation is made with another object, and thus the angle between them, or their bearing, is given.

The sight-vane is connected with a lever, and can be turned down when it presses the magnet on the pivot, thus keeping it rigid, so that the compass can be transported in any position.

As the image is seen through the convex face of the prism it is magnified, and as it is seen by reflection it is reversed, so that in order to read the figures correctly they must be reversed on the card; the reflection being total there is little loss of light.

698. **Inclination. Magnetic equator.**—It might be supposed from the northerly direction which the magnet needle takes that the force acting upon it is situated in a point of the horizon. This is not the case, for if the needle be so arranged that it can move freely in a vertical plane about a horizontal axis, it will be seen that, although the centre of gravity of the needle coincides with the centre of suspension, the north pole in our hemisphere dips downwards. In the other hemisphere the south pole is inclined downwards.

The angle which the magnetic needle makes with the horizon, when the vertical plane, in which it moves, coincides with the magnetic meridian, is called the *inclination* or *dip* of the needle. In any other plane than the magnetic meridian the inclination *increases*, and is 90° in a plane at right angles to the magnetic meridian. For the magnetic inclination represents the direction of the total magnetic force, and may be resolved into two forces, one acting in a horizontal and the other in a vertical plane. When the needle is moved so that it is at right angles to the magnetic meridian, the horizontal component can only act in the direction of the axis of suspension, and therefore cannot affect the needle, which is then solely influenced by the vertical component, and stands vertically. The following considerations will make this clearer :—

Let NS (fig. 657) represent a magnetic needle capable of moving in a vertical plane. Let NT represent, in direction and intensity, the entire force of the earth's magnetism acting on the pole N. Then NT can be resolved into the forces N*h* and NV; TN*h* being the angle of inclination or dip.

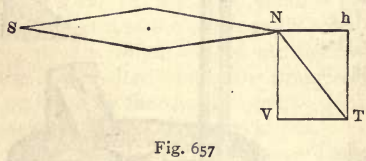


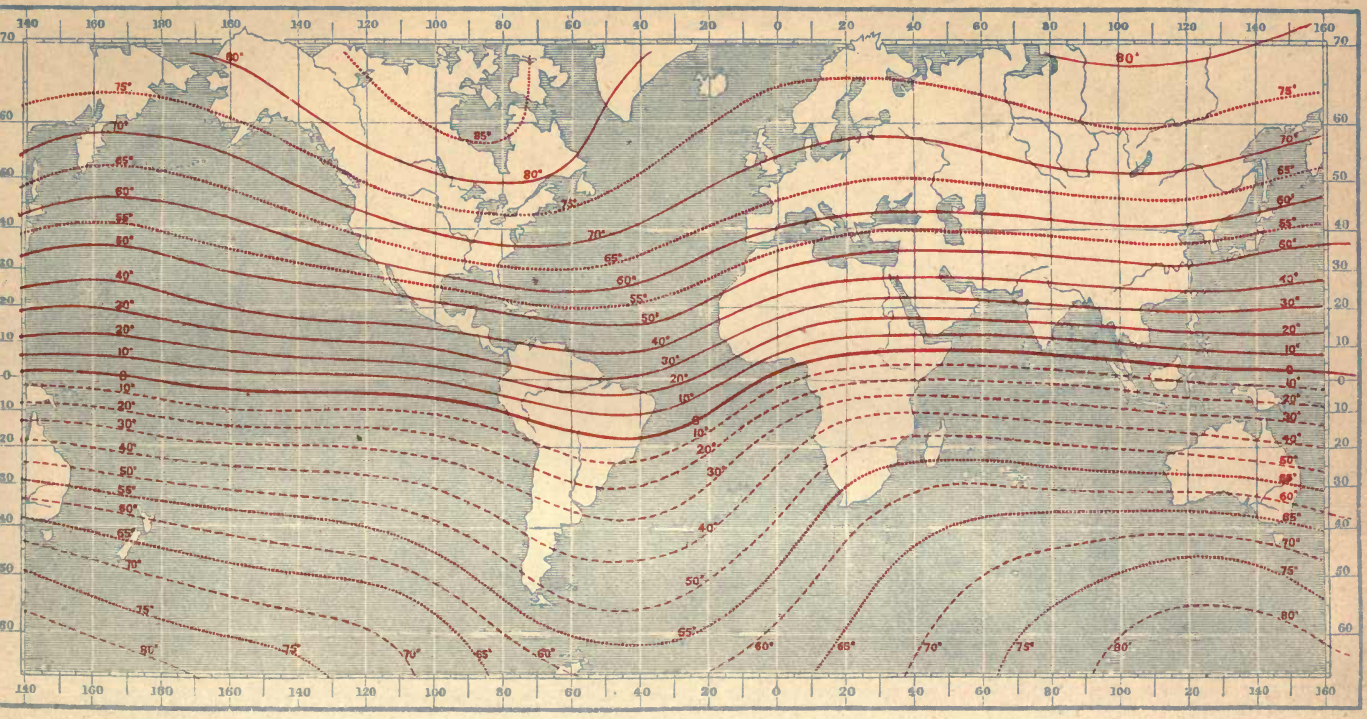
Fig. 657

NT is termed the *total force* M; and its components are N*h*, or the *horizontal force* H, and NV, or the *vertical force* Z.

Now, it is clear that the greater the angle of dip, TN*h*, the less becomes N*h*, or the horizontal force, and the greater NV, or the vertical force. Hence, in high latitudes, the directive force of a compass, which depends on the horizontal force, is less than in low latitudes. At the magnetic poles the horizontal force will be *nil*, and the vertical force a maximum; here, therefore, the needle will be vertical. At the magnetic equator the reverse is the

LINES OF EQUAL MAGNETIC DIP 1882.

No. 2.



case, and the needle will be horizontal. Hence, the oscillations of a *compass* needle, by which, as will presently be explained, the strength of the earth's magnetism is measured, become fewer and fewer in a given time as the magnetic poles are approached, although there is really an increase in the total force of the earth.

Again, the reason why a dip needle stands vertical when placed E. and W. is clearly because in those positions the horizontal force now acting at right angles to the plane of motion of the needle is ineffectual to move it, and therefore merely produces a pressure on the pivot which supports the needle. But the vertical component of the total force remains unaffected by the new position of the needle. Acting, therefore, entirely alone when the dip needle is exactly E. and W., this vertical component drags the needle into a line with itself; that is, 90° from the horizontal plane.

The value of the dip, like that of the declination, differs in different localities. It is greatest in the polar regions, and decreases with the latitude to the equator, where it is approximately zero. In London at the present time (1892) the dip is 67° 22′, reckoning from the horizontal line. In the southern hemisphere the inclination is again seen, but in a contrary direction; that is, the south pole of a needle dips below the horizontal line.

The *magnetic poles* are those places in which the dipping-needle stands vertical; that is, where the inclination is 90°. In 1831 the first of these, the terrestrial north pole, was found by Sir James Ross in 96° 43′ west longitude of 70° north latitude. The same observer found in the South Sea, in 76° south latitude and 168° east longitude, that the inclination was 88° 37′. From this and other observations, it has been calculated that the position of the magnetic south pole was at that time in about 154° east longitude and 75½° south latitude. The line of no declination passes through these poles, and the lines of equal declination converge towards them.

The *magnetic equator*, or *aclinic line*, is the line which joins all those places on the earth where there is no dip; that is, all those in which the dipping-needle is quite horizontal. It is a somewhat sinuous line, not differing much from a great circle inclined to the equator at an angle of 12°, and cutting it on two points almost exactly opposite each other—one in the Atlantic and one in the Pacific. These points appear to be gradually moving their position, and travelling from east to west.

Lines connecting places in which the dipping-needle makes equal angles are called *isoclinic lines*. They have a certain analogy and parallelism with the parallels of latitude, and the term *magnetic latitude* is sometimes used to denote positions on the earth with reference to the magnetic dip. Plate IV. is an inclination map for the year 1882, the construction of which is quite analogous to that of the map of declination.

The inclination is subject to secular variations, like the declination, as is readily seen from a comparison of maps of inclination for different epochs. At Paris, in 1671, the inclination was 75°; since then it has been continually decreasing: in 1835 it was 67° 24′; in 1849, 67°; in 1859, 66° 16′; in 1869, 65° 43′; in 1879, 65° 32′; in 1883, 65° 17′; and in 1888, 65° 14′; in 1891, 65° 11′.

The following table gives the alterations in the inclination at London, from which it will be seen that since 1723, in which it was at its maximum,

it has continually diminished by an average of something more than three minutes in each year :—

Year	Inclination	Year	Inclination
1576	71° 50′	1828	69° 47′
1600	72°	1838	69° 17′
1676	73° 30′	1854	68° 31′
1723	74° 42′	1859	68° 21′
1773	72° 19′	1874	67° 43′
1780	72° 8′	1876	67° 39′
1790	71° 33′	1878	67° 36′
1800	70° 35′	1880	67° 35′
1821	70° 31′	1881	67° 35′

699. **Inclination compass.**—An inclination compass, or *dip needle*, is an instrument for measuring the magnetic inclination or dip. One form, represented in fig. 658, though not adapted for the most accurate measurements, is well suited for illustrating the principle. It consists of a graduated horizontal brass circle *m*, supported on three legs, provided with levelling screws. Above this circle there is a plate A, movable about a vertical axis, and supporting, by means of two columns, a second graduated circle M, which measures the inclination. The needle rests on a frame *r*, and the diameter passing through the two zeros of the circle N can be ascertained to be perfectly horizontal by means of the spirit-level *n*.

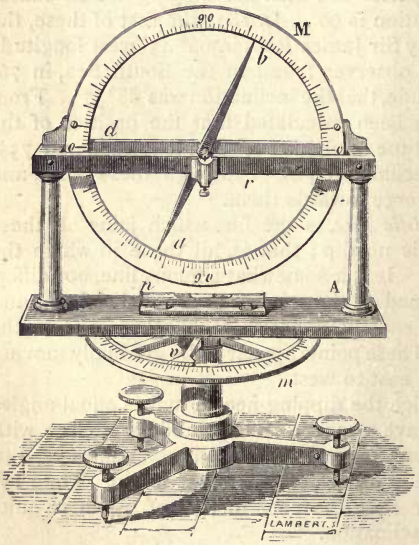


Fig. 658.

To observe the inclination, the magnetic meridian must first be determined, which is effected by turning the plate A on the circle *m*, until the needle is vertical, which is the case when it is in a plane at right angles to the magnetic meridian (698). The plate A is then turned 90° on the circle *m*, by which the vertical circle M is brought into the magnetic meridian. The angle *dca*, which the magnetic needle makes with the horizontal diameter, is the angle of inclination.

There are here several sources of error, which must be allowed for. The most important are these :—i. The magnetic axis of the needle may not coincide with its axis of figure : hence an error which is corrected by a

method of reversion analogous to that already described (696). ii. The centre of gravity of the needle may not coincide with the axis of suspension, and then the angle *dca* is too great or too small, according as the centre of gravity is below or above the centre of suspension ; for in the first case the action of gravity is in the same direction as that of magnetism, and in the second it is in the opposite direction. To correct this error, the poles of the needle must be reversed by first demagnetising it, and then imparting a contrary magnetism to what it had at first. The inclination is now re-determined, and the mean taken of the results obtained in the two groups of operations. iii. The plane of the ring may not concide with the true magnetic meridian. It should be in that plane when the needle has its minimum deviation ; an observation of this kind should therefore be taken along with that previously described, by which the needle is moved 90° from its maximum deviation.

The dip needle may be used to determine the inclination in another way. It is first allowed to oscillate in the magnetic meridian, and then in a plane at right angles to it. If the number of oscillations in a given time in the first position be n, and in the second position n_1, then in the first position the whole force of the earth's magnetism E acts, and in the second position only the vertical component, which is E sin x, x being the angle of dip. Now, since the forces acting on the needle are, from the laws of the pendulum (55), as the squares of the number of oscillations in a given time, we have $\frac{E}{E \sin x} = \frac{n^2}{n_1^2}$, from which $\sin x = \frac{n_1^2}{n^2}$.

700. **Astatic needle and astatic system.**—An *astatic needle* is one which is uninfluenced by the earth's magnetism. A needle movable about an axis in the plane of the magnetic meridian and parallel to the inclination would be one of this kind ; for the terrestrial magnetic couple, acting then in the direction of the axis, cannot impart to the needle any determinate direction.

An *astastic system* is a combination of two needles of the same force joined parallel to each other with the poles in contrary directions, as shown in fig. 659. If the two needles have exactly the same magnetic force, the opposite actions of the earth's magnetism on the poles a' and b and on the poles a and b' counterbalance each other ; the system is then completely astatic, and sets at right angles to the magnetic meridian.

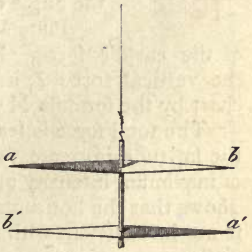


Fig. 659.

A single magnetic needle may also be rendered astatic by placing a large magnet near it. By repeated trials a certain position and distance can be found at which the action of the magnet on the needle just neutralises that of the earth's magnetism, and the needle is free to obey any third force ; in other words, the field due to the magnet just neutralises the earth's field.

701. **Force of the earth's magnetism.**—If a magnetic needle be moved from its position of equilibrium, it will revert to it after a series of oscillations, which follow laws analogous to those of the pendulum (80). If

the magnet be removed to another place, and caused to oscillate during the same length of time as the first, a different number of oscillations will be observed. And the earth's magnetic force in the two places will be respectively proportional to the squares of the number of oscillations.

If at M the number of oscillations in a minute had been $25 = n$, and at another place M′, $24 = n'$, we should have—

$$\frac{\text{Force of the earth's magnetism at M}}{\text{Force of the earth's magnetism at M}'} = \frac{n^2}{n_,^2} = \frac{625}{576} = 1{\cdot}085.$$

That is, if the force of the magnetism at the second place is taken as unity, that of the first is 1·085. If the magnetic condition of the needle had not changed in the interval between the two observations, this method would give the relation of the force at the two places.

In these determinations of the force, it would be necessary to have the oscillations of the dip-needle, which are produced by the total force of the earth's magnetism. These, however, are difficult to obtain with accuracy, and therefore those of the declination needle are usually taken. The force which makes the declination needle oscillate is only a portion of the total magnetic force, and is smaller in proportion as the inclination is greater. If a line $ac = \mathrm{M}$ (fig. 660) represent the total force, the angle i the inclination, then the horizontal component $ab = \mathrm{H}$ is $\mathrm{M} \cos i$. Hence, to express the total force in the two places by the oscillations of the declination needle, we must substitute the values $\mathrm{M} \cos i$ and $\mathrm{M}' \cos i'$ for M and M′ in the preceding equation, and we have—

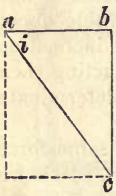


Fig. 660.

$$\frac{\mathrm{M} \cos i}{\mathrm{M}' \cos i'} = \frac{n^2}{n'^2}\text{; hence } \frac{\mathrm{M}}{\mathrm{M}'} = \frac{n^2 \cos i'}{n'^2 \cos i}.$$

That is to say, having observed in two different places the number of oscillations, n and n', that the same needle makes in the same time, the ratio of the magnetic force in the two places will be found by multiplying the ratio of the square of the number of oscillations by the inverse ratio of the cosine of the angle of dip.

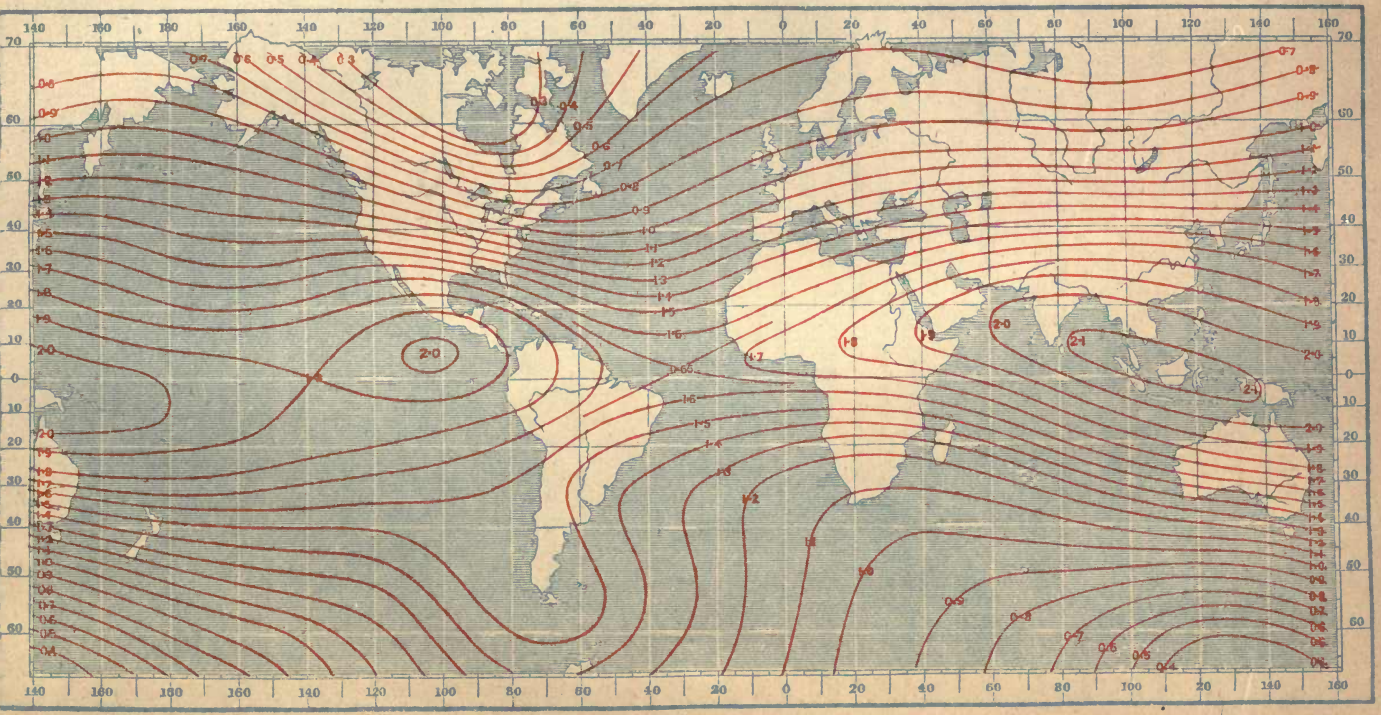
Plate V. is a chart representing the horizontal component of the earth's force. Knowing the angle of dip i, the total force M, or the vertical force Z, in any place, may be obtained from the values in the chart by the formula $\mathrm{M} = \mathrm{H} \sec i$; and $\mathrm{Z} = \mathrm{H} \tan i$.

The total force is least near the magnetic equator, and, increasing with the latitude, is greatest near, but not quite at, the magnetic poles; the places of maximum intensity are conveniently named the *magnetic foci*. The chart shows that the horizontal force diminishes as we go towards the poles: this is not inconsistent with the above statement if we take the dip into account (698).

The lines connecting places of equal force are called *isodynamic lines*. They are not parallel to the magnetic equator, but seem to have about the same direction as the isothermal lines. According to Kuppfer, the force appears to diminish as the height of the place is greater; a needle which made one oscillation in 24″ vibrated more slowly by 0·01″ at a height of 1,000 feet: but, according to Forbes, the force is only $\frac{1}{1000}$ less at a height

LINES OF EQUAL HORIZONTAL FORCE 1882

No. 3.



of 3,000 feet. There is, however, some doubt as to the accuracy of these observations, owing to the uncertainty of the correction for temperature.

The intensity varies in the same place with the time of day: it attains its maximum between 4 and 5 in the afternoon, and is at its minimum between 10 and 11 in the morning.

According to Gauss, the total magnetic action of the earth is the same as that which would be exerted if in each cubic yard there were eight steel bars magnetised to saturation, each weighing a pound.

It is probable, though it has not yet been ascertained with certainty, that the force undergoes secular variations. From measurements made at Kew, it appears that, on the whole, the total force experiences a very slight annual increase (692).

702. **Magnetic observatories.**—During the last few years great attention has been devoted to the observation of the magnetic elements, and observatories for this purpose have been fitted up in different parts of the globe. These observations have led to the discovery that the magnetism of the earth is in a state of constant fluctuation, like the waves of the sea. And in studying the variations of the declination, &c., the mean of a great number of observations must be taken, so as to eliminate the irregular disturbances, and bring out the general laws.

The principle on which magnetic observations are automatically recorded is as follows:—Suppose that in a dark room a bar magnet is suspended horizontally, and at its centre is a small mirror; suppose further that a lamp sends a ray of light to this mirror, the inclination of which is such that the ray is reflected, and is received on a horizontal drum placed underneath the lamp. The axis of the drum is at right angles to the axis of the magnet; it is covered with sensitive photographic paper, and is rotating uniformly by clockwork. If now the magnet is quite stationary, as the drum rotates, the reflected spot of light will trace a straight line on the paper with which the revolving drum is covered. But if, as is always the case, the position of the magnet varies during the twenty-four hours, the effect will be to trace a sinuous line on the paper. These lines can afterwards be fixed by ordinary photographic methods. Knowing the distance of the mirror from the drum, and the length of the paper band which comes under the influence of the spot of light in a given time—twenty-four hours, for instance—the angular deflection at any given moment may be deduced by a simple calculation (522).

The observations made in the English magnetic observatories were reduced by Sabine, and revealed some curious facts in reference to magnetic storms (694). He found that there is a certain periodicity in their appearance, and that they attain their greatest frequency about every ten years. Independently of this, Schwabe, who for many years studied the subject, found that the spots on the sun, seen on looking at it through a coloured glass, vary in their number, size, and frequency, but attain their maximum about every ten or eleven years. Now Sabine established the interesting fact that the period of their greatest frequency coincides with the period of greatest magnetic disturbance. Other remarkable connections between the sun and terrestrial magnetism have been observed; one, especially, of recent occurrence has attracted considerable attention. It was the flight of a large luminous mass across a vast sun-spot, while a simul-

taneous perturbation of the magnetic needle was observed in the observatory at Kew: subsequent examination of magnetic observations in various parts of the world showed that within a few hours one of the most violent magnetic storms ever known had prevailed.

It seems, however, that these accidental variations in the declination cannot be due to changes in any *direct* action of a possible magnetic condition of either the sun or the moon. For it can be shown that if the magnetisation of the latter were as powerful as that of the earth, the deflection which it could produce would not amount to the $\frac{1}{20}$th of a second, a quantity which cannot be measured. In order to produce a variation of 10′, such as is frequently met with, the magnetisation of the sun or of the moon must be 12,000 times that of the earth; in other words, a more powerful degree of magnetisation than that of powerfully magnetised steel bars.

Magnetic storms are nearly always accompanied by the exhibition of the aurora borealis in high latitudes; that this is not universal may be due to the fact that many auroras escape notice. The converse of this is true, that no great display of the aurora takes place without a violent magnetic storm.

The centre or focus towards which the rays of the aurora converge lies approximately in the prolongation of the direction of the dipping-needle.

CHAPTER III

LAWS OF MAGNETIC ATTRACTION AND REPULSION

703. **Law of decrease with distance.**—Coulomb discovered the remarkable law in reference to magnetism, *that magnetic attractions and repulsions are inversely as the squares of the distances.* He proved this by means of two methods :—(i.) that of the torsion balance, and (ii.) that of oscillations.

704. i. **The torsion balance.**—This apparatus depends on the principle that, when a wire is twisted through a certain space, the angle of torsion is proportional to the force of torsion (89). It consists (fig. 661) of a glass case closed by a glass top, with an aperture near the edge, to allow the introduction of a magnet, A. In another aperture in the centre of the top a glass tube fits, provided at its upper extremity with a micrometer. This consists of two circular pieces : *d*, which is fixed, is divided on the edge into 360°, while on one *e*, which is movable, there is a mark, *c*, to indicate its rotation. D and E represent the two pieces of the micrometer on a larger scale. On E there are two uprights connected by a horizontal axis, on which is a very fine silver wire supporting a magnetic needle, *ab*. On the side of the case there is a graduated scale, which indicates the angle of the needle *ab*, and hence the torsion of the wire.

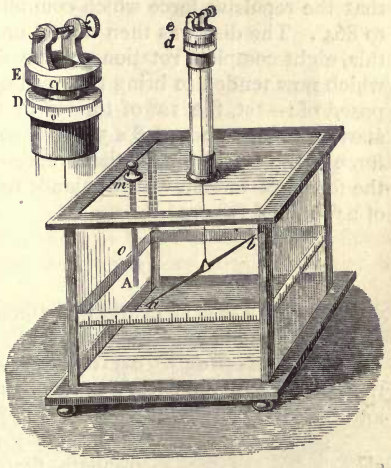


Fig. 661.

When the mark *c* of the disc E is at zero of the scale D, the case is so arranged that the wire supporting the needle and the zero of the scale in the case are in the magnetic meridian. The needle is then removed from its stirrup, and replaced by an exactly similar one of copper, or any unmagnetic substance ; the tube, and with it the pieces D and E, are then turned so that the needle stops at zero of the graduation. The magnetic needle *ab*, being now replaced, is exactly in the magnetic meridian, and the wire exerts no torsion.

Before introducing the magnet A, it is necessary to investigate the action of the earth's magnetism on the needle *ab*, when the latter is removed out of

the magnetic meridian. This will vary with the dimensions and force of the needle, with the dimensions and nature of the particular wire used, and with the intensity of the earth's magnetism in the place of observation. Accordingly, the piece E is turned until *ab* makes a certain angle with the magnetic meridian. Coulomb found in his experiments that E had to be turned 36° in order to move the needle through 1°; that is, the earth's magnetism was equal to a torsion of the wire corresponding to 35°. As the force of torsion is proportional to the angle of torsion, when the needle is deflected from the meridian by 2, 3 . . . degrees, the directive action of the earth's magnetism is equal to 2, 3 . . . times 35°.

The action of the earth's magnetism having been determined, the magnet A is placed in the case so that similar poles are opposite each other. In one experiment Coulomb found that the pole *a* was repelled through 24°. Now the force which tended to bring the needle into the magnetic meridian was represented by $24° + 24 \times 35 = 864$, of which the part 24° was due to the torsion of the wire, and $24 \times 35°$ was the equivalent in torsion of the directive force of the earth's magnetism. As the needle was in equilibrium, it is clear that the repulsive force which counterbalanced those forces must be equal to 864°. The disc was then turned until *ab* made an angle of 12°. To effect this, eight complete rotations of the disc were necessary. The total force which now tended to bring the needle into the magnetic meridian was composed of:—1st, the 12° of torsion by which the needle was distant from its starting point; 2nd, of $8 \times 360° = 2880$, the torsion of the wire; and 3rd, the force of the earth's magnetism, represented by a torsion of $12 \times 35°$. Hence the forces of torsion which balance the repulsive forces exerted at a distance of 24° and of 12° are—

24°	864
12°	3312

Now, 3312 is very nearly four times 864; hence for half the distance the repulsive force is four times as great.

705. ii. **Method of oscillations.**—A magnetic needle oscillating under the influence of the earth's magnetism may be considered as a pendulum, and the laws of pendulum motion apply to it (55). The method of oscillations consists in causing a magnetic needle to oscillate first under the influence of the earth's magnetism alone, and then successively under the combined influence of the earth's magnetism and of a magnet placed at unequal distances.

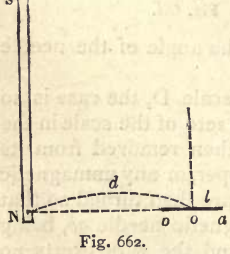


Fig. 662.

The following determination by Coulomb will illustrate the use of the method. A magnetic needle was used which made 15 oscillations in a minute under the influence of the earth's magnetism alone. A magnetic bar about 2 feet long was then placed vertically in the plane of the magnetic meridian, so that its north pole was downwards and presented to the south pole *o* of the oscillating needle (fig. 662), so as to concur in its action with that of the earth. He found that at a distance of 4 inches the needle made 41 oscillations in a minute, and at a distance of 8 inches 24

oscillations. Now, from the laws of the pendulum (55), the intensities of the forces are inversely as the squares of the times of oscillation. Hence, if we call M the force of the earth's magnetism, m the attractive force of the magnet at the distance of 4 inches, m' at the distance of 8 inches, we have

$$M : M + m = 15^2 : 41^2, \text{ and}$$
$$M : M + m' = 15^2 : 24^2,$$

eliminating M

$$m : m' = 41^2 - 15^2 : 24^2 - 15^2 = 1456 : 351 = 4 : 1 \text{ nearly,}$$

or $$m : m' = 4 : 1.$$

In other words, the force acting at 4 inches is quadruple that which acts at double the distance.

The above results do not quite agree with the numbers required by the law of inverse squares. But this could only be expected to apply in the case in which the repulsive or attractive force is exerted between two points, and not, as is here the case, between the resultant of a system of points. And it is to this fact that the discrepancy between the theoretical and observed results is due.

When a magnet acts upon a mass of soft iron, the law of the variation with the distance is modified. The attraction in this case is inversely proportional to the distance between the magnet and the iron.

When the distance between the magnet and the iron is small, Tyndall found that the attraction is directly proportional to the square of the strength of the magnet; but when the iron and the magnet are in contact, then the attraction is directly proportional to the strength of the magnet.

706. **Magnetic curves.**—If a stout sheet of paper stretched on a frame be held over a horseshoe magnet, and then some very fine iron filings be strewn on the paper, on tapping the frame the filings will be found to arrange themselves in thread-like curved lines, stretching from pole to pole (fig. 663). These lines form what are called *magnetic curves*. The direction of the curve at any point represents the direction of the lines of magnetic force at this point.

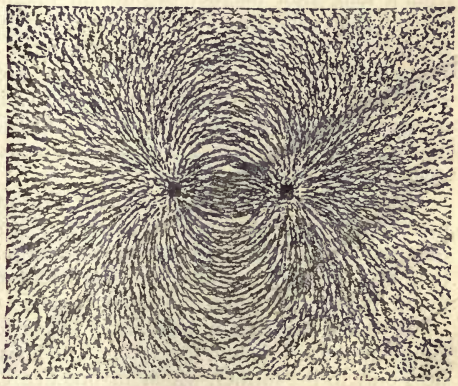


Fig. 663.

To render these curves permanent, the paper on which they are formed should be waxed; if then a hot iron plate be held over them, this melts the wax, which rises by capillary attraction (131) between the particles of filings, and on subsequent cooling connects them together. They may also be fixed by carefully placing on them a sheet of paper coated with paste, which is then

gently pressed and lifted off; it should be quickly dried to prevent the iron from rusting.

These curves are a graphic representation of the law of magnetic attraction and repulsion with regard to distance; for under the influence of the two poles of the magnet, each particle becomes itself a minute magnet, the poles of which arrange themselves in a position dependent on the resultant of the forces exerted upon them by the two poles, and this resultant varies with the distance of the two poles respectively. A small magnetic needle placed in any position near the magnet will take a direction which is the tangent to the curve at this place.

707. **Magnetic definitions.**—The space in the immediate neighbourhood of any magnet undergoes some change in consequence of the presence of this magnet, and such a space is spoken of as a *magnetic field*; it is indeed the sphere of action of the magnet; the effect produced by the magnet itself is often spoken of as due to the magnetic field. Magnets of different powers produce magnetic fields of different intensities. The strength of the field diminishes with the distance from the magnet.

The direction which represents the resultant of the magnetic forces at any position in a magnetic field is spoken of as the direction of the *lines of force* of this field. In fig. 663 the magnetic curves represent the direction of the lines of force in the field due to the two opposite poles. The expression 'lines of force' or 'lines of magnetic force' is used in much the same sense as that in which we speak of 'rays of light.' And just as we may express the illumination of a surface by the number of rays of light which fall upon it, so also we may say that the strength of any magnetic surface is proportional to the number of lines of force which it cuts.

A uniform magnetic field is one in which the lines of force are parallel. This is practically the case with a small portion of a field at some distance from a long thin magnet of uniform magnetisation. The dipping-needle, when free to oscillate in a vertical plane in the magnetic meridian, represents the direction of the lines of force due to the *terrestrial magnetic field*. The strength of the field due to this in any one place is uniform in much the same sense in which gravity is uniform in any place. A field of unit strength is one which acts on a unit pole with a force equal to that of a dyne (709). The strength of any magnetic field is measured by the number of lines of magnetic force present in the field.

The value of the earth's magnetic field in any locality is apt to be greatly modified by the presence of masses of iron or magnets in the place of observation, and the more so the more largely iron enters into the construction of a building. Thus in a particular case the value in the open was found to be 0·189, while in a room it was 0·114.

We have seen that in speaking of the pendulum we distinguish between a simple and a compound one (79). The laws of the pendulum apply in strictness only to the former, which in practice cannot be realised, although we possess arrangements which produce the same effect as a simple pendulum, and are equivalent to it. So too in magnetism we may discriminate between an ideal and an actual magnet; the former being considered as a long, infinitely thin, bar of magnetised molecules, to which only do the laws of magnetic action in strictness apply, although they can be realised with

ordinary magnets with sufficient approximation. Thus in the action of magnets at a distance we may assume that all the magnetism is concentrated in the poles, provided that the length is three to four hundred times the diameter, or provided the fourth power of half the length of the magnet may be disregarded in comparison with the distance at which it acts (708).

In a magnet the *magnetic moment* M is the product of the length l of the magnet into the strength of one pole m, that is $M = ml$.

If a magnetic body be placed in a magnetic field, the intensity of the magnetisation A which it acquires will be proportional to the strength of the field F, and to a coefficient k, which depends on the material itself, and which is called the *coefficient of magnetisation*, $A = kF$. Bodies such as soft iron, which are readily magnetised, are said to have great *susceptibility* to magnetisation.

The magnetic moment of a bar divided by its mass represents the *specific magnetism*.

The *intensity of magnetisation* A in a bar, assumed to be uniformly magnetised, is the magnetic moment divided by the volume $\frac{M}{V}$; that is to say, the weight in grammes of the bar divided by the specific gravity.

The ratio of the total magnetic induction B, to the force producing it, has been called by Sir W. Thomson the *magnetic permeability* of a substance, and is represented by the symbol μ; $B = \mu F$. Magnetic permeability represents in magnetism the specific inductive capacity of dielectrics (748), and may be regarded as expressing the magnetic inductive capacity, or magnetic conductivity for lines of force.

708. **Total action of two magnets on each other.**—In the above case of the torsion balance one pole of the magnet to be tested was at so great a distance that it could not appreciably modify the influence of the other. When, however, the conditions are such that both poles act, then they follow a different law, as will now be demonstrated.

Let *ns* (fig. 664) be a small magnetic needle, free to move in a horizontal plane, and let NS be a bar magnet placed at right angles to the magnetic meridian, at a distance which is great compared with its own dimensions—not less than ten times as great—and so that the straight line drawn through its middle point and that of the needle coincides with the magnetic meridian. In this position the magnet NS is said to be 'broadside on.' The two poles S and *s* will repel each other in the direction *sa*; if mm_1 is the repellent force which these two poles would exert at a unit distance, then $\frac{mm_1}{r^2}$ is the force which they would exert at the distance $Ss = r$; let this force be represented in direction and strength by the line *sa*. Similarly, the pole N will act on *s*, with a force represented by the line *sc*; S and N being at the same distance r from *s*, *sa* and *sc* are equal, and their resultant may be represented by the line *sb*. From the similarity of the triangles *bsa* and NS*s* we have the proportion $Ss : SN = as : bs$; if

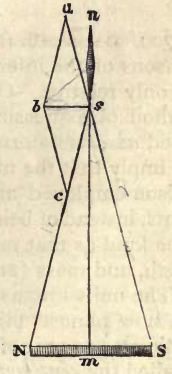


Fig. 664.

f is the value of the resultant bs, that is the total action of the magnet SN on the pole s, and if l be half the length of the magnet SN, we have $r : 2l = \frac{mm_{,}}{r^2} : f$, from which $f = \frac{2mm_{,}l}{r^3}$; that is, the total action of the magnet NS upon another magnet is inversely as the cube of the distance r.

If the magnet be placed 'end on' as represented in fig. 665, the needle being in the magnetic meridian, and the deflecting magnet at right angles thereto, and so that the prolongation of its axis bisects the needle, then if $mm_{,}$ is the force with which the pole N attracts the pole s at the unit distance, m and $m_{,}$ being the strength of the poles in the bar magnet and the magnetic needle respectively, the attracting force at the distance Ns will be $\frac{mm}{(r \times l)^2}$, l being, as before, the half-length of the magnet, and r the distance of the pole s from the middle of the magnet NS; in like manner the repellent force with which S acts upon s will be $\frac{mm'}{(r-l)^2}$. If ns is small compared with the distance of the bar magnet NS, the direction of these forces may be assumed to be parallel, and at right angles to ns. Since S is nearer than N, the repulsion will predominate, and the total force with which the magnet NS acts on the pole s is

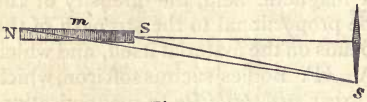


Fig. 665.

$$F = \frac{mm_{,}}{(r-l)^2} - \frac{mm_{,}}{(r+l)^2},$$

which, assuming that l is so small in comparison with r that its square and higher powers may be neglected, gives approximately

$$F = \frac{4\,mm_{,}l}{r^3},$$

so that compared with the first position of the magnet

$$F = 2f.$$

709. **Determination of magnetism in absolute measure.**—The comparisons of the intensity of the earth's magnetism in different places (701) are only relative. Of late years much attention has been devoted to the method of expressing not only this, but all other magnetic forces in what is called *absolute measure*. This term is used as opposed to *relative*, and does not imply that the measure is absolutely accurate, or that the units of comparison employed are of perfect construction; it means that the measurements, instead of being a simple comparison with an arbitrary quantity of the same kind as that measured, are referred to the fundamental units of time, length, and mass (21).

The units originally adopted on the proposal of the British Association, and now almost universally received, are the second as unit of time, the centimetre as unit of length, and the gramme as unit of mass. This system is called the *centimetre-gramme-second*, or *C.G.S.* system, and units referred to this system are spoken of as *C.G.S.* units (61 *a*).

The manner in which this determination is made in the case of magnet-

ism depends essentially on the observation of the oscillation of a horizontal bar magnet under the influence of the earth's magnetism ; and in the second place, on observing the deflection of a magnetic needle under the influence of this same magnet.

When a bar magnet suspended by a thread without torsion, free to oscillate in a horizontal plane, is deflected from its position of equilibrium and then left to itself, it vibrates backwards and forwards through its position of equilibrium, making oscillations which, if small, are isochronous like those of the pendulum. The number of these oscillations in a given time depends on the mass and dimensions of the bar, on its magnetic power, and on the intensity of the earth's magnetism in the place of observation. The time, t, of a complete oscillation of such a magnet is represented by the formula $t = 2\pi \sqrt{\frac{k}{\mathrm{H\,M}}}$ where k is the moment of inertia of the magnet ; that is, the mass which must be concentrated at the unit of distance from the centre of suspension, to present the same resistance to change of angular velocity, about this centre, as the magnet itself actually does. The moment of inertia of a magnet may be determined theoretically if it be homogeneous in structure, and of a regular geometrical shape ; or it may be determined experimentally by first observing the time of oscillation of the magnet under the influence of the earth's magnetism, and then the time when it has been loaded with a mass the inertia of which is known, and which does not alter the magnetic moment of the bar. M is the magnetic moment of the bar itself, and H is the force of the earth's magnetism. Hence

$$\mathrm{H\,M} = \frac{4\pi^2 k}{t^2}. \quad . \quad . \quad . \quad . \quad . \quad (1).$$

This expression gives the force which, applied in opposite directions at the ends of a lever of unit length, placed at right angles to the direction of this force, would have the same effect in tending to turn the lever, as the magnetic force of the earth has in tending to turn the magnet about a vertical axis when it is set at right angles to the magnetic meridian.

Now the value of HM depends on the nature of the bar, and on the force of the earth's magnetism in the place in question. If the bar were magnetised more or less strongly, or if the same bar were removed to a different locality, the product would have a different value. We must, therefore, find some independent relation between H and M, which will give rise to a new equation, and thus M, the magnetic moment of the bar, would be got rid of, and an absolute value be obtained for H.

Such a relation exists in the deflection from the magnetic meridian, which a bar magnet produces in a magnetic needle.

If, in the formula in the preceding article, we put $\mathrm{M} = 2ml$, then $\frac{2\mathrm{M}m_{,}}{r^3} =$ the + or − force acting on either pole of the magnetic needle, and, as both poles are acted on, the magnet will be subject to the action of a couple, the moment of which will be expressed by $\frac{2\mathrm{M}m_{,}}{r^3} 2l_{,} \cos a$, where a is the angle of deflection, l the half-length of the small magnetic needle ; let $\mathrm{M}_{,} = 2m_{,}l_{,}$ In like manner the earth's magnetism will act upon the magnetic needle

with a couple, the moment of which is expressed by $Hm, 2l \sin a = HM' \sin a$. Now when the needle is in equilibrium these forces are equal; that is—

$$\frac{2MM_{,}}{r^3} \cos a = HM_{,} \sin a,$$

from which

$$\frac{2M}{H} = r^3 \tan a \quad . \quad . \quad . \quad . \quad . \quad . \quad (2).$$

Combining (1) and (2) we get the expression

$$H = \frac{\pi}{t}\sqrt{\frac{k}{r^3 \tan a}},$$

an expression which involves no other physical units than those of length (involved in k and r), mass (involved in k), and time (involved in t), so that the value of H can be expressed in absolute measure.

The value for H in this expression only gives the horizontal component of the earth's magnetism; the total force is obtained by dividing the value of H by the cosine of the angle of dip for the place and time of observation (701). This varies on the earth's surface from 0·3 to 0·7.

The numerical value of H will depend, moreover, on the units taken. On the C.G.S. system the unit of force is called a *dyne*. It is (as we have seen, 61 *a*) the force which acting upon a gramme for a second generates a velocity of a centimetre per second. The value of H at Greenwich for the year 1877, expressed in this unit, is 0·18079 of a dyne; that is, the horizontal component of the earth's magnetism at this place acting on the unit of magnetism, associated with one gramme of matter, would produce a velocity of 0·18079 centimetre at the end of a second. The angle of dip at this time and place being 67° 37′, we get the total force = 0·4745 unit. If British units—namely, the foot, grain, second—be employed, the unit of force is that which by acting for a second on a grain gives to it a velocity of a foot per second, and the unit magnetic pole is such that if placed one foot from a second equal pole it will repel it with a force equal to the unit just defined. To convert the value of H, when expressed in centimetres, grammes, and seconds, into the equivalent value referred to British units, we must multiply by 21·69. In like manner, to convert magnetic forces referred to British units into the corresponding values expressed in centimetres, grammes, and seconds we must multiply by 0·0461 $= \frac{1}{21{\cdot}69}$.

If once the value of H in any locality has been determined it is easy to determine the value of M for any magnet; it is by experiments of this kind that the magnetic moment of minerals is arrived at. It has been found, for instance, that magnetite can acquire twice the magnetisation of steel.

CHAPTER IV

METHODS OF MAGNETISATION

710. **Magnetisation.**—The various methods of magnetisation are the influence of natural or artificial magnets, of terrestrial magnetism, and of electricity. This last method will be described under voltaic electricity. The three principal methods of magnetisation by magnets are known by the technical names of *single touch*, *separate touch*, and *double touch*.

711. **Method of single touch.**—This consists in moving the pole of a powerful magnet from one end to the other of the bar to be magnetised, and repeating this operation several times always in the same direction. The neutral magnetism is thus gradually decomposed throughout all the length of the bar, and that end of the bar which was touched last by the magnet is of opposite polarity to the end of the magnet by which it has been touched. This method only produces a feeble magnetic power, and is, accordingly, only used for small magnets. It has further the disadvantage of frequently developing consequent poles.

712. **Method of separate touch.**—This method, which was first used by Dr. Knight in 1745, consists in placing the two opposite poles of two magnets of equal force in the middle of the bar to be magnetised, and in moving each of them simultaneously towards the opposite ends of the bar. Each magnet is then placed in its original position and the operation repeated. After several rubbings on both faces of the bar it is magnetised.

In Knight's method the magnets are held vertically. Duhamel improved the method by inclining the magnets, as represented in fig. 667 ; and still

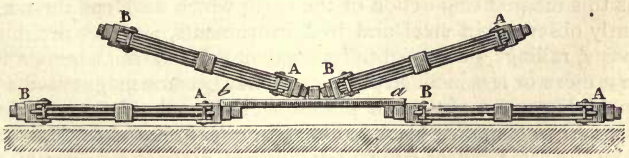


Fig. 666.

more by placing the bar to be magnetised on the opposite poles of two fixed magnets, the action of which strengthens that of the movable magnets. The relative position of the poles of the magnets is indicated in the figure. This method produces the most regular magnets.

713. **Method of double touch.**—In this method, which was invented by Mitchell, the two magnets are placed with their poles opposite each other in the middle of the bar to be magnetised. But, instead of moving them in opposite directions towards the two ends, as in the method of separate touch,

they are kept at a fixed distance by means of a piece of wood placed between them (fig. 667), and are simultaneously moved first towards one end, then from this to the other end, repeating this operation several times, and finishing in the middle, taking care that each half of the bar receives the same number of frictions.

Epinus, in 1758, improved this method by supporting the bar to be magnetised, as in the method of separate touch, on the opposite poles of two powerful magnets, and by inclining the bars at an angle of 15° to 20°. In practice, instead of two bar magnets, it is usual to employ a horseshoe magnet which has its poles conveniently close together.

By this method of double touch, powerful magnets are obtained, but they have frequently consequent poles. As this would be objectionable in compass needles, these are best magnetised by separate touch.

714. **Magnetisation by the action of the earth.**—The action of the earth on magnetic substances resembles that of a magnet, and hence the terrestrial magnetism is constantly tending to separate the two magnetisms in soft iron and in steel. But as the coercive force is very considerable in the latter substance, the action of the earth is inadequate to produce magnetisation, except when continued for a long time. This is not the case with perfectly soft iron. When a bar of this metal is held in the magnetic meridian parallel to the inclination, the bar becomes at once endowed with feeble magnetic polarity. The lower extremity is a north pole, and if the north pole of a small magnetic needle be approached, it will be repelled. This magnetism is of course unstable, for if the bar be turned the poles are inverted, as pure soft iron is destitute of coercive force.

While the bar is in this position, a certain amount of coercive force may be imparted to it by giving it several smart blows with a hammer, and the bar retains for a short time the magnetism which it has thus obtained. But the coercive force thus developed is very small, and after a time the magnetism disappears.

If a bar of soft iron be twisted while held vertically, or, better, in the plane of the dip, it acquires a feeble permanent magnetism.

It is this magnetising action of the earth which develops the magnetism frequently observed in steel and iron instruments, such as fireirons, rifles, lamp-posts, railings, gates, lightning conductors, &c., which remain for some time in a more or less inclined position. They become magnetised with their north pole downwards, just as if placed over the pole of a powerful magnet. The magnetism of native black oxide of iron (680) has doubtless been produced by the same causes ; the very different magnetic power of different specimens being partly attributable to the different positions of the veins of ore with regard to the line of dip. The ordinary irons of commerce are not quite pure, and possess a feeble coercive force ; hence a feeble magnetic polarity is generally found to be possessed by the tools in a smith's shop. Cast-iron, too, has usually a great coercive force, and can be permanently magnetised. The turnings, also, of wrought iron and of steel produced by the powerful lathes of our ironworks are found to be magnetised.

715. **Magnetism of iron ships.**—The inductive action of terrestrial magnetism upon the masses of iron always found in ships exerts a disturbing action upon the compass needle. The *local attraction*, as it is called, may

be so considerable as to render the indications of the needle almost useless if it be not guarded against. A full account of the manner in which local attraction is produced, and in which it is compensated, is inconsistent with the limits of this book, but the most important points are the following :—

i. A vertical mass of soft iron in the vessel, say in the bows, would become magnetised under the influence of the earth ; in the northern hemisphere, the lower end would be a north pole, and the upper end a south pole ; and as the latter may be assumed to be nearer the north pole of the compass needle, its action would preponderate. So long as the vessel was sailing in the magnetic meridian this would have no effect ; but in any other direction the needle would be drawn out of the magnetic meridian, and a little consideration will show that when the ship was at right angles to the magnetic meridian the effect would be greatest. This *vertical induction* would disappear twice in swinging the ship round, and would be at its maximum twice ; hence the deviation due to this cause is known as *semicircular deviation.*

ii. Horizontal masses again, such as deck beams, are also acted upon inductively by the earth's magnetism, and their induced magnetism exerts a disturbing influence upon the magnetic needle. The effect of this horizontal induction will disappear when the ship is in the magnetic meridian and also when it is at right angles thereto. In positions intermediate to the above the disturbing influence will attain its maximum. Hence in swinging a ship round there would be four positions of the ship's head in which the influence would be at a maximum, and four in which it would be at a minimum. The effect of horizontal induction is accordingly spoken of as *quadrantal deviation.*

The influence of both these causes, vertical and horizontal induction, may be remedied in the process of 'swinging the ship.' This consists in comparing the indications of the ship's compass with those of a standard compass placed on shore. The ship is then swung round in various positions, and by arranging small vertical and horizontal masses of soft iron in proximity to the steering compass, positions are found for them in which the inductive action of the earth upon them quite neutralises the influence of the earth's magnetism upon the ship ; and in all positions of the ship, the compass points in the same direction as the one on shore.

iii. The extended use of iron in ship-building, more especially when the frames are entirely of iron, has increased the difficulty. In the process of building a ship, the hammering and other mechanical operations to which it is subject, while under the influence of the earth's magnetism, will cause it to become to a certain extent permanently magnetised. The distribution of the magnetism, the direction of its magnetic axis, will depend on the position in which it has been built ; it may or may not coincide with the direction of the keel. The vessel becomes, in short, a huge magnet, and will exert an influence of its own upon the compass quite independently of vertical or horizontal induction. The influence is *semicircular* ; that is, it disappears when the magnetic axis of the ship is in the magnetic meridian, and is greatest at right angles to it. It may be compensated by two permanent magnets placed near the compass in suitable positions found by trial during the process of swinging the ship. Supposing the inherent magnetism of the ship to have the power of drawing the compass a point to the east, the com-

pensating magnets may be so arranged as to tend to draw it a point to the west, and thus keep it in the magnetic meridian. If, however, the inherent magnetism be destroyed, from whatever cause, it is clear that the magnets will now draw it aside a point too much to the west. This is the source of a new difficulty. It has been found that a ship which at the time of sailing was properly compensated, would, on returning from a long voyage, have its compasses over-compensated. The buffeting which the ship had experienced had destroyed its inherent magnetism, and numerous instances are known where the loss of a vessel can be directly traced to this cause. Fortunately, it has been found that after some time a ship's magnetic condition is virtually permanent, and is unaltered by any further wear and tear. The magnetism which it then retains is called its *permanent* magnetism, in opposition to the *sub-permanent* which it loses.

The difficulty of adequately compensating compasses, which is greatly increased by the armour-plated and turret ships now in use, has induced one school to throw over any attempt at correction ; but by careful observation of the magnetic condition of a ship, and tabulating the errors to construct a table, and comparing this with the indications of the compass at any one time, the true course can be made out.

In the Royal Navy, the plan now adopted is to combine both methods : compensate the errors to a considerable extent, and then construct a table of the residual errors.

716. **Magnetic saturation.**—Experiment has shown that with feeble magnetising power the magnetic force which can be imparted to a steel bar increases with the magnetising force used. It depends also on the number of strokes or movements of the magnetising magnets or coils : on the form and dimensions of the bar, on its density, on the quantity of carbon it contains, on its hardness, and on the manner in which it is tempered. Yet there is a limit to the magnetic force which can be imparted to iron or steel, and when this is attained, the bar is said to be *saturated or magnetised to saturation.* A bar may indeed be magnetised beyond this point, but this excess is *temporary* ; it gradually diminishes until the magnet has sunk to its point of saturation.

This is intelligible, for the magnetisms once separated tend to reunite, and when their attractive force is equal to that which opposes their separation—that is, the coercive force of the metal—a higher degree of equilibrium is attained, and the magnet is saturated. Hence more magnetisation ought to be developed in bars than they can retain, in order that they may decline to their permanent state of saturation. To increase the magnetisation of an unsaturated bar, a more feeble magnet must not be used than that by which it was originally magnetised.

In order to attain a stationary condition, the magnet should be heated to boiling for some time after being magnetised ; it should then be remagnetised and again heated to boiling, and so forth ; and after the last magnetisation it should be boiled for six hours or more. Such magnets are far more durable than ordinary ones.

717. **Magnetic battery.**—A *magnetic battery* or *magazine* consists of a number of magnets joined together by their similar poles. Sometimes they have the form of a horseshoe, and sometimes a rectilinear form. The

battery represented in fig. 668 consists of five superposed steel-plates. That in fig. 669 consists of twelve plates, arranged in three layers of four each. The horseshoe form is best adapted for supporting a weight, for then both poles are used at once. In both the bars are magnetised separately, and then fixed by screws.

The force of a magnetic battery consisting of n similar plates equally magnetised is not n times as great as that of a single one, but is somewhat smaller. These magnets mutually enfeeble each other; manifestly because, each north pole evokes south magnetism in the adjacent north pole, and thereby diminishes some of its north polarity. At the same time the strength is greater than if the steel is in one coherent mass; the reason doubtless is that thin plates of steel are more easily magnetised to saturation than thick ones, as the inducing action does not extend deep. The separate plates should not be in contact, as the enfeeblement of the magnetism is thereby less. It is also advisable to connect the pieces by a mass of soft iron as shown in fig. 669. The magnetism of a plate which has formed part of such a battery will be found to be materially less than it was originally. Thus Jamin found that six equal plates, which separately had each the portative force 18 kilos, only lifted 64 kilos when arranged as a battery, instead of 108; and when removed from the battery, each of them had only the portative force 9 to 10 kilos. The force is increased by making the lateral plates 1 or 2 centimetres shorter than the one in the middle (fig. 668).

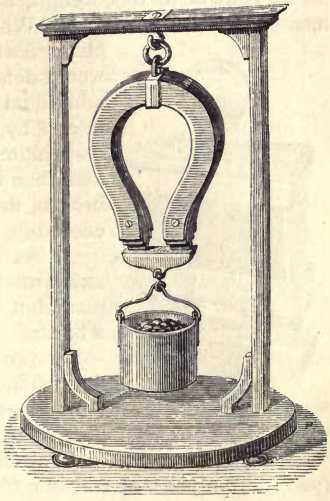


Fig. 667.

718. **Armatures.**—When even a steel bar is at its limit of saturation, it gradually loses its magnetism. To prevent this, *armatures* or *keepers* are used; these are pieces of soft iron, A and B (fig. 668), which are placed in contact with the poles. Acted on inductively, they become powerful temporary magnets, possessing opposite polarity to that of the inducing pole; they thus react in turn on the permanent magnetism of the bars, preserving and even increasing it.

Fig. 668.

Fig. 669.

When the magnets are in the form of bars, they are arranged in pairs, as shown in fig. 669, with opposite poles in juxtaposition, and the circuit is completed by two small bars of soft iron, AB. Movable magnetic needles,

if not clamped down, set spontaneously towards the magnetic poles of the earth, the influence of which acts as a keeper.

A horseshoe magnet has a keeper attached to it, which is usually arranged so as to support a weight. The keeper becomes magnetised under the influence of the two poles, and adheres with great force: the weight which it can support being more than double that which a single pole would hold.

In respect to this weight, a singular and hitherto inexplicable phenomenon has been observed. When contact is once made, and the keeper is charged with its maximum weight, any further addition would detach it: but if left in contact for a day, an additional weight may be added without detaching it, and by slightly increasing the weight every day it may ultimately be brought to support a far greater load than it would originally. But if contact be once broken, the weight it can now support does not much exceed its original charge.

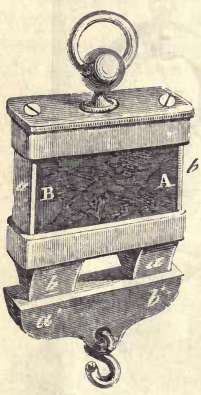


Fig. 670.

It is advantageous that the surface of the magnet and armatures which are in contact should not be plane but slightly cylindrical, so that they touch along a line.

In providing a natural magnet with a keeper, the line joining the two poles may first be approximately determined by means of iron filings; it may also be determined by bringing it near a magnetic needle, and ascertaining the positions in which its action is greatest (708). Two poles of soft iron (fig. 670), each terminating in a massive shoe, are then applied to the faces corresponding to the poles. Under the influence of the natural magnet, these plates become magnetised, and if the letters A and B represent the position of the poles of the natural magnet, the poles of the armature are *a* and *b*.

719. **Portative force. Power of magnets.**—The *portative force* is the greatest weight which a magnet can support. It can be determined by suspending to the keeper a vessel to which shot or sand is gradually added, until the keeper is detached (fig. 668). Häcker found that the portative force of a saturated horseshoe magnet, which, by repeatedly detaching the keeper, had become constant, may be represented by the formula

$$P = a^3/p^2,$$

in which P is the portative force of the magnet, p its own weight, and a a coefficient which varies with the nature of the steel and the mode of magnetising. Hence a magnet which weighs 1000 ounces only supports 25 times as much as one weighing 8 ounces or $\frac{1}{125}$ as heavy, and 25 such bars would support as much as a single one which is as heavy as 125 of them. It appears immaterial whether the section of the bar is quadratic or circular, and the distance of the legs is of inconsiderable moment; it is important, however, that the magnet be suspended vertically, and that the load be exactly in the middle. In Häcker's magnets the value of a was 10·33, while in Logemann's it was 23. By arranging together several thin magnetised

plates Jamin constructed bar magnets which support 15 times their own weight.

The strength of two bar magnets may be compared by the following simple method, which is known as Külp's *compensation method* :—A small magnetic compass needle is placed in the magnetic meridian. One pole of one of the magnets to be tested is then placed at right angles to the magnetic meridian in the same plane as the needle, and so that its axis prolonged would bisect the needle. The compass needle is thereby deflected through a certain angle. The similar pole of the other magnet is then placed similarly on the other side of the needle, and a position found for it in which it exactly neutralises the action of the first magnet ; that is, when the needle is again in the magnetic meridian. If the magnets are not too long, compared with their distance from the needle, their strengths are approximately as the cubes of the distance of the acting poles from the magnetic needle.

720. **Circumstances which influence the power of magnets.**—All bars do not attain the same state of saturation, for their coercive force varies. Twisting or hammering imparts to iron or steel a considerable coercive force. But the most powerful of these influences is the operation of tempering (94). Coulomb found that a steel bar tempered at dull redness, and magnetised to saturation, made ten oscillations in 93 seconds. The same bar tempered at a cherry-red heat, and similarly magnetised to saturation, only took 63 seconds to make ten oscillations.

Hence it would seem that the harder the steel the greater is its coercive force ; it undergoes magnetisation with much greater difficulty, but retains it more effectually. It appears, however, from Jamin's experiments that no general rule of this kind can be laid down ; for each specimen of steel there seems, according to the proportion of carbon which it contains, to be a certain degree of tempering which is most favourable for the development of permanent magnetisation.

Very hard steel bars have the disadvantage of being very brittle, and in the case of long thin bars a hard tempering is apt to produce consequent poles. Compass needles are usually tempered at the blue heat—that is, about 300° C.—by which a high coercive force is obtained without great fragility. Steel is magnetised with difficulty even when placed for some time in a coil through which a powerful current is passing ; soft iron under these circumstances is magnetised at once. If a short coil covering only a portion of the steel bars be moved backwards and forwards the magnetisation is more complete.

The hardness of steel, and the proportion of carbon which it contains, exert an important influence on the degree to which it can be magnetised. For the same degree of hardness, the magnetisation increases with the proportion of carbon in the steel, and more markedly the smaller this proportion ; with the same proportion of carbon it increases with the hardness of the steel. It appears probable that the compound of iron and carbon in steel is the carrier of the permanent magnetisation, and the interjacent particles of iron the carriers of the temporary magnetisation. Holtz magnetised plates of English corset steel to saturation and determined their magnetic moment ; they were then placed in dilute hydrochloric acid, by

which the iron was eaten away, and the magnetic moment determined when the plate had been magnetised to saturation after each such treatment. It was thus found that, with a diminution in the proportion of iron, there was an increase in the magnetic moment for the unit of weight. Holtz found, however, that perfectly pure iron prepared by electrolysis can acquire permanent magnetism. Ferro-manganese, or iron alloyed withabout 12 per cent. of manganese, is quite destitute of magnetic properties.

In ordinary bar magnets the intensity of magnetisation (707) varies from 200 to 400 C.G.S. units, and in very thin long ones may attain 800, or about half the maximum of soft iron. Taking the specific gravity of steel at 7·8, the specific magnetism is 25 to 50 for ordinary magnets. It is here assumed that the magnetisation is uniform, which is not the case. The magnetisation of nickel is about one-third that of soft iron.

Jamin investigated the distribution of force in magnets by suspending from one arm of a delicate balance a small iron ball, and then ascertaining what force, applied at the other arm, was required to detach the ball when placed in contact with various parts of the magnet to be investigated.

Taking thus a thin plate magnetised to saturation, it was found that the magnetisation increased with the thickness, but did not materially vary with the breadth of the plate. The magnetic force was developed almost exclusively at the ends. The curve representing the magnetic force (721) was convex towards the poles at the ends. If now several similar plates are superposed, the corresponding curves become steeper and prolonged towards the middle ; the magnetic force thus becomes increased. When the curves run into each other in the middle the maximum of the combination is reached ; any additional plates produce no increase in the strength. Steel bars may also be magnetised so as to show the same curves, and such bars and combinations of plates are called by Jamin *normal* magnets.

Jamin found that magnetisation extends deeper in a bar than has been usually supposed ; in soft and annealed steel it penetrates deeply. The depth diminishes with the hardness of the steel and the proportion of carbon it contains. If plates of varying thickness are so thin that the magnetisation can entirely penetrate them, the thicker of these plates are more strongly magnetised by the same force, for the magnetisation extends through a thicker layer than the thinner ones ; if, however, the plates are very thick, they are magnetised to the same extent by one and the same force. With equal bars the thickness of the magnetic layer varies with the strength of the magnetising force. Jamin proved this by placing the plates in dilute sulphuric acid ; he found magnetisation in bars which had been exposed to the stronger force, while those which had been more feebly magnetised exhibited none when they had been eaten away by the acid to the same extent. He also showed that the magnetisation which had penetrated was as strong as that on the surface.

Holtz made some experiments on the influence of solid bars as against hollow tubes in the construction of permanent steel magnets. The latter are to be preferred ; they are decidedly cheaper, as they need not be bored, but may be bent from steel plates. A bar and a tube of the same steel, 125 mm. in length by 13 mm. diameter, the tube being 1·75 mm. thick, were magnetised

to saturation, and their magnetic moments determined by the method of oscillations (705), the tube being loaded with copper. The magnetism of the tube was to that of the bar as 1·6 : 1. The tubes also retained their magnetisation better. After the lapse of six months the ratio of the magnetisation of the tube was to that of the bar as 2·7 : 1. A magnetised steel tube filled with a soft iron core has scarcely any directive force. Holtz considers that it acts as a keeper.

Temperature.—Increase of temperature always produces a diminution of magnetisation. If the changes of temperature are small—those of the atmosphere, for instance—the magnet is not permanently altered. Kuppfer allowed a magnet to oscillate at different temperatures, and found a definite decrease in its power with increased temperature, as indicated by its slower oscillations. In the case of a magnet 2½ inches in length, he observed that with an increase of each degree of temperature the duration of 800 oscillations was 0·4″ longer. If n be the number of oscillations at zero, and n_1 the number at t, then

$$n = n_1 (1 - ct),$$

where c is a constant depending in each case on the magnet used. This formula has an important application in the correction of the observations of magnetic force which are made at different places and at different temperatures, and which, in order to be comparable, must first be reduced to a uniform temperature.

When a magnet has been more strongly heated, it does not regain its original force on cooling to its original temperature ; and when it has been heated to redness, it is demagnetised. This was first shown by Coulomb, who took a saturated magnet, heated it to progressively higher temperatures, and noted the number of oscillations after each heating. The higher the temperature to which it had been heated the slower its oscillations.

A magnet heated to bright redness, or about 785°, suddenly loses its magnetism, and so completely that it is quite indifferent not only towards iron, but also towards another magnet, and this holds so long as this high temperature continues. Incandescent iron also does not possess the property of being attracted by the magnet. Hence there is in the case of iron a *magnetic limit*, beyond which it is unaffected by magnetism. Such a magnetic limit exists in the case of other magnetic metals. With *cobalt*, for instance, it is far beyond a white heat, for at the highest temperatures hitherto examined it is still magnetic ; the magnetic limit of *chromium* is somewhat below red heat ; that of *nickel* at about 350° C., and of *manganese* at about 15° to 20° C.

A change of temperature, whether from 16° to 100°, or from 100° to 16°, increases the strength of temporary or induced magnetisation both in the case of iron and of steel.

Percussion and torsion.—When a steel bar is hammered while being magnetised it acquires a much higher degree of magnetisation than it would without this treatment. Conversely when a magnet is let fall, or is otherwise violently disturbed, it loses much of its magnetisation. Wiedemann has investigated in a very complete manner the relations of torsion and magnetisation. Torsion exerts a great influence on the magnetisation of a bar, and the interesting phenomenon has been observed that torsion influences magnetism in

the same manner as magnetism does torsion. Thus the permanent magnetisation of a steel bar is diminished by torsion, but not proportionately to the increase of torsion. In like manner the torsion of twisted iron wires is diminished by their being magnetised, though less so than in proportion to their magnetisation. Repeated torsions in the same direction scarcely diminish magnetisation, but a torsion in the opposite direction produces a new diminution of the magnetism. In a perfectly analogous manner, repeated magnetisations in the same direction scarcely diminish torsion, but a renewed magnetisation in the opposite direction does so.

721. **Distribution of free magnetism.**—Coulomb investigated the distribution of magnetic force by placing a large magnet in a vertical position in the magnetic meridian; he then took a small magnetic needle suspended by a cocoon thread, and fixed at right angles to a stout copper wire so as to retard the oscillations (fig. 654); and having ascertained the number of its oscillations under the influence of the earth's magnetism alone, he presented it to different parts of the magnet. The oscillations were fewer as the needle was nearer the middle of the bar, and when they had reached that position their number was the same as under the influence of the earth's magnetism alone. For saturated bars of more than 7 inches in length the distribution could always be expressed by a curve whose abscissæ were the distances from the ends of the magnet, and whose ordinates were the force of magnetism at these points. With magnets of the above dimensions the poles are at the same distance from the end; Coulomb found the distance to be 1·6 inch in a bar 8 inches long. He also found that, with shorter bars, the distance of the poles from the end is $\frac{1}{8}$ of the length; thus with a bar of three inches it would be half an inch. These results presuppose that the other dimensions of the bar are very small as compared with its length, that it has a regular shape, and is uniformly magnetised. When these conditions are not fulfilled, the positions of the poles can only be determined by direct trials with a magnetic needle. With lozenge-shaped magnets the poles are nearer the middle. Coulomb found that these lozenge-shaped bars have a greater *directive* force than rectangular bars of the same weight, thickness, and hardness.

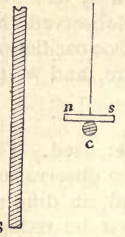


Fig. 671.

A short magnet is defined by Coulomb as one whose length is less than 50 times its diameter.

Kohlrausch found that the pole of a magnet, as far as its action at a distance is concerned, is $\frac{1}{12}$ of its length from the end.

722. **Mayer's floating magnets.**—The reciprocal action of magnetic poles may be conveniently illustrated by an elegant method devised by Prof. A. M. Mayer. Steel sewing-needles are magnetised so that their points are north poles, and their eyes, which are thus south poles, just project through minute cork discs, so that when placed in water the magnets float in a vertical position. If the north pole of a strong magnet is brought near a number of these floating magnets, they are attracted by it, and take up definite positions, forming figures which depend on the reciprocal repulsion of the floating magnets, and on their number. Some of them are represented in fig. 672. The more complex produce more than one arrangement

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which are not equally stable, the letters *a*, *b*, and *c* indicating the decreasing order of stability. A slight shock often causes one form to pass into another and more stable form.

These figures not only illustrate magnetic actions, but they suggest an

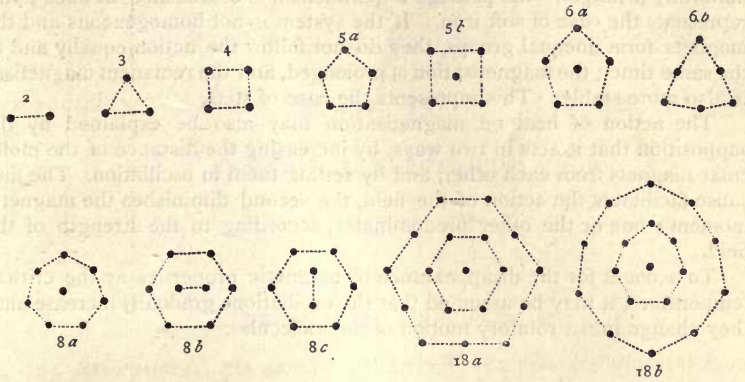


Fig. 672.

image of the manner in which alteration of molecular groupings may give rise to physical phenomena, such as those of superfusion (345).

Such floating magnets as are here described are delicate tests of magnetisation, and are convenient for investigating the distribution of the poles in bodies of irregular shape.

722 *a*. **Professor Ewing's experiments.**—We may here give an account of Professor Ewing's experiments in illustration of magnetic properties, which explain them without requiring any other assumption than that magnetism is a molecular property, each molecule being a perfect magnet with a north and south pole.

By placing a number of small magnetic needles near each other, he found that when left to themselves they set in various configurations each of which is stable, but which has no external action. If one part of such a system is disturbed, a new arrangement is established. Each separate magnet takes up a position of stable equilibrium, after oscillations of greater or less amplitude.

If now, any given configuration is submitted to the action of a gradually increasing magnetic field, by placing a series of parallel wires conveying a voltaic current, the magnets are at first deflected progressively, but if the current is stopped these magnets revert to their original arrangement, and there is no permanent effect on the system.

When, however, the strength of the field is gradually increased, a certain stage is reached at which equilibrium is broken, and a fresh configuration is formed where all the elementary magnets are in the direction of the field. From this stage any increase in the strength of the field only makes the observation more marked.

On gradually weakening the field, the system does not return through

the same stages ; it tends towards that configuration in which the magnets are parallel to the field. This illustrates the case of permanent magnetisation.

If the system is quite homogeneous—that is, if the molecular magnets are uniformly arranged—the passage to parallelism is established at once ; this represents the case of soft iron. If the system is not homogeneous and the magnets form unequal groups, they do not follow the action equally and at the same time ; the magnetisation is prolonged, and the remanent magnetism is also more stable. This represents the case of steel.

The action of heat on magnetisation may also be explained by the supposition that it acts in two ways, by increasing the distance of the molecular magnets from each other, and by setting them in oscillation. The first cause facilitates the action of the field, the second diminishes the magnetic moment ; one or the other predominates, according to the strength of the field.

To account for the disappearance of magnetic properties at the critical temperature, it may be assumed that the oscillations gradually increase until they change into a rotatory motion of the molecules.

BOOK IX

FRICTIONAL ELECTRICITY

CHAPTER I

FUNDAMENTAL PRINCIPLES

723. **Electricity. Its nature.**—Electricity is a powerful physical agent which manifests itself mainly by attraction and repulsion, but also by luminous and heating effects, by violent shocks, by chemical decomposition, and many other phenomena. It is evoked in bodies by a variety of causes, among which are friction, pressure, chemical action, heat, and magnetism.

Thales, 600 B.C., knew that when *amber* was rubbed with silk it acquired the property of attracting light bodies; and from the Greek form of this word (*ἤλεκτρον*) the term *electricity* has been derived. This is nearly all the knowledge left by the ancients; it was not until towards the end of the sixteenth century that Dr. Gilbert, physician to Queen Elizabeth, showed that this property was not limited to amber, but that other bodies, such as sulphur, wax, glass, &c., also possessed it in a greater or less degree.

724. **Development of electricity by friction.**—When a glass rod, or a stick of sealing-wax, or shellac, is held in the hand, and is rubbed with a piece of flannel, or with the skin of a cat, the parts rubbed will be found to have the property of attracting light bodies, such as pieces of silk, wool, feathers, paper, bran, gold leaf, &c., which, after remaining a short time in contact, are again repelled. They are then said to have become *electrified.* In order to ascertain whether bodies are electrified or not, instruments called *electroscopes* are used. The simplest of these, the *electric pendulum* (fig. 673), consists of a pith ball attached by means of a silk thread to a glass support. When an electrified body is brought near the pith ball, the latter is instantly attracted, but after momentary contact is again repelled (fig. 674).

A solid body may also be electrified by friction with a liquid or with a gas. In the Torricellian vacuum a movement of the mercury against the sides of the glass produces a disengagement of electric light visible in the dark; a tube exhausted of air, but containing a few drops of mercury, becomes also luminous when agitated in the dark.

If a quantity of mercury in a dry glass vessel be connected with a gold-leaf electroscope by a wire, and a dry glass rod be immersed in it, no indica-

tions are observed during the immersion, but on smartly withdrawing the rod, the leaves increasingly diverge, attaining their maximum when the rod leaves the mercury.

Some substances, particularly metals, do not seem capable of receiving the electric excitement. When a rod of metal is held in the hand, and

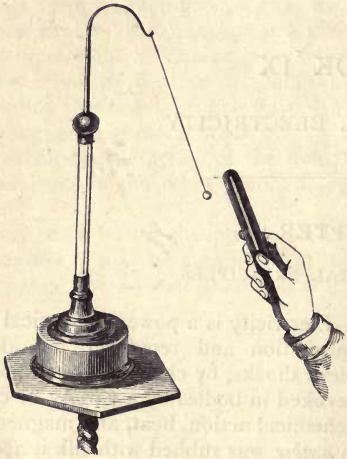


Fig. 674.

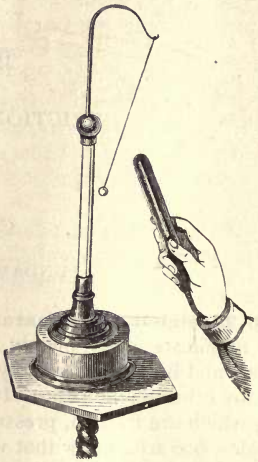


Fig. 675.

rubbed with silk or flannel, no electrical effects are produced in it ; and bodies were divided by Gilbert into *ideoelectrics*, or those which become electrical by friction ; and *anelectrics*, or those which do not possess this property. These distinctions no longer obtain in any absolute sense ; under appropriate conditions, all bodies may be electrified by friction (726).

725. **Conductors and nonconductors.**—When a dry glass rod, rubbed at one end, is brought near an electroscope, that part only will be electrified which has been rubbed ; the other end will produce neither attraction nor repulsion. The same is the case with a rod of shellac or of sealing-wax. In these bodies electricity does not pass from one part to another—they do not *conduct* electricity. Experiment shows that, when a metal has received electricity in any of its parts, the electricity instantly spreads over its entire surface. Metals are hence said to be good *conductors* of electricity.

Bodies have, accordingly, been divided into *conductors* and *nonconductors* or *insulators*. This distinction is not absolute, and we may advantageously consider bodies as offering a *resistance* to the passage of electricity which varies with the nature of the substance. Those bodies which offer little resistance are conductors, and those which offer great resistance are non-conductors or insulators: electrical *conductivity* is accordingly the inverse of electrical *resistance*. There is no such thing as an absolute nonconductor of electricity, any more than there is an absolute nonconductor of heat. We are to consider that between conductors and nonconductors there is a *quantitative* and not a *qualitative* difference ; there is no conductor so good

but that it offers some resistance to the passage of electricity, nor is there any substance which insulates so completely but that it allows some electricity to pass. The transition from conductors to nonconductors is gradual, and no line of sharp demarcation can be drawn between them.

In this sense we are to understand the following table, in which bodies are conveniently classed as *conductors*, *semiconductors*, and *nonconductors*; those bodies being designated as conductors which, when applied to a charged electroscope, discharge it almost instantaneously; semiconductors being those which discharge it in a short but measurable time—a few seconds, for instance; while nonconductors effect no perceptible discharge in the course of a minute.

Conductors	*Semiconductors*	*Nonconductors*
Metals.	Alcohol and ether.	Dry oxides.
Well-burnt charcoal.	Powdered glass	Ice at $-25°$ C.
Graphite.	Flour of sulphur.	Lime.
Acids.	Dry wood.	Caoutchouc.
Aqueous solutions.	Paper.	Air and dry gases.
Water.	Ice at $0°$.	Dry paper.
Snow.		Silk.
Vegetables.		Diamonds and precious stones.
Animals.		Glass.
Soluble salts.		Wax.
Linen.		Sulphur.
Cotton.		Resins.
		Amber.
		Shellac.

This list is arranged in the order of decreasing conductivity, or, what is the same thing, of increasing resistance. The arrangement, however, is not invariable. Conductivity depends on many physical conditions. Glass, for example, which does not conduct at ordinary temperatures, does so at $200°$ to $300°$ C. To show this, platinum wire is coiled on a glass rod to within a couple of inches from the end. If the finger touches the coiled part, and the free end when at the ordinary temperature is applied to a charged electroscope, it does not affect it; but if the free end be heated by placing it in a Bunsen's flame, it will now be found to discharge the electroscope. Shellac and resin do not insulate so well when they are heated. Water, which is a good conductor, conducts but little in the state of ice at $0°$, and very badly at $-25°$. Powdered glass and flour of sulphur conduct very well, while in large masses they are nonconductors; probably because in a state of powder each particle becomes covered with a film of moisture that acts as a conductor. The nonconducting power of glass is also greatly influenced by its chemical composition. Some specimens have an appreciable conductivity even if dry and at the ordinary temperature.

Heat acts indirectly by drying, by which many bodies lose their conductivity either partially or wholly.

According to Said Effendi, if the conducting power of water be taken at 1,000, the conducting power of petroleum is 72; alcohol 49; ether 40;

turpentine 23; and benzole 16. Domalip obtained the following numbers for the respective conductivities: Water 1,000; ether 44; turpentine 13; and benzole 7.

726. **Insulating bodies. Common reservoir.**—Bad conductors are called *insulators*, for they are used as supports for bodies in which electricity is to be retained. A conductor remains electrified only so long as it is surrounded by insulators. If this were not the case, as soon as the electrified body came in contact with the earth, which is a good conductor, the electricity would pass into the earth, and diffuse itself through its whole extent. On this account, the earth has been named the *common reservoir*. A body is insulated, by being placed on a support with glass feet, or on a cake of resin, or by being suspended by silk threads. No bodies, however, insulate perfectly; all electrified bodies lose their electricity more or less rapidly by means of the supports on which they rest. Glass is always somewhat hygroscopic, and the aqueous vapour which condenses on it affords a passage for the electricity; the insulating power of glass is materially improved by coating it with shellac or copal varnish. Dry air is a good insulator; but when the air contains moisture it conducts electricity, and this is the principal source of the loss of electricity. Hence it is necessary, in electrical experiments, to rub the supports with cloths dried at the fire, and to surround electrified bodies by glass vessels, containing substances which absorb moisture, such as chloride of calcium, or pumice soaked with sulphuric acid.

From their great conductivity metals do not seem to become electrified by friction. But if they are insulated, by being held in the hand by an india-rubber glove or a silk handkerchief and then rubbed, they give good indications. This may also be seen by the following experiment (fig. 676). A brass tube is provided with a glass handle by which it is held, and then rubbed with silk or flannel. On approaching the metal to an electrical pendulum (fig. 674), the pith ball will be attracted. If the metal is held in the hand electricity is indeed produced by friction—but it immediately passes through the body into the ground.

Fig. 676.

If, too, the cap of a gold-leaf electroscope be briskly flapped with a dry silk handkerchief, the gold leaves will diverge.

727. **Distinction of the two kinds of electricity.**—If electricity be developed on a glass rod by friction with silk, and the rod be brought near an electrical pendulum, the ball will be attracted to the glass, and after momentary contact will be again repelled. By this contact the ball becomes electrified, and so long as the two bodies retain their electricity, repulsion follows whenever they are brought near each other. If a stick of sealing-wax, electrified by friction with flannel or silk, be approached to another electrical pendulum, the same effects will be produced—the ball will fly towards the wax, and after contact will be repelled. Two bodies which have been charged with electricity repel one another. But the electricities respectively developed in the preceding cases are not the same. If, after the pith ball had been touched with an electrified glass rod, an electrified stick of sealing-wax, and then an electrified glass rod, be alternately approached to it, the

pith ball will be *attracted* by the former and *repelled* by the latter. Similarly, if the pendulum be charged by contact with the electrified sealing-wax, it will be *repelled* when this is approached to it, but *attracted* by the approach of the excited glass rod.

On experiments of this nature, Dufay first made the observation that there are two different electricities: the one developed by the friction of glass under certain circumstances, the other by the friction of resin or shellac. To the first the name *vitreous* electricity is given; to the second the name *resinous* electricity.

728. **Theories of electricity.**—To account for the different effects of electricity, Franklin supposed that there exists a peculiar, subtle, imponderable fluid, which acts by repulsion on its own particles, and pervades all matter. This fluid is present in every substance in a quantity peculiar to it, and when it contains this quantity it is in the natural state, or in a state of equilibrium. By friction certain bodies acquire an additional quantity of the fluid, and are said to be *positively* electrified; others by friction lose a portion, and are said to be *negatively* electrified. The former state corresponds to *vitreous* electricity, and the latter to *resinous* electricity. Positive electricity is represented by the sign +, and negative electricity by the sign −; a designation based on the algebraical principle, that when a plus quantity is added to an equal minus quantity zero is produced. So when a body containing a quantity of positive electricity is touched with a body possessing an equivalent quantity of negative electricity, a neutral or zero state is produced.

The *theory of Symmer* assumes that every substance contains an indefinite quantity of a subtle, imponderable matter, which is called the electric fluid. This fluid is formed by the union of two fluids—the *positive* and the *negative*. When they are combined they neutralise one another, and the body is then in the natural or neutral state. By friction, and by several other means, the two fluids may be separated, but one of them can never be excited without a simultaneous production of the other. There may, however, be a greater or less excess of the one or the other in any body, and it is then said to be electrified *positively* or *negatively*. As in Franklin's theory, *vitreous* corresponds to *positive* and *resinous* to *negative* electricity. This distinction is merely conventional: it is adopted for the sake of convenience, and there is no other reason why resinous electricity should not be called positive electricity.

Electricities of the same name repel one another, and electricities of opposite kinds attract each other. The electricities can circulate freely on the surface of certain bodies, which are called conductors, but remain confined to certain parts of others, which are called nonconductors.

It must be added that this theory is quite hypothetical; but for purposes of instruction its general adoption is for the present justified by the convenient explanation which it gives of electrical phenomena.

729. **Action of electrified bodies on each other.**—Admitting the two-fluid hypothesis, the phenomena of attraction and repulsion may be enunciated in the following law:—

Two bodies charged with the same electricity repel each other; two bodies charged with opposite electricities attract each other.

These attractions and repulsions take place in virtue of the action which the two electricities exert on themselves, and not in virtue of their action on the particles of matter.

730. **Law of the development of electricity by friction.**—Whenever two bodies are rubbed together, the neutral electricity is decomposed. Two electricities are developed at the same time and in equal quantities—one body takes positive and the other negative electricity. This may be proved by the following experiment devised by Faraday :—

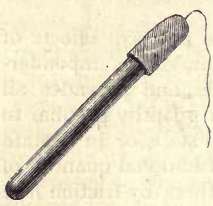


Fig. 677.

A small flannel cap provided with a silk thread (fig. 677) is fitted on the end of a stout rod of shellac, and rubbed round a few times. When the cap is removed by means of the silk thread, and presented to a pith ball pendulum charged with positive electricity, the latter will be repelled, proving that the flannel is charged with positive electricity ; while if the shellac is presented to the pith ball, it will be attracted, showing that the shellac is charged with negative electricity. Both electricities are present in equal quantities ; for if the rod be presented to the electroscopes before removing the cap, no action is observed.

The electricity developed on a body by friction depends on the rubber as well as the body rubbed. Thus glass becomes negatively electrified when rubbed with catskin, but positively when rubbed with silk. In the following list, which is mainly due to Faraday, the substances are arranged in such an order that each becomes positively electrified when rubbed with any of the bodies following, but negatively when rubbed with any of those which precede it :—

1. Catskin.	5. Glass.	9. Wood.	13. Resin.
2. Flannel.	6. Cotton.	10. Metals.	14. Sulphur.
3. Ivory.	7. Silk.	11. Caoutchouc.	15. Guttapercha.
4. Rock crystal.	8. The hand.	12. Sealing-wax.	16. Gun-cotton.

The nature of the electricity set free by friction depends also on the degree of polish, the direction of the friction, and the temperature. If two glass discs of different degrees of polish are rubbed against each other, that which is most polished is positively, and that which is least polished is negatively, electrified. If two silk ribbons of the same kind are rubbed across each other, that which is transversely rubbed is negatively and the other positively electrified. If two bodies of the same substance, of the same polish, but of different temperatures, are rubbed together, that which is most heated is negatively electrified. Generally speaking, the particles which are most readily displaced are negatively electrified.

Poggendorff has observed that many substances which have hitherto been regarded as highly negative, such as gun-paper, gun-cotton, and ebonite, yield positive electricity when rubbed with leather coated with amalgam. It must be added that the results of experiments on the kind of electricity produced by rubbing bodies together are somewhat uncertain, as slight differences in the surfaces of the bodies rubbed may completely alter their deportment. A valuable source of negative electricity is a strip of pyroxyline or gun-paper drawn through the fingers.

731. **Development of electricity by pressure and cleavage.**—Electrical excitement may be produced by other causes than friction. If a disc of wood, covered with silk, on which some amalgam has been rubbed, and a metal disc, each provided with an insulating handle, be placed in contact, and then suddenly separated, the metal disc is negatively electrified. A crystal of Iceland spar pressed between the fingers becomes positively electrified, and retains this state for some time. The same property is observed in several other minerals, even though conductors, provided they be insulated. If cork and caoutchouc be pressed together, the first becomes positively and the latter negatively electrified. A disc of wood pressed on an orange and separated carries away a good charge of electricity if the contact be rapidly interrupted. But if the disc is slowly removed the quantity is smaller, for the two fluids recombine at the moment of their separation. For this reason there is no apparent effect when the two bodies pressed together are good conductors.

The contact of heterogeneous bodies is no doubt the source of electricity. Pressure and friction are but particular cases ; in the former case the contact is closer, and in the latter case the surfaces are being continually renewed ; the effect is the same as if there were a series of rapidly succeeding contacts.

Cleavage also is a source of electricity. If a plate of mica be rapidly split in the dark, a slight phosphorescent light is perceived. Becquerel fixed glass handles to each side of a plate of mica, and then rapidly separated them. On presenting each of the plates thus separated to an electroscope, he found that one was negatively and the other positively electrified. If a stick of sealing-wax be broken, the ends exhibit different electricities.

All badly conducting crystalline substances exhibit electrical indications by cleavage. The separated plates are always in opposite electrical conditions, provided they are not good conductors : for if they were, the separation would not be sufficiently rapid to prevent the recombination of the two electricities. To the phenomena here described is due the luminous appearance seen in the dark when sugar is broken. If sulphur or resin be melted in glass vessels and a glass rod be placed in the melted mass, on cooling the solid mass can be lifted out, and will be found to be negatively electrified.

732. **Pyroelectricity.**—Certain minerals, when warmed, acquire electrical properties ; a phenomenon to which the name *pyroelectricity* is given. It is best studied in *tourmaline*, in which it was first discovered from the fact that this mineral has the power of first attracting and then repelling hot ashes when placed among them.

To observe this phenomenon, a crystal of tourmaline (fig. 678) is suspended horizontally by a silk thread, in a glass cylinder placed on a heated metal plate, or in an ordinary hot-air bath. On subsequently investigating the electric condition of the ends by approaching to them successively an electrified glass rod, one end will be found to be positively electrified, and the other end negatively electrified, and each end shows this polarity as long as the temperature rises. The arrangement of the electricity is thus like that of the magnetism in a magnet. The points at which the intensity of free electricity is greatest are called the *poles*, and the line connecting them is the *electric axis*. When a tourmaline, while thus electrified, is broken in the middle, each of the pieces has its two poles, and the polarity of the

broken ends is opposite, resembling thus the experiment of the broken magnets (685). The quantities of electricity produced when tourmaline is heated are equal as well as opposite, for if a heated crystal be suspended by an insulating support inside an insulated metal cylinder, the outside of which is connected with an electroscope (745), no divergence in its leaves is produced.

Fig. 678.

These polar properties depend on the *change* of temperature. When a tourmaline, which has become electrical by being warmed, is allowed to cool slowly, it first loses electricity, and then its polarity becomes reversed; that is, the end which was positive now becomes negative, and that which was negative becomes positive, and the position of the poles now remains unchanged so long as the temperature sinks. Tourmaline only becomes pyroelectric within certain limits of temperature; these vary somewhat with the length, but are usually between 10° and 150° C. Below and above these temperatures it behaves like any other body, and shows no polarity.

Tourmaline belongs to the hexagonal system, and usually crystallises in hemihedral forms; those, that is to say, which are differently modified at the ends of their crystallographical principal axis. The name *analogous* pole is given to that end A of the crystal which shows positive electricity when the temperature is rising, and negative electricity when it is sinking; *antilogous* pole to the end B which becomes negative by being heated, and positive by being cooled.

Besides tourmaline the following minerals are found to be pyroelectric, though not so markedly—boracite, topaz, prehnite, silicate of zinc, scolezite, axenite. And the following organic bodies are pyroelectric—cane-sugar, Pasteur's salt (racemate of sodium and ammonium), tartrate of potassium, &c.

Lord Kelvin supposes that every portion of tourmaline and other hemihedral crystals possesses a definite electrical polarity, the intensity of which depends on the temperature. When the surface is passed through a flame every part becomes electrified to such an extent as to exactly neutralise, for all external points, the effect of the internal polarity. The crystal thus has no external action, nor any tendency to change its mode of electrification. But if it be heated or cooled the internal polarisation of each particle of the crystal is altered, and can no longer be balanced by the superficial electrification, so that there is a resultant external action.

A very convenient, and at the same time sensitive, means of investigating the action of heat on crystals is to sift on these, after having been warmed, a mixture of flour of sulphur and red lead through a small cotton sieve. By the friction in sifting the sulphur acquires negative and the red lead positive electricity, and the powders thus charged attach themselves to those parts of the crystal which have the opposite electricity, and thus by their different colours give at once an image of its distribution.

Crystals of fluor-spar are not only electrified by heat, but also when they are exposed to radiation from the sun and from the electric light. This phenomenon is known as *photo-electricity*.

CHAPTER II

QUANTITATIVE LAWS OF ELECTRICAL ACTION

733. **Electrical quantity.**—In the experiment with the flannel cap, described above (730), each time the experiment is made, the quantity of positive electricity produced, which remains on the flannel, is equal to that of the negative electricity, which remains on the sealing-wax. The flannel, with its charge of positive electricity, may be detached, and if we work under precisely uniform conditions, equal *quantities* of electricity can thus be separated.

If we fill a cask with water by means of a measure, the quantity added would be directly proportional to the number of such measures. Now, although in the above experiment the quantities of electricity produced each time are equal, yet when the flannel cap is applied each time to an insulated conductor it does not necessarily follow that the quantity of electricity imparted is directly proportional to the number of such applications.

On the C.G.S. system the *unit quantity of electricity* is that amount which, acting, at a distance of one centimetre across air, on a quantity of electricity of the same kind equal to itself, would repel it with a force equal to one dyne (709), and is called a *Coulomb*.

734. **Laws of electrical attraction and repulsion.**—The laws which regulate the attraction and repulsion of electrified bodies may be thus stated :—

I. *The repulsions or attractions between two electrified bodies are in the inverse ratio of the squares of their distance.*

II. *The distance remaining the same, the force of attraction or repulsion between two electrified bodies is directly as the product of the quantities of electricity with which they are charged.*

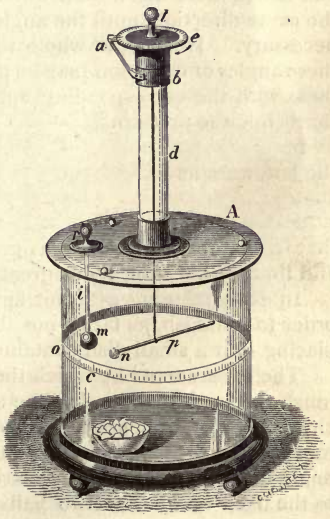


Fig. 679.

These laws were established by Coulomb, by means of the torsion balance, used in determining the laws of magnetic attractions and repulsions (704), modified in accordance with the

requirements of the case. The wire, on the torsion of which the method depends, is so fine that a foot weighs only $\frac{1}{10}$ of a grain. At its lower extremity there is a fine shellac rod, *np* (fig. 679), at one end of which is a small disc of copper-foil, *n*. Instead of the vertical magnetic needle, there is a glass rod, *i*, terminated by a gilt pith ball, *m*, which passes through the aperture *r*. The scale *oc* is fixed round the sides of the vessel, and during the experiment the ball, *m*, is opposite the zero point *o*. The micrometer consists of a small graduated disc, *e*, movable independently of the tube *d*, and of a fixed index, *a*, which shows by how many degrees the disc is turned. In the centre of the disc there is a small button, *t*, to which is fixed the wire which supports *np*.

i. The micrometer is turned until the zero point is opposite the index, and the tube *d* is turned until the knob *n* is opposite zero of the graduated circle; the knob *m* is in the same position, and thus presses against *n*. The knob *m* is then removed and electrified, and replaced in the apparatus, through the aperture *r*. As soon as the electrified knob *m* touches *n*, the latter becomes electrified, and is repelled, and after a few oscillations remains constant at a distance at which the force of repulsion is equal to the force of torsion. In a special experiment Coulomb found the angle of torsion between the two to be 36°; and as the force of torsion is proportional to the angle of torsion, this angle represents the repulsive force between *m* and *n*. In order to reduce the angle to 18° it was necessary to turn the disc through 126°. The wire was twisted 126° in the direction of the arrow at its upper extremity, and 18° in the opposite direction at its lower extremity, and hence there was a total torsion of 144°. On turning the micrometer in the same direction, until the angle of deviation was $8\frac{1}{2}$°, 567° of torsion was necessary. Hence the whole torsion was $575\frac{1}{2}$°. Without sensible error these angles of deviation may be taken at 36°, 18°, and 9°; and on comparing them with the corresponding angles of torsion, 36°, 144°, and 576°, we see that while the first are as

$$1 : \tfrac{1}{2} : \tfrac{1}{4},$$

the latter are as

$$1 : 4 : 16;$$

that is, that for a distance $\frac{1}{2}$ as great the angle of torsion is 4 times as great, and that for a distance $\frac{1}{4}$ as great the repulsive force is 16 times as great.

In experimenting with this apparatus the air must be thoroughly dry, in order to diminish, as far as possible, loss of electricity. This is effected by placing in it a small dish containing chloride of calcium.

The experiments by which the law of attraction is proved are made in much the same manner, but the two balls are charged with opposite electricities. A certain quantity of electricity is imparted to the movable ball, by means of an insulated pin, and the micrometer moved until there is a certain angle below. A charge of electricity of the opposite kind is then imparted to the fixed ball. The two balls tend to move towards each other, but are prevented by the torsion of the wire, and the movable ball remains at a distance at which there is equilibrium between the force of attraction, which draws the balls together, and that of torsion, which tends to separate them. The micrometer screw is then turned to a greater extent, by which more

torsion and a greater angle between the two balls are produced. And it is from the relation which exists between the angle of deflection on the one hand and the angle which expresses the force of torsion on the other, that the law of attraction has been deduced.

ii. To prove the second law let a charge be imparted to m; n being in contact with it becomes charged, and is repelled to a certain distance. The angle of deflection being noted, let the ball m be touched by an insulated but unelectrified ball of exactly the same size and kind. If in this way half the charge on one of the balls is removed, it will be found that the amount of torsion necessary to maintain the balls at their original angular distance is half what it was before.

The two laws are included in the formula $F = \frac{ee'}{d^2}$, where F is the force, e and e' the quantities of electricity on any two surfaces, and d the distance between them. If e and e' are of opposite electricities the action is one of attraction, while if they are the same it is a repulsive action.

Coulomb also established the law by the method of oscillations which is particularly applicable to the case of attraction, as there are difficulties in experimenting with the torsion balance. An apparatus for this purpose consists of an insulated metal sphere (fig. 680), and at a little distance a short thin rod of shellac hung by a silk thread and with a disc of metal foil at one end, the whole being enclosed in a glass cylinder, which rests on an insulating plate. If now the disc is charged with the opposite electricity to that of the sphere, and is removed from its position of equilibrium, it will make a series of oscillations before coming to rest. It can be proved that the charge on the sphere acts as if it were concentrated at the centre, and if the needle is short, the distance at which the force acts will be that from the centre of the sphere to the thread of suspension. As in the case of magnetic oscillations we may use a formula for the time of a single oscillation analogous to that of the pendulum (55); that is $t = \pi \sqrt{\frac{M}{FL}}$, in which M is the moment of inertia of the needle, L its length, and F the force of attraction. Now, all other things being the same, it is found that when the sphere is placed at varying

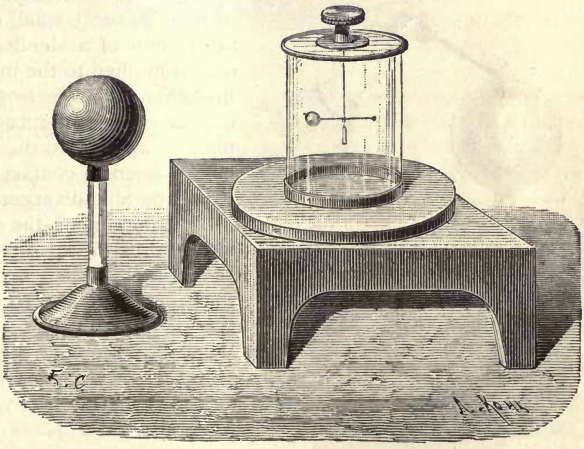


Fig. 680.

distances, d and $d_{,}$, the times of oscillations, t and $t_{,}$, vary, and therefore the force varies, and the relation is established that $F : F_{,} = d^2_{,} : d^2$.

735. **Distribution of electricity.**—When an insulated sphere of conducting material is charged with electricity, the electricity passes to the surface of the sphere, and forms there an extremely thin layer. If, in Coulomb's balance, the fixed ball be replaced by another electrified sphere, a certain repulsion will be observed. When this sphere is touched with an insulated sphere identical with the first, but in the neutral state, the first ball will be found to have lost half its electricity, and only half the repulsion will be observed. By repeating this experiment with spheres of various substances solid and hollow, but all having the same superficies, the result will be the same, excepting that, with imperfectly conducting materials, the time required for the distribution will be greater. From this it is concluded that the distribution of electricity depends on the extent of the surface, and not on the mass, and, therefore, that electricity does not penetrate into the interior, but is confined to the surface. This conclusion is further established by the following experiments :—

i. A thin hollow copper sphere provided with an aperture of about an inch in diameter (fig. 681), and placed on an insulating support, is charged in the interior with electricity. When the *carrier* or *proof plane* (a small disc of copper-foil at the end of a slender glass or shellac rod) is applied to the interior, and is then brought near an electroscope, no electrical indications are produced. But if the proof plane is applied to the electroscope after having been in contact with the exterior, a considerable divergence ensues.

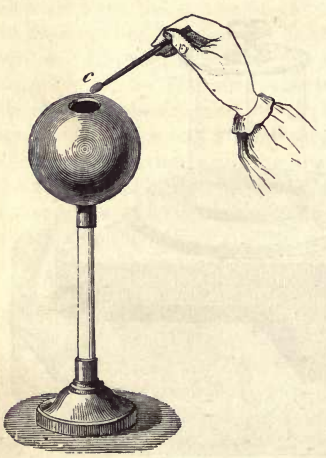


Fig. 681.

The action of the proof plane as a measure of the quantity of electricity is as follows :—When it touches any surface the proof plane becomes confounded with the element touched ; it takes in some sense its place relatively to the electricity, or rather, it becomes itself the element on which the electricity is diffused. Thus when the proof plane is removed from contact we have in effect cut away from the surface an element of the same thickness and the same extent as its own, and have transferred it to the balance without its losing any of the electricity which covered it.

ii. A hollow globe, fixed on an insulating support, is provided with two hemispherical envelopes which fit closely and can be separated by glass handles. The interior is now electrified and the two hemispheres brought in contact. On then rapidly removing them (fig. 682), the coverings will be found to be electrified, while the sphere is in its natural condition.

This may also be illustrated by the experiment represented in fig. 682, in which A is a hollow brass hemisphere resting on a support of ebonite, and is electrified by striking it with silk ; a similar hemisphere B provided with a

glass handle G is placed over it. A metal spring on the inside of B is brought in contact with A by pressing the ebonite button E, and on afterwards examining the two hemispheres all the electricity is found on the outer one B.

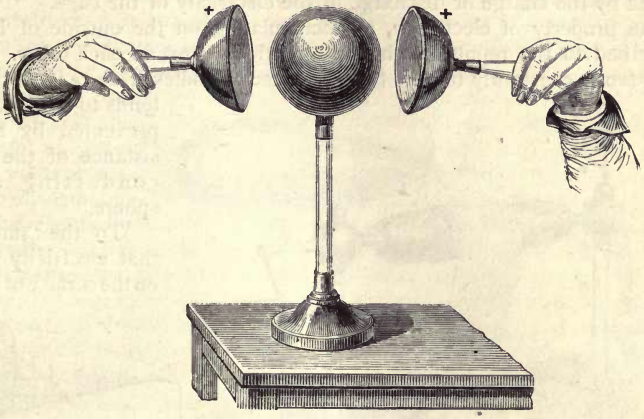


Fig. 682.

iii. The distribution of electricity on the surface may also be shown by means of the following apparatus:—It consists of a metal cylinder on insulating supports, on which is fixed a long strip of tinfoil which can be rolled up by means of a small insulating handle (fig. 684). A quadrant electrometer is fitted in metallic communication with the cylinder. When the strip is rolled up, a charge is imparted to the cylinder, by which a certain divergence is produced. On unrolling the tinfoil this divergence gradually diminishes, and increases as it is again rolled up. The quantity of electricity remaining the same, the electrical force, on each unit of surface, is therefore less as the surface is greater.

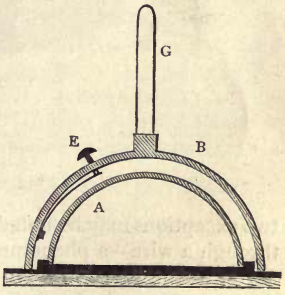


Fig. 683.

iv. The following ingenious experiment by Faraday further illustrates this law:—A metal ring is fitted on an insulated support, and a conical gauze bag, such as is used for catching butterflies, is fitted to it (fig. 685).

By means of a silk thread, the bag can be drawn inside out. After electrifying the bag, it is seen by means of a proof plane that the electricity is on the exterior; but if the positions are reversed by drawing the bag inside out, so that the interior has now become the exterior, the electricity will still be found on the exterior.

v. The same point may be further illustrated by an experiment due to Prof. Terquem. A bird-cage, preferably of metal wire, is suspended by insulators, and contains either a gold-leaf electroscope or pieces of Dutch metal, feathers,

pith balls, &c. When the cage is connected with an electrical machine, the articles in the interior are quite unaffected, although strong sparks may be taken from the outside. Bands of paper may be fixed to the inside; while those fixed to the outside diverge widely. A bird in the inside is quite unaffected by the charge or discharge of the electricity of the cage.

The property of electricity, of accumulating on the outside of bodies, is ascribed to the repulsion which the particles exert on each other. Electricity tends constantly to pass to the surface of bodies, whence it continually tends to escape, but is prevented by the resistance of the feebly conducting atmosphere.

To the statement that electricity resides on the surface of bodies,

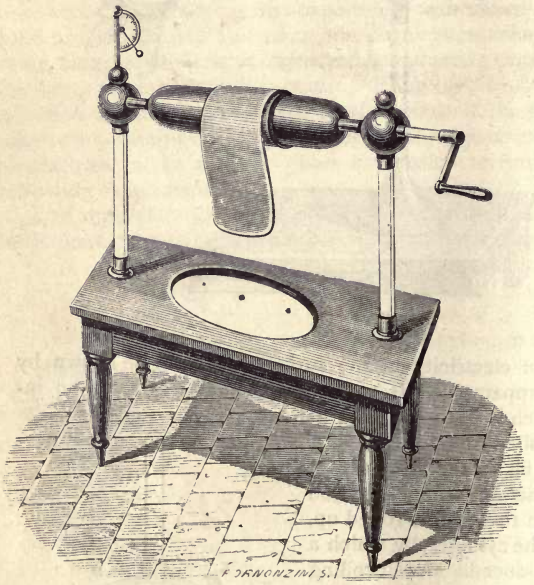


Fig. 684.

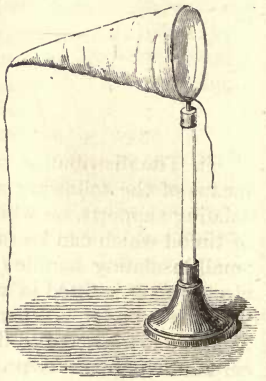


Fig. 685.

two exceptions may be noted. When two opposite electricities are discharged through a wire—a phenomenon which, when continuous, forms an electrical current—the discharge is effected throughout the whole mass of the conductor. Also a body placed inside another may, if insulated from it, receive charges of electricity. On this depends the possibility of electrical experiments in ordinary rooms.

736. **Electric density.**—On a metal sphere the distribution of the electricity is everywhere the same, simply from its symmetry. This can be demonstrated by means of the proof plane and the torsion balance. A metal sphere placed on an insulating support is electrified, and touched at different parts of its surface with the proof plane, which each time is applied to the movable needle of the torsion balance. As in all cases the torsion observed is sensibly the same, it is concluded that the proof plane each time receives the same quantity of electricity. In the case of an elongated ellipsoid (fig. 686) it is found that the distribution of electricity is different at different

points of the surface. The electricity accumulates at the most acute points. This is demonstrated by successively touching the ellipsoid at different parts with the proof plane, and then bringing this into the torsion balance. By this means Coulomb found that the greatest deflection was produced when the proof plane had been in contact with the point *a*, and the least by contact with the middle space *e*.

The *electric density* or *electric thickness* is the term used to express the quantity of electricity found at any moment on a given surface. If S represents the surface and Q the quantity of electricity on that surface, then, assuming that the electricity is equally distributed, its electrical density is equal to $\frac{Q}{S}$.

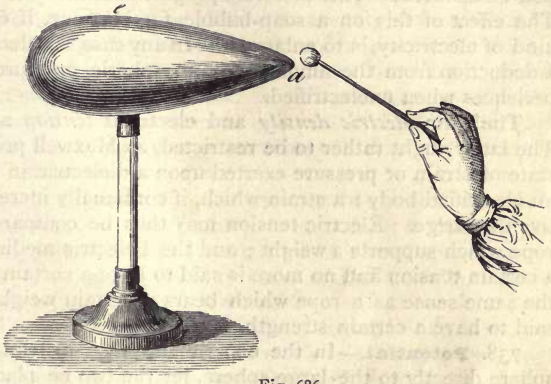


Fig. 686.

Coulomb found by quantitative experiments, that in an ellipsoid the density of the electricity, at the equator of the ellipsoid, is to that at the ends in the same ratio as the length of the minor to the major axis. On an insulated cylinder, terminated by two hemispheres, the density of the electrical layer at the ends is greater than in the middle. In one case, the ratio of the two densities was found to be as 2·3 : 1. On a circular disc the density is greatest at the edges.

737. **Force outside an electrified body.**—The force F which a sphere, charged with a quantity of electricity Q, exerts on a point at a distance *d* from its centre, is $\frac{Q}{d^2}$; this is equal to $\frac{\rho S}{d^2}$ if S is the area of the sphere, and ρ the density of electricity on the unit of surface. Now the area of the sphere is $4\pi^2R$; and if the distance *d* is equal to the radius R, then the force at the surface $S = \frac{4\pi\rho R^2}{R^2} = 4\pi\rho$.

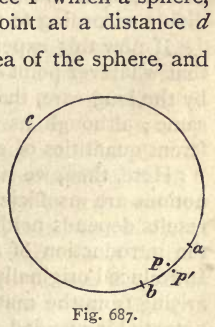


Fig. 687.

This holds also if the point considered is at a very small distance just outside the sphere. Let a small segment *ab* be cut in a sphere (fig. 687). Then its action on a point *p* just inside the sphere will be exactly neutralised by the action of the rest of the sphere *acb* on this point, since there is no electrical force inside a sphere (735); that is, the action of the two portions is equal, but in opposite directions. Now for a point *p'*, just outside the sphere, the actions will also be equal, but in the same directions. But the total action of the whole sphere is $4\pi\rho$: hence the action of each portion is half of this; that is, $2\pi\rho$.

It may be shown in like manner that the whole force of any closed conductor is $4\pi\rho$ per unit area.

On an insulated conductor, where the electricity is in equilibrium, a particle of electricity will have no tendency to move along the surface, for otherwise there would be no equilibrium. But the electricity does exert a pressure on the external non-conducting medium, which is always directed outwards, and is called the *electrostatic tension* or *pressure*. The amount of this pressure is $2\pi\rho^2$ for unit area, ρ being the electrical density at the point considered. It is therefore proportional to the square of the density. The effect of this on a soap-bubble, for instance, if electrified with either kind of electricity, is to enlarge it. In any case the electrification constitutes a deduction from the amount of atmospheric pressure which the body experiences when unelectrified.

The term *electric density* and electrical *tension* are often confounded. The latter ought rather to be restricted, as Maxwell proposed, to express the state of strain or pressure exerted upon a dielectric in the neighbourhood of an electrified body ; a strain which, if continually increased, tends to disruptive discharge. Electric tension may thus be compared to the strain on a rope which supports a weight ; and the dielectric medium which can support a certain tension and no more is said to have a certain *electrical strength* in the same sense as a rope which bears a certain weight without breaking is said to have a certain strength.

738. **Potential.**—In the experiment (736), instead of applying the test sphere directly to the large sphere, let the two be placed at a considerable distance from each other, and let them be connected by a long thin wire and then, detaching the small sphere, let the quantity upon it be measured by the torsion balance : the angle of deflection will show that this quantity is the same whatever part of the large sphere be touched, as must indeed be the case, owing to symmetry ; but the amount of this charge will be materially different from the amount when the small sphere is placed in direct contact with the larger one. Hence the quantity of electricity removed differs according to the mode in which connection is made.

If now this experiment be repeated with the ellipsoid, it will be found that whatever point of this is put in distant connection with the proof sphere by the long wire, the charge which the small sphere acquires is always the same ; although, as we have seen, the proof sphere would remove very different quantities of electricity according to the part where it touches.

Here, then, we are dealing with experimental facts which our previous notions are insufficient to explain. It is manifest that the difference in the results depends neither on the total charge nor on the density. We require the introduction of a new conception, which is that of *electrical potential*. Introduced originally into electrical science by Green, out of considerations arising from the mathematical treatment of the subject, the use of the term potential is justified and recommended by the clearness with which it brings out the relations of electricity to work.

We have already seen, that in order to lift a certain mass against the attraction of gravitation (59–62) there must be a definite expenditure of work, and the equivalent of this work is met with in the energy which the lifted mass retains, or what is called the potential energy of position.

Let us now suppose that we have a large insulated metal sphere charged with positive electricity, and that, at a distance which is very great in comparison with the size of the sphere, there is a small insulated sphere charged with the same kind of electricity. If now we move the small sphere to any given point nearer the larger one, we must do a certain amount of work upon it to overcome the repulsion of the two electricities.

The work required to be done against electrical forces, in order to move the unit of positive electricity from an infinite distance to a given point in the neighbourhood of an electrified conductor, is called the *potential* at this point. If, in the above case, the larger sphere were charged with negative electricity, then instead of its being needful to do work in order to bring a unit of positive electricity towards it, work would be done by electrical attraction, and the potential of the point near the charged sphere would thus be negative. The potential at any point may also be said to be the work done against electrical force, in moving unit charge of negative electricity from that point to an infinite distance.

The amount of work required to move the unit of positive electricity against electrical force, from any one position to any other, is equal to the excess of the electrical potential of the second position over the electrical potential of the first. This is, in effect, the same as has been said above, for at an infinite distance the potential is zero.

We cannot speak of potential in the abstract, any more than we can speak of any particular height, without at least some tacit reference to a standard of level. Thus, if we say that such and such a place is 300 feet high, we usually imply that this height is measured in reference to the level of the sea. So, too, we refer the longitude of a place to some definite meridian, such as that of Greenwich, either expressly or by implication.

In like manner we cannot speak of the potential of a mass of electricity without, at least, an implied reference to a standard of potential. This standard is usually the earth, which is taken as being at zero potential. If we speak of the potential at a given point, the difference between the potential at this point and the earth is referred to.

If, in the imaginary experiment described above, we move the small sphere round the large electrified one always at the same distance, no work is done by or against it for the purpose of overcoming or of yielding to electrical attraction or repulsion, just as if we move a body at a certain constant level above the earth's surface, no work is done upon it as respects gravitation. An imaginary surface drawn in the neighbourhood of an electrified body, such that a given charge of electricity can be moved from any one point of it to any other without any work being done either by or against electrical force, is said to be an *equipotential surface*. Such a surface may be described as having everywhere the *same electrical level*; and the notion of bodies at different electrical levels, in reference to a particular standard, is analogous to that of bodies at different potentials. In the case of an insulated electrified sphere the successive equipotential surfaces would be successive shells of gradually increasing radii, like the coats of an onion. The space about an electrified body or electrified system is called *the electrical field*. The fall of potential from one equipotential surface to another is most rapid in the direction of the perpendiculars to the two surfaces.

These perpendiculars represent the lines of electrical force, the 'lines of force' of Faraday, or the 'lines of induction' of Maxwell. On the surface of an insulated electrified sphere at a distance from other conductors, these lines of force are perpendicular to the surface of the sphere.

The lines of electrical force may be made visible in the dark by placing two small balls at a distance from each other in conducting communication with an electrical machine at work, and then sifting lycopodium powder through a fine sieve while the space is at the same time illuminated by the lime or the electric light.

As water only flows from places at a higher level to places at a lower level, so also electricity only passes from places at a higher to places at a lower potential. If an electrified body is placed in conducting communication with the earth, electricity will flow from the body to the earth, if the body is at a higher potential than the earth; and from the earth to the body, if the body is at a lower potential, and its flow will be proportional to the difference of potential. If the potential of a body is higher than that of the earth, it is said to have a positive potential; and if at a lower potential, a negative potential. A body charged with *free negative electricity* is one at a *lower* potential than the earth; one charged with *free positive electricity* is at a *higher* potential.

739. **Electrical capacity.**—The capacity of any conductor may be measured by the quantity of electricity which it can acquire when placed in contact with a body which charges it to unit electrical potential.

We may illustrate the relation between capacity and potential by reference to the analogous phenomenon of heat. In the interchange of heat between bodies of different temperatures the final result is that heat only passes from bodies of higher to bodies of lower temperature. So also electricity only passes from bodies of higher to bodies of lower potential. Potential is as regards electricity what *temperature* is as regards heat, and might indeed be called *electrical temperature.* We may have a small quantity of heat at a very high temperature. Thus a short thin wire heated to incandescence has a far higher heat potential, or temperature, than a bucket of warm water. But the latter will have a far larger quantity. A flash of lightning represents electricity at a very high potential, but the quantity is small.

The relation between electrical potential and density may be further illustrated by reference to the head of water in a reservoir. The pressure is proportional to the depth; the potential is everywhere the same. For suppose we want to introduce an additional pound of water into the reservoir, the same amount of work is required whether the water be forced in at the bottom or be poured in at the top.

If a hole be made very near the top of the reservoir, a quantity of water in falling to the ground would generate an amount of heat proportional to the fall. If the same quantity escaped through a hole near the bottom, it would not produce so much heat by direct fall; but it will possess a certain velocity, the destruction of which will produce a quantity of heat which, added to that produced by the fall, will give exactly as much as the other.

When the charge or quantity of electricity imparted to a body increases, the potential increases in the same ratio; so that, calling Q the quantity of

electricity, C the capacity, and V the potential, we have $Q = CV$; that is to say, the charge, or quantity of electricity, that any body possesses, is the product of the potential into the capacity.

Now for a sphere whose radius is R the potential $V = \frac{Q}{R}$, from which we get $C = R$; that is, the *capacity of a sphere is equal to its radius.*

While there is a close analogy between heat and electricity, as regards capacity, there are important differences ; thus the capacity of a body for heat is influenced by the temperature (457), being greater at higher temperatures, while the capacity of a body for electricity does not depend on the potential. Again, the calorific capacity depends solely on the mass of a body, and in bodies of the same material and shape is proportional to the cube of homologous dimensions ; the capacity for electricity is directly proportional to such dimensions, and not to the weight or volume. Calorific capacity is proportional to a specific coefficient, which varies with the material, but is independent of its shape ; while electrical capacity varies with the shape of a body, but not with its material, provided the electricity can move freely upon it. Calorific capacity is unaffected by the proximity of other bodies, while the electrical capacity depends on the position and shape of all the adjacent conductors.

If we have a series of bodies at a considerable distance from each other, whose capacities and potentials are respectively c, c', c'', &c., and v, v', v'', &c., then, if they are all connected by fine wires of no capacity, they all instantly acquire the same potential V, which is determined by the equation

$$V = \frac{cv + c'v' + c''v''}{c + c' + c''}.$$

The analogy of this to the equalisation of temperature which takes place when bodies at different temperatures are mixed together is directly apparent (449). It may be further illustrated by supposing a series of tubes of different diameters, and connected by very narrow tubes, but in which are stopcocks to cut off communication. If, while in this state, water be poured into the tubes to different heights, it will be manifest that they will hold very various quantities of water. If, however, the stopcocks are opened, the tubes will still contain quantities of water proportional to their capacities, but the level or potential in all will be the same.

740. **Measurement of capacity and potential.**—We may use Coulomb's balance for the purpose of measuring the capacity C, or the potential V, of a body charged with electricity. For this purpose the body in question is placed, by means of a long fine wire of no capacity, in distant contact with a small neutral insulated sphere of known radius r. This small sphere is then applied to the torsion balance, and its charge $q = rv$ is measured. Now since the original charge on the sphere is $Q = CV$, after contact with the small sphere, which is neutral, the system will have a new potential or electrical level, v, such that $CV = (C + r)\, v$. Restoring now the small sphere to the neutral state, and repeating the experiment and the measurement, we shall then get a second value rv', from which we have the equation $Cv = (C + r)\, v'$. Combining and reducing, we get the ratio $V = \frac{v^2}{v'}$, which,

seeing that rv and rv' are numerical values, leads directly to the desired result. In like manner it is easy to determine the capacity by obvious transformations of these equations.

It will thus be seen that this process of determining potential is analogous to that of determining temperature by means of a thermometer; and the proof sphere plays the part, as it were, of an *electrical thermometer*. It may be observed that in the case of heat we pass from the conception of *temperature* to that of *quantity* of heat, while with electricity, starting with the fact of quantity, or charge of electricity, we arrive at the conception of potential of electricity.

741. **Potential of a sphere.**—If q, q', and q'' are any masses of electricity on the surface of an insulated conducting sphere, and d, d', and d'' their respective distances from any point of the interior of the sphere, then $\frac{q}{d}, \frac{q'}{d'}$ and $\frac{q''}{d''}$ are the values of the potentials v, v', and v'' which they would severally produce at this point. Let the point in question be the centre, and let Q be the sum of all the quantities; then V, the potential of the sphere, equals $\frac{Q}{R}$, R being the radius.

If there be a sphere, or uniform spheroidal shell of matter, which acts according to the inverse square of the distance, the total action of this sphere is the same as if the whole matter were concentrated at the centre. This was first proved by Newton in the case of gravitation; but it also applies to electricity, and hence, in calculating the potential at any point outside a sphere possessing a uniform charge, we need only consider its distance from the centre, and for such a case we may write the value of the potential $V = \frac{Q}{d}$.

If a charge of electricity, Q, be imparted to two insulated conducting spheres whose radii are respectively r and r', and which are connected by a long fine wire, the capacity of which may be neglected, the electricity will distribute itself over the two spheres, which will possess the charges q and q'; that is, $q + q' = Q$. (1) The whole system will be at the same potential V, such that $V = \frac{q}{r} = \frac{q'}{r'}$. (2) Combining these two equations and reducing, we get for the quantities q and q' on each sphere $q = \frac{Qr}{r + r'}$ and $q' = \frac{Qr'}{r + r'}$. Now, since the diameter of any sphere with which we can experiment is infinitely small compared with that of the earth, it follows that when a sphere is connected with the earth by a fine wire the quantity of electricity which it retains is infinitely small.

The densities on the two spheres are $d = \frac{q}{4\pi r^2}$ and $d' = \frac{q'}{4\pi r'^2}$, from which by equation (2) it is readily deduced that $d : d' = r' : r$; that is, the electrical densities on two spheres in distant connection are inversely as the radii. If, for instance, a fine wire be connected with a charged insulated sphere, the distant pointed end of the wire may be regarded as a sphere

with an infinitely small radius, and thus the density upon it would be infinitely great.

742. **Action of points.**—We have just seen that the density on a point in connection with a conductor charged with electricity may be considered to be infinitely great, but the greater the density the greater will be the tendency of electricity to overcome the resistance of the air, and escape, for the electrostatic pressure is proportional to the square of the density (737). If the hand be brought near a point on an electrified conductor a slight wind is felt; and if the disengagement of electricity takes place in the dark a luminous brush is seen. If an electrified conductor is to retain its electricity all sharp points and edges must be avoided; on the other hand, to facilitate the outflow of electricity in apparatus and experiments (764), frequent use is made of this action of points. A flame acts like a very fine point in diffusing electricity.

743. **Loss of electricity.**—Experience shows that electrified bodies gradually lose their electricity, even when placed on insulating supports. This loss is mainly due to the insulating supports. The charge is gradually dissipated in consequence of the electricity either passing through the supports or creeping over the surface. All substances conduct electricity in some degree; those which are termed insulators are simply very bad conductors. An electrified conductor resting on supports must therefore lose a certain quantity of electricity—either by penetration into its mass or along the surface. This loss of electricity is a main cause of difficulty in experiments on the quantitative laws of electricity; it varies with the electric density, and increases with the hygrometric state of the air, though it does not seem that the loss from this cause is due to a direct conductivity by moist air. Lord Kelvin ascribes the greater part of the loss to the conducting layer of moisture which covers the supports; and he finds that, in comparison with this, the direct loss by even moist air is inconsiderable.

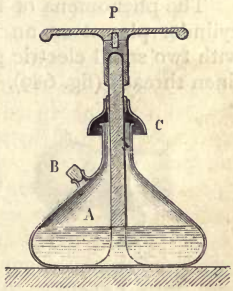


Fig. 688.

Brown shellac or ebonite is the best insulator; glass is a hygroscopic substance, and must be dried with great care. It is best covered with a thin layer of shellac varnish, as has already been stated.

Mascart's insulator (fig. 688) is admirably adapted for supporting bodies charged with electricity. It consists of a glass vessel of special shape (fig. 688), to the glass vase of which is fused the stem. This passes through the neck and supports the plate, P; the neck is closed by an ebonite stopper, and inside the vessel is sulphuric acid, so that the stem A is always dry.

CHAPTER III

ACTION OF ELECTRIFIED BODIES ON BODIES IN THE NATURAL STATE. INDUCED ELECTRICITY. ELECTRICAL MACHINES

744. **Electrical influence or induction.**—An insulated conductor, charged with either kind of electricity, acts on bodies in a neutral state placed near it in a manner analogous to that of the action of a magnet on soft iron; that is, it decomposes the neutral electricity, attracting the opposite and repelling the like kind of electricity. The action thus exerted is said to take place by *influence* or *induction*.

The phenomena of induction may be demonstrated by means of a brass cylinder placed on an insulating support, and provided at its extremities with two small electric pendulums, which consist of pith balls suspended by linen threads (fig. 689). If this apparatus is placed near an insulated con-

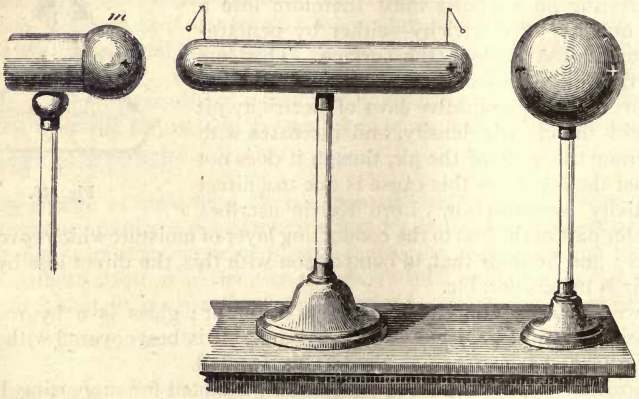


Fig. 689.

ductor *m*, charged with either kind of electricity—for instance, the conductor of an electrical machine, which is charged with positive electricity—the natural electricity of the cylinder is decomposed, free electricity will be developed at each end, and both pendulums will diverge. If, while they still diverge, a stick of sealing-wax, excited by friction with flannel, be approached to that end of the cylinder nearest the conductor, the corresponding pith ball will be repelled, indicating that it is charged with the same kind of electricity as the sealing-wax—that is, with negative electricity; while if the excited sealing-wax is brought near the other ball it will be attracted, showing that it is charged with positive electricity. If, further, a glass rod excited

by friction with silk, and therefore charged with positive electricity, be approached to the end nearest the conductor, the pendulum will be attracted; while if brought near the other end, the corresponding pendulum will be repelled. If the influence of the charged conductor be suppressed, either by removing it or placing it in communication with the ground, the separated electricities will recombine, and the pendulums exhibit no divergence.

The cause of this phenomenon is obviously a decomposition of the neutral electricity of the cylinder, by the free positive electricity of the conductor; the opposite or negative electricity being attracted to that end of the cylinder nearest the conductor, while the similar electricity is repelled to the other end. Between these two extremities there is a space destitute of free electricity. This is seen by arranging on the cylinders a series of pairs of pith balls suspended by threads. The divergence is greatest at each extremity, and there is a line at which there is no divergence at all, which is called the *neutral* line. The two electricities, although equal in quantity, are not distributed over the cylinder in a symmetrical manner; the attraction which accumulates the negative electricity at one end is, in consequence of the greater nearness, greater than the repulsion which drives the positive electricity to the other end, and hence the neutral line is nearer one end than the other. Nor is the electricity induced at the two ends of the cylinder under the same conditions. That which is repelled to the distant extremity is free to escape if a communication be made with the ground; whilst, on the other hand, the unlike electricity which is attracted is held bound or captive by the inducing action of the electrified body. Even if contact be made with the ground on the face of the cylinder adjacent to the inducing body, the electricity induced on that face will not escape. The repelled electricity, however, on the distant surface is not thus bound; it is free to escape by any conducting channel, and hence will immediately disappear wherever contact be made between the ground and the cylinder. Both the pith balls will collapse, and all signs of electricity on the cylinder depart with the escape of the repelled or free electricity. But now, if communication with the ground be broken, and the inducing body be discharged or removed to a considerable distance, the attracted or bound electricity is itself set free, and diffusing over the whole cylinder causes the pith balls again to diverge, but now with the opposite electricity to that of the original inducing body. The reason for the escape of the repelled electricity is as follows:—If the cylinder be placed in connection with the ground, by metallic contact with the posterior extremity, and the charged conductor be still placed near the anterior extremity, the conductor will exert its inductive action as before. But it is now no longer the cylinder alone which is influenced. It is a conductor consisting of the cylinder itself, the wire, and the whole earth. The neutral line will recede indefinitely, and, since the conductor has become infinite, the quantity of neutral fluid decomposed will be increased. Hence, when the posterior extremity is placed in contact with the ground, the pendulum at the anterior extremity diverges more widely. If the connecting-rod be now removed, neither the quantity nor the distribution will be altered; and if the conductor be removed or be discharged, a charge of negative electricity will be left on the cylinder. It will, in fact, remain charged with electricity, the opposite of that of the charged conductor. Even

if, instead of connecting the posterior extremity of the cylinder with the ground, any other part had been so connected, the general result would have been the same. All the parts of the cylinder would be charged with negative electricity, and, on breaking the connection with the earth, would remain so charged.

Thus a body can be charged with electricity by induction as well as by conduction. But, in the latter case, the charging body loses part of its electricity, which remains unchanged in the former case. The electricity imparted by conduction is of the same kind as that of the electrified body, while that excited by induction is of the opposite kind. To impart electricity by conduction, the body must be quite insulated; while in the case of induction it must be in connection with the earth—at all events momentarily.

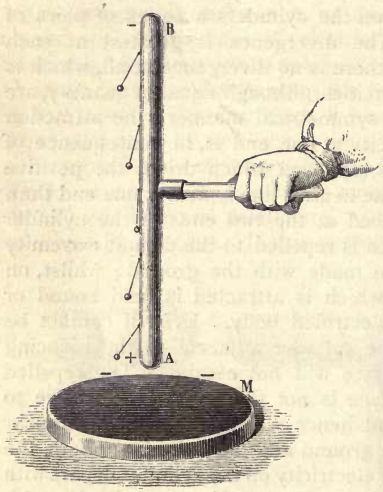


Fig. 690.

A body electrified by induction acts in turn on bodies placed near it, separating the two fluids in a manner shown by the signs on the sphere.

What has here been said has reference to the inductive action exerted on good conductors. Bad conductors are not so easily acted upon by induction, owing to the great resistance they present to the circulation of electricity; but, when once charged, the electric state is more permanent.

This is analogous to what is met with in magnetism; a magnet instantaneously magnetises a piece of soft iron, but this is only temporary, and depends on the continuance of the action of the magnet; a magnet magnetises steel with far greater difficulty, but this magnetisation is permanent.

The fundamental phenomena of induction may be conveniently investigated and demonstrated by means of the apparatus represented in fig. 690, which consists of a narrow cylindrical brass tube BA, supported by an insulating glass handle, and held over the excited cake of an electrophorus (752).

745. **Faraday's experiments.**—The following experiments of Faraday, which are often known as 'the ice-pail experiments,' from the vessels with which they were originally made, are excellent illustrations of the operation of induction, and are of great theoretical importance:—

A carefully insulated metal cylinder, A, fig. 691, is connected by a wire with an electroscope E, at some distance. On slowly placing inside the cylinder an insulated brass ball C, charged with positive electricity, which is small in comparison with the size of the cylinder, the leaves of the electroscope diverge, and, as can be shown, with positive electricity, and the

divergence increases until a certain depth is attained, when there is no further increase. The divergence now remains constant, whatever be the position of the ball, and when the inside and outside are tested with the proof plane they are found to be charged with negative and positive respectively. If the ball is withdrawn the leaves of the electroscope collapse, and there is no electrification on the cylinder; the quantities of negative and positive electricity developed on the two surfaces are accordingly equal to each other.

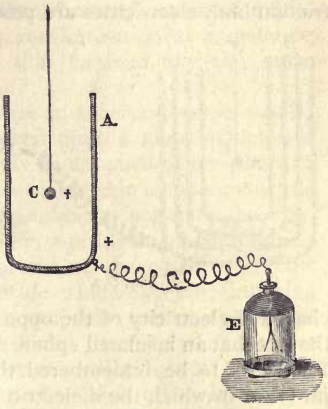


Fig. 691.

If now the ball, while still charged with positive electricity, be brought as before into the cylinder, and be allowed to touch the inside, there is no alteration, not even a momentary one, in the divergence of the leaves of the electroscope; but if the ball be withdrawn it will now be found to be neutral, as is also the inside of the cylinder, while the outside is charged with positive electricity. When the ball touches the interior, the system forms only a single conductor, and all the electricity passes to the outside; but since the charge as indicated by the electroscope does not alter, it follows that the positive of the ball and the negative of the inside of the cylinder are equal to each other.

If while the ball charged with positive electricity is inside the cylinder, the latter is momentarily put to earth, the gold leaves collapse, and the proof plane, if applied to the outside, removes no trace of electricity; the cylinder behaves towards all external bodies as if it were neutral. The internal surface is, however, covered with a layer of negative electricity, and this is equivalent to the positive charge of the ball, for all trace of electricity disappears if the ball is made to touch the side.

If the ball, after the cylinder has been momentarily connected to earth, be removed without having touched the sides, the negative passes to the outside and forms there a layer which is distributed as was the layer of positive electricity before being connected with the ground. The cylinder is thus finally charged with a quantity of electricity equal and of opposite sign to the inducing body.

Four such cylinders (fig. 692) are placed concentrically within each other, and are insulated from each other by discs of shellac, and the outer one is connected with the electroscope. On introducing the charged ball into the central cavity the leaves diverge just as if the intermediate ones did not exist. Each of these is charged with equal quantities of opposite electricities, all equal in value to that of the sphere. The internal charge of the cylinder is the same as if all the intermediate cylinders were suppressed, and the charge does not vary even when the intermediate ones are connected with each other or are touched by the electrified ball C.

If, while C is in its original condition, the internal cylinder, 4, is con-

nected with the ground, the leaves collapse, and the other cylinders are in the neutral state ; the two layers which remain, positive on C, and negative on the adjacent cylinder, are without action on an external point. If any other cylinder be thus treated, the external ones are reduced to the neutral state.

With the aid of the cylinder (fig. 692) it is easy to demonstrate that by friction both electricities are produced at the same time, and in equal quantities. For if the flannel and sealing-wax in fig. 677 after being rubbed are placed simultaneously in the cylinder no divergence is produced, while if each is introduced separately, they produce equal divergence but of opposite sign.

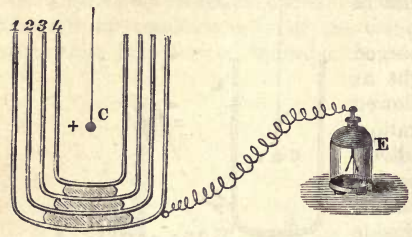


Fig. 692.

Whenever a charge of electricity exists there is somewhere an equivalent and corresponding charge of electricity of the opposite kind. This may seem inconsistent with the fact that an insulated sphere may have a charge of one kind of electricity. But it is to be remembered that this is in effect the case of a Leyden jar (770) in which the dielectric is the layer of air between the sphere and the sides of the room which form the outer coating.

746. **Limit to the action of induction.**—The inductive action which an electrified body exerts on an adjacent body in decomposing its neutral electricity is limited. On the surface of the insulated cylinder, which we have considered in the preceding paragraph, let there be at *n* any small quantity of neutral electricity (fig. 693). The positive electricity of the source *m* first decomposes by induction the neutral electricity in *n*, attracting its negative towards A, and repelling its positive towards B ; but in the degree in which the extremity A becomes charged with negative electricity, and the extremity B with positive electricity, there are developed at A and B two forces, *f* and *f'*, which act in the opposite direction to the original force. For the forces *f* and *f'* concur in driving towards B the negative of *n*, and towards A its positive. But as the inducing force F which is exerted at *m* is constant, while the forces *f* and *f'* are increasing, a time arrives at which the force F is balanced by the forces *f* and *f'*. All decomposition of the neutral condition then ceases ; the inducing action has attained its limit.

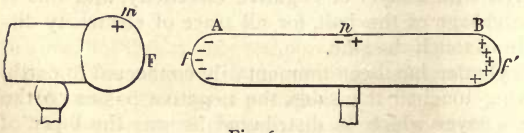


Fig. 693.

If the cylinder be removed from the source of electricity, as the inducing action decreases, a portion of the free electricities at A and B recombine to form the neutral fluid. If, on the other hand, they are brought nearer, as the force F now exceeds the forces *f* and *f'*, a new decomposition of the neutral fluid takes place, and fresh quantities of positive and negative electricities are respectively accumulated at A and B.

747. **Faraday's theory of induction.**—Hitherto any possible influence of the medium which separates the electrified from the unelectrified body in the case of induction has been disregarded. It has been tacitly assumed that electrical actions are exerted at a distance, and the medium has been looked upon as an inert mass through which the forces can act, but which itself is destitute of any active properties. The researches of Faraday, however, prove that this is not the case; that the medium is of fundamental importance, and that the action is not an action at a distance, or at any rate at no greater distance than that between any two molecules.

According to Faraday's views conductors are in a certain sense qualitatively different from non-conductors. He looked upon a non-conductor as consisting of a number of molecules which may be assumed to be spherical, and which are perfect conductors, and are disseminated in a non-conducting medium. The action of an electrified body is either to separate the electricities within the molecule and arrange them in a polar chain, or to impart to the molecules which are themselves polarised at the outset a definite polar arrangement; those ends of the molecule which face the inducing body having electricity of the opposite kind, and those which are turned away from it having electricity of the same kind. In the interior of the medium, where successively the positive end of one molecule faces the negative end of the next, the two electricities neutralise each other as regards electric action; but where the non-conductor is bounded by a conductor the free electrification is no longer neutralised, but constitutes the charge which is perceived. The action is therefore analogous to that of the pole of a magnet on a piece of soft iron; and Faraday called it *dieletric polarisation.* Electrification is accordingly rather to be regarded as a property of the dielectric medium itself than as a property of the conductors by which that medium is bounded.

The following experiment was devised by Faraday to illustrate this *polarisation of the medium*, as he called it. He placed small filaments of silk in a vessel of turpentine (fig. 694), and, having placed two conductors in the liquid on opposite sides, he charged one by connecting it with an electrical machine at work, and placed the other in connection with the ground. The particles of silk immediately arranged themselves end to end, and adhered closely together, forming a continuous chain between the two sides. If the chain is broken it again forms, while when the electrical action ceases the particles disperse. An experiment by Matteucci also supports Faraday's theory. He placed several thin plates of mica closely together, and provided the outside ones with metallic coatings, like a fulminating pane (769). Having electrified the system, the coatings were removed by insulating handles, and on examining the plates of mica successively, each was found charged with positive electricity on one side and negative electricity on the other.

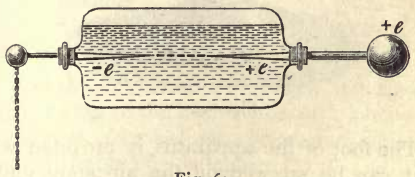


Fig. 694.

748. **Specific inductive capacity.**—Faraday named the property which bodies have of transmitting electrical induction, the *specific inductive capacity*, or, as it is often called, the *inductive power*. If the dielectric does play the essential part in the phenomena of induction it is not likely that all insulating bodies possess it in the same degree. This seems to have been known to Cavendish. To examine and compare the inductive power Faraday used the apparatus represented in fig. 695, and of which fig. 696 represents a vertical section. It consists of a brass sphere made up of two halves, P and Q, which fit accurately into each other, like the Magdeburg hemispheres. In the interior of this spherical envelope there is a smaller brass sphere C, connected with a metal rod, terminating in a ball B. The rod is insulated from the envelope PQ by a thick layer of shellac A. The space *mn* receives the substance whose inductive power is to be determined.

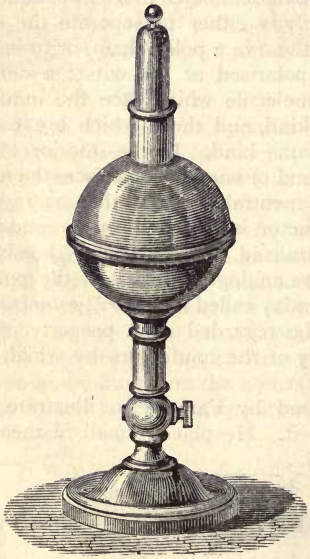


Fig. 695.

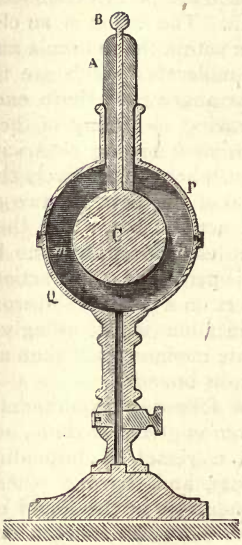


Fig. 696.

The foot of the apparatus is provided with a screw and stopcock, so that it can be screwed on the air-pump, and the air in *mn* either rarefied or exhausted.

Two such apparatus perfectly identical are used, and at first they only contain air. The envelopes PQ are connected with the ground, and the knob B of one of them receives a charge of electricity. The sphere C thus becomes charged like the inner coating of a Leyden jar (770). The layer *mn* represents the insulator which separates the two coatings. By touching B with the proof plane, which is then applied to the torsion balance, the quantity of free electricity is measured. In one experiment Faraday observed a torsion of 250°, which represented the free electricity on B. The knob B was then placed in metallic connection with the knob B′ of the

other apparatus, and the torsion was now found to be 125°, showing that the electricity had become equally distributed on the two spheres, as might have been anticipated, since the pieces of apparatus were quite equal, and each contained air in the space *mn*.

This experiment having been made, the space *mn* in the second apparatus was filled with the substance whose inductive power was to be determined: for example, shellac. The other apparatus, in which *mn* is filled with air, having been charged, the density of the free electricity on C was measured. Let it be taken at 290°, the number observed by Faraday in a special case. When the knob B of the first apparatus was connected with the knob B′ of the second, the density was not found to be 145°, as would be expected. The apparatus containing air exhibited a density of 114°, and that with shellac of 113°. Hence the former had lost 176°, and had retained 114°, while the latter ought to have exhibited a density of 176° instead of 113°. The second apparatus had taken more than half the charge, and hence a larger quantity of electricity had been condensed by the shellac. Of the total quantity of electricity, the shellac had taken 176° and the air 114°; hence the specific inductive capacity of air is to that of shellac as 114 : 176; or as 1 : 1·55. That is, the inductive power of shellac is more than half as great again as air.

By the following simple experiment the influence of the dielectric may be shown:—At a fixed distance above a gold-leaf electroscope let an electrified sphere be placed, by which a certain divergence of the leaves is produced. If, now, the charges remaining the same, a disc of sulphur or of shellac be interposed, the divergence increases, showing that inductive action takes place through the sulphur to a greater extent than through a layer of air of the same thickness.

The following are the mean values which have been obtained by various improved methods for the specific inductive capacity of *dielectrics*, as they are called, in opposition to *anelectrics*, or conductors:—

Air	1·00	Shellac	3·04
Paraffine	2·02	Ebonite	3·42
India-rubber	2·22	Sulphur	3·97
Gutta-percha	2·46	Glass	5 to 6

These values are known as the *dielectric constants*; and their determination presents considerable difficulty, owing to the occurrence of a phenomenon to which Faraday gave the name of *electrical absorption*, and which is due to the same cause as the residual charge of condensers (773).

A condenser with a glass plate would thus have five or six times the capacity of an air condenser of the same dimensions, or the same capacity as an air condenser of the same surface, but five or six times as thin.

Boltzmann divides dielectrics into two classes: to one of which belong shellac, paraffine, sulphur and resin, which act like perfect insulators; that is, in using them the maximum charge is attained, if not instantaneously, at all events after a very short time: in others, such as gutta-percha, stearine, and glass, the charge increases appreciably with the time.

A very interesting relation exists between the dielectric constant, k, and

the refractive index, n, of certain substances. Thus the following numbers have been found :—

	k	$\sqrt{k}$	n
Sulphur	3·84	1·96	2·04
Resin	2·55	1·59	1·54
Paraffine	2·32	1·52	1·53
Oil of Turpentine . .	2·23	1·49	1·47

where n is the refractive index (550), and $\sqrt{k}$ the square root of the dielectric constant.

In crystallised bodies the dielectric constant varies with the direction of the axes. Thus with a native crystal of sulphur Boltzmann found the values 4·77, 3·97, and 3·81 for the direction of the longest, mean, and shortest axes respectively.

Hopkinson found the following numbers for the dielectric constants of certain liquids. Petroleum 2·10, oil of turpentine 2·23, olive oil 3·16, and castor oil 4·78.

Faraday was not able to detect any difference in the dielectric constants of various gases. Boltzmann has shown, however, that there are differences among them, and that there is a very close agreement between the square root of their dielectric constants and their refractive indices, thus :—

	k	$\sqrt{k}$	n
Vacuum . . .	1·00000	1·00000	1·0000
Air	1·00059	1·000295	1·000294
Carbonic acid . .	1·00095	1·000473	1·000449
Hydrogen . . .	1·00026	1·000132	1·000138
Olefiant gas . .	1·00131	1·000656	1·000678

The accurate determination of the dielectric constant is attended with great experimental difficulties ; it is a matter of great theoretical importance, especially from its bearing on Maxwell's electro-magnetic theory of light. According to this theory, the medium in which both electrical and luminous actions are transmitted is the same, and is in fact the luminiferous ether (637), and it is a necessary consequence of this theory that the above relation must exist between the refractive index of a substance and its dielectric constant.

749. **Communication of electricity at a distance.**—In the experiment represented in fig. 689 the opposite electricities of the conductor and the separated cylinder tend to unite, but are prevented by the resistance of the air. If the density is increased, or if the distance of the bodies be diminished, the opposed electricities at length overcome this obstacle ; they rush together and combine, producing a spark, accompanied by a sharp sound. The negative electricity separated on the cylinder being thus neutralised by the positive electricity of the charged body, a charge of positive electricity remains on the cylinder. The same phenomenon is observed when a finger is presented to a strongly electrified conductor. The latter decomposes by induction the neutral electricity of the body, the opposite electricities combine with the production of a spark, while the electricity of the same kind as the electrified conductor, which is left on the body, passes off into the ground.

The striking distance varies with the density, the shape of the bodies, their conducting power, and with the resistance and pressure of the interposed medium.

750. **Motion of electrified bodies.**—The various phenomena of attraction and repulsion, which are among the most frequent manifestations of electrical action, may all be explained by reference to the laws of induction. If M (fig. 697) be a fixed insulated conductor charged with positive electricity, and N be a movable insulated body—for instance, an electrical pendulum—there are three cases to be considered :—

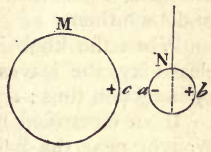


Fig. 697.

i. *The movable body is unelectrified and is a conductor.*—In this case M, acting inductively on N, attracts the negative and repels the positive electricity, so that the maxima of density are respectively at the points *a* and *b*. Now *a* is nearer *c* than *b* is ; and, since attractions and repulsions are inversely as the square of the distance, the attraction between *a* and *c* is greater than the repulsion between *b* and *c* ; and, therefore, N will be attracted to M by a force equal to the excess of the attractive over the repulsive force.

ii. *The movable body is a conductor and is electrified.*—If the electricity of the movable body is different from that of the fixed body, there is always attraction ; but if they are of the same kind, there is at first repulsion and afterwards attraction. This anomaly may be thus explained : Besides its charge of electricity, the neutral electricity is decomposed by the induction of the positive electricity on M ; and consequently the hemisphere *b* obtains an additional supply of positive electricity, while *a* becomes charged with negative electricity. There is thus attraction and repulsion, as in the foregoing case. The force of repulsion is at first greater, because the quantity of positive electricity on N is greater than that of negative ; but as the distance *ac* diminishes, the attractive force increases more rapidly than the repulsive force, and finally exceeds it.

iii. *The movable body is a bad conductor.*—If N is charged, repulsion or attraction takes place, according as the electricity is of the same or opposite kind to that of the fixed body. If it is in the natural state, the body M will decompose the neutral electricity of N, and attraction will take place as in the first case, since a powerful and permanent source of electricity can more or less decompose the neutral electricity even of bad conductors.

751. **Gold-leaf electroscope.**—The name *electroscope* is given to instruments for detecting the presence and determining the kind of electricity in any body. The original pith-ball pendulum is an electroscope ; but, though sometimes convenient, it is not sufficiently delicate. Many successive improvements have been made in it, and have resulted in the form used, which is due to Bennett.

Bennett's, or the gold-leaf electroscope.—This consists of a tubulated glass shade B (fig. 698), standing on a metal foot, which thus communicates with the ground. A metal rod terminating at its upper extremity in a knob C, and holding at its lower end two narrow strips of gold-leaf, *n n*, fits in the tubulure of the shade, the neck of which is coated with an insulating

varnish. The air in the interior is dried by quicklime, or by chloride of calcium, and on the insides of the shade there are two strips of gold-leaf *a*, communicating with the ground. These, being charged by induction with the opposite electricity to that of the gold leaves, increase the divergence, and therefore the delicacy of the apparatus. They also prevent the leaves when diverging too suddenly from adhering to the sides, from which it is difficult to detach them.

When the knob is touched with a body charged with either kind of electricity, the leaves diverge; usually, however, the apparatus is charged by induction thus :—

If an electrified body—a stick of rubbed sealing-wax, for example—be brought near the knob, it will decompose the neutral electricity of the system, attracting to the knob the electricity of the opposite kind, and retaining it there, and repelling the electricity of the same kind to the gold leaves, which consequently diverge. In this way the presence of an electrical charge is ascertained, but not its quality.

To ascertain the *kind* of electricity the following method is pursued :—If, while the instrument is under the influence of the body A, which we will suppose has a negative charge, the knob be touched by the finger, the negative electricity produced by induction passes off into the ground, and the previously divergent leaves will collapse; there only remains positive electricity, retained in the knob by induction from A. If now the finger be first removed, and then the electrified body the positive electricity previously retained by A will spread over the system, and cause the leaves to diverge. If now, while the system is charged with positive electricity, a positively electrified body—as, for example, an excited glass rod—be approached, the leaves will diverge more widely; for the electricity of the same kind will be repelled to the ends. If, on the contrary, an excited shellac rod be presented, the leaves will tend to collapse, the electricity with which they are charged being attracted by the opposite electricity. Hence we may ascertain the kind of electricity, either by imparting to the electroscope electricity from the body under examination, and then bringing near it a rod charged with positive or negative electricity; or the electroscope may be charged with a known kind of electricity, and the electrified body in question brought near the electroscope.

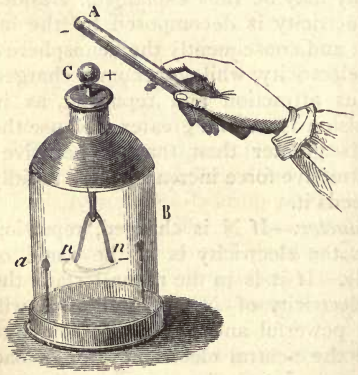


Fig. 698.

The gold-leaf electroscope is sometimes used as an *electrometer*, or measurer of electricity, by measuring the angle of divergence of the leaves; this is done by placing behind them a graduated scale; for small angles the quantity of electricity is nearly proportional to the sine of half the angle of divergence.

ELECTRICAL MACHINES

752. **Electrophorus.**—It will now be convenient to describe the various electrical machines, or apparatus for generating and collecting large quantities of statical electricity. One of the most simple and inexpensive of these is the *electrophorus*, which was invented by Volta. It consists of a *cake* of resin, B (fig. 700), say about 12 inches in diameter, and an inch thick, which is placed on a metal surface, or frequently fits into a wooden mould lined with tinfoil, which is called the *form*. Besides this there is a metal disc A (fig. 700), of a diameter somewhat less than that of the cake, and provided with an insulating glass handle; this is the *cover*. The mode of working is as follows: All the parts of the apparatus having been well dried, the cake, which is placed in the form, or rests on a metal surface, is briskly flapped with silk, or, better, with catskin, by which it becomes charged with negative electricity. The cover is then placed on the cake. Owing, how-

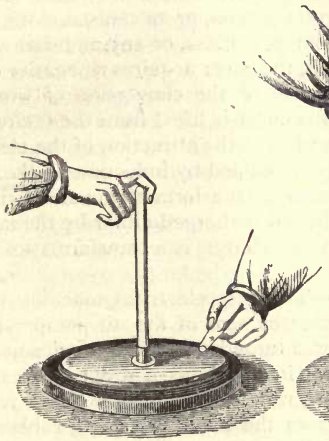


Fig. 699.

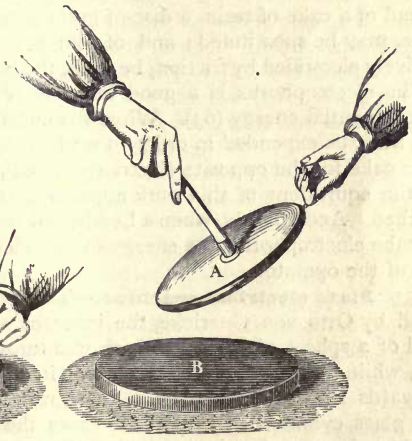


Fig. 700.

ever, to the minute rugosities of the surface of the resin, the cover only comes in contact with a few points, and, from the non-conductivity of the resin, the negative electricity of the cake does not pass off to the cover. On the contrary, it acts by induction on the neutral electricity of the cover, and decomposes it, attracting the positive electricity to the under surface, and repelling the negative electricity to the upper. If the upper surface be now touched with the finger, the negative electricity passes off, and the cover remains charged with positive electricity, held, however, by the negative electricity of the cake; the two electricities do not unite, in consequence of the non-conductivity of the cake (fig. 699). If now the cover be raised by its insulating handle, the charge diffuses itself over the surface; and if a conductor be brought near it (fig.700), a smart spark passes.

The metal form on which the cake rests plays an important part in the action of the electrophorus, as it increases the quantity of electricity, and

makes it more permanent. For the negative electricity of the upper surface of the resin, acting inductively on the neutral electricity of the lower, decomposes it, retaining on the under surface the positive electricity, while the negative electricity passes off into the ground. The positive electricity thus developed on the under surface reacts on the negative electricity of the upper surface, binding it, and causing it to penetrate into the badly conducting mass, on the surface of which fresh quantities of electricity can be excited far beyond the limits possible without the action of the form. For this reason the electrophorus, once charged, retains its state for a considerable time, and sparks can be taken even after a long interval. If the form be insulated, the charge obtained from it is far less than if it is on a conducting support. For the negative electricity developed by induction on the lower surface being now unable to escape, the condensing action referred to cannot take place, and only a feeble charge can be given to the resin. The retention of electricity is greatly promoted by keeping the cake on the form, and placing the cover upon it, by which the access of air is hindered. Instead of a cake of resin, a disc of gutta-percha, or vulcanised cloth, or vulcanite, may be substituted; and, of course, if glass, or any material which is positively electrified by friction, be used, the cover acquires a negative charge.

The electrophorus is a good instance of the conversion of work into electropotential energy (63). When the cover is lifted from the excited cake work must be expended in order to overcome the attraction of the electricity in the cake for the opposite electricity developed by induction on the cover; and the equivalent of this work appears in the form of the electricity thus detached. Accordingly, when a Leyden jar is charged either by the machine or by the electrophorus, the energy of the charge is a transformation of the work of the operator.

753. **Plate electrical machine.**—The first electrical machine was invented by Otto von Guericke, the inventor also of the air-pump. It consisted of a sphere of sulphur, which was turned on an axis by means of the hand, while the other, pressing against it, served as a rubber. Resin was afterwards substituted for the sulphur, which, in turn, Hawksbee replaced by a glass cylinder. In all these cases the hand served as rubber; and Winckler, in 1740, first introduced cushions of horsehair, covered with silk, as rubbers. At the same time Bose collected electricity, disengaged by friction, on an insulated cylinder of tin plate. Lastly, Ramsden, in 1760, replaced the glass cylinder by a circular glass plate, which was rubbed by cushions. The form which the machine has now is but a modification of Ramsden's original machine.

Between two wooden supports (fig. 701) a circular glass plate P is suspended by an axis passing through the centre, and which is turned by means of a handle M. The plate revolves between two sets of *cushions* or *rubbers*, F, of leather or of silk, one set above the axis and one below, which, by means of screws, can be pressed as tightly against the glass as may be desired. The plate also passes between two brass rods, shaped like a horseshoe, and provided with a series of points on the sides opposite the glass; these rods are fixed to larger metallic cylinders C C, which are called the *prime conductors*. The latter are insulated by being supported on glass feet, and are connected with each other by a smaller rod *r*.

The action of the machine is thus explained. By friction with the rubbers the glass becomes positively and the rubbers negatively electrified. If now the rubbers were insulated, they would receive a certain charge of negative electricity which it would be impossible to exceed, for the tendency of the opposed electricities to reunite would be equal to the power of the friction to decompose the neutral state. But the rubbers communicate with the ground by means of a chain ; and, consequently, as fast as the negative electricity is generated, it is continually reduced to zero by contact with the ground. The positive electricity of the glass acts then by induction on the conductor, attracting the negative electricity. This negative electricity collects on the points opposite to the glass. Here its tendency to discharge becomes so high that it passes across the intervening space of air, and neutralises the positive electricity on the glass. The conductors thus lose their negative electricity and remain charged with positive electricity. The plate accordingly gives up nothing to the prime conductors ; in fact, it only abstracts from them their negative electricity.

If the hand be brought near the conductor when charged, a spark follows, which is renewed as the machine is turned. In this case the positive electricity decomposes the neutral electricity of the body, attracting its negative electricity, and combining with it when the two have a sufficient tension. Thus, with each spark, the conductor reverts to the neutral state, but becomes again electrified as the plate is turned.

754. **Precautions in reference to the machine.**—The glass, of which the plate is made, must be as little hygroscopic as possible. Of late ebonite has been frequently substituted for glass ; it has the advantage of being neither hygroscopic nor fragile, and of readily becoming electrified by friction. It cannot, however, be relied on, as its surface in time undergoes a change, especially if exposed to the light, whereby it becomes a conductor. The plate is usually from $\frac{1}{12}$ to $\frac{1}{5}$ of an inch in thickness, and from 20 to 30 inches in diameter, though these dimensions are not unfrequently exceeded.

The rubbers require great care, both in their construction and their preservation. They are commonly made of leather, stuffed with horsehair. Before use they are coated either with powdered *aurum musivum* (sulphuret of tin), graphite, or amalgam. The action of these substances is not very clearly understood. Some consider that it merely consists in promoting friction. Others, again, believe that a chemical action is produced, and assign in support of this view the peculiar smell noticed near the rubbers when the machine is worked. Amalgams, perhaps, promote most powerfully the disengagement of electricity. *Kienmayer's amalgam* is the best of them. It is prepared as follows : One part of zinc and one part of tin are melted together and removed from the fire, and two parts of mercury stirred in. The mass is transferred to a wooden box containing some chalk, and then well shaken. The amalgam, before it is cold, is powdered in an iron mortar, and preserved in a stoppered glass vessel. For use a little cacao butter or lard is spread over the cushion, some of the powdered amalgam sprinkled over it, and the surface smoothed by a ball of flattened leather.

In order to avoid a loss of electricity, two quadrant-shaped pieces of oiled silk are fixed to the rubbers, so as to cover the plate on both sides : one at the upper part from *a* to F, and the other in the corresponding part

of the lower rubbers. These flaps are not represented in the figure. Yellow oiled silk is the best, and there must be perfect contact between the plate and the cloth.

Ramsden's machine, as represented in fig. 701, only gives positive electricity. But it may be arranged so as to give negative electricity by placing it on a table with insulating supports. The conductor is connected with the ground by a chain, and the machine worked as before. The positive electricity passes off by the chain into the ground, while the negative electricity

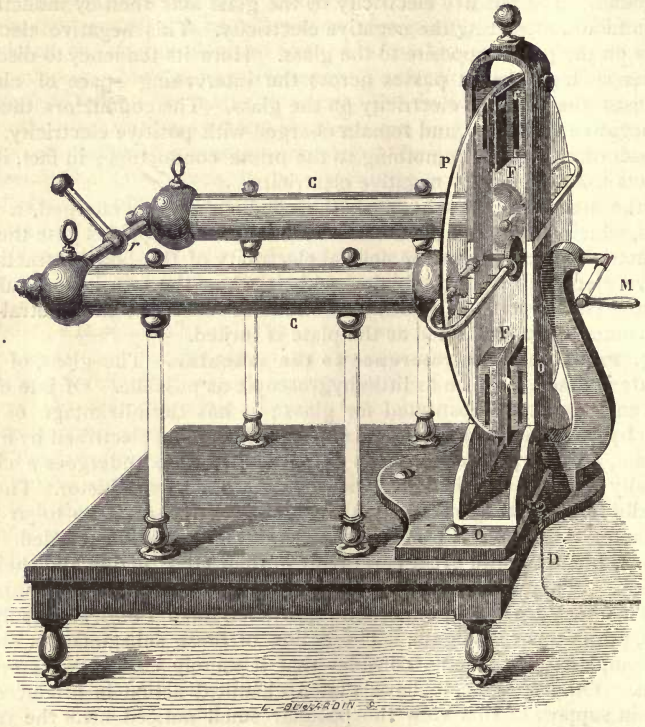


Fig. 701.

remains on the supports and on the insulated table. On bringing the finger near the uprights, a sharper spark than the ordinary one is obtained.

Winter compared the working of an electrical machine directly with the indications of a hygrometer, and found that the length of the spark obtainable is inversely as the hygrometric state.

755. **Maximum of charge.**—It is impossible to exceed a certain limit of electrical charge with the machine, whatever precautions are taken, or however rapidly the plate is turned. This limit is attained when the loss of electricity equals its production. The loss depends on three causes : i. The loss by the atmosphere, and the moisture it contains. ii. The loss by the supports. iii. The recombination of the electricities of the rubbers and the glass.

The first two causes have been already mentioned. With reference to the last, it must be noticed that the electrical charge increases with the rapidity of the rotation, until it reaches a point at which it overcomes the resistance presented by the non-conductivity of the glass. At this point, a portion of the two electricities separated on the rubbers and on the glass recombines, and the charge remains constant. It is, therefore, ultimately independent of the rapidity of rotation.

756. **Quadrant electrometer.**—The electrical charge is roughly measured by the *quadrant* or *Henley's electrometer*, which is attached to the conductor. This is a small electric pendulum, consisting of a wooden rod *d*, to which is attached an ivory or cardboard scale (fig. 702). In the centre of this is a small index of straw, movable on an axis, and terminating in a pith ball. Being attached to the conductor, the index diverges as the machine is charged, ceasing to rise when the limit is attained. When the rotation is discontinued the index falls rapidly if the air is moist; but in dry air it only falls slowly, showing, therefore, that the loss of electricity in the latter case is less than in the former.

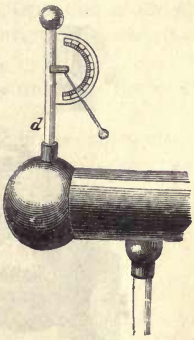


Fig. 702.

757. **Cylinder electrical machine.**—The construction of the cylinder machines, as ordinarily used in England, is due to Nairne. They are well adapted for obtaining either kind of electricity. In Nairne's machine (fig. 703) the cylinder is rubbed by only one cushion C, which is made of leather stuffed with horsehair, and is screwed to an insulated conductor A. On the opposite side of the cylinder there is a similar insulated conductor B, provided with a series of points on the sides next the glass. To the lower part of the cushion C is attached a piece of

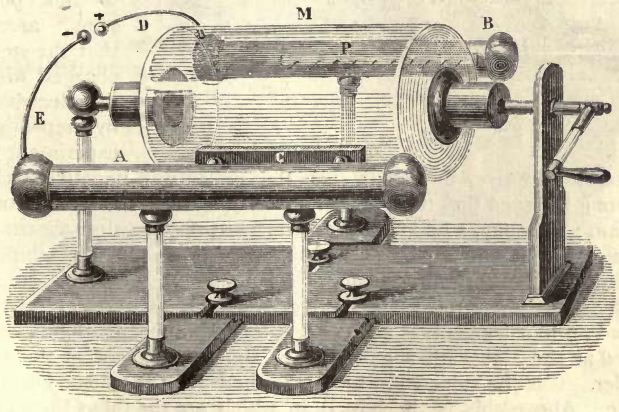


Fig. 703.

oiled silk, which extends over the cylinder to just above the points. This is not represented in the figure When the cylinder is turned, A becomes

charged with negative and B with positive electricity by the loss of its negative from the points P. The two opposite electricities will now unite by a succession of sparks across D and E. If use is to be made of the electricity, either the rubber or the prime conductor must be connected with the ground. In the former case positive electricity is obtained; in the latter, negative.

758. **Armstrong's hydro-electric machine.**—In this machine electricity is produced by the disengagement of aqueous vapour through narrow orifices. The discovery of the machine was occasioned by an accident. A workman having accidentally held one hand in a jet of steam, which was issuing from an orifice in a steam boiler at high pressure, while his other hand grasped the safety-valve, was astonished at experiencing a smart shock. Lord Armstrong (then Mr. Armstrong, of Newcastle), whose attention was drawn to this phenomenon, ascertained that the steam was charged with positive electricity, and, by repeating the experiment with an insulated locomotive, he found that the boiler was negatively charged. Armstrong believed that the electricity was due to a sudden expansion of the steam; Faraday, who afterwards examined the question, ascertained its true cause, which will be best understood after describing a machine which Armstrong devised for reproducing the phenomenon.

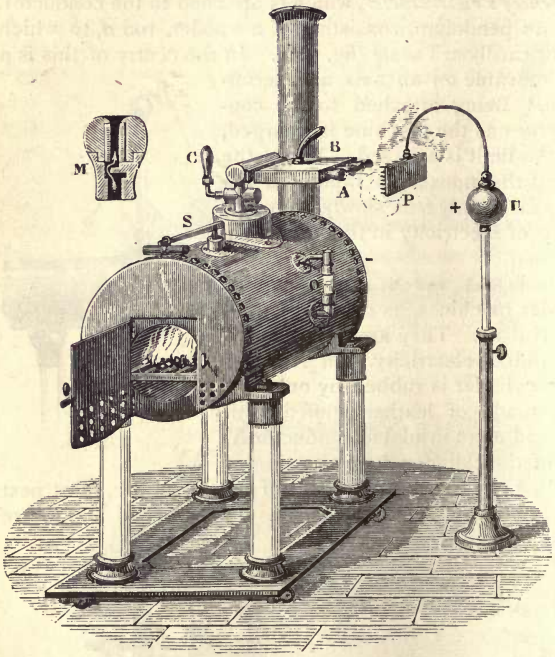


Fig. 704.

It consists of a wrought-iron boiler (fig. 704), with a central fire, and insulated on four legs. It is about 5 feet long by 2 feet in diameter, and is provided at the side with a gauge O, to show the height of the water in the boiler. C is the stopcock, which is opened when the steam has sufficient pressure. Above this is the box B, in which are the tubes through which the steam is disengaged. On these are fitted jets of a peculiar construction, which will be understood from the section of one of them, M, represented on a larger scale. They are lined with hard wood in a manner represented

by the diagram. The box B contains cold water. Thus the steam, before escaping, undergoes partial condensation, and becomes charged with vesicles of water—a necessary condition, for Faraday found that no electricity is produced when the steam is perfectly dry.

The development of electricity in the machine was at first attributed to the condensation of the steam; but Faraday found that it is solely due to the friction of the globules of water against the jet. For if the little cylinders which line the jets are changed, the kind of electricity is changed; and if ivory is substituted, little or no electricity is produced. The same effect is produced if any fatty matter is introduced into the boiler. In this case the linings are of no use. It is only in case the water is pure that electricity is disengaged, and the addition of acid or saline solutions, even in minute quantity, prevents any disengagement of electricity. If turpentine is added to the boiler, the effect is reversed—the steam becomes negatively, and the boiler positively, electrified.

With a current of moist air Faraday obtained effects similar to those of this apparatus, but with dry air no effect is produced.

759. **Holtz's electrical machine.**—Before the end of last century electrical machines were known in this country in which the electricity was not

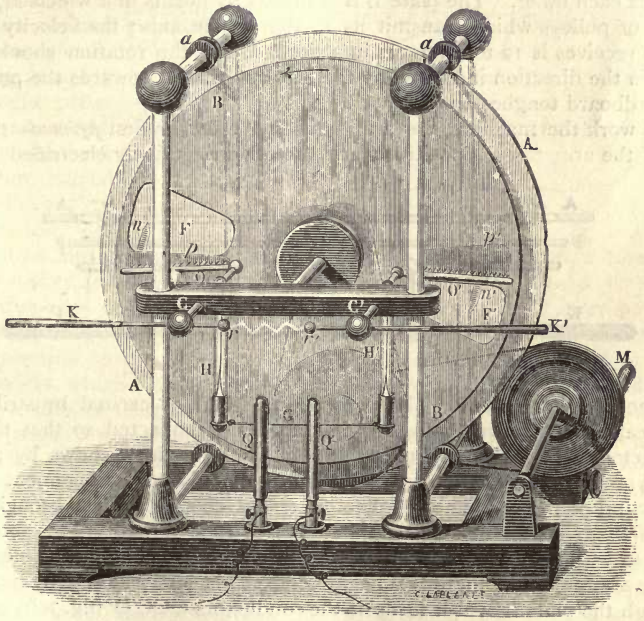


Fig. 705.

developed by friction, but by the continuous inductive action of a body already electrified, as the electrophorus; within the last few years such machines have been re-invented and come into use. The form represented in fig. 705 was invented by Holtz, of Berlin.

It consists of two circular plates of thin glass at a distance of 3 mm. from each other; the larger one, AA, which is 2 feet in diameter, is fixed by means of 4 wooden rollers *a*, resting on glass axes and glass feet. The diameter of the second plate, BB, is 2 inches less; it turns on a horizontal glass axis, which passes through a hole in the centre of the large fixed plate without touching it. In the plate A, on the same diameter, are two large apertures or *windows*, FF′. Along the lower edge of the window F, on the posterior face of the plate, a band of paper, *p*, is glued, and on the anterior face a sort of *tongue* of thin cardboard, *n*, joined to *p* by a thin strip of paper, and projecting into the window. At the upper edge of the window, F′, there are corresponding parts, *p′* and *n′*. The papers *p* and *p′* constitute the *armatures*. The two plates, the armatures, and their tongues are covered with shellac varnish, but more especially the edges of the tongues.

In front of the plate B, at the height of the armatures, are two brass *combs*, OO′, supported by two conductors of the same metal, CC′. In the front end of these conductors are two moderately large brass knobs, through which pass two brass rods terminated by smaller knobs, *r r′*, and provided with ebonite handles, K K′. These rods, besides moving with gentle friction in the knobs, can also be turned so as to be more or less near and inclined towards each other. The plate B B is turned by means of a winch M, and a series of pulleys which transmit its motion to the axis; the velocity which it thus receives is 12 to 15 turns in a second, and the rotation should take place in the direction indicated by the arrows—that is, towards the points of the cardboard tongues *n n′*.

To work the machine, the armatures *p p′* must be first *primed*—that is, one of the armatures is positively and the other negatively electrified. This

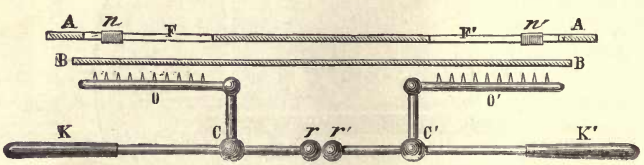


Fig. 706.

is effected by means of a plate of ebonite, which is excited by striking it with catskin; the two knobs *r r′* having been connected so that the two conductors C C only form one, as seen in fig. 706, which shows by a horizontal section, through the axis of rotation, the relative arrangement of the plates and of the conductors. The electrified ebonite is then brought near one of them—*p*, for instance—and the plate B is turned. The ebonite is charged with negative electricity, and this withdraws the positive electricity of the armature and charges it negatively. This latter acting by induction through the plate BB, as it turns on the conductors OCC′O′ (fig. 706), attracts through the *comb* O the positive electricity which collects on the front face of the movable plate; while at the same time negative electricity, repelled on the comb O′, collects, like the former, on the front face of the plate B. Hence, the two electricities being carried along by the rotation, at the end of half a turn all the lower half of the plate B, from *p* to F′ (fig. 707), is posi-

tively electrified, and its upper surface from p' to F negatively. But the two opposite electricities above and below the window F′ concur in decomposing the electricity of the armature $p'n'$; the part p' is positively electrified, while negative electricity is liberated by the tongue n', and is deposited on the inner face of the plate B B, which from its thinness almost completely neutralises the positive electricity on the anterior face.

The two armatures are then primed, and the same effect as at F′ is produced at F on the armature $p\,n$—that is, that the opposite electricities above and below $p\,n$, decomposing a new quantity of neutral electricity, the negative charge of the part p increases, while the positive electricity which is liberated by the tongue n neutralises the negative electricity which comes from F′ towards F; and so forth, until, the machine having attained its maximum charge, there is equilibrium in all its parts. From that point it only keeps itself up, and in perfectly dry air it may work for a long time without its being necessary to employ the ebonite plate. If this be removed, and the knobs r and r' are moved apart (fig. 707) to a distance dependent on the power of the machine, on continuing to turn, a torrent of sparks strikes across from one knob to the other.

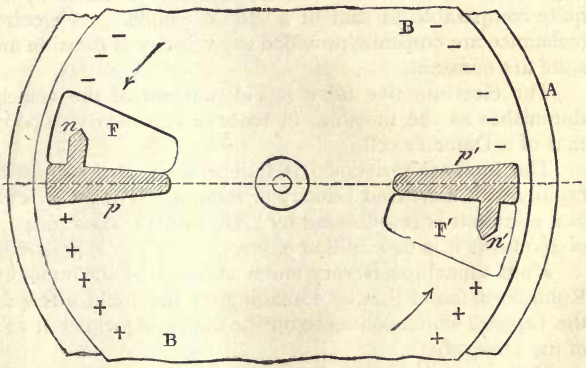


Fig. 707.

With plates of equal dimensions Holtz's machine is far more powerful than the ordinary electrical machine (753). The power is still further increased by suspending to the conductors C C′ two *condensers*, H H′ (765), or small Leyden jars, which consist of two glass tubes coated with tinfoil, inside and out, to a fifth of their height. Each of them is closed by a cork through which passes a rod, communicating at one end with the inner coating, and suspended to one of the conductors by a crook at the other end. The two external coatings are connected by a conductor, G. They are, in fact, only two small Leyden jars (770), one of them, H, becoming charged with positive electricity on the inside and negative on the outside; the other, H′, with negative electricity on the inside and positive on the outside. Becoming charged by the play of the machine, and being discharged at the same rate by the knobs $r\,r'$, they strengthen the spark, which may attain a length of 6 or 7 inches.

The current of the machine is utilised by placing in front of the frame two brass uprights, Q Q′, with binding screws in which are copper wires; then, by means of the handles K K′, the rods which support the knobs $r\,r'$ are inclined, so that they are in contact with the uprights. The current being

then directed by the wires, a battery of six jars can be charged in a few minutes, water can be decomposed, a galvanometer deflected, and Geissler's tubes illuminated as with the voltaic battery.

Kohlrausch found that a Holtz machine with a plate 16 inches in diameter, and making 5 turns in three seconds, produced a constant current capable of decomposing water at the rate of $3\frac{1}{2}$ millionths of a milligramme in a second. This is equal to the effect produced by a single Grove's cell in a circuit of 45,000 ohms resistance.

Rossetti, who made a series of measurements with a Holtz machine, found that the strength of the current is nearly proportional to the velocity of the rotation; it increases a little more rapidly than the rotation. The ratio of the velocity of rotation to the strength of the current is greater when the hygrometric state increases. The current produced by a Holtz machine is quite comparable to that of a voltaic couple. Its electromotive force and resistance are constant, provided the velocity of rotation and the hygrometric state are constant.

The electromotive force is independent of the velocity of rotation, but diminishes as the moisture increases; it is nearly 52,000 times as great as that of a Daniell's cell.

The internal resistance is independent of the moisture, but diminishes rapidly with increased velocity of rotation. Thus with a velocity of 120 turns in a minute it is represented by 2,810 million ohms (964), and with a velocity of 450 turns it is 646 million ohms.

Holtz's machine is very much affected by the moisture of the air; but Ruhmkorff found that by spreading on the table a few drops of petroleum, the vapours which condense on the machine protect it against the moisture of the atmosphere.

Holtz's machine affords a means of making a curious experiment on *reversibility*. If the two combs of a machine in the ordinary state are connected with the poles of a second similar one, which is then set in action, the combs of the first become luminous, and the plate begins to rotate, but in the opposite direction to its ordinary course; the electricity thus transmits the motion of the second machine to the first; the one expends what the other produces. It may also be observed that the two machines are connected by opposite poles, and the system constitutes a circuit which is traversed in a definite direction by a continuous electrical current.

A very simple and efficient machine of this kind is made by Voss of Berlin. One with a plate of 10 inches diameter produces a spark of 4 to 5 inches.

760. **Wimshurst's machine.**—This is the simplest and most efficient of all induction machines.

It consists (fig. 708) of two circular glass discs mounted on a fixed horizontal spindle in such a way as to be rotated in opposite directions at a distance of not more than a quarter of an inch apart. Both discs are well varnished, and attached to the outer surface of each are narrow radial sections of tinfoil arranged at equal angular distances apart.

Attached to the fixed spindle on which the discs rotate is a bent conducting rod, at the ends of which are two fine wire brushes; twice during each revolution two diametrically opposite conductors are put in connection with each other by means of this conductor, as they just graze the tips of the

brushes. At the back is a similar one at right angles to that in front, and there is a position of maximum efficiency, which is when they make an angle of 45° with the fixed collectors. There are two forks provided with combs directed towards one another, and towards the two discs which rotate between them ; they are supported horizontally on glass Leyden jars, to which are also attached the terminal electrodes or dischargers, the distance apart of which can be varied by turning the Leyden jar from which they rise.

The machine is quite self-exciting, and requires neither friction, nor the spark from any outside exciter, to start it. This is one of the most remark-

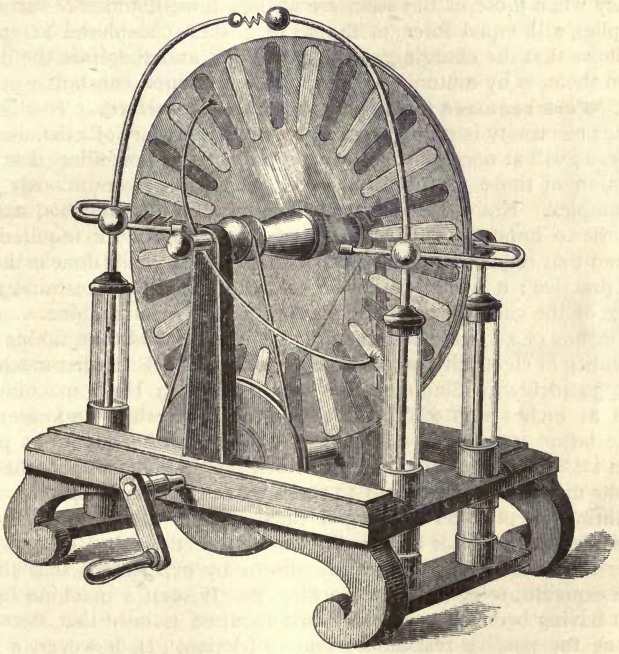


Fig. 708.

able features of this machine, that under ordinary conditions it attains its full power after the second or third turn. The initial charge is probably obtained from the electricity of the air, or from the frictional resistance against it.

With a machine having plates 17 inches in diameter, a powerful spark discharge passes between the two electrodes when they are 4 to 5 inches apart, in regular succession, at the rate of 2 or 3 for every turn of the handle. A machine with 12 plates, 30 inches in diameter, when driven at a speed of 200 turns per minute, produces sparks between the terminals of $13\frac{1}{2}$ inches in length ; and when the terminals are closed by a wire of 3,000 ohms

resistance (964) a current of two-thirds of a millampere is produced. With these machines the increase of electricity has been found proportional to the speed of rotation up to 5,000 turns in a minute.

It is not easy to give a satisfactory account of the theory of the machine. Its inventor considers that its remarkable efficiency may be partly due to the duplex action of the apparatus, both plates being active and contributing electricity to the collecting combs, the sector-shaped plates of tinfoil acting as *inductors* when in their position of lowest efficiency as *carriers*, and as *carriers* when in the positions at which their inductive effect is at a minimum, and *vice versâ*; and as it follows from the construction of the instrument that the inductors of the one disc are at a position of highest efficiency when those of the other are at their lowest, and *vice versâ*, and as this applies with equal force to the sectors when considered as carriers, it also follows that the charging of the electrodes, and therefore the discharge between them, is by mutual compensation maintained constant.

761. **Work required for the production of electricity.**—In all electrical machines electricity is only produced by the expenditure of a definite amount of force, as will at once be seen by a perusal of the preceding descriptions. The action of those machines, however, which work continuously is somewhat complex. Not only is electricity produced, but heat also; and it has been hitherto impossible to estimate separately the work required for the heat from that required for the electricity. This is easily done in theory, but not in practice: it would be, for instance, difficult to determine the temperature of the cushion, or of the plate of a Ramsden machine.

By means of a Lane electrometer (777) it was found that, taking as unity the quantity of electricity produced by one turn of a Ramsden machine with a plate 39 inches in diameter, that produced by a Holtz machine with a plate of 21 inches was 0·86; but as for the same work the former made 1 and the latter 10 turns in a second, it follows that the quantities produced were as 1 : 8·6. Comparing the quantities per unit of surface, the yield of the Holtz machine is more than 12 times that of the Ramsden.

In lifting the plate off a charged electrophorus a certain expenditure of force is needed, though it be too slight to be directly estimated (752). With a Holtz machine it may be readily shown by experiment that there is a definite expenditure of force in working it. If such a machine be turned without having been charged, the work required is only that necessary to overcome the passive resistances due to friction. If, however, a charged ebonite plate is approached to one of the sectors, as soon as the peculiar sound indicates that the machine is at work, it will be observed that there must be a distinct increase in the mechanical effort necessary to work the machine.

The work required to charge an unelectrified conductor to a given potential may be deduced from the following considerations:—To impart to a body which is at potential V a quantity of electricity Q would require an amount of work represented by QV (739). But in the case of an unelectrified body it is neutral at the outset—that is, at zero potential; and we may conceive the electricity imparted to it in a series of n very small charges of q each, such that $nq = Q$; and as the potential rises proportionally to the number of charges, it may be assumed that the work done is equal to that required to

charge the body to an average potential of $\frac{1}{2}V$; hence the work in question $W = \frac{1}{2}QV$.

From the relation between the quantity of heat produced by the current of a Holtz machine working under definite conditions, and the amount of work expended in producing the rotation of the plate, Rossetti made a determination of the mechanical equivalent of heat, which gave the number 1,397, agreeing therefore very well with the numbers obtained by other methods (497).

761*a*. **Thomson's water-dropping collector.**—This may be given as an illustration of an arrangement by which a known charge may be almost indefinitely multiplied. In fig. 709 I is an insulated metal cylinder called the *inductor*, and water falls in drops from an uninsulated metal tap the nozzle of which is in the centre of the cylinder. Directly below the inductor is a second similar insulated metal cylinder R, with a funnel the nozzle of which is also in the centre. This second cylinder is called the *receiver*. If now a very feeble positive charge be given to the inductor I, the drops of water as they issue will be charged with positive electricity, and will repel each other as they issue. Falling on the funnel of the receiver, they will give up to this the whole of their charge, and the water as it issues will be neutral. The charge thus imparted to R will go on increasing until the loss equals the production, or until the drops issuing from the inductor are repelled by the receiver, so that they do not fall into the funnel.

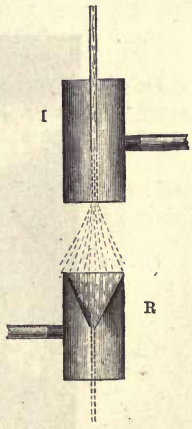


Fig. 709.

Suppose two such apparatus I I′ and R R′ be arranged near each other, and in such a manner that the inductor I of the one is in metallic connection with the receiver R′ of the other, and conversely the inductor I′ in connection with the receiver R of the other. By this means they will act on each other and reciprocally increase their charges. If a feeble charge be given to one of the inductors, the charges will go on increasing until sparks pass between. It is not even necessary to give a charge at the outset, the ordinary electricity of the atmosphere is sufficient.

The energy in this apparatus is derived from that of the falling body, and would be exactly equivalent to it if there were no loss, and if the drops reached the funnel without any velocity.

EXPERIMENTS WITH THE ELECTRICAL MACHINE

762. **Electrical spark.**—One of the most curious phenomena observed with the electrical machine is the spark drawn from the conductor when a finger is presented to it. The positive electricity of the conductor, acting inductively on the neutral electricity of the body, decomposes it, repelling the positive and attracting the negative. When the attraction of the opposite electricities is sufficiently great to overcome the resistance of the air, they recombine

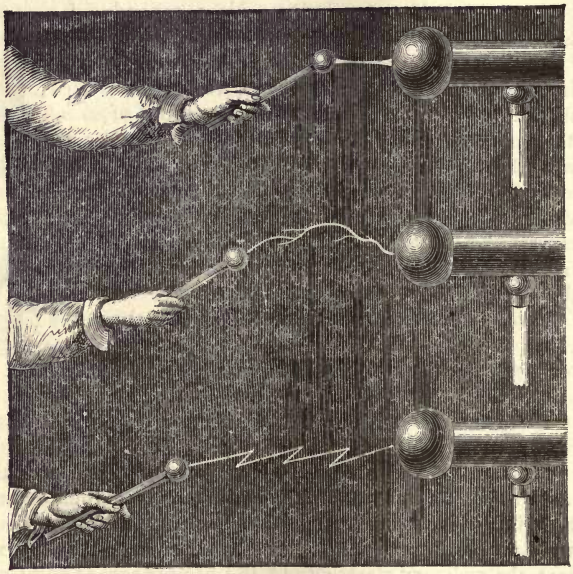
with a smart crack and a spark. The spark is instantaneous, and is accompanied by a sharp prickly sensation, more especially with a powerful machine Its shape varies. When it strikes at a short distance it is rectilinear, as seen in fig. 710. Beyond two or three inches in length the spark becomes irregular, and has the form of a sinuous curve with branches (fig. 711). If the discharge is very powerful, the spark takes a zigzag shape (fig. 712). These two latter appearances are seen in the discharge of lightning.

A spark may be taken from the human body by aid of the *insulating stool*, which is simply a low stool with stout glass legs. The person standing on this stool touches the prime conductor, and, as the human body is a conductor, the electricity is distributed over its surface as over an ordinary insulated metallic conductor. The hair diverges in consequence of repulsion,

Fig. 710.

Fig. 711.

Fig. 712.



a peculiar sensation is felt on the face, and if another person, standing on the ground, presents his hand to any part of the body, a smart crack with a pricking sensation is produced.

A person standing on an insulated stool may be positively electrified by being struck with a catskin. If the person holding the catskin stands on an insulated stool, the striker becomes positively and the person struck negatively electrified.

763. **Electrical chimes.**—The *electrical chimes* is a piece of apparatus consisting of three bells suspended to a horizontal metal rod (fig. 713). Two of them, A and B, are in metallic connection with the conductor; the middle bell hangs by a silk thread, and is thus insulated from the conductor, but is connected with the ground by means of a chain. Between the bells are

small copper balls suspended by silk threads. When the machine is worked, the bells A and B, being positively electrified, attract the copper balls, and after contact repel them. Being now positively electrified, they are in turn attracted by the middle bell, C, which is charged with negative electricity by induction from A and B. After contact they are again repelled, and this process is repeated as long as the machine is in action.

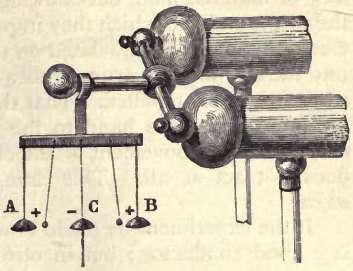


Fig. 713.

Fig. 714 represents an apparatus originally devised by Volta for the purpose of illustrating what he supposed to be the motion of hail between two clouds oppositely electrified. It consists of a tubulated glass shade, with a metal base, on which are some pith balls. The tubulure has a metal cap, through which passes a brass rod, provided with a metal disc or sphere at the lower end, and at the upper with a ring, which touches the prime conductor.

When the machine is worked, the sphere becoming positively electrified attracts the light pith balls, which are then immediately repelled, and, having

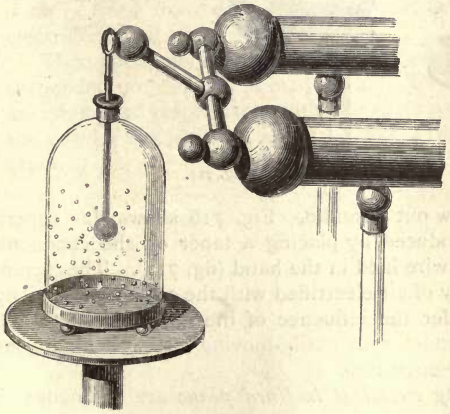


Fig. 714.

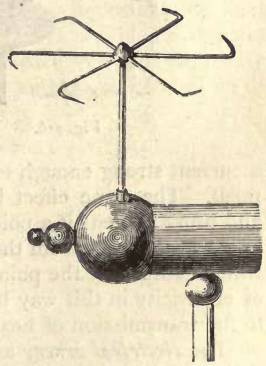


Fig. 715.

lost their charge of positive electricity, are again attracted, again repelled, and so on, as long as the machine continues to be worked. An amusing modification of this experiment is frequently made by placing between the two plates small pith figures, somewhat loaded at the base. When the machine is worked, the figures execute a regular dance.

764. **Electrical whirl or vane.**—The electrical *whirl* or *vane* consists of 5 or 6 wires, terminating in points, all bent in the same direction, and fixed in a central cap, which rotates on a pivot (fig. 715). When the apparatus is placed on the conductor, and the machine worked, the whirl begins

to revolve in a direction opposite that of the points. This motion is not analogous to that of the hydraulic tourniquet (149). It is not caused by a flow of material fluid, but is owing to a repulsion between the electricity of the points and that which they impart to the adjacent air by conduction. The electricity, being accumulated on the points in a high state of density, passes into the air, and, imparting thus a charge of electricity, repels this electricity, while it is itself repelled. That this is the case is evident from the fact that on approaching the hand to the whirl while in motion, a slight draught is felt, due to the movement of the electrified air, while in vacuo the apparatus does not act at all. This draught or wind is known as the electrical *aura*.

If the experiment be made in water, the fly remains stationary, for water is a good conductor ; but in olive oil, which is a bad conductor, the whirl rotates.

When the electricity thus escapes by a point, the electrified air is repelled so strongly as not only to be perceptible to the hand, but also to engender

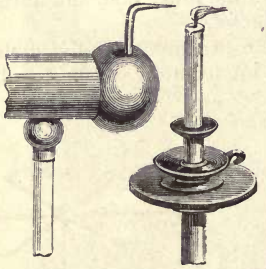


Fig. 716.

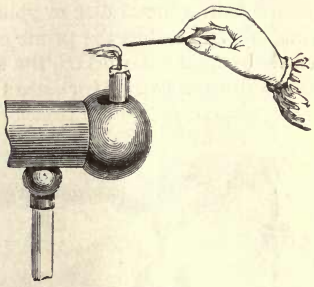


Fig. 717.

a current strong enough to blow out a candle. Fig. 716 shows this experiment. The same effect is produced by placing a taper on the conductor and bringing near it a pointed wire held in the hand (fig. 717). The current arises in this case from the flow of air electrified with the contrary electricity which escapes by the point under the influence of the machine. The loss of electricity in this way by contact with easily-moving bodies is analogous to the transmission of heat by convection.

The *electrical orrery* and the *electrical inclined plane* are analogous in their action to these pieces of apparatus.

The velocity of the electrical aura has been determined by placing a wire gauze connected with earth at a fixed distance from the point, and an anemometer at varying distances behind the gauze. The velocity of the wind was found to diminish with the distance, but not in direct proportion ; at a distance of 22 inches it was $5\frac{1}{2}$ feet per second, while at 60 inches its velocity was 2 feet per second.

The production of the electrical aura is accompanied by luminous phenomena which can be seen in the dark. If positive electricity escapes from the point a violet aigrette is formed ; while when the electricity is negative a small brilliant star forms on the point.

It is pretty certain that in these experiments it is not the air itself, but the particles in it, whether of dust or of moisture, which become electrified. This may be illustrated by the following simple experiment. A glass globe is filled with dense smoke of turpentine or petroleum or of sal-ammoniac (fig. 718), and the bared end of a guttapercha-covered wire is held in it while the other end is connected with an electrical machine. On giving two or three turns to the machine the smoke is rapidly deposited, and the inside becomes quite clear. Here the smoke consists of solid particles, which become polarised by induction and attract each other like the particles of silk in fig. 694. They thereby become agglomerated, and fall to the bottom of the globe. Nahrwold proves that if air is freed from dust by filtration, it takes little if any charge from an electrified point.

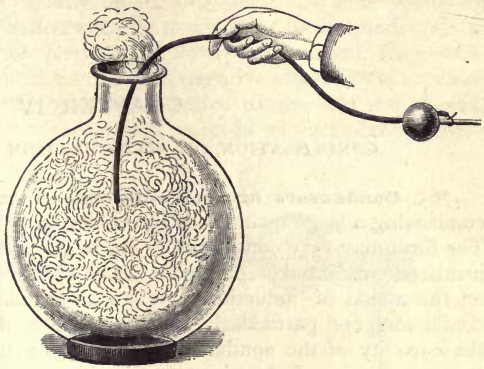


Fig. 718.

This phenomenon is employed industrially in the deposition of finely suspended powders, as in lead works. Two conductors provided with points are connected respectively with a positive and negative source of electricity; the powder electrified by the one point is repelled and is precipitated on the other.

CHAPTER IV

CONDENSATION OR ACCUMULATION OF ELECTRICITY

765. **Condensers or accumulators.**—A *condenser* is an apparatus for condensing a large quantity of electricity on a comparatively small surface. The form may vary considerably, but in all cases consists essentially of two insulated conductors, separated by a non-conductor, and the working depends on the action of induction. When an insulated conductor is near other conductors, and particularly when these latter are connected with the earth, the capacity of the conductor is increased; that is to say, it requires a greater quantity of electricity to raise it to a given potential than when the other conductors are away. An arrangement of this kind is called a *condenser* or *accumulator*, the latter term, though less usual, being preferable, as the former tacitly implies some hypothesis of the nature of electricity.

Epinus's condenser consists of two circular brass plates, A and B (fig. 719), with a sheet of glass, C, between them. The plates, each provided with a pith-ball pendulum, are mounted on insulated glass legs, and can be moved along a support and fixed in any position. When electricity is to be accumulated, the plates are placed in contact with the glass, and then one of them, B for instance, is connected with the electrical machine, and the other placed in connection with the ground, as shown in fig. 720.

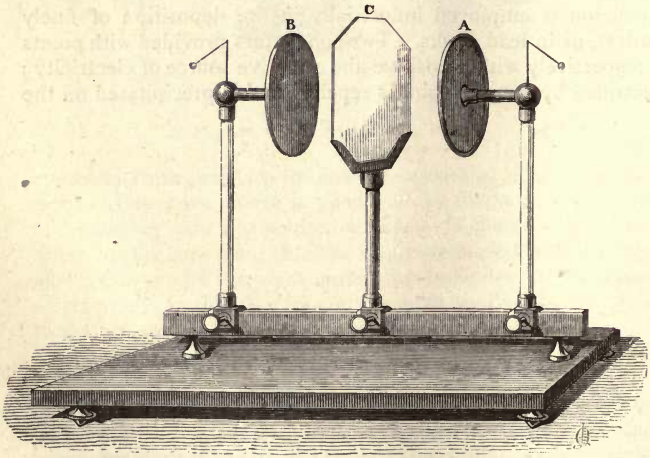


Fig. 719.

In explaining the action of the condenser, it will be convenient in each case to call that side of the metal plate nearest the glass the *anterior* and the other the *posterior* side. And first let A be at such distance from B

as to be out of the sphere of its action. The plate B, which is then connected with the conductor of the electrical machine, takes its maximum charge, which is distributed equally on its two faces, and the pendulum diverges widely. If the connection with the machine be interrupted, nothing would be changed; but if the plate A be slowly approached, its neutral state being decomposed by the influence of B, negative electricity is accumulated on its anterior face, *n* (fig. 721), and positive passes into the ground. But as the negative electricity of the plate A reacts in its turn on the positive

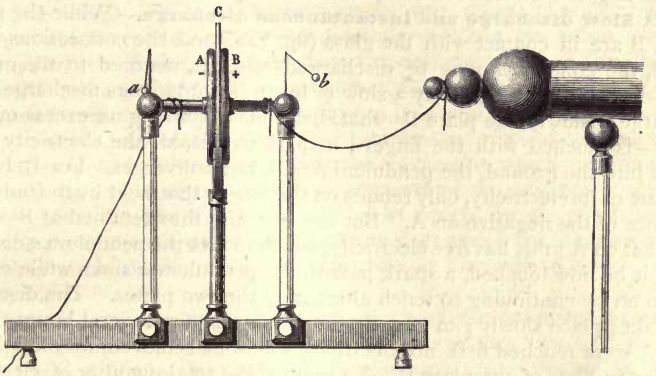


Fig. 720.

of the plate B, the latter ceases to be equally distributed on both faces, and is accumulated on its anterior face, *m*. The posterior face, *p*, having thus lost a portion of its electricity, its density has diminished, and is no longer equal to that of the machine, and the pendulum *b* diverges less widely. Hence B can receive a fresh quantity from the machine, which, acting as just described, decomposes by induction a second quantity of neutral electricity on the plate A. There is then a new accumulation of negative electricity on the face *n*, and consequently of positive electricity on *m*. But each time that the machine gives off electricity to the plate, only a part of this passes to the face *m*, the other remaining on the face *p*; the density here, therefore, continues to increase until it equals that of the machine. From this moment equilibrium is established, and a limit to the charge is attained which cannot be exceeded. The quantity of electricity accumulated now on the two faces *m* and *n* is very considerable, and yet the pendulum diverges just as much as it did when A was absent, and no more; in fact, the density at *p* is just what it was then—namely, that of the machine.

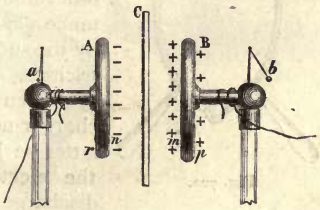


Fig. 721.

When the condenser is charged—that is, when the opposite electricities are accumulated on the anterior faces—connection with the ground is broken by raising the wires. The plate A is charged with negative electricity, but

simply on its anterior face (fig. 721), the other side being neutral. The plate B, on the contrary, is electrified on both sides, but unequally; the accumulation is only on its anterior face, while on the posterior, p, the density is simply equal to that of the machine at the moment the connections are interrupted. In fact, the pendulum b diverges, and a remains vertical. But if the two plates are removed, the two pendulums diverge (fig. 719), which is owing to the circumstance that, as the plates no longer act on each other, the positive electricity is equally distributed on the two faces of the plate B, and the negative on those of the plate A.

766. **Slow discharge and instantaneous discharge.**—While the plates A and B are in contact with the glass (fig. 720), and the connections interrupted, the condenser may be discharged—that is, restored to the neutral state—in two ways; either by a slow or by an instantaneous discharge. To discharge it slowly, the plate B—that is, the one containing an excess of electricity—is touched with the finger; a spark passes, all the electricity on p passes into the ground, the pendulum b falls, but a diverges. For B, having lost part of its electricity, only retains on the face m that held by the inductive influence of the negative on A. But the quantity thus retained at B is less than that on A; this has free electricity, which makes the pendulum a diverge; and if it be now touched, a spark passes, the pendulum a sinks while b rises, and so on by continuing to touch alternately the two plates. The discharge only takes place slowly; in very dry air it may require several hours. If the plate A were touched first, no electricity would be removed, for all it has is retained by that of the plate B. To remove the total quantity of electricity by the method of alternate contacts, an infinite number of such contacts would theoretically be required.

An instantaneous discharge may be effected by means of the *discharging rod* (fig. 722). This consists of two bent brass rods, terminating in knobs and joined by a hinge. When provided with glass handles, as in fig. 722, it forms a *glass discharging rod.* In using this apparatus one of the knobs is pressed against one plate of the condenser, and the other knob brought near the other. At a certain distance a spark strikes from the plate to the knob, caused by the sudden recomposition of the two opposite electricities.

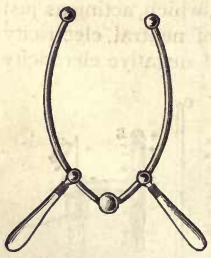


Fig. 722.

When the condenser is discharged by the discharger no sensation is experienced, even though the latter be held in the hand; of the two conductors, the electricity chooses the better, and hence the discharge is effected through the metal, and not through the body. But if, while one hand is in contact with one plate the other touches the second, the discharge takes place through the breast and arms, and a considerable shock is felt; and the larger the surface of the condenser, and the greater the electric density, the more violent is the shock.

767. **Condensing force.**—The *condensing force* is the relation between the whole charge, which the collecting plate can take while under the influence of the second plate, and that which it would take if alone; in other words, it is the ratio of the capacities under the two conditions.

768. **Limit of the charge of condensers.**—The quantity of electricity which can be accumulated on each plate is, *cæteris paribus*, proportional to the potential of the electricity on the conductor, and to the surface of the plates ; it decreases as the insulating plate is thicker, and it differs with the specific inductive capacity of the substance. There is, however, a limit in the case of each condenser beyond which it cannot be charged. The effect of dielectric polarisation (747) is to put the medium into a state of strain from which it is always trying to release itself, and which is the equivalent of the work done in charging a condenser. This is, indeed, the seat of the electrical energy. It is as if two surfaces were pulled together by elastic threads which repelled each other laterally. When the strain exceeds a certain limit, a discharge takes place through the mass of the dielectric, generally accompanied by light and sound, and with a temporary or permanent rupture of the dielectric according as it is fluid or solid. This is what occurs when a substance—glass, for instance—is exposed to a continually increasing weight ; a point is ultimately reached at which the glass gives way, and the weight at that point is a measure of the resistance to fracture of the glass. In like manner, the point at which the electrical discharge takes place is a measure of the electrical strength of the dielectric. This electrical strength is greater in glass than in air, and in dense than in rarefied air.

Thus to produce a spark of 0·5 cm. in air at the pressures 20, 180, and 685 mm. respectively, the potentials required were 3·23, 12·2, and 36.

We may, following Maxwell, further illustrate this point by the twisting of a wire : a wire in which a small force produces a permanent twist corresponds to the case of the conduction of electricity in a good conductor ; one which having been twisted, reverts to its former shape when the twisting force is removed, is completely elastic, and corresponds to a perfect insulator with respect to the charge employed. If no permanent twist can be given to the wire by a force which does not break it, the wire is brittle. A dielectric such as air, which does not transmit electricity except by disruptive discharge, may be said to be electrically brittle.

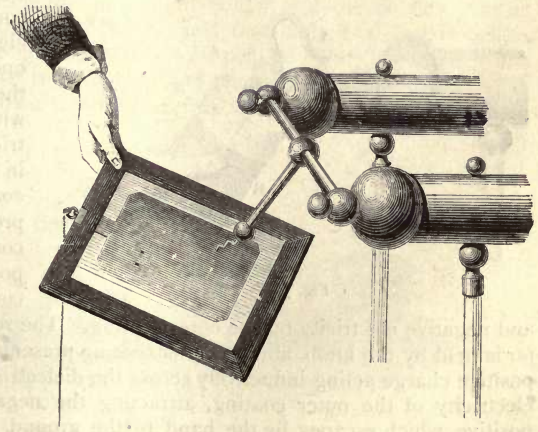


Fig. 723.

769. **Fulminating pane. Franklin's plate.**—This is a simple form of the condenser, and consists of a glass plate fixed in a wooden frame (fig. 723) ; on each side of the glass, pieces of tinfoil are fastened opposite each other,

leaving a space free between the edge and the frame. It is well to cover this part of the glass with an insulating layer of shellac varnish. One of the sheets of tinfoil is connected with the ring on the frame by a strip of tinfoil, so that it can be connected with the ground by means of a chain. To charge the pane the insulated side is connected with the machine. As the other side communicates with the ground, the two coatings play exactly the part of the condenser. On both plates there are accumulated large quantities of contrary electricities.

The pane may be discharged by touching one knob of the discharger against the lower surface, while the other is brought near the upper coating. A spark ensues, due to the recombination of the two electricities ; but the operator experiences no sensation, for the discharge takes place through the wire. But if the connection between the two coatings be made by touching them with the hands, a violent shock is felt in the hands and breast, for the combination then takes place through the body.

770. **Leyden jar.**—The *Leyden jar*, so named from the town of Leyden, where it was invented, is essentially a modified condenser, or fulminating pane rolled up. Fig. 724 represents a Leyden jar of the usual French shape in the process of being charged. It consists of a glass jar of any convenient size, the interior of which is either coated with tinfoil or filled with thin leaves of copper, or with gold-leaf. Up to a certain distance from the neck the outside is coated with tinfoil. The neck is provided with a cork, through which passes a brass rod, which terminates at one end in a knob, and communicates with the metal in the interior. The metallic coatings are called respectively the *inner* and *outer coatings* or *armatures*. Like any other condenser, the jar is charged by connecting one of the coatings with the ground, and the other with the source of electricity. When it is held in the hand by the outer coating, and the knob presented to the positive conductor of the machine, positive electricity is accumulated on the inner and negative electricity on the outer coating. The reverse is the case if the jar is held by the knob, and the outer coating presented to the machine. The positive charge acting inductively across the dielectric glass, decomposes the electricity of the outer coating, attracting the negative and repelling the positive, which escapes by the hand to the ground. Thus it will be seen that the action of the jar is the same as that of the condenser, and all that has been said of this applies to the jar, substituting the two coatings for the two plates A and B of fig. 720.

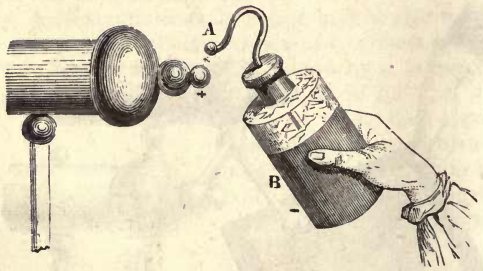


Fig. 724.

Like any other condenser, the Leyden jar may be discharged either slowly or instantaneously. For the latter purpose it is held n the hand by

the outside coating (fig. 725), and the two coatings are then connected by means of the simple discharger. Care must be taken to touch *first* the external coating with the discharger, otherwise a smart shock will be felt. To discharge it slowly the jar is placed on an insulated plate, and first the inner and then the outer coating touched, either with the hand or with a metallic conductor. A slight spark is seen at each discharge.

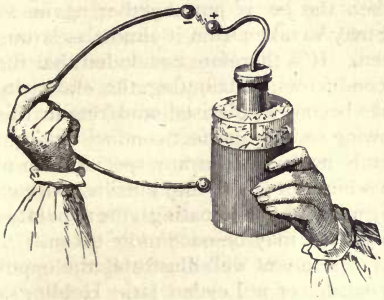


Fig. 725.

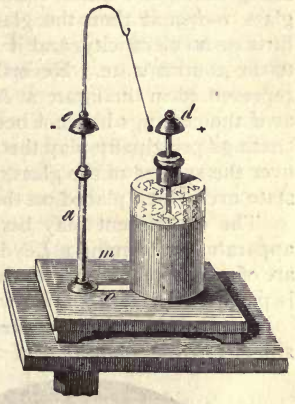


Fig. 726.

Fig. 726 represents a very pretty experiment for illustrating the slow discharge. The rod terminates in a small bell, *d*, and the outside coating is connected with an upright metal support, on which is a similar bell, *e*. Between the two bells a light brass ball is suspended by a silk thread. The jar is then charged in the usual manner and placed on the support *m*. The internal coating contains a quantity of free electricity; the pendulum is

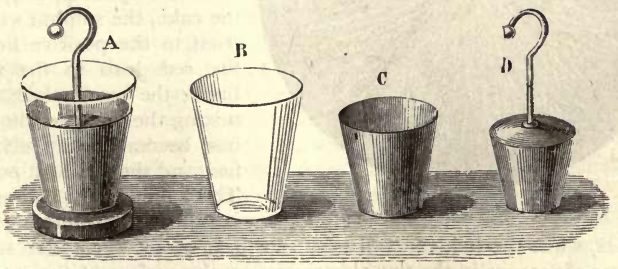


Fig. 727.

attracted and immediately repelled, striking against the second bell, to which it imparts its free electricity. Being now neutralised, it is again attracted by the first bell, and so on for some time, especially if the air be dry, and the jar somewhat large.

771. **Leyden jar with movable coatings.**—This apparatus (fig. 727) is used to demonstrate that in the Leyden jar the opposite electricities are no

accumulated on the coatings merely, but are stored up in a state of strain into which the glass is put, and this state of strain is the mechanical equivalent of the work done in charging the jar. It consists of a somewhat conical glass vessel, B, with movable coatings of zinc or tin, C and D. These separate pieces placed one in the other, as shown in figure A, form a complete Leyden jar. After having charged the jar, it is placed on an insulating cake; the inner coating is first removed by the hand, or better by a glass rod, and then the glass vessel. The coatings are found to contain little or no electricity, and if they are placed on the table they are restored to the neutral state. Nevertheless, when the jar is put together again, as represented in the figure at A, a shock may be taken from it almost as strong as if the coatings had not been removed. It is therefore concluded that the coatings principally play the part of conductors, distributing the electricity over the surface of the glass, which thus becomes polarised, and retains this state even when placed on the table, owing to its imperfect conductivity.

The experiment may be conveniently made without any special form of apparatus by forming a Leyden jar, of which the inside and outside coatings are of mercury, charging it; then, having mixed the two coatings, the apparatus is put together again, upon which a discharge may be once more taken.

772. **Lichtenberg's figures.**—This experiment well illustrates the opposite electrical conditions of the two coatings of a Leyden jar. Holding a jar charged with positive electricity by the hand, a series of lines are drawn with the knob on a cake of resin or vulcanite; then having placed the jar on an insulator, it is held by the knob, and another series traced by means of the outer coating. If now a mixture of red-lead and flour of sulphur be projected on the cake, the sulphur will attach itself to the positive lines, and the red lead to the negative lines; the reason being that in mixing the powders the sulphur has become negatively electrified, and the red lead positively. The sulphur will arrange itself in tufts with numerous diverging branches, while the red lead will take the form of small circular spots, indicating a difference in the two electricities on the surface of the resin (787). These figures form, in short, a very sensitive electroscope for investigating the distribution of electricity on an insulating surface (732).

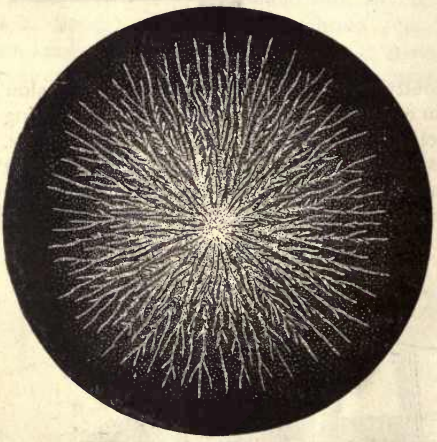


Fig. 728.

Fig. 728 represents the appearance of a plate of resin, which has been touched by the knob of a Leyden jar charged with positive electricity, and has then been dusted with lycopodium powder.

773. **Residual charge.**—Not only do the electricities adhere to the two surfaces of the insulating medium which separates them, but they penetrate

to a certain extent into the interior, as is shown by the following experiment:—A condenser is formed of a plate of shellac and movable metal plates. It is then charged, retained in that state for some time, and afterwards completely discharged. On removing the metal coatings and examining both surfaces of the insulator, they show no signs of electricity. After some time, however, each face exhibits the presence of some electricity of the same kind as that of the plate with which it was in contact while the apparatus was charged. This is explained, by some, as a kind of *electrical absorption.*

A phenomenon frequently observed in Leyden jars is of the same nature. When a jar has been completely discharged by bringing the inner and outer coatings in metallic contact, and allowed to stand a short time, it exhibits a second charge, which is called the *electric residue.* The jar may be again discharged, and a second residue will be left, feebler than the first, and so on, for three or four times. Indeed, with a delicate electroscope a long succession of such residues may be demonstrated. The residue is greater the longer the jar has remained charged. The magnitude of the residue further depends on the amount of the charge, and also on the degree in which the metal plates are in contact with the insulator. It varies with the nature of the substance, but there is no residue with either liquids or gaseous insulators. Faraday found that with paraffine the residue was greatest, then with shellac, while with glass and sulphur it was least of all. Kohlrausch has found that the residue is nearly proportional to the thickness of the insulator. If successive small charges, alternately positive and negative, be imparted to the jar, it is found that the residual charges come out in the reverse order to that in which the original charges go in. This residue is not to be confounded with that observed when a Leyden jar is discharged at the greatest striking distance (788), and which residue Reiss found to be always in a constant proportion, $\frac{2}{13}$, of the entire charge.

Maxwell proved that a dielectric composed of strata of different materials may exhibit the phenomena of the residual charge, even though none of the substances composing it exhibit it when alone.

From what has been said as to the state of mechanical strain in which the dielectric of a condenser is thrown when charged with electricity (771), it is not difficult to account for the phenomenon of the residual charge. An elastic body, such as a steel plate, which has been twisted or bent, reverts to its original state when the force which brought about the deformation ceases to act, but not at once quite completely. A certain length of time is required for this alteration to take place, but the change is promoted by any gentle mechanical action, such as tapping, which gives the molecules a certain freedom of motion. Dr. Hopkinson has made an experiment with a Leyden jar which is quite analogous to this. A glass vessel (fig. 729) contains sulphuric acid, and in it is placed a thinner one, about half full of the same liquid. Platinum wires dip in the two liquids, one of which is in connection with the prime conductor of an electrical machine, while the other is connected with the earth. The arrangement forms, in short, a condenser, the coatings of which are sulphuric acid. When after being thus

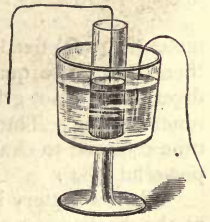


Fig. 729.

charged, the jar is discharged, a residual discharge may be taken after some time by again connecting the wires ; if, however, the inner jar be gently struck with a piece of wood, the residue makes its appearance much more rapidly. The same observer draws a parallel between the phenomena of the residual charge and those of residual magnetism (715).

774. **Electric batteries.**—The charge which a Leyden jar can take depends on the extent of the coated surface, and for small thicknesses is inversely proportional to the thickness of the insulator. Hence, the larger and thinner the jar the more powerful the charge. But very large jars are expensive, and liable to break ; and when too thin, the accumulated electricities discharge themselves through the glass, especially if it is not quite homogeneous. Leyden jars have usually from $\frac{1}{2}$ to 3 square feet of coated surface. For more powerful charges electric batteries are used.

An *electric battery* consists of a series of Leyden jars, whose internal and external coatings are respectively connected with each other (fig. 730). They are usually placed in a wooden box lined on the bottom with tinfoil. This lining is connected with two metal handles in the sides of the box. The inner coatings are connected with each other by metal rods, and the battery is charged by placing the inner coatings in connection with the prime conductor, while the outer coatings are connected with the ground by means of a chain fixed to the handles. A quadrant electrometer (fig. 702) fixed to one jar indicates the charge of the battery. Although there is a large quantity of electricity accumulated in the apparatus, the divergence is not great, for it is simply due to the free electricity on the inner coating. The larger and more numerous they are, the longer is the time required to charge the battery, but the effects are so much the more powerful (784).

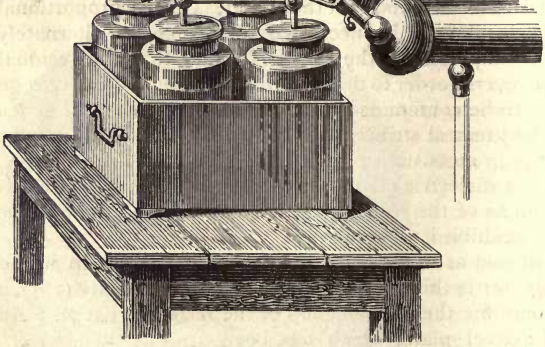


Fig. 730.

When a battery is to be discharged, the coatings are connected by means of the discharging rod, the outside coating being touched first. Great care is required, for with large batteries serious and even fatal accidents may occur.

775. **The universal discharger.**—This is an almost indispensable apparatus in experiments with the electric battery. On a wooden stand (fig. 731) are two glass legs, each provided with universal joints, in which movable

brass rods are fitted. Between these legs is a small ivory table, on which is placed the object under experiment. The two metal knobs being directed towards the objects, one of them is connected with the outer coating of the battery, and the moment communication is made between the outer and the inner coating by means of the glass discharging rod, a violent shock passes through the object on the table.

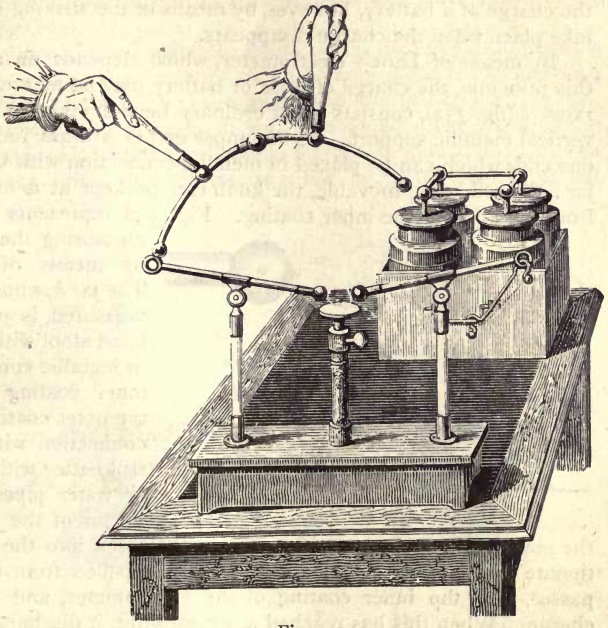


Fig. 731.

776. **Charging by cascade.**—A series of Leyden jars are placed each separately on insulating supports. The knob of the first is in connection with the prime conductor of the machine, and its outer coating joined to the knob of the second, the outer coating of the second to the knob of the third, and so on, the outer coating of the last communicating with the ground. The inner coating of the first receives a charge of positive electricity from the machine, and the corresponding positive electricity set free by induction on its outer coating, instead of passing to the ground, gives a positive charge to the inner coating of the second, which, acting in like manner, develops a charge in the third jar, and so on to the last, where the positive electricity developed by induction on the outer coating passes to the ground. The jars may be discharged either singly by connecting the inner and outer coatings of each jar, or simultaneously by connecting the inner coating of the first with the outer of the last. In this way the quantity of electricity necessary to charge one jar is available for charging a series of jars. Fig. 732 represents the arrangement applied to two jars.

777. **Measurement of the charge of a battery. Lane's electrometer.**—When the outer and inner coatings of a charged Leyden jar are gradually brought nearer each other, at a certain distance a spontaneous discharge ensues. The distance is called the *striking* or *sparking distance*. For the same charge it is inversely proportional to the pressure of the air (768), and, with the same jar, but different charges, directly proportional to the

electric density of that point of the inner coating at which the discharge takes place. As the density of any point of the inner coating, other things remaining the same, is proportional to the entire charge, the striking distance is proportional to the quantity of electricity in a jar. The measurement of the charge of a battery, however, by means of the striking distance, can only take place when the charge disappears.

By means of Lane's electrometer, which depends on an application of this principle, the charge of a jar or battery may be measured. This apparatus, *c* (fig. 732), consists of an ordinary Leyden jar, near which there is a vertical metallic support. At the upper end is a brass rod, with a knob at one end, which can be placed in metallic connection with the outside of the jar: the rod being movable, the knob can be kept at a measured distance from the knob of the inner coating. Fig. 732 represents the operation of measuring the charge of a jar by means of this apparatus. The jar *b*, whose charge is to be measured, is placed on an insulated stool with its outer coating in metallic connection with the inner coating of Lane's jar *c*, the outer coating of which is in connection with the ground, or still better with a system of gas or water pipes; *a* is the conductor of the machine. When the machine is worked, positive electricity passes into the jar *b*; a proportionate quantity of positive electricity is repelled from its outer coating, passes into the inner coating of the electrometer, and there produces a charge. When this has reached a certain limit, it discharges itself between the two knobs, and as often as such a discharge takes place, the same quantity of positive electricity will have passed from the machine into the battery; hence its charge is proportional to the number of discharges of the electrometer.

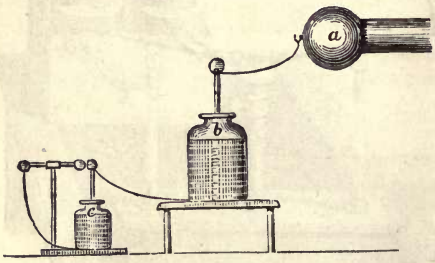


Fig. 732.

778. **Harris's unit jar.**—Harris's unit jar (fig. 733) is an application of the same principle, and is often convenient for measuring quantities of electricity. It consists of a small Leyden phial, 4 inches in length and $\frac{3}{4}$ inch in diameter, coated to about an inch from the end, so as to expose about 6 inches of coated surface. It is fixed horizontally on a long insulator, and the charging rod connected at P with the conductor of the machine, while the outer coating is connected with the jar or battery by the rod *t p*. When the accumulation of electricity in the interior has reached a certain height depending on the distance of the two balls *m* and *n*, a discharge ensues, and marks a certain quantity of electricity received as a charge by the battery, in terms of the small jar.

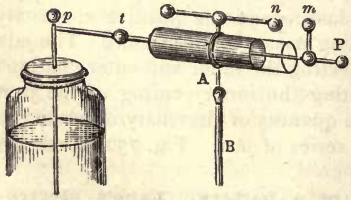


Fig. 733.

779. **Volta's condensing electroscope.**—The condensing electroscope invented by Volta is a modification of the ordinary gold-leaf electroscope (751). The rod to which the gold-leaves are affixed terminates in a disc instead of in a knob, and there is another disc of the same size provided with an insulating glass handle. The discs are covered with a layer of insulating shellac varnish (fig. 734).

To render very small quantities of electricity perceptible by this apparatus, one of the plates, which thus becomes the *collecting plate*, is touched with the body under examination. The other plate, the *condensing plate*, is connected with the ground by touching it with the finger. The electricity of the body, being diffused over the collecting plate, acts inductively through the varnish on the other plate, attracting the opposite electricity, but

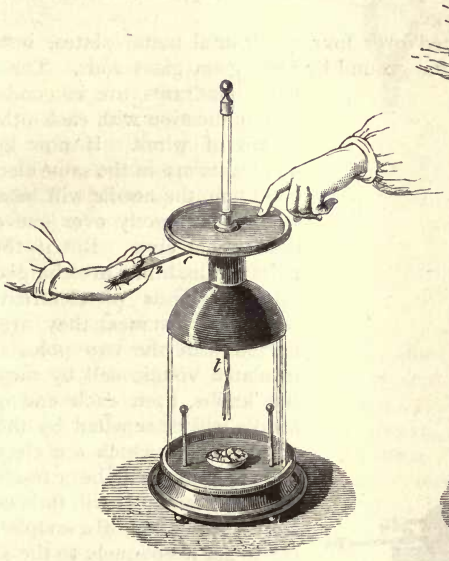


Fig. 734.

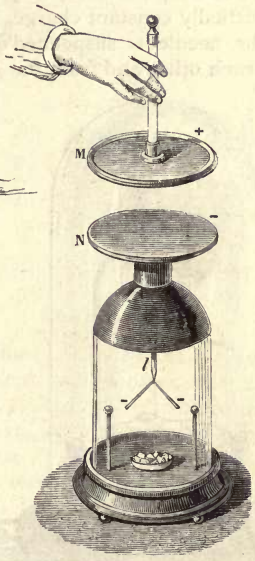


Fig. 735.

repelling that of like kind. The two electricities thus become accumulated on the two plates just as in a condenser, but there is no divergence of the leaves, for the opposite electricities counteract each other. The finger is now removed, and then the source of electricity, and still there is no divergence ; but if the upper plate be raised (fig. 735) the neutralisation ceases, and the electricity being free to move diffuses itself over the rod and the leaves, which then diverge widely. The delicacy of this electroscope is increased by adapting to the foot of the apparatus two metal rods, terminating in knobs ; for these knobs, being excited by induction from the gold-leaves, react upon them.

A still further degree of delicacy is attained if the rods be replaced by two Bohnenberger's dry piles (818), one of which presents its positive and the

other its negative pole. Instead of two gold-leaves there is only one; the least trace of electricity causes it to oscillate either to one side or to the other, and at the same time shows the kind of electricity.

780. **Thomson's quadrant electrometer.**—Lord Kelvin devised a new and delicate form of electrometer, by which accurate measurements of the amount of electrical charge may be made. The principle of this instrument may be understood from the following description of a form of it constructed for lecture purposes by Messrs. Elliott.

A light flat broad aluminium needle (fig. 736) hangs by a very fine wire from the inner coating of a charged Leyden jar, the outer coating being in conducting communication with the earth. The whole apparatus is enclosed within a glass shade, and the air is kept dry by means of a dish of sulphuric acid; there is, therefore, very little loss of electricity, and the needle remains at a virtually constant charge.

The needle is suspended over four quadrantal metal plates, insulated from each other and from the ground by resting on glass rods. The alternate quadrants are in conducting communication with each other by means of wires. If now all the quadrants are in the same electrical condition, the needle will be at rest when it is directly over one of the diametrical slits. But if the two pairs of quadrants are charged with opposite kinds of electricity, as when, for instance, they are connected with the two poles of an insulated voltaic cell by means of the knobs, then each end of the needle will be repelled by the pair of quadrants which are electrified like itself, and will be attracted by the other pair. It will thus be subject to the action of a couple tending to set it obliquely to the slit.

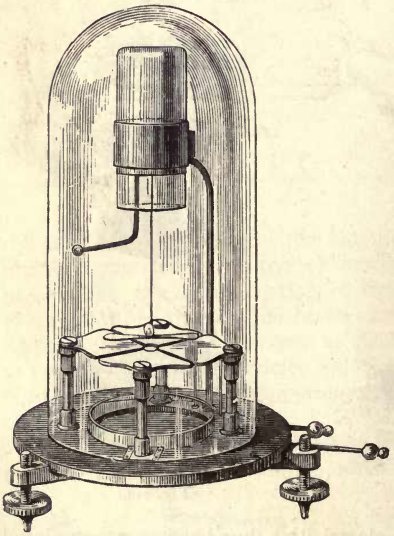


Fig. 736.

In order to render the slightest motion of the needle visible, a small silver concave mirror with a radius of about a metre is fixed above it. The light of a petroleum lamp, not represented in the figure, strikes against this, and is reflected as a spot on a horizontal scale. Any deflection of the needle, either on one side or the other, is indicated by the motion of the spot of light on the scale (520).

781. **Thomson's absolute electrometer.**—Another class of electrometers, also invented by Lord Kelvin, have the advantage of furnishing a direct measure of electrical constants in absolute measure. Fig. 737 represents the essential features of a modified form of the electrometer, which has been devised by Professor Foster for class experiments.

Two plane metal discs A and B, about 10 cm. in diameter, are kept at a

distance from each other, which is small in proportion to their diameters, but which can be very accurately measured. Out of the centre of the upper one is cut a disc *c*; this is suspended by insulating threads from one end of the arm *a b* of a balance, at the other end of which is a counterpoise, or a scalepan *p*. At the end of the arm is a fork, across which is stretched a fine wire; when the disc is exactly in the plane of the circular band or ring which surrounds it, and which is called the *guard ring*, this fine wire is exactly across the interval between two marks in the upright, and the position of which can be accurately determined by means of the lens C. The disc and the guard ring are kept at a constant potential, being connected by a wire with a constant source of electricity, while the other can be kept at any potential.

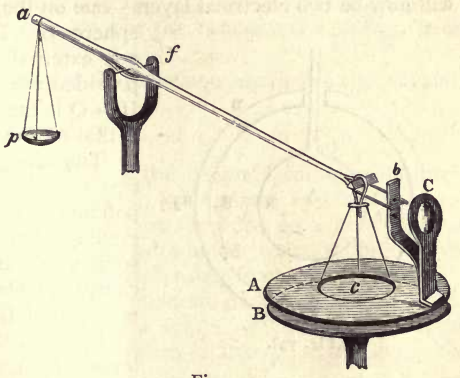


Fig. 737.

Suppose now that the whole system is at the same potential, and that the disc is exactly balanced so as to be in the plane of the guard ring. If now A be electrified to a given potential, while the plate B is connected with the earth, then the body charged with electricity of higher potential—that is, the disc—will be urged towards the body of lower potential, the fixed plate; and in order to retain it exactly in the plane of the guard ring the force applied at the other end of the lever must be increased. This may be done by altering the distance of the counterpoise, or by adding weights to a scalepan, and the additional weight thus applied is a measure of the attractive force.

Now, it can be shown that the attractive force between any two plates electrified to different potentials is proportional to the square of the difference of potentials, provided the distance between them is small in comparison with their area, and that the portions of the plates opposite each other are at some distance from the edge. These conditions are fulfilled in the above case. If S is the area of the disc, d the distance of the plates, $V - V_1$ the difference of potentials, and F the force required to balance a certain attraction, then

$$F = \frac{(V - V_1)^2 S}{8\pi d^2}$$

for $V = 0$, this is $\frac{V^2 S}{8\pi d^2}$ and $V = d\sqrt{\frac{8\pi F}{S}}$. Now as F is expressed by a weight, and S and d depend on measures of length, we have a means of expressing difference of potentials in absolute measure (709). It is also clear that the experiments may be modified by making the weight constant, and the distance variable. By means of micrometric arrangements the distance of the plates may be varied and measured with very great accuracy. This principle is applied in a portable form of this electrometer.

782. **Potential and capacity of a Leyden jar.**—These may be most conveniently investigated by considering the case of a spherical jar. Let us suppose A (fig. 738) to represent an insulated metal sphere, and let us consider it placed in conducting communication with a source of, say, positive electricity, which is supposed to be at a constant potential V. Then its potential V is $\frac{q}{R}$, and its charge $q = VR$, R being the radius of the sphere A.

Suppose now it be possible to surround this sphere by an external conducting shell or envelope B, which is in connection with the ground; movements of electricity will take place; a new equilibrium will be established, and there will now be two electrical layers—one on the sphere A, and the other on the sphere B. These will have no action on any external point, which is only possible provided the charges are equal and contrary. If $+Q$ is the charge on the inner, then $-Q$ is that on the outer sphere (745).

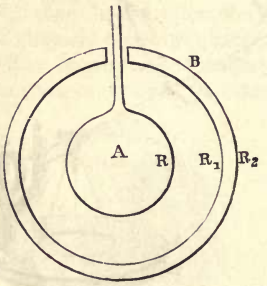


Fig. 738.

The charge of the original sphere is at first not altered by this operation, but its potential is less, its capacity being now greater; but, as it is in contact with the source, which is constant, it receives fresh charges of electricity until it is again at the potential of the source V.

Now let us suppose that the insulating layer which separates the inner from the outer coating is air, and that its thickness is t; then the potential V of the whole system is made up of two parts, the first due to the electrical charge of the inner sphere $V = +\frac{Q}{R}$, and the second due to the charge of the outer sphere $= -\frac{Q}{R'}$; that is, $V = \frac{Q}{R} - \frac{Q}{R'} = \frac{Q(R'-R)}{RR'}$, or $Q = \frac{VRR'}{R'-R}$.

The charge of the insulated sphere $q = VR$; hence $\frac{q}{Q} = \frac{R'-R}{R'}$. But $R'-R$ is the thickness of the dielectric, which, for the sake of simplicity, we will suppose is air, and calling this t, we have $\frac{Q}{q} = \frac{R'}{t}$; that is, the charge is inversely as the thickness of the dielectric.

It is to be observed that the results here obtained apply strictly only to the supposed case in which the inner conductor is completely surrounded by the outer one (745), which is not the case with the ordinary form of a Leyden jar. It may, however, be applied to them if we compare homologous jars; in the above formula $Q = \frac{VRR'}{R'-R}$, now if R and R′ are nearly equal, then $Q = \frac{VR^2}{t} = \frac{4\pi VR^2}{4\pi t}$, which may be written $\frac{VS}{4\pi t}$, where $4\pi R^2 = S$ is the coated surface of one side and t the thickness of the dielectric. In this formula $\frac{S}{4\pi t}$ is a constant for a Leyden jar of given dimensions, and represents the *capacity* of the jar.

If instead of air there be a solid or liquid dielectric, whose specific inductive capacity is κ, the formula becomes $Q = \frac{VS}{4\pi t} = \frac{VS\kappa}{4\pi t}$. If the dielectric be partly air and partly some other material, such as glass, then if the thickness of this latter is θ, $Q = \frac{VS}{4\pi\left(t - \theta + \frac{\theta}{\kappa}\right)}$. The expression θ is sometimes written t', and represents the thickness of the layer of air equivalent to it in specific inductive capacity. It is also called the *reduced thickness*.

From the expression $Q = \frac{VRR'}{R'-R}$ we get the capacity $C = \frac{RR'}{R'-R}$; or as above $= \frac{RR'}{t}$, so that the presence of the envelope multiplies the capacity of the sphere by $\frac{R'}{t}$.

If R′ is so great that the value of R in the denominator may be disregarded, we get C = R, which is the expression for the capacity of an insulated sphere (739); such a sphere may indeed be regarded as a condenser, in which the layer of air, between it and the sides of the room, represents the dielectric. This represents in effect the condensing force (767).

When n identical jars are joined in surface, we have a condenser whose capacity is equal to the n-fold capacity of a single jar. If these n jars are joined in cascade, the capacity of the system is that of a single jar, the dielectric of which is n times as thick.

A cylindrical Leyden jar with the diameter 10 cm. and coated to a height of 20 cm. will have a total coated surface of 706·5; if the glass has the dielectric constant 5, and its thickness is 3 mm., the capacity of the jar will be 937·5; and as the capacity of a sphere is equal to its radius (739), it will be equal to the capacity of a freely suspended sphere 18·75 metres in diameter (748).

THE ELECTRIC DISCHARGE

783. **Effects of the electric discharge.**—The recombination of the two electricities which constitutes the electrical discharge may be either continuous or sudden: *continuous*, or of the nature of a current, as when the two conductors of a Holtz's machine are joined by a chain or a wire; and *sudden* or *disruptive*, as when the opposite electricities accumulate on the surface of two adjacent conductors, till their mutual attraction is strong enough to overcome the intervening resistances, whatever they may be. But the difference between a sudden and a continuous discharge is one of degree, and not of kind, for there is no such thing as an absolute non-conductor, and the very best conductors, the metals, offer an appreciable resistance to the passage of electricity. Still the difference at the two extremes of the scale is sufficiently great to give rise to a wide range of phenomena.

Riess showed that the discharge of a battery does not consist in a simple union of the positive with the negative electricity, but that it consists

of a series of successive partial discharges. The direction of the discharge depends mainly on the length and nature of the circuit.

Feddersen examined the discharge of a Leyden jar by the arrangement represented in fig. 739, in which the spark of the Leyden jar passes between the knobs *a* and *b*. On the axis *xx*, which by means of clockwork is rotated at a known and uniform rate, are two long focus concave mirrors, *d* and *c*; to

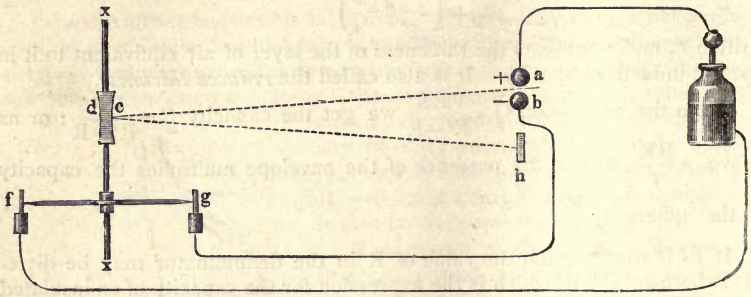


Fig. 739.

the axis is also attached a brass strip, the ends of which just touch the bare ends of the insulated wires *f* and *g* at the moment the spark passes between *a* and *b*; at this instant the spark is in the vertical plane of the mirror. The image of the spark is reflected on the ground-glass plate *h*.

Observed in this manner the spark is seen as a narrow band of light the length of which varies with the duration of the discharge. The duration increases with the striking distance, and with the number of jars.

When the resistance of the circuit through which the discharge takes place is small, it is found that the discharge is an *oscillatory* one, consisting of a series of separate discharges in alternating directions; the image is traversed by a number of dark lines.

When the resistance is greater the discharge is a single *continuous* one, and its image is that of a continuous band of light. With very great resistance the discharge is an *intermittent* one, and consists of sparks following each other at irregular intervals.

These oscillatory discharges may be illustrated by means of a simple hydrostatical experiment. Suppose that in the U-tube (fig. 740) is a valve, *s*, by which the two tubes are separated, and that water is poured in one so that it is at the height L above the level *oo*, and in the other in the corresponding distance L′ below the level. When the valve is suddenly opened, the water passes through, and only comes to rest in the position *oo* after several oscillations about this level. Suppose the valve to be suddenly closed during the oscillation, it may easily happen that the water is higher in that limb in which it was previously lower. This would represent the case observed by Oettingen with the electrical residues, who found them to be sometimes negative and sometimes positive.

Fig. 740.

Again, if the valve be only slightly opened, so that great resistance is

offered, the water slowly sinks to its level, the discharge is continuous, and there are no oscillations; this corresponds to the case in which the electrical resistance is very great.

We may further compare the dielectric in a state of strain, like the glass of a charged Leyden jar, to a narrow steel plate, clamped at one end; if the free end is pulled aside, the plate is in a state of strain, and when this strain is removed the plate comes to rest after making a series of oscillations. To prevent these oscillations the plate must be exposed to a great resistance, by being placed, for instance, in a viscous liquid; in like manner, as we have seen, by offering a great resistance to the electrical discharge, it becomes continuous. The rate at which a vibrating steel plate or a stretched string comes to rest will depend on its mass and on its inertia. In like manner the period of oscillation of an electrical discharge depends on a certain coefficient of self-induction (907), which represents the electro-magnetic inertia of the medium about the circuit.

The oscillatory nature of the discharge has been confirmed by the observations of Paalzow on the luminous phenomena seen in highly rarefied gases when it takes place in them, as well as by the manner in which a magnet affects the phenomena. Von Helmholtz had already deduced the necessity of such an oscillating motion from the laws of the conservation of energy, and Lord Kelvin and Kirchhoff deduced the conditions under which it occurs.

784. **Work effected by the discharge of a Leyden jar.**—The work required to charge a Leyden jar is $W = \frac{1}{2} QV = \frac{CV^2}{2} = \frac{Q^2}{2C} = \frac{1}{2} \frac{S}{4\pi t} V^2$; that is, is proportional to the surface and to the square of the potential, and is inversely as the thickness of the insulator. From the principle of the conservation of energy, this stored-up energy reappears when the jar is discharged. This occurs partly in the form of a spark, partly in the heating effect of the whole system of conductors through which the discharge takes place. When the armatures are connected by a thick short wire, the spark is strong and the heating effect small; if, on the contrary, the jar is discharged through a long fine wire, this becomes more heated, but the spark is weaker.

If a series of identical jars are each separately charged from the same source, they will each acquire the same potential, which will not be altered if all the jars are connected by their inner and outer coatings respectively. The total charge will be the same as if the battery had been charged directly from the source, and its energy will be $W = \frac{1}{2} VnQ = \frac{1}{2} VQ$; that is, the energy of a battery of n equal jars is the same as that of a single jar of the same thickness but of n times the surface.

Let us consider two similar Leyden jars having respectively the capacities c and c'; let one of them be charged to potential V, and let the other remain uncharged. Suppose now that the inner and outer coatings of the jars are respectively connected with each other. Then the energy of the charged jar alone is $W = \frac{1}{2} \frac{Q^2}{c}$, and when it is connected with the other, the original charge will spread itself over the two, so that the energy of the charge in the two jars is $W' = \frac{Q^2}{2(c + c')}$. Hence $W : W' = c + c' : c$; and there-

fore, since $c + c'$ is always greater than c, there must be a loss of energy. In point of fact, when a charged jar is connected with an uncharged one, a spark passes which is the equivalent of this loss of energy.

It follows, further, that when two jars at different potentials are united there is always a loss of energy.

If a series of n similar jars are joined in surface, and a given charge of electricity is imparted to them, the energy is inversely as the number of jars; but, when they are charged from a source of constant potential, the energy is proportional to the number of jars. If, however, the jars are arranged in cascade, then for a given charge the energy is n times that of a single jar, while for a given potential it is n times smaller. It is sometimes convenient to arrange the jars in a combination of the two systems.

785. **Physiological effects.**—The shock from the electrical machine has been already noticed (770). The shock taken from a charged Leyden jar by grasping the outer coating with one hand and touching the inner with the other is much more violent, and has a peculiar character. With a small jar the shock is felt in the elbow; with a jar of about a quart capacity it is felt across the chest, and with jars of still larger dimensions in the stomach.

A shock may be given to a large number of persons simultaneously by means of the Leyden jar. For this purpose they must form a chain by joining hands. If, then, the first touches the outside coating of a charged jar, while the last at the same time touches the knob, all receive a simultaneous shock, the intensity of which depends on the charge, and on the number of persons receiving it. Those in the centre of the chain are found to receive a less violent shock than those near the extremities. The Abbé Nollet discharged a Leyden jar through an entire regiment of 1,500 men, who all received a violent shock in the arms and shoulders.

With large Leyden jars and batteries the shock is sometimes very dangerous. Priestley killed rats with batteries of 7 square feet coated surface, and cats with a battery of about $4\frac{1}{2}$ square yards coating.

Experience shows that the physiological effect varies with the electrical energy; thus a discharge from an ordinary electrical machine which gives a spark of nearly a foot may be taken without danger, while one from a battery of large capacity of a few millimetres could not be borne. The duration of the discharge has also an influence; a battery which gives a violent shock when discharged in ordinary conditions, only gives a feeble one when discharged through a moist cord, which only delays the rapidity of the discharge.

786. **Luminous effects.**—The recombination of two electricities of high potential (738) is always accompanied by a disengagement of light, as is seen when sparks are taken from a machine, or when a Leyden jar is discharged. The better the conductors on which the electricities are accumulated, the more brilliant is the spark; its colour varies not only with the nature of the bodies, but also with the nature of the surrounding medium and with the pressure. The spark between two charcoal points is yellow, between two balls of silvered copper it is green, between knobs of wood or ivory it is crimson. In air at the ordinary pressure the electric spark is white and brilliant; in rarefied air it is reddish; and in vacuo it is violet.

In oxygen, as in air, the spark is white; it is reddish in hydrogen, and green in the vapour of mercury; in carbonic acid it is also green, while in nitrogen it is blue or purple, and accompanied by a peculiar sound. Generally speaking, the higher the potential, the greater is the lustre of the spark.

When these sparks are examined by the spectroscope (576) they show the lines characteristic of the metals between which the spark passes, and also of the gas in which it takes place. If the knobs are of different metals the lines of both are seen. Part of the energy is accordingly consumed in detaching and volatilising the metal particles on the two electrodes; when a powerful discharge takes place between a knob of gold and one of silver, the latter metal is found on the gold knob, while some gold is found on the silver.

787. **Spark and brush discharge.**—The shapes which luminous electric phenomena assume may be classed under two heads—the *spark* and the *brush*. The brush forms when the electricity leaves the conductor in a continuous flow; the spark, when the discharge is discontinuous. The formation of one or the other of these depends on the nature of the conductor and on the nature of the conductors in its vicinity; and small alterations in the position of the surrounding conductors transform the one into the other.

The spark which at short distances appears straight, at longer distances has a zigzag shape with diverging branches. Its length depends on the density at the part of the conductor from which it is taken; and to obtain the longest sparks the electricity must be of as high a density as possible, but not so high as to discharge spontaneously. With long sparks the luminosity is different in different parts of the spark.

The brush derives its name from the radiating divergent arrangement of the light, and presents the appearance of a luminous cone, whose apex touches the conductor. Its size and colour differ with the nature and form of the conductor; it is accompanied by a peculiar hissing noise, very different from the sharp crack of the spark. Its luminosity is far less than that of the spark; for while the latter can easily be seen by daylight, the former is only visible in a darkened room.

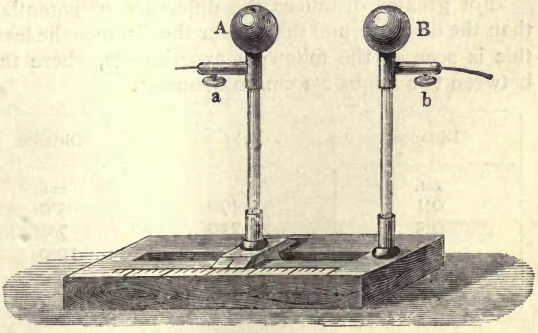


Fig. 741.

The brush discharge may be obtained by placing on the conductor a wire filed round at the end, or, with a powerful machine, by placing a small bullet on the conductor. The brush from a negative conductor is less than from a positive conductor; the cause of this differ-

ence has not been satisfactorily made out, but may originate in the fact, which Faraday has observed, that negative electricity discharges into the air at a somewhat lower density than positive electricity; so that a negatively charged knob sooner attains that density at which spontaneous discharge takes place, than does a positively charged one, and therefore discharges the electricity at smaller intervals and in less quantities.

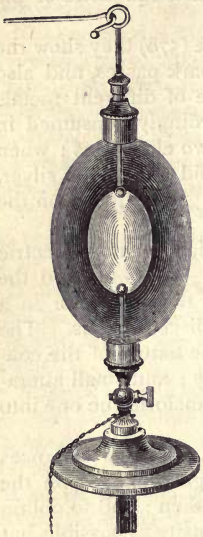


Fig. 742.

When electricity, in virtue of its high density, issues from a conductor, no other conductor being near, the discharge takes place without noise, and at the places at which it appears there is a pale blue luminosity called the *electrical glow*, or on points a star-like centre of light. It is seen in the dark by placing a point on the conductor of the machine. It may be regarded as a very short brush.

788. **Striking distance.**—Sir W. Harris by means of experiments with his unit jar suitably modified, and Riess by independent researches, found that for small distances the striking distance is directly proportional to the quantity of electricity, and inversely proportional to the coated surface; in other words, it is proportional to the potential. For his experiments Riess used the *spark micrometer*, which consists of two metal knobs, A and B (fig. 741), provided with binding screws, *a* and *b*, and on insulating supports, the distance of which from each other could be varied by a micrometric screw.

The striking distance varies slightly with the shape of the electrodes; thus for the same distance the difference of potential required is slightly greater for two spheres than for two plates.

For greater distances the difference of potential increases less rapidly than the distance, and the greater the distance the less is the rate of increase; this is seen in the following experiments, where the discharge took place between two knobs 2·2 cm. in diameter.

Distance	Volts	Distance	Volts
cm.		cm.	
0·1	5,490	5·0	94,800
0·5	26,730	7·0	107,700
1·0	48,600	10·0	119,100
2·0	64,800	12·0	124,200
3·0	76,800	15·0	127,800

The striking distance in air is virtually the same for the spark proper as for the brush.

The influence of pressure on the electric discharge may be studied by means of the *electric egg*. This consists of an ellipsoidal glass vessel (fig. 742) with metal caps at each end. The lower cap is provided with a stopcock, so that it can be screwed into an air-pump, and also into a heavy

metallic foot. The upper metal rod moves up and down in a leather stuffing-box ; the lower one is fixed to the cap. A vacuum having been made, the stopcock is turned, and the vessel screwed into its foot ; the upper part is then connected with a powerful electrical machine, and the lower one with the ground. On working the machine, the globe becomes filled with a feeble violet light continuous from one end to the other, and resulting from the recomposition of the positive electricity of the upper cap with the negative of the lower. If the air be gradually allowed to enter by opening the stop-cock, the light now appears white and brilliant, and is only seen as an ordinary intermittent spark.

Some beautiful effects of the electric discharge are obtained by means of Geissler's tubes (518), which will be noticed under Dynamical Electricity.

Fig. 743.

789. **Luminous tube and square.**—The *luminous tube* (fig. 743) is a glass tube about a yard long, round which are arranged in a spiral form a series of lozenge-shaped pieces of tinfoil, between which are very short intervals. There is a brass cap with hooks at each end, in which the spiral terminates. If one end be presented to a machine in action, while the other is held in the hand, sparks appear simultaneously at each interval, and produce a brilliant luminous appearance, especially in the dark.

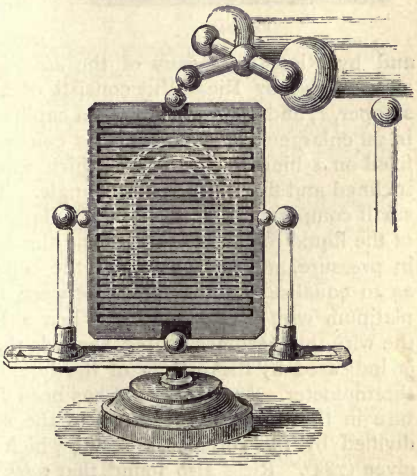


Fig 744.

The *luminous pane* (fig. 744) is constructed on the same principle, and consists of a square of ordinary glass, on which is fastened a narrow strip of tinfoil folded parallel to itself for a great number of times. Spaces are cut out of this strip so as to represent any figure, a portico for example. The pane being fixed between two insulating supports, the upper extremity of the strip is connected with the electrical machine, and the lower part with the ground. When the machine is in operation, a spark appears at each interval, and reproduces in luminous flashes the object represented on the glass

790. **Heating effects.**—Besides being luminous, the electric spark is a source of great heat. When it passes through inflammable liquids, as ether or alcohol, it inflames them. An arrangement for effecting this is represented in fig. 745. It is a small glass cup through the bottom of which passes a metal rod, terminating in a knob and fixed to a metal foot. A quantity of liquid sufficient to cover the knob is placed in the vessel. The outer coating of the jar having been connected with the foot by means of a chain, the spark which passes when the two knobs are brought near each other inflames the liquid. With ether the experiment succeeds very well, but alcohol requires to be first warmed.

Coal gas may also be ignited by means of the electric spark. A person standing on an insulated stool places one hand on the conductor of a machine which is then worked, while he presents the other to the jet of gas issuing from a metallic burner. The spark which passes ignites the gas. When a battery is discharged through an iron or steel wire it becomes heated, and even made incandescent or melted if the discharge is very powerful.

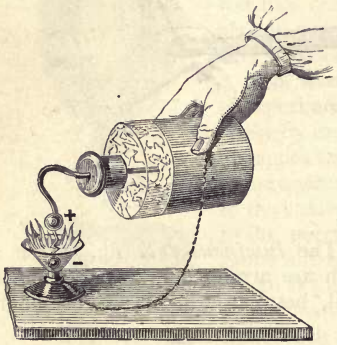


Fig. 745.

If, in discharging a jar, the discharge does no other work, then the whole of the energy of the charge (784) appears in the form of heat; and if we divide this by Joule's equivalent (497), we have the total heating due to any charge.

The laws of this heating effect were investigated independently by Harris and by Riess by means of the *electric thermometer*. In its later forms as modified by Riess, this consists of a glass bulb (fig. 746), closed by a stopper, *c*, and to which is fixed a capillary tube bent twice, and terminating in an enlargement; this contains coloured liquid. The whole apparatus is fixed on a hinged support, A, which works on the base B, so that it can be inclined and fixed at any given angle. The diameter of the tube being very small compared with that of the enlargement, a considerable displacement of the liquid may take place along the scale without any material alteration in pressure, and before making the experiment the stopper *c* is opened so as to equalise the pressure. Between the binding screws *a* and *b* a fine platinum wire is stretched. When a Leyden jar is discharged through the wire this becomes heated, expands the air in the bulb, and the expansion is indicated by the motion of the liquid along the graduated stem of the thermometer. In this way it has been found that the increase in temperature in the wire is proportional to the square of the quantity of electricity divided by the surface—a result which follows from the formula already given (784). Riess also found that *with the same charge, but with wires of different dimensions, the rise of temperature is inversely as the fourth power of the diameter.* Thus, compared with a given wire as unity, the *rise of temperature* in a wire of double or treble the diameter would be $\frac{1}{16}$ or $\frac{1}{81}$ as small; but as the masses of these wires are four and nine times

as great, *the heat produced* would be respectively $\frac{1}{4}$ and $\frac{1}{9}$ as great as in a wire of unit thickness.

If a jar charged to a given potential be discharged through the electrical thermometer, the discharge will take place at a certain striking distance, and a certain depression will be produced which is a measure of the heating effect in the thermometer. If now a card be interposed in the path of the discharge, a certain proportion of its energy will be expended in the mechanical perforation of the card, and the proportion in the thermometer will be less. Thus Riess found that that charge which when passed through air produced a depression of 15·9, when passed in addition through one card, two cards, and a plate of mica, produced depressions of 11·7, 8·0, and 6·8 respectively; showing that the heating effect was less according as more of the energy of the discharge was used for other purposes.

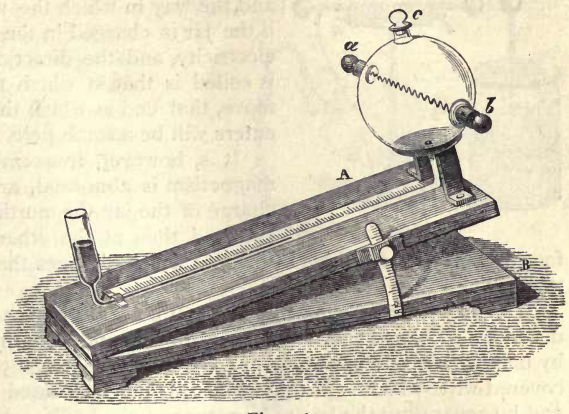


Fig. 746.

When an electric discharge is sent through gunpowder placed on the table of a Henley's discharger, it is not ignited, but is projected in all directions. But if a wet string be interposed in the circuit, a spark passes which ignites the powder. This arises from the retardation which electricity experiences in traversing a semi-conductor, such as a wet string; for the heating effect is proportional to the duration of the discharge.

When a charge is passed through sugar, heavy spar, fluor-spar, and other substances, they afterwards become phosphorescent in the dark. Eggs, fruit, &c., may be made luminous in the dark in this way.

When a battery is discharged through a gold leaf pressed between two glass plates or between two silk ribbons, the gold is volatilised in a violet powder which is finely divided gold. In this way what are called *electric portraits* are obtained.

Siemens showed that when a jar is charged and discharged several times in succession the glass becomes heated. Hence during the discharge there must be movements of the molecules of the glass, as Faraday supposed (747); we have here, probably, something analogous to the heating produced in iron when it is rapidly magnetised and demagnetised.

791. **Magnetic effects.**—By the discharge of a large Leyden jar or battery, a steel wire may be magnetised if it is laid at right angles to a conducting wire through which the discharge is effected, either in contact with the wire or at some distance. And even a steel rod or needle may be

magnetised by placing it inside a spiral of insulated copper wire, A (fig. 747), and passing one or more discharges through it. The polarity depends on the direction in which the electricity enters the coil and the way in which the wire is coiled. Thus if the jar is charged in the inside with positive electricity, and the direction in which the wire is coiled is that in which the hands of a watch move, that end at which the positive electricity enters will be a south pole.

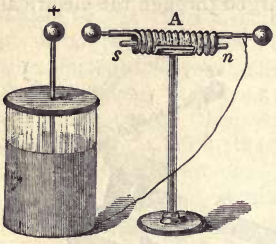


Fig. 747.

It is, however, frequently observed that the magnetism is abnormal, and that for the same charge of the jar the north pole is first at one end and then at the other. This is to be referred (783) to the residue in the jar, which changes the sign in an irregular manner.

To effect a deflection of the magnetic needle by the electric current produced by frictional electricity is more difficult. It may be accomplished by making use of a galvanometer consisting of 400 or 500 turns of fine silk-covered wire, which is further insulated by being coated with shellac varnish, and by separating the layers by means of oiled silk. When the prime conductor of a machine in action is connected with one end of the galvanometer wire, and the other with the ground, a deflection of the needle is produced.

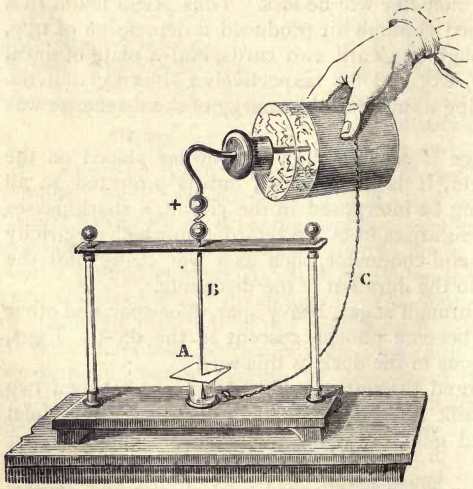


Fig. 748.

792. **Mechanical effects.**—The mechanical effects are the violent lacerations, fractures, and sudden expansions which ensue when a powerful discharge is passed through a badly conducting substance. Glass is perforated, wood and stones are fractured, and gases and liquids are violently disturbed. The mechanical effects of the electric spark may be demonstrated by a variety of experiments.

Fig. 748 represents an arrangement for perforating a piece of glass or card. It consists of two glass columns, with a horizontal cross-piece, in which is a pointed conductor, B. The piece of glass, A, is placed on an insulating glass support, in which is placed a second conductor, terminating also in a point, which is connected with the outside of the battery, while the knob of the inner coating is brought near the knob of B. When the discharge passes between the two conductors the glass is perforated. The experiment

only succeeds with a single jar when the glass is very thin; otherwise a battery must be used.

When the discharge takes place through a piece of cardboard between two points exactly opposite each other, the line of perforation is quite straight; but if not exactly opposite a slight hole is seen near the negative point. This phenomenon, which is known as *Lullin's experiment*, is probably connected with the fact that negative electricity discharges into air more readily than positive (787); in other words, that positive electricity must be raised to a higher potential in order to discharge.

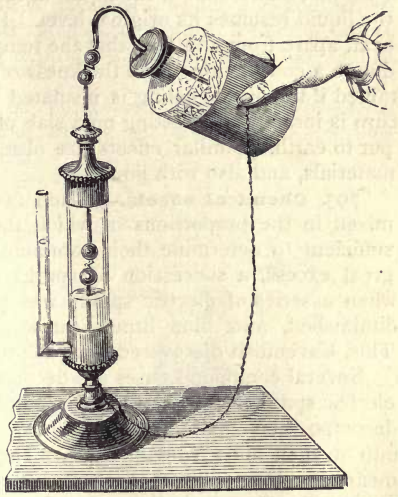


Fig. 749.

The perturbation and sudden expansion which the discharge produces may be illustrated by means of what is known as *Kinnersley's thermometer*. This consists of two glass tubes (fig. 749), which fit into metallic caps and communicate with each other. At the top of the large tube is a rod terminating in a knob, and moving in a stuffing-box, and at the bottom there is a similar rod with a knob. The apparatus contains water up to the level of the lower knob. When the electric discharge passes between the two knobs, the water is driven out of the larger tube and rises to a slight extent in the small one. The level is immediately re-established, and therefore the phenomenon is not due to a rise of temperature.

If the upper knob inside a Kinnersley's thermometer be replaced by a point, and the outside knob is connected with the prime conductor of a machine at work, the electricity discharges itself in the form of a brush, and a permanent displacement of the liquid in the stem shows that this is due to the heating effect of the brush discharge.

For the production of mechanical effects the universal discharger (fig. 731) is of great service. A piece of wood, for instance, placed on the table between the two conductors, is split when the discharge passes.

When a Leyden jar is charged it undergoes a true expansion which is not that due to heat. This was shown by Quincke, one of whose experiments is represented in fig. 750. It consists of a glass bulb A about 2 inches in diameter at the end of a narrow capillary tube, K, on an enlargement in which a platinum wire, B, is fused. The bulb and a portion of the stem

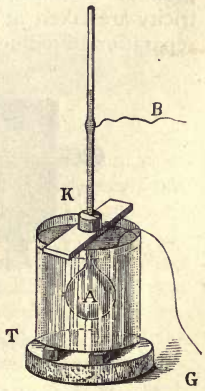


Fig. 750.

contains a conducting liquid, such as water or sulphuric acid, and it is placed in a vessel of ice-cold water, K, which can be connected with the earth by a conducting wire, G. If now this condenser is charged by connecting the wire B with an electrical machine, while G is in connection with the earth, there is a distinct depression of the liquid in the tube. When the jar is discharged the liquid resumes its original level. Hence this cannot have been due to heat, apart from the fact that the temperature was kept constant ; nor is it due to a contraction of the thickness of the glass. The same results are obtained if the outer coating is insulated by resting it on shellac, T, which in turn is insulated by resting on a slab of india-rubber, the inner coating being put to earth. Similar effects are observed with solid condensers of other materials, and also with liquids.

793. **Chemical effects.**—When two gases which act on each other are mixed in the proportions in which they combine, a single spark is often sufficient to determine their combination; but when either of them is in great excess, a succession of sparks is necessary. Priestley found that when a series of electric sparks was passed through moist air, its volume diminished, and blue litmus introduced into the vessel was reddened. This, Cavendish discovered, was due to the formation of nitric acid.

Several compound gases are decomposed by the continued action of the electric spark. With olefiant gas, sulphuretted hydrogen, and ammonia, the decomposition is complete ; while carbonic acid is partially decomposed into oxygen and carbonic oxide. The electric discharge also by suitable means can feebly decompose water, oxides, and salts ; but, though the same in kind, the chemical effects of statical electricity are by no means so powerful and varied as those of dynamical electricity. The chemical action of the spark is easily demonstrated by means of a solution of iodide of potassium. A small lozenge-shaped piece of filtering paper, impregnated with iodide of potassium, is placed on a glass plate, and one corner connected with the ground. When a few sparks from a conductor charged with positive electricity are taken at the other corner, brown spots are produced, due to the separation of iodine.

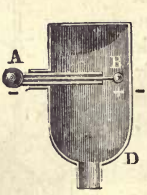


Fig. 751.

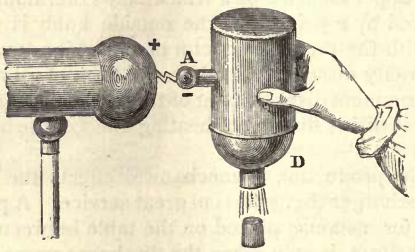


Fig. 752.

The *electric pistol* is a small apparatus which serves to demonstrate the chemical effects of the spark. It consists of a brass vessel (fig. 751), in which is introduced a detonating mixture of two volumes of hydrogen and one of oxygen, and which is then closed with a cork. In a tubulure in the

side there is a glass tube, in which fits a metal rod, terminated by the knobs A and B. The vessel is held as represented in fig. 752, and brought near the machine. The knob A becomes negatively, and B positively, electrified by induction from the machine, and a spark passes between the conductor and A. Another spark passes at the same time between the knob B and the side; this determines the combination of the gases, which is accompanied by a great disengagement of heat, and the vapour of water formed acquires such an expansive force that the cork is projected with a report like that of a pistol.

Among the chemical effects must be enumerated the formation of *ozone*, which is recognised by its peculiar odour, and by certain chemical properties. The odour is perceived when electricity issues from a conductor into the air through a series of points. It has been established that ozone is an allotropic modification of oxygen.

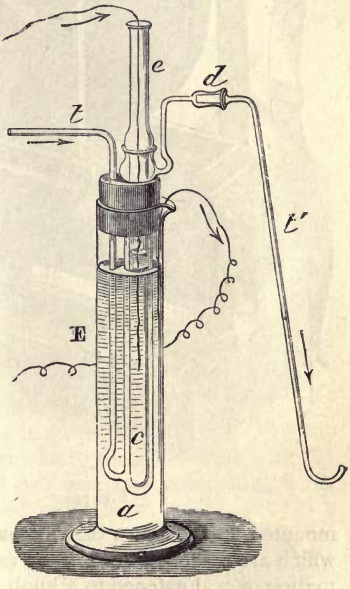


Fig. 753.

With these effects may be associated a certain class of phenomena observed when gases are made to act as the dielectric in a charged Leyden jar. An apparatus by which this is effected is represented in fig. 753; it is a modification of one invented by Siemens. It consists of a glass cylinder, E, containing dilute sulphuric acid; *a* is a glass tube closed at the bottom, and also containing sulphuric acid, in an enlargement of which at the top the inner tube *ec* fits. There is a tube, *t*, by which gas enters, and one, *dt'*, by which it emerges. When the acids in E and *e* are respectively connected with the two combs of a Holtz machine, or with the two terminals of a Ruhmkorff's coil, a certain condition or strain (747) is produced in the dielectric, which is known as the *silent discharge* or the *electric effluvium*. What that condition is cannot be definitely stated; but it gives rise to powerful and characteristic chemical actions, often differing from those produced by the spark.

By this apparatus large quantities of ozone may be produced.

794. **Application of the electrical discharge to firing mines.**—By the labours of Sir F. Abel in this country, and of Baron von Ebner in Austria, the electrical discharge has been applied to firing mines for military purposes, and the methods have acquired a high degree of perfection. The principle on which they are based may be understood from the following statement:—

One end of an insulated wire in which is a small break is placed in contact with the outside of a charged Leyden jar, the other end being placed near the inner coating. If now this end be brought in contact with the inner

coating the jar is discharged, and a spark strikes across the break; and if there be here some explosive compound it is ignited, and this ignition may of course be communicated to any gunpowder in which it is placed. If on one side of the break, instead of having an insulated wire direct back to the outer coating of the Leyden jar, an uncovered wire be led into the ground, the outside of the jar being also connected with the ground, the result is unchanged, the earth acting as a return wire. Moreover, if there be several breaks, the explosion will still ensue at each of them, provided the charge be sufficiently powerful.

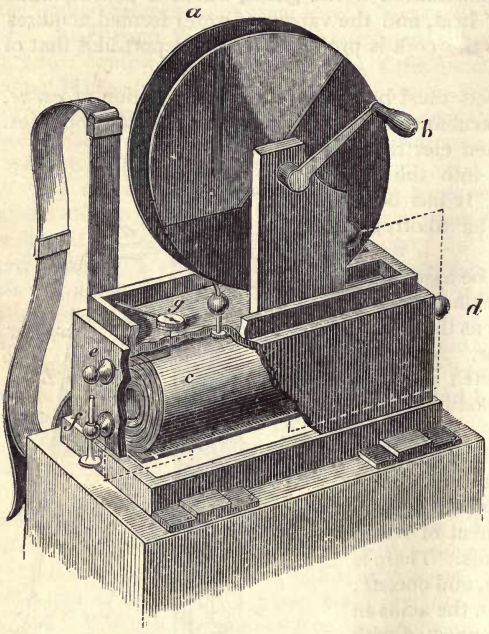


Fig. 754.

In the actual application it is of course necessary to have an arrangement for generating frictional electricity which shall be simple, portable, powerful, and capable of working in any weather. Fig. 754 represents a view of Von Ebner's instrument as constructed by Messrs. Elliott, part of the case being removed to show the internal construction.

It consists of two circular plates of ebonite, *a*, mounted on an axis so that they are turned by a handle, *b*, between rubber, which are so arranged as to be easily removed for the purposes of amalgamation, &c. Fastened to a knob on the base of the apparatus and projecting between the plates is a pointed brass rod, which acts as a collector of the electricity. The condenser or Leyden jar arrangement is inside the case, part of which has been removed to show the arrangement. It consists of india-rubber cloth coated on each side with tinfoil, and formed into a roll for the purpose of greater compactness. By means of a metal button the knob is in contact with one tinfoil coating, which thus receives the electricity of the machine, and corresponds to the inner coating of the Leyden jar. Another button, connected with the other tinfoil coating, rests on a brass band at the base of the apparatus which is in metallic contact with the cushions, the knob *d*, and the perforated knob in which slides a rod at the front of the apparatus. These are all in connection with the earth. The knob *e* is in metallic connection with a disc, *g*, provided with a light arm. By means of a flexible chain this is so connected with a trigger on the side of the apparatus not represented in the figure, that when the trigger is

depressed, the arm, and therewith the knob *e*, is brought into contact with the inner coating of the condenser.

On depressing the trigger, after a certain number of turns, a spark passes between the knob *e* and the sliding rod, and the striking distance is a measure of the working condition of the instrument.

The fuse used is known as *Abel's electrical fuse*, and has the following construction :—The ends of two fine copper wires (fig. 755) are imbedded in a thin solid gutta-percha rod, parallel to each other, but at a distance of about 1·5 mm. At one end of the gutta-percha a small cap of paper, *c c*, is fastened, in which is placed a small quantity of the priming composition, which consists of an intimate mixture of subsulphide of copper, subphosphide of copper, and chlorate of potassium. The paper is fastened down so that the exposed ends of the wires are in close contact with the powder.

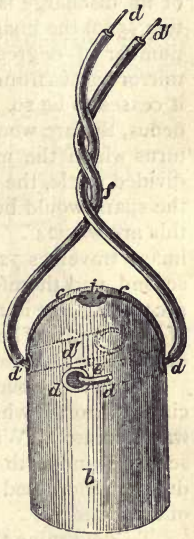


Fig. 755.

This is the actual fuse ; for service the capped end of the fuse is placed in a perforation in the rounded head of a wooden cylinder so as to project slightly into the cavity *g* of the cylinder. This cavity is filled with meal powder, which is well rammed down, so that the fuse is firmly imbedded. It is afterwards closed by a plug of gutta-percha, and the whole is finally coated with black varnish.

The free ends of the wire *a a* are pressed into small grooves in the head of the cylinder (fig. 756), and each end is bent into one of the small channels with which the cylinder is provided, and which are at right angles to the central perforation. They are wedged in here by driving in small copper tubes, the ends of which are then filed flush with the surface of the cylinder. The bared ends of two insulated conducting wires are then pressed into one of the small copper tubes or eyes, and fixed there by bending the wire round on to the wood, as shown at *e*.

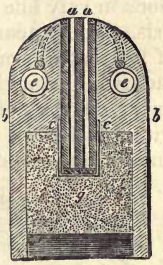


Fig. 756.

The conducting wire used in firing may be thin, but it must be well insulated. One end which is bared, having been pressed into the hole *d* of the fuse (fig. 755), the other is placed near the exploder. In the other hold *d'* of the fuse a wire is placed which serves as earth wire, care being taken that there is no connection between the two wires. The fuse having been introduced into the charge, the earth wire is placed in good connection with the ground. The knob *f* of the exploder is also connected with the earth by leading the bare wire into water or moist earth, and the condition of the machine tested. The end of the insulated wire is then connected with the knob *e* and the rod drawn down ; at the proper signal the handle is turned the requisite number of times, and when the signal is given the trigger is depressed, and the explosion ensues.

When a number of charges are to be fired they are best placed in a single circuit, care being taken that the insulation is good.

795. **Duration of the electric spark.**—Wheatstone measured the duration of the electric spark by means of the rotating mirror which he invented for this purpose. At some distance from this instrument, which can be made to rotate with a measured velocity, a Leyden jar is so arranged that the spark of its discharge is reflected from the mirror. Now, from the laws of reflection (520) the image of the luminous point describes an arc of double the number of degrees which the mirror describes, in the time in which the mirror passes from the position in which the image is visible to that in which it ceases to be so. If the duration of the image were absolutely instantaneous, the arc would be reduced to a mere point. Knowing the number of turns which the mirror makes in a second, and measuring, by means of a divided circle, the number of degrees occupied by the image, the duration of the spark would be determined. In one experiment Wheatstone found that this arc was 24°. Now, in the time in which the mirror traverses 360° the image traverses 720° ; but in the experiment the mirror made 800 turns in a second, and therefore the image traversed 576,000° in this time ; and as the arc was 24°, the image must have lasted the time expressed by $\frac{24}{576000}$, or $\frac{1}{24000}$ of a second. Thus the discharge is not instantaneous, but has a certain duration, which, however, is excessively short.

Feddersen found that when greater resistances were interposed in the circuit through which the discharge was effected, the duration of the spark was increased. With a tube of water 9 mm. in length, the spark lasted 0·0014 second ; and with one of 180 mm. its duration was 0·0183 second. The duration increased also with the striking distance, and with the dimensions of the battery.

To determine the duration of the electric spark Lucas and Cazin used a method by which it may be measured in millionths of a second. The method is an application of the vernier (10). A disc of mica 15 centimetres in diameter is blackened on one face, and at the edge are traced 180 equal divisions in very fine transparent lines. The disc is mounted on a horizontal axis, and by means of a gas engine it may be made to turn with a velocity of 100 to 300 turns in a second. A second disc of silvered glass of the same radius is mounted on the same axis as the other and very close to it ; at its upper edge six equidistant transparent lines are traced, forming a vernier with the lines on the mica. For this, the distance between two consecutive lines on the two discs is such that five divisions of the mica disc D C correspond to six divisions of the glass disc AB, as seen in fig. 757. Thus the vernier gives the sixths of a division of the mica disc (10). In the apparatus the lines AB are not above the lines CD, but are at the same distance from the axis, so that the latter coincide successively with the former.

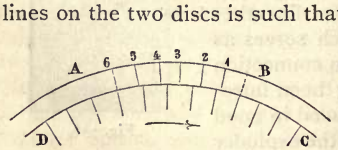


Fig. 757.

The mica disc is contained in a brass box, D (fig. 758), on the hinder face of which is fixed the vernier. In the front face is a glass window, O, through which the coincidence of the two sets of lines can be observed by means of a magnifying lens, L.

The source of electricity is a battery of 2 to 8 jars, each having a coated surface of 1,243 square centimetres, and charged continuously by a Holtz machine. The spark strikes between two metal balls *a* and *b*, 11 millimetres in diameter. Their distance can be varied, and at the same time measured, by means of a micrometric screw, *r*. The two opposite electricities arrive by wires *m* and *n*, and the sparks strike at the principal focus of a condensing lens placed in the collimator C, so that the rays which fall on the vernier are parallel.

The motion is transmitted to the toothed wheels and to the mica disc by means of an endless band, which can be placed on any one of three pulleys

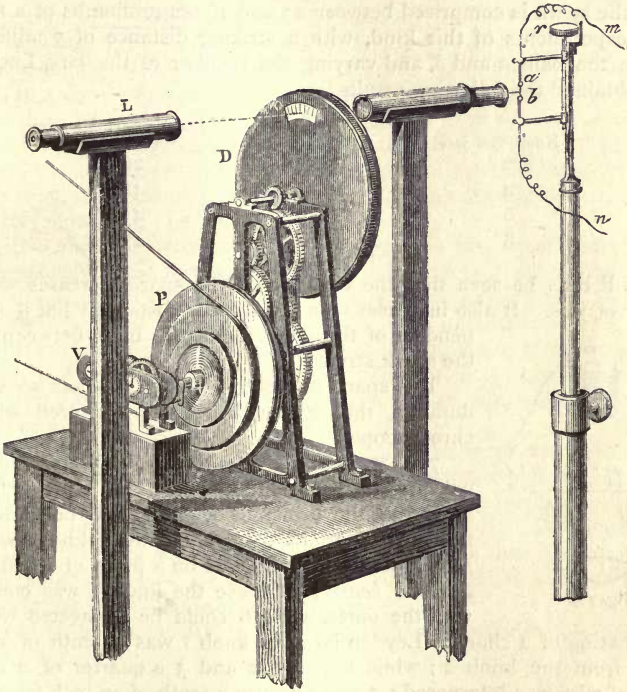


Fig. 758.

P, so that the velocity may be varied. At the end of the axis of the pulleys is a bent wire which moves a counter, V, that marks on three dials the number of turns of the disc.

These details being premised, suppose the velocity of the disc is 400 turns in a second. In each second 400 + 180, or 72,000 lines pass before the observer's eye in each second; hence an interval of $\frac{1}{72000}$ of a second elapses between two consecutive lines. But as the spark is only seen when one of the lines of the disc coincides with one of the six lines of the vernier, and as this gives sixths of a division of the movable disc, when the

latter has turned through a sixth of a division, a second coincidence is produced; so that the interval between two successive coincidences is

$$\frac{1}{72000 \times 6} = 0{\cdot}0000023 \text{ of a second.}$$

That being the case, let the duration of a spark be something between 23 and 46 ten-millionths of a second; if it strikes exactly at the moment of a coincidence, it will last until the next coincidence; and owing to the persistence of impressions on the retina (625) the observer will see two luminous lines. But if the spark strikes between two coincidences and has ceased when the third is produced, only one brilliant line is seen. Thus, if with the above velocity sometimes 1 and sometimes 2 bright lines are seen, the duration of the spark is comprised between 23 and 46 ten-millionths of a second.

By experiments of this kind, with a striking distance of 5 millimetres between the balls *a* and *b*, and varying the number of the jars, Lucas and Cazin obtained the following results:—

Number of jars	Duration in millionths of a second
2	26
4	41
6	45
8	47

It will thus be seen that the duration of the spark increases with the number of jars. It also increases with the striking distance; but it is independent of the diameter of the balls between which the spark strikes.

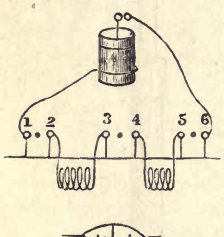


Fig. 759.

The spark of electrical machines has so short a duration that it could not be measured with the chronoscope.

796. **Velocity of electricity.**—To determine the velocity of electricity Wheatstone constructed an apparatus the principle of which will be understood from fig. 759. Six insulated metal knobs were arranged in a horizontal line on a piece of wood called a *spark board*; of these the knob 1 was connected with the outer, while 6 could be connected with the inner coating of a charged Leyden jar; the knob 1 was a tenth of an inch distant from the knob 2; while between 2 and 3 a quarter of a mile of insulated wire was interposed; 3 was likewise a tenth of an inch from 4, and there was a quarter of a mile of wire between 4 and 5; lastly, 5 was a tenth of an inch from 6, from which a wire led directly to the inner coating of the Leyden jar. Hence, when the jar was discharged by connecting the wire from 6 with the inner coating of the jar, sparks would pass between 1 and 2, between 3 and 4, and between 5 and 6. Thus the discharge, supposing it to proceed from the inner coating, has to pass in its course through a quarter of a mile of wire between the first and second spark, and through the same distance between the second and third.

The spark board was arranged at a distance of 10 feet from the rotating mirror, and at the same height, both being horizontal; and the observer

looked down on the mirror. Thus the sparks were visible when the mirror made an angle of 45° with the horizon.

Now, if the mirror were at rest, or had only a small velocity, the images of the three spots would be seen as three dots ⋮, but when the mirror had a certain velocity these dots appeared as lines, which were longer as the rotation was more rapid. The greatest length observed was 24°, which, with 800 revolutions in a second, can be shown to correspond to a duration of $\frac{1}{24000}$ of a second. With a slow rotation the lines present the appearance ═══; they are quite parallel, and the ends in the same line. But with greater velocity, and when the rotation took place from left to right, they presented the appearance ═══, and when it turned from right to left the appearance ═══, because the image of the centre spark was formed after the lateral ones. Wheatstone found that this displacement amounted to half a degree before or behind the others; accordingly this arc corresponds to a duration of about the $\frac{1}{115\frac{1}{2}000}$ of a second; the space traversed in this time being a quarter of a mile, gives for the velocity of electricity 288,000 miles in a second, which is greater than that of light. The velocity obtained from experiments with dynamical electricity is far less; and, owing to induction, the transmission of a current through submarine wires is comparatively slow.

In the above experiment the images of the two outer sparks appear simultaneously in the mirror, from which it follows that the electric current issues simultaneously from the two coatings of the Leyden jar.

From theoretical considerations based upon measurements of constant electrical currents Kirchhoff concluded that the motion of electricity in a wire in which it meets with no resistance is like that of a wave in a stretched string, and has the velocity of 192,924 miles in a second, which is about that of light in vacuo (507).

According to Fizeau and Gounelle it is 62,100 miles in iron, and 111,780 in copper wire. These measurements, however, were made with telegraph wires, which induce opposite electricities in the surrounding media; there is thus produced a resistance which diminishes the velocity. The velocity is less in insulated wires in water than in air. The nature of the conductor appears to have some influence on the velocity; but not the thickness of the wire nor the potential of the electricity.

For atmospheric electricity, reference must be made to the chapter on Meteorology.

BOOK X

DYNAMICAL ELECTRICITY

CHAPTER I

VOLTAIC PILE. ITS MODIFICATIONS

797. **Galvani's experiment and theory.**—The fundamental experiment which led to the discovery of dynamical electricity is due to Galvani, Professor of Anatomy in Bologna. Occupied with investigations on the influence of electricity on the nervous excitability of animals, and especially of the frog, he observed that when the lumbar nerves of a dead frog were connected with the crural muscles by a metallic circuit, the latter became briskly contracted.

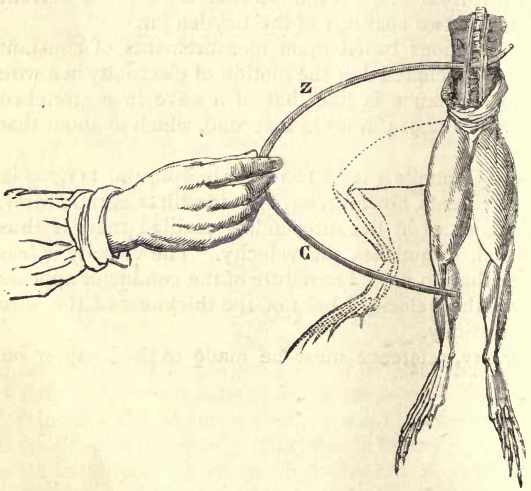


Fig. 760.

To repeat this celebrated experiment, the legs of a recently killed frog are prepared, and the lumbar nerves on each side of the vertebral column are exposed in the form of white threads. A metal conductor, composed of zinc and copper, is then taken (fig. 760), and one end introduced between the nerves and the vertebral column, while the other touches one of the muscles of the thighs or legs; at each contact a smart contraction of the muscles ensues.

Galvani had some time before observed that the electricity of machines produced in dead frogs analogous contractions, and he attributed the phenomena first described to an electricity inherent in the animal. He assumed

that this electricity, which he called *vital fluid*, passed from the nerves to the muscles by the metallic arc, and was thus the cause of contraction. This theory met with great support, especially among physiologists, but it was not without opponents. The most considerable of these was Alexander Volta, Professor of Physics in Pavia.

798. **Volta's fundamental experiment.**—Galvani's attention had been exclusively devoted to the nerves and muscles of the frog; Volta's was directed upon the connecting metal. Resting on the observation, which Galvani had also made, that the contraction is more energetic when the connecting arc is composed of two metals than when there is only one, Volta attributed to the metals the active part in the phenomenon of contraction. He assumed that the disengagement of electricity was due to their contact, and that the animal parts only officiated as conductors, and at the same time as a very sensitive electroscope.

By means of the condensing electroscope, which he had then recently invented, Volta devised several modes of showing the disengagement of electricity on the contact of metals, of which the following is the easiest to perform :—

The moistened finger being placed on the upper plate of a condensing electroscope (fig. 734), the lower plate is touched with a plate of copper, *c*, soldered to a plate of zinc, *z*, which is held in the other hand. On breaking the connection and lifting the upper plate (fig. 735), the gold leaves diverge, and, as may be proved, with negative electricity. Hence, when soldered together, the copper is charged with negative electricity, and the zinc with positive electricity. The electricity could not be due either to friction or pressure; for if the condensing plate, which is of copper, is touched with the zinc plate *z*, the copper plate to which it is soldered being held in the hand, no trace of electricity is observed.

A memorable controversy arose between Galvani and Volta. The latter was led to give greater extension to his contact theory, and propounded the principle that when *two heterogeneous substances are placed in contact, one of them always assumes the positive and the other the negative electrical condition.* In this form Volta's theory obtained the assent of the principal philosophers of his time. Galvani, however, made a number of highly interesting experiments with animal tissues. In some of these he obtained indications of contraction, even though the substances in contact were quite homogeneous.

799. **Disengagement of electricity in chemical actions.**—The contact theory which Volta had propounded, and by which he explained the action of the pile, soon encountered objectors. Fabroni, a countryman of Volta, having observed that, in the pile, the discs of zinc became oxidised in contact with the acidulated water, thought that this oxidation was the principal cause of the disengagement of electricity. In England Wollaston soon advanced the same opinion, and Davy supported it by many ingenious experiments.

It is true that in the fundamental experiment of the contact theory (798) Volta obtained signs of electricity. But De la Rive showed that if the zinc be held in a wooden clamp, all signs of electricity disappear, and that the same is the case if the zinc be placed in gases, such as hydrogen or nitrogen, which exert upon it no chemical action. De la Rive accordingly concluded

that in Volta's original experiment the disengagement of electricity is due to the chemical actions which result from the perspiration and from the oxygen of the atmosphere.

The development of electricity in chemical actions may be demonstrated in the following manner by means of the condensing electroscope (779) :—A disc of moistened paper is placed on the upper plate of the condenser, and on this a zinc capsule, in which some very dilute sulphuric acid is poured. A platinum wire, communicating with the ground, but insulated from the sides of the vessel, is immersed in the liquid, and at the same time the lower plate of the condenser is also connected with the ground by touching it with the moistened finger. On breaking contact and removing the upper plate, the gold leaves are found to be positively electrified, proving that the upper plate has received a charge of negative electricity.

By a variety of analogous experiments it may be shown that various chemical actions are accompanied by a disturbance of the electrical equilibrium ; though of all chemical actions those between metals and liquids are the most productive of electricity. All the various resultant effects are in accordance with the general rule, that when a liquid acts chemically on a metal the liquid assumes the positive, and the metal the negative, condition. In the above experiment the sulphuric acid, by its action on zinc, becomes positively electrified, and its electricity passes off through the platinum wire into the ground, while the negative electricity excited on the zinc acts on the condenser just as an excited rod of sealing-wax would do.

In many cases the electrical indications accompanying chemical actions are but feeble, and require the use of a very delicate electroscope to render them apparent. Thus, one of the most energetic chemical actions, that of sulphuric acid upon zinc, gives no more free electricity than water alone does with zinc.

Opinion—which in this country, at least, had, mainly by the influence of Faraday's experiments, tended in favour of the purely chemical origin of the electricity produced in voltaic action—has of late inclined more and more towards the contact theory. The following experiments, due to Lord Kelvin, afford perhaps the most conclusive arguments hitherto adduced in favour of the latter view :—

A very light metal bar is suspended by fine wire, so as to be movable about an axis perpendicular to the plane of a disc made up of two half discs, one of zinc, Z, and the other of copper, C (fig. 761). The light bar is counterpoised so as to be exactly over one half of the line of separation of the two discs. When the discs are placed in contact and the bar is charged positively by being connected with a Leyden jar, the bar moves from the zinc towards the copper ; if the jar, and therefore the bar, is charged negatively, its motion is in the opposite direction. The same results are obtained when the discs are connected by a wire, thus showing that the contact of the two metals causes them to assume different electrical conditions, the zinc taking the positive, and the copper the negative electricity.

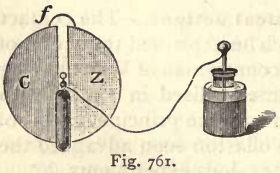


Fig. 761.

When, however, the two halves, instead of being in metallic contact, are

connected by a drop of water, no change is produced in the position of the bar by altering its electrification, provided it hangs quite symmetrically relative to the two halves of the ring. This result shows that, under the circumstances mentioned, no difference is produced in the electrical condition of the two metals. Hence the conclusion has been drawn by Sir W. Thomson and others, that the movement of electricity in the galvanic circuit is entirely due to the electrical difference produced at the surfaces of contact of the dissimilar metals. These results have been confirmed by some recent very careful experiments by Professor Clifton.

There are, however, other facts which are not easily harmonised with this view ; and indeed the last-mentioned experiment can hardly be regarded as proving that in *all* cases two different metals, connected by an electrolytic liquid (842), assume the same electrical condition. It may, therefore, still be regarded as possible, or even probable, that the contact between the metals and the liquids of a cell contributes, at least in some cases, to the production of the current.

A most complete discussion of the question as to the seat of electromotive forces in the voltaic cell is published in a series of papers by Prof. Lodge in the nineteenth volume of the ' Philosophical Magazine.'

800. **Current electricity.**—When a plate of zinc and a plate of copper are partially immersed in dilute sulphuric acid, no electrical or chemical change is apparent beyond perhaps a slight disengagement of hydrogen from the surface of the zinc plate. If now the plates are placed in direct contact, or, more conveniently, are connected by a metal wire, the chemical action sets in, a large quantity of hydrogen is disengaged ; but this hydrogen is no longer disengaged at the surface of the zinc, but at the surface of the copper plate. Here then we have to deal with something more than mere chemical action, for chemical action would be unable to explain either the increase in the quantity of hydrogen disengaged when the metals touch, or the fact that this hydrogen is now given off at the surface of the copper plate. At the same time, if the wire is examined it will be found to possess many remarkable thermal, magnetic, and other properties which will be afterwards described.

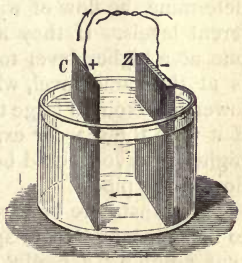


Fig. 762.

In order to understand what here takes place, let us suppose that we have two insulated metal spheres, and that one is charged with positive and the other with negative electricity, and that they are momentarily connected by means of a wire. Electricity will pass from a place of higher to a place of lower potential—that is, from the positive along the wire to the negative—and the potentials become equal. This is, indeed, nothing more than an electrical discharge taking place through the wire ; and during the infinitely short time in which this is accomplished, it can be shown that the wire exhibits certain heating and magnetising effects, of which the increase of temperature is perhaps the easiest to observe. If now we can imagine some agency by which the different electrical conditions of the two spheres are renewed as fast as they are discharged, which is what very nearly takes

place when the two spheres are respectively connected with the two conductors r and r_1 of a Holtz machine (figs. 705, 706), this equalisation of potentials, thus taking place, is virtually continuous, and the phenomena above mentioned are also continuous.

Now this is what takes place when the two metals are in contact in a liquid which acts upon them unequally. This is independent of hypothesis as to the cause of the phenomena—whether the electrical difference is only produced at the moment of contact of the metals, or whether it is due to the chemical action, or tendency to chemical action, between the metal and the liquid. The rapidly succeeding series of equalisations of potential, which takes place in the wire, being continuous, so long as the chemical action continues, is what is ordinarily spoken of as the *electrical current.*

If we represent by $+e$ the potential of the copper plate, and by $-e$ the potential of the zinc, then the electrical difference—that is, the difference of potentials—is $+e-(-e)=2e$. And this is general; the essential point of any such combination as the above is, that it maintains, or tends to maintain, a difference of potentials, which difference is constant. If, for instance, the zinc plate be connected with the earth which is at zero potential, its potential also becomes zero; and since the electrical difference remains constant, we have for the potential of the copper plate $+2e$. Similarly, if the copper be connected with the earth the potential of the zinc plate is negative and is $-2e$.

The conditions under which a current of electricity is formed in the above experiment may be further illustrated by reference to the conditions which determine the flow of water between two reservoirs containing water at different levels. If they are connected by a pipe, water will flow from the one at a higher level to the one at a lower level until the water in the two is at the same level, when of course the flow ceases. If we imagine the lower reservoir so large that any water added to it would not affect its level—if it were the sea, for example—that would represent zero level, and if the higher reservoir could be kept at a constant level there would be a constant flow in the pipe.

We must here be careful not to dwell too much on this analogy. It is not to be supposed that in speaking of *current* of electricity we mean to assert that anything actually flows—that there is any actual transfer of matter. We say 'electricity flows' or 'a current is produced,' in the same sense as that in which we say 'sound or light travels.'

801. **Voltaic couple. Electromotive series.**—The arrangement just described, consisting of two metals in metallic contact, and a conducting liquid in which they are placed, constitutes a *simple voltaic element or couple.* So long as the metals are not in contact, the couple is said to be *open*, and when connected it is *closed.*

According to the chemical view, to which we shall for the present provisionally adhere, it is not necessary for the production of a current that one of the metals be unaffected by the liquid, but merely that the chemical action upon the one be greater than upon the other. For then we may assume that the current produced would be due to the difference between the differences of potential which each of the metals separately produces by its contact with the liquid. If the differences of potentials were absolutely equal—a condition, however, impossible of realisation with two distinct metals—we

must assume that when the metals are joined no current would be produced. The metal which is most attacked is called the *positive* or *generating* plate, and that which is least attacked the *negative* or *collecting* plate. The positive metal determines the direction of the current, which proceeds *in* the liquid from the positive to the negative plate, and *out* of the liquid through the connecting wire from the negative to the positive plate.

In speaking of the *direction of the current* the direction of the positive electricity is always understood.

In the fundamental experiment, not only the connecting wire, but also the liquid and the plates, are traversed by the electrical current—are the scene of electrical actions.

The mere immersion of two different metals in a liquid is not alone sufficient to produce a current; there must be chemical action. When a platinum and a gold plate are connected with a delicate galvanometer, and immersed in pure nitric acid, no current is produced; but on adding a drop of hydrochloric acid a strong current is excited, which proceeds in the liquid from the gold to the platinum, because the gold is attacked by the nitro-hydrochloric acid, while the platinum is less so, if at all.

As a voltaic current is produced whenever two metals are placed in metallic contact in a liquid which acts more powerfully upon one than upon the other, there is a great choice in the mode of producing such currents. In reference to their electrical deportment, the metals have been arranged in what is called an *electromotive series*, in which the most *electropositive* are at one end, and the most *electronegative* at the other. Hence when any two of these are placed in contact in dilute acid, the current in the connecting wire proceeds from the one lower in the list to the one higher. The principal metals are as follows :—

1. Zinc
2. Cadmium
3. Tin
4. Lead
5. Iron
6. Nickel
7. Bismuth
8. Antimony
9. Copper
10. Silver
11. Gold
12. Platinum
13. Graphite

It will be seen that the electrical deportment of any metal depends on the metal with which it is associated. Iron, for example, in dilute sulphuric acid is electronegative towards zinc, but is electropositive towards copper; copper in turn is electronegative towards iron and zinc, but is electropositive towards silver, platinum, or graphite.

802. **Electromotive force.**—The force in virtue of which continuous electrical effects are produced throughout a circuit consisting of two metals in metallic contact in a liquid which acts unequally upon them, is usually called *the electromotive force.* Electromotive force and *difference of potentials* are commonly used in the same sense. It is, however, more correct to regard difference of potentials as a particular case of electromotive force; for as we shall afterwards see, there are cases in which electrical currents are produced without the occurrence of that particular condition which we have called difference of potentials. The electromotive force is greater in proportion to the distance of the two metals from one another in the series. That is to say, it is greater the greater the difference between the chemical action upon

the two metals immersed. Thus the electromotive force between zinc and platinum is greater than that between zinc and iron, or between zinc and copper. The law established by experiment is, that *the electromotive force between any two metals is equal to the sum of the electromotive forces between all the intervening metals.* Thus the electromotive force between zinc and platinum is equal to the sum of the electromotive forces between zinc and iron, iron and copper, and copper and platinum.

The electromotive force is influenced by the condition of the metal; rolled zinc, for instance, is negative towards cast zinc. It also depends on the degree of concentration of the liquid; in dilute nitric acid zinc is positive towards tin, and mercury positive towards lead; while in concentrated nitric acid the reverse is the case, mercury and zinc being respectively electro-negative towards lead and tin.

The nature of the liquid also influences the direction of the current. If two plates, one of copper and one of iron, are immersed in dilute sulphuric acid, a current is set up proceeding through the liquid from the iron to the copper; but if the plates, after being washed, are placed in solution of potassium sulphide, a current is produced in the opposite direction—the copper is now the positive metal. Other examples may be drawn from the following table, which shows the electric deportment of the principal metals with three different liquids. It is arranged like the preceding one; each metal being electropositive towards any one lower in the list, and electro-negative towards any one higher.

Caustic potass	Hydrochloric acid	Sulphide of potassium
Zinc	Zinc	Zinc
Tin	Cadmium	Copper
Cadmium	Tin	Cadmium
Antimony	Lead	Tin
Lead	Iron	Silver
Bismuth	Copper	Antimony
Iron	Bismuth	Lead
Copper	Nickel	Bismuth
Nickel	Silver	Nickel
Silver	Antimony	Iron

A voltaic current may also be produced by means of two liquids and one metal. This may be shown by the following experiment:—In a beaker containing strong nitric acid is placed a small porous pot (fig. 763), containing strong solution of caustic potass. If now two platinum wires connected with the two ends of a galvanometer (821) are immersed respectively in the alkali and in the acid, a voltaic current is produced, proceeding in the wire from the nitric acid to the potass, which thus correspond respectively to the negative and positive plates in ordinary couples.

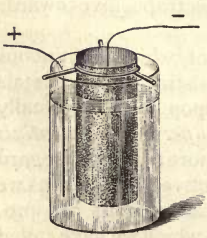


Fig. 763.

A metal which is acted upon by a liquid can be protected from solution by placing in contact with it a more electropositive metal, and thus forming

a simple voltaic circuit. This principle is the basis of Davy's proposal to protect the copper sheathings of ships, which are rapidly acted upon by sea-water. If zinc or iron be connected with the copper, these metals are dissolved and the copper protected. Davy found that a piece of zinc the size of a nail was sufficient to protect a surface of forty or fifty square inches; unfortunately the proposal has not been of practical value, for the copper must be attacked to a certain extent to prevent the adherence of marine plants and shellfish.

803. **Poles and electrodes.**—If the wire connecting the two terminal plates of a voltaic couple be cut, it is clear, from what has been said about the origin and direction of the current, that positive electricity will tend to accumulate at the end of the wire attached to the copper or negative plate, and negative electricity on the wire attached to the zinc or positive plate. These terminals have been called the *poles* of the battery. For experimental purposes, more especially in the decomposition of salts, plates of platinum are attached to the ends of the wires. Instead of the term poles, the word *electrodes* (ἤλεκτρον, and ὁδός, a way) is now commonly used; for these are the ways through which the respective electricities emerge. It is important not to confound the positive *plate* with the positive *pole* or *electrode*. The positive electrode is that connected with the negative plate, while the negative electrode is connected with the positive plate.

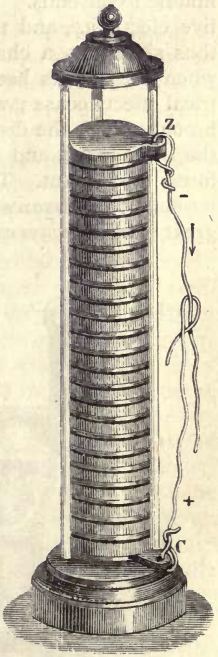


Fig. 764.

804. **Voltaic pile. Voltaic battery.**—When a series of voltaic elements or pairs is arranged so that the zinc of one element is connected with the copper of another, the zinc of this with the copper of another, and so on, the arrangement is called a *voltaic battery*; and by its means the effects produced by a single element are capable of being very greatly increased.

The earliest of these arrangements was devised by Volta himself. It consists (fig. 764) of a series of discs piled one over the other in the following order:—At the bottom, on a frame of wood, is a disc of copper, then a disc of cloth moistened by acidulated water, or by brine, then a disc of zinc; on this a disc of copper, and another disc of moistened cloth, to which again follow as many sets of copper-cloth-zinc, always in the same order, as may be convenient, the highest disc being of zinc. The discs are kept in a vertical position by glass rods.

It will be readily seen that we have here a series of simple voltaic couples, the moisture in the cloth acting as the liquid in the cases already mentioned, and that the terminal zinc is the negative and the terminal copper the positive pole. From the mode of its arrangement, and from its discoverer, the apparatus is known as the *voltaic pile*, a term applied to all apparatus of this kind for accumulating the effects of dynamical electricity.

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The distribution of electricity in the pile varies according as it is in connection with the earth by one of its extremities, or as it is insulated by being placed on a non-conducting cake of resin or glass.

In the former case, the end in contact with the ground is neutral, and the rest of the apparatus contains only one kind of electricity; this is negative if the copper disc, and positive if the zinc disc, is in contact with the ground.

In the insulated pile the electricity is not uniformly distributed. By means of a proof-plane and electroscope it may be demonstrated that the middle part is in a neutral state, and that one-half is charged with positive and the other with negative electricity, the potential increasing from the middle to the ends. The half terminated by a zinc disc is charged with negative electricity, and that by a copper with positive electricity. The pile is thus similar to a charged Leyden jar; with this difference, however, that when the jar has been discharged by connecting its two coatings, the electrical effects cease; while in the case of the pile, the cause which originally brought about the distribution of electricity restores this state of charge after the discharge; and the continuous succession of charges and discharges forms the current. The effects of the pile will be discussed in other places.

805. **Wollaston's battery.**—The original form of the voltaic pile has a great many inconveniences, and possesses now only an historical interest.

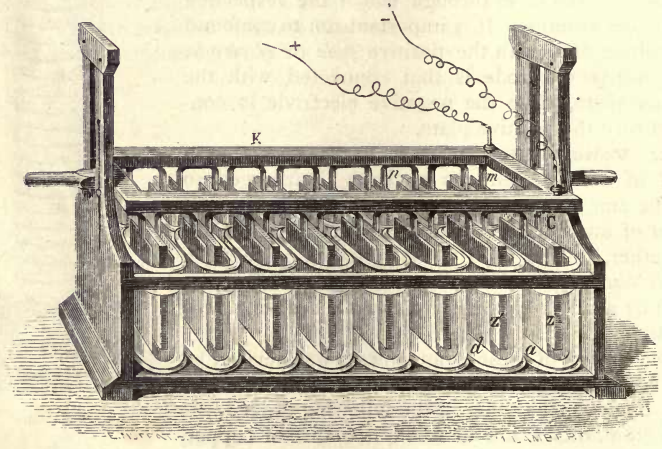


Fig. 765.

It has received a great many improvements, the principal object of which has been to facilitate manipulation, and to produce greater electromotive force.

One of the earliest of these modifications was the crown of cups, or *couronne des tasses*, invented by Volta himself. An improved form of this is known as *Wollaston's battery* (fig. 765); it is arranged so that when the current is not wanted, the action of the battery can be stopped.

The plates Z are of thick rolled zinc, and usually about eight inches in length by six in breadth. The copper plates, C, are of thin sheet, and bent

so as to surround the zincs without touching them, contact being prevented by small pieces of cork. To each copper plate a narrow strip of copper, *o*, is soldered, which is bent twice at right angles and is soldered to the next zinc plate ; and the first zinc, Z, is surrounded by the first copper C ; these two constitute a couple, and each couple is immersed in a glass vessel, containing acidulated water. The copper, C, is soldered to the second zinc by the strip *o*, and this zinc is in turn surrounded by a second copper, and so on.

Fig. 765 represents a pile of sixteen couples united in two parallel series of eight each. All these couples are fixed to a cross frame of wood, by which they can be raised or lowered at pleasure. When the battery is not wanted, the couples are lifted out of the liquid. The water in these vessels is usually acidulated with $\frac{1}{10}$ sulphuric and $\frac{1}{20}$ nitric acid.

Hare's deflagrator.—This is a simple voltaic arrangement, consisting of two large sheets of copper and zinc rolled together in a spiral, but preserved from direct contact by bands of leather or horsehair. The whole is immersed in a vessel containing acidulated water, and the two plates are connected outside the liquid by a conducting wire.

806. **Enfeeblement of the current in batteries. Secondary currents.** —The various batteries already described—Volta's, Wollaston's, and Hare's, which consist essentially of two metals and one liquid—labour under the objection that the currents produced rapidly diminish in strength.

This is due principally to three causes : the first is the decrease in the chemical action owing to the neutralisation of the sulphuric acid by its combination with the zinc. This is a necessary action, for upon it depends the current ; it therefore occurs in all batteries, and is without remedy except by replacement of acid and zinc. The second is due to what is called *local action* ; that is, the production of small closed circuits in the active metal, owing to the impurities it contains. These local currents rapidly wear away the active plate, without contributing anything to the continuance of the general current. They are remedied by amalgamating the zinc with mercury, by which chemical action is prevented until the circuit is closed, as will be more fully explained (816). The third arises from the production of an inverse electromotive force, which tends to produce a current in a contrary direction to the principal current, and therefore to destroy it either totally or partially. In the fundamental experiment (fig. 762), when the circuit is closed, zinc sulphate is formed, which dissolves in the liquid, and at the same time a layer of hydrogen gas is gradually formed on the surface of the copper plate. This diminishes the activity of the combination in more than one way. In the first place, it interferes with the contact between the metal and the liquid ; in the second place, in proportion as the copper becomes coated with hydrogen, we have virtually a plate of hydrogen instead of a plate of copper opposed to the zinc, and in addition, the hydrogen, by reacting on the zinc sulphate, which accumulates in the liquid, gradually causes a deposition of zinc on the surface of the copper ; hence, instead of having two different metals unequally attacked, the two metals become gradually less different, and, consequently, the total effect and the current become weaker and weaker.

The *polarisation* of the plate (as this phenomenon is termed) may be destroyed by breaking the circuit and exposing the copper plate to the air ;

the deposited hydrogen is thus more or less completely got rid of, and on again closing the circuit the current has nearly its original strength. The same result is obtained when the current of another battery is transmitted through the battery in a direction opposite to that of the first.

When platinum electrodes are used to decompose water, a similar phenomenon is produced, called *polarisation of the electrodes*, which may be illustrated by an arrangement represented in fig. 766, in which B is a constant element, V a voltameter (846), G a galvanometer (821), and H a mercury cup. The wire L being disconnected from H, a current is produced in the voltameter, the direction of which is from P to P′; if now the wire F be detached from H, and L be connected therewith, a current is produced through the galvanometer the direction of which is from P′ to P; that is, the opposite of that which the element had previously produced. Becquerel and Faraday have shown that this polarisation of the metals results from the deposits caused by the passage of the current, and an important application of this phenomenon will be found described farther on (849).

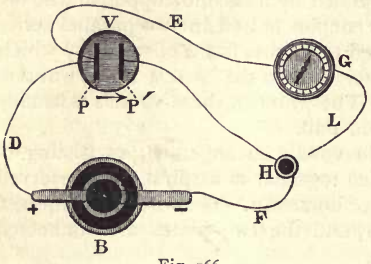


Fig. 766.

CONSTANT CURRENTS

807. **Constant currents.**—With few exceptions, batteries composed of elements with a single liquid have almost gone out of use, in consequence of the rapid enfeeblement of the current produced. They have been replaced by batteries with two liquids, which are called *constant batteries* because their action continues without material alteration for a considerable period of time. The essential point to be attended to in securing a constant current is to prevent the polarisation of the inactive metal; in other words, to hinder any permanent deposition of hydrogen on its surface. This is effected by placing the inactive metal in a liquid upon which the deposited hydrogen can act chemically.

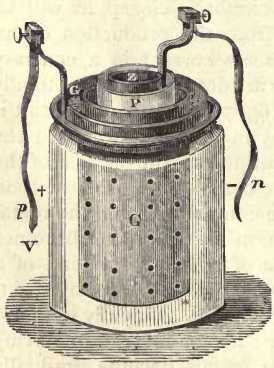


Fig. 767.

808. **Daniell's battery.**—This was the first form of the constant battery, and was invented by Daniell in the year 1836. As regards the constancy of its action, it is perhaps still the best of all constant batteries. Fig. 767 represents a single element. A glass or porcelain vessel, V, contains a saturated solution of copper sulphate, in which is immersed a copper cylinder, G, open at both ends, and perforated by holes. At the upper part of this cylinder there is an annular shelf, G, also perforated by small holes, and below the level of the solution; this is intended to support crystals of

copper sulphate to replace that decomposed as the electrical action proceeds. Inside the cylinder is a thin porous vessel, P, of unglazed earthenware. This contains either water, or solution of common salt, or dilute sulphuric acid, in which is placed the cylinder of amalgamated zinc, Z. Two thin strips of copper *p* and *n*, fixed by binding screws to the copper and to the zinc, serve for connecting the elements in series.

When a Daniell's element is closed, the hydrogen resulting from the action of the dilute acid on the zinc is liberated on the surface of the copper plate, but meets there the copper sulphate, which is reduced, forming sulphuric acid and metallic copper, which is deposited on the surface of the copper plate. In this way copper sulphate in solution is taken up ; and if it were all consumed, hydrogen would be deposited on the copper, and the current would lose its constancy. This is prevented by the crystals of copper sulphate which keep the solution saturated. The sulphuric acid produced by the decomposition of the sulphate permeates the porous cylinder, and tends to replace the acid used by its action on the zinc ; and as the quantity of sulphuric acid formed in the solution of copper sulphate is regular, and proportional to the acid used in dissolving the zinc, the action of this acid on the zinc is regular also, and thus a constant current is produced.

In order to join together several of these elements to form a battery, the zinc of one is connected either by a copper wire or strip with the copper of the next, and so on from one element to another, as shown in fig. 772, for another kind of battery.

Fig. 768 represents one form of a standard Daniell. The vessel A contains dilute sulphuric acid of specific gravity 1·075, and in it is a plate, Z, of amalgamated zinc. A_1 contains saturated solution of copper sulphate, and in it is a plate of copper, K. The syphon tube, C C_1, which connects the two vessels, is closed at both ends by bladder, and is filled with dilute sulphuric acid.

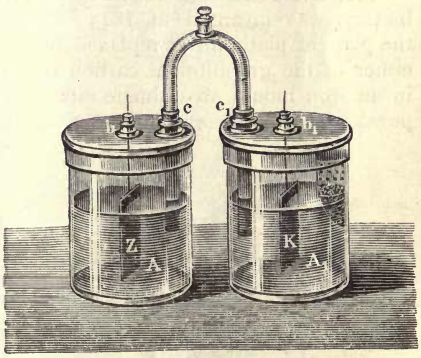


Fig. 768.

The current produced by a Daniell's battery is constant for some hours ; its action is stronger when it is placed in hot water. Its electromotive force is about 1·08 volt.

Taking the cost of the materials consumed in working a Daniell's cell, but allowing for the copper deposited, and assuming that 70 per cent. of the electrical energy is obtained in mechanical work, it appears that the expense per horse power per hour is at least two shillings and sixpence.

809. **Grove's battery.**—In this battery the copper sulphate solution is replaced by nitric acid, and the copper by platinum, by which greater electromotive force is obtained. Fig. 769 represents one of the forms of a couple of this battery. It consists of a glass vessel, A, partially filled with dilute sulphuric acid (1 : 8) ; of a cylinder of zinc, Z, open at both ends ; of a

vessel, V, made of porous earthenware, and containing ordinary nitric acid ; of a plate of platinum, P (fig. 770), bent in the form of an S, and fixed to a cover, *c*, which rests on the porous vessel. The platinum is connected with a binding screw, *b*, and there is a similar binding screw on the zinc. In this battery the hydrogen, which would be disengaged on the platinum, meeting the nitric acid, decomposes it, forming hyponitrous acid, which dissolves, or is disengaged as nitrous fumes. Grove's battery is the most convenient, and one of the most powerful of the two fluid batteries. It is, however, expensive, owing to the high price of platinum ; besides which the platinum is liable, after some time, to become brittle and break very easily. But as the platinum is not consumed, it retains most of its value, and when the plates which have been used in a battery are heated to redness they regain their elasticity.

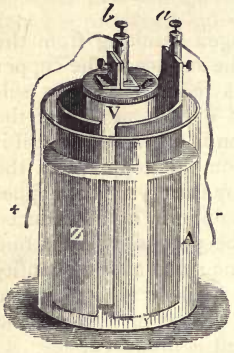


Fig. 769.

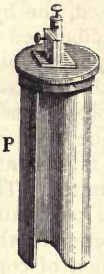


Fig. 770.

810. **Bunsen's battery.**—*Bunsen's*, also known as the *zinc carbon* battery, was invented in 1843 ; it is in effect a Grove's battery, where the plate of platinum is replaced by a cylinder of carbon. This is made either of the graphitoidal carbon deposited in gas retorts, or by calcining in an iron mould an intimate mixture of coke and bituminous coal, finely powdered and strongly compressed. Both those modifications of carbon

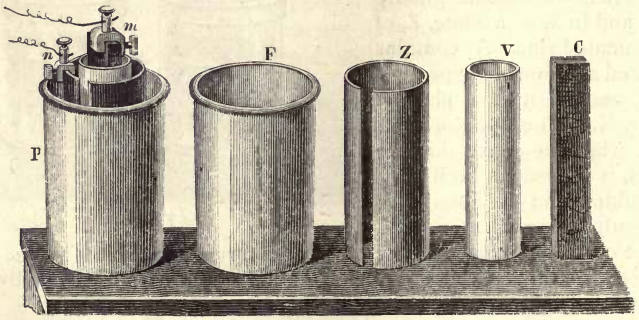


Fig. 771.

are good conductors. Each element consists of the following parts : 1, a vessel, F (fig. 771), either of stoneware or of glass, containing dilute sulphuric acid ; 2, a hollow cylinder, Z, of amalgamated zinc ; 3, a porous vessel, V, in which is ordinary nitric acid ; 4, a rod of carbon, C, prepared in the above manner. In the vessel F the zinc is first placed, and in it the carbon C in the porous vessel V as seen in P. To the carbon is fixed a binding screw, *m*, to which a copper wire is attached, forming the positive

pole. The zinc is provided with a similar binding screw, *n*, and wire, which is thus a negative pole.

A single cell of the ordinary dimensions, 20 cm. in height and 9 cm. in diameter, has a resistance of about 0·14 ohm, and taking its E.M.F. at 1·82 (814), gives a current of 12 to 13 amperes when on *short circuit*, that is, when it is closed without measurable external resistance.

The elements are arranged to form a battery (fig. 772) by connecting each carbon to the zinc of the following one by means of the clamps *mn*, and a strip of copper, *c*, represented in the top of the figure. The copper is pressed at one end between the carbon and the clamp, and at the other it is soldered to the clamp *n*, which is fitted on the zinc of the following element, and so forth. The clamp of the first carbon and that of the last zinc are alone provided with binding screws, to which are attached the wires.

The chemical action of Bunsen's battery is the same as that of Grove's, and being equally powerful, while less costly, is very generally used on the

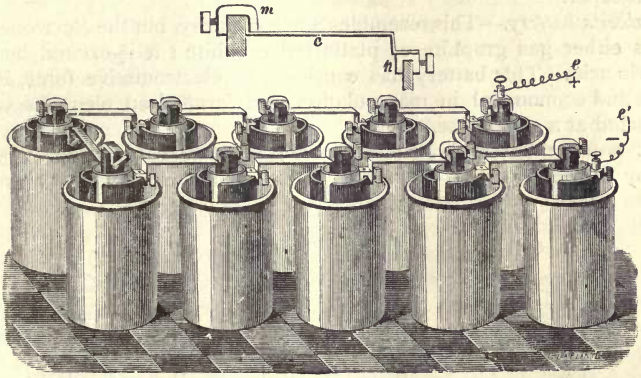


Fig. 772.

Continent. But though its first cost is less than that of Grove's battery, it is more expensive to work, and is not so convenient to manipulate.

Callan's battery is a modified form of Grove's. Instead of zinc and platinum, zinc and platinised lead are used; and instead of pure nitric acid Callan used a mixture of sulphuric acid, nitric acid, and saturated solution of nitre. The battery is said to be equal in its action to Grove's, and is much cheaper.

Callan also constructed a battery in which zinc in dilute sulphuric acid forms the positive plate, and cast iron in strong nitric acid the negative. Under these circumstances the iron becomes passive; it is strongly electro-negative, and does not dissolve. If, however, the nitric acid becomes too weak, the iron is dissolved with simultaneous disengagement of nitrous fumes.

After being in use some time, all the batteries in which the polarisation is prevented by nitric acid disengage nitrous fumes in large quantities, and this is a serious objection to their use, especially in closed rooms. To prevent this, nitric acid is frequently replaced by chromic acid, or by a mixture of 4 parts potassium bichromate, 4 parts sulphuric acid, and 18 water. The liberated hydrogen reduces the chromic acid to the state of

oxide of chromium, which remains dissolved in sulphuric acid. With the same view, sesquichloride of iron is sometimes substituted for nitric acid; it becomes reduced to protochloride. But the action of the elements thus modified is considerably less than when nitric acid is used, owing to the increased resistance.

811. **Smee's battery.**—In this battery the polarisation of the negative plate is prevented by mechanical means. Each element consists of a sheet of platinum placed between two vertical plates of zinc, as in Grove's battery; but as there is only a single liquid, dilute sulphuric acid, the elements have much the form of those in Wollaston's battery. The adherence of hydrogen to the negative plate is prevented by covering the platinum with a deposit of finely divided platinum. In this manner the surface is roughened, which facilitates the disengagement of hydrogen to a remarkable extent, and consequently diminishes the resistance of a couple. Instead of platinum, silver covered with a deposit of finely divided platinum is frequently substituted, as being cheaper.

Walker's battery.—This resembles Smee's battery, but the electronegative plate is either gas graphite or platinised graphite; it is excited by dilute sulphuric acid. This battery has considerable electromotive force, is convenient and economical in manipulation, and large-sized elements can be constructed at a cheap rate.

812. **Recent batteries.**—The *mercury sulphate* battery (fig. 773), devised by Marié Davy, is essentially a zinc-carbon element, but of smaller

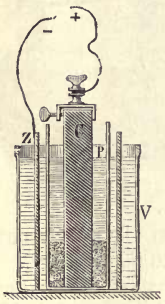


Fig. 773.

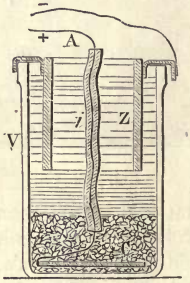


Fig. 774.

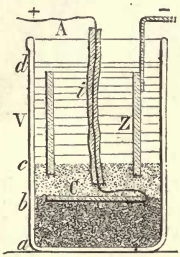


Fig. 775.

dimensions than those elements usually are. In the outer vessel, V, ordinary water or brine is placed, and in the porous vessel mercury sulphate. This salt is agitated with about three times its volume of water, in which it is difficultly soluble, and the liquid poured off from the pasty mass. The carbon being placed in the porous vessel, the spaces are filled with the residue, and then the decanted liquid poured into it.

Chemical action takes place when the cell is closed. The zinc then decomposes the water, liberating hydrogen, which, traversing the porous vessel, reduces the mercury sulphate, forming metallic mercury, which collects at the bottom of the vessel, while the sulphuric acid formed at the same time traverses the diaphragm to act on the zinc, and thus increases the action.

The mercury which is deposited may be used to prepare a quantity of

sulphate equal to that which has been consumed. A small quantity of the solution of mercury sulphate may also pass through the diaphragm ; but this is rather advantageous, as its effect is to amalgamate the zinc.

The electromotive force of this element is about a quarter greater than that of Daniell's element, but it has greater resistance ; it is rapidly exhausted when continuously worked, though it appears well suited for discontinuous work, as with the telegraph, and with alarums.

Gravity batteries.—The use of porous vessels is open to many objections, more especially in the case of Daniell's battery, in which they gradually become encrusted with copper, which destroys them. A kind of battery has been devised in which the porous vessel is entirely dispensed with, and the separation of the liquids is effected by the difference of density. Such batteries are called *gravity batteries.* Fig. 774 represents a form devised by Callaud. V is a glass or earthenware vessel in which is a copper plate soldered to a wire insulated by gutta-percha. On the plate is a layer of crystals of copper sulphate, C ; the whole is then filled with water, and the zinc cylinder, Z, is immersed in it. The lower part of the liquid becomes saturated with copper sulphate ; the action of the battery is that of a Daniell, and the zinc sulphate, which gradually forms, floats on the solution of copper sulphate owing to its lower density. This battery is easily manipulated, the consumption of copper sulphate is economical, and when not agitated it works constantly for some time, provided care be taken to replace the water lost by evaporation.

Meidinger's element, which is much used in Germany, is essentially a gravity battery of special construction, with zinc in solution of magnesium sulphate, and copper in solution of copper sulphate.

Minotto's battery.—This may be described as a Daniell's element, in which the porous vessel is replaced by a layer of sawdust or of sand. At the bottom of an earthenware vessel (fig. 775) is placed a layer of coarsely-powdered copper sulphate *a*, and on this a copper plate provided with an insulated copper wire *i*. On this there is a layer of sand or of sawdust *bc*, and then the whole is filled with water, in which rests a zinc cylinder Z. The action is just that of a Daniell ; the sawdust prevents the mixture of the liquids, but it also offers great resistance, which increases with its thickness. From its simplicity and economy, and the facility with which it is constructed, the battery merits increased attention.

De la Rue and Müller's element consists of a glass tube about 6 inches long by 0·75 inch in diameter, closed by a vulcanised india-rubber stopper through which passes a zinc rod 0·18 inch in diameter and 5 inches long. A flattened silver wire also passes through the stopper to the bottom of the tube, in which is placed about half an ounce of silver chloride, the greater part of the cell being filled with solution of sal-ammoniac. The hydrogen evolved at the negative plate reduces the chloride to metallic silver, which is thereby recovered. Since there is only one liquid, and the solid electrolyte is not acted upon when the circuit is open, the element is easily worked and requires little attention. It is very compact, 1,000 elements occupying a space of less than a cubic yard ; De la Rue and Müller have used as many as 14,400 such cells in investigations on the stratification of the electric light. A battery of 8,040 of these cells gave a spark $\frac{1}{3}$ of an inch in length in air

under the ordinary atmospheric pressure; while under a pressure of a quarter of an atmosphere the striking distance was $1\frac{1}{8}$ inch.

The electromotive force of a silver chloride cell is 1·03 of a volt, and that of one made with silver bromide is 0·908; hence a series of three of the silver chloride cells with one of bromide gives an average electromotive force of 1 volt (814).

Latimer Clark's element consists of perfectly pure mercury as a negative plate covered with a paste obtained by boiling sulphate of mercury in a saturated solution of zinc sulphate. The positive metal is a plate of zinc resting on this paste of sulphate. Insulated wires, leading to the mercury and the zinc respectively, form the connections. This battery is not well adapted for continuous work, but it furnishes a standard of electromotive force, which is constant and can be relied upon. Its electromotive force is 1·435 volt at 15°, and it diminishes by 0·00078 for an increase of 1° C.

A convenient form of element for many purposes is the *potassium bichromate*, or, as it is frequently termed, the *bichromate of potass* element (fig. 776). It consists of a zinc plate Z, attached to a brass rod, which slides up and down in a brass tube in an ebonite or porcelain cover, so that it can be wholly or partially immersed in the liquid. Two graphite plates, C C, are similarly fitted in the cover, and by means of strips of brass the carbon and the zinc plates are respectively in connection with the binding screws, which thus form the poles. The exciting liquid is a mixture of 1 part of potassium bichromate, 2 of sulphuric acid, and 10 of water.

Fig. 776.

The electromotive force is about 1·8 or 1·9 that of a Daniell; when the element is closed by a wire of small resistance its E.M.F. increases slightly at first, then remains constant for some time, after which it rapidly sinks to half its original amount.

Instead of potassium bichromate, chromic acid, which is now prepared industrially at a cheap rate, is often used.

In *Niaudet's element* a zinc cylinder dips in a solution of common salt and surrounds a porous cell, in which is a carbon plate surrounded by pieces of carbon and filled with chloride of lime, which does not act on the zinc even when the circuit is closed. The electromotive force is 1·6 that of a Daniell.

The element of *Lalande and Chaperon* is zinc in a 30 per cent. solution of caustic potass and copper in contact with oxide of copper which acts as depolariser. The E.M.F. is 0·85 volt, and there is no action unless the circuit is closed. To prevent the absorption of carbonic acid the solution is covered with paraffine oil.

813. **Leclanché's element.**—This consists (fig. 777) of a rod of carbon, C, placed in a porous pot, which is then very tightly packed with a mixture of pyrolusite (peroxide of manganese) and gas graphite, M. This is covered over with a layer of pitch. At the top of the carbon is soldered a mass of lead, L, to which is affixed a binding screw. The positive plate

is a rod of zinc, Z, in which is fixed a copper wire. The exciting liquid consists of a strong solution of sal-ammoniac, contained in a glass vessel, G, which is not more than one-third full. The electromotive force of the element is said to be 1·48, or about one-third greater than that of a Daniell's element; its internal resistance varies of course with the size, from 4 to 8 ohms. The battery is not adapted for continuous work as in heavy telegraphic circuits, or in electro-plating, since it soon becomes polarised; it has, however, the valuable property of quickly regaining its original strength when left at rest, and is extremely well adapted for discontinuous work, such as that of electrical bells.

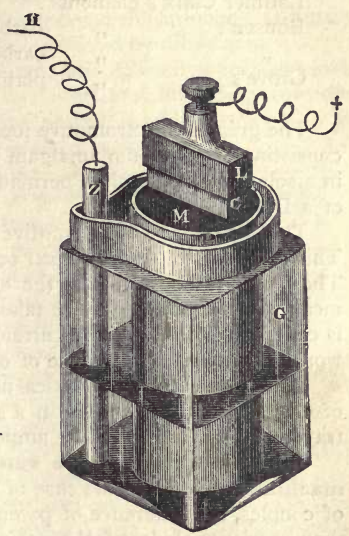


Fig. 777.

A modification of this element by von Beetz for therapeutic purposes consists of a test tube in the bottom of which is fused a platinum wire; this is then covered to one-third the height with a layer of a mixture of bruised gas coke and pyrolusite. In other respects the element is constructed like that of De la Rue and Müller.

A rod of carbon $4\frac{3}{4} \times 1\frac{7}{8} \times \frac{5}{16}$ inches should have a maximum resistance of 1 ohm; but good plates made from the carbon of gas retorts do not average more than 0·5, and in some cases 0·1 ohm. If the resistance equals an ohm, the conducting power of carbon is about 0·003 that of mercury.

A drawback to the use of carbon is that, from its porosity, the exciting liquid rises, and forms local currents at the junction with the binding screw, which injure or destroy contact. This may be remedied to a very great extent by soaking the plates before use in hot melted paraffine, which penetrates into the pores, expelling the air. On cooling, it solidifies and prevents the capillary action mentioned above. By carefully scraping the paraffine from the outside, a surface is exposed which is as good a conductor as if the pores were filled with air. Measurements have shown that the resistance of a rod thus prepared is not altered.

In a recent modification of his element Leclanché dispensed with the porous cell, and placed the carbon plate C between two similar flat prisms, made by compressing a mixture of 55 parts of graphite, 40 parts of pyrolusite, and 5 parts of shellac in steel moulds at a temperature of 100° under a pressure of 300 atmospheres. The resistance of this form of element is from 0·9 to 1·8 ohm.

814. **Electromotive force of different elements.**—The following numbers represent the electromotive force of some of the elements most frequently used, compared with that of an ordinary Daniell's cell charged as above described; they are the means of many careful determinations:—

Daniell's element	set up with water	1·00
„ „	pure zinc and pure water, with pure copper and pure saturated solution of copper sulphate	1·02
Leclanché's „	zinc in saturated solution of ammonium chloride	1·32
Latimer Clark's element		1·436
Bunsen's „	carbon in nitric acid	1·77
„ „	carbon in chromic acid	1·87
Grove's „	platinum in nitric acid	1·82

The greatest electromotive force as yet observed is by Beetz in a couple consisting of potassium amalgam in caustic potash, combined with pyrolusite in a solution of potassium permanganate. It is three times as much as that of a Daniell's element.

The standard of electromotive force on the C. G. S. system is the *Volt.* This is equal to 1,000,000,000 or 10^8 absolute electromagnetic units (709). The *volt* is rather less than the electromotive force of a Daniell's cell, the mean value of which may be taken at 1·08 volt. The unit of current, which is called an *Ampere*, is the current due to an electromotive force of one volt working through a resistance of one ohm.

The *Coulomb* is the practical unit of electrical quantity; it is that quantity of electricity which passes in a second through the section of a conductor traversed by a current of an ampere.

815. **Comparison of the voltaic battery with a frictional electrical machine.**—Except in the case of batteries consisting of a very large number of couples, the difference of potentials between the terminals is far weaker than in frictional electrical machines, and is insufficient to give any visible spark. With De la Rue and Müller's great battery the striking distance between two terminals was found to increase with the potential, but for high potentials rather more rapidly than in direct ratio. Thus while the striking distance was 0·012 in., with the potential due to 1,200 of their cells, it was 0·049 in. with 4,800 cells, and 0·133 in. with 11,000 cells.

In the case of a small battery or of a single cell, very delicate tests are required to detect any signs of free electrification. But by means of a delicate condensing electroscope, and by careful insulation, it can be shown that one pole possesses a positive and the other a negative charge. For this purpose one of the plates of the electroscope is connected with one pole, and the other with the other pole or with the ground. The electroscope thus becomes charged, and on breaking the connection electroscopic indications are observed.

On the other hand, the strength of current which a voltaic element can produce in a good conductor is much greater than that which can be produced by a machine. Faraday immersed two wires—one of zinc, and the other of platinum, each $\frac{1}{13}$ of an inch in diameter—in acidulated water for $\frac{3}{20}$ of a second. The effect thus produced on a magnetic needle in this short time was greater than that produced by 23 turns of the large electrical machine of the Royal Institution.

Nystrom has ascertained by quantitative measurements that the potential

of the charge of the cover of an ordinary electrophorus is not less than 50,000 times as great as the potential of a Meidinger's cell (812); that is, that not less than 50,000 of those elements would be required to produce the same potential as the electrophorus. In practice, a far greater number would be needed, owing to the difficulty of getting good insulation.

816. **Amalgamated zinc. Local currents.**—Perfectly pure distilled zinc is not attacked by dilute sulphuric acid, but becomes so when immersed in that liquid in contact with a plate of copper or of platinum. Ordinary commercial zinc, on the contrary, is rapidly dissolved by dilute acid.

This is due to what is called *local action* (806), arising from impurities which are always present in commercial zinc. To understand this effect consider as two portions *a* and *b* of a plate of zinc placed in dilute acid (fig. 778), *a* representing pure zinc, while *b* is supposed to represent such an impurity as a piece of lead or iron. Here are all the conditions for the production of an electrical current, two different metals in metallic connection which acts upon them unequally; the effect is that a current is produced from *a* to *b* through the liquid, and the zinc is eaten away.

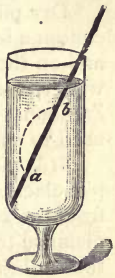


Fig. 778.

All ordinary zinc contains metallic impurities, such as lead and iron, which realise the above conditions, forming innumerable *local electrical currents*, which rapidly wear away the active plate without contributing anything to the general current.

Zinc, when amalgamated, acquires the properties of perfectly pure zinc, and is unaltered by dilute acid, so long as it is not in contact with a copper or platinum plate immersed in the same liquid. To amalgamate a zinc plate, it is first immersed in dilute sulphuric or hydrochloric acid so as to obtain a clean surface, and then a drop of mercury is placed on the plate and spread over it with a brush. The amalgamation takes place immediately, and the plate has the brilliant aspect of mercury. Zinc as well as other metals are readily amalgamated by dipping them in an amalgam of one part sodium and 200 parts of mercury. Zinc plates may also be amalgamated by dipping them in a solution of mercury prepared by dissolving one pound of mercury in five pounds of aqua regia (one part of nitric to three of hydrochloric acid), and then adding five parts more of hydrochloric acid.

Zinc may also be amalgamated in the mass by melting it in a closed vessel with 4 per cent. of mercury, and running it into moulds.

The amalgamation of the zinc removes from its surface all the impurities, especially the iron. The mercury effects a solution of pure zinc, which covers the surface of the plate, as with a liquid layer. The process was first applied to electrical batteries by Kemp. Amalgamated zinc is not attacked so long as the circuit is not closed—that is, when there is no current; when closed the current is more regular, and at the same time stronger, for the same quantity of metal dissolved.

817. **Dry piles.**—In *dry piles* the liquid is replaced by a solid hygrometric substance, such as paper. They are of various kinds; in Zamboni's, which is most extensively used, the electromotors are tin or silver, and binoxide of manganese. To construct one of these a piece of paper silvered or tinned on one side is taken; the other side of the paper is coated with finely-

powdered binoxide of manganese by slightly moistening it, and rubbing the powder on with a cork. Having placed together seven or eight of these sheets, they are cut by means of a punch into discs an inch in diameter. These discs are then arranged in the same order, so that the tin or silver of each disc is in contact with the manganese of the next. Having piled up 1,200 or 1,800 couples, they are placed in a glass tube, which is provided with a brass cap at each end. In each cap there is a rod and knob, by which the leaves can be pressed together, so as to produce better contact. The knob in contact with the manganese corresponds to the positive pole, while that at the other end, which is in contact with the silver or tin, is the negative pole.

Dry piles are remarkable for the permanence of their action, which may continue for several years. Their action depends greatly on the temperature and on the hygrometric state of the air. It is stronger in summer than in winter, and the action of a strong heat revives it when it appears extinct. A Zamboni's pile of 2,000 couples gives neither shock nor spark, but can be used to charge a Leyden jar and other condensers.

818. **Bohnenberger's electroscope.**—Bohnenberger constructed a dry-pile electroscope of great delicacy. It is a condensing electroscope (fig. 737), from the rod of which is suspended a single gold leaf. This is at an equal distance from the opposite poles of two dry piles placed vertically, inside the bell jar, on the plate of the apparatus. When the gold leaf has any free electricity it is attracted by one of the poles and repelled by the other, and its electricity is obviously contrary to that of the pole towards which it moves.

CHAPTER II

DETECTION AND MEASUREMENT OF VOLTAIC CURRENTS

819. **Detection and measurement of voltaic currents.**—The remarkable phenomena of the voltaic battery may be classed under the heads physiological, chemical, mechanical, and physical effects ; and these latter may be again subdivided into the thermal, luminous, and magnetic effects. For ascertaining the existence and measuring the strength of voltaic currents, the magnetic effects are more suitable than any of the others, and, accordingly, the fundamental magnetic phenomena will be described here, and the description of the rest postponed to a special chapter on Electro-magnetism.

820. **Oersted's experiment.**—Oersted published in 1819 a discovery which connected magnetism and electricity in a most intimate manner, and became, in the hands of Ampère and of Faraday, the source of a new branch of physics. The fact discovered by Oersted is the directive action which a fixed current exerts at a distance on a magnetic needle.

To make this experiment a copper wire is suspended horizontally in the direction of the magnetic meridian over a movable magnetic needle, as represented in fig. 779. So long as the wire is not traversed by a current, the needle remains parallel to it ; but as soon as the ends of the wire are respectively connected with the poles of a battery or of a single element, *the needle is deflected, and tends to take a position which is the more nearly at right angles to the magnetic meridian in proportion as the current is stronger.*

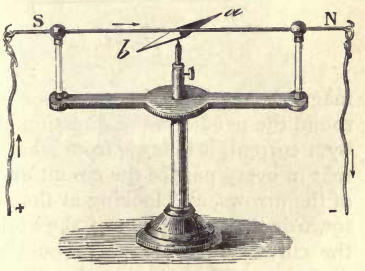


Fig. 779.

In reference to the direction in which the poles are deflected, there are several cases which may, however, be referred to a single principle. Remembering our assumption as to the direction of the current in the connecting wire (803), the preceding experiment presents the following four cases :—

i. If the current passes above the needle, and goes from south to north, the north pole of the magnet is deflected towards the west ; this arrangement is represented in the above figure.

ii. If the current passes below the needle, also from south to north, the north pole is deflected towards the east.

iii. When the current passes above the needle, but from north to south, the north pole is deflected towards the east.

iv. Lastly, the deflection is towards the west when the current goes from north to south below the needle.

Ampère has given the following *memoria technica* by which all the various directions of the needle under the influence of a current may be remembered. If we imagine an observer placed in the connecting wire in such a manner that the current entering by his feet issues by his head, and that his face is always turned towards the needle, we shall see that in the above four positions the north pole is always deflected towards the left of the observer. By thus personifying the current, the different cases may be comprised in this general principle : *In the directive action of currents on magnets, the north pole is always deflected towards the left of the current.*

821. **Galvanometer or multiplier.**—The name *galvanometer*, or sometimes *multiplier* or *rheometer*, is given to a very delicate apparatus by which the existence, direction, and intensity of currents may be determined. It was invented by Schweigger a short time after Oersted's discovery.

In order to understand its principle, let us suppose a magnetic needle suspended by a filament of silk (fig. 780), and surrounded in the plane of the

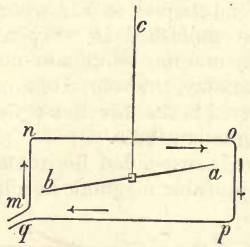


Fig. 780.

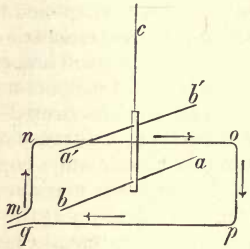


Fig. 781.

magnetic meridian by a copper wire, *mnopq*, forming a complete circuit round the needle in the direction of its length. When this wire is traversed by a current, it follows, from what has been said in the previous paragraph, that in every part of the circuit an observer lying in the wire in the direction of the arrows, and looking at the needle *ab*, would have his left always turned towards the same point of the horizon, and consequently, that the action of the current in every part would tend to turn the north pole in the same direction ; that is to say, that the actions of the four branches of the circuit concur to give the north pole the same direction. By coiling the copper wire in the direction of the needle, as represented in the figure, the action of the current has been *multiplied*. If, instead of a single one, there are several circuits, provided they are insulated, the action becomes still more multiplied, and the deflection of the needle increases. Nevertheless, the action of the current cannot be multiplied indefinitely by increasing the number of windings, for, as we shall presently see, the strength of a current diminishes as the length of the circuit is increased.

As the directive action of the earth continually tends to keep the needle in the magnetic meridian, and thus opposes the action of the current, the effect of the latter is increased by using an astatic system of two needles,

as shown in fig. 781. The action of the earth on the needle is then very feeble, and, further, the actions of the current on the two needles become accumulated. In fact, the action of the circuit, from the direction of the current indicated by the arrows, tends to deflect the north pole of the lower needle towards the west. The upper needle, *a'b'*, is subjected to the action of two contrary currents, *no* and *qp*, but as the first is nearer, its action preponderates. Now this current, passing below the needle, evidently tends to turn the pole *a'* towards the east, and, consequently, the pole *b'* towards the west; that is to say, in the same direction as the pole *a* of the other needle.

From these principles it will be easy to understand the action of the *multiplier*. The apparatus represented in fig. 782 consists of a thick copper plate, D, resting on levelling screws; on this is a rotating plate, P, of the same metal, to which is fixed a copper frame, the breadth of which is almost equal to the length of the needles. On this is coiled a great number of turns of wire covered with silk. The two ends terminate in binding screws, *i* and *o*. Above the frame is a graduated circle, C, with a central slit parallel to the direction in which the wire is coiled. The zero corresponds to the position of this slit, and there are two graduations on the scale, the one on the right and the other on the left of zero, but they only extend to 90°. By means of a very fine filament of silk, an astatic system is suspended; it consists of two needles *ab* and *a'b'*, one above the scale, and the other within the circuit itself. These needles, which are joined together by copper wire, like those in fig. 659 and fig. 781, and cannot move separately, must not have exactly the same magnetic intensity; for if they are exactly equal, every current, strong or weak, would always put them at right angles with itself.

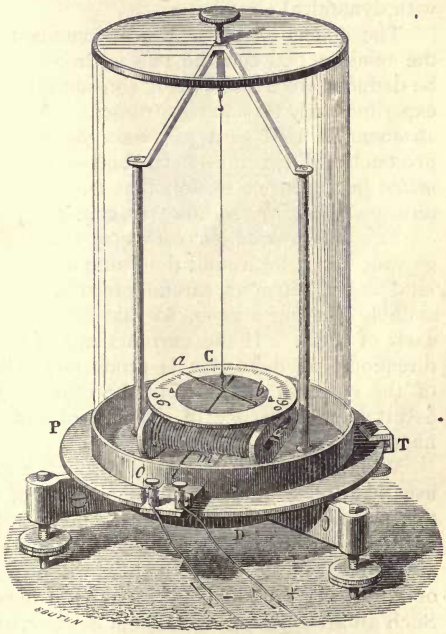


Fig. 782.

In using this instrument the diameter, to which corresponds the zero of the graduation, is brought into the magnetic meridian by turning the plate P until the end of the needle *ab* corresponds to zero. The instrument is fixed in this position by means of the screw-clamp T.

The length and diameter of the wire vary with the purpose for which the galvanometer is intended. For one which is to be used in observing the

currents due to chemical actions, a wire about $\frac{1}{8}$ millimetre in diameter, and making about 800 turns, is well adapted. Those for thermo-electric currents, which have low intensity, require a thicker and shorter wire ; for example, thirty turns of a wire $\frac{2}{3}$ millimetre in diameter. For very delicate experiments, as in physiological investigations, galvanometers with as many as 30,000 turns have been used.

By means of a delicate galvanometer consisting of 2,000 or 3,000 turns of fine wire, the coils of which are carefully insulated by means of silk and shellac, currents of high potential, as those of the electrical machine (791), may be shown. One end of the galvanometer is connected with the conductor, and the other with the ground, and on working the machine the needle is deflected, affording thus an illustration of the identity of statical with dynamical electricity.

The deflection of the needle increases with the strength of the current ; the relation between the two is, however, so complex, that it cannot well be deduced from theoretical considerations, but requires to be determined experimentally for each instrument. And in the majority of cases the instrument is used as a *galvanoscope* or *rheoscope*—that is, to ascertain the presence and direction of currents—rather than as a *galvanometer* or *rheometer* in the strict sense ; that is, as a measurer of their intensity. The term *galvanometer* is, however, commonly used.

The *differential galvanometer* consists of a needle, as in an ordinary galvanometer, but round the frame of which are coiled two wires of the same kind and dimensions, carefully insulated from each other, and provided with suitable binding screws, so that separate currents can be passed through each of them. If the currents are of the same strength but in different directions, no deflection is produced ; where the needle is deflected one of the currents differs from the other. Hence the apparatus is used to ascertain a difference in strength of two currents, and to this it owes its name.

When a current is passed through a galvanometer, the needle does not usually at once attain its final position of equilibrium, but oscillates about this position, which in observations causes much loss of time. If such a needle is surrounded by a mass of a good conductor such as copper, currents are induced in the mass which, as will afterwards be explained (905), impede, or *damp*, the motion of the magnetic needle and tend to bring it to rest. Such an arrangement is called a *damper*, and in practice is frequently used ; the copper frame on which the wires of the galvanometer are coiled, and the wires themselves, act in this way. The natural logarithm of the ratio of the amplitudes of two successive oscillations of the needle is called the *logarithmic decrement*. The logarithmic decrement λ is proportional to the product of the damping power ϵ and the time of a single oscillation t; that is, $\lambda = \epsilon t$. By diminishing the directive power of the earth on the magnet by making it astatic, the logarithmic decrement becomes infinite, and the needle attains its position of equilibrium without oscillations. Galvanometers in which the needle acquires at once this final deflection are known as *aperiodic*, or *dead-beat galvanometers*.

To this class belong that of Deprez and D'Arsonval represented in fig. 783, which is a development of Thomson's syphon recorder (892).

Between the branches of a strong horse-shoe magnet is a light iron cylinder supported independently and which becomes magnetised by induction. Between this and the magnet is a light rectangular wire coil, supported by wires conveying the current which are in connection with binding screws. When the current passes, the coil is deflected at right angles to the field, and equilibrium is established when the electro-magnetic action is equalled by the torsion of the wire. The motion of the coil can be read off by a spot of light reflected from a mirror (822) attached to it, and for small angles the current is proportional to tangent of the angle of deflection (823). Induction currents due to the motion of the coil in the field are produced, and as this is very powerful, the galvanometer is virtually dead-beat when closed by a small resistance.

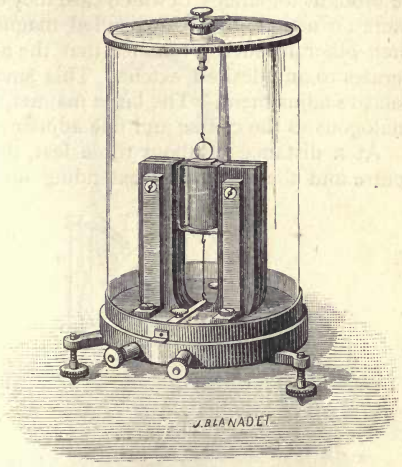


Fig. 783.

When a current of very small duration is passed through a galvanometer, a momentary deflection or *swing* or *throw* of the needle will be produced. The product of a constant into the sine of half the angle of the first swing is then a measure of the strength of the current, so that if momentary currents of different strengths are passed through one and the same galvanometer, they will be measured by the sines of the corresponding angles of deflection, or by the angles themselves where these are small. This is known as the *ballistic method* of measuring currents, and the galvanometers adapted for the purpose are known as *ballistic galvanometers.*

822. **Thomson's marine galvanometer.**—In laying submarine cables the want was felt of a galvanometer sufficiently sensitive to test insulation, which at the same time was not affected by the pitching and rolling of the ship. For this purpose, Lord Kelvin invented his marine galvanometer. B (fig. 784) represents a coil of many thousand turns of the finest copper wire, carefully insulated throughout, terminating in the binding screws, EE. In the centre of this coil is a slide, which carries the magnet, the arrangement of which is represented on a larger scale in D. The magnet itself is made of a piece of fine watch-spring about $\frac{3}{8}$ of an inch in length, and does not weigh more than a grain; it is attached to a small and very slightly concave mirror of very thin silvered glass. A single fibre of silk is stretched across the slide, and the mirror and magnet are attached to it in such a manner that the fibre passes exactly through the centre of gravity in every position. As the mirror and magnet weigh only a few grains, they retain their position relatively to the instrument, however the ship may pitch and roll. The slide fits in a groove in the coil, and the whole is enclosed within a wrought-iron case

with an aperture in front and a wrought-iron lid on the top. The effect of this is to act as a *magnetic screen* and thereby counteract the influence of terrestrial magnetism when the ship changes its course.

Underneath the coil is a large bent steel magnet N, which compensates the earth's directive action upon the magnet D (700) ; and in the side of the case, and on a level with D, a pair of magnets, C, are placed with opposite poles together. By a screw, suitably adjusted, the poles of the magnets may be brought together ; in which case they quite neutralise each other, and thus exert no action on the suspended magnet, or they may be slid apart from each other in such a manner that the action of either pole on D preponderates to any desired extent. This small magnet is thus capable of very delicate adjustment. The large magnet, N, and the pair of magnets, C, are analogous to the coarse and fine adjustment of a microscope.

At a distance of about three feet, there is a scale with the zero in the centre and the graduation extending on each side. Underneath this zero

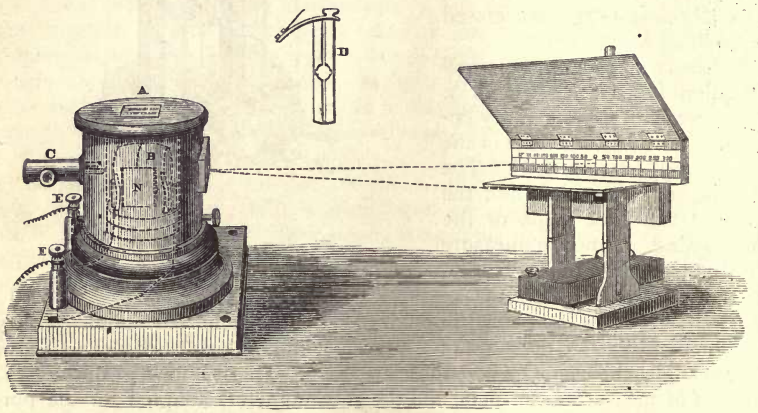


Fig. 784.

point is a narrow slit, through which passes the light of a paraffine lamp, and which, traversing the window, is reflected from the bent mirror against the graduated scale. By means of the adjusting magnets the image of the slit is made to fall on the centre of the graduation.

This being the case, if any arrangement for producing a current, however weak, be connected with the terminal, the spot of light is deflected either to one side or the other, according to the direction of the current ; the stronger the current the greater the deflection of the spot ; and if the current remains of constant strength for any length of time, the spot is stationary in a corresponding position.

The movement, on a screen, of a spot of light reflected from a body, is the most delicate and convenient means of observing motions which of themselves are too small for direct measurement or observation. Hence this principle is frequently applied in experimental investigations and in lecture illustrations (522). It is used in observing the motion of oscillating bodies,

in measuring the variations of magnetism, in determining the expansion of solids, &c.

It will be seen from the article on the Electric Telegraph how alternate deflections of the spot of light may be utilised in forming a code of signals.

823. **Tangent compass, or tangent galvanometer.**—When a magnetic needle is suspended in the centre of a voltaic current in the plane of the magnetic meridian, it can be proved that the strength of a current is directly proportional to the tangent of the angle of deflection, provided the dimensions of the needle are sufficiently small as compared with the diameter of the circuit. An instrument based on this principle is called the *tangent galvanometer* or *tangent compass*. It consists of a copper ring, 12 inches in diameter (fig. 785), and about an inch in breadth, mounted vertically on a stand; the lower half of the ring is generally fitted in a semicircular frame of wood to keep it steady. In the centre of the ring is suspended a delicate magnetic needle, whose length must not exceed $\frac{1}{12}$ or $\frac{1}{10}$ of the diameter of the circle. Underneath the needle there is a graduated circle. The ends of the ring are prolonged in copper wires, fitted with mercury cups, *ab*, by which it can be connected with a battery or element.

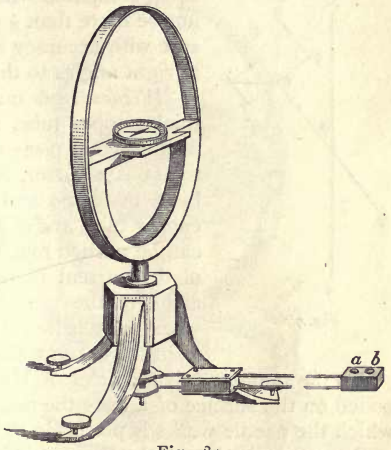


Fig. 785.

The circle is placed in the plane of the magnetic meridian, and the deflection of the needle is directly read off on the circle, and its corresponding value obtained from a table of tangents.

On account of its small resistance, the tangent galvanometer is well adapted for currents of low potential, but in which a considerable quantity of electricity is set in motion.

To prove that the intensities of various currents are proportional to the tangents of the corresponding angles of deflection, let NS, fig. 786, represent the wire of the galvanometer and *ns* the needle, and let ϕ be the angle of deflection produced when a current C is passed. Two forces now act upon the needle—the force of the earth's magnetism, which we will denote by H, which tends to place the needle in the magnetic meridian, and the strength of the current C, which strives to place it at right angles to the magnetic meridian. Let the magnitudes of these forces be represented by the corresponding lines *an* and *bn*. Now the whole intensities of these forces do not act so as to turn the point of the needle round, but only those components which are at right angles to the needle. Resolving them, we have *ng* and *nf* as the forces acting in opposite directions on the needle; and since the needle is at rest these forces must be equal.

The angle *nag* is equal to the angle ϕ, and therefore $ng = an \sin \phi$; and in like manner the angle *bnf* is equal to ϕ and $nf = bn \cos \phi$; and therefore

since $nf = ng$, $bn \cos \phi = an \sin \phi$, or $bn = an \dfrac{\sin \phi}{\cos \phi} = an \tan \phi$; that is, $C = H \tan \phi$.

If any other current be passed through the galvanometer we shall have similarly $C' = H \tan \phi'$; and since the earth's magnetism does not appreciably alter in one and the same place, $C : C' = \tan \phi : \tan \phi'$.

In this reasoning it has been assumed that the action of the current on the needle is the same whatever be the angle by which it is deflected. This is only the case when the dimensions of the needle are small compared with the diameter of the ring: it should not be more than $\frac{1}{8}$ or $\frac{1}{10}$ the diameter. In order to measure with accuracy the deflection a light index is placed at right angles to the needle.

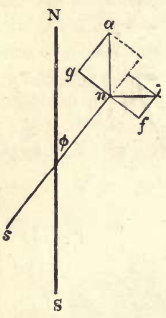


Fig. 786.

Wiedemann's tangent galvanometer consists of a short thick copper tube, in which is suspended, instead of a needle, a thin piece of soft iron, silvered on one side so as to act as a mirror, the position of which can be observed by a telescope and scale (522). On each side of the copper tube, and sliding in grooves, are coils of wire which can be pushed over the tube. By this lateral arrangement of the current in reference to the magnetic needle, the error of the tangent galvanometer is diminished; for when the needle is deflected, though one end moves away from the current, the other approaches it.

In the tangent galvanometer of Helmholtz and of Gaugain the wires are coiled on the surface of a cone the angle of which is 120°, and the point on which the needle works is placed in the position of the corresponding apex of the cone: the law of the tangent holds then even with longer needles, and especially if the wire is divided between two such cones, one on opposite sides of the needle.

If the ring of the tangent galvanometer is so constructed that it can turn about its axis, which is in the magnetic meridian, the action of the current on the needle is inversely proportional to the cosine of the angle θ, through which the ring is turned. Hence by increasing θ, the action of any current on the needle may be made as small as we please, and thus even very powerful currents may be measured by this instrument.

824. **Sine galvanometer.**—This is another form of galvanometer for measuring powerful currents. Round the circular frame M (fig. 787), several turns of stout insulated copper wire are coiled, the two ends of which, *i*, terminate on the binding screws at E. On a table in the centre of the ring there is a magnetic needle, *m*; a second light needle, *n*, fixed to the first, serves as pointer along the graduated circle N. Two copper wires, *a*, *b*, from the source of electricity to be measured, are connected with E. The circles M and N are supported on a foot O, which can move about a vertical axis passing through the centre of a fixed horizontal circle H.

The circle M being then placed in the magnetic meridian, and therefore in the same plane as the needle, the current is allowed to pass. The needle being deflected, the circuit M is turned until it coincides with the vertical plane passing through the magnetic needle *m*. The directive action of the

current is now exerted perpendicularly to the direction of the magnetic needle, and it may be shown that the strength of the current is proportional to the sine of the angle of deflection: this angle is measured on the circle H by means of a vernier on the piece C. This piece C, fixed to the foot O, turns it by means of a knob A. This angle of deflection, and hence its sine, being known, the intensity of the current may be thus deduced: let *mm'* be the direction of the magnetic meridian, *d* the angle of deflection, C the strength of the current, and H the directive action of the earth. If the direction and intensity of this latter force be represented by *ak*, it may be replaced by two components, *ah* and *ac* (fig. 788). Now, as the first has no directive action on the needle, the component *ac* must alone counterpoise the force C; that is, $C = ac$. But in the triangle *ack*, $ac = ak \cos cak$, from which

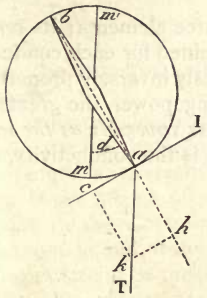


Fig. 788.

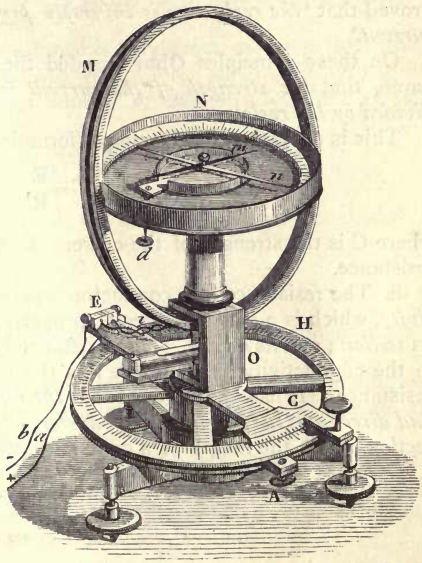


Fig. 787.

$ac = H \sin d$, for the angle *cak* is the complement of the angle *d*, and *ak* is equal to H; hence, lastly, $C = H \sin d$, which was to be proved. In like manner for any other current C', which produces a deflection *d'*, we shall have $C' = H \sin d'$, whence $C : C' = \sin d : \sin d'$.

825. **Ohm's law.**—For a knowledge of the conditions which regulate the action of the voltaic current, science is indebted to the late G. S. Ohm. His results were at first deduced from theoretical considerations; but by his own researches as well as by those of Fechner, Pouillet, Daniell, De la Rive, Wheatstone, and others, they received the fullest confirmation, and their great theoretical and practical importance has been fully established.

i. The force or cause by which electricity is set in motion in the voltaic circuit is called the *electromotive force.* The quantity of electricity which in any unit of time flows through a section of the circuit is called the *intensity*, or, perhaps better, *the strength of the current.* Ohm found that this strength is the same in all parts of one and the same circuit, however heterogeneous they were; one and the same magnetic needle is deflected to the same

extent over whatever part of the circuit it is suspended; and the same voltameter, wherever interposed in the circuit, indicates the same disengagement of gas; he also found that the strength is proportional to the electromotive force.

It has further been found that when the current from the same element is passed respectively through a short and through a long wire of the same material, its action on the magnetic needle is less in the latter case than in the former. Ohm accordingly supposed that in the latter case there was a greater *resistance* to the passage of the current than in the former; and he proved that '*the resistance is inversely proportional to the strength of the current.*'

On these principles Ohm founded the celebrated law which bears his name, that *the strength of the current is equal to the electromotive force divided by the resistance.*

This is expressed by the simple formula

$$C = \frac{E}{R},$$

where C is the strength of the current, E the electromotive force, and R the resistance.

ii. The resistance of a conductor depends on three elements; its *conductivity*, which is a constant special property, determined for each conductor; its *section*; and its *length*. The resistance is obviously inversely proportional to the conductivity; that is, the less the conducting power, the greater the resistance. It has been proved that *the resistance is inversely as the section and directly as the length of a conductor.* If then κ is the conductivity, ω the section, and λ the length of a conductor, we have

$$R = \frac{\lambda}{\kappa\omega} \text{ and } C = \frac{E}{\frac{\lambda}{\kappa\omega}} = \frac{\kappa\omega E}{\lambda};$$

that is, *the strength of a current is inversely proportional to the length of the conductor, and directly proportional to its section and conductivity.*

iii. In a voltaic battery composed of different elements, the strength of the current is equal to the sum of the electromotive forces of all the elements divided by the sum of the resistances. Usually, however, a battery is composed of elements of the same kind, each having, in intention at least, the same electromotive force and the same resistance.

In an ordinary element there are essentially two resistances to be considered: 1. That offered by the liquid conductor between the two plates, which is frequently called the *internal or essential resistance*; and 2. That offered by the interpolar conductor which connects the two plates outside the liquid; this conductor may consist either wholly of metal, or may be partly of metal and partly of liquids to be decomposed; it is the *external* or *non-essential resistance.* Calling the former R and the latter r, Ohm's formula becomes

$$C = \frac{E}{R + r}.$$

iv. If any number, n, of similar elements are joined together, there is

n times the electromotive force, but at the same time n times the internal resistance, and the formula becomes $\frac{nE}{nR+r}$. If the resistance in the interpolar, r, is very small—which is the case, for instance, when it is a short, thick copper wire—it may be neglected in comparison with the internal resistance, and then we have

$$C=\frac{nE}{nR}=\frac{E}{R};$$

that is, a battery consisting of several elements produces in this case no greater effect than a single element.

v. If, however, the external resistance is very great, as when the current has to produce the electric light, or to work a long telegraphic circuit, advantage is gained by using a large number of elements, for then we have the formula

$$C=\frac{nE}{nR+r}.$$

If r is very great as compared with nR, the latter may be neglected, and the expression becomes

$$C=\frac{nE}{r};$$

that is, the strength, within certain limits, is proportional to the number of elements.

In a thermo-electric pile (942), which consists of very short metallic conductors, the internal resistance R is so small that it may be neglected, and the strength is inversely as the length of the connecting wire.

vi. If the plates of an element be made m times as large, there is no increase in the electromotive force, for this depends solely on the nature of the metals and of the liquid (802); but the resistance is m times as small, for the section is m times larger: the expression becomes then

$$C=\frac{E}{\frac{R}{m}+r}=\frac{mE}{R+mr}.$$

Hence, an increase in the size of the plate—or, what is the same thing, a decrease in the internal resistance—does not increase the strength to an indefinite extent; for ultimately the resistance of the element R vanishes in comparison with the resistance r, and the strength continually approximates to the value $C=\frac{E}{r}$.

vii. Ohm's law enables us to arrange a battery so as to obtain the greatest effect in any given case. For instance, with a battery of six elements there are the following four ways of arranging them:—1. In a single series (fig. 789), in which the zinc Z of one element is united with the copper C of the second, the zinc of this with the copper of the third, and so on. 2. Arranged in a system of three double elements, each element being formed by joining two of the former (fig. 790). 3. In a system of two elements, each of which consists of three of the original elements joined, so as to form one of triple

the surface (fig. 791). Lastly, of one large element, all the zincs and all the coppers being joined, so as to form a pair of six times the surface (fig. 792).

With a series of twelve elements there may be six different combinations, and so on for a larger number.

Now let us suppose that in the particular case of a battery of six elements the internal resistance R of each element is 3, and the external resistance $r = 12$. Then in the first case where there are six elements arranged in series we have the value

$$C = \frac{6E}{6R + r} = \frac{6E}{6 \times 3 + 12} = \frac{6E}{30} \quad . \quad . \quad . \quad (1)$$

If they were united so as to form three elements, each of double the surface as in the second case (fig. 790), the electromotive force would then

Fig. 789.

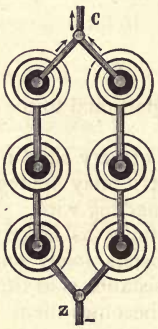


Fig. 790.

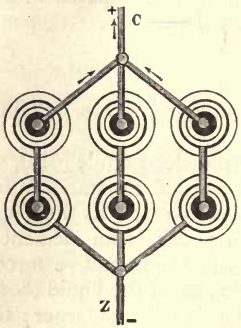


Fig. 791.

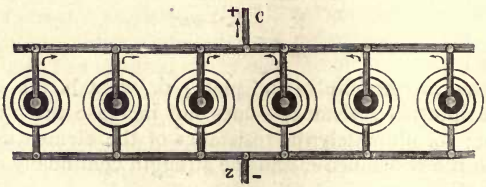


Fig. 792.

be three times the electromotive force in each element: there would also be a resistance R in each element, but this would be only half as great, for the section of the plate is now double; hence the strength in this case would be

$$C' = \frac{3E}{\frac{3R}{2} + r} = \frac{3E}{\frac{9}{2} + 12} = \frac{6E}{33}; \quad . \quad . \quad . \quad . \quad . \quad (2)$$

accordingly this change would lessen the strength.

If, with the same elements, the resistance in the connecting wire were only $r = 2$, we should have the values in the two cases respectively—

$$C = \frac{6 \times E}{6 \times 3 + 2} = \frac{6E}{20},$$

$$\text{and } C' = \frac{3E}{\frac{3R}{2} + r} = \frac{6E}{9 + 4} = \frac{6E}{13}.$$

The result in the latter case is, therefore, more favourable. If the resistance r were 9, the strength would be the same in both cases. Hence, then, by altering the size of the plates or their arrangement, favourable or unfavourable results are obtained according to the relation between R and r.

826. **Arrangement of multiple battery for maximum current.**—It can be shown that *in any given combination the maximum effect is obtained when the total resistance in the elements is equal to the resistance of the interpolar.* For let N be the total number of cells available for a given combination, and let n be the number of cells arranged *tandem*, or in series—that is, when the zinc of one is connected with the copper of the next, and so on; then there will be $\frac{N}{n}$ elements arranged *abreast.* If e be the electromotive force and r the resistance of one cell, while l is the external resistance, then the strength of the current will be

$$C = \frac{ne}{\frac{nr}{N} + l} = \frac{ne}{\frac{n^2 r}{N} + l} = \frac{e}{\frac{nr}{N} + \frac{l}{n}}$$

Therefore C is a maximum when $\frac{nr}{N} + \frac{l}{n}$ is a minimum. But $\frac{nr}{N} \times \frac{l}{n} = \frac{rl}{N}$ is a constant, therefore the sum $\frac{nr}{N} + \frac{l}{n}$ is a minimum when $\frac{nr}{N} = \frac{l}{n}$; that is, when $\frac{n^2 r}{N} = l$, or when the total internal resistance is equal to the external resistance.

For if x and $\frac{A^2}{x}$ are any two quantities whose product is A^2, then

$$x + \frac{A^2}{x} = \frac{x^2 + A^2 - 2Ax + 2Ax}{x} = \frac{(x - A)^2}{x} + 2A.$$

This is greater than 2A unless $x - A = 0$, in which case it is equal to 2A, and is a minimum. In that case $x = A$, and therefore

$$x = \frac{A^2}{x}.$$

It follows thus from the above formula that the best effect is obtained when $n = \sqrt{\frac{Nl}{r}}$.

If in a given case we have 8 elements, each offering a resistance 15, and an interpolar with the resistance 40, we get $n = 4{\cdot}6$. But this is an

impossible arrangement, for it is not a whole number, and the nearest whole number must be taken. This is 4 ; and it will be found, on making a calculation analogous to that above, that when arranged so as to form 4 elements, each of double surface, the greatest effect is obtained.

The formula for the strength of current from several elements, $C = \frac{nE}{R}$, may also be applied to the currents produced by a magneto-electrical machine (920). In that case n stands for the number of coils which in a given time cut the lines of force of a magnetic field.

The principle that the best effect is obtained when the total internal is equal to the total external resistance, holds also for the currents produced by these machines.

CHAPTER III

EFFECTS OF THE CURRENT

827. **Physiological actions.**—Under this name are included the effects produced by a battery current on living organisms or tissues.

When the electrodes of a battery of many cells are held in the two hands a violent shock is felt, especially if the hands are moistened with acidulated water, which increases the conductivity. The violence of the shock increases with the number of elements used, and with a large number—as 200 Bunsen's cells—is even dangerous.

The power of contracting upon the application of a voltaic current seems to be a very general property of *protoplasm*—the physical basis of both animal and vegetable life ; if, for example, a current of moderate strength be passed through such a simple form of protoplasm as an amœba, it immediately withdraws its processes, ceases its changes of form, and contracts into a rounded ball—soon, however, resuming its activity upon the cessation of the current. Essentially similar effects of the current have been observed in the protoplasm of young vegetable cells.

If a frog's fresh muscle (which will retain its vitality for a considerable time after removal from the body of the animal) be introduced into a galvanic circuit, no apparent effect will be observed during the steady passage of the current, but every opening or closure of the circuit will cause a muscular contraction, as will also any sudden and considerable alteration in its intensity. By very rapidly interrupting the current, the muscle can be thrown into a state of uninterrupted contraction, or physiological *tetanus*, each new contraction occurring before the previous one has passed off. Other things being equal, the amount of shortening exhibited by the muscles increases, up to a certain limit, with the intensity of the current. These phenomena entirely disappear with the life of the muscle ; hence the experiments are somewhat more difficult with warm-blooded animals, the vitality of whose muscles, after exposure or removal from the body, is maintained with more difficulty ; but the results of careful experiment are exactly the same here as in the case of the frog.

The influence of an electric current upon living nerves is very remarkable ; as a general rule, it may be stated that its effect is to throw the nerve into a state of activity, whatever its special function may be : thus, if the nerve be one going to a muscle, the latter will be caused to contract ; if it be one of common sensation, pain will be produced ; if one of special sense, the sensation of a flash of light, or of a taste, &c., will be produced, according to the nerve irritated. These effects do not manifest themselves during the even passage of the current, but only when the circuit is either opened or

closed, or both. Of course the continuity of the nerve with the organ where its activity manifests itself must be maintained intact. The changes set up by the current in the different nerve-trunks are probably similar, the various sensations, &c., produced depending on the different terminal organs with which the nerves are connected.

Professor Burdon Sanderson has ascertained that the movement which causes the *Dionœa muscipula* (Venus's fly-trap), one of those which are called *carnivorous plants*, to close its hairy leaves and thereby entrap insects which alight upon it, is accompanied by an electrical current in a manner analogous to that manifested in muscular contraction. The manner in which the irritation is caused seems immaterial.

828. **Electrotonus.**—In a living nerve, as will be stated more fully in Chapter X., certain parts of the surface are electropositive to certain other parts, so that if a pair of electrodes connected with a galvanometer be applied to these two points, a current will be indicated; if now another part of the nerve be interposed in a galvanic circuit, it will be found that, if this extraneous current be passing in the same direction as the proper nerve-current, the latter is increased, and *vice versâ*; and this although it has previously been demonstrated experimentally that none of the battery current escapes down the nerve, so as to exert any influence of its own on the galvanometer. This alteration of its natural electromotive condition, produced through the whole of a nerve by the passage of a constant current through part of it, is known as the *electrotonic state*; it is most intense near the extraneous, or, as it is called, the *exciting current*. It continues as long as the latter is passing, and is attended with important changes in the *excitability* of the nerve, or, in other words, the readiness with which the nerve is thrown into a state of functional activity by any stimulus applied to it. Pflüger, who has investigated these changes, has named the part of the nerve through which the exciting current is passing the *intrapolar region*: the condition of the nerve close to the positive pole is called *anelectrotonus*; that near the negative pole, *kathelectrotonus*. The excitability of the nerve is diminished in the anelectrotonic region, so that with a motor nerve, for example, a stronger stimulus than before would need to be applied at this part in order to obtain a muscular contraction; in the kathelectrotonic region, on the contrary, the excitability of the nerve is heightened. Moreover, with an exciting current of moderate strength, the power of the nerve to conduct a stimulus is lowered in the anelectrotonic region, and increased in the kathelectrotonic; with strong currents it is said to be diminished in both.

These facts have to be taken into account in the scientific application of galvanism to medical purposes. If, for instance, it is wished to diminish the excitability of the sensory nerves of any part of the body, the current should be passed in such a direction as to throw the nerves of that part into a state of anelectrotonus—and similarly in other cases.

If a powerful electric current be passed through the body of a recently killed animal, violent movements are produced, as the muscles ordinarily retain their vitality for a considerable time after general systematic death: by this means, also, life has been re-established in animals which were apparently dead—a properly applied current stimulating the respiratory muscles to contract.

829. **Heating effects.**—When a voltaic current is passed through a metal wire the same effects are produced as by the discharge of an electric battery (790); the wire becomes heated, and even incandescent if it is very short and thin. With a powerful battery all metals are melted, even iridium and platinum, the least fusible of metals. Carbon is the only element which has not hitherto been fused by it. Despretz, however, with a battery composed of 600 Bunsen's elements joined in six series (825), raised rods of very pure carbon to such a temperature that they were softened and could be welded together, yielding an incipient fusion.

A battery of 30 to 40 Bunsen's elements is sufficient to melt and volatilise fine wires of lead, tin, zinc, copper, gold, silver, iron, and even platinum, with differently coloured sparks. Iron and platinum burn with a brilliant white light; lead with a purple light; the light of tin and of gold is bluish-white; the light of zinc is a mixture of white and gold; finally, copper and silver give a green light.

The thermal effects of the voltaic current are used for firing mines for military purposes and for blasting operations. The following arrangement was devised by Colonel Schaw, and serves to illustrate the principle :—Fig. 793 represents a small wooden box provided with a lid. Two moderately stout copper wires, *bb'*, insulated by being covered with gutta-percha, are deprived of this coating at the ends, which are then passed through and through the box in the manner represented in the figure. The distance between them is $\frac{3}{8}$ of an inch, and a very fine platinum wire is soldered across. The object of arranging the wires in this manner is that they shall not be in contact, and that the strain which they exert may be spent on the box, and not on the platinum wire joining them, which, being extremely thin, would be broken by even a very slight pull. The box is then filled with fine grained powder, and the lid tied down. The wires of the fuse are then carefully joined to the long conducting wires which lead to the battery: these should be of copper, and as thick as is convenient, so as to offer very little resistance. The fuse is then introduced into the charge to be fired: if it is for a submarine explosion the powder is contained in a canister, the neck of which, after the introduction of the fuse, is carefully fastened by means of cement. When contact is made with the battery, which is effected through the inter-

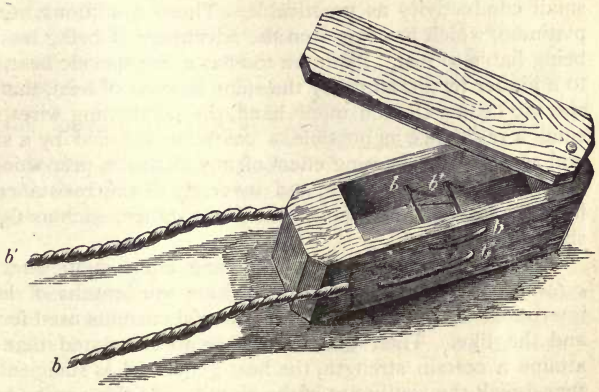


Fig. 793.

vention of mercury cups, the current traversing the platinum wire renders it incandescent, which fires the fuse ; and thus the ignition is communicated to the charge in which it is placed.

The heating effect depends more on the size than on the number of the plates of a battery, for the resistance in the connecting wires is small (825). An iron wire may be melted by a single Wollaston's element, the zinc of which is 8 inches by 6. Hare's battery (805) received its name *deflagrator* on account of its greater heating effect, produced by the great surface of its plates.

When any circuit is closed, a definite amount of heat, H, is produced throughout the entire circuit ; and the amount of heat, h, produced in any particular part of the circuit bears to the total heat, H, the same ratio which the resistance, r, of this part bears to R, that of the entire circuit. Hence, in firing mines, the wire to be heated should be of as small section and of as small conductivity as practicable. These conditions are well satisfied by platinum, which has over iron the advantage of being less brittle and of not being liable to rust. Platinum too has a low specific heat, and is thus raised to a higher temperature, by the same amount of heat, than a wire of greater specific heat. On the other hand, the conducting wires should present as small a resistance as possible, a condition satisfied by a stout copper wire ; and again, as the heating effect of any circuit is proportional to the square of the electromotive force, and inversely as the resistance, a battery with a high electromotive force and small resistance, such as Grove's or Bunsen's, should be selected.

Another application of the heating effect is to what are called *safety catches* or *automatic cut-nuts*. These are lengths of lead wire or strips interposed in the circuit of the powerful currents used for electrical lighting and the like. Their dimensions are so calculated that when the current attains a certain strength, the heat generated is sufficient to melt them and thus break the continuity of the circuit. As this can be arranged with great accuracy, it is possible so to regulate the circuit that it shall not exceed a certain limit.

By means of a heated platinum wire, parts of the body may be safely cauterised which could not be got at by a red-hot iron ; the removal of tumours and the like may be effected by drawing a loop of cold platinum wire round their base, which is then made hot by pressing the button of a contact arrangement, and gradually pulled together. It has been observed that when the temperature of the wire is about 600° C., the combustion of the tissues is so complete that there is no hæmorrhage ; while at 1500° the action of the wire is like that of a sharp knife. For other purposes of this *galvanic cauterisation*, platinum wire coiled in grooves cut in a porcelain rod is used.

830. **Laws of heating effects. Galvanothermometer.**—Although the thermal effects are most obvious in the case of thin wires, they are by no means limited to them. The laws of the heating effect were investigated by Lenz, by means of an apparatus called the *Galvanothermometer* (fig. 794). A wide-mouthed stoppered bottle was fixed upside down, with its stopper, b, in a wooden box ; the stopper was perforated so as to give passage to two thick platinum wires, connected at one end with binding screws, ss, while their free ends were provided with platinum cones by which the wires under

investigation could be readily affixed ; the vessel contained alcohol, the temperature of which was indicated by a thermometer fitted in a cork inserted in a hole made in the bottom of the vessel. The current is passed through the platinum wires, and its strength measured by means of a tangent compass interposed in the circuit. By observing the increase of temperature in the thermometer in a given time, and knowing the weight of the alcohol, the mass of the wire, the specific heat, and the calorimetric values (453) of the vessel, and of the thermometer, compared with alcohol, the heating effect which is produced by the current in a given time can be calculated.

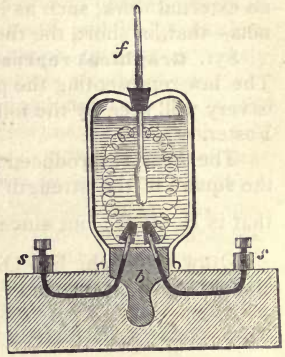


Fig. 794.

By apparatus of this kind the truth of the following law may be established.

The heat disengaged in a given time, t, is directly proportional to the square of the strength of the current, and to the resistance.

This is known as *Joule's law* (831), and is expressed in the formula $H = C^2Rt = \frac{E^2t}{R} = EC t$. If we divide the values E, C, R expressed in ergs by the mechanical equivalent of a water-gramme degree, that is, by $4{\cdot}16 \times 10^7$, we get the value H in *water-gramme degrees.*

If the current passes through a chain of platinum and silver wire of equal sizes, the platinum becomes more heated than the silver from its greater resistance ; and with a suitable current the platinum may become incandescent while the silver remains dark. This experiment was devised by Children.

If a long thin platinum wire be raised to dull redness by passing a voltaic current through it, and if part of it be cooled down by ice, the resistance of the cooled part is diminished, the strength of the current increases, and the rest of the wire becomes brighter than before. If, on the contrary, a part of the feebly incandescent wire be heated by a spirit-lamp, the resistance of the heated part increases ; the effect is the same as that of introducing additional resistance, the strength of the current diminishes, and the wire ceases to be incandescent in the non-heated part.

The cooling by the surrounding medium exercises an important influence on the phenomenon of ignition. A round wire is more heated by the same current than the same wire which has been beaten out flat : for the latter with the same section offers a greater service to the cooling medium than the other. For the same reason, when a wire is stretched in a glass tube on which two brass caps are fitted air-tight, and the wire is raised to dull incandescence by the passage of a current, the incandescence is more vivid when the air has been pumped out of the tube, because it now simply loses heat by radiation, and not by communication to the surrounding medium.

Similarly, a current which will melt a wire in air will only raise it to dull redness in ether, and in oil or in water will not heat it to redness at all, for the liquids conduct heat away more readily than air does.

From the above laws it follows that the heating effect is the same in a wire whatever be its length, provided the current is constant; but it must be remembered that by increasing the length of the wire we increase the resistance, and consequently diminish the current; further, in a long wire there is a greater surface, and hence more heat is lost by radiation and by conduction.

It must be added that Joule's law only holds provided the current does no external work, such as inductive actions on adjacent conductors, or magnets—that, in short, the thermal is the only action of the current.

831. **Graphical representation of the heating effects in a circuit.**—The law representing the production of heat in a circuit in the unit of time is very well seen by the following geometrical construction, due to Professor Foster.

The heat H produced in a circuit in the unit of time is proportional to the square of the strength of the current C, and to the resistance R (830), that is $H = C^2R$; but since $C = \frac{E}{R}$ (825), we have $H = \frac{E^2}{R}$.

Draw a straight line DAB (fig. 795), and from any point A in it draw a line AC, at right angles to DAB, and of a length proportional to the electromotive force of the cell. Lay off a length AB proportional to the resistance of the circuit. Join CB, and at C draw a line at right angles to BC, and let D be the point where this line cuts the line DAB. Then the length AD is proportional to the *heat* produced in the whole circuit in unit time. For the triangles ADC and ACB are similar, and therefore AD : AC = AC : AB; that is, $AD = \frac{AC^2}{AB}$; that is, $H = \frac{E^2}{R}$.

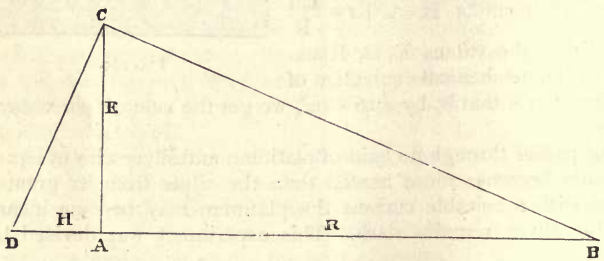


Fig. 795.

By drawing figures similar to the above it will be found that for a given electromotive force the heat is inversely proportional to the resistance, and for a given resistance directly proportional to the square of the electromotive force. That is, if the resistance is doubled, the heat is reduced to one-half; if the electromotive force is doubled, the heat is quadrupled.

832. **Relation of heating effect to work of a battery.**—In every closed circuit chemical action is continuously going on; in ordinary circuits, the most common action is the solution of zinc in sulphuric acid, which may be regarded as an oxidation of the zinc to form oxide of zinc, and a combination of this oxide of zinc with sulphuric acid to form water and zinc sulphate. It is a true combustion of zinc, and this combustion serves to maintain all the actions which the circuit can produce, just as all the work which a steam-engine can effect has its origin in the combustion of fuel (473).

By independent experiments it has been found that, when a given weight of zinc is dissolved in sulphuric acid, a certain definite measurable *quantity*

of heat is produced, which, as in all cases of chemical action, is the same, whatever be the rapidity with which this solution is effected. If this solution takes place while the zinc is associated with another metal so as to form a voltaic couple, the rapidity of the solution will be altered and the whole circuit will become heated—the liquid, the plates, the containing vessel as well as the connecting wire. But although the distribution of the heat is thus altered, its quantity is not. If the values of all the several heating effects in the various parts of the circuit be determined, it will still be found that, however the resistance of the connecting wire be varied, this sum is exactly equivalent to that produced by the solution of a certain weight of zinc.

If the couple be made to do external mechanical work the case is different. Joule made the following remarkable experiment :—A small zinc and copper couple was arranged in a calorimeter, and the amount of heat determined while the couple was closed for a certain length of time by a short thick wire. The couple still contained in the calorimeter was next connected with a minute electromagnetic engine (899), by which a weight was raised. It was thus found that the heat produced in the calorimeter in a given time—while, therefore, a certain amount of zinc was dissolved—was less while the couple was doing work than when it was not ; and the amount of this diminution was the exact thermal equivalent of the work performed in raising the weight (497).

That the whole of the chemical work and disengagement of heat in the circuit of an ordinary cell has its origin in the solution of zinc in acid is confirmed by the following experiment, due to Favre :—

In the muffle of his calorimeter (463), five small zinc platinum elements were introduced ; the other muffle contained a voltameter. Now when the element was closed until one equivalent of zinc was dissolved in the whole of the cells, $\frac{1}{5}$ of an equivalent of water should be decomposed in the voltameter (846), which was found to be the case. In one case the current of the battery was closed without inserting the voltameter, and the heat disengaged during the solution of one equivalent of zinc was found to be 18,796 thermal units ; when, however, the voltameter was introduced, the quantity disengaged was only 11,769 thermal units. Now the difference, 7,027, is represented by the chemical work of decomposing $\frac{1}{5}$ of an equivalent of water : this agrees very well with the number, $6{,}892 = \frac{34{,}462}{5}$, which represents the heat disengaged during the formation of $\frac{1}{5}$ of an equivalent of water.

However complicated may be the chemical processes involved in a voltaic combination, the total heat produced in it is the sum of the quantities of heat which are produced and absorbed in these various chemical processes.

We may illustrate this important principle by reference to the element of De la Rue and Müller (812), the chemical actions in which are perhaps the simplest of all constant elements. The normal action is that, when the element is closed, zinc decomposes ammonium chloride with the formation of zinc chloride, while the liberated ammonium unites with the chlorine of the silver chloride, re-forming ammonium chloride and depositing silver. The heat of decomposition and of re-formation of the ammonium chloride compensate one another, and the net result is the formation of zinc chloride, and the decomposition of silver chloride. Now the heat produced in the

formation of a molecule of zinc chloride ($ZnCl_2$) is 112,840 gramme units, and that of the equivalent silver chloride (2Ag Cl) is 58,760. The difference is 54,080, which is less than 58,360, the heat required to decompose a molecule of water. Hence it is that one such element will not effect a continuous decomposition of water, but at least two are required for the purpose. In like manner the heat available by the substitution of zinc for copper in one Daniell's cell is represented by 47,300, and accordingly at least two of these elements are also required.

In some cases, however, the current of a single cell does produce a feeble but continuous decomposition of water. This arises from the fact that the water of the voltameter contains air in solution, and the hydrogen as it is liberated unites with the dissolved oxygen. This process is known as *electrolytic convection.*

833. **Luminous effects.**—Luminous effects are obtained when the battery is sufficiently powerful, by bringing the two electrodes very nearly in contact; a succession of bright sparks springs sometimes across the interval, which follow each other with such rapidity as to produce continuous light. Although the quantity of electricity put in motion by the voltaic current is very great, the distance across which the spark passes is very small. Jacobi found that with a battery of 12 Grove's elements the electrodes could be approached within 0·0013 mm. before the spark passed.

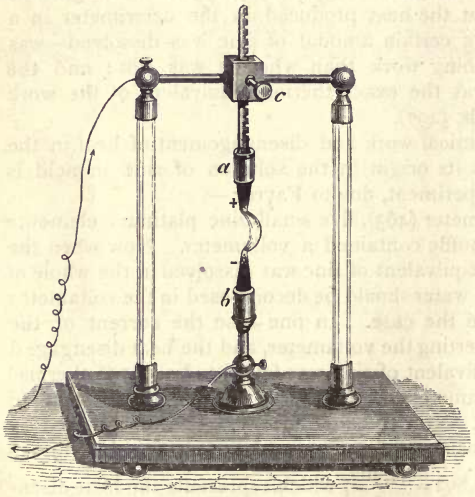


Fig. 796.

When one terminal of a battery of a few elements is connected with a file, and an iron wire connected with the other is moved over the file, a stream of brilliant luminous sparks is obtained, which obviously arises from a combustion.

The most beautiful effect of the electric light is obtained when two pencils of charcoal are connected with the terminals of the battery in the manner represented in fig. 796. The charcoal *b* is fixed, while the charcoal *a* can be raised and lowered by means of a rack and pinion motion, *c*. The two charcoals being placed in contact, the current passes, and their ends soon become incandescent. If they are then removed to a distance of about the tenth of an inch, according to the strength of the current, a luminous arc extends between the two points, which has an exceedingly brilliant lustre, and is called the *voltaic arc.*

The length of this arc varies with the force of the current. In air it

may exceed 2 inches, with a battery of 600 elements, arranged in six series of 100 each, provided the positive pole is uppermost, as represented in the figure; if it is undermost, the arc is about one-third shorter. In a partial vacuum the distance of the charcoals may be greater than in air; in fact, as the electricity meets with no resistance, it springs between the two charcoals, even before they are in contact. The voltaic arc can also be produced in liquids, but it is then much shorter, and its brilliancy is greatly diminished.

The voltaic arc has the property that it is attracted when a magnet is presented to it—a case of the action of magnets on currents (865).

The voltaic arc may be considered as formed of a very rapid succession of bright sparks. Its colour and shape depend on the nature of the conductors between which it is formed, and it is probably due to the incandescent particles of the conductor, which are volatilised and transported in the direction of the current; that is, from the positive to the negative pole. The more easily the electrodes are disintegrated by the current, the greater is the distance at which the electrodes can be placed. Charcoal, which is a very friable substance, is one of the bodies which give the largest luminous arc.

Davy first made the experiment of the electric light, in 1801, by means of a battery of 3,000 plates, each four inches square. He used charcoal points made of light wood charcoal which had been heated to redness, and immersed in a mercury bath; the mercury penetrating into the pores of the charcoal increased its conductivity. When any substance was introduced into the voltaic arc produced by this battery, it became incandescent; platinum melted like wax in the flame of a candle; sapphire, magnesia, lime, and most refractory substances were fused. Fragments of diamond, of charcoal, and of graphite rapidly disappeared without undergoing any previous fusion.

As charcoal rapidly burns in air, it was necessary to operate in vacuo, and hence the experiment was for a long time made by fitting the two points in an electric egg, like that represented in fig. 742. At present the electrodes are made of gas graphite, a modification of charcoal deposited in gas retorts; this is hard and compact, and only burns slowly in air; hence it is unnecessary to operate in vacuo. When the experiment is made in vacuo, there is no combustion, but the charcoal wears away at the positive pole, while it is somewhat increased on the negative pole, indicating that there is a transport of solid matter from the positive to the negative pole.

It appears from the researches of Edlund that the disintegration of the electrodes which takes place when the voltaic arc is formed give rise to a counter-electromotive force which is analogous to the polarisation which takes place in the decomposition of water (806), and the existence of which can be demonstrated by similar experiments. The magnitude of this force varies with the nature of the electrodes; it is greatest with carbon, amounting to 35 volts; with iron it is 25; copper, 24; zinc, 19; and cadmium 10 volts.

The resistance of the arc itself, due to the medium, increases like other resistances with the distance of the terminals; it diminishes as the strength of the current increases, for then the temperature increases. Working with carbon electrodes, it was found to amount to 1·3 ohm for each mm. of distance.

This counter-electromotive force explains how it is that a continuous arc can only be obtained with a current of considerable electromotive force.

834. **Foucault's experiment.**—This consists in projecting on a screen the image of the charcoal points produced in the camera obscura at the moment at which the electric light is formed (fig. 797). By means of this experiment which is made by the photo-electric microscope already described (fig. 565), the two charcoals can be readily distinguished, and the positive charcoal is seen to become somewhat hollow and diminished, while

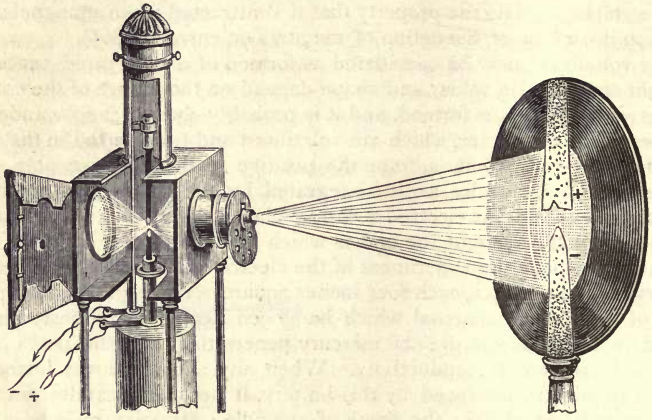


Fig. 797.

the other increases. The globules represented on the two charcoals arise from the fusion of a small quantity of silica contained in the charcoal. When the current begins to pass, the negative charcoal first becomes luminous, but the light of the positive charcoal is the brightest; as it also wears away about twice as rapidly as the negative electrode, it ought to be rather the larger.

835. **Regulator of the electric light.**—When the electric light is to be used for illumination, it must be as continuous as other modes of lighting. For this purpose, not only must the current be constant, but the distance of the charcoals must not alter, which necessitates the use of some arrangement for bringing them nearer together in proportion as they wear away. One of the best modes of effecting this is by an apparatus invented by Duboscq.

In this regulator the two charcoals are movable, but with unequal velocities, which are virtually proportional to their waste. The motion is transmitted by a drum placed on the axis *xy* (fig. 798). This turns, in the direction of the arrows, two wheels, *a* and *b*, the diameters of which are as 1 : 2, and which respectively transmit their motion to two rackworks, C′ and C. C lowers the positive charcoal, *p*, by means of a rod sliding in the tube H, while the other C′ raises the negative charcoal, *n*, half as rapidly. By means of the milled head *y* the drum can be wound up, and at the same time the positive charcoal moved by the hand; the milled head *x* moves the negative charcoal also by the hand, and independently of the first. For this purpose the axis, *xy*, consists of two parts pressing against each other with

some force, so that, holding the milled head *x* between the fingers, the other, *y*, may be moved, and by holding the latter the former can be moved. But the friction is sufficient when the drum works to move the two wheels *a* and *b* and the two rackworks.

The two charcoals being placed in contact, the current of a powerful battery of 40 to 50 elements reaches the apparatus by means of the wires E and E′. The current rising in H descends by the positive charcoal, then by the negative charcoal, and reaches the apparatus, but without passing into the rackwork C, or into the part on the right of the plate N ; these pieces being insulated by ivory discs placed at their lower part. The current ultimately reaches the bobbin B, which forms the foot of the regulator, and passes into the wire E′. Inside the bobbin is a bar of soft iron, which is magnetised as long as the current passes in the bobbin, and demagnetised when it does not pass, and this temporary magnet is the regulator. For this purpose it acts attractively on an armature of soft iron, A, open in the centre so as to allow the rackwork C′ to pass, and fixed at the end of a lever, which works on two points, *mm*, and transmits a slight oscillation to a rod, *d*, which, by means of a catch, *i*, seizes the wheel *z*, as is seen on a larger scale in fig. 799. By an endless screw, and a series of toothed wheels, the stop is transmitted to the drum, and the rackwork being fixed, the same is the case with the carbons. This is what takes place so long as the magnetisation in the bobbin is strong enough to keep down the armature A ; but in proportion as the carbons wear away, the current becomes feebler, though the voltaic arc continues, so that ultimately the attraction of the magnet no longer counterbalances a spring *r*, which continually tends to raise the armature. It then ascends, the piece *d* disengages the stop *i*, the drum works, and the carbons come

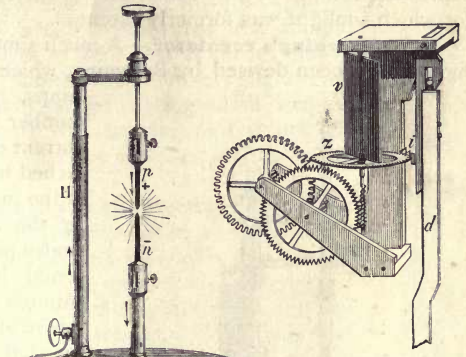


Fig. 799.

Fig. 798.

nearer; they do not, however, touch, because the strength of the current gains the upper hand, the armature A is attracted, and the carbons remain fixed. As their distance only varies within very narrow limits, a regular and continuous light is obtained with this apparatus until the carbons are quite used.

By means of a regulator, Duboscq illuminates the photogenic apparatus represented in fig. 585, by which all optical experiments may be performed for which sunlight was formerly essential.

836. **Browning's regulator.**—A much simpler apparatus, represented in fig. 800, has been devised by Browning, which is less costly than the other lamps, and also requires a smaller number of elements to work it. The current enters the lamp by a wire attached to a binding screw on the base of the instrument, passing up the pillar by the small electro-magnet to the centre pillar along the top of the horizontal bar, down the left-hand bar through the two carbons, and away by a wire attached to a binding screw on the left hand. A tube holding the upper carbon slides freely up and down a tube at the end of the cross-piece, and would by its own weight rest on the lower carbon, but the electro-magnet is provided with a keeper, to which is attached a rest that encircles the carbon tube and grasps it. When the electro-magnet works and attracts the keeper, the rest tightens, and thereby prevents the descent of the carbon. When the keeper is not attracted the rest loosens, and the carbon-holder descends.

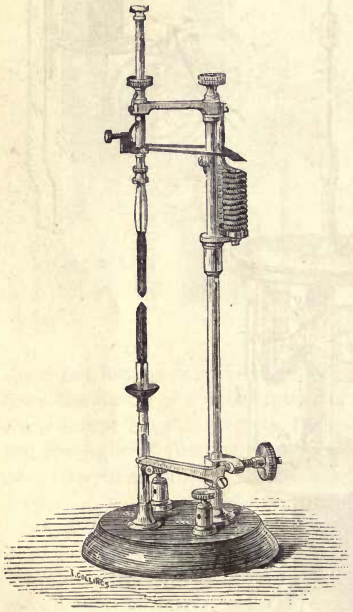


Fig. 800.

When the two carbons are at rest, on making contact with a battery the current traverses both carbons and no light is produced. But if the upper carbon be raised ever so little, a brilliant light is emitted. When the lamp is thus once set to work, the rod attached to the upper carbon may be let go, and the magnet will afterwards keep the lamp at work. For when some of the carbon is consumed, and the interval between the two is too great for the current to pass, the magnet loses some of its power, the keeper loosens its hold on the carbon, and this descends by its own weight. When they are sufficiently near, but before they are in contact, the current is re-established; the magnet again draws on the keeper, and the keeper again checks the descent of the carbon, and so forth. Thus the points are retained at the right distances apart, and the light is continuous and brilliant.

Stohrer has devised a regulator for the electrical light which is very

simple in principle, and which also only requires a few elements. Its essential features are represented in fig. 801, in which *b* is a cylinder containing vaseline and surrounded by the wire of the circuit *f*. In this is a hollow cylindrical floater *a*, nearly as wide as the vessel; at its top is a copper tube *c*, in which the carbon point *d* can be fixed. A stout copper wire fixed to the bottom of the float dips in an iron tube filled with mercury, with which is connected one pole of the battery; the other pole is connected with the carbon *d'*, which is supported in a suitable manner. The size of the float is such that it moves slowly upwards, so that the carbon *d* presses with but very slight force against *d'*. This can be regulated by placing small weights in the collar on *c*. An insulated wire forming part of the circuit is coiled in a spiral *k* round the cylinder, and aids the regulation.

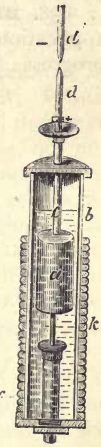


Fig. 801.

837. **Properties and intensity of the electric light.**—The electric light has similar chemical properties to sunlight; it effects the combination of chlorine and hydrogen, acts chemically on chloride of silver, and can be applied in photography.

Passed through a prism, the electric light, like that of the sun, is decomposed and gives a spectrum. Wollaston, and more especially Fraunhofer, found that the spectrum of the electric light differs from that of other lights, and of sunlight, by the presence of several very bright lines, as has been already stated (578). Wheatstone was the first to observe that, by using electrodes of different metals, the spectrum and the lines are modified.

Masson, who experimented upon the light of the electric machine, that of the voltaic arc, and that of Ruhmkorff's coil, found the same colours in the electric spectrum as in the solar spectrum, but traversed by very brilliant luminous bands of the same shades as that of the colour in which they occur. The number and position of these bands do not depend on the intensity of the light, but, as we have seen (833), upon the substances between which the voltaic arc is formed.

With carbon the lines are remarkable for their number and brilliancy; with zinc the spectrum is characterised by a very marked apple-green tint; silver produces a very intense green; with lead a violet tint predominates, and so on with other metals.

Bunsen, in experimenting with 48 couples, and removing the charcoals to a distance of a quarter of an inch, found that the intensity of the electric light is equal to that of 572 candles.

Fizeau and Foucault compared the chemical effects of the sun and the electric lights by investigating their action on iodised silver plates. Representing the intensity of the sun's light at midday at 1,000, these physicists found that the light from a battery of 46 Bunsen's elements was 235, while that from one of 80 elements was only 238. It follows that the intensity does not increase to any material extent with the number of the couples; but experiment shows that it increases considerably with their surface. For with a battery of 46 elements, each consisting of three elements, with their zinc and copper respectively united so as to form one element of triple surface

(825), the intensity was 385, the battery working for an hour ; that is to say, more than a third of the intensity of the solar light.

Too great precautions cannot be taken against the effects of the electric light when they attain a certain intensity. The light of 100 couples may produce very painful affections of the eyes. With 600, a single moment's exposure to the light is sufficient to produce very violent headaches and pains in the eye, and the whole frame is affected as by a powerful sunstroke.

838. **Electric lighting.**—Great progress has of late been made in the application of the electric light to purposes of ordinary illumination. This progress has been mainly due to the improvements which have been made in the means of generating electricity, for which some form of magneto- or dynamo-electrical machine (916), driven by steam or water power or by gas engines (476), is used. So long as the electricity from the voltaic battery was alone available for the production of the electric light, no great extension was possible, for the cost and inconvenience were far too great to permit it to be used for anything more than lecture purposes and occasional scenic illumination.

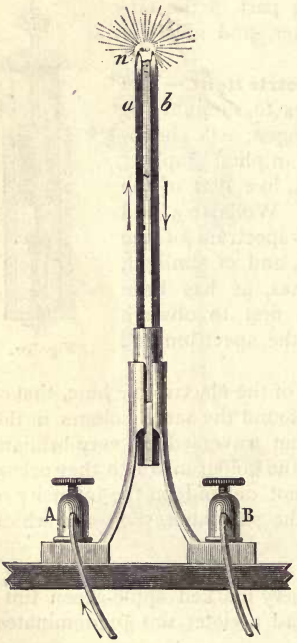


Fig. 802.

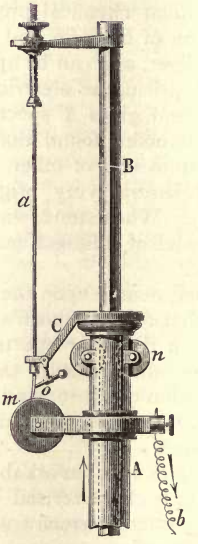


Fig. 803.

Very considerable improvements have also been made in the lamps, which are ordinarily divided into *arc* lamps, in which the light is produced between carbon points automatically kept at a constant distance by the action of the current itself, and *incandescent* lamps, in which the light is produced by the incandescence of a thin continuous solid conductor. To this may be added the *electrical candles*, of which the best known is the *Jablochkoff candle.* It consists (fig. 802) of two rods of gas carbon, *a* and *b*, from 2 to 4 mm. in diameter, separated by a layer of kaolin or Chinese clay about 2 mm. thick, fixed respectively in the supports, to which the positive and negative electrodes A B are respectively attached. The rods are insulated from each other by the whole being bound by some insulating material.

The current is started by a small piece of carbon, *n*, placed across the top. As the arc passes, the kaolin melts away, and the arrangement may therefore fitly be called a candle. The positive electrode wears away twice as fast as the negative, which would soon destroy the arc, but by using *alternating* currents the unequal waste of the carbons is prevented.

Fig. 797, which represents one of the forms of an arc lamp, may be taken as an example of the manner in which the regulation of the arc is effected.

Reynier's electric lamp, fig. 803, consists of a rectangular copper rod, B, moving in a copper tube A, guided by four pulleys, *n*, of which only two are shown; to B is fixed a cross-piece holding a thin carbon pencil, *a*, the lower part of which passes through a silver guide, and its end presses, but not quite over the centre, against a carbon disc, *m*, which moves about a horizontal axis. The piece supporting this is insulated from A, but is connected with the negative pole by a wire, *b*. The positive current, entering by A, passes by C to a small block of carbon, *o*, which presses against the pencil. Thus the current only passes through a very small portion of this pencil, and it is this small portion which becomes incandescent and forms the arc. The rod, as it burns away and sinks by its own weight, rotates the disc *m* slowly, and prevents its being irregularly worn away.

When either of the carbon electrodes which produce the electric light is increased in size, its increase of temperature is lessened, while that of the other is greater. When the negative electrode is large the light of the positive electrode is very bright. This is seen in *Werdermann's electric lamp*, which consists essentially of a carbon disc about 2 inches in diameter and an inch in thickness, which is connected with the negative pole of the battery; the positive pole is a rod of carbon about 3 mm. in diameter, of any suitable length; it slides vertically in a copper tube, which serves both as a guide and as a contact for it; this is pressed upwards against the centre by a weight passing over a pulley. The current can be passed *abreast* through as many as ten of such lamps, though it seems that the total illuminating power of this arrangement is not so great as when only two parallel lights are employed.

The electrical arc has had a very useful application to the *welding* or *autogenous soldering* of metals, that is to say, joining them without the use of a solder; a method which is of great service, particularly in the case of iron. The two plates to be joined are placed in contact, and having been connected with the negative pole, the positive carbon fixed in a suitable holder is held at such distance that the arc passes, which then melts one plate on the other. In other cases the two pieces of metal are pressed against each other, and the current passed through the line of contact. For these operations accumulators (849) are used charged by dynamos, which yield very powerful currents; by means of a commutator the electromotive force and the strength of the current can be varied within very wide limits at the will of the operator.

Von Hefner's *differential lamp* is represented in fig. 804; the current arriving by A divides at *i* (961); one portion passing through a fine wire coil, R, offering a large resistance, and the other through a short thick coil *r*,

whence it passes to a lever which turns about d; to this is connected at one end, m, a soft iron core which plays in the two coils, and at the other end is the positive carbon C_1.

When the carbons are apart a great resistance is presented, and the current passes chiefly through R, so that the core is drawn within R, and the lever, and with it the carbon C_1 falls; the fastening in the holder is such that at a certain angle the carbon C_1 slips in the holder a, and, touching the lower one, the current now passes by $r\ d\ C_1\ C_2\ B$; the iron core is then drawn down, but the holder a moves up, grips the carbon, which it moves with it, and the arc is reproduced; when its normal length is attained its resistance increases to an amount such that the currents passing through the two coils now balance themselves, and their attraction on the iron being equal the core is stationary. Several such lamps may be arranged in a circuit, and if one of them is extinguished it does not affect the others.

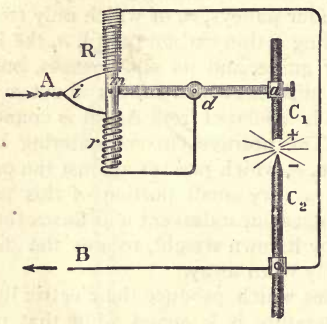


Fig. 804.

Schwendler devised a new unit of luminous intensity, which he calls the *platinum light standard*, specially for use with the electric light. It is the incandescence produced by a current of known strength passing through a U-shaped strip of platinum foil 36·28 mm. in length, 2 mm. in breadth, and 0·017 mm. in thickness. The circuit contains a rheostat and a galvanometer, by which the constancy of the current can be ensured and observed. When the strength of the current is constant the intensity of the light, radiated by the platinum, is constant also, and fulfils all the conditions of a standard measure of light, as it can always be reproduced in exactly the same form from pure platinum.

The standard of light adopted by the International Congress of Electricians in 1884 is the light emitted by a square centimetre of melted platinum when on the point of solidifying.

According to Rosetti the temperature of the positive carbon in the electric arc is between 2400° and 3900° C.; it is higher the smaller is the radiating surface. The temperature of the negative electrode lies between 2138° and 2530°.

The resistance of the heated air in the arc is from 1 to 12 ohms (833).

Incandescent lamps, though not so economical as arc lights, lend themselves best to the distribution of the electric light. We have seen that when a strong current of electricity is passed through a wire of small conductivity (829), its temperature is raised to incandescence; if the strength of the current is increased, the brightness of the light increases, but in a greater ratio than the strength of the current. At such high temperatures, however, wires even of the most difficultly fusible metals fuse or are disintegrated; and the only material which does not fuse at the highest temperature is carbon. The first lamps in which this was applied were constructed inde-

pendently by Edison in America and Swan in this country. Fig. 805 is a representation of *Swan's lamp.* Inside the globular glass vessel with a neck, and fused to it, is a glass rod, through which pass two platinum wires, bent outside in loops. These loops can be easily fitted in the two bent wires in the holder (fig. 806), which are in contact with the binding screws, and thus allow a current to be transmitted. The spring wire exerts an upward pressure, so as to always ensure good contact. To the other ends of the platinum is fixed the characteristic part, the carbon filament ; this is about 0·25 mm. in diameter, and is bent in the form of a double loop. It is prepared by immersing crochet cotton in sulphuric acid of a certain strength, by which it is converted into what is known as vegetable parchment. This is then carbonised by heating it to a high temperature in closed vessels. Before sealing the bulb it is exhausted of air by means of a Sprengel pump, and the vacuum is so perfect that electricity does not pass in it. The carbon of such a lamp, which is a thread about 12·7 cm. in length, and 0·013 cm. in diameter, has a resistance of 143 ohms in its normal incandescence.

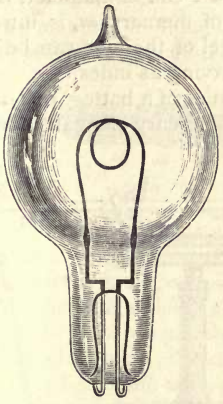


Fig. 805.

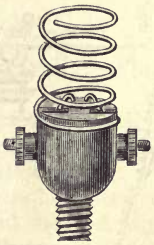


Fig. 806.

In Edison's lamp the carbon filament is made of a special kind of bamboo carbonised at high temperatures in closed nickel moulds. In the Maxim lamp, and in that of Lane Fox, the carbon filaments, after being carbonised and mounted, are heated by the current itself in an atmosphere of coal gas or the vapour of a hydrocarbon ; in this way carbon is deposited on the filament, by which it is rendered more uniform and durable.

If we surround an electric light in one case by an opaque calorimeter, which therefore absorbs the entire radiation, and then by a transparent one, which allows the light to pass, it will be found that the luminous radiation is about 10 per cent. in the case of arc lamps and 5 in incandescent lamps.

The lighting power varies in different lamps according to the strength of the currents. Edison's lamp, giving 16-candle power, requires a current of 0·6 ampere ; taking its resistance when hot at 170 ohms, the potential difference at the connections would be from Ohm's law (825) $0{\cdot}6 \times 170 = 102$ volts. For the same standard of light, Swan's lamp requires a current of 1·28 amperes, its resistance is 40, and hence the potential difference is 52 volts.

The electric effect VA, or the number of watts divided by the light expressed in candles, gives the electric effect required for one candle, and the number 746, divided by the number thus obtained, gives the number of candles which can be obtained from one electrical horse-power. In the above cases these are 198 and 180 respectively. It may be taken that the light of one candle represents about 4 watts. Lamps are usually classed according

to the number of volts they require. The greater this number the more brilliant is the light. Whatever care may be exerted in their manufacture, the carbons at last give way; their life, however, ought to be from 1,000 to 2,000 hours.

839. **Mechanical effects of the battery.**—Under this head may be included the motion of solids and liquids effected by the current. An example of the former is found in the voltaic arc, in which there is a passage of the molecules of carbon from the positive to the negative pole (833).

The mechanical action of the current may be shown by means of the following experiment (fig. 807). A glass tube, AB, bent at the two ends, about 50 cm. in length and 1 cm. in diameter, is almost filled with dilute sulphuric acid, and a globule of mercury, *m*, is introduced. The whole is fixed in a support, and the level of the tube can be adjusted by the screw *n*, the drop of mercury itself serving as index.

When the two poles of a battery of 4 or 5 cells are introduced into the two ends, the globule of mercury elongates and moves towards the negative pole with a velocity which increases with the number of elements. With 24, a long column of mercury can be moved through a tube a metre in length; with 50, the velocity is greater and the mercury divides into globules, all moving in the same direction. If the direction of the current is reversed, the mercury first remains stationary, and then moves in the opposite direction.

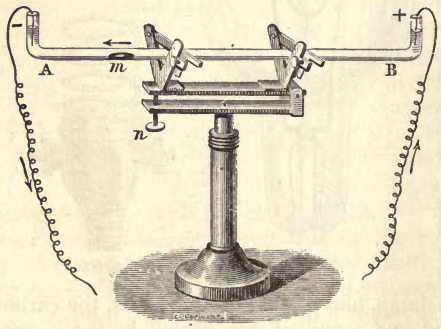


Fig. 807.

If the tube is gently inclined towards the positive pole, the mercury is still moved with the current; and a moment is at length reached at which there is equilibrium between the force of the current and the weight of the mercury. The component of this weight parallel to the plane may then be taken as representing the mechanical action of the current which traverses the globule of mercury.

A similar phenomenon, known as *electrical endosmose*, is observed in the following experiment, due to Porret. Having divided a glass vessel into two compartments by a porous diaphragm, he poured water into the two compartments to the same height, and immersed two platinum electrodes in connection with a battery of 80 elements. As the water became decomposed, part of the liquid was carried in the direction of the current through the diaphragm, from the positive to the negative compartment, where the level rose above that in the other compartment. A solution of copper sulphate is best for these experiments, because then the disturbing influence of the disengagement of gas at the negative electrode is avoided.

A porous vessel is necessary, for otherwise the transport by the liquid would be at once hydrostatically equalised.

According to Zöllner earth currents (894) are analogous to diaphragm currents; there are currents in the liquid mass in the interior of the earth, and

these currents coming in contact with the solidified masses produce electrical currents.

The converse of these phenomena is observed when a liquid is forced through a diaphragm by mechanical means. Such currents, which were discovered by Quincke, are called *diaphragm currents.* A porous diaphragm, *p*, is fixed in a glass tube (fig. 808), in which are also fused two platinum wires terminating in platinum electrodes, *a* and *b*; on forcing a liquid through the diaphragm the existence of a current is evidenced by a galvanometer with which the wires are connected, the direction of which is that of the flow of the liquid. The difference of potential due to this flow is proportional to the pressure.

According to Zöllner, all circulatory motions in liquids, especially when they take place in partial contact with solids, are accompanied by electrical currents, which have generally the same direction as that in which the current flows.

Fig. 808.

Wertheim found that the elasticity of metal wires is diminished by the current, and not by the heat alone, but by the electricity; he has also found that the cohesion is diminished by the passage of a current.

To the mechanical effects of the current may be assigned the sounds produced in soft iron when submitted to the magnetising action of a discontinuous current—a phenomenon which will be subsequently described.

840. **Electrocapillary phenomena.**—If a drop of mercury be placed in dilute sulphuric acid containing a trace of chromic acid, and the end of a bright iron wire be so fixed that it dips in the acid and just touches the edge of the mercury, the latter begins a series of regular vibrations which may last for hours. The explanation of this phenomenon, which was first observed by Kühne, is as follows:—When the iron first touches the mercury, an iron-mercury couple is formed, in consequence of which the surface of the mercury is polarised by the deposition of an invisible layer of hydrogen; this polarisation (806) increases the surface-tension of the mercury (138), it becomes rounder, and contact with the iron is broken; the chromic acid present depolarises the mercury, its original shape is restored, the couple is again formed, and the process repeats itself continuously.

Lippmann was led by the observation of this phenomenon to a series of interesting experimental results, which have demonstrated a relation between capillary and electrical phenomena. Of these results the most important is the construction of a *capillary electrometer.*

A glass tube, A (fig. 809), is drawn out to a fine point, and is filled with mercury: its lower end dips in a glass vessel, B, containing mercury at the bottom and dilute sulphuric acid at the top. Platinum wires are fused in the tubes A and B, and terminate in the binding screws *a* and *b* respectively.

Now at the beginning of the experiment, the position of the mercury in the drawn-out tube is such that the capillary action due to the surface-tension at the plane of separation of the mercury in the tube and the liquid is sufficient to counterbalance the pressure of the column A. This position is observed by means of a microscope, the focus of which is at the fiducial

mark on the glass at which the mercury stops. If now a difference of potential be established, by connecting the poles of a cell with the wires *a* and *b*, the surface-tension is increased, the mercury ascends in the capillary tube, and in order to bring the meniscus back to its former position the pressure on A must be increased. This is most simply effected by means of a thick caoutchouc tube, T, connected with the top of A, and with a manometer, H ; and which can be more or less compressed by means of a screw, E. The difference in level of the two legs of the manometer is thus a measure of the increase of the surface-tension, and therewith of the difference of potential. Lippmann found by special experiments, that this increase is almost directly proportional to the electromotive force, up to about 0·9 of a Daniell's element. Each electrometer requires a special table of graduation, but when once this is constructed it can be directly used for determining electromotive forces. It should not be used for greater electromotive forces than 0·6 of a Daniell ; but it can estimate the one-thousandth part of this quantity, and, as its electrical capacity is very small, it shows rapid changes of potential, which ordinary electrometers cannot do. For very small electromotive forces, the pressure is kept constant, and the displacement of the meniscus is measured by the microscope. Its use is especially convenient with zero methods.

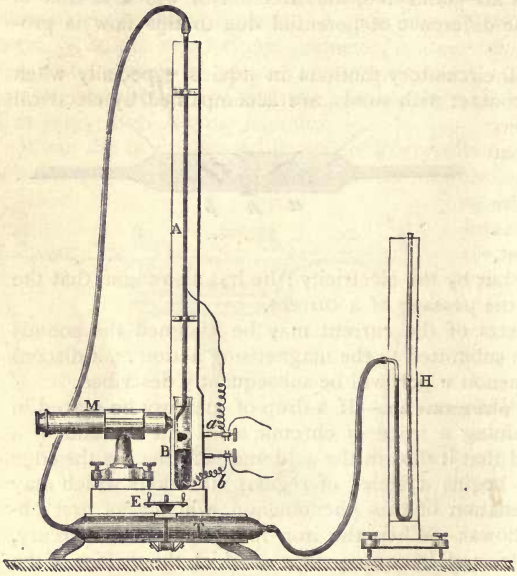


Fig. 809.

841. **Chemical effects.**—The first decomposition effected by electricity was that of water, in 1800, by Carlisle and Nicholson, by means of a voltaic pile. Water is rapidly decomposed by 4 or 5 Bunsen's cells ; the apparatus (fig. 810) is convenient for the purpose. It consists of a glass vessel fixed on a wooden base. In the bottom of the vessel two platinum electrodes, *p* and *n*, are fitted, communicating by means of copper wires with the binding screws. The activity of these electrodes is increased by covering them with a deposit of pulverulent platinum by electrolysis. The vessel is filled with water to which some sulphuric acid has been added to increase its conductivity, for pure water is a very imperfect conductor ; two glass tubes filled with water are inverted over the electrodes, and on interposing the apparatus in the circuit of a battery, decomposition is rapidly set up, and gas bubbles

rise from the surface of each pole. The volume of gas liberated at the negative pole is about double that at the positive, and on examination the former gas is found to be hydrogen and the latter gas oxygen. This experiment accordingly gives at once the qualitative and quantitative analysis of water. The oxygen thus obtained has the peculiar and penetrating odour observed when an electrical machine is worked (793), and which is due to ozone. The water contains at the same time peroxide of hydrogen, in producing which some oxygen is consumed. Moreover, oxygen is somewhat more soluble in water than hydrogen. Owing to these causes the volume of oxygen is less than that required by the composition of water, which is two volumes of hydrogen to one of oxygen. Hence voltametric measurements are most exact when the hydrogen alone is determined, and when this is liberated at the surface of a small electrode.

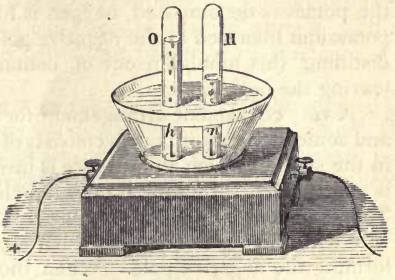


Fig. 810.

842. **Electrolysis.**—The term *electrolyte* was applied to those substances which, like water, are resolved into their elements by the voltaic current, by Faraday, to whom the principal discoveries in this subject and the nomenclature are due. *Electrolysis* is the decomposition by the voltaic battery;

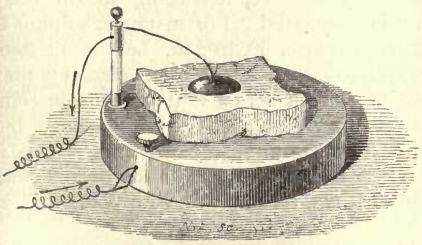


Fig. 811.

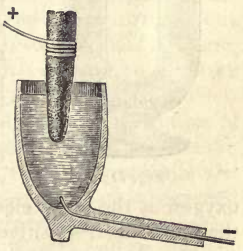


Fig. 812.

the positive electrode, or that by which positive electricity enters, Faraday called the *anode*, and the negative electrode the *kathode*. The products of decomposition are *ions*; *kation*, that which appears at the kathode; and *anion*, that which appears at the anode.

By means of the battery, the compound nature of several substances which had previously been considered as elements has been determined. By means of a battery of 250 couples, Davy, shortly after the discovery of the decomposition of water, succeeded in decomposing the alkalies potass and soda, and proved that they were the oxides of the hitherto unknown metals *potassium* and *sodium*. The decomposition of potass may be demonstrated, with the aid of a battery of 4 to 6 elements, in the following manner: a small cavity is made in a piece of solid caustic potass, which is moistened,

and a drop of mercury placed in it (fig. 811). The potass is placed on a piece of platinum connected with the positive pole of the battery. The mercury is then touched with the negative pole. When the current passes, the potass is decomposed, oxygen is liberated at the positive pole, while the potassium liberated at the negative pole amalgamates with the mercury. On distilling this amalgam out of contact with air, the mercury passes off, leaving the potassium.

A very convenient arrangement for the preparation of metallic magnesium and some of the rarer metals consists of an ordinary clay tobacco pipe (fig. 812), in the stem of which an iron wire is inserted just extending to the bowl, which is nearly filled with a mixture of the chlorides of potassium and magnesium. This is melted by a Bunsen's burner, and a piece of graphite connected by a wire with the positive pole of a battery is dipped in it, the wire in the stem forming the negative pole. When the current passes, chlorine gas is liberated at the positive pole, while metallic magnesium collects about the end of the iron wire in the bowl.

The decomposition of binary compounds—that is, bodies containing two elements—is quite analogous to that of water and of potass; one of the elements goes to the positive and the other to the negative pole. The bodies separated at the positive pole are called *electronegative* elements, because at the moment of separation they are considered to be charged with negative electricity, while those separated at the negative pole are called *electropositive* elements. One and the same body may be electronegative or electropositive, according to the body with which it is associated For instance, sulphur is electronegative towards hydrogen, but is electropositive towards oxygen. The various elements may be arranged in such a series that any one in combination is electronegative to any following, but electropositive towards all preceding ones. This is called the *electrochemical series*, and begins with oxygen as the most electronegative element, terminating with potassium as the most electropositive.

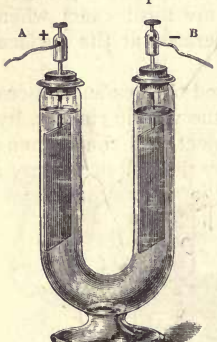


Fig. 813.

The decomposition of hydrochloric acid into its constituents, chlorine and hydrogen, may be shown by means of the apparatus represented in fig. 813. Carbon electrodes must, however, be substituted for those of platinum, which is attacked by the liberated chlorine: a quantity of common salt also must be added to the hydrochloric acid, in order to diminish the solubility of the liberated chlorine. The decomposition of potassium iodide may be demonstrated by means of a single element. For this purpose a piece of bibulous paper is soaked with a solution of starch, to which potassium iodide has been added. On touching this paper with the electrodes, a blue spot is produced at the positive pole, due to the action of the liberated iodine on the starch.

One of the best methods of determining whether a body is, or is not, an electrolyte, is to place it between the two platinum electrodes of a battery, and then, disengaging the electrodes from the battery, connect it with a

galvanometer, and observe whether a reverse current, due to polarisation of the electrodes (806), passes through the galvanometer. Such a current, being due to the accumulation of different substances on the two electrodes, is a proof that the substance has been electrolytically decomposed by the original current from the battery. This method can often be applied when it is difficult, by direct chemical methods, to detect the presence of products of decomposition at the electrodes.

843. **Decomposition of salts.**—Ternary salts in solution are decomposed by the battery, and then present effects varying with the chemical affinities and the intensity of the current. In all cases the acid, or the body which is chemically equivalent to it, is electronegative in its action towards the other constituent. The decomposition of salts may be readily shown by means of the bent tube represented in fig. 813. This is nearly filled with a saturated solution of a salt, say sodium sulphate, coloured with syrup of violets. The platinum electrodes of a battery of four Bunsen's elements are then placed in the two legs of the tube. After a few minutes the liquid in the positive leg, A, becomes of a red, and that in the negative leg, B, of a green colour, showing that the salt has been resolved into acid which has passed to the positive, and into a base which has gone to the negative pole, for these are the effects which a free acid and a free base respectively produce on syrup of violets.

In a solution of copper sulphate free acid and oxygen gas appear at the positive electrode, and metallic copper is deposited at the negative electrode. In like manner, with silver nitrate, metallic silver is deposited on the negative, while free acid and oxygen appear at the positive electrode.

This decomposition of salts was formerly explained by saying that *the acid was liberated at the positive electrode and the base at the negative.* Thus potassium sulphate, K_2OSO_3, was considered to be resolved into sulphuric acid, SO_3, and potash, K_2O. This view regarded salts composed of three elements as different in their constitution from binary or haloid salts. Their electrolytic deportment has led to a mode of regarding the constitution of salts which brings all classes of them under one category. In potassium sulphate, for instance, the electropositive element is potassium, while the electronegative element is a complex of sulphur and oxygen, which is regarded as a single group, SO_4, and to which the name *oxy-sulphion* may be assigned. The formula of potassium sulphate would thus be K_2SO_4, and its decomposition would be quite analogous to that of potassium chloride, KCl, lead chloride, $PbCl_2$, potassium iodide, KI. The electronegative group SO_4 corresponds to a molecule or two atoms of chlorine or iodine. In the decomposition of potassium sulphate, the potassium liberated at the negative pole decomposes water, forming potash and liberating hydrogen. In like manner the electronegative constituent SO_4, which cannot exist in the free state, decomposes into oxygen gas, which is liberated, and into anhydrous sulphuric acid, SO_3, which immediately combines with water to form ordinary sulphuric acid, H_2SO_4. In fact, where the action of the battery is strong, these gases are liberated at the corresponding poles ; in other cases they combine in the liquid itself, reproducing water. The constitution of copper sulphate, $CuSO_4$, and of silver nitrate, $AgNO_3$, and their decomposition, will be readily understood from these examples.

844. **Transmissions effected by the current.**—In chemical decompositions effected by the battery there is not merely a separation of the elements, but a passage of the one to the positive and of the other to the negative electrode. This phenomenon was demonstrated by Davy by means of several experiments, of which the two following are examples :—

i. He placed solution of sodium sulphate in two capsules connected by a thread of asbestos moistened with the same solution, and immersed the positive electrode in one of the capsules, and the negative electrode in the other. The salt was decomposed, and at the expiration of some time all the sulphuric acid was found in the first capsule, and the soda in the second.

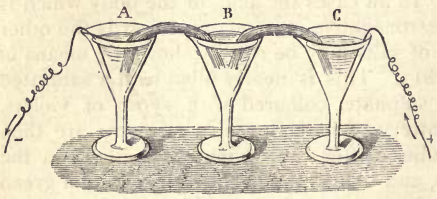


Fig. 814.

ii. Having taken three glasses, A, B, and C (fig. 814), he poured into the first solution of sodium sulphate, into the second dilute syrup of violets, and into the third pure water, and connected them by moistened threads of asbestos. The current was then passed in the direction from C to A. The sulphate in the vessel A was decomposed, and in the course of time there was nothing but soda in this glass, which formed the negative end, while all the acid had been transported to the glass C, which was positive, B containing only pure water. If, on the contrary, the current passed from A to C, the soda was found in C, while all the acid remained in A ; but in both cases the remarkable phenomenon was seen that the syrup of violets in B neither became red nor green by the passage of the acid or base through its mass, a phenomenon the explanation of which is based on the hypothesis enunciated in the following paragraph.

845. **Grothüss's hypothesis.**—Grothüss has given the following explanation of the chemical decompositions effected by the battery. Adopting the hypothesis that in every binary compound, or body which acts as such, one of the elements is electropositive, and the other electronegative, he assumes that, under the influence of the contrary electricities of the electrodes, there is effected, in the liquid in which they are immersed, a series of successive decompositions and recompositions from one pole to the other. Hence it is only the elements of the terminal molecules which do not recombine, but, remaining free, appear at the electrodes. Water, for instance, is formed of one atom of oxygen and two atoms of hydrogen ; the first gas being electronegative, the second electropositive. Hence when the liquid is traversed by a sufficiently powerful current, the molecule *a* in contact with the positive pole arranges itself as shown in fig. 815—that is, the oxygen is attracted and the hydrogen repelled. The oxygen of this molecule is then given off at the positive electrode, the liberated hydrogen immediately unites with the oxygen of the molecule *b*, the hydrogen of this with the oxygen of the molecule *c*, and so on, to the negative electrode, where the last atoms of hydrogen become free and appear on the poles. The same theory applies to the metallic oxides, to the acids and salts, and explains why in the experiment mentioned in the preceding paragraph the syrup of violets in the vessel B

becomes neither red nor green. The reason why, in the fundamental experiment, the hydrogen is given off at the negative pole when the circuit is closed will be readily understood from a consideration of this hypothesis.

Clausius objected that, according to this theory, a very great force must be required for overcoming the affinity for each other of the oppositely electrolysed particles of the compound; and that below a certain minimum strength of current no decomposition could occur. Now Buff showed that the action of even the feeblest currents, $\frac{1}{30000}$ of an ampere, for instance, continued for a long time can produce decomposition. Again, when the necessary potential is obtained, it should be sudden and complete; whereas we know that it is proportional to the strength of the current.

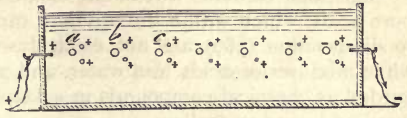


Fig. 815.

To overcome this difficulty Clausius applied the theory now generally admitted of the constitution of liquids (292), and which was originally propounded by Williamson on the basis of purely chemical considerations. On this theory the particles of a compound liquid have not the rigid unalterable condition of a solid body; they are in a perpetual state of separation and reunion, so that we must suppose compound bodies and their elementary constituents to coexist with each other in a liquid. Water, for instance, contains particles of water, together with particles of oxygen and of hydrogen; the former are being continually decomposed and the latter continually reunited.

The theory of Van 't Hoff on the nature of solutions (139), and the experimental researches to which it has led, support the present explanation of electrolytic phenomena. In the case of a solution of potassium chloride in water, a certain proportion, probably considerable, of the molecules of the salt is in a state of dissociation, which proportion increases with the dilution of the solution; so that along with molecules of the undecomposed salt there are present the free ions potassium and chlorine. These latter are exclusively the carriers of the positive and negative electricity respectively. They may in this respect be regarded as performing a function analogous to that of the pith ball in the convective discharge (770). When the voltaic current passes it acts on the motion of the molecules in such a manner that the negatively electrical particles of chlorine pass to the positive electrode, and the positively electrical particles of potassium to the negative electrode, and there give up their charges and are liberated in the free state. Hence the current does not bring about the decomposition, but utilises it, to give definite direction to the particles which are already separated.

These considerations explain why the conductivity of a liquid increases with the temperature (958); for this increases the velocity of the molecules (294) and also the dissociation, that is, the number of partial molecules.

It also shows that the conductivity should increase with the concentration of the liquid, seeing that a great number of decomposable molecules must be favourable to the movement of electricity. On the other hand, an increase in the number must give rise to an increased number of collisions;

hence it is that, though for very dilute solutions the conductivity increases with the concentration, it does so more slowly than in direct ratio, and it is not difficult to understand that for some liquids there is a concentration which corresponds to a maximum conductivity, and this in a great many cases is below the point of saturation of the solution.

This also explains why stable chemical compounds, such as water and pure acids, which within the ordinary range of temperatures are not subject to dissociation (389), are not electrolysed and therefore not decomposed, while mixtures of acids and water, and solutions of salts, which may be regarded as chemical compounds in a state of dissociation, are easily electrolysed and conduct well.

In dealing with molecular magnitudes, theoretical investigations make it probable that the electrolytic resistance, which the molecules experience in being moved by the current, is of the same order of magnitude as the capillary resistance which results from their friction in the liquid (147). Nothing is opposed to the idea that electrolysis is a purely mechanical process. Decomposition occurs in the first place by dissociation; the difference of potential is the force in virtue of which the previously united molecules are urged in contrary directions. The moving molecules, the ions, are the carriers of the motion of electricity and produce the current; the resistance which they thereby experience is the electrical resistance of the liquid. This, therefore, is the cause of the development of heat in the circuit.

846. **Laws of electrolysis.**—The laws of electrolysis were discovered by Faraday; the most important of them are as follows:—

I. *Electrolysis cannot take place unless the electrolyte is a conductor.* Hence ice is not decomposed by the battery, because it is a bad conductor. Other bodies, such as lead oxide, silver chloride, &c., are only electrolysed in a fused state—that is, when they can conduct the current. The converse of this is true; if a liquid transmits a current it must be an electrolyte. From the fact that he was able to obtain a current in liquids which deflected a galvanometer without producing any visible decomposition, Faraday inferred that liquids had a slight conductivity like that of metals independently of their electrolytic conductivity. This apparent conductivity is, however, to be assigned to *electrical convection* (832).

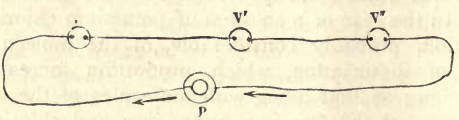


Fig. 816.

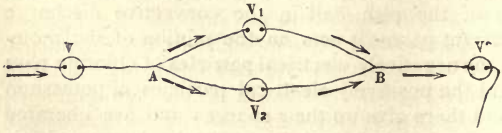


Fig. 817.

II. *The energy of the electrolytic action of the current is the same in all its parts.*

For if a number of voltameters V, V′, V″ (*vide sup.*), are arranged in series so that they are all traversed by the same current (fig. 816), it is found that the weight of hydrogen in each of them in the same time is the same,

whatever may be the nature and distance of the electrodes, the proportion and nature of the acid.

If the current from the battery divides at A into two branches (fig. 817), in which are two equal voltameters V_1 and V_2, then the quantities of gas liberated in V and V″ will still be equal to each other ; and the quantities in V_1 and V_2 will be equal to each other, but each will only have half the quantity which passes in either of the voltameters V and V″.

III. *The same quantity of electricity—that is, the same electric current—aecomposes chemically equivalent quantities of all the bodies which it traverses* ; from which it follows, *that the weights of elements separated in these electrolytes are to each other as their chemical equivalents.*

In a circuit containing a voltameter, V, Faraday introduced a tube, AB, containing tin chloride kept in a state of fusion by the heat of a spirit lamp (fig. 818). In the bottom of this the negative pole was fused, while the positive electrode consisted of a rod of a graphite ; when the current passed chlorine was liberated at the positive, while tin collected at the negative pole ; in like manner lead oxide was electrolysed and yielded lead at the

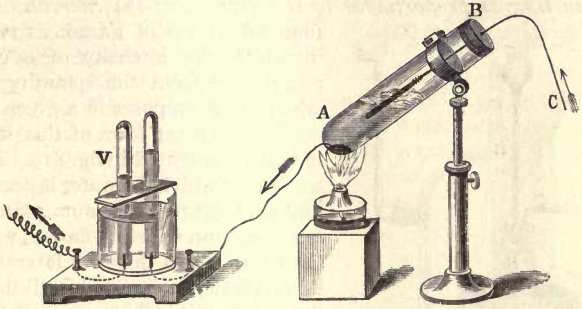


Fig. 818.

negative and oxygen at the positive pole. Comparing the quantities of substances liberated, they are found to be in a certain definite relation. Thus for every 18 parts of water decomposed in the voltameter there will be liberated 2 parts of hydrogen, 207 parts of lead, and 117 of tin at the respective negative electrodes, and 16 parts of oxygen and 71 (or $2 \times 35{\cdot}5$) parts of chlorine at the corresponding positive electrodes. Now these numbers are exactly as the equivalents (not as the atomic weights) of the bodies.

It will further be found that in each of the cells of the battery 65 parts by weight of zinc have been dissolved for every two parts by weight of hydrogen liberated ; that is, that for every equivalent of a substance decomposed in the circuit one equivalent of zinc is dissolved. This is the case whatever be the number of cells. An increase in the number only has the effect of overcoming the great resistance which many electrolytes offer, and of accelerating the decomposition. It does not increase the quantity of electrolyte decomposed. If in any of the cells more than 65 parts of zinc are dissolved for every two parts of hydrogen liberated, this arises from a disadvantageous local action ; and the more perfect the battery, the more nearly does it approach this ratio.

Chemistry takes account of the *valency* of an element, and divides them into *monads*, *dyads*, *triads* and *tetrads*—a classification based on their equivalence to and their power of replacing other elements; thus one atom of the monad hydrogen (H = 1), the basis of this classification, or one atom of monad silver (Ag = 108), would combine with one atom of chlorine (Cl = 35·5) or one atom of iodine (I = 127). One atom of oxygen (O = 16) unites with two atoms of hydrogen to form water, or with two atoms of silver to form silver oxide, Ag_2O; one atom of the dyad zinc (Zn = 65) unites with one atom of the dyad oxygen to form ZnO, or with the dyad sulphur (S = 32) to form ZnS Again, gold is a triad, and one atom (Au = 196) can combine with three atoms of chlorine to form $AuCl_3$, and, accordingly, one monad is equivalent to one-third of the atom of the triad. Now electrolysis proceeds according to the *equivalence*; that is, the same quality of electricity which liberates one atom of a monad liberates half a one of a dyad, and a third of a triad. This remark applies to the compound groups, such as NO_3, which acts as a monad, and SO_4, which acts as a dyad.

IV. It follows from the above law, that *the quantity of a body decomposea in a given time is proportional to the strength of the current.* On this is founded the use of Faraday's *voltameter*, in which the intensity of a current is ascertained from the quantity of water which it decomposes in a given time.

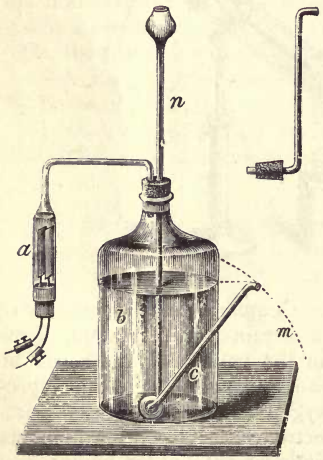


Fig. 819.

A convenient form of this instrument is that represented in fig. 819. The vessel *a* is that in which the water is decomposed, and contains two platinum plates, and is in connection with the flask *b*, which contains water. In this is a lateral delivery tube, *c*, which is inclined until the level of the liquid in it is the same as in the funnel tube *n*. The air is then under the same pressure as the atmosphere. When the battery is connected with the decomposing cell *a*, the gases disengaged expel a corresponding volume of water through the delivery tube *c*; at the conclusion of the experiment, this tube is inclined until the liquid is at the same level as in the tube *n* and in the flask. The weight of the liquid expelled is then a direct measure of the volume of the disengaged gases.

The use of this voltameter appears simple and convenient; Jacobi proposed *as unit of the strength of current, that current which in one minute yields a cubic centimetre of mixed gas reduced to the temperature* 0° *and the pressure* 760 *mm.* This is equal to 0·09567 ampere, that is, an ampere liberates 10·44 ccm. mixed gas in a minute. Yet, for reasons mentioned before (841), the measurements should be based on the volume of hydrogen liberated.

Poggendorff's *silver voltameter* (fig. 820) is an instrument for measuring

the strength of the current. A solution of silver nitrate of known strength is placed in a platinum dish which rests on a brass plate that can be connected with the negative pole of the battery by means of the binding screw *b*. In this solution dips the positive pole, which consists of a rod of silver wrapped round with muslin, and suspended to an adjustable support. When the current passes, silver separates at the negative pole, and is washed, dried, and weighed; and the weight thus produced in a given time is a very accurate measure of the strength of the current. Some silver particles which are apt to become detached from the positive pole are retained in the muslin. Edison has used a zinc voltameter for measuring the powerful currents employed for technical purposes.

It has been found by experiment that, when water is decomposed, a current of 1 ampere liberates 0·000010386 gramme or 0·1168 cc. of hydrogen in a second; this, then, is the *electrochemical equivalent of hydrogen*, and from this we can deduce the weight of any element liberated in the same time by unit current, if we multiply it by the equivalent weight of the element referred to hydrogen. The equivalent of silver is usually taken at 108; hence, if any of its salts are decomposed, the weight of silver liberated by an ampere in a second is 0·0011217 gramme; this is the electrochemical equivalent of silver, and similarly that of copper is 0·0003271 and that of zinc 0·0003375.

The quantity of electricity which passes through a conductor with a current of one ampere is called a *coulomb* (733), and thus we may say that a coulomb of electricity, in traversing an electrolyte, carries with it a weight of a metal which is represented by its electrochemical equivalent.

The current from the electrical machine, which is of very high potential, is capable of traversing any electrolyte, but the quantity which it can decompose is extremely small as compared with even the smallest voltaic apparatus, and the quantity of electricity developed by the frictional machine is very small as compared with that developed by chemical action. It has been calculated by Weber that if the quantity of positive electricity required to decompose a grain of water were accumulated on a cloud at a distance of 3,000 feet from the earth's surface, it would exert an attractive force upon the earth of upwards of 1,500 tons.

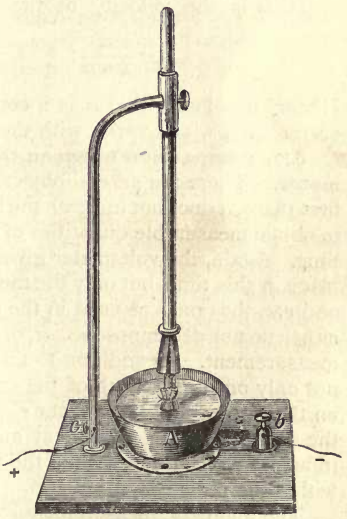


Fig. 820.

846*a*. **Migration of the Ions.**—From what has been said, it would seem that when a solution of copper sulphate is electrolysed between copper electrodes, for every equivalent of copper deposited at the negative electrode an equivalent weight should be dissolved at the positive, and, the transfer taking place as described, the concentration of the solution should remain unchanged. This, however, is not the case; when the operation takes

place without any agitation of the solution, the liquid about the negative pole becomes lighter in colour, indicating that the solution there is weaker.

This phenomenon, which was investigated by Hittorf, is ascribed by him to the fact that in electrolysis both electricities, together with their *ions* or products of electrolytical decomposition, travel in the liquid towards their respective electrodes, but with unequal velocities, and this transference is called the *migration of the ions*. Each ion has a special velocity in the liquid independently of the compound of which it forms part; thus in the same time SO_4 travels twice as fast as Cu.

The number which expresses this rate of travel is called n, and has this meaning: let us conceive a vertical layer in the liquid the concentration of which remains unchanged by what takes place on each side; then, if after electrolysis we determine the quantity of the constituents on each side, there is an increase of the positive on one side and of the negative on the other. These increases correspond to the quantities of the two constituents which have been driven through.

The number n expresses the ratio of the number of molecules of the anion which passes through the imaginary layer in a given time to that of the electrolyte decomposed.

If k is the velocity of the kation, and a that of the anion, then

$$n = \frac{a}{k+a}; \qquad 1-n = \frac{k}{k+a}; \qquad \frac{1-n}{n} = \frac{k}{a}.$$

Hittorf has shown that n is a constant independent of the strength of the current, but which varies with the concentration of the liquid.

847. **Comparison between the tangent galvanometer and the voltameter.**—There are several objections to the use of the voltameter. In the first place, it does not indicate the strength at any given moment, for in order to obtain measurable quantities of gas the current must be continued for some time. Again, the voltameter gives no indications of the changes which take place in this time, but only the mean intensity. It offers also great resistance, and can thus only be used in the case of strong currents; for weak currents either do not decompose water, or only yield quantities too small for accurate measurement. In addition to this, the indications of the voltameter depend not only on the strength of the current, but on the acidity of the water, and on the distance and size of the electrodes. But although it does not measure the strength of the current at any one time, it does, apart from accidental influences, give a measure of the total quantity of electricity that has passed within the period of observation.

The magnetic measurements are preferable to the chemical ones. Not only are they more delicate and offer less resistance, but they give the strength at any moment. On the other hand, indications furnished by the tangent galvanometer hold only for one special instrument. They vary with the diameter of the ring and the number of turns; moreover, one and the same instrument will give different indications on different places, seeing that the force of the earth's magnetism varies from one place to another (701).

The indications of the two instruments may, however, be readily compared with one another. For this purpose the voltameter and the tangent

galvanometer are *simultaneously* inserted in the circuit of a battery, and the deflection of the needle and the amount of gas liberated in a given time are noted. In one set of experiments the following results were obtained :—

Number of elements	Deflection	Gas liberated in three minutes
12	28·5°	125 cc.
8	24·8	106
6	22·0	93
3	13·75	56
2	6·9	24

If we divide the tangents of the angles into the corresponding volumes of gas liberated in *one* minute, we should obtain a constant magnitude which represents how much gas is developed in a minute by a current which could produce on the tangent galvanometer the deflection 45°, for tang. 45° = 1. Making this calculation with the above observations, we obtain a set of closely agreeing numbers the mean of which is 76·5. The gas was measured under a pressure of 737 mm. and at a temperature of 15°, and therefore under normal conditions (332) its volume would be 70 cubic centimetres. That is to say, this is the volume of gas which corresponds to a deflection of 45°. Hence in chemical measure the strength C of a current which produces in *this* particular tangent galvanometer a deflection of $\phi°$ is

$$C = 70 \text{ tang. } \phi.$$

For instance, supposing a current produced in this tangent galvanometer a deflection of 54°, this current, if it passed through a voltameter, would liberate in a minute 70 × tang. 54° = 70 × 1·376 = 96·32 cubic centimetres of gas.

If once the *reduction factor* for a tangent galvanometer has been determined, the strength of any current may be readily calculated in chemical measure by a simple reading of the angle of deflection. This reduction factor of course only holds for one special instrument, and for experiments in the same place, seeing that the force of the earth's magnetism varies in different places.

The indications of the sine-compass may be compared with those of the galvanometer in a similar manner.

848. **Polarisation.**—When the platinum electrodes, which have been used in decomposing water, are disconnected from the battery, and connected with a galvanometer, the existence of a current is indicated which has the opposite direction to that which had previously passed. This phenomenon is explained by the fact that oxygen has been condensed on the surface of the positive plate, and hydrogen on the surface of the negative plate, analogous to what has been already seen in the case of the non-constant batteries (806). The effect of this is to produce two different electromotors, which produce a current opposed in direction to the original one, and which, therefore, must weaken it. As the two electrodes thus become the poles of a new current, they are said to be *polarised*, and the current is called a *polarisation current*. The polarisation is not instantaneous, but may increase continuously from zero to a certain maximum limit which may be considerable ; it increases with the strength of the current, attaining the force of 2·6 volts with platinum

plates in dilute sulphuric acid. It constitutes a negative electromotive force, e, and must be allowed for in Ohm's formula (825), which then becomes

$$C = \frac{E - e}{R + r}.$$

The quantity of electricity required to produce a given state of polarisation depends on the condition and dimensions of the plate, and is often called the *capacity of polarisation* relative to the given system.

849. **Secondary batteries. Accumulators.**—Ritter was the first to show that on this principle batteries might be constructed of pieces of metal of the same kind—for instance, platinum—which otherwise give no current. A piece of moistened cloth is interposed between each pair, and each end of this system is connected with the poles of a battery. After some time the apparatus has received a charge, and if separated from the battery can itself produce all the effects of a voltaic battery. Such batteries are called *secondary* batteries or, also, *accumulators*. Their action depends on an alteration of the surface of the metal produced by the electric current, the constituents of the liquid with which the cloth is moistened having become accumulated on the opposite plates of the circuit.

Planté first showed the practical importance of these batteries. His element (fig. 821) is constructed as follows: A broad strip of sheet lead with a tongue is laid upon a second similar sheet, contact being prevented by narrow strips of felt; and two similar strips having been laid on the upper piece, the sheets are rolled together so as to form a compact cylinder. This is placed in a vessel containing dilute sulphuric acid, and, being connected by wires attached to the tongues with a battery of two Grove's cells, a current is passed through it. The effect of this is that water is decomposed, oxygen being liberated at the anode, or plate, which serves as positive pole, and there unites with the lead, forming peroxide of lead, while hydrogen is accumulated at the other plate. If now the plates are detached from the charging battery and are connected with each other, a powerful polarisation current is produced in the opposite direction to the primary; the oxygen of the peroxide at the anode decomposes the dilute acid, combining with its hydrogen, and so travels through to the other plate, where it combines with the lead. When these operations are repeated several times the activity of the element increases, owing in great measure to the alteration in the surfaces which is thereby produced. The element does, in fact, require a considerable expenditure of energy and time to *form* it, which is a source of expense, even when the energy of the discharge is expended in *forming* new plates.

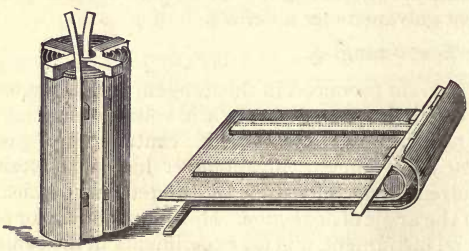


Fig. 821.

Faure made a great improvement in this direction. It consists in coating the lead plates with a thick paste of red lead, Pb_3O_4, so as to have about one

gramme to the square centimetre. This is kept in its place by a sheet of parchment paper and slips of felt, and is then coiled up as in Planté's (fig. 822). When the current is passed, the ultimate effect is that the red lead at the one electrode is oxidised to Pb_2O_4, while the other is reduced to granular porous grey metallic lead, both which coatings present a large surface. Such coatings, however, are liable to become detached, and a considerable advance was made in the introduction of *grids*, or gratings of lead in which square or round holes are filled with compressed oxide of lead; the object being to store firmly as much of the porous material as possible, consistently with strength, lightness, and compactness.

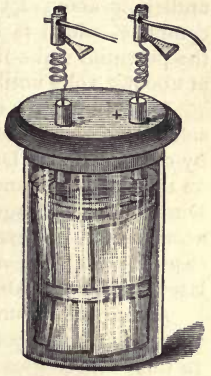


Fig. 822.

There are many plans by which this may be effected. Fig. 823 represents one of the batteries of the Electric Power Storage Company; it will be seen that the whole of one set of six plates, forming the negative electrode, are fixed together, and a corresponding set of five plates, also joined together, can be placed between the other set, being kept from touching each other by staples or studs of some insulating material. Each set of plates forms in effect a single large one, which is thus placed with its coated face opposite the coated faces of the other plates. The object is by bringing the plates near each other to diminish the internal resistance.

The inverse electromotive force of such a cell is about $2\frac{1}{2}$ times that of a Daniell's cell, so that three Daniell's or two Grove's cells are required to charge it. In charging, a considerable number of elements are joined together by their similar poles, and connected with the respective electrodes of the charging battery or of the dynamo; the effect is the same as that of using a single element of a surface equal to the sum of the surfaces of all the elements. By means of a specially contrived commutator a given number of such batteries may be combined so as to produce at will the effects either of high potential or of quantity.

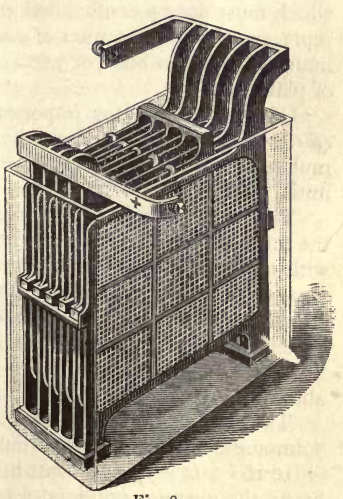


Fig. 823.

So long as such batteries could be charged only from a voltaic battery they could never be economical; but the fact that after having been once charged they retain the charge for a considerable time, has led to their use in what is called 'storing electricity' produced by mechanical power through the agency of powerful dynamo and magneto-electrical machines. What they do is to store the products of chemical decomposition, and that in a form in which they are immediately available for electrical effects.

They are usually charged by shunt wound dynamos (919*a*), whereby about 75 per cent. of the energy is available. An accumulator of a given size can only consume in each interval of time a definite quantity of gas for its formation by oxidation and reduction. If more gas is developed it escapes uselessly. The charging current must be neither too strong nor too weak. For each accumulator there is a special rate of charge, which is most advantageous.

During the charge the E.M.F. of a secondary battery at first rises rapidly until it is about 2·3 volts, and then remains constant until the charge is complete, which is known by the disengagement of gas. In discharging the potential sinks rapidly the first few minutes, and then remains constant at about 2 volts until towards the end of the charge, when it again sinks.

An accumulator of great capacity is obtained by placing a zinc plate in a solution of zincate of potass or soda, and a porous plate of copper obtained by compression. During the charge the zinc in the solution is precipitated on the zinc plate, and the copper absorbs an equivalent quantity of oxygen. During the discharge the copper is reduced and the zinc redissolves. This accumulator, however, does not retain its charge, and is only suitable for cases in which the discharge rapidly succeeds the charge. So far accumulators with lead plates have alone proved to be of practical utility.

A charged accumulator gradually loses its charge by leakage, and the efficiency of an accumulator depends on the power of retaining its charge. In this respect great improvement has been made by attention to a number of minute points ; the durability now extends to years, whereas it was formerly measured by months or weeks.

The efficiency further depends on the *capacity*, which is the quantity of electrical energy which can be stored for unit weight of the accumulator, and which must not be confounded with the electrostatic capacity. It is usually represented by the number of *ampere-hours* ; that is, a current of an ampere maintained for an hour, or 3,600 coulombs of electricity, for each kilogramme of plates.

Of perhaps greater importance in judging of an accumulator is the *efficiency*, by which is meant the ratio of the electrical work which is accumulated in order to charge it to that which it gives out in sinking to its initial condition.

The energy stored up in an accumulator is measured by the potential at the terminals during the charging, multiplied by the strength of the current with the time. This gives the energy in volt-amperes or watts. In like manner the energy given out in the discharge is the potential into the current strength into the time of discharge. The whole charge which can be imparted to an accumulator cannot be advantageously utilised, for it is found to injure the accumulator if this is done, and in practice the charge is only allowed to run down until the potential is 10 per cent. less.

Thus a given accumulator was charged for 10·16 hours with a current of 5 amperes, the average potential being 2·15 volts ; hence the energy stored is $10{\cdot}16 \times 2{\cdot}15 \times 5 = 109$ watt-hours. In the discharge which lasted 7·35 hours the average potential was 1·88, and the current 6·5 amperes, representing therefore 90 watt-hours ; the ratio of the two is 0·826—that is, the efficiency of the accumulator is 82·6 per cent. ; a number which is now required for a good accumulator.

It cannot be said that the reactions which take place during the charge and discharge of an accumulator are thoroughly understood; they are undoubtedly more complicated than has been represented above, in which no account has been taken of the sulphuric acid. During the charge, the strength of the dilute sulphuric acid, and therewith its conductivity, gradually diminish, while during the discharge both increase. Hence a determination of the specific gravity of the solution at any time is a convenient practical method of measuring the state of the charge. This is effected by flat densimeters (130) which float between the plates. The density may vary between 1·12 and 1·22, representing respectively about 16 and 30 per cent. of sulphuric acid, SH_2O_4.

As an example, one cell of the Electric Power Storage Company had an internal resistance of 0·0012 ohm at the beginning, and 0·0028 at the end, and weighed 50 kilos. In such a cell 880 watt-hours could be accumulated, and 680 watt-hours, or about 79 per cent., obtained in the discharge. Thus each kilo represents 13·6 watt-hours of available energy, or $\frac{1}{54}$ of a horse-power; that is, it could yield $\frac{1}{54}$ H.P. for an hour, or 1 H.P. for 1·1 minute. As a horse-power is equal to 270,000 kilogrammetres (472), this gives 5,000 kilogrammetres for each kilo, sufficient, therefore, to raise the battery through a height of 5,000 metres.

In accumulators which are to be used as motors, in such cases as tramcars, electrical boats, &c., the *capacity* is of first importance, while with stationary accumulators, as in electric lighting, the efficiency is the chief point.

Many instructive comparisons may be made between a secondary battery and a charged Leyden jar. Thus, for instance, when the poles of a secondary battery have been connected until no current passes, and are then disconnected for a while, a current in the same direction as the first is obtained on again connecting them; this is the *residual discharge*. The capacity of a secondary battery depends on the area of the electrodes, on their nature, and on that of the interposed liquid, but not on the distance between them. The energy of the Leyden jar is stored in that state of mechanical strain which is called polarisation of the dielectric; in the secondary battery the energy consists in the products which are stored up on the surface of the electrodes in a state ranging from chemical combination to mechanical adherence or simple juxtaposition.

A dry pile which has become inactive may be used as a secondary battery. When a current is passed through it, in a direction contrary to that which the active battery would itself yield, it then regains its activity.

850. **Grove's gas battery.**—On the property, which metals have, of condensing gases on their surfaces, Grove constructed his *gas battery* (fig. 824.) A single cell consists of two glass tubes, B and A, in each of which is fused a platinum electrode, provided on the outside with binding screws. These electrodes are made more efficient by being covered with finely divided platinum. One of the tubes is partially filled with hydrogen, and the other partially with oxygen, and they are inverted over dilute sulphuric acid, so that half the platinum is in the liquid and half in gas. On connecting the electrodes with a galvanometer, the existence of a current is indicated whose direction in the connecting wire is from the platinum in oxygen to that in

hydrogen ; so that the latter is negative towards the former. As the current passes through water this is decomposed : oxygen is separated at the positive plate and hydrogen at the other. These gases unite with the gases condensed on their surface, so that the volume of gas in the tubes gradually diminishes, but in the ratio of one volume of oxygen to two volumes of hydrogen. These elements can be formed into a battery (fig. 824) by joining the dissimilar plates with one another just as they are joined in an ordinary battery. One element of such a battery is sufficient to decompose potassium iodide, and four will decompose water.

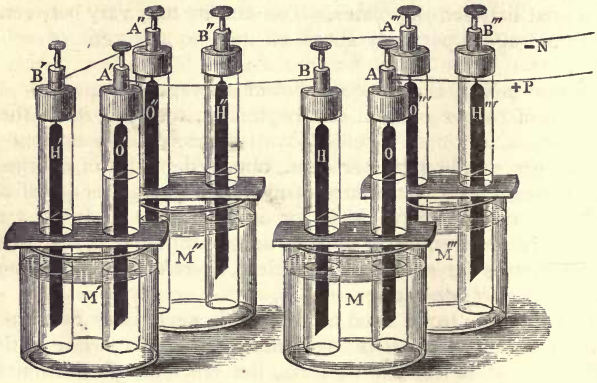


Fig. 824.

Mond and Langer have constructed a battery on this principle. On each side of a plate of plaster is a lead grid, the holes of which are filled with platinum black. Air and hydrogen are forced through these holes and combine, forming water, and also an electric current, the electromotive force of which is one volt.

851. **Passive state of iron.**—With polarisation is probably connected a very remarkable chemical phenomenon, which many metals exhibit, but more especially iron. When this is immersed in concentrated nitric acid it is unattacked. This condition of iron is called the *passive state*, and upon it depends the possibility of the zinc-iron battery (810). It is probable that in this experiment a thin superficial layer of protosesquioxide of iron is formed ; on the one hand this protects the iron from further attack, and on the other it acts as an electromotor, like the layer of peroxide of lead in Planté's element (849). The position of passive iron in the electromotive series is near that of platinum.

852. **Nobili's rings.**—When a drop of acetate of copper is placed on a silver plate, and the silver is touched in the middle of a drop with a piece of zinc, there are formed around the point of contact a series of copper rings alternately dark and light. These are *Nobili's coloured rings*. They may be obtained in beautiful iridescent colours by the following process : A solution of lead oxide in potash is obtained by boiling finely powdered litharge in a solution of potash. In this solution is immersed a polished plate of silver or of German silver, which is connected with the positive electrode of a battery of eight Bunsen's elements. With the negative pole is connected a fine platinum wire fused in glass, so that only its point projects ; and this is placed in the liquid at a small distance from the plate. Around this point

binoxide of lead is separated on the plate in very thin concentric layers, the thickness of which decreases from the middle. They show the same series of colours as Newton's coloured rings in transmitted light (650). The binoxide of lead owes its origin to a secondary decomposition ; by the passage of the current some lead oxide is decomposed into metallic lead, which is deposited at the negative pole, and oxygen which is liberated at the positive ; and this oxygen combines with some oxide of lead to form binoxide, which is deposited on the positive pole as the decomposition proceeds. This process is used for the metallic coloration of objects of domestic use and ornamentation.

The effects are also well seen if a solution of copper sulphate is placed on a silver plate, which is touched with a zinc rod, the point of which is in the solution ; for then a current is formed by these metals and the liquid.

853. **Arbor Saturni, or lead tree. Arbor Dianæ.**—When in a solution of a salt is immersed a metal which is more oxidisable than the metal of the salt, the latter is precipitated by the former, while the immersed metal is substituted, equivalent for equivalent, for the metal of the salt. This precipitation of one metal by another is partly attributable to the difference in their affinities, and partly to the action of a current which is set up as soon as a portion of the less oxidisable metal has been deposited. The action is promoted by the presence of a slight excess of acid in the solution.

A remarkable instance of the precipitation of one metal by another is the *Arbor Saturni.* This name is given to a series of brilliant ramified crystallisations obtained by zinc in solutions of lead acetate. A glass flask is filled with a clear solution of this salt, and the vessel closed with a cork, to which is fixed a piece of zinc in contact with some copper wire. The flask, being closed, is left to itself. The copper wire at once begins to be covered with a moss-like growth of metallic lead, out of which brilliant crystallised laminæ of the same metal continue to form ; the whole phenomenon has great resemblance to the growth of vegetation, from which indeed the old alchemical name is derived. For the same reason the name *Arbor Dianæ* has been given to the metallic deposit produced in a similar manner by mercury in a solution of silver nitrate.

If a rod of zinc be dipped in an acid solution of stannous chloride, crystallised tin is formed upon it ; the experiment is rendered more beautiful by dipping the platinum electrodes of a battery in the solution ; if the poles are reversed, the crystallised laminæ disappear at one pole to reappear at the other.

ELECTROMETALLURGY

854. **Electrometallurgy.**—The decomposition of salts by the battery has received a most important application in *electrometallurgy*, or *galvanoplastics*, or the art of precipitating certain metals from their solutions by the action of a voltaic current. The processes are twofold; in the one, *electrotyping* or *galvanoplastics* proper, a mould is used, on which a metal, usually copper, is more or less thickly deposited, which can afterwards be detached, and gives a copy of the original object; in the other, which is known as *electroplating*, a thin coherent coating of metal, gold or silver—for instance—is deposited on objects and remains adherent to them. The art was discovered independently by Spencer in England and by Jacobi in St. Petersburg.

In order to obtain a galvanoplastic reproduction of a medal or any other object, a mould must first be made, on which the layer of metal is deposited by the electric current.

For this purpose several substances are in use, and one or the other is preferred according to circumstances. For medals and similar objects which can be submitted to pressure, gutta-percha may be used with advantage. The gutta-percha is softened in hot water, pressed against the object to be copied and allowed to cool, when it can be detached without difficulty. For the reproduction of engraved woodblocks or type, wax moulds are now commonly used. They are prepared by pouring into a narrow flat pan a suitable mixture of wax, tallow, and Venice turpentine, which is allowed to set, and is then carefully brushed over with very finely powdered graphite. While this composition is still somewhat soft, the wood block or type is pressed upon it either by a screw press or, still better, by hydraulic pressure. If plaster of Paris moulds are to be made use of, it is essential that they be first thoroughly saturated with wax or tallow, so as to become impervious to water.

In all cases, whether the moulds be of gutta-percha or wax, or any non-conducting substance, it is of the highest importance that the surface be brushed over very carefully with graphite, and so made a good conductor. The conducting surface thus prepared must also be in metallic contact with a wire or a strip of copper by which it is connected with the negative electrode. Sometimes the moulds are made of a fusible alloy (340), which may consist of 5 parts of lead, 8 of bismuth, and 3 of tin. Some of the melted alloy is poured into a shallow box, and just as it begins to solidify, the medal is placed horizontally on it in a fixed position. When the alloy has become cool, a slight shock is sufficient to detach the medal. A copper wire is then bound round the edge of the mould, by which it can be connected with the negative electrode of the battery, and then the edge and the back are covered with a thin non-conducting layer of wax, so that the deposit is only formed on the mould itself.

The most suitable arrangement for producing an electro-deposit of copper consists of a trough of glass, slate, or of wood, lined with india-rubber or coated with marine glue (fig. 825). This contains an acid solution of copper sulphate, and across it are stretched copper rods, B and D, connected

respectively with the negative and positive poles of a battery. By their copper conductors the moulds, *m*, are suspended in the liquid from the negative rod B, whilst a sheet of copper, C, presenting a surface about equal to that of the moulds to be covered, is suspended from the positive rod D, at the distance of about two inches, directly opposite to them.

The copper plate suspended from the positive pole acts not only as a conductor of electricity, but it keeps the solution in a state of concentration, for the acid liberated at the positive pole dissolves the copper, and reproduces a quantity of copper sulphate equal to that decomposed by the current.

The battery employed for the electric deposition of metals ought to be one of great constancy, and Daniell's and Smee's are mostly in use. These batteries have in large establishments been supplanted by accumulators or by dynamo machines (913), which furnish the electricity at one quarter the expense, and which are specially constructed so as to furnish a current which has small E.M.F., but great quantity.

The *density* of a current is the strength divided by the surface of the electrodes, or the number of amperes per square decimetre, and a statement of this density in conjunction with a knowledge of the composition and strength of the bath is a succinct way of defining the conditions of electric deposition.

Another, and very simple, process for producing the electric deposit of copper consists in making use of what is in effect a Daniell's cell. A porous pot or a glass cylinder, covered at the bottom with bladder or with vegetable parchment, is immersed in a vessel of larger capacity containing a concentrated solution of copper sulphate. The porous vessel contains acidulated water, and in it is suspended a piece of amalgamated zinc of suitable form, and having a surface about equal to that of the mould. The latter is attached to an insulated wire connected with the zinc, and is immersed in the solution of copper sulphate in such a position that it is directly opposite to the diaphragm. The action commences by the mould becoming covered with copper, commencing at the point of contact with the conductor, and gradually increasing in thickness in proportion to the action of the Daniell's element thus formed. It is, of course, essential in the process to keep the solution of copper sulphate at a uniform strength, which is done by agitating the liquid and suspending in it bags filled with crystals of this salt. How great is the delicacy which such electric deposits can attain appears

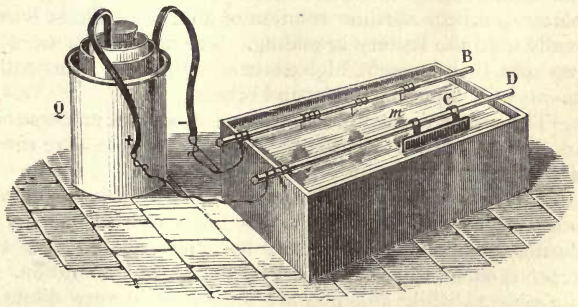


Fig. 825.

from the fact that galvanoplastic copies can be made of daguerreotypes, which are of the greatest accuracy.

An important industrial application is made of electrolysis in the *refining of copper*. The metal is extracted by the ordinary metallurgical processes so as to obtain plates containing 95 per cent. of pure copper. These plates are then used as positive electrodes in a bath of copper sulphate, and the metal is deposited in a state of perfect purity on thin sheets of pure copper, which form the negative electrode, while the impurities fall to the bottom. As the electrodes are practically identical, there is no polarisation (848), and the work of the current is solely employed in overcoming the resistance of the baths. The application of electrolysis to the extraction of metals was of limited use until the powerful currents of dynamos became available. In mountainous countries, where water-power can be had, it may in many cases be practicable to deal *in situ* with the extraction of metals from their ores.

855. **Electrogilding.**—The old method of gilding was by means of mercury. It was effected by an amalgam of gold and mercury, which was applied on the metal to be gilt. The objects thus covered were heated in a furnace, the mercury volatilised, and the gold remained in a very thin layer on the objects. The same process was used for silvering ; but they were expensive and unhealthy methods, and have now been entirely replaced by electrogilding and electrosilvering. Electrogilding only differs from the process described in the previous paragraph in that the layer is thinner and adheres more firmly. Brugnatelli, a pupil of Volta, appears to have been the first, in 1803, to observe that a body could be gilded by means of the battery and an alkaline solution of gold : but De la Rive was the first who really used the battery in gilding. The methods both of gilding and silvering owe their present high state of perfection principally to the improvements of Elkington, Ruolz, and others.

The pieces to be gilt have to undergo three processes before gilding.

The first consists in heating them so as to remove the fatty matter which has adhered to them in previous processes.

As the objects to be gilt are usually of what is called *gilding metal* or red brass, which is a special kind of brass rich in copper, and their surface during the operation of heating becomes covered with a layer of cupric or cuprous oxide, this is removed by the second operation. For this purpose the objects, while still hot, are immersed in very dilute nitric acid, where they remain until the oxide is removed. They are then rubbed with a hard brush, washed in distilled water, and dried in gently heated sawdust.

To remove all spots they must undergo the third process, which consists in rapidly immersing them in ordinary nitric acid, and then in a mixture of nitric acid, bay salt, and soot.

When thus prepared, the objects are attached to the negative pole of a battery consisting of three or four Bunsen's or Daniell's elements. They are then immersed in a bath of gold as previously described. They remain in the bath for a time, which depends on the thickness of the desired deposit. There is a great difference in the composition of the baths. That most in use consists of 1 part of gold chloride and 10 parts of potassium cyanide, dissolved in 200 parts of water. In order to keep the bath in a state of concentration, a piece of gold is suspended from the positive electrode, which

dissolves in proportion as the gold dissolved in the bath is deposited on the objects attached to the negative pole. The density of the current should not exceed 0·8 ampere for each square decimetre of the surface of the kathode.

The method which has just been described can also be used for silver, bronze, German silver, &c. But other metals, such as iron, steel, zinc, tin, and lead, are very difficult to gild well. To obtain a good coating, they must first be covered with a layer of copper, by means of the battery and a bath of copper sulphate ; the copper with which they are coated is then gilded, as in the previous case.

The tint of the deposit is modified by adding solutions of copper or of silver to the gold bath ; the former gives a reddish and the latter a greenish tint.

856. **Electrosilvering.**—What has been said about gilding applies exactly to the process of electrosilvering. The difference is in the composition of the bath, which consists of 2 parts of silver cyanide and 2 parts of potassium cyanide, dissolved in 250 parts of water. To the positive electrode is suspended a plate of silver, which prevents the bath from becoming poorer ; its surface should be equal to the total surface of the objects to be silvered ; the pieces to be silvered, which must be well cleaned, are attached to the negative pole. It may here be observed that these processes succeed best with hot solutions, and when the baths are old. The density of the current should be one-third of an ampere per square decimetre.

Knowing the weight of any given metal which is transported by unit of electricity (846), it is easy to calculate the weight deposited in a given time by a current of known strength. Thus the current just specified would deposit 1·46 gramme of silver in an hour. A deposit of one ounce of silver on a square foot of surface gives a good coating ; its thickness, $\frac{1}{864}$ inch or 0·03 mm., is about half that of thin writing paper.

857. **Electric deposition of iron, nickel, cobalt, and platinum.**—One of the most valuable applications of the electric deposition of metals is to what is called the *steeling* (*acierage*) of engraved copper plates. The bath required for this purpose is obtained by suspending a large sheet of iron, connected with the positive pole of a battery, in a trough filled with a saturated solution of sal-ammoniac ; whilst a thin strip of iron, also immersed, is connected with the negative pole. By this means iron from the large plate is dissolved in the sal-ammoniac, while hydrogen is given off on the surface of the small one. When the bath has thus taken up a sufficient quantity of iron, an engraved copper plate is substituted for the small negative strip. A bright deposit of iron begins to form on it at once, and the plate assumes the colour of a polished steel plate. The deposit thus obtained in the course of half an hour is exceedingly thin, and an impression of the plate thus covered does not seem different from an uncovered plate ; it possesses, however, an extraordinary degree of hardness, so that a very large number of impressions can be taken from such a plate before the thin coating of iron is worn off. When, however, this is the case, the film of iron is dissolved off by dilute nitric acid, and the plate is again covered with the deposit of iron.

An indefinite number of perfect impressions may, by this means, be obtained from one copper plate, without altering the original sharp condition of the engraving.

The covering of metals by a deposit of *nickel* has of late come into use. The process is essentially the same as that just described. The bath used for the purpose can, however, be made more directly by mixing, in suitable proportions, salts of nickel with those of ammonia. The positive pole consists of a plate of pure nickel. A special difficulty is met with in the electric deposition of nickel, owing to the tendency of this metal to deposit in an uneven manner, and then to become detached. This is got over by frequently removing the articles from the bath, and submitting them to a polishing process.

Objects coated with nickel show a highly polished surface of the characteristic bright colour of this metal ; this is moreover very hard and durable, and is not affected either by the atmosphere or even by sulphuretted hydrogen. A deposit of 2 grammes of nickel on the square decimetre represents a coating 0·023 mm. in thickness.

The deposit of *cobalt* has a brighter tint than that of nickel. Professor Silvanus Thompson uses a bath of cobalt sulphate or chloride, to which magnesium sulphate is added.

To obtain a deposit of *platinum*, the hydrate of this metal is dissolved in syrupy phosphoric acid, and this solution diluted with water so that it contains 1·2 to 1·5 per cent. of the hydrate. An anode of platinum or of carbon is used, and the strength of the bath is kept constant by the addition of the hydrate. Objects of iron, nickel, and zinc must previously be coated with copper.

CHAPTER IV

ELECTRODYNAMICS. ATTRACTION AND REPULSION OF CURRENTS BY CURRENTS

858. **Electrodynamics.**—By the term *electrodynamics* is understood the laws of electricity in a state of motion, or the action of electric currents upon each other and upon magnets, while *electrostatics* deals with the laws of electricity in a state of rest.

The action of one electrical current upon another was first investigated by Ampère, shortly after the discovery of Oersted's celebrated fundamental

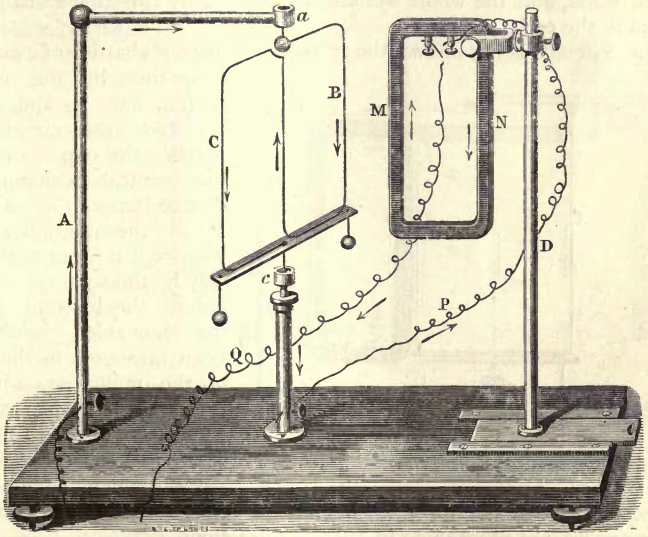


Fig. 826.

experiment (820). All the phenomena, even the most complicated, follow from two simple laws, which are—

I. *Two currents which are parallel, and in the same direction, attract one another.*

II. *Two currents parallel, but in contrary directions, repel one another.*

In order to demonstrate these laws, the circuit which the current traverses must consist of two parts, one fixed and the other movable. This is effected

by the apparatus (fig. 826), which is a modified and improved form of one originally devised by Ampère.

It consists of two brass columns, A and D, between which is a shorter one. The column D is provided with a multiplier (821) of 20 turns, MN (fig. 828), which greatly increases the sensitiveness of the instrument. This can be adjusted at any height, and in any position, by means of a universal screw clamp (see figs. 828–830).

The short column is hollow, and in its interior slides a brass tube terminating in a mercury cup, *c*, which can be raised or lowered. On the column A is another mercury cup represented in section at fig. 827 in its natural size. In the bottom is a capillary aperture through which passes the point of a sewing-needle fixed to a small copper ball. This point extends as far as the mercury, and turns freely in the hole. The movable part of the circuit consists of a copper wire proceeding from a small ball, and turning in the direction of the arrows from the cup *a* to the cup *c*. The two lower branches are fixed to a thin strip of wood, and the whole system is balanced by two copper balls, suspended to the ends.

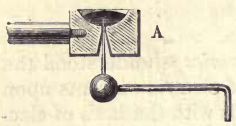


Fig. 827.

These details being known, the current of a Bunsen's battery of 4 or 5 cells ascending by the column A (fig. 828) to the cup *a*, traverses the circuit BC, reaches the cup *c*, descends the central column, and thence passes by a wire, P, to the multiplier MN, whence it returns to the battery by the wire Q. Now, if, before the current passes, the movable circuit has been arranged in the plane of the multiplier, with the sides B and M opposite each other, when the current passes, the side B is repelled, which demonstrates the second law; for in the branches B and M the currents, as indicated by the arrows, are proceeding in opposite directions.

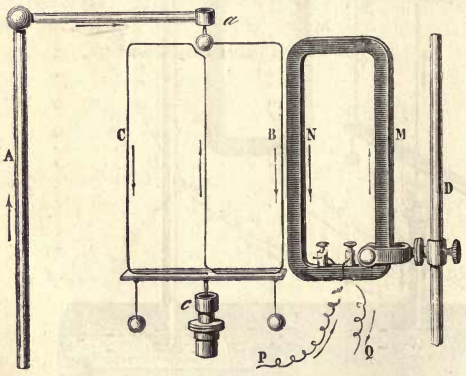


Fig. 828.

To demonstrate the first law the experiment is arranged as in fig. 828—that is, the multiplier is reversed; the current is then in the same direction both in the multiplier and in the movable part; and when the latter is removed out of the plane of the multiplier, so long as the current passes it tends to return to it, proving that there is attraction between the two parts.

859. **Roget's vibrating spiral.**—The attraction of parallel currents may also be shown by an experiment known as that of *Roget's vibrating spiral.* A copper wire about 0·7 mm. in diameter is coiled in a spiral of about 30

coils of 25 mm. in diameter. At one end it is hung vertically from a binding screw, while the other just dips in a mercury cup. On passing the current of a battery of 3 to 5 Grove's cells through the spiral by means of the mercury cup and the binding screw, its coils are traversed by parallel currents; they therefore attract one another, and rise, and thus the contact with the mercury is broken. The current having thus ceased, the coils no longer attract each other, they fall by their own weight, contact with the mercury is re-established, and the series of phenomena are indefinitely produced. The experiment is still more striking if a magnetised rod the thickness of a pencil is introduced into the interior. This will be intelligible if we consider the action between the parallel Ampèrian currents (879) of the magnet and of the helix.

860. **Laws of angular currents.**—I. *Two rectilinear currents, the directions of which form an angle with each other, attract one another when both approach or recede from the apex of the angle.*

II. *They repel one another if one approaches and the other recedes from the apex of the angle.*

These two laws may be demonstrated by means of the apparatus above described, replacing the movable circuit by the circuit BC (fig. 829). If then the multiplier is placed horizontally, so that its current is in the same direction as in the movable current, on removing the latter it quickly approaches the multiplier, which verifies the first law.

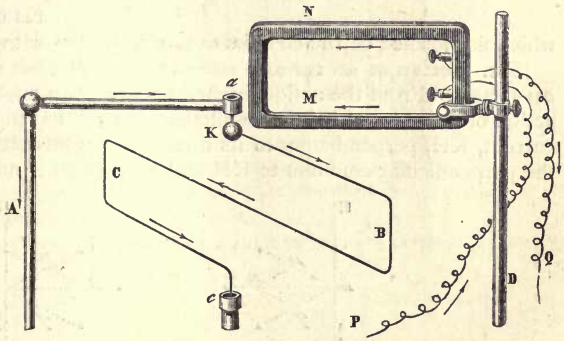


Fig. 829.

To prove the second law, the multiplier is turned so that the currents are in opposite directions, and then repulsion ensues (fig. 829).

In a rectilinear current each element of the current repels the succeeding one, and is itself repelled.

This is an important consequence of Ampère's law, and may be experimentally demonstrated by the following arrangement, which was devised by Faraday. A U-shaped piece of copper wire, the ends of which dip in two separate deep mercury cups, is suspended from one end of a delicate balance and suitably equipoised. When the mercury cups are connected with the two poles of a battery, the wire rises very appreciably, and sinks again to its original position when the current ceases to pass. The current passes into the mercury and into the wire; but from the construction of the apparatus the former is fixed, while the latter is movable, and is accordingly repelled.

861. **Laws of sinuous currents.**—*The action of a sinuous current is equal to that of a rectilinear current of the same length in projection.*

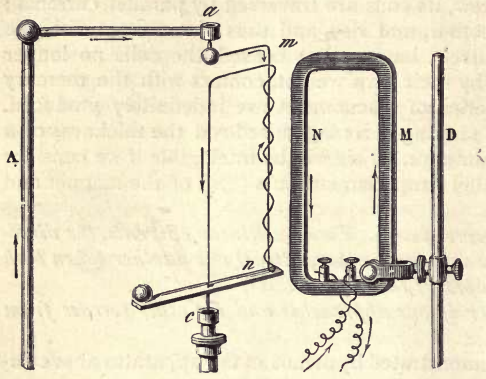


Fig. 830.

This principle is demonstrated by arranging the multiplier vertically and placing near it a movable circuit of insulated wire half sinuous and half rectilinear (fig. 830). It will be seen that there is neither attraction nor repulsion, showing that the action of the sinuous portion *mn* is equalled by that of the rectilinear portion.

An application of this principle will presently be met with in the apparatus called *solenoids* (874), which are formed of the combination of a sinuous with a rectilinear current.

862. **Action of an infinite current on a current perpendicular to its direction.**—From the action exerted between two angular currents (860) the action of a fixed and infinite rectilinear current, PQ (fig. 831), on a movable current, KH, perpendicular to its direction can be determined. Let OK be the perpendicular common to KH and PQ, which is null if the two lines PQ

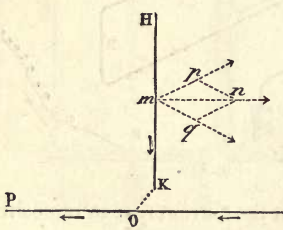


Fig. 831.

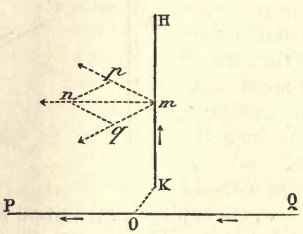


Fig. 832.

and KH meet. The current PQ flowing from Q to P in the direction of the arrows, let us first consider the case in which the current KH approaches the current QP. From the first law of angular currents (860) the portion OQ of the current PQ attracts the current KH, because they both flow towards the summit of the angle formed by their directions. The portion PO, on the contrary, will repel the current KH, for here the two currents are in opposite directions at the summit of the angle. If then *mq* and *mp* stand for the two forces, one attractive and the other repulsive, which act on the current KH, and which are necessarily of the same intensity, since they are symmetrically arranged in reference to the two sides of the point O, these two forces may be resolved into a single force, *mn*, which tends to move the current KH parallel to the current QP, but in a contrary direction.

A little consideration will show that when the current KH is below the current PQ, its action will be the opposite of what it is when above.

On considering the case in which the current KH moves away from PQ (fig. 832), it will be readily seen from similar considerations that it moves parallel to this current, but in the same direction.

Hence follows this general principle. *A finite movable current which approaches a fixed infinite current is acted on so as to move in a direction parallel and opposite to that of the fixed current; if the movable current tends from the fixed current, it is acted on so as to move parallel to the current and in the same direction.*

It follows from this, that if a vertical current is movable about an axis, XY, parallel to its direction (figs. 833 and 834), any horizontal current PQ

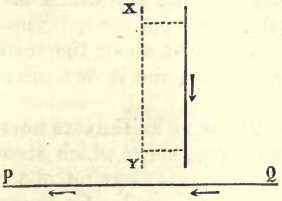


Fig. 833.

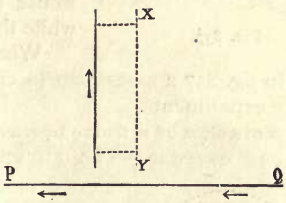


Fig. 834.

will have the effect of turning the movable current about its axis, *until the plane of the axis and of the current have become parallel* to PQ; the vertical current stopping, in reference to its axis, *on the side from which the current* PQ *comes* (fig. 833), *or on the side towards which it is directed* (fig. 834), *according as the vertical current descends or ascends*—that is, according as it approaches or moves from the horizontal axis.

It also follows from this principle that a system of two vertical currents rotating about a vertical axis (figs. 835 and 836) is directed by a horizontal current, PQ, in a plane parallel to this current when one of the vertical currents is ascending and the other descending (fig. 835); but that if they are both ascending or both descending (fig. 836), they are not directed.

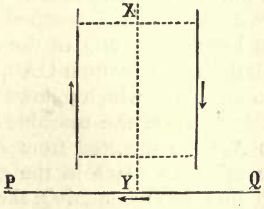


Fig. 835.

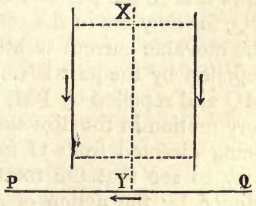


Fig. 836.

863. **Action of an infinite rectilinear current on a rectangular or circular current.**—It is easy to see that a horizontal infinite current exercises the same directive action on a rectangular current movable about a vertical axis (fig. 837) as that which has been above stated. For from the direction of the currents indicated by the arrows, the part QY acts by attraction not only on the horizontal portion YD (*law of angular currents*), but also on the vertical portion AD (*law of perpendicular currents*). The same action

evidently takes place between the part PY and the parts CY and BC. Hence, *the fixed current* PQ *tends to direct the movable rectangular current* ABCD *into a position parallel to* PQ, *and such that in the wires* CD *and* PQ *the direction of the two currents is the same.*

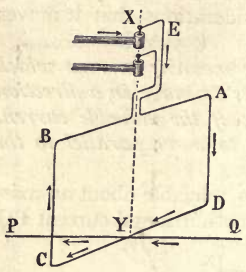


Fig. 837.

This principle is readily demonstrated by placing the circuit ABCD on the apparatus with two supports (fig. 837), so that at first it makes an angle with the plane of the supports. On passing a somewhat powerful current below the circuit in the same plane as the supports, the movable part passes into that plane. It is best to use the circuit in fig. 845, which is astatic, while that of fig. 837 is not.

What has been said about the rectangular current in fig. 837 applies also to circular currents, and is demonstrated by the same experiments.

864. **Rotation of a finite horizontal current by an infinite horizontal rectilinear current.**—The attractions and repulsions which rectangular currents exert on one another may readily be transformed into a continuous circular motion. Let OA (fig. 838) be a current movable about the point O in a horizontal plane, and let PQ be a fixed infinite current also horizontal. As these two currents flow in the direction of the arrows, it follows that in the position OA the movable current is attracted by the current PQ, for they are in the same direction. Having reached the position OA′, the movable current is attracted by the part NQ of the fixed current, and repelled by the part PN. Similarly in the position OA″, it is attracted by MQ and repelled by PM, and so on ; from which follows a continuous rotatory motion in the direction AA′A″A‴. If the movable current, instead of being directed from O towards A, were directed from A towards O, it is easy to see that the rotation would take place in the contrary direction. Hence, by the action of a fixed infinite current, PQ, the movable current OA tends to a continuous motion *in a direction opposite to that of the fixed current.*

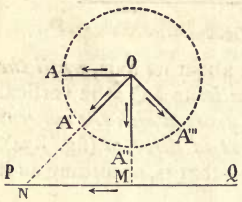


Fig. 838.

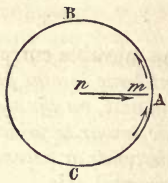


Fig. 839.

If, both currents being horizontal, the fixed current were circular instead of being rectilinear, its effect would still be to produce a continuous circular motion. For, let ABC (fig. 839) be a fixed circular current, and *mn* a rectilinear current movable about the axis *n*, both currents being horizontal. These currents, flowing in the direction of the arrows, would attract one another in the angle *n*AC, for they both flow towards the summit (860). In the angle *n*AB, on the contrary, they repel one another, for one goes towards the summit and the other moves from it. Both effects coincide in moving the wire *mn* in the same direction ACB.

865. **Rotation of a vertical current by a horizontal circular current.** A horizontal circular current, acting on a rectilinear vertical, also imparts to it a continuous rotatory motion. In order to show this, the apparatus represented in fig. 840 is used.

It consists of a brass vessel, round which are rolled several coils of insulated copper wire, through which a current passes. In the centre of the vessel is a brass support, *a*, terminated by a small cup containing mercury. In this dips a pivot supporting a copper wire, *bb*, bent at its ends in two vertical branches, which are soldered to a very light copper ring immersed in acidulated water contained in the vessel. A current entering through the wire *m*, reaches the wire A, and, having made several circuits, terminates at B, which is connected by a wire underneath with the lower part of the column *a*. Ascending in this column, it passes by the wires *bb* into the copper ring, into the acidulated water, and into the sides of the vessel, whence it returns to the battery by the strip D. The current being thus closed, the circuit *bb* and the ring tend to turn in a direction contrary to that of the fixed current, a motion due to the action of the circular current on the current in the vertical branches *bb* ; for, as follows from the two laws of angular currents, the branch *b* on the right is attracted by the portion A of the fixed current, and the branch *b* on the left is attracted in the contrary direction by the opposite part, and these two motions coincide in giving the ring a continuous rotatory motion in the same direction. The action of the circular current on the horizontal part of the circuit *bb* would tend to turn it in the same direction ; but from its distance it may evidently be neglected.

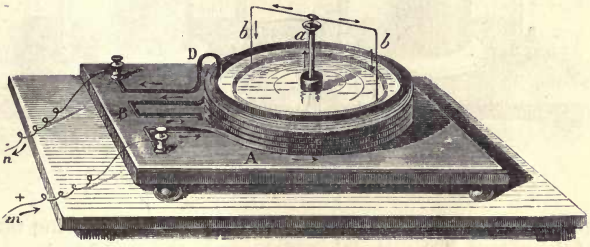


Fig. 840.

866. **Rotation of magnets by currents.**—Faraday proved that currents impart the same rotatory motions to magnets which they do to currents. This may be shown by means of the apparatus represented in fig. 841. It consists of a large glass vessel, almost filled with mercury. In the centre of this is immersed a magnet, A, about eight inches in length, which projects a little above the surface of the mercury, and is loaded at the bottom with a platinum cylinder. At the top of the magnet is a small cavity containing mercury ; the current ascending the column *m* passes into this cavity by the rod C. From the magnet it passes by the mercury to a copper ring, G, whence it emerges by the column *n*. When this takes place the magnet begins to rotate round its own axis with a velocity depending on its magnetic power and on the intensity of the current.

Instead of making the magnet rotate on its axis, it may be caused to rotate round a line parallel to its axis by arranging the experiment as shown in fig. 842.

This rotatory motion is readily intelligible on Ampère's theory of magnetism (879), according to which, magnets are traversed on their surface by an infinity of circular currents in the same direction, in planes perpendicular to the axes of the magnet. At the moment at which the current passes from the magnet into the mercury, it divides on the surface of the mercury into an infinity of rectilinear currents proceeding from the axis of the magnet to the circumference of the glass. Figs. 843 and 844, which correspond respectively to figs. 841 and 842, give on a larger scale, and on a horizontal plane passing through the surface of the mercury, the direction of the currents to which the rotation is due. In fig. 843, the north pole being at the top, the Ampèrian currents pass round the magnet in the reverse direction to that of the hands of a watch, as indicated by the arrow *i* (879), while the currents which radiate from the rod C towards the metal ring GG′, have the direction CD, CE. Thus (860) any given element *e* of the magnetic current of the bar A is attracted by the current CE and repelled by the current CD; hence results a rotation of the bar about its axis in the same direction as the hands of a watch.

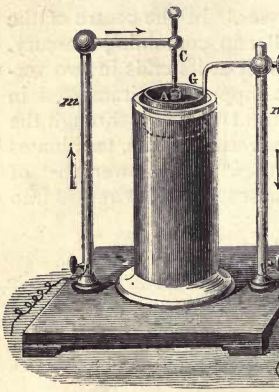


Fig. 841.

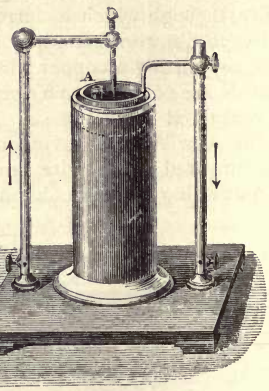


Fig. 842.

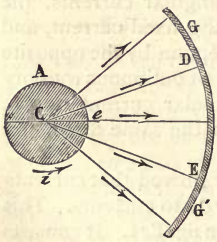


Fig. 843.

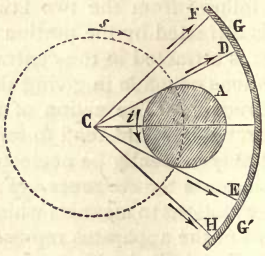


Fig. 844.

In fig. 844 the currents CD, CF, being in the opposite direction to those of the bar, would repel the latter, which would be attracted by the currents CE, CH. Hence the bar rotates in a circular direction, shown by the arrow *s*, about the vertical axis which passes through the rod C.

If the north pole is below, or if the direction of the current be altered, the rotation of the magnet is in the opposite direction.

ACTION OF THE EARTH AND OF MAGNETS ON CURRENTS

867. **Directive action of magnets on currents.**—Not only do currents act upon magnets, but magnets also act upon currents. In Oersted's fundamental experiment (fig. 779), the magnet being movable while the current is fixed, the former is directed and sets at right angles with the current. If, on the contrary, the magnet is fixed and the current movable, the latter is directed and sets across the direction of the magnet. This may be illustrated by the apparatus represented in fig. 845. This is the original form of *Ampère's stand*, and is frequently used in experimental demonstration. It needs no explanation. The circuit which the current traverses is movable, and below its lower branch a powerful bar magnet is placed ; the circuit immediately begins to turn, and stops after some oscillations in a plane perpendicular to the axis of the magnet.

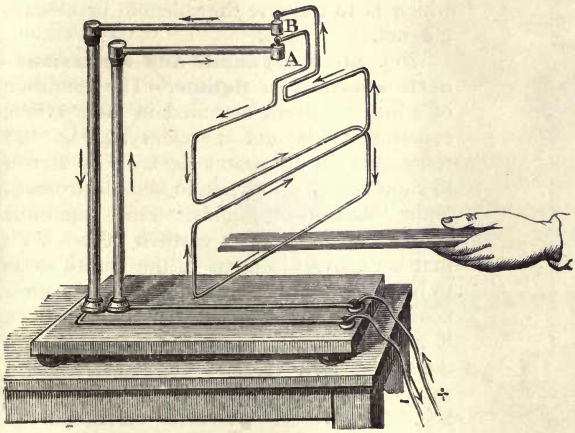


Fig. 845.

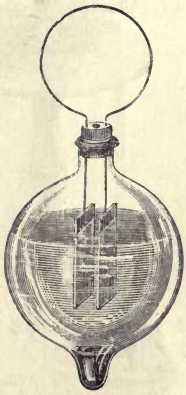


Fig. 846.

For demonstrating the action of magnets upon currents, De la Rive's *floating battery* (fig. 846) is well adapted. It consists of plates of zinc and copper which are immersed in dilute sulphuric acid contained in a glass bulb slightly loaded with mercury to keep it upright, and which can float freely on water. With the plates can be connected either circular or rectangular wires, coils, or solenoids ; they are then traversed by a current, and can be subjected to the action either of magnets or of currents.

868. **Rotation of currents by magnets.**—Not merely can currents be directed by magnets, but they may also be made to rotate, as is seen from the following experiment, devised by Faraday (fig. 847). On a base with levelling screws, and resting on an ivory support, is a copper rod, BD. It is surrounded in part of its length by a bundle of magnetised wires, AB, and at the top is a mercury cup. A copper circuit, EF, balanced on a steel

point, rests in the cup, and the other ends of the circuit, which terminate in steel points, dip in an annular trough full of mercury.

The apparatus being thus arranged, the current from 4 or 5 Bunsen's elements enters at the binding screw *b* ; it thence rises in the rod D, descends by the two branches, reaches the mercury by the steel points, whence it passes by the framework, which is of copper, to the battery by the binding screw *a*. If now the magnetised bundle be raised, the circuit EF rotates, either in one direction or the other, according to the pole by which it is influenced. This rotation is due to currents assumed to circulate round magnets ; currents which act on the vertical branches EF in the same way as the circular current on the branches *bb* in fig. 840.

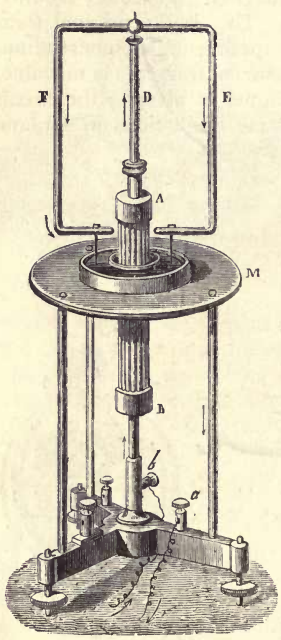


Fig. 847.

In this experiment the magnetised bundle may be replaced by a solenoid (874) or by an electromagnet, in which case the two binding screws in the base of the apparatus on the left give entrance to the current which is to traverse the solenoid or electromagnet.

869. **Electrodynamic and electromagnetic rotation of liquids.**—The condition of a linear current assumed in the previous experiments is not necessary. Fig. 848 represents an apparatus devised by Bertin to show the electrodynamic and electromagnetic rotation of liquids. This apparatus consists of an annular earthen vessel, VV ; that is to say, it is open in the centre so as to be traversed by a coil, H. It rests on a board which can be raised along two columns, E and I, and which are fixed by means of the screws KK. Round the vessel VV is a second larger coil, G, fixed on the columns SS′. The vessel VV rests on the lower plane. In the centre of the coil is a bar of soft iron, *x*, which makes an electromagnet.

The vessel VV contains acidulated water, and in the liquid are two cylindrical copper plates *e* and *i*, soldered to copper wires, *e′* and *i′*, which convey the current of a battery of four cells through the rods E and I. The whole system is arranged on a larger base, on the left of which is a commutator represented afterwards on a larger scale (fig. 849). With the base of the columns E, I, S, and S′ are connected four copper strips, three of which lead to the commutator and the fourth to the binding screw A, which receives the wire from the positive pole.

The following three effects may be obtained with this apparatus :—(1) the action of the coil G alone ; (2) the action of the electromagnet H alone ; (3) the simultaneous action of the coil and of the electromagnet.

I. Fig. 848 represents the apparatus arranged for the first effect. The

current coming by the binding screw A attains the column S′, which leads it to the coil G, with regard to which it is *left*—that is, in a contrary direction to the hands of a watch. Then descending by the column S, it reaches the commutator, which leads it by the plate marked *centripete* to the column E and to the electrode *e′*. The current here traverses the liquid from the circumference to the centre, attains the electrode *i*, the column I, and by the intervention of the plate *centrifuge* the central piece of the commutator. This transmits it finally to the negative binding screw, which leads it to the battery. The liquid then commences a *direct* rotatory motion—that is to say, in the same direction as the coil. If the direction of the current in the liquid is *centrifugal*—that is, proceeds from the centre to the circumference—the rotation is *inverse* ; that is, in the opposite direction to that of the coil. In both cases the rotations may be shown to those at a distance by means of small flags, *f*, *f′*, fixed on discs of cork which float on the liquid, and which are coated with lampblack to prevent adherence by capillary attraction between the discs and the electrodes *e* and *i*.

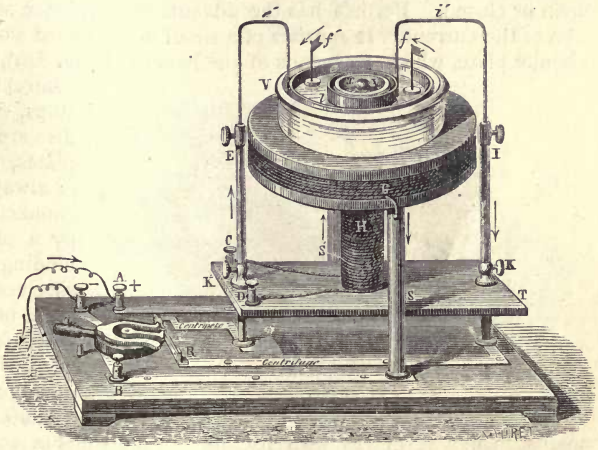


Fig. 848.

II. To experiment with the electromagnet alone, the positive wire of the battery is connected with the binding screw C, and the binding screws D and B are joined by a copper wire. The current first passes into the electromagnet H, then, reaching the commutator by the binding screw B, passes into the centripetal plate, whence it rises in the column E, traverses the liquid in the same direction as at first, reascends by the column I, and from thence to the centre of the commutator and the negative binding screw, which leads it to the battery. If the north pole of the electromagnet is at the same height as the glass vessel, as in the figure, the Ampèrian currents move in the opposite direction to the hands of a watch, and the floats then move in the same direction as above ; and if the electromagnet is raised until the neutral line is at the same height as the vessel, the floats stop ; if it is above them, the floats move again, but in the opposite direction.

III. To cause the coil and the electromagnet to act simultaneously, the positive wire of the battery is attached at C, and the binding screws D and A are connected by a conductor. Hence, after having traversed the coil H,

the current arrives from D, and the binding screw A, whence it traverses exactly the same circuit as in the first experiments. The effects are the same, though more intense, the action of the coil and the electromagnet being in the same direction.

A simpler form of this experiment was devised by Clerk Maxwell. At the bottom of a small beaker, a copper disc is placed with an insulated tongue bent at right angles, and connected with a similar zinc disc supported about an inch above the copper. Dilute acid is placed so as to cover both discs, and, some fine sawdust having been added to the liquid, the whole is placed on the pole of an electromagnet. The rotation of the liquid is then shown by that of the sawdust.

870. **Bertin's commutator.**—*Commutators* are apparatus by which the direction of currents may be changed at pleasure, or by which they may be open or closed. Bertin's has the advantage of at once showing the direction of the current. It consists of a small base of hard wood on which is an ebonite plate, which, by means of the handle *m* (fig. 849), is turned about a central axis, between two stops, *c* and *c'*. On the disc are fixed two copper plates, one of which, *o*, is always positive, being connected by the axis and by a plate, +, with the binding screw P, which receives the positive electrode of the battery; the other, *ie*, bent in the form of a horseshoe, is in metallic connection with a plate below the disc against which it moves with friction; this plate is in connection with the negative electrode N. On the opposite side of the board are two binding screws, *b* and *b'*, to which are adapted two elastic metal plates, *r* and *r'*.

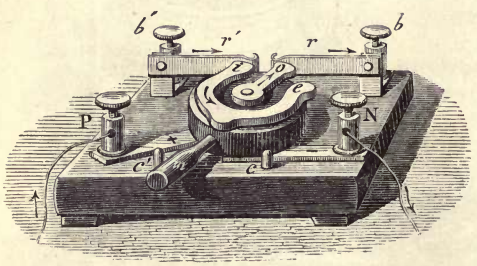


Fig. 849.

These details being premised, the disc being turned as shown in the figure, the current coming by the binding screw P passes into the piece *o*, the plate *r* and the binding screw *b*, which by a second plate, or by a copper wire, leads it to the apparatus shown in fig. 848, or any other. Then returning to the binding screw *b'*, the current attains the plate *r'*, the piece *ie*, and ultimately the binding screw N, which returns it to the battery.

If the disc is turned so that the handle is half-way between *c* and *c'*, the pieces *o* and *ie* being no longer in contact with the plates *r* and *r'*, the current does not pass. If *m* is turned as far as *c*, the plate *o* touches *r'*, and *r* touches *e*; the current thus passes first to *b'* and returns by *b*; it is therefore reversed.

871. **Directive action of the earth on vertical currents.**—The earth, which exercises a directive action on magnets (689), acts also upon currents, giving them in some cases a fixed direction, in others a continuous rotatory motion.

The first of these two actions may be thus enunciated: *Every vertical*

current movable about an axis parallel to itself, places itself under the directive action of the earth in a plane through this axis perpendicular to the magnetic meridian, and stops after some oscillations, on the east of its axis of rotation when it is descending, and on the west when it is ascending.

This may be demonstrated by means of the apparatus represented in fig. 851, which consists of two brass vessels of somewhat different diameters. The larger, *a*, about 13 inches in diameter, has an aperture in the centre, through which passes a brass support, *b*, insulated from the vessel *a*, but communicating with the vessel K. This column terminates in a small cup,

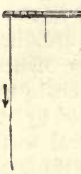


Fig. 850.

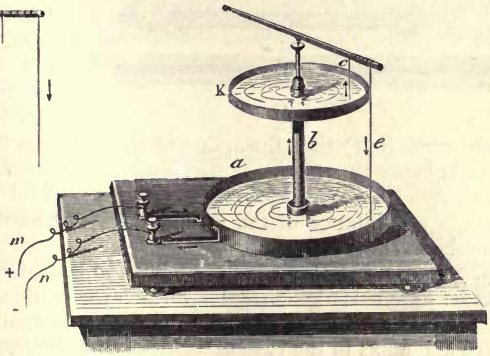


Fig. 851.

in which a light wooden rod rests on a pivot. At one end of this rod a fine wire is coiled, each end of which dips in acidulated water, with which the two vessels are respectively filled.

The current arriving by the wire *m* passes to a strip of copper, which is connected underneath the base of the apparatus with the bottom of the column *b*. Ascending in this column, the current reaches the vessel K, and the acidulated water which it contains; it ascends from thence in the wire *c*, redescends by the wire *e*, and, traversing the acidulated water, it reaches the sides of the vessel *a*, and so back to the battery through the wire *n*.

The current being thus closed, the wire *e* moves round the column *b*, and stops to the east of it, when it descends, as is the case in the figure; but if it ascends, which is effected by transmitting the current by the wire *n*, the wire *e* stops to the west of the column *b*, in a position directly opposite to that which it assumes when it is descending.

If the rod with a single wire, in fig. 851, be replaced by one with two wires as in fig. 852, the rod will not move, for as each wire tends to place itself on the east of the column *a*, two equal and contrary effects are produced, which counterbalance one another.

872. **Action of the earth on horizontal currents movable about a vertical axis.**—The action of the earth on horizontal currents is not directive, but *gives them a continuous rotatory motion from the east round by the north to the west, when the horizontal current moves away from the axis of rotation, and from the west to the east when it is directed towards this axis.*

This may be illustrated by means of the apparatus represented in fig. 852, which only differs from that of fig. 851 in having but one vessel. The current, ascending by the column *a*, traverses the two wires *cc*, and descends by the wires *bb*, from which it regains the pile; the circuit *bccb* then begins a continuous rotation either from the east to the west, or from the west to the east, according as in the wires *cc* the current goes from the centre, as is the case in the figure, or goes towards it, which is the case when the current enters by the wire *m* instead of by *n*. But we have seen (871) that the action of the earth on the vertical wires *bb* is destroyed; hence the rotation is that produced by the action on the horizontal branches *cc*. This rotatory action of the terrestrial current on horizontal currents is an instance of the rotation of a finite horizontal by an infinite horizontal current (864).

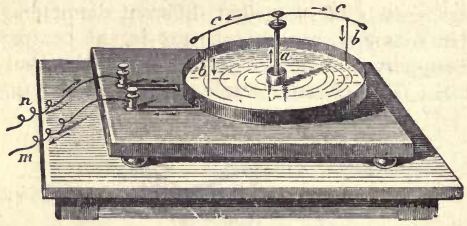


Fig. 852.

873. **Directive action of the earth on closed currents movable about a vertical axis.**—If the current on which the earth acts is closed, whether it be rectangular or circular, the result is not a continuous rotation, but a directive action, as in the case of vertical currents (871), in virtue of which *the current places itself in a plane perpendicular to the magnetic meridian, so that it is descending on the east of its axis of rotation, and ascending on the west.*

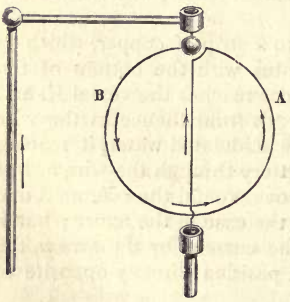


Fig. 853.

This property, which can be shown by means of the apparatus represented in fig. 830, is a consequence of what has been said about horizontal and vertical currents. For in the closed circuit BA, the current in the upper and lower parts tends to turn in opposite directions, from the law of horizontal currents (872), and hence is in equilibrium; while in the lateral parts the current on the one side tends towards the east, and on the other side to the west, from the law of vertical currents.

From the directive action which the earth exerts on currents, it is necessary, in many experiments, to neutralise this action. This is effected by arranging the movable circuit symmetrically about its axis of rotation, so that the directive action of the earth tends to turn the two branches in opposite directions. This condition is fulfilled in the circuit in fig. 853. Such circuits are hence called *astatic circuits.*

874. **Structure of a solenoid.**—A solenoid is a system of equal and parallel circular currents formed of the same piece of covered copper wire and coiled in the form of a helix or spiral, as represented in fig. 854. A

solenoid, however, is only complete when part of the wire BC passes in the direction of the axis in the interior of the helix. With this arrangement, when the circuit is traversed by a current, it follows from what has been said about sinuous currents (861) that the action of a solenoid in a longitudinal direction, AB, is counterbalanced by that of the rectilinear current BC. This action is accordingly null in the direction of the length, and the *action of a solenoid in a direction perpendicular to its axis is exactly equivalent to that of a series of equal parallel currents.*

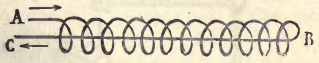


Fig. 854.

875. **Action of currents on solenoids.**—What has been said of the action of fixed rectilinear currents on finite rectangular or circular currents (864) applies evidently to each of the circuits of a solenoid, and hence a rectilinear current must tend to direct these circuits parallel to itself. To demonstrate this fact experimentally, a solenoid is constructed as shown in fig. 855, so that it can be suspended by two pivots in the cups *a* and *c* of the apparatus represented in fig. 853. The solenoid is then movable about a vertical axis, and if a rectilinear current QP be passed beneath it, which at the same time traverses the wires of the solenoid, the latter is seen to turn and set at right angles to the lower current—that is, in such a position that its circuits are parallel to the fixed current; and, further, the current in the lower part of each of the circuits is in the same direction as in the rectilinear wire.

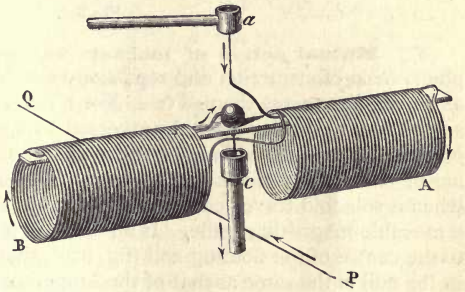


Fig. 855.

If, instead of passing a rectilinear current below the solenoid, it is passed vertically on the side, an attraction or repulsion will take place, according as the two currents in the vertical wire, and in the nearest part of the solenoid, are in the same or in contrary directions.

876. **Directive action of the earth on solenoids.**—If a solenoid be suspended in the two cups (fig. 856), not in the direction of the magnetic meridian, and a current be passed through the solenoid, the latter will begin to move, and will finally set in such a position that its axis is in the direction of the magnetic meridian. If the solenoid be removed, it will, after a few oscillations, return, so that its axis is in the magnetic meridian. Further, it will be found that in the lower half of the coils of which the solenoid consists the direction of the current is from east to west; in other words, the current is *descending* on that side of the coil turned towards the east and *ascending* on the west. The directive action of the earth on solenoids is accordingly a consequence of that which it exerts on circular currents. In this experiment the solenoid is directed like a magnetic needle, and the *north pole*, as in magnets, is that end which points towards the

north, and the *south pole* that which points towards the south. This experiment may be made by means of a solenoid fitted on a De la Rive's floating battery (867).

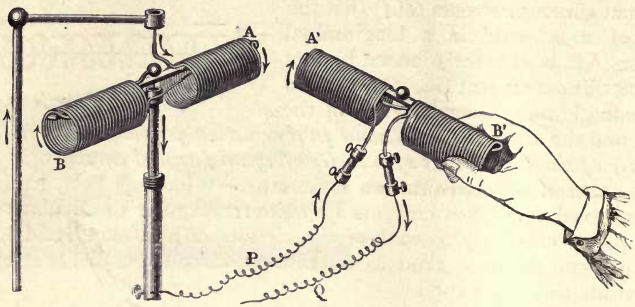


Fig. 856.

877. **Mutual action of magnets and solenoids.**—Exactly the same phenomena of attraction and repulsion exist between solenoids and magnets as between magnets themselves. For if one of the poles of a magnet be presented to a movable solenoid, traversed by a current, attraction or repulsion will take place, according as the poles of the magnet and of the solenoid are of contrary or of the same name. The same phenomenon takes place when a solenoid traversed by a current and held in the hand is presented to a movable magnetic needle. If one pole of a long bar magnet be presented to the centre of the floating coil (fig. 856), then if the direction of the current in the coil is the same as that of the Ampèrian current (879) in that pole of the magnet, the coil will be attracted to the magnet, and, encircling it, will move towards the middle, where it is stationary; if the currents are opposite, then the coil will first of all be repelled, it will then turn round, and proceed as before.

878. **Mutual action of solenoids.**—When two solenoids traversed by a powerful current are allowed to act on each other, one of them being held in the hand and the other being movable about a vertical axis, as shown in fig. 856, attraction and repulsion will take place just as in the case of two magnets. These phenomena are readily explained by reference to what has been said about the mutual action of the currents, bearing in mind the direction of the currents in the extremities presented to each other.

879. **Ampère's theory of magnetism.**—Ampère propounded a theory, based on the analogy between solenoids and magnets, by which all magnetic phenomena may be referred to electrodynamical principles.

Instead of attributing magnetic phenomena to the existence of two fluids, Ampère assumed that each individual molecule of a magnetic substance is traversed by a closed electric current, and further that these molecular currents are free to move about their centres. The coercive force, however, which is little or nothing in soft iron, but considerable in steel, opposes this motion, and tends to keep the molecular currents in any position in which they happen to be. When the magnetic substance is not magnetised, these currents, under the influence of their mutual attractions, occupy such positions that their total action on any external substance is *nil*. Magnetisation consists

in giving to these molecular currents a parallel direction, and the stronger the magnetising force the more perfect the parallelism. The *limit of magnetisation* is attained when the currents are completely parallel.

The resultant of the actions of all the molecular currents is equivalent to that of a single current which traverses the outside of a magnet. For by inspection of fig. 857, in which the molecular currents are represented by a series of small internal circles in the two ends of a cylindrical bar, it will be seen that the adjacent parts of the currents oppose one another and cannot exercise any external electrodynamic action. This is not the case with the surface; there the molecular currents at *ab* are not neutralised by other currents, and as the points *abc* are infinitely near, they form a series of elements in the same direction situated in planes perpendicular to the axis of the magnet, and which constitute a true solenoid.

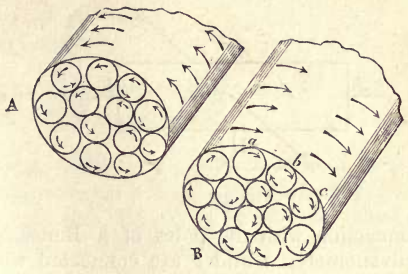


Fig. 857.

The direction of these currents in magnets can be ascertained by considering the suspended solenoid (fig. 855). If we supposed it traversed by a current, and in equilibrium in the magnetic meridian, it will set in such a position that in the lower half of each coil the current flows from *east to west*. We have then the following rule :—*When the north pole of a magnet is looked at, the direction of the amperian currents is opposite to that of the hands of a watch; and when the south pole is looked at, the direction is the same as that of the hands.*

880. **Terrestrial current.**—In order to explain terrestrial magnetic effects on this supposition, the existence of electrical currents is assumed, which continually circulate round our globe from east to west perpendicular to the magnetic meridian. The resultant of their action is a single current traversing the magnetic equator from east to west. They are supposed by some to be thermo-electric currents due to the variations of temperature caused by the successive influence of the sun on the different parts of the globe from east to west.

These currents direct magnetic needles; for a suspended magnetic needle comes to rest when the molecular currents on its under-surface are parallel and in the same direction as the terrestrial currents. As the molecular currents are at right angles to the direction of its length, the needle places its greatest length at right angles to east and west, or north and south. Natural magnetisation is probably imparted in the same way to iron minerals.

881. **Hall's experiment.**—In the action of magnets on currents which has been described in the foregoing sections, we have been concerned with the action of the magnet on the body which conveys the current.

Professor Hall of Baltimore has made the following experiment to determine whether the path of a current itself in the body of a conductor is

or is not deflected when it is exposed to the direct action of a magnetic field. A strip of gold leaf AB, 9 centimetres in length by 2 centimetres broad (fig. 858), is fastened on a glass plate, placed between the poles of an electromagnet in such a manner that the plane of the strip is at right angles to the lines of force of the magnetic field. The ends of this strip A and B are in

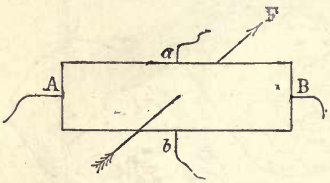


Fig. 858.

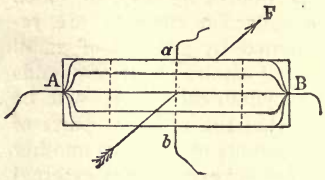


Fig. 859.

connection with the poles of a Bunsen's cell. Two wires leading to a galvanometer *a* and *b* are connected with two equipotential points at the opposite edges of the strip ; that is to say, at two points, found by trial, in which there is no deflection of the galvanometer (738). Fig. 859 shows the general direction of the lines of force of the current when the electromagnet is not excited, the dotted lines being equipotential lines. When the

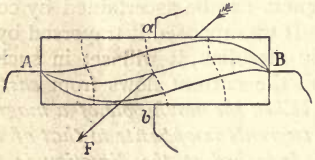


Fig. 860.

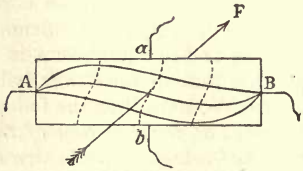


Fig. 861.

electromagnet is excited by passing a current through it, a distinct deflection is produced in the galvanometer, showing that the position of the equipotential lines is varied (fig. 860), and that the path of the current in the conducting strip has been deflected. This deflection is permanent, and cannot therefore be due to induction, and its direction is reversed when the current in the magnet is reversed (fig. 861).

The magnetic field acts thus upon the current in the gold leaf in such a manner as to displace it towards one edge or the other, and to cause a small portion to pass through the circuit of the galvanometer.

The electricity is displaced in the direction of the electromagnetic force, due to the magnet, from *a* to *b* through the galvanometer in the case of iron, zinc, and cobalt, but from *b* to *a* through the galvanometer, with nickel, gold, and bismuth. Of all metals, bismuth shows the phenomenon in far the highest degree.

CHAPTER V

MAGNETISATION BY CURRENTS. ELECTROMAGNETS. ELECTRIC TELEGRAPHS

882. **Magnetisation by currents.**—When a wire traversed by a current is immersed in iron filings, they adhere to it in large quantities (fig. 862), each particle setting perpendicularly to the wire; they become detached as soon as the current ceases, and there is no action on any non-magnetic metal.

In like manner an iron or steel bar is magnetised when placed at right angles, and near to a current; the effect is increased by coiling an insulated

Fig. 862.

copper wire round a glass tube, in which there is an unmagnetised steel rod. If a current be passed through the wire, even for a short time, the bar becomes strongly magnetised.

If, as we have already seen (791), the discharge of a Leyden jar be transmitted through the wire, by connecting one end with the outer coating, and the other with the inner coating, the bar is also magnetised. This is a convenient way of illustrating the identity between frictional and voltaic electricity.

If in this experiment the wire be coiled on the tube in such a manner that when it is held vertically the downward direction of the coils is from right to left on the side next the observer, this constitutes a *right-handed* or *dextrorsal spiral* or *helix* (fig. 863), of which the ordinary screw is an

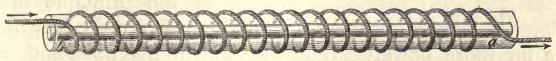


Fig. 863.

example. In a *left-handed* or *sinistrorsal helix* the coiling is in the opposite direction—that is, from left to right (fig. 864).

In a right-handed spiral the north pole is at the end at which the current emerges, and the south pole at the end at which it enters; the reverse is the case in a left-handed spiral. But whatever the direction of the coiling, the polarity is easily found by the following rule: *If a person swimming in the current look at the axis of the spiral, the north pole is always on his left.* If the wire be not coiled regularly, but if its direction be reversed at each change of direction, a consequent pole (681) is formed in the magnet. The

simplest method of remembering the polarity produced is as follows : Whatever be the nature of the helix, either right or left handed, if the end facing the observer has the current flowing in the direction of the hands of a watch,

Fig. 864.

it is a *south* pole, and *vice versâ*. The same polarity is produced whether or not there is an iron core with the helix.

In order to magnetise a steel bar by means of electricity, it need not be placed in a tube, as shown in figs. 863 and 864. It is sufficient to coil round it a copper wire, covered with silk cotton, or gutta-percha, in order to insulate the circuits from one another. The action of the current is thus multiplied, and a feeble current is sufficient to produce a powerful magnetising effect.

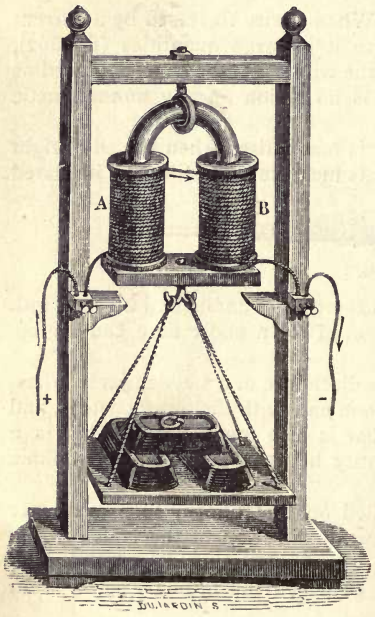


Fig. 865.

883. **Electromagnets.**—The arrangement which has been described above of a long length of insulated wire coiled on a bar or *core* of soft iron, and traversed by a current, forms what is called an *electromagnet*. In the case of soft iron the magnetisation is only temporary ; when the current ceases the magnetisation of the bar ceases also, and the iron reverts almost wholly to its ordinary magnetic but unmagnetised condition.

From its property of producing a powerful magnetic field, the coil in this experiment constitutes a *magnetising coil or spiral* ; and the magnetisation of the iron by its means is an application of the principle of magnetic induction. From the fact that electromagnets are far more powerful than permanent magnets, and, still more, that their magnetisation can be instantaneously evoked and destroyed, they have met with a host of applications of the very greatest importance ; and the form, dimensions, and strength of such electromagnets vary greatly with the purpose for which they are intended.

There are, however, two principal types ; *bar magnets*, as in fig. 867, or *horseshoe magnets*, either in one piece, as shown in fig. 865, or else formed of two straight electromagnets, each joined to a cross-piece of soft iron or *yoke*, as shown in fig. 880. It is better to have the iron in one piece, but the bending of large masses is difficult, and is apt to leave much residual magnetisation, so that the other plan is generally adopted, great care being taken that the surfaces in contact be very accurately fitted to each other.

The part on which the attraction is exerted is called the *armature*, and the space between the poles is the *air-gap*. The direction of the winding is such that the current flows in the same direction as the hands of a watch at the south pole, and in the opposite direction at the north pole.

The strength of the magnetisation in the core depends on the strength of the field due to the magnetising coil, and on the nature and dimensions of the core. For a short bar placed in a coil of considerable length it is represented by the formula $I = 4\pi ni\mu s$, where n is the number of turns per unit length, i the current, s the cross section of the bar, and μ the coefficient of permeability (707).

If a soft iron ring or *tore* be coiled round with insulated wire through which a current is passed, it is the seat of a very powerful magnetic induction, though it has no poles, and therefore no external action. Such a system forms a closed *magnetic circuit*; the arrangement represented in fig. 865, where the two poles of an electromagnet are connected by an armature, also forms such a circuit, and an interesting analogy may be made between it and a closed *electrical circuit*.

If for n in the above expression we write $\frac{n}{l}$, where n is the total number of turns of the wire, and l the length of the bar, the formula may be put

$$I = \frac{4\pi ni}{\frac{l}{\mu s}}$$

in which it is quite analogous to Ohm's formula for the electric current (825), $4\pi ni$ being the magnetic equivalent of the electromotive force, and may accordingly be called the *magnetomotive force*; while $\frac{l}{\mu s}$ is the analogue of electrical resistance R. It is, in fact, the magnetic resistance or *reluctance* R, as it is called. This, it will be seen, is directly as the length of the bar, and inversely as the cross section and the *permeability* μ or the equivalent in magnetism, of conductivity in electricity (707); it represents conductivity for lines of magnetic force. This analogy also holds if we consider the magnetic circuit as made up of bodies of different permeabilities, and also in the case of divided magnetic circuits.

The analogy fails, however, in one important respect; electrical resistance is quite independent of electromotive force, while permeability differs in value with the value of the magnetising force. Hence the analogy is rather formal than real; it is, however, useful in dealing with calculations about electromagnets.

The expression $4\pi ni$ shows that we get the same effect whether we have a small number of turns of wire with a strong current, or a great number of turns with a weak current. Thus with a given bar the same effect is produced by one turn conveying a current of one ampere as by ten turns with a current of one-tenth of an ampere. Electromagnets are usually defined by the number of *ampere-turns* used.

With a given battery the greatest magnetising force is obtained when the resistance in the magnetising spiral is equal to the other resistances in the circuit, those of the battery included, and the length and diameter of the wire must be so arranged as to satisfy these conditions.

Taking the permeability of air as unity, that of iron is many hundred times as great, hence the introduction of a layer of air in a magnetic circuit is analogous to the introduction of a bad conductor in an electrical circuit. Iron being the most permeable of all substances, a magnetic circuit should have as much of this as possible. The junctions also should be made close and true, since each joint increases the magnetic reluctance.

If we take a given magnetising spiral, at right angles to the magnetic meridian, and place at some distance from it, and in the line of its axis, a small magnetic needle, on passing a weak current through the spiral the needle is deflected, and this deflection is a measure of the magnetising force ; if the current be gradually increased the deflection will also be increased, and in proportion to the strength of the current.

If, however, the spiral contains a bar of soft iron, the case is not so simple. If we plot the curve which represents the ratio of the magnetising force to the magnetisation, as measured by the deflection of the needle, it will be found that at first the magnetisation increases more rapidly than in direct proportion to the magnetising force, but the rate of increase gradually becomes less, and it ultimately approaches a limit which is not materially exceeded, even by a considerable increase in the magnetising force. This represents a state of saturation (716), and on the theory of magnetism which has been laid down (879), it corresponds to the case in which the molecular magnets or currents have acquired their final positions.

The intensity of the magnetisation which can be imparted to a bar is 1700 CGS units in the case of wrought iron, 1240 with cast iron, and 515 in the case of nickel. Steel can acquire pretty much the same intensity of magnetisation as wrought iron, and retains about one half in the form of permanent or residual magnetism. Soft iron almost wholly loses its magnetisation when the current ceases, and the more so the purer the iron, and the more carefully it is annealed. In many applications it is of great importance that the cessation of the magnetisation with the current should be as complete as possible.

Residual magnetism is greater in long magnets, that is to say, those in which the diameter is small in comparison with the length. Hence for rapid demagnetisation the cores should be short and thick. A bundle of soft iron wires is more rapidly demagnetised than a massive bar of the same size. Residual magnetism is greater when the magnetising current is not opened suddenly, as is usually the case, but is gradually brought back to zero by successively introducing increasing resistances. By suddenly opening the current it has sometimes been found with thick rods of very soft iron, a reversed magnetisation is met with which is called *abnormal magnetisation.* This is easily understood from the tendency of molecular magnets to revert to their primitive condition (519). In doing this they experience a certain friction or resistance, and when the magnetisation gradually diminishes, this hinders any reversal of the molecules ; but when the cessation is sudden, the molecules, from the greater *vis viva* of their reversal, will sooner come back to their original position, or even pass it, and come to rest on the opposite side.

The effects of residual magnetism are lessened by preventing the armature from coming in direct contact with the magnet, either by interposing a thin paper or providing the cores with brass studs.

With uniform magnetisation the portative force of a magnet may be readily calculated. It is expressed by the formula

$$gP = 2\pi A^2 S$$

where P is the weight in grammes, $g = 981$, A is the induction, and S the available surface. If 800 is the value in CGS units, we get for the portative force 4 kilos per square centimetre, which is looked for in good specimens of permanent magnets. With soft iron in a state of saturation values of 10 to 12 kilos are obtained.

When an armature is not in contact with the poles the attraction diminishes very rapidly with the distance; for in the first place the attraction is inversely proportional to the distance, and then the fact of this distance is to introduce a layer of air which from its very great reluctance greatly lessens the strength of the magnetic circuit itself. Hence at a small distance comparatively, the attraction may be neglected.

According to the researches of Bidwell, it appears that for low degrees of magnetisation the portative force increases less rapidly than the current strength up to a certain point, at which the field represents about 10,800 grammes per square centimetre. From this point the magnetising current and the load increased in the same ratio as the strength of the current. When the field had an intensity of 1,074 units, the greatest weight supported was 150,000 grammes per sq. cm., or 52 pounds per square inch.

During magnetisation the volume of a magnet does not vary. This has been established by placing the bar to be magnetised with its spiral in a sort of water thermometer, consisting of a glass flask provided with a capillary tube. Allowing for the expansion of the water due to the heat of the voltaic current, the magnetisation of the bar produces no increase in volume. The dimensions vary, however; the diameter is somewhat lessened, and the length increases, according to Joule to the extent of $\frac{1}{2700000}$ if the bar is magnetised to saturation.

Bidwell has worked in this direction with great magnetising forces; he has found that if the magnetising force is increased beyond the point at which the magnetic elongation of the rod is greatest, a change sets in; the length of the rod, instead of remaining unchanged, slightly but steadily diminishes; the curve which expresses the ratio of the length to the magnetising force descends in a straight line, which shows no tendency to become horizontal.

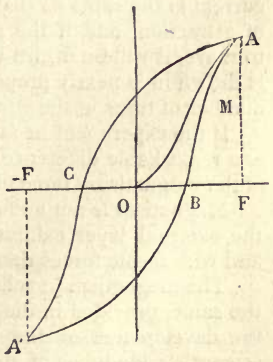


Fig. 866.

In fig. 866 the abscissæ represent the magnetising force, and the ordinates the corresponding magnetisation in a soft iron bar. The curve A represents the magnetisation starting from the magnetising force O and rising to F. The curve ACA′ represents decreasing values of the force from F to $-F'$, that is, an equal current in the opposite direction; and, lastly, the curve A′BA represents increasing values from $-F$ to $+F$.

It will thus be seen that the magnetisation of a bar for a given force

depends not only on its existing condition, but also on its previous state. The magnetisation is greater in the descending than in the ascending period for the same value of the magnetising force. This is due to residual magnetisation ; there is a *retardation* or *lag* of the magnetisation in respect of the magnetising force to which Prof. Ewing has applied the term *hysterisis*. This is equivalent to the coercive force ; it is greater the wider the difference in the two curves. Hysterisis is diminished if the body is submitted to vibrations during the process of magnetisation. If the current be broken while a horseshoe electromagnet is supporting even a heavy weight attached to the keeper, it frequently happens that the keeper does not at once become detached ; if now the magnet is gently tapped so as to set the molecules in vibration, the keeper immediately drops, and is no longer attracted when again placed in contact.

To destroy residual magnetism as, for instance, in a watch spring which has accidentally become magnetised, it should be submitted to a series of magnetising forces in alternate directions gradually decreasing to zero. A bar may also be rendered neutral by being heated to redness.

When a soft iron bar is submitted to magnetisations and demagnetisations in rapid successions its temperature rises. This represents the work of magnetisation converted into heat. If in the above curve the magnetising force and the magnetisation are expressed in CGS units, the area AB A′C represents the value in ergs of the work so transformed. The loss may attain 15,000 ergs per cubic centimetre of soft iron for each complete cycle. One erg represents $4{\cdot}17 \times 10^{-7}$ calories (122), and since the density of iron is 7·8, and its specific heat 0·11 (457), the calorific capacity of a cubic centimetre is 0·858. So that, taking the above number, we have 0·0004° as the rise in temperature for each complete cycle.

If a bar magnet be suspended by a spring so that its axis is in the prolongation of that of the spiral, and the current be now passed, it will be seen that the magnet will be attracted or repelled according as the direction of the current is the same as that of the current in the spiral or not. In the case of attraction, and if the magnet be not too long and be sufficiently free to move, it will be drawn within the spiral. The force with which the magnet is drawn in is nearly proportional to the strength of the current and to the number of turns of the wire.

If the experiment be made with a bar of soft iron, it is drawn in and there is a remarkable difference in the strength, which is proportional to the square of the magnetising force of the spiral.

Magnetism is not uniformly distributed in the section of electromagnets ; the external layer exhibits a stronger magnetisation than the inner ones, and with feeble forces there is only a magnetic excitation in the outer layer.

The magnetism in solid and in hollow cylinders of the same diameters is the same, provided in the latter case there is sufficient thickness of iron for the development of the magnetisation. With currents below a certain strength, wide tubes of sheet-iron are far more powerfully magnetised than solid rods of the same length and weight ; but with more powerful currents the magnetism of the latter preponderates.

This may be illustrated by the following experiment : Two identical magnetising spirals are joined by a wire and placed vertically a little dis-

tance apart; from one end of the beam of an ordinary balance a solid soft iron rod is suspended so that it is half way within the spiral, and this is counterpoised by a sheet-iron cylinder of the same length and weight, but of greater diameter, which is also half way within the other spiral.

If now the same weak current is transmitted through both spirals the cylinder is drawn down, but if a stronger one is passed, it is the rod which is sucked in.

884. **Vibratory motion and sounds produced by currents.**—When a rod of soft iron is magnetised by a strong electric current, it gives a very distinct sound, which, however, is only produced at the moment of closing or opening the current. This phenomenon, first observed by Page in America, and by Delezenne in France, was particularly investigated by De la Rive, who attributed it to a vibratory motion of the molecules of iron in consequence of a rapid succession of magnetisations and demagnetisations.

When the current is broken and closed at very short intervals, De la Rive observed that, whatever be the shape or magnitude of the iron bars, two sounds may always be distinguished; one, which is musical, corresponds to that which the rod would give by vibrating transversely; the other, which consists of a series of harsh sounds, corresponding to the interruptions of the current, was compared by De la Rive to the noise of rain falling on a metal roof. The most marked sound is that obtained by stretching, on a sounding-board, pieces of soft iron wire, well annealed, from 1 to 2 mm. in diameter and 1 to 2 yards long. These wires, being placed in the axis of one or more bobbins traversed by powerful currents, send forth a number of sounds, which produce a surprising effect, and much resemble that of a number of church bells heard at a distance. Rods of zinc, copper, or brass give no note even with strong currents.

Wertheim also obtained the same sounds by passing a discontinuous current, not through the bobbins surrounding the iron wires, but through the wires themselves. The musical sound is then stronger and more sonorous in general than in the previous experiment. The hypothesis of a molecular movement in the iron wires at the moment of their magnetisation, and of their demagnetisation, is confirmed by the researches of Wertheim, who found that their elasticity is then diminished.

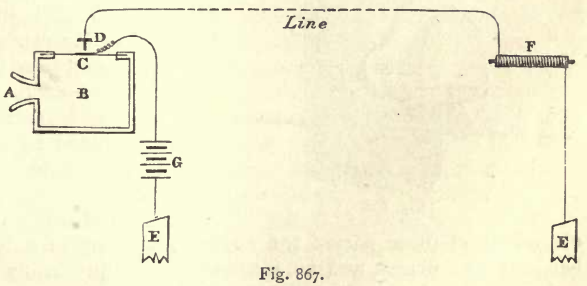


Fig. 867.

885. **Reis's telephone.**—The essential features of this instrument (fig. 867) are a sort of box, B, one side of which is closed by a membrane C, while there is a mouthpiece, A, in another side. On the membrane is a piece

of thin metal-foil C, which is connected with a wire leading to one pole of the battery G, the other pole of which is put to earth. Just above the foil, and almost touching it, is a metal point D, which is connected by the line wire (886) with one end of a coil of insulated wire surrounding an iron wire, the other end of which is put to earth.

When the mouthpiece is spoken or sung into, the sounds set the membrane in vibration ; this alternately opens and closes the current, and these makes and breaks being transmitted through the circuit to the electromagnet F, produce the corresponding sounds.

ELECTRIC TELEGRAPH

886. **Electric telegraphs.**—These are apparatus by which signals can be transmitted to considerable distances by means of voltaic currents propagated in metallic wires. Towards the end of the last century, and at the beginning of the present, many philosophers proposed to correspond at a distance by means of the effects produced by electrical machines when propagated in insulated conducting wires. In 1811, Sœmmering invented a telegraph, in which he used the decomposition of water for giving signals. In 1820, at a time when the electromagnet was unknown, Ampère proposed to correspond by means of magnetic needles, above which a current was sent, as many wires and needles being used as letters were required. In 1834, Gauss and Weber constructed an electromagnetic telegraph, in which a voltaic current transmitted by a wire acted on a magnetised bar, the oscillations of which under its influence were observed by a telescope. They succeeded in thus sending signals from the Observatory to the Physical Cabinet in Göttingen, a distance of a mile and a quarter, and to them belongs the honour of having first demonstrated experimentally the possibility of electrical communication at a considerable distance. In 1837, Steinheil in Munich, and Wheatstone in London, constructed telegraphs in which several wires each acted on a single needle ; the current in the first case being produced by an electromagnetic machine, and in the second by a constant battery.

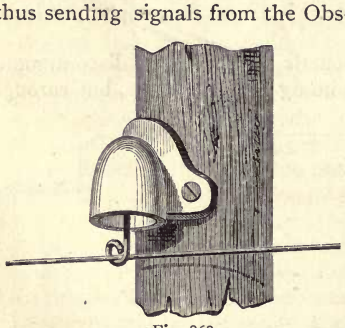


Fig. 868.

Every electric telegraph consists essentially of three parts : 1, a *circuit* consisting of a metallic connection between two places, and an *electromotor* for producing the current ; 2, a *communicator* for sending the signals from the one station ; and, 3, an *indicator* for receiving them at the other station. The manner in which these objects, more especially the last two, are effected can be greatly varied, and we shall limit ourselves to a description of the three principal methods.

One form of electromotor still sometimes used in England is a modification of Wollaston's battery. It consists of a trough divided into compartments in each of which is an amalgamated zinc plate and a copper plate ;

these plates are usually about 4½ inches in height by 3½ in breadth. The compartments are filled with sand, which is moistened with dilute sulphuric acid. This battery is inexpensive and easily worked, only requiring from time to time the addition of a little acid; but it has very low electromotive force and considerable resistance, and when it has been at work for some time the effects of polarisation begin to be perceived. On the telegraphs of the South-Eastern Railway, the platinised graphite (811) battery, invented by Mr. C. V. Walker, has been used with success. On circuits on which there is constant work some form of Daniell's battery is used, and for other circuits Leclanché's cell is coming into more extended use. In France, Daniell's battery is used for telegraphic purposes.

The connection between two stations is made by means of galvanised iron wire suspended by porcelain supports (fig. 868), which insulate and protect them against the rain, either on posts or against the sides of buildings. In England and other moist climates special attention is required to be paid to the perfection of the insulation. In towns, wires covered with gutta-percha are placed in tubes laid in the ground. Submarine cables, where great strength is required combined with lightness and high conducting power, are formed on the general type of one of the Atlantic cables, a longitudinal view of which is given in fig. 869, while fig. 870 represents a cross section.

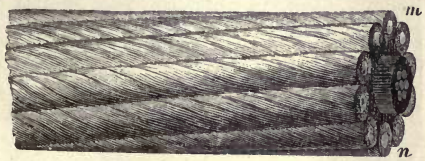


Fig. 869.

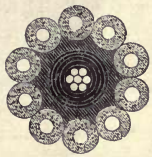


Fig. 870.

In the centre is the *core*, which is the conductor; it consists of seven copper wires, each 1 mm. in diameter, twisted in a spiral strand and covered with several layers of gutta-percha, between each of which is a coating of *Chatterton's compound*—a mixture of tar, resin, and gutta-percha. This forms the *insulator* proper, and it should have great resistance to the passage of electricity, combined with low specific inductive capacity (748). Round the insulator is a coating of hemp, and on the outside is wound spirally a protecting *sheath* of steel wire, spun round with hemp.

At the station which sends the despatch, the line is connected with the positive pole of a battery, the current passes by the line to the other station, and if there were a second return line, it would traverse it in the opposite direction to return to the negative pole. In 1837, Steinheil made the very important discovery that the earth might be used for the return conductor, thereby saving the expense of the second line. For this purpose the end of the conductor at the one station, and the negative pole of the battery at the other, are connected with large copper plates, which are sunk to some depth in the ground. The action is then the same as if the earth acted as a return wire. The earth is, indeed, far superior to a return wire; for the added resistance of such a wire would be considerable, whereas the resistance of the earth beyond a short distance is absolutely *nil*. The earth really

dissipates the electricity, and does not actually return the same current to the battery.

887. **Wheatstone and Cooke's single-needle telegraph.**—This consists essentially of a vertical multiplier (821) with an astatic needle, the arrangement of which is seen in fig. 872, while fig. 871 gives a front view of the case in which the apparatus is placed. A (fig. 872) is the bobbin, consisting of about 400 feet of fine copper wire, wound in a frame in two connected coils. Instead of an astatic needle, Mr. Walter has found it advantageous to use a single needle formed of several pieces of very thin steel

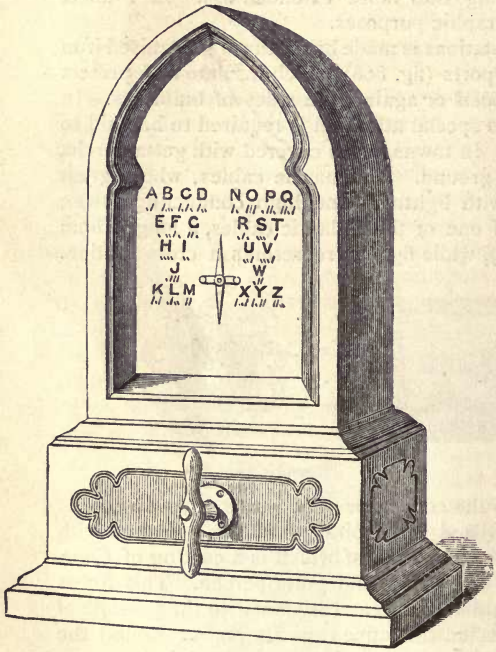


Fig. 871.

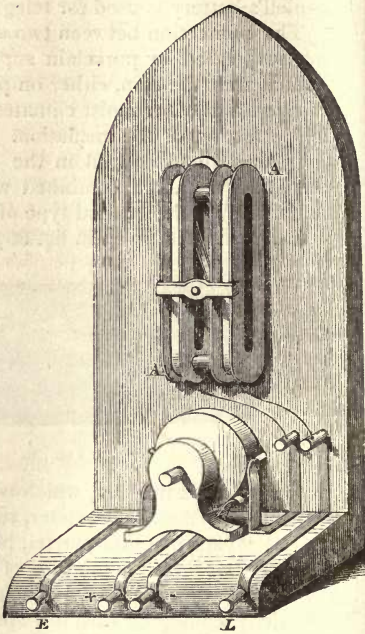


Fig. 872.

strongly magnetised ; it works with the bobbin, and a light index joined to it by a horizontal axis indicates the motion of the needle on the dial.

The signs are made by transmitting the current in different directions through the multiplier, by which the needle is deflected either to the right or left, according to the will of the operator. The instrument by which this is effected is a *commutator* or *key*, G, fig. 873 ; its action is shown in fig. 874, while fig. 873 shows on a large scale how two stations are connected. It consists of a cylinder of boxwood with a handle, which projects in front of the case (fig. 871). On its circumference parallel to the axis are seven brass strips (fig. 873), the spaces between which are insulated by ivory ; these strips are connected at the end by metallic wires, also insulated from each other, in the following manner : *a* with *b* and *c*, *f* with *d*, and *e* with *g*. Four

springs press against the cylinder; x and y are connected with the poles of the battery, m, with the earth plate, and n with one end of the multiplier, N.

When not at work the cylinder and the handle are in a vertical position, as seen on the left of the diagram. The circuit is thus *open*, for the pole springs, x and y, are not connected with the metal of the commutator. But

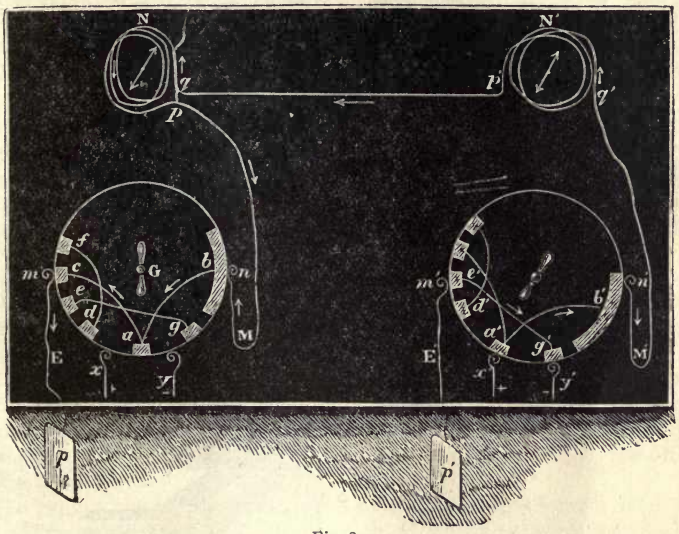


Fig. 873.

if, as in the figure on the right, the key is turned to the right, the battery is brought into the circuit, and the current passes in the following direction: + pole, $x'a'b'n'$M$'q'$N$'$, conductor qpM$nacm$Ep, earth p'E$'m'e'g'y'$, − pole. The coils N and N$'$ are so arranged that by the action of the current the motion of the needle corresponds to the motion of the handle. By turning the handle to the left the current would have the following direction: + pole $x'df'm'$E$'p'$, earth pE$mcabn$Mq, conductor $p'q'$M$'n'b'a'y'$, − pole, and thus the needle would be deflected in the opposite direction.

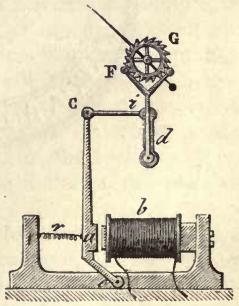


Fig. 874.

The signs are given by differently combined deflections of the needle as represented in the alphabet on the dial (fig. 871). ╲ denotes a deflection of the upper end of the needle to the left, and ╱ a deflection to the right; I, for instance, is indicated by two deflections to the left, and M by two to the right. D is expressed by right-left-left, and C by right-left-right-left, &c.

These signs are somewhat complicated and require great practice; usually not more than 12 to 20 words can be sent in a minute. The single-

needle telegraph was formerly sometimes replaced by the double-needle one, which is constructed on the same principle, but there are two needles and two wires instead of one.

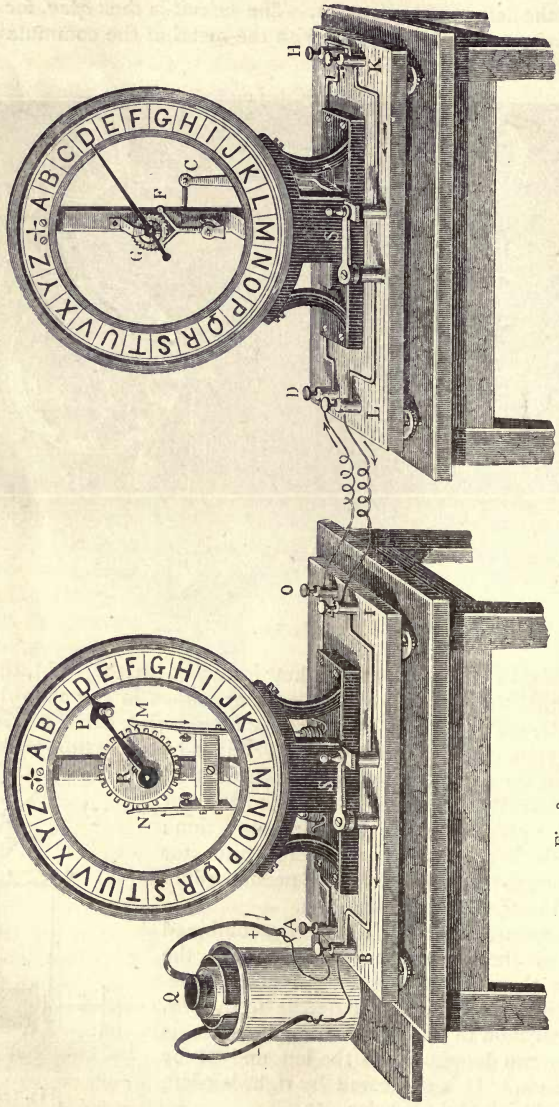


Fig. 876.

Fig. 875.

888. **Dial telegraphs.**—Of these many kinds exist. Figs. 875 and 876 represent a lecture-model of one form, constructed by Froment, and which

will serve to illustrate the principle. It consists of two parts—the *key* for transmitting signals (fig. 875), and the *indicator* (fig. 876), for receiving them. The first apparatus is connected with a battery, Q, and the two apparatus are in communication by means of metal wires, one of which, AOD (fig. 875), goes from the departure to the arrival station, and the other, HKLI (fig. 876), from the arrival to the departure. In practice, the latter is replaced by the earth circuit. Each apparatus is furnished with a dial with 25 of the letters of the alphabet, on which a needle moves. The needle at the departure station is moved by hand, that of the arrival by electricity.

The path of the current and its effects are as follows: from the battery it passes through a copper wire, A (fig. 875), into a brass spring, N, which presses against a metal wheel, R, then by a second spring, M, into the wire O, which joins the other station. Thence the current passes into the bobbin of an electromagnet, *b*, not fully shown in fig. 876, but of which fig. 874 represents a section, showing the front of the apparatus. This electromagnet is fixed horizontally at one end, and at the other it attracts an armature of soft iron, *a*, which forms part of a bent lever, movable about its axis, *o*, while a spring, *r*, draws the lever in the opposite direction.

When the current passes, the electromagnet attracts the lever *a*C, which by a rod, *i*, acts on a second lever, *d*, fixed to a horizontal axis, itself connected with a fork, F. When the current is broken the spring *r* draws the lever *a*C, and therewith all the connected pieces; a backward-and-forward motion is produced, which is communicated to the fork F; this transmits it to a toothed wheel, G, on the axis of which is the needle. From the arrangement of its teeth, the wheel G is always moved in the same direction by the fork.

To explain the intermittent action of the magnet, we must refer to fig. 875. The toothed wheel, R, has 26 teeth, of which 25 correspond to letters of the alphabet, and the last to the interval reserved between the letters Z and A. When holding the knob P in the hand the wheel R is turned, the end of the plate N from its curvature is always in contact with the teeth; the plate M, on the contrary, terminates in a catch cut so that contact is alternately made and broken. Hence, the connections with the battery having been made, if the needle P is advanced through four letters, for example, the current passes four times in N and M, and is four times broken. The electromagnet of the arrival station will then have attracted four times, and have ceased to do so four times. Lastly, the wheel G will have turned by four teeth, and as each tooth corresponds to a letter, the needle of the arrival station will have passed through exactly the same number of letters as that of the departure station. The piece S, represented in the two figures, is a copper plate, movable on a hinge, which serves to make or to break the current at will.

From this explanation it will be readily intelligible how communications are made between different places. Suppose, for example, that the first apparatus being at London and the second at Brighton, there being metallic connection between the two towns, it is desired to send the word *signal* to the latter town: as the needles correspond on each apparatus to the interval retained between Z and A, the person sending the despatch moves the needle P to the letter S, where it stops for a very short time; as the needle

in Brighton accurately reproduces the motion of the London needle, it stops at the same letter, and the person who receives the despatch notes this letter. The one at London, always continuing to turn in the same direction, stops at the letter I, the second needle immediately stops at the same letter ; and continuing in the same manner with the letters, G, N, A, L, all the word is soon transmitted to Brighton. The attention of the observer at the arrival station is attracted by means of an electric alarum. Each station must further be provided with the two apparatus (figs. 875 and 876), without which it would be impossible to answer.

889. **Morse's telegraph.**—The telegraphs hitherto described leave no trace of the despatches sent, and if any errors have been made in copying the signals there is no means of remedying them. These objections are now met in the case of the *writing telegraphs*, in which the signs themselves are printed on a strip of paper at the time at which they are transmitted.

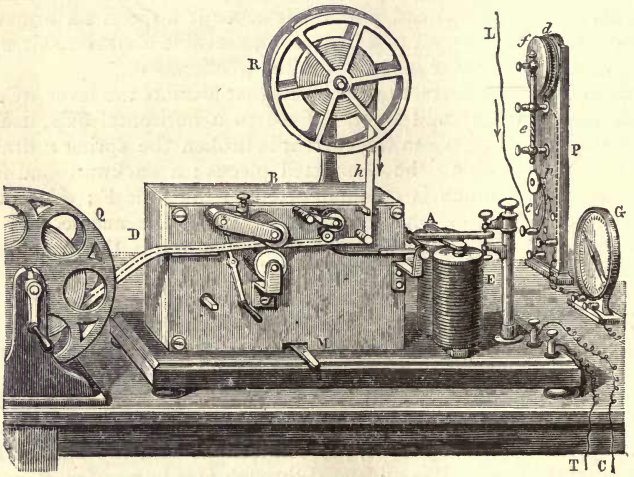


Fig. 877.

Of the numerous printing and writing telegraphs which have been devised, that of Morse, first brought into use in North America, is best known. It has been almost universally adopted on the Continent. In this instrument there are three distinct parts : the *receiver*, the *sender*, and the *relay* ; figs. 877, 878, 879, and 880 represent these apparatus.

Receiver. We will first describe the receiver (fig. 877), leaving out of sight for the moment the accessory pieces, G and T, placed on the right of the figure. The current which enters the indicator by the wire, C, passes into an electromagnet, E, which when the current is closed attracts an armature of soft iron, A, fixed at the end of a horizontal lever movable about an axis, x ; when the current is open the lever is raised by a spring r. By means of two screws, m and v, the amplitude of the oscillations is regulated. At the other end of the lever there is a pencil, o, which writes the signals. For this purpose a long band of strong paper, hp, rolled round a drum, R, passes

between two copper rollers with a rough surface, *u*, and turning in contrary directions. Drawn in the direction of the arrows, the band of paper becomes rolled on a second drum, Q, which is turned by hand. A clockwork motion placed in the box, BD, works the rollers, between which the band of paper passes.

The paper being thus set in motion, whenever the electromagnet works, the point *o* strikes the paper, and, without perforating it, produces an indentation the shape of which depends on the time during which the point is in contact with the paper. If it only strikes it instantaneously, it makes a *dot* (–) or short stroke ; but if the contact has any duration, a *dash* (—) of corresponding length is produced. Hence, by varying the length of contact of the transmitting key at one station, a combination of dots and dashes may be produced at another station, and it is only necessary to give a definite meaning to these combinations.

In order to make an indentation a considerable pressure is required, which necessitates the employment of a strong current, and the newer instruments (fig. 878) are based on the use of *ink-writers*. The paper band passes close to, but not touching, a metal disc with a fine edge, *c*, which turns against a small *ink-roller*, *a*, all being rotated by the same mechanism. When the end A is attracted, the bent plate at the other end presses the paper against the disc which is inked by contact with the ink-roller, and thus produces a mark on the paper, which is either short or long according to the duration of the contact. The signs are thus more legible, and are produced by far weaker currents.

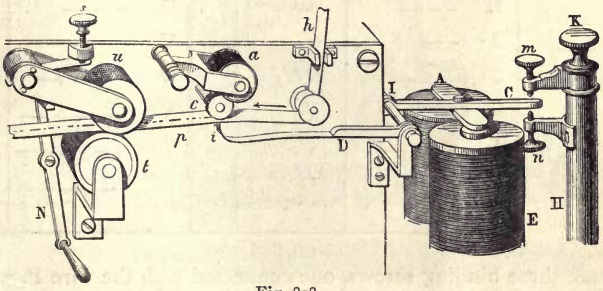


Fig. 878.

The same telegraphic alphabet is now universally used wherever telegraphic communication exists ; and the signals for the single-needle instrument (fig. 871) as well as those used for printing have been modified, so that they now correspond to each other. Thus a beat of the top of the needle to the left ⟍ is equivalent to a dot : and a beat to the right ⟋ to a dash. The figure on the next page gives the alphabet.

The *flag signals* used in military operations are similarly used. A swing of the flag from its upright vertical position to the right or left has the same meaning as the corresponding motion of the top end of the needle. So too long or short obscurations of the limelight used in signalling by night, or of the heliograph (523), correspond to dashes and dots.

Sender or key. This consists of a small mahogany base, which acts as support for a metal lever *ab* (fig. 879), movable about a horizontal axis which passes through its middle. The extremity *a* of this lever is always pressed

upwards by a spring beneath, so that it is only by pressing with the finger on the key B that the lever sinks and strikes the button *x*. Round the base

PRINTING.	SINGLE NEEDLE.	PRINTING.	SINGLE NEEDLE.
A – —	\/	N — -	/\
B — – – –	/\\\	O — — —	///
C — – — –	/\/\	P – — — –	\/\
D — – –	/\\	Q — — – —	//\/
E –	\	R – — –	\/\
F – – — –	\\/\	S – – –	\\\
G — — –	//\	T —	/
H – – – –	\\\\	U – – —	\\/
I – –	\\	V – – – —	\\\/
J – — — —	\///	W – — —	\//
K — – —	/\/	X — – – —	/\\/
L – — – –	\/\\	Y — – — —	/\//
M — —	//	Z — — – –	//\\

are three binding screws, one connected with the wire P, which comes from the positive pole of the battery; the second connected with L, the line wire; and the third with the wire A, which passes to the indicator; for of course two places in communication are each provided with an indicator and communicator.

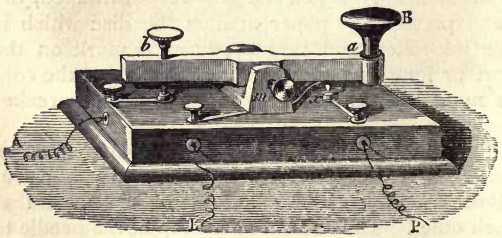


Fig. 879.

These details known, there are two cases to be considered. 1. The key arranged so as to receive a message from a distant station; the end *b* is then down, as represented in the figure, so that the current which arrives by the line wire L, and ascends in the metal piece *m*, descends in the wire A, which leads it to the indicator of the station at which the apparatus is placed. 2. A message is to be transmitted; in this case the key B is pressed so that the lever comes in contact with the button *x*. The current of the local battery, which comes by the wire P, ascending then in the lever, descends by *m* and joins the wire L, which conducts it to the station to which the despatch is addressed. According to the length of time

during which B is pressed, a dot or a line is produced in the receiver to which the current proceeds.

Relay. In describing the receiver we have assumed that the current of the line coming by the wire C (fig. 877) entered directly into the electromagnet, and worked the armature A, producing a despatch; but when the current has traversed a distance of a few miles its strength has diminished so greatly that it cannot act upon the electromagnet with sufficient force to print a despatch. Hence it is necessary to have recourse to a relay—that is, to an auxiliary electromagnet which is still traversed by the current of the line, but which serves to introduce into the communicator the current of a *local battery* of four or five elements placed at the station, and which is only used to print the signals transmitted by the wire.

For this purpose the current entering the relay by the binding screw, L (fig. 880), passes into an electromagnet, E, whence it passes into the earth by the binding screw T. Now, each time that the current of the line passes into the relay, the electromagnet attracts an armature, A, fixed at the bottom of a vertical lever, *p*, which oscillates about a horizontal axis.

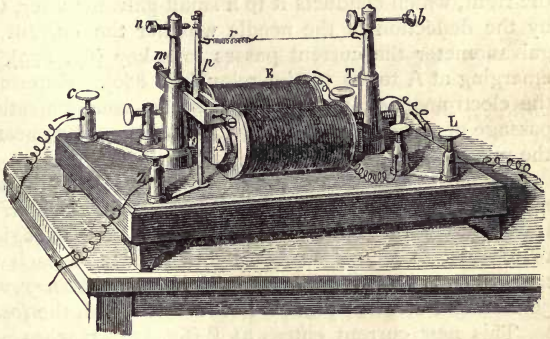


Fig. 880.

At each oscillation the top of the lever *p* strikes against a button *n*, and at this moment the current of the local battery, which enters by the binding screw *c*, ascends the column *m*, passes into the lever *p*, descends by the rod *o*, which transmits it to the screw Z: thence it enters the electromagnet of the indicator, whence it emerges by the wire Z, to return to the local battery from which it started. Then, when the current of the line is open, the electromagnet of the relay does not act, and the lever *p*, drawn by a spring *r*, leaves the button *n*, as shown in the drawing, and the local current no longer passes. Thus the relay transmits to the indicator exactly the same phases of passage and intermittence as those effected by the manipulator in the station which sends the despatch.

With a general battery of 25 Daniell's elements the current is usually strong enough at upwards of 90 miles from its starting-point to work a relay. For a longer distance a new current must be taken, as will be seen in the paragraph on the change of current (*vide infra*).

Working of the three apparatus. The three principal pieces of Morse's apparatus being thus known, the following is the actual path of the current.

The current of the line coming by the wire L (fig. 877) passes at first to the piece T intended to serve as lightning-conductor, when, from the influence of atmospheric electricity in time of storm, the conducting wires

become so highly charged with electricity as to give dangerous sparks. This apparatus consists of two copper discs, *d* and *f*, provided with teeth on the sides opposite each other, but not touching. The disc *d* is connected with the earth by a metal plate at the back of the stand which supports this lightning-conductor, while the disc *f* is in the current. The latter coming by the line L enters the lightning-conductor by the binding screw fixed at the lower part of the stand on the left ; then rises to a commutator, *n*, which conducts it to a button, *c*, whence it reaches the disc *f* by a metal plate at the back of the stand; in case a lightning discharge should pass along the wire, it would now act inductively on the disc *d*, and emerge by the points without danger to those about the apparatus. Moreover, from the disc *f*, the current passes into a very fine wire insulated on a tube, *e*. As the wire is melted when the discharge is too strong, the electricity does not pass into the apparatus, which still further removes any danger.

Lastly, the current proceeds from the foot of the support to a screw on the right, which conducts it to a small galvanometer, G, serving to indicate by the deflection of the needle whether the current passes. From this galvanometer the current passes to a key (fig. 879), which it enters at L, emerging at A to go to the relay (fig. 880). Entering this at L, it works the electromagnet, and establishes the communication necessary for the passage of the current of the local battery, as has been said in speaking of the relay.

Change of current. To complete this description of Morse's apparatus it must be observed that in general the current which arrives at L, after having traversed several miles, has not sufficient force to register the despatch, nor to proceed to a new distant point. Hence in each telegraphic station a new current must be taken, that of the *postal battery*, which consists of 20 to 30 Daniell's elements, and is not identical with the *local battery*.

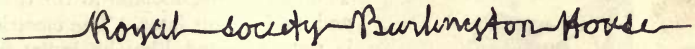
This new current enters at P (fig. 877), reaches a binding screw which conducts it to the column H, and thence only proceeds further when the armature A sinks. A small contact placed under the lever then touches the button *v*; the current proceeds from the column H to the metallic mass BD, whence by a binding screw and a wire, not represented in the figure, it reaches, lastly, the wire of the line, which sends it to the following post, and so on from one point to another.

890. **Cowper's writing telegraph.**—This very remarkable invention is a true telegraph, in that it faithfully reproduces at a distance an exact facsimile of a person's handwriting. The following is a general idea of the principle of the instrument.

Two line wires are required, which are severally connected at the receiving station with two galvanometers, whose coils are at right angles to each other. At the sending station is a vertical pencil with two light rods, jointed to it at right angles to each other. One of these contact rods guides a contact piece which is connected by a wire with one pole of a battery, the other pole of which is to earth. This contact piece slides over the edges of a series of contact plates insulated from each other, between each of which a special resistance is interposed, and the last of the contact plates is connected with one line wire. The other contact piece slides over a second series of such plates connected with the other line wire.

Let us consider one contact alone ; as it moves over the contact plates in one direction or the other, it brings less or more resistance into the circuit, and thereby alters the strength of the current. The effect of this is that the needle of the corresponding galvanometer is less or more deflected. Now the end of this needle is connected by a light thread with a receiving pen, which is a capillary tube full of ink. An oscillation of the needle would produce an up-and-down motion of the pen, and if simultaneously a band of paper passed under the pen, being moved regularly by clockwork, there would be produced on it a series of up-and-down strokes. A corresponding effect would be produced by the action of the needle of the other galvanometer, except that its strokes would be backwards and forwards instead of up and down.

Now the action of the writing pen is that it varies simultaneously the strengths of the two currents, and they produce a motion of the receiving pen which is compounded of the two movements described above, and which is an exact reproduction, on a smaller scale, of the original motion. The following line is a facsimile.



Both the paper written in pencil at the sending station and that written in ink at the receiving station move along as the writing proceeds, and the messages have only to be cut off from time to time.

Experiments made with this instrument show that it will write through resistances equal to 36 miles.

891. **Induction in telegraph cables.**—In the earliest experiments on the use of insulated subterranean wires for telegraphic communication it was found that difficulties occurred in their use which were not experienced with overhead wires. This did not arise from defective insulation, for the better the insulation the greater the difficulty. It was suspected by Siemens and others that the retardation was due to statical induction, taking place between the inner wire through the insulator and the external moisture ; and Faraday proved that this was the case by the following experiments among others. A length of about 100 miles of gutta-percha-covered copper wire was immersed in water, the ends being led into the observing-room. When the pole of a battery containing a large number of cells was momentarily connected with one end of the wire, the other end being insulated, and a person simultaneously touched the wire and the earth contact, he obtained a violent shock.

When the wire, after being in momentary contact with the battery, was placed in connection with a galvanometer, a considerable deflection was observed ; there was a feebler one 3 or 4 minutes after, and even as long as 20 or 30 minutes afterwards.

When the insulated galvanometer was permanently connected with one end of the wire, and then the free end of the galvanometer wire joined to the pole of the battery, a rush of electricity through the galvanometer into the wire was perceived. This speedily diminished and the needle ultimately came to rest. When the galvanometer was detached from the battery and put to earth, the electricity flowed as rapidly out of the wire, and the needle was momentarily deflected in the opposite direction.

These phenomena are not difficult to explain. The wire with its thin insulating coating of gutta-percha becomes statically charged with electricity from the battery. The coating of gutta-percha through which the inductive action takes place is only $\frac{1}{12}$ of an inch in thickness, and the extent of the coatings is very great. The surface of the copper wire amounts to 8,300 square feet and that of the outside coating is four times as much. The potential can only be as great as that of the battery, but from the enormous surface the capacity, and therefore the quantity, is very great. Thus the wires, after being detached from the battery, showed all the actions of a powerful electric battery. These effects take place, but to a less extent, with wires in air; the external coating is here the earth, which is so distant that induction and charge are very small, more especially in the long lines.

Hence the difficulty in submarine telegraphy. The electricity which enters the insulating wire must first be used in charging the large Leyden jar which it constitutes, and only after this has happened can the current reach the distant end of the circuit. The current begins later at the distant end, and ceases later. The electricity is not projected like the bullet from a gun, but rather like a quantity of water flowing from a large reservoir into a canal in connection with large basins which it has to fill as well as itself. If the electrical currents follow too rapidly, an uninterrupted current will appear at the other end, which indicates small differences in strength, but not with sufficient clearness differences in duration or direction. Hence in submarine wires the signals must be slower than in air wires to obtain clear indications. By the use of alternating currents—that is, of currents which are alternately positive and negative—these disturbing influences may be materially lessened, and communication be accelerated and made more certain, but they can never be entirely obviated.

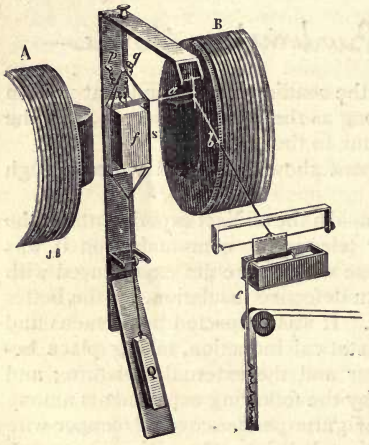


Fig. 881.

In the Atlantic Cable, instruments on the principle of Thomson's reflecting galvanometer (822) are used for the reception of signals; the motions of the spot of light to the right and left forming the basis of the alphabet.

892. **Syphon recorder.**—Lord Kelvin has invented an extremely ingenious instrument called the *syphon recorder*, by which the very feeble signals transmitted through long lengths of submarine cables are observed and also recorded.

A light rectangular coil of iron *s* (fig. 881), connected with the line wire by the screws *p* and *q*, hangs by a bipolar suspension between the two poles of a powerful electromagnet AB, so that its plane is in the right line joining the poles. The space inside the coil is occupied by a mass of soft iron *f*, by which the strength of the fluid is greatly increased. When a current

is passed this coil thereby becomes a magnet, and is deflected either to the right or the left according to the direction of the current; its movements are almost deadbeat (821), as the damping is considerable.

A very light capillary tube *c* dips with its short end in a reservoir of ink, while the other end is in front of a paper ribbon which is moved along at a uniform rate like the ribbon in a Morse's recorder. In order to get rid of friction against the paper, this ink is electrified, and spurts out in a continuous series of fine drops against the paper, marking on it a straight line so long as no current passes in the coil. This syphon is, however, connected by a system of silk threads with the coil, and according as this is deflected

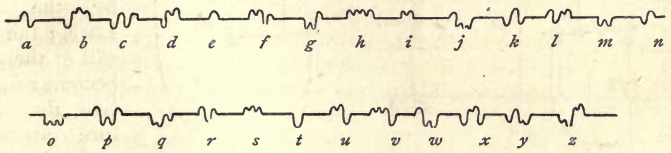


Fig. 882.

either to the right or the left the end of the syphon is deflected too, and traces a wavy line (fig. 882) on the paper, which represents deflections right or left of the central line, that are, in short, the Morse signals (889).

The electrification of the ink is effected by a small electrostatic induction machine; this is worked by clockwork, which at the same time pays out the paper ribbon.

893. **Duplex telegraphy.**—By this is meant a system of telegraphy by which messages may be simultaneously sent in opposite directions on one and the same wire, whereby the working capacity of a line is practically doubled.

Several plans have been devised for accomplishing this very important improvement; no more can here be attempted than to give a general account of the principle of the method in one or two cases.

Let *m* (fig. 883) represent the electromagnet of a Morse's instrument which is wound round with two equal coils in opposite directions; these coils are represented by the full and dotted lines, and one of them, which may be called the *line coil*, is joined to the line LL′, which connects the two stations. The other coil, that represented by the dotted line, which may be called the *equating coil*, is in connection with the earth at E by means of an adjustable resistance, or *artificial line*, R. By this means the resistance of the branch *a*RE may be made equal to that of the branch *a*LL′*a*. The battery *b* has one pole to earth at E, and the other pole, by means of a make-and-break key, *c*, can be connected at *a*, where the two oppositely wound coils bifurcate. The back contact of the key is also connected with earth.

The station at B is arranged in a similar manner, as is represented by corresponding letters with affixes.

Now when B depresses his key and sends a current into the line, inasmuch as the electromagnet of his instrument is wound with equal coils in opposite directions, the armature is not attracted, for the core is not magnetised because

the currents in the two coils counteract one another. Thus, although a current passes from B, there is no indication of it in his own instrument—a condition essential in all systems of duplex telegraphy.

But with regard to the effect on A, there are two cases, according as he is or is not sending a message at the same time. If A's key is not down, then the current will circulate round the core of the electromagnet and will reach the earth by the path L*ac*E ; the core will therefore become magnetised, the armature attracted, and a signal produced in the ordinary way.

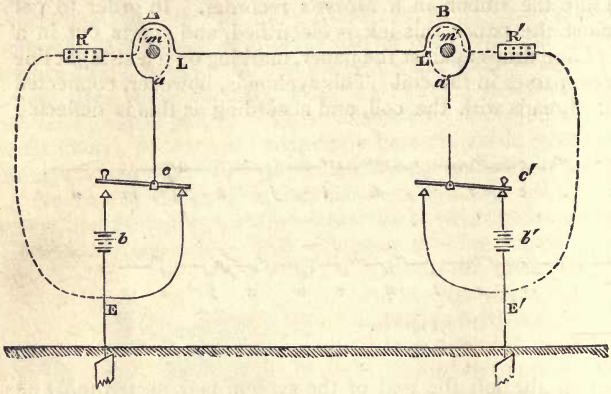


Fig. 883.

If, however, at the moment at which B has his key down, A also depresses his, then it will be seen that, as currents are sent in opposite directions from both A and B, they neutralise one another, and no current passes in the line *a*LL′*a*′ : it is, as it were, blocked. But though no current passes in the line coil, a current does pass at each station to earth, through the equating coil, which, being no longer counterbalanced by any opposite current in the line coil, magnetises the core of the electromagnet, which thus attracts the armature and produces a signal.

We have here supposed that A and B both send, for instance, the same currents to line : the final effect is not different if they send opposite currents at the same time. For then, as they neutralise each other in the line LL′, the effect is the same as if the resistance of the line were diminished. More electricity flows to line from each station through the line coil being no longer balanced by the equating coil ; the current of the line coil preponderates and then works the electromagnet.

Hence, in both these cases, each station, so to speak, produces the signal which the other one wishes to send.

Another method is based on the principle of Wheatstone's bridge (955). At each station is a battery P (fig. 884), one pole of which is to earth while the other is connected with the key M. The wire from M bifurcates at A into the two branches B and C, between which is connected the galvanometer or the receiving instrument. The branch AB goes to line and AC to earth. There are exactly corresponding parts at the other station. Now, from the principle of the bridge, the resistances AB and AC may be adjusted in such a manner that the potentials at the points B and C are equal when the key is depressed and the current sent ; accordingly, no current passes in the bridge, and the galvanometer is at rest ; but the current from A passing

to line bifurcates at B′, traversing the galvanometer and going to earth; hence a signal is received at that station.

Other methods of duplex telegraphy are based on the principle of leakage; but for these, as well as for quadruplex telegraphy, special manuals must be consulted.

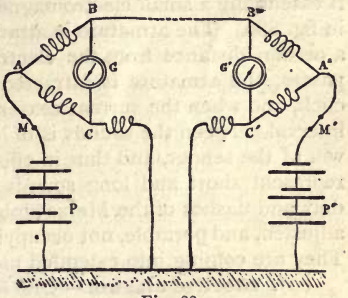


Fig. 884.

894. **Earth currents.**—In long telegraph circuits more or less powerful currents are produced, even when the battery is not at work. This arises from a difference of potential being established in the earth at the two places between which the communication is established. These currents are sometimes in one direction and sometimes in another, and are at times so powerful and irregular as quite to interfere with the working of the lines. Lines running NE and SW are most frequently affected; lines running NW and SE are less so, and the currents are far weaker. Their strength often amounts to as much as 40 millamperes (964), which is a stronger current than is required for working ordinary telegraph instruments.

These currents do not seem to be due to atmospheric electricity, for they cease if a wire be disconnected at one of its ends, and they appear in underground wires.

According to Wild, they are the prime cause of magnetic storms, but not of the periodical variations in the magnetic elements.

895. **Bain's electrochemical telegraph.**—If a strip of paper be soaked in a solution of ferrocyanide of potassium and be placed on a metal surface connected with the negative pole of a battery, on touching the paper with a steel pointer connected with the positive pole, a blue mark due to the formation of some Prussian blue will be formed about the iron, so long as the current passes. The first telegraph based on this principle was invented by Bain. The alphabet is the same as Morse's, but the despatch is first composed at the departure station on a long strip of ordinary paper. It is perforated successively by small round and elongated holes, which correspond respectively to the dots and marks. This strip of paper is interposed between a small metal wheel and a metal spring, both forming part of the circuit. The wheel, in turning, carries with it the paper strip, all parts of which pass successively between the wheel and the plate. If the strip were not perforated, it would, not being a conductor, constantly offer a resistance to the passage of the current; but, in consequence of the holes, every time one of them passes, there is contact between the wheel and the plate. Thus the current works the relay of the station to which it is sent, and traces in blue, on a paper disc, impregnated with ferrocyanide of potassium, the same series of points and marks as those on the perforated paper.

896. **The sounder.**—The sound produced when the armature of the electromagnet in a Morse's instrument is attracted by the passage of the current is so distinct and clear that many telegraph operators have been in the habit of reading the messages by the sounds thus produced, and at most of

checking their reading by comparison with the signs produced on the paper.

Based on this fact a form of instrument invented in America has come into use for the purpose of reading by sound. The *sounder*, as it is called is essentially a small electromagnet on an ebonite base, resembling the relay in fig. 880. The armature is attached to one end of a lever, and is kept at a certain distance from the electromagnet by a spring. When the current passes, the armature is attracted against the electromagnet with a sharp click, and when the current ceases it is withdrawn by the spring. Hence the interval between the sounds is of longer or shorter duration according to the will of the sender, and thus in effect a series of short or long intervals which represent short and long sounds can be produced which correspond to the dots and dashes of the Morse alphabet. Such instruments are simple, easily adjusted, and portable, not occupying more space than an ordinary field-glass. They are coming into extended use, especially for military telegraph work.

897. **Electric alarum.**—One form of these instruments is represented in fig. 885. On a wooden board arranged vertically is fixed an electromagnet, E ; the line wire is connected with the binding screw, *m*, with which is also connected one end of the wire of the electromagnet ; the other end is connected with a spring, *c*, to which is attached the armature, *a* ; this again is pressed against by a spring, C, which in turn is connected with the binding screw *n*, from which the wire leads to earth.

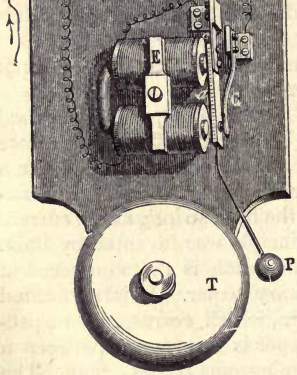


Fig. 885.

Whenever the current passes, the armature *a* is attracted, carrying with it a hammer, P, which strikes against the bell T and makes it sound. The moment this takes place, contact is broken between the armature *a* and the spring C, and the current being stopped the electromagnet does not act ; the spring *c*, however, in virtue of its elasticity, brings the armature in contact with the spring C, the current again passes, and so on as long as the current continues to pass.

898. **Electrical clocks.** — Electrical clocks are clockwork machines, in which an electromagnet is both the motor and the regulator, by means of an electric current regularly interrupted, in a manner resembling that described in the preceding paragraph. Fig. 886 represents the face of such a clock, and fig. 887 the mechanism which works the needles.

An electromagnet, B, attracts an armature of soft iron, P, movable on a pivot, *a*. The armature P transmits its oscillating motion to a lever, *s*, which by means of a ratchet, *n*, turns the wheel A. This, by the pinion, D, turns the wheel C, which by a series of wheels and pinions moves the hands. The small one marks the hours, the large one the minutes ; but as the latter does not move regularly, but by sudden starts from second to second, it follows that it may also be used to indicate the seconds.

It is obvious that the regularity of the motion of the hands depends on the regularity of the oscillations of the piece P. For this purpose, the oscillations of the current, before passing into the electromagnet B, are regulated by a standard clock, which itself has been previously regulated by a seconds pendulum. At each oscillation of the pendulum there is an arrangement by which it opens and closes the current, and thus the armature P beats seconds exactly.

To illustrate the use of these clocks, suppose that on the railway from London to Birmingham each station has an electric clock, and that from the London station a conducting wire connects all the clocks on the line to Birmingham. When the current passes in this wire all the clocks will simultaneously indicate the same hour, the same minute, and the same second; for electricity takes an inappreciable time to go from London to Birmingham.

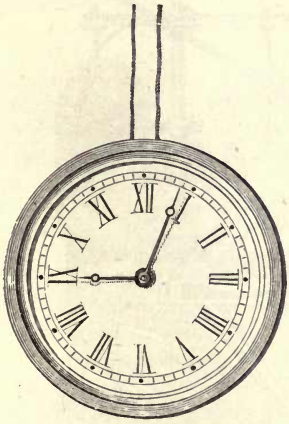


Fig. 886.

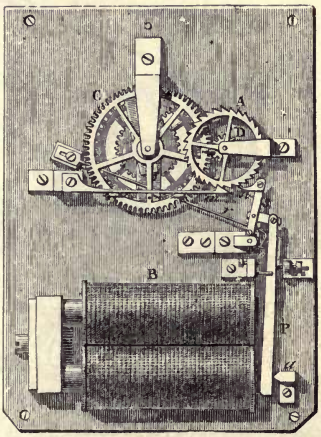


Fig. 887.

899. **Electromagnetic motors.**—Numerous attempts have been made to apply electromagnetism as a motive power in machinery. Fig. 888 represents an engine of this kind constructed by Froment. It consists of four powerful electromagnets, ABCD, fixed on an iron frame, X. Between these electromagnets is a system of two iron wheels movable on the same horizontal axis, with eight soft iron armatures, M, on their circumference.

The current arrives at K, ascends in the wire E, and reaches a metallic arc, O, which serves to pass the current successively into each electromagnet, so that the attractions exerted on the armatures M shall always be in the same direction. Now this can only be the case provided the current is broken in each electromagnet just when an armature comes in front of the axis of the bobbin. To produce this interruption the arc O has three branches *e*, each terminating with a steel spring, to which a small sheave is attached. Two of these establish the communication respectively with one electromagnet, and the third with two. On a central wheel, *a*, there are cogs, on which the sheaves alternately rest. Whenever one of them rests on a cog,

the current passes into the corresponding electromagnet, but ceases to pass when there is no longer contact. On emerging from the electromagnets the current passes to the negative pole of the battery by the wire H.

In this manner, the armatures M being successively attracted by the four electromagnets, the system of wheels which carries them acquires a rapid rotatory motion, which by the wheel P and an endless band is transmitted to a sheave, Q, which sends it finally to any machine, a grinding-mill for example.

In his workshops Froment had an electromotive engine of one-horse power. But, though an interesting application of the transformation of

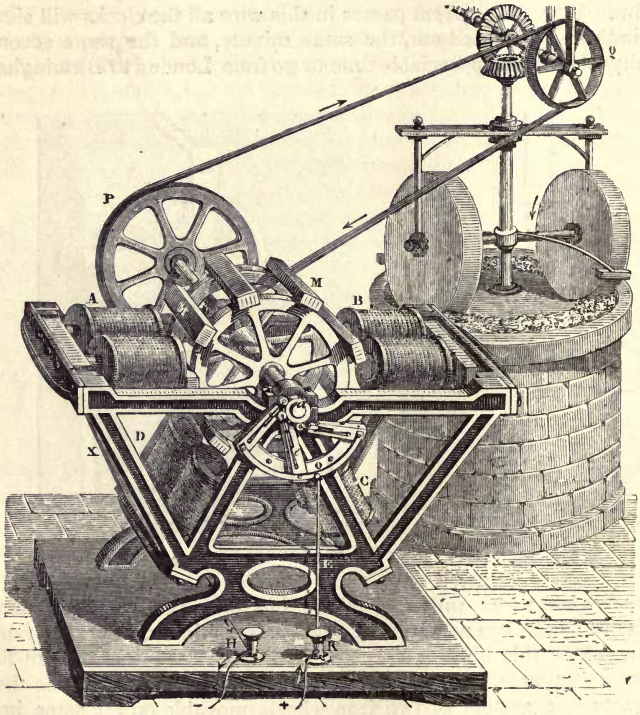


Fig. 888.

energy, these machines will never be practically applied on the large scale in manufactures, for the expense of the acids and the zinc which they use very far exceeds that of the coal in steam-engines of the same force.

Thus a machine devised by Kravogl produces about 17 per cent. of the useful effect due to the chemical combination of the zinc with the acid in the battery, and therefore in utilising this force they are about equal to the best steam-engines. But a pound of coal yields 7,200 thermal units, and a pound of zinc only 1,200 (484); and as zinc is ten times as dear as coal, motors worked by electricity, independently of any question as to the cost of con-

struction, or of the cost of the acids, are at least sixty times as dear to work as steam-engines.

Hence, considering the great cost of producing electricity by the battery, the most advantageous employment has hitherto been in the rapid transmission of a small power to great distances, as in the electric telegraph, rather than in the relatively slow movement of large masses. But with the great development in the construction of magneto-electrical machines the cost of electrical currents is greatly reduced, and along with this the wider distribution of such powerful currents for electric lighting, makes it probable that electromotors will be used for a number of the smaller branches of manufacturing industry.

CHAPTER VI

VOLTAIC INDUCTION

900. **Induction by currents.**—We have already seen (744) that by *induction* is meant the action which electrified bodies exert at a distance on bodies in the natural state. Hitherto we have only had to deal with electrostatical induction; we shall now see that dynamical electricity produces analogous effects.

Faraday discovered this class of phenomena in 1832, and he gave the name of *currents of induction* or *induced currents* to instantaneous currents developed in conductors under the influence of metallic conductors traversed by electric currents, or by the influence of powerful magnets, or even by the magnetic action of the earth; and the currents which give rise to them he called *inducing currents.*

The inductive action of a current at the moment of opening or closing may be shown by means of a bobbin with two wires. This consists (fig. 889)

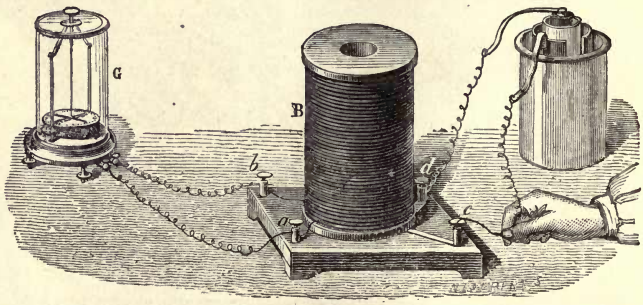


Fig. 889.

of a cylinder of wood or of cardboard, on which a quantity of silk-covered No. 16 copper wire is coiled; on this is coiled a considerably greater length of fine copper wire, about No. 35, also insulated by being covered with silk. This latter coil, which is called the *secondary coil*, is connected by its ends with two binding screws, *a*, *b*, from which wires pass to a galvanometer, while the thicker wire, the *primary* coil, is connected by its extremities with two binding screws, *c* and *d*. One of these, *d*, being connected with one pole of a battery, when a wire from the other pole is connected with *c*, the current passes in the primary coil, and in this alone. The following phenomena are then observed :—

i. At the moment at which the thick wire is traversed by the current, the galvanometer, by the deflection of the needle, indicates the existence in the *secondary coil* of a current *inverse* to that in the primary coil, that is, in the contrary direction; this is only instantaneous, for the needle immediately reverts to zero, and remains so as long as the inducing current passes through *cd*.

ii. At the moment at which the current is opened—that is, when the wire *cd* ceases to be traversed by a current—there is again produced in the wire *ab* an induced current instantaneous like the first, but *direct*, that is, in the same direction as the inducing current.

901. **Production of induced currents by continuous ones.**—Induced currents are also produced when a primary coil traversed by a current is approached to or removed from a secondary one; this may be shown by the

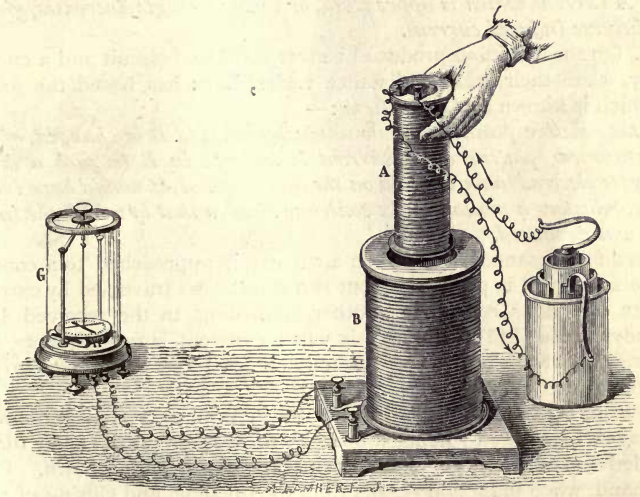


Fig. 890.

following apparatus (fig. 890), in which B is a hollow coil consisting of a great length of fine wire, and A a coil consisting of a shorter and thicker wire, and of such dimensions that it can be placed in the secondary coil. The coil A being traversed by a current, if it is suddenly placed in the coil B, a galvanometer connected with the latter indicates by the direction of its deflection the existence in it of an *inverse* current; this is only instantaneous; the needle rapidly returns to zero, and remains so as long as the small bobbin is in the large one. If it is rapidly withdrawn, the galvanometer shows that the wire is traversed by a direct current. If, instead of rapidly introducing or replacing the primary coil, this is done slowly, the galvanometer only indicates a weak current, which is the feebler the slower the motion.

If, instead of varying the distance of the inducing current, its intensity be varied—that is, either increased by bringing additional battery power into

the circuit, or diminished by increasing the resistance—an induced current is produced in the secondary wire, which is inverse if the intensity of the inducing current increases, and direct if it diminishes.

902. **Conditions of induction. Lenz's law.**—From the experiments which have been described in the previous paragraphs the following principles may be deduced :—

I. The distance remaining the same, *a continuous and constant current does not induce any current in an adjacent conductor.*

II. *A current, at the moment of being closed, produces in an adjacent conductor an inverse current.*

III. *A current, at the moment it ceases, produces a direct current.*

IV. *A current which is removed, or whose strength diminishes, gives rise to a direct induced current.*

V. *A current which is approached, or whose strength increases, gives rise to an inverse induced current.*

VI. On the induction produced between a closed circuit and a current in activity, when their relative distance varies, Lenz has based the following law, which is known as *Lenz's Law* :—

If the relative position of two conductors A and B be changed, of which A is traversed by a current, a current is induced in B in such a direction that, by its electrodynamic action on the current in A, it would have imparted to the conductors a motion of the contrary kind to that by which the inducing action was produced.

Thus, for instance, in V, when a current is approached to a conductor, an inverse current is produced ; but two conductors traversed by currents in opposite directions *repel* one another, according to the received laws of electrodynamics (858). Conversely when a current is *moved away from* a conductor, a current of the same direction is produced ; now two currents in the same direction *attract* one another.

On bringing the inducing wire near the induced as well as in removing it away, work is required ; hence a quantity of heat proportional to the work expended must result, as Edlund's investigations have shown. On the other hand, when induction results from the opening and closing of the circuit (II. and III.) no work is lost, but the inducing current loses as much heat as is produced in the induced circuit.

903. **Inductive action of the Leyden discharge.**—Fig. 891 represents an apparatus devised by Matteucci, which is very well adapted for showing the development of induced currents produced either by the discharge of a Leyden jar or by the passage of a voltaic current.

It consists of two glass plates about 12 inches in diameter, fixed vertically on the two supports A and B. These supports are on movable feet, and can either be approached or removed at will. On the anterior face of the plate A are coiled about 30 yards of copper wire C, a millimetre in diameter. The two ends of this wire pass through the plate, one in the centre, the other near the edge, terminating in two binding screws, like those represented in *m* and *n* on the plate B. To these binding screws are attached two copper wires, *c* and *d*, through which the inducing current is passed.

On the face of the plate B, which is towards A, is enrolled a spiral of finer copper wire than the wire C. Its extremities terminate in the binding

screws *m* and *n*, on which are fixed two wires, *h* and *i*, intended to transmit the induced current. The two wires on the plates are not only covered with silk, but each circuit is insulated from the next one by a thick layer of shellac varnish.

In order to show the production of the induced current by the discharge of a Leyden jar, one end of the wire C is connected with the outer coating, and the other end with the knob of the Leyden jar, as shown in the figure. When the spark passes, the electricity traversing the wire C acts by induction on the wire on the plate B, and produces an instantaneous current in this wire. A person holding two copper handles connected with the wire *i* and *h* receives a shock, the intensity of which is greater in proportion as the plates A and B are nearer.

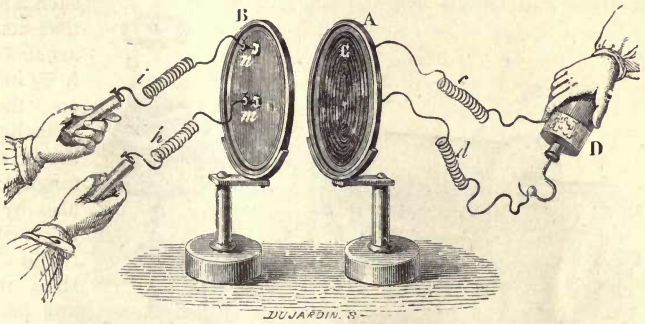


Fig. 891.

The experiment may also be made by simply twisting together two lengths of a few feet of gutta-percha-covered copper wire. The ends of one length being held in the hand, an electric discharge is passed through the other length.

The above apparatus can also be used to show the production of induced currents by the influence of voltaic currents. For this purpose the current of a battery is passed through the inducing wire C, while the ends of the other wire, *h* and *i*, are connected with a galvanometer. At the moment at which the current commences or finishes, or when the distance of the two conductors is varied, the same phenomena are observed as in the case of the apparatus represented in fig. 890.

904. **Induction by magnets.**—It has been seen that the influence of a current magnetises a steel bar ; in like manner a magnet can produce induced currents in metal circuits. Faraday showed this by means of a coil with a single wire of 200 to 300 yards in length. The two ends of the wire being connected with a galvanometer, as shown in fig. 892, a strongly magnetised bar is suddenly inserted in the bobbin, and the following phenomena are observed :—

i. At the moment at which the magnet is introduced, the galvanometer indicates in the wire the existence of a current, the direction of which is opposed to that which circulates round the magnet, considering the latter as a solenoid on Ampère's theory (879).

ii. When the magnet is withdrawn, the needle of the galvanometer, which has returned to zero, indicates the existence of a direct current.

The inductive action of magnets may also be illustrated by the following experiment : a bar of soft iron is placed in the above bobbin and a strong magnet suddenly brought in contact with it ; the needle of the galvanometer is deflected, but returns to zero when the magnet is stationary, and is deflected in the opposite direction when it is removed. The induction is here produced by the magnetisation of the soft iron bar in the interior of the bobbin under the influence of the magnet.

The same inductive effects are produced in the wires of an electromagnet, if a strong magnet be made to rotate rapidly in front of the extremities of the wire in such a manner that its poles act successively by influence on the two branches of the electromagnet ; or also by forming two coils round a horseshoe magnet, and passing a plate of soft iron rapidly in front of the poles of the magnet ; the soft iron becoming magnetic reacts by influence on the magnet, and induced currents are produced in the wire alternately in different directions.

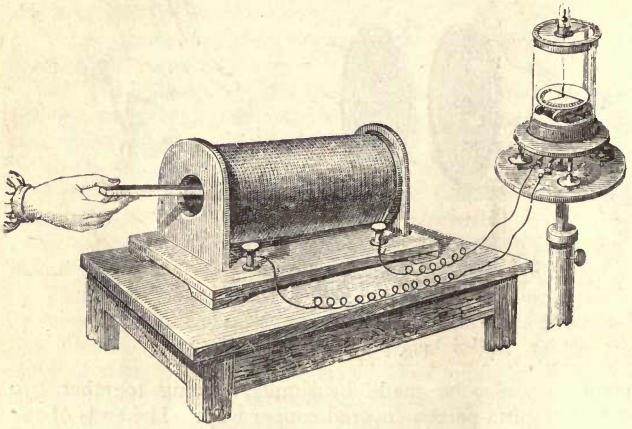


Fig. 892.

The inductive action of magnets is a confirmation of Ampère's theory of magnetism. For as, on this theory, magnets are solenoids, all the experiments which have been mentioned may be explained by the inductive action of currents which traverse the surface of magnets ; the induction of magnets is, in short, an induction of currents. And it is a useful exercise to see how on this view the inductive action of magnets falls under Lenz's law (902).

905. **Inductive action of magnets on bodies in motion.**—Arago was the first to observe, in 1824, that the number of oscillations which a magnetised needle makes in a given time, under the influence of the earth's magnetism, is very much lessened by the proximity of certain metallic masses, and especially of copper, which may reduce the number in a given time from 300 to 4. This observation led Arago in 1825 to the discovery of an equally unexpected fact—that of the rotative action which a plate of copper in motion exercises on a magnet.

This phenomenon may be shown by means of the apparatus represented

in fig. 893. It consists of a copper disc, M, movable about a vertical axis. On this axis is a sheave, B, round which is coiled an endless cord, passing also round the sheave A. By turning this with the hand, the disc M may be rotated with great rapidity. Above the disc is a glass plate, on which is a small pivot supporting a magnetic needle, *ab*. If the disc be now rotated with a slow and uniform velocity, the needle is deflected in the direction of the motion, and stops at an angle of from 20° to 30° with the direction of the magnetic meridian, according to the velocity of the rotation of the disc. But if this velocity increases, the needle is ultimately deflected more than 90° ; it is then carried along, describes an entire revolution, and follows the motion of the disc until this stops.

Babbage and Herschel modified Arago's experiment by causing a horse-shoe magnet placed vertically to rotate below a copper disc suspended on

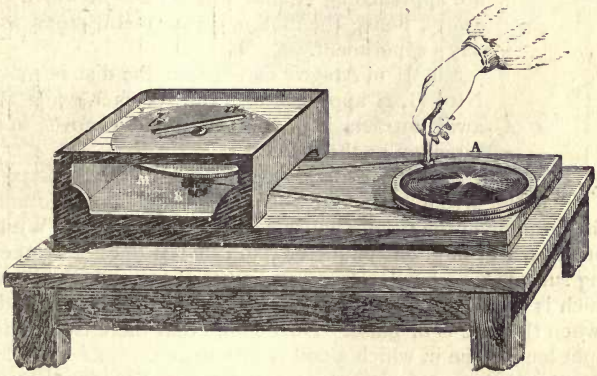


Fig. 893.

silk threads without torsion ; the disc rotated in the same direction as the magnets. The effect decreases with the distance of the disc, and varies with its nature. The maximum effect is produced with metals : with wood, glass, water, &c., it disappears. Babbage and Herschel found that, representing this action on copper at 100, the action on other metals is as follows : zinc 95, tin 46, lead 25, antimony 9, bismuth 2. Lastly, the effect is enfeebled if there are non-conducting breaks in the disc, especially in the direction of the radii ; but it is the same if these breaks are soldered with any metal.

Faraday made an experiment the reverse of Arago's first observation ; since the presence of a metal at rest stops the oscillations of a magnetic needle, the neighbourhood of a magnet at rest ought to stop the motion of a rotating mass of metal. Faraday suspended a cube of copper to a twisted thread, which was placed between the poles of a powerful electromagnet. When the thread was left to itself, it began to spin round with great velocity, but stopped the moment a powerful current was passed through the electromagnet (fig. 962).

Faraday was the first to give an explanation of all these phenomena of magnetism by rotation. They depend on the circumstance that a magnet or a solenoid can induce currents in a solid mass of metal. In the above case

the magnet induces currents in the disc when the latter is rotated ; and conversely when the magnet is rotated while the disc is primarily at rest. Now these induced currents, by their electrodynamic action, tend to destroy the motion which gave rise to them ; they are simple illustrations of Lenz's law ; they act in the same way as friction would do.

i. For instance, let AB (fig. 894) be a needle oscillating over a copper disc, and suppose that in one of its oscillations it goes in the direction of the arrows from N to M. In approaching the point M, for instance, it develops there a current in the opposite direction, and which therefore repels it ; in moving away from N it produces currents which are of the same kind, and which therefore attract, and both these actions concur in bringing it to rest.

ii. Suppose the metallic mass turns from N towards M, and that the magnet is fixed ; the magnet will repel by induction points such as N which are approaching A, and will attract M which is moving away ; hence the motion of the metal stops, as in Faraday's experiment.

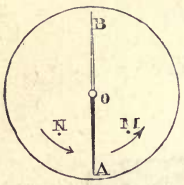


Fig. 894.

iii. If in Arago's experiment the disc is moving from N to M, N approaches A and repels it, while M, moving away, attracts it ; hence the needle moves in the same direction as the disc.

If this explanation is true, all circumstances which favour induction will increase the dynamic action ; and those which diminish the former will also lessen the latter. We know that induction is greater in good conductors, and that it does not take place in insulating substances ; but we have seen that the needle is moved with a force which is less, the less the conducting power of the disc, and it is not moved when the disc is of glass. Dove found that there is no induction on a tube split lengthwise in which a coil is introduced.

906. **Induction by the action of the earth.**—Faraday discovered that terrestrial magnetism can develop induced currents in metallic bodies in motion, acting like a powerful magnet placed in the interior of the earth in the direction of the dipping-needle, or, according to the theory of Ampère, like a series of electrical currents directed from east to west parallel to the magnetic equator. He first proved this by placing a long helix of copper wire covered with silk (such as A, fig. 890) in the plane of the magnetic meridian parallel to the dipping-needle ; by turning this helix 180° about an axis perpendicular to its length in its middle, he observed that at each turn a galvanometer connected with the two ends of the helix was deflected. The apparatus depicted in fig. 895, and known as *Delezenne's circle*, serves well for showing the currents produced by the inductive action of the earth. It consists of a wooden ring, RS, about two feet in diameter, fixed to an axis, *oa*, about which it can be turned by means of a handle, M. The axis *oa* is itself fixed in a frame PQ, movable about a horizontal axis. By pointers fixed to these two axes the inclination towards the horizon of the frame PQ, and therefore of the axis *oa*, is indicated on a dial, *b*, while a second dial, *c*, gives the angular displacement of the ring. This ring has a groove in which is coiled a great length of insulated copper wire. The two ends of the wire terminate in a *commutator* analogous to that in Clarke's apparatus (912), the object of which is to pass the current always in the same sense, although its

direction SR, changes at each half-turn of the ring. On each of the rings of the commutator are two brass plates, which transmit the current to two wires in contact with the galvanometer. Suppose that the ends of the wire on the coil are directly connected with wires leading to a galvanometer at some distance, and the apparatus so placed that its axis of rotation *oa* is at right angles to the magnetic meridian, and the plane of the ring, RS, at right angles to the line of dip. If, now, the frame be quickly turned through 180°, the needle will be momentarily deflected, to the right for instance; if, while the needle on its return is just passing its position of rest, the frame is rapidly turned to its original position, it will be deflected to the left to a greater angle than at first, for the needle is already in motion; by repeating the operation, that is, reversing the swing when the needle is passing its position of rest, the deflections will increase to a maximum, which is a measure of the earth's magnetism. This method of amplifying an originally small motion is known as *the method of multiplication.*

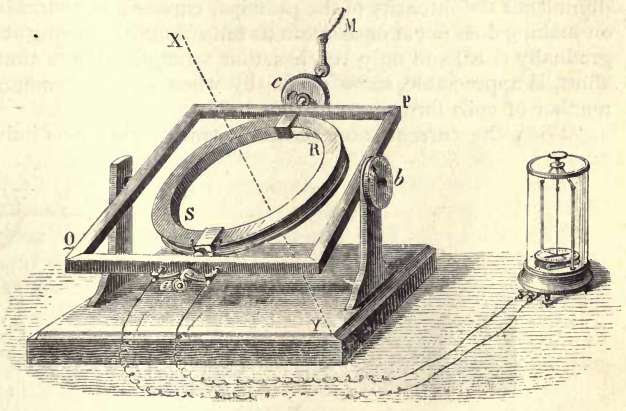


Fig 895.

If the axis of rotation, *oa*, is vertical and the ring is rotated as above described, only the horizontal component of the earth's magnetism can act, and the angular deflection is then a measure of the horizontal component H. Similarly, if the axis is horizontal and in the plane of the magnetic meridian, and if the rotation is made through 180° from the horizontal position, only the vertical component V acts, and is thus measured by the deflection.

Hence, from two such sets of observations we may determine the inclination in any place, for $\tan i = \frac{V}{H}$.

To experiment with the currents produced by continuous rotation the wires are connected with the commutator.

907. **Current of self-induction. Extra current.**—If a closed circuit traversed by a voltaic current be broken, a scarcely perceptible spark is obtained if the wire joining the two poles be short. Further, if the observer himself form part of the circuit by holding a pole in each hand, no shock is perceived unless the current is very strong. If, on the contrary, the wire is long, and especially if it makes a great number of turns so as to form a coil with very close folds, and still more if a soft iron bar be inserted in the coil, the spark, which is inappreciable when the current is closed, acquires a

great intensity when it is opened, and an observer who forms part of the circuit receives a shock which is the stronger the greater the number of turns.

Faraday, who discovered these phenomena, showed that they were effects of induction ; not only does a current act inductively on a neighbouring circuit, but each winding acts inductively on the next winding of its own circuit, both on making and on breaking the circuit, by a process which is called *self-induction.* On making the circuit, each winding induces in the next one a current in the opposite direction to its own—that is, an *inverse current.* This which is the *extra current on making* or the *inverse extra current* diminishes the intensity of the principal current ; the effect is that the current on making does not at once attain its full strength ; starting at zero its intensity gradually rises, and only reaches that strength after a time which, though short, is appreciable, more especially when an electromagnet with a great number of coils forms part of the circuit.

When the current is opened each winding then acts inductively on each

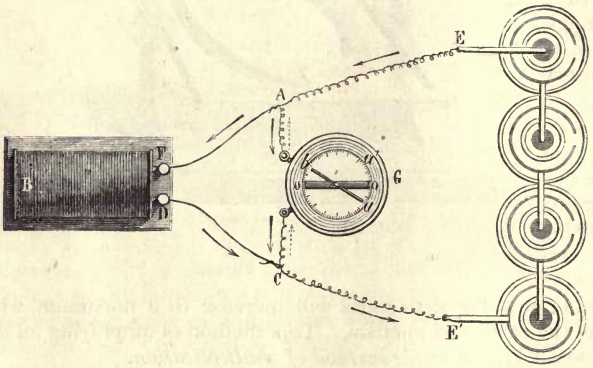


Fig. 896.

succeeding one, producing a current in the same direction as its own, and forming, as it were, a prolongation of it, but of far higher intensity.

To show the existence of this current on breaking, Faraday arranged the experiment as seen in fig. 896. Two wires from the poles E E′ of a battery are connected with two binding screws, D and F, with which are also connected the two ends of a coil B, with a long fine wire. On the path of the wires at the points A and C are two other wires, which are connected with a galvanometer, G. Hence the current from the pole E branches at A into two currents, one of which traverses the galvanometer, the other the bobbin, and both joining the negative pole E′.

The needle of the galvanometer being then deflected from G to *a′* by the current which goes from A to C, it is brought back to zero, and kept there by an obstacle which prevents it from turning in the direction G*a′*, but leaves it free in the opposite direction. On breaking contact at E, it is seen that the moment the circuit is open the needle is deflected in the direction G*a* ; showing a current contrary to that which passed during the existence of the current—that is, the current from C to A. But the battery current having

ceased, the only remaining one is the current AFBDCA ; and since in the part CA the current goes from C to A, it must traverse the entire circuit in the direction AFBDC—that is, the same as the principal current. This is the *extra current on opening*, or *direct extra current.*

908. **Direct extra current.**—To observe the direct extra current, the conductor on which its effect is to be tried may be introduced into the circuit, by being connected in any suitable manner with the binding screws A and C in the place of the galvanometer. It can thus be shown that the direct extra current gives violent shocks and bright sparks, decomposes water, melts platinum wires, and magnetises steel needles. The shock produced by the current may be tried by attaching the ends of the wire to two files, which are held in the hands. On moving the point of one file over the teeth of the other, a series of shocks is obtained, due to the alternate opening and closing of the current.

The above effects acquire greater intensity when a bar of soft iron is introduced into the bobbin, or, what is the same thing, when the current is passed through the bobbin of an electromagnet ; and still more is this the case if the core, instead of being massive, consists of a bundle of straight wires. Faraday explained this strengthening action of soft iron as follows : If inside the spiral there is an iron bar, on opening the circuit when the principal current disappears, the magnetism which it evokes in the bar disappears too ; but the disappearance of this magnetism acts like the disappearance of the electrical current, and the disappearing magnetism induces a current in the same direction as the disappearing principal current, the effect of which is thus heightened.

In the experiments just described the effects of the two extra currents accompany those of the principal current. Edlund devised an ingenious arrangement of apparatus by which the action of the principal current on the measuring instruments can be completely avoided so that only that of the extra current remains.

The plan of this experiment is represented in fig. 897, in which A is a battery the poles of which are connected with *b* and *c*. M is a differential reflecting galvanometer the exactly equal coils of which terminate in *eþ* and *fh*. Wires connect *b* with *h* and *þ*, and in like manner *c* is connected with *e* and *f*. The current from A divides at *c*, passing round the galvanometer the adjustment of the resistances is such that the primary current A does not deflect the needle of the galvanometer. This current is denoted by the unfeathered arrow. A coil being introduced at S and an equivalent resistance T between *c* and *e* ; in order that this latter might have no self-induction, it was coiled on two glass rods 10 feet apart.

When the resistances had been adjusted so that they were equal, the current at *q* was broken and an extra current was produced, which, circulating in the same direction round both windings in the galvanometer as represented by the feathered arrows, deflected it. When the current was closed, the extra current passed through both coils in the same direction, which was opposite that of the feathered arrows, and as the deflections were the same, it followed that the currents on opening and closing are equal and opposite. Edlund also showed that the electromotive force of the extra current is proportional to the strength of the primary current.

Induced currents can themselves, by their action on closed circuits, give rise to new induced currents, these again to others, and so on, producing *induced currents of different orders.* These currents, discovered by Henry, may be obtained by causing to act on each other a series of bobbins, each formed of a copper wire covered with silk, and coiled spirally in one plane, like that represented in plate A, fig. 891. The currents thus produced are alternately in opposite directions, and their intensity decreases in proportion as they are of a higher order.

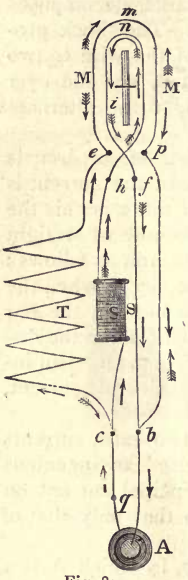


Fig. 897.

909. **Energy of the current of self-induction.**—During the steady period of the current the energy in it, whether displayed in the form of heat or otherwise, is expressed by Joule's law (830). During the variable periods, a portion of the energy is expended apparently without producing any corresponding work; it manifests itself, however, when the current is opened in the form of the spark. The expression for this is $\frac{LI^2}{2}$, when L is the *coefficient of self-induction.* This expression is analogous to that for the energy of a Leyden jar (784), and is deduced by a similar process.

The greater the self-induction of a conductor, the higher is the potential of the extra currents, which changes in the primary current produce in it.

In what form this energy is stored cannot be exactly stated; but according to modern views it is expended in producing that modification of the medium (965), whatever it may be, which constitutes the electromagnetic field; it is in fact the work required for the creation of the field, and is known as the *intrinsic energy of the current.* The spark, like that which accompanies the discharge of the Leyden jar, is due to a sudden release of the strain of the surrounding dielectric, and is the equivalent of this energy.

The electromotive force of currents of self-induction, like that of all induction currents, depends on the strength of the primary current, and secondly on the rate at which this current charges; it is greater the shorter the time in which it is made. The change in the current on making is slow and gradual, and its electromotive force is small; but on opening this is sudden and its potential is so high that it discharges across a considerable length of air, which the current on closing cannot do.

The effects of self-induction on extra currents can be annulled almost entirely by folding the covered wire conveying the current closely on itself. The current then flows in two adjacent windings in opposite directions, and, therefore, the extra currents in the individual coils neutralise each other.

A striking experiment on self-induction is to introduce a coil of wire into the circuit of an alternate current dynamo machine feeding incandescent lamps, and, when the current is steady, to insert suddenly a thick iron bar into the coil. The greater self-induction is at once seen by the diminished light in the lamps.

910. **Alternate currents.**—We have seen that when a coil such as that represented in fig. 895 is placed with its plane in the direction of the earth's magnetic lines of force, and this be turned rapidly through 180°, a momentary deflection is produced in a galvanometer connected with the coil. If the motion of the coil is reversed until it is in its original position, an equal deflection is produced, but in the opposite direction. If the apparatus is rotated continuously the coil will be traversed during each complete revolution by two equal currents in opposite directions; a series of such currents following in rapid succession are known as *alternate* or *alternating* currents, and a circuit may be traversed by such currents just as it may by steady currents in one direction. They follow one another with such rapidity that the ordinary galvanometer is not affected, but their existence is revealed by heating and other effects.

If we represent these currents graphically, taking the times as abscissæ and the corresponding intensities as ordinates, a curve is obtained the general form of which is that represented in fig. 39; it is a curve of sines, and such a current is called a *sinusoidal current.*

The change which takes place in the direction of the current in the coil does not take place exactly when the coil is in the position corresponding to the minimum, but owing to the effect of self-induction it is somewhat

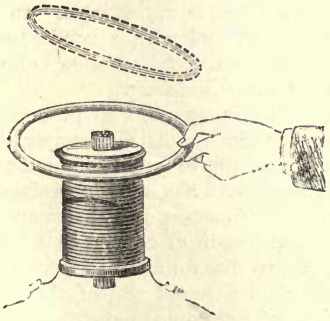


Fig. 898.

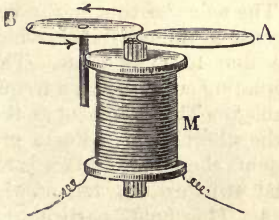


Fig. 899.

behind it; this is called the *retardation* or *lag*, and the fraction of the whole rotation at which it occurs is called the phase. This retardation increases with the *frequency* of the alternation—that is, with the speed of rotation; it also increases when the resistance diminishes, or in other words when the current increases.

An alternating sinusoidal current in a primary coil induces a sinusoidal current in a secondary coil, but which is different in phase owing to the effects of self-induction. If the difference in phase between the two currents is 90°, the electrodynamic attractions and repulsions between them are equal both in intensity and as to the time at which they act, and accordingly the resultant action is small. The difference in phase between the two circuits can in fact never be less than 90° nor more than 180°, but is between the two, and accordingly the repulsive actions always more or less preponderate over the

attractive ones. An experimental illustration of this is seen in the apparatus represented in fig. 898. An electromagnet is traversed by a powerful alternating current, and when a metal ring is placed over the coil, the ring is violently repelled. If a metal plate is interposed between the ring and the coil no repulsion is exerted on the ring ; the plate is itself the seat of an inductive action and plays the part of a screen. Several arrangements have been devised by which the effect of repulsion may give rise to a continuous current. In fig. 899 a copper disc, B, mounted on a pivot rapidly rotates when placed eccentrically in reference to an alternating pole, and with a second disc, A, between, which partly screens the first. If the disc A is itself movable, it rotates in the opposite direction to the disc B.

When a current is in the steady state its density is uniform throughout the whole mass of the conductor, but this is not the case during the variable condition—that is, when it is being made or broken. The *density*—that is, the intensity divided by the section of the part considered—is greater at the surface than at the centre. This is due to a self-induction of the current in the mass of the wire itself. To understand this let us imagine the wire split up into a number of parallel filaments ; the current which passes through one of these filaments induces an inverse current in the adjacent ones. The current is formed first on the outside, and only reaches the inner layers after a time which, though excessively short, is appreciable. In the case of alternating currents of great frequency, and with a conductor of sufficient diameter, it does not reach the centre at all, and a hollow cylinder has the same resistance as a solid one of the same diameter.

The effect of this, in the case of alternating currents, is to produce an increase in the resistance, which with stout conductors is far greater than that due to Ohm's law. Thus, with a conductor of 5 cm. diameter, an alternating current with a frequency of 80 per second has its *ohmic* resistance doubled. This is known as the *impedance*, and increases with the frequency of the alternations. For a given weight and length of copper, that which presents the larger surface is to be preferred ; so that for alternating currents a flat strip or a strand of wires is better than a round rod of the same mass. This holds particularly in the case of lightning-conductors, as we may consider the discharge as formed of a series of rapid alternate discharges.

911. **Magneto-electrical machine.**—After the discovery of magneto-electrical induction, several attempts were made to produce an uninterrupted series of sparks by means of a magnet. Apparatus for this purpose were devised by Pixii and Ritchie, and subsequently by Saxton, Ettinghausen, and Clarke. Fig. 900 represents that invented by Clarke. It consists of a powerful horseshoe magnetic battery, A, fixed against a vertical wooden support. In front of this are two coils, B B′, movable round a horizontal axis. These coils are wound on two cylinders of soft iron joined at one end by a plate of soft iron, V, and at the other by a similar plate of brass. These two plates are fixed on a copper axis, terminated at one end by a commutator, *qi*, and at the other by a pulley, which is moved by an endless band passing round a large wheel, which is turned by a handle.

Each coil consists of about 1,500 turns of very fine copper wire covered with silk. One end of the wire of the coil B is connected on the axis of rotation with one end of the wire of the coil B′, and the two other ends of these wires terminate in a copper washer, *q*, which is fixed to the axis, but is insulated by a cylindrical envelope of ivory. In order that in each wire the induced current may be in the same direction, it is coiled on the two coils in different directions—that is, one is right-handed, the other left-handed.

When now the electromagnet turns, its two branches become alternately magnetised in contrary directions under the influence of the magnet A, and

Fig. 901.

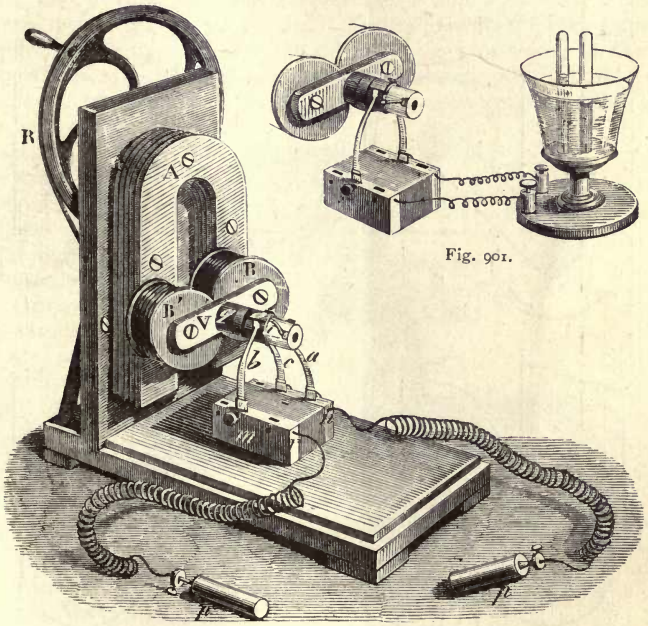


Fig. 900.

in each wire an induced current is produced, the direction of which changes at each half-turn.

Let us follow one of the coils—B, for instance—while it makes a complete revolution in front of the poles *a* and *b* of the magnet ; calling the poles of the electromagnet successively *a′* and *b′*. Let us further consider the latter when it passes in front of the north pole of the magnetic battery (fig. 900). The iron has then a south pole, in which, as we know, the Ampèrian currents move like the hands of a watch. The contrary seems to be represented in fig. 902, but it must be remembered that the coils are seen here as they are in fig. 900 ; and hence, when viewed at the end which faces the magnet, the Ampèrian currents seem to turn like the hands of a

watch. These currents act inductively on the wire of the bobbin, producing a current in the same direction (902, iii.), for the bobbin moves away from the pole *a*, its soft iron is demagnetised, and the Ampèrian currents cease. The intensity of the induced currents in the coil decreases, until the right line joining the axes of the two coils is perpendicular to that which joins the poles *a* and *b* of the magnet. There is now no magnetisation in the bar, but quickly approaching the pole *b*, its soft iron is then magnetised in the opposite direction—that is, becomes a north pole (fig. 903). The Ampèrian currents are then in the direction of the arrow *a'* ; and as they are commencing, they develop in the wire of the bobbin an inverse current (901)

Fig. 902.

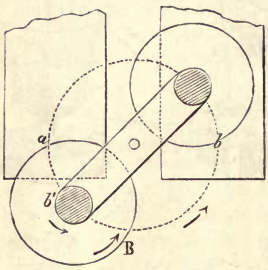
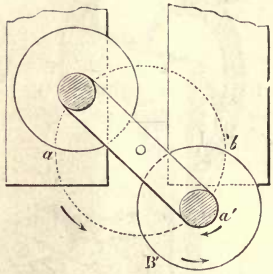


Fig. 903.



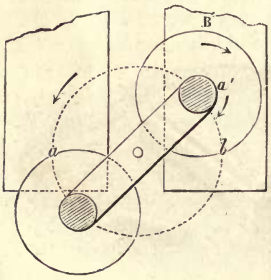


Fig. 904.

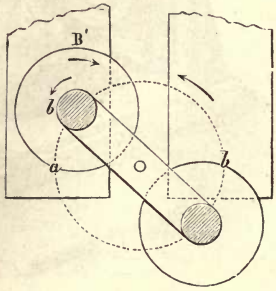


Fig. 905.

which is in the same direction as that developed in the first quarter of the turn. Moreover, this second current adds itself to the first ; for while the coil moves away from *a*, it approaches *b*. Hence, during the lower half-turn from *a* to *b*, the wire was successively traversed by two induced currents in the same direction, and if the rotatory motion is sufficiently rapid, we might admit during this half-turn the existence of a single current in the wire.

The same reasoning applied to the figures 904 and 905 will show that during the upper half-turn the wire of the coil B is still traversed by a single current, but in the opposite direction to that of the lower half-turn.

What has been said about the coil B applies obviously to the coil B′; yet, as one of these is right-handed and the other left-handed, the currents are constantly in the same direction in the two coils during each upper or lower half-revolution. At each successive half-turn they both change, but are in the same direction as regards each other; the term 'direction' having here reference to figs. 902-905.

912. **Commutator.**—The object of this apparatus (fig. 906), of which fig. 907 is a section, is to bring the two alternating currents always in the same direction. It consists of an insulating cylinder of ivory or ebony, J, in the axis of which is a copper cylinder, *k*, of smaller diameter, fixed to the armature V, and turning with the coils. On the ivory cylinder is first a brass ferrule, *q*, and in front of it two half-ferrules, *o* and *o′*, also of brass, and completely insulated from one another. The half-ferrule *o* is connected with the ferrule *q* by a tongue, *x*. On the sides of a block of wood, M, there are two brass plates, *m*, *n*, on which are screwed two elastic springs, *b* and *c*, which press successively on the half-ferrules *o* and *o′*, when rotation takes place.

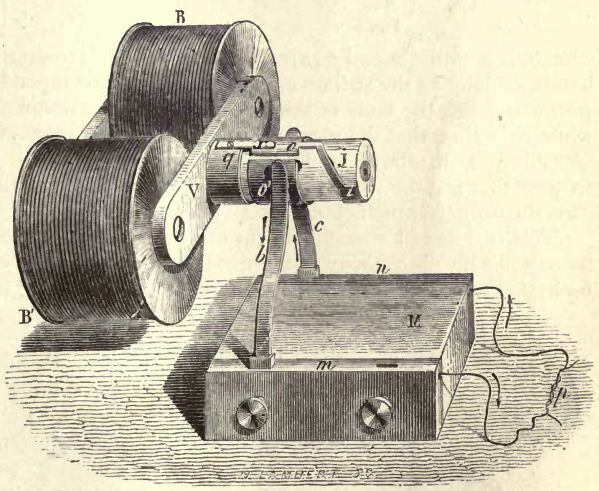


Fig. 906.

We have already seen that the two ends of the wire of the coil, those in the same direction with respect to the currents passing through them at any time, which will be found to be those farthest away from the armature V, terminate in the metallic axis *k*, and therefore on the half-ferrule *o′*; while the other two ends, both in the same direction with respect to the current, are joined to the ferrule *q*, and therefore to the half-ferrule *o*. It follows that the pieces *o o′* are always poles of alternating currents which are developed in the coils: and, as these are alternately in contrary directions, the pieces *o* and *o′* are alternately positive and negative. Now, taking the case in which the half-ferrule *o′* is positive, the current descends by the spring *b*, follows the plate *m*, arrives at *n* by the wire *p*, ascends in *c*, and is closed by contact with the piece *o*; then when, in consequence of rotation, *o* takes the place of *o′*, the current retains the same direction; for, as it is then reversed in the coils, *o* has become positive and *o′* negative, and so forth, as long as the coil is turned.

With the two springs *b* and *c* alone, the opposite currents from the two

pieces *o* and *o'* could not unite when *m* and *n* are not joined ; this is effected by means of a third spring, *a* (fig. 909), and of two appendices, *i*, only one of which is visible in the figure. These two pieces are insulated from one another on an ivory cylinder, but communicate respectively with the pieces *o* and *o'*. As often as the spring *a* touches one of these pieces it is connected with the spring *b*, and the current is closed, for it passes from *b* to *a*, and then reaches the spring *c* by the plate *n*. On the contrary, as long as the spring *a* does not touch one of these appendices the current is broken.

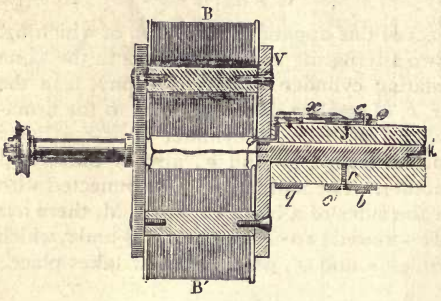


Fig. 907.

For physiological effects the use of the spring *a* greatly increases the intensity of the shocks. For this purpose two long spirals of copper wire with handles, *p* and *p'*, are fixed at *n* and *m*. Holding the handles in the hands, so long as the spring *a* does not touch the appendices *i*, the current passes through the body of the experimenter, but without appreciable effect ; while each time that the plate *a* touches one of the appendices *i*, the current, as we have seen above, is closed by the pieces *b*, *a*, and *c*, and ceasing then to pass through the wires *np*, *mp'*, produces in this and through the body a direct extra current which causes a violent shock.

This is renewed at each half-turn of the electromagnet, and its intensity increases with the velocity of the rotation. The muscles contract with such force that they do not obey the will, and the two hands cannot be detached. With an apparatus of large dimensions a continuance of the shock is unendurable.

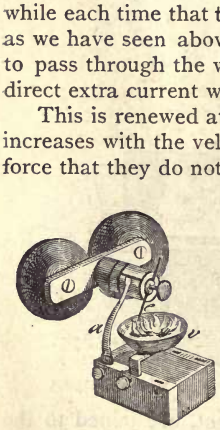


Fig. 908.

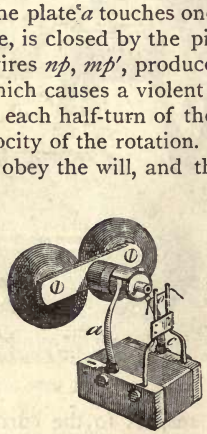


Fig. 909.

All the effects of voltaic currents may be produced by the induced current of Clarke's machine. Fig. 901 shows how the apparatus is to be arranged for the decomposition of water. The spring *a* is suppressed, the current being closed by the two wires which represent the electrodes.

For physiological and chemical effects the wire rolled on the coils is fine, and each about 500 or 600 yards in length. For heating effects, on the contrary, the wire is thick, and there are about 25 to 35 yards on each coil. Figs. 908 and 909 represent the arrangement of the bobbins and the commutator in each case. The first represents the inflammation of ether, and the second the incandescence of a wire, *o*, in which the current from the plate *a* to the plate *c* always passes in the same direction.

Wheatstone devised a compendious form of the magneto-electrical machine, for the purpose of using the induced spark in firing mines (794).

Breguet's apparatus for the same purpose consists of a powerful horse-shoe magnetic battery, to the ends of which are screwed soft iron cores, round which are coils of fine wires ; to these are connected the wires leading to the mine to be fired. The ends of the soft cores are connected by a soft iron keeper ; and when, by a suitable mechanism, this is suddenly detached from the cores, a powerful momentary induction current is produced in the coils, which is sufficient to fire more than one fuse, through even a considerable length of wire.

913. **Nollet's magneto-electrical machine.**—The principle of Clarke's apparatus has received in the last few years a remarkable extension in large

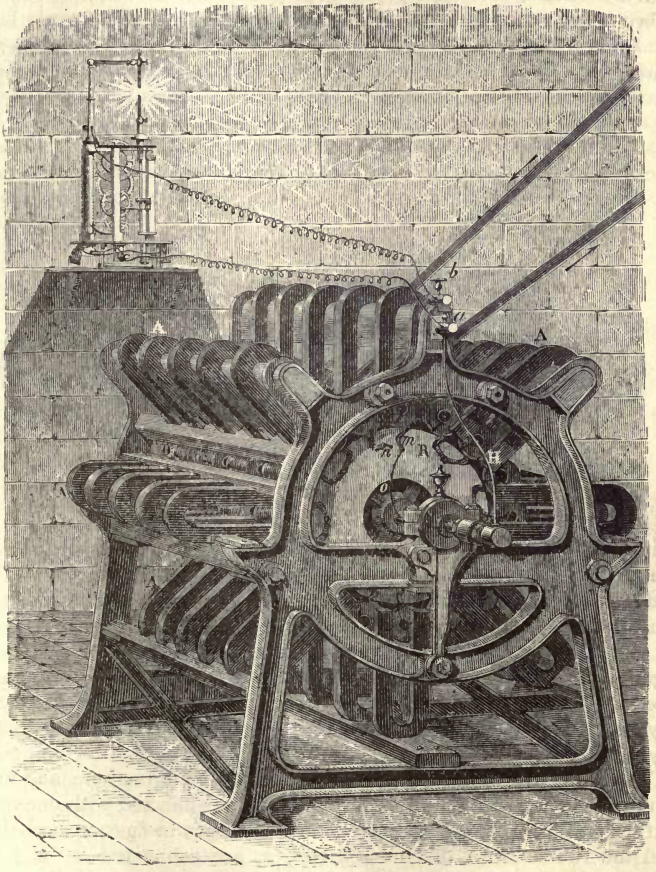


Fig. 910

magneto-electrical machines, by means of which mechanical work is transformed into powerful electrical currents by the inductive action of magnets on coils in motion.

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The first machine of this kind was invented by Nollet, in Brussels, in 1850. It consists (fig. 910) of a cast-iron frame, 5½ feet in height, on the circumference of which eight series of five powerful horseshoe magnetic batteries, A, A, A, are arranged in a parallel order on wooden cross-pieces. These batteries, each of which can support from 120 to 130 pounds, are so arranged that if they are considered either parallel to the axis of the frame, or in a plane perpendicular to this axis, opposite poles always face one another. In each series the outside batteries consist of three magnetised plates, while the three middle ones have six plates, because they act by both faces, while the first only acts by one.

On a horizontal iron axis going from one end to the other of the frame four bronze wheels are fixed, each corresponding to the intervals between the magnetic batteries of two vertical series. There are 16 coils on the circumference of each of these—that is, as many as there are magnetic poles in each vertical series of magnets. These coils, represented in fig. 911, differ from those of Clarke's apparatus in having 12 wires, each 11½ yards in length, instead of a single wire, by which the resistance is diminished. The wires of these coils are insulated by means of bitumen dissolved in oil of turpentine. They are not wound upon solid cylinders of iron, but on iron tubes, split

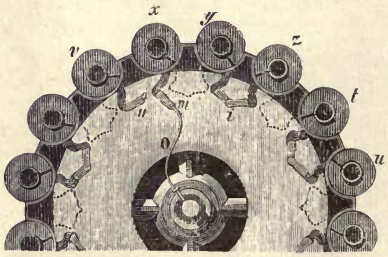

Fig. 911.

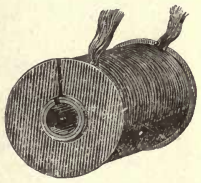

Fig. 912.

longitudinally; this device renders the magnetisation and demagnetisation more rapid when the coils pass in front of the poles of the magnet. Further, the discs of copper which terminate the coils are slit in the direction of the radius, in order to prevent the formation of induced currents in these discs. The four wheels being respectively provided with 16 coils each, there are altogether 64 coils arranged in 16 horizontal series of four, as seen at D on the left of the frame. The length of the wire on each coil being 12 times 11½ yards, or 138 yards, the total length in the whole apparatus is 64 times 138 yards, or 8,832 yards.

The wires are wound on all the coils in the same direction; and not only on the same wheel, but on all four, all wires are connected with one another. For this purpose the bobbins are joined, as shown in fig. 912; on the first wheel the twelve wires of the first coil, *x*, are connected on a piece of mahogany fixed on the front face of the wheel with a plate of copper, *m*, connected by a wire, O, with the centre of the axis which supports the wheels. At the other end, on the other face of the wheel, the same wires are soldered to a plate indicated by a dotted line which connects them with the

coil y; from this they are connected with the coil z by a plate, i, and so on, for the coils t, u, . . . up to the last, v. The wires of this coil terminate in a plate, n, which traverses the first wheel, and is soldered to the wires of the first coil of the next wheel, on which the same series of connections is repeated; these wires pass to the third wheel, thence to the fourth, and so on to the end of the axis.

The coils being thus arranged, one after another, like the elements of a battery connected in a series (825), the electricity is of high potential. But they may also be arranged by connecting the plates alternately, not with each other, but with two metal rings, in such a manner that all the ends of the same name are connected with the same ring. Each of these rings is then a pole, and this arrangement may be used where a high degree of potential is not required.

From these explanations it will be easy to understand the manner in which electricity is produced and propagated in this apparatus. An endless band, receiving its motion from a steam-engine, passes round a pulley fixed at the end of the axis which supports the wheels and the coils, and moves

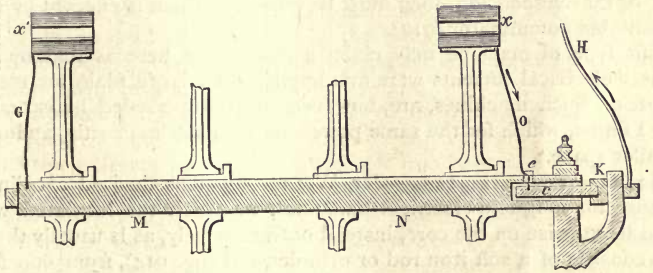

Fig. 913.

the whole system with any desired rapidity. Experience has shown that to obtain the greatest degree of light the most suitable velocity is 235 revolutions in a minute. During this rotation, if we at first consider a single coil, the tube of soft iron on which it is coiled, in passing in front of the poles of the magnet, undergoes at its two ends an opposite induction, the effects of which are added, but change from one pole to another. As these tubes, during one rotation, pass successively in front of sixteen poles alternately of different names, they are magnetised eight times in one direction and eight times in the opposite direction. In the same time there are thus produced in the bobbin eight direct induced currents and eight inverse induced currents; in all, sixteen currents in each revolution. With a velocity of 235 turns in a minute, the number of currents in the same time is $235 \times 16 = 3{,}760$ alternately in opposite directions. The same phenomenon is produced with each of the 64 coils; but as they are all wound in the same direction, and are connected with each other, their effects accumulate, and there is the same number of currents, but they are more intense.

To utilise these currents in producing the electric light, the connections are made as shown in fig. 913. On the posterior side the last coil, x', of the fourth wheel terminates by a wire, G, on the axis MN, which supports

the wheels : the current thus passes to the axis, and thence over all the machine, so that it can be taken from any desired point. In the front the first coil, *x*, of the first wheel communicates, by the wire O, not with the axis itself but with a steel cylinder, *c*, fitted in the axis, from which, however, it is insulated by an ivory collar. The screw *e*, to which the wire O is attached, is likewise insulated by a piece of ivory. From the cylinder *c* the current passes to a fixed metal piece, K, from which it passes to the wire H, which transmits it to the binding screw *a* of fig. 912. The binding screw *b* communicates with the framework, and therefore with the wire of the last coil *x'* (fig. 913). From the two binding screws *a* and *b* the current passes by two copper wires to two carbons, the distance of which is regulated by means of an apparatus analogous in principle to that already described (835).

In this machine the currents are not rectified so as to be in the same direction—it produces *alternate currents* ; hence each carbon is alternately positive and negative, and in fact they are consumed with equal rapidity if a suitable lamp be used ; but when they are to be used for electro-metallurgy, or for magnetising, they must be rectified, which is effected by means of a suitable commutator (912).

This type of machine may claim a description here as that by which magneto-electrical currents were first applied on a large scale for technical purposes. Such machines, are, however, being superseded by various improved forms, which for the same power are simpler, less costly, and occupy a smaller space.

914. **Siemens' armature.**—Dr. Werner Siemens devised a cylindrical armature for magneto-electrical machines, in which the insulated wire is wound lengthwise on the core, instead of transversely, as is usually the case.

It consists of a soft iron rod or cylinder, AB (fig. 914), from one foot to three feet in length. A deep groove is cut in this cylinder and on the ends,

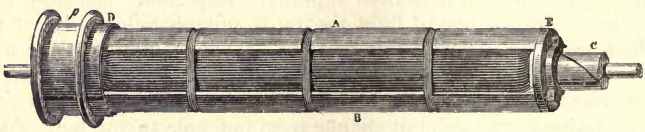

Fig. 914.

in which is coiled the insulated wire, as shown in section in fig. 916. To the two ends of the cylinder, brass discs, E and D, are secured. With E is connected a commutator C, consisting of two pieces of steel insulated from each other, and connected respectively with the two ends of the wire. On the other disc is a pulley, *p*, round which passes a cord, so that the bobbin moves very rapidly on the two pivots.

When a voltaic current circulates in the wire, the two cylindrical segments A and B are immediately magnetised, one with one polarity and the other with the opposite. On the other hand, if instead of passing a voltaic current through the wire of the coil, the coil itself be made to rotate rapidly between the opposite poles of magnetised masses, as the segments A and

B become alternately magnetised and demagnetised, their induction produces in the wire a series of currents alternately positive and negative, as in Clarke's apparatus (910). When these currents are collected in a commutator which adjusts them (912)—that is, sends all the positive currents on one spring and all the negative on another—these springs become electrodes, from one of which positive electricity starts, and from the other negative. If these springs are connected by a conductor, the same effects are obtained as when the two poles of a voltaic battery are united.

This armature has the great advantage that a large number of comparatively small magnets may be used instead of one large one. As, weight for weight, the former possess greater magnetic force than the latter, they can be made more economically. And as the armature is enclosed by and is very near the magnets, it experiences the action of the field in its greatest strength.

915. **Wild's magneto-electrical machine.**—Mr. Wild constructed a magneto-electrical machine in which Siemens' armature is used along with a new principle—that of the *multiplication of the current.* Instead of utilising directly the current produced by the introduction of a magnet, Mr. Wild passes it into an electromagnet, and by the induction of this latter a more energetic current is obtained ; the electromagnet thus excited plays the part of the permanent magnets, but is more powerful.

This machine consists first of a battery of 12 to 16 magnets, P (fig. 915), each of which weighs about 3 pounds, and can support about 20 pounds. Between the poles of the magnets two soft iron keepers CC are arranged, separated by a brass plate, O. These three pieces are joined by bolts, and the whole compound keeper is perforated longitudinally by a cylindrical cavity, in which works a Siemens' armature, *n*, about 2 inches in diameter. The wire of this armature terminates in a commutator, which leads the positive and negative currents to two binding screws, *a* and *b*. This commutator is represented on a larger scale in fig. 916. At the other end is a pulley by which the armature can be turned at the rate of 25 turns in a second. The wire on the armature is 20 yards long.

Below the support for the magnets and their armatures are two large electromagnets, BB, which are called the *field magnets*, since to them is due the production of the magnetic field. Each consists of a rectangular soft iron plate, 36 inches in length by 26 in breadth and $1\frac{1}{4}$ inch thick, on which are coiled about 1,610 feet of insulated copper wire. The wires of these electromagnets are joined at one end, so as to form a single circuit of 3,200 feet. One of the other ends is connected with the binding screw *a*, and the other with *b*. At the top the two plates are joined by a transverse plate of iron, so as to form a single electromagnet.

At the bottom of the electromagnets BB are two iron armatures, separated by a brass plate, O, and in the entire length is a cylindrical channel in which works a Siemens' armature, *m*, as above : this armature, however, is above a yard in length, nearly 6 inches in diameter, and its wire is 100 feet long. The ends are connected with a commutator, from which the adjusted currents pass to two wires, *r* and *s*. The armature *m* is rotated at the rate of 1,700 turns in a minute.

Fig. 916 shows on a larger scale a cross section of the coil of the armatures

CC, and of the plates AA, on which the wire of the electromagnets BB is coiled.

These details being premised, the following is the working of the machine :—When the armatures *n* and *m* are rotated by means of a steam-

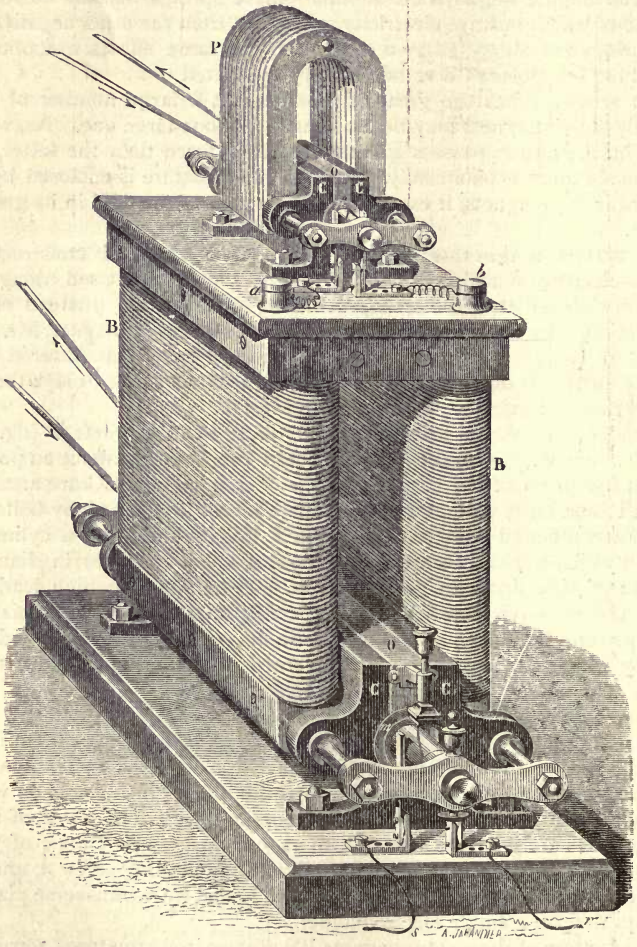

Fig. 915.

engine with the velocity mentioned, the magnets produce in the first armature induced currents, which, adjusted by the commutator, pass into the electromagnet BB, and magnetise it. But as these impart to the lower armatures, CC, opposite polarities, the induction of these latter produces in the armature *m* a series of positive and negative currents far more powerful

than those of the upper armature; so that when these are adjusted by a commutator and directed by the wires *r* and *s*, very powerful effects are obtained.

These effects are still further intensified if, as Mr. Wild has done, the

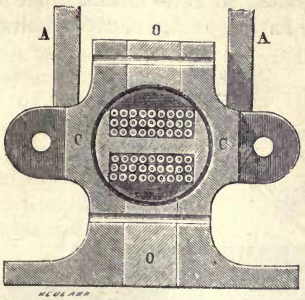

Fig. 916.

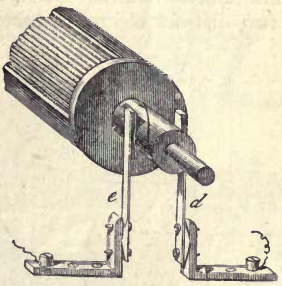

Fig. 917.

adjusted current of the armature *m* is passed into a second electromagnet, whose armatures surround a third and larger Siemens' armature turning with the two others. Mr. Wild thus produced currents of a strength far exceeding anything which up to that time had been attained; he was able, for instance, to melt easily an iron wire a foot long and more than 0·2 inch in diameter.

916. **Dynamo-electrical machines.**—A great advance was made by the discovery of the principle of the reaction of a current on itself—a discovery made by Dr. Werner Siemens and Sir C. Wheatstone independently of each other, and almost simultaneously. If a momentary voltaic current be passed through the wires of the rotating armature of such a machine as the above, a trace of residual magnetism (883) will be left in the core. The rotation of this armature induces a current in the electromagnets BB; this in turn reacts on the armature, increases its magnetism, which again increases the strength of the electromagnets, and so forth. We have in this an analogy with Holtz's machine (759), in which the electricity of the plate and the conductors reciprocally strengthen each other. It is not even necessary to specially magnetise the iron at the outset; the trace of residual magnetism always present in iron (883) is sufficient to start the apparatus, which then goes on increasing with the velocity of the rotation, and which indeed is only limited by the heating of the wires and the bearings, and by the difficulty of properly insulating the coils when such powerful currents are used.

Apparatus which transform mechanical work into electricity without the use of permanent magnets, or of extraneous electromagnets, are known as *dynamo-electrical machines*, or briefly *dynamos*, in contradistinction to *magneto-electrical* machines, in which the magnetism is not furnished by the play of the machine itself, but is got from permanent magnets. It must not, however, be supposed that in the one the electricity is produced at the expense of the magnetism, and in the other at the expense of the work. There is really no distinction of this kind between them; in both kinds of machine electricity is produced at the cost of work; and for this reason

both are indeed dynamo-electrical machines, and the distinction of the two kinds is only one of convenience.

The earliest machine of this kind was that invented by Mr. Ladd. It consists essentially of two Siemens' armatures, rotating with great velocity, and of two iron plates, AA (fig. 918), surrounded by an insulated copper wire.

The electromagnets BB are not joined so as to form a single one, but are two distinct electromagnets, each having at the end two hollow cylin-

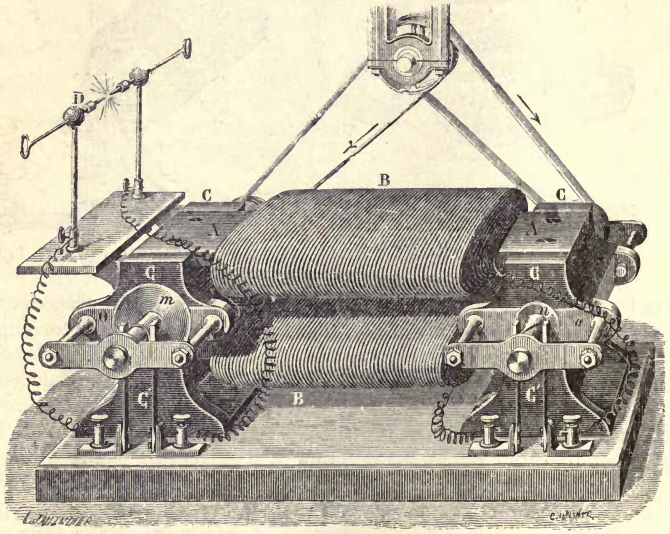

Fig. 918.

ders, CC′, in which are fitted two Siemens' armatures, *m* and *n*: the current of the armature *n* passing round the electromagnets reverts to itself. The wire of the armature *m* passes into the apparatus which is to utilise the current—for instance, two carbon points, D.

The residual magnetism in the armature plates and their keepers is sufficient to start the machine. If, then, the armatures *m* and *n* be rotated by means of two bands passing round a common drum, the magnetism of the hollow cylinders CC′, acting upon the armature *n*, excites induction currents, which, adjusted by a commutator, pass round the electromagnets BB, and more strongly magnetise the cylinders or *shoes* CC′. These in their turn reacting more powerfully on the armature *n*, strengthen the current; we thus see that *n* and B continually and mutually strengthen each other as the velocity of the rotation increases. Hence, as the iron of the armature *m* becomes more and more strongly magnetised under the influence of the electromagnets BB, a gradually more powerful induced current is developed in this armature, which is directed, commutated or not, according to the use for which it is designed.

In a machine exhibited at the Paris Exhibition of 1867 the plates AA were only 24 inches in length by 12 inches in width. With these small

dimensions the current is equal to that of 25 to 30 Bunsen's cells. It can work the electric light and keep incandescent a platinum wire a metre in length and 0·5 mm. in diameter.

The above form of the machine is worked by steam power. Mr. Ladd devised a more compact form, which may be worked by hand. This is represented in fig. 919. The two armatures are fixed end to end, and the coils are wound on it at right angles to each other, as shown in the figure. The current from this can raise to white heat 18 in. of platinum wire 0·01 in. in thickness, and with an inductorium (921) containing 3 miles of secondary wire 2-in. sparks can be obtained.

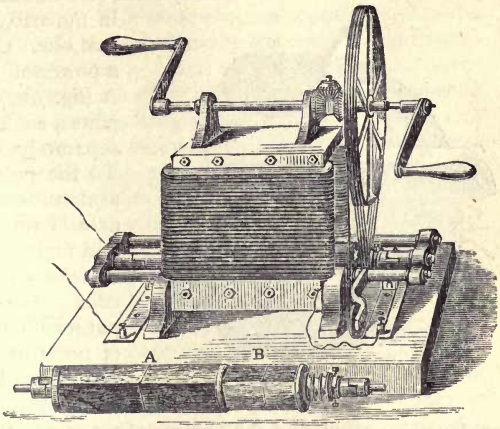

Fig. 919.

917. **Pacinotti's ring. Gramme's magneto-electrical machine.**—A remarkable improvement in magneto- and dynamo-electric machines is the application of a *ring inductor*. This was invented by Prof. Pacinotti in 1862, and is known as *Pacinotti's ring*. It was first applied by him to an electromagnetic motor (899), that is to say, one in which the power was furnished by a battery, but he showed that it could be used as a magneto-electrical motor. The same principle was discovered several years later, it would appear quite independently, by M. Gramme, and utilised by him in the construction of a new form of magneto-electrical machine. This differed from all previous forms in giving

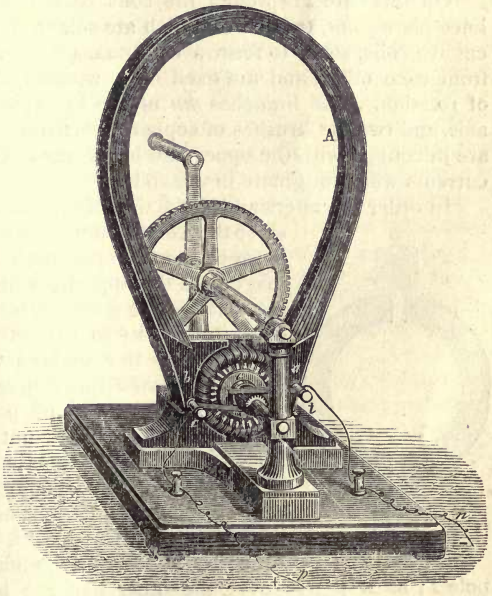

Fig. 920.

at once direct, and what are practically continuous currents, and which, having regard to the size of the machine, were more powerful than any hitherto obtained. A laboratory form of Gramme's machine is represented in fig. 920, in about one-sixth of the real size. On a base is fixed vertically a powerful Jamin's magnetic battery, A (fig. 920), constructed of 24 steel plates, each 1 mm. in thickness, then separately magnetised to saturation. To the poles are affixed two soft iron armatures *a* and *b*, between which an axle is rotated by means of a wheel and rackwork. On this axle is a ring on which are wound a series of thirty coils. The ring or core is not solid, but itself consists of a coil of a number of turns of soft iron wire, as seen in fig. 921, and in this way the changes in its magnetisation which take place are far more rapid, and the heating effect due to these rapid changes is less; the wire is continuous, and the two ends are soldered together.

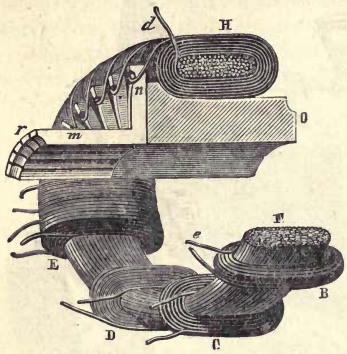

Fig. 921.

On this core are wound the coils BCD; they are joined by thin brass knee-plates, *mn*, to each of which are soldered the copper wires of two successive coils, so as to form a continuous whole. The plates are insulated from each other, and are fixed on a wooden block *o*, mounted on the axis of rotation. The branches *mn* of the knee-plates form a sheath about this axis, and two flat brushes of copper wire, fixed to the binding screws *c* and *i*, are in contact with the upper and lower parts of this sheath, and receive the currents which originate in the coils.

In order to understand the action of Gramme's machine, let us now consider the condition of a soft iron ring which is placed between the two opposite poles of a powerful permanent magnet, at the opposite ends of a diameter of the ring (fig. 922). The parts nearest the magnet will be of the opposite polarity to that of the inducing magnet. We may consider that under its influence each half of the ring is converted into a magnet with its two poles and neutral line. The same poles of the ring face each other, and the effect is not altered if the ends touch. Let us now suppose the ring fixed, and that a thin coil moves round it, starting from the neutral line. As it nears the pole *s*, a current on approach will be induced in the coil in the opposite direction to that which, on Ampère's hypothesis, circulates round the end of the pole *s*; as it passes over the other half *s*, a leaving current is produced, which is in the *same* direction as that which circulates round *s*; but it must be remembered that as these poles face one another, their Ampèrian currents are in opposite directions, the result of which is that the currents induced on approaching *s* and on leaving *s* are in the same direction; in other words,

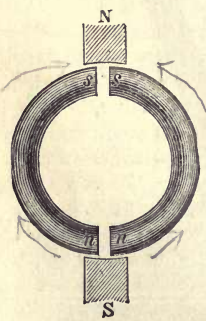

Fig. 922.

as the coil circulates in front of the double pole it will be traversed by a continuous current in the same direction, the strength of which increases from the neutral point till it comes in front of the poles, and then diminishes until it is at the neutral point again. The same process repeats itself in the coil as it approaches the other pole, except that the current is negative, so that if the collectors are adjusted one on each side of the neutral point, they will collect the opposite currents, and they can be utilised in an external circuit. What is here true of one coil is true of all others as they pass in front of the poles ; and as they are all connected together we get, not so much a series of separate impulses, as a continuous series of currents. This continuous character of the currents is improved by the fact that the collector brushes are so arranged as to touch more than one of the knee-pieces at once.

The ring, of course, does actually rotate with the coils, and the polarity of each part is continually changing ; but, although this is the case, the position of the poles remains fixed in space, and the effect is as we have said. It must be added that the poles of the magnet also act directly on the coils ; and if we consider the ring as non-magnetic, and only the direct action of the poles on the coils to operate, it will be seen to be in the same direction as the action of the ring. Both effects concur then in increasing the strength and also continuity of the currents.

This apparatus is very powerful ; the smallest size made can decompose water, and heat to redness an iron wire 20 centimetres in length and a millimetre in diameter. Mascart and Angot determined the electromotive force of different Gramme's machines by placing in the circuit of the machine, but in opposition to it, a number of Daniell's elements. The velocity of rotation was then increased until a galvanometer in the circuit was not deflected. When this was the case, seeing that the resistance traversed by the opposing currents was the same, it is clear that the electromotive force due to the machine rotating at the given speed is exactly equivalent to that of the corresponding number of elements. Thus, for instance, the current from 3 Daniell's cells was found to neutralise that of a particular hand Gramme's machine rotating with a velocity of 10·2 turns per second. The average electromotive force due to this machine was found equal to 0·27 of a Daniell for a velocity of 1 turn per second. With another the ratio was 0·31, and with others again as much as 0·8 of a Daniell.

It will be seen from the description that the action of the ring inductor is not inconsistent with the application of the dynamo-electrical principle ; and in the larger machines it is applied, and the rotation effected, by steam or gas engines or by water power. The dimensions and details of the construction vary with the purpose for which the machine is designed. Thus in a machine which is to be used for electrolysis, the coils in the ring inductor are made up of a comparatively short length of insulated upper bands, while for the electric light a long length of fine insulated wire is used.

Gramme's machine is reversible ; for while by its means motion is converted into electricity, it can in like manner convert electricity into motion. This may be seen by connecting the binding screws *c* and *i* with the poles of a Grove's battery. This iron core then becomes magnetised by the

action of the current passing through the coils; the whole system rotates rapidly under the influence of the magnetised bundle.

918. **Siemens' dynamo-electrical machines.**—Fig. 923 represents the essential features of one of the small-sized vertical machines made by Siemens & Co. A characteristic is the cylindrical or *drum armature*, which may be regarded as an extension of that already described (914). The electromagnets MM and M′M′ with double poles feed the magnetism of the soft iron armatures NN, which are bent so as to almost completely encircle the inductor; they are in detached pieces, so that air can freely circulate between them, and thereby the temperature be kept down.

The inductor itself, D, consists of a drum-shaped frame of soft iron wire covered with a layer of insulating material, and fixed to an axle which rests in the strong upright supports, and is rotated by means of power transmitted to the sheave A. The wire is coiled on this; one end is attached to a plate which forms part of the collector, as in Gramme's machine; it passes lengthwise round the drum in several turns, and the other end is attached to a similar piece on the collector, which is diametrically opposite the first. The wire is continuous, the connection of the individual strands being effected by means of the collector. On the collector rest two pairs of brushes, *b b* and *b′ b′*; they are connected respectively with insulated binding screws; from these the current passes through the wires of the electromagnet, and thence to the terminals, *p p′*, where it may be utilised in the external circuit.

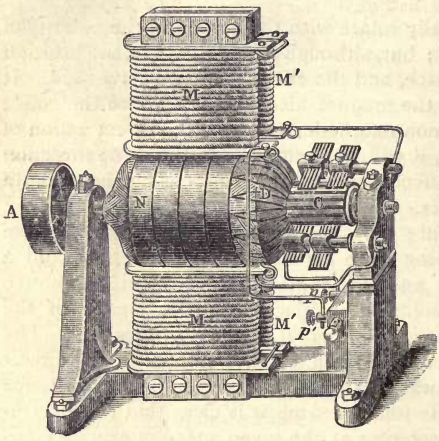

Fig. 923.

The advantage of this construction is that from the length of the inductor the wires are moving in a more extended field; and being on the surface, and quite close to the armature of the field magnets, are more under their influence.

A small machine of this kind, which does not occupy a space of more than three cubic feet, and rotating with a velocity of 15 turns in a second, which is effected by 1½ horse-power, can produce a light of 1,400 candles. The larger sizes produce far more powerful effects, but require of course greater power to work them.

Machines of this class give continuous currents, but alternators (910) are also constructed.

919. **Brush dynamo-electrical machine.**—The armature of this machine (fig. 924) is ring-shaped, and has some resemblance to Gramme's, but the coiling is different. The section of the ring is rectangular (fig. 925), and there are deep rectangular grooves in it, in which are the coils of wire

eight in number. The projected cheeks thus formed between the coils form polar appendices, which are intended to act laterally on the coils. These cheeks are traversed by deep horizontal grooves, and also by a large and deep vertical groove, which almost divides the ring into two parts. By this means the formation of local currents is hindered, and a greater cooling surface is obtained.

The ring rotates between the four poles of two very powerful electro-

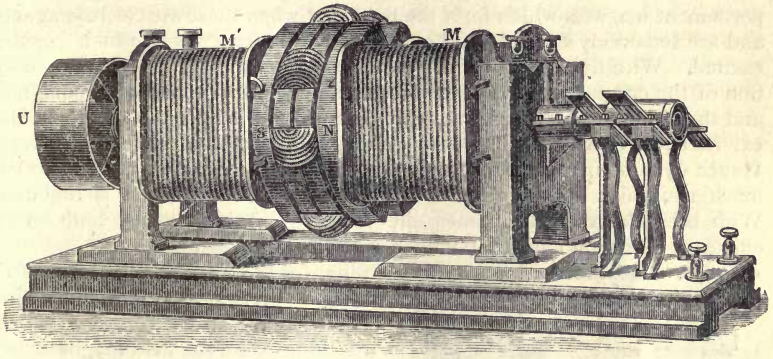

Fig. 924.

magnets, M and M′, whose soft iron armatures are prolonged in pole plates, N and S, double poles being adjacent.

On the collector are four rings (fig. 926). Each ring consists of two segments, A B, separated from each other at one end by an air space, while between the others is a smaller segment, C, called the 'insulator.' During the rotation one pair of coils is in the neutral position, in which no electromotive force is being developed in it. In this position the coils only represent a resistance, and their presence in the circuit is a pure loss. The contacts are so arranged that the moment the pair is in this position, which is at each quarter of a rotation, one of the brushes touches the insulator, and the coil is thus not only removed from the circuit, but, not being closed, no current can circulate in it.

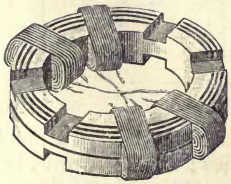

Fig. 925.

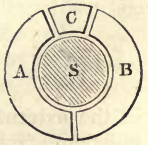

Fig. 926.

One end of each coil is connected with one end of the coil exactly opposite it, the other ends being connected with one of the four commutator rings where they are connected to isolated segments. From these segments the current of the two coils is taken off by brushes arranged horizontally and in connection with curved spring bands, which lead it to the binding screws, from which it passes into the external circuit.

In a machine of this kind which gives 16 arc lights the ring is half a metre in diameter, and each of the 8 coils contains 275 metres of cotton-

covered copper wire 2 mm. in diameter, and weighing 10 kg. Each pair of coils has a resistance of 1·5 ohm, and the electromagnets have a resistance of 6 ohms, so that the total internal resistance is 12 ohms.

In all such machines, the strength of the current which it produces is proportional to the strength of the magnetic field, and with a given armature to the speed of rotation, or to the number of lines of force cut in a given time (826); and is inversely as the resistance of the circuit. The strength of the magnetic field in a magneto machine depends on the strength of the permanent magnets which form the field, and when these are electromagnets and are separately excited, on the strength of the batteries by which they are excited. With dynamo machines the strength of the field magnets is a function of the current which it itself produces in the coils of the electromagnet, and the strength of this current depends on the resistance of the circuit, the external part of which is liable to frequent variations from accidental causes. Hence dynamo machines are more irregular in their action than magneto machines, which are therefore to be preferred where steadiness is required. With both classes of machines the most favourable results, both as to efficiency and economy, are obtained with the larger sizes.

919*a*. **Classification of dynamo machines.**—The principal types of dynamo machines are depicted in figs. 927–930. Fig. 927 represents a machine in which the wires from a separate machine excite the field magnets, and this type is known as the *separately excited* machine; Wild's machine (fig. 915) is an example of this class.

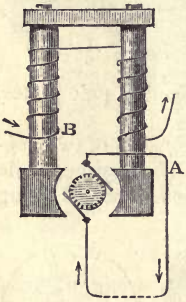

Fig. 927.

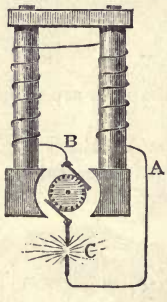

Fig. 928.

Fig. 928 represents the original form of the dynamo; the current from the armature passes directly from one brush into the wire of the field magnet, from thence into the external circuit, returning to the armature by the other brush; such machines are said to be *series wound*. This mode of winding has the defect that variations or breaks in the external resistance have a much greater effect on the current than in magneto machines; for the E.M.F. in these is constant for a given speed of rotation, and alterations in the external resistance only affect the current in accordance with Ohm's law. With the series machine the E.M.F. itself is lessened if the external resistance is doubled, for instance; for a weaker current now circulates in the field magnets, and the magnetic field in which the armature rotates is thereby weaker. Accordingly the current becomes much less than half what it was. If, further, the current is completely stopped, the field magnets almost entirely lose their magnetism, and a considerable time elapses before their magnetism is again excited. Complete stoppages also reverse the polarity of the magnets in consequence of the production of polarisation currents, and accordingly when a steady current is required as in electroplating, or in charging an accumulator (849), such machines are not used.

A third type is that represented in fig. 929, and is known as the *shunt*

wound dynamo; the current at the armature divides at B, one portion passes through the wire of the field magnet, which is long and thin, and the other through the external circuit—for instance, an electroplating bath. If a total break occurs in this circuit, the effect is that a more powerful current passes through the field magnets, which are thus again in readiness to act when the circuit is restored. An increase in the resistance of the external circuit has but small effect; for if the E.M.F. remained constant, the current would only diminish in accordance with Ohm's law, and as a relatively larger proportion now goes through the field magnets, the latter are more strongly excited, and the current again increased; the latter is, in short, lessened in a smaller degree than that in which the resistance is increased. Such machines are used for electroplating and other electrolytic work.

The *compound wound* dynamo is represented in fig. 930. Consider one wire as in the ordinary series wound machine, and in addition to this a second long thin wire from B passing round the field magnets to the other brush at B′. This machine is used for feeding a number of glow lamps which are inserted in parallel, and for which it is essential that the difference of potential should be constant. If now a number of these lamps are removed, the resistance in the circuit of the stout wire is increased, and the current would be lessened, partly from Ohm's law and partly from the weaker magnetism, whereby the difference of potential would be less, and possibly to such an extent that the lamps would not glow; but with the compound winding a greater proportion of the current now passes through the thin wire, and thus acts more strongly on the magnetism. By a suitable choice of the resistances, and the relative number of turns of the wires, the increase of the magnetisation can be made so great that the diminution in the difference of potential is thereby compensated.

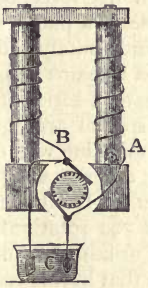

Fig. 929.

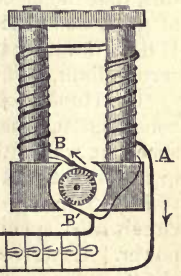

Fig. 930.

920. **Applications of magneto and dynamo-electrical machines.**—Magneto-electrical machines with constant currents are a triumph of modern times; from their discovery, together with that of the dynamo principle (946), is dated the introduction of electricity for industrial purposes. Great improvements have of late been made in magneto-electrical machines, both in the economy and simplicity of their construction, and also in their power; for details on these matters we must refer to special technical works.

The energy of any electrical current or portion of an electrical current is measured by the product of the electromotive force, E, or difference of potentials at the ends of the portion considered, into the strength of the current itself. The magnitude represented by an electromotive force of a volt, V, multiplied by a current strength of an ampere, A, is called a *volt-ampere*, and from its great practical utility has got a special name, that of *Watt* (964). A volt-ampere is *equivalent* to a watt, but the two are not identical;

the former is the measure of the *electrical*, and the latter of the *mechanical* effect which can result from the electrical, that is, can be transformed into it. A horse-power is equal to 746 watts, or a watt is 0·0134 of a horse-power. Hence, if we know the number of watts produced in any circuits, this divided by 746 gives at once the equivalent in horse-power. The *kilowatt* is the Board of Trade unit of electrical energy; it is 1000 watts or 1·3 horse-power.

A magneto-electrical machine may be compared to a pump forcing water through a pipe against friction; the electrical current corresponds to the volume of water passing in a second, and the electromotive force corresponds to the difference in pressure on the two sides of the pump. Just as the power of a pump is measured by the product of the pressure, and volume of water per second, so the product of the electromotive force and current is power, and the ratio of this power to the mechanical power expended in driving the magneto-electrical machine is the *efficiency* of the magneto-electrical machine. The peculiarity of the dynamo-electrical machine is this, that the electromotive force, or the element corresponding to difference of pressure in the case of a pump, depends directly on the current passing. It does not increase indefinitely with increase of current, but increases to a certain limit, and then remains constant.

Dr. Hopkinson made a series of experiments with a machine of Siemens' construction, where special arrangements were made for determining the speed at which the machine was driven, the driving power, the resistances in the circuit, and the difference in potential between the two ends of a known resistance in the circuit. He thus found that to drive the machine in open circuit at a speed of 720 rotations required an expenditure of 0·28 horse-power. Exclusive of friction, the efficiency of the machine was found to be about 90 per cent., so that in this respect little improvement can be expected.

If the relation between the electromotive force measured in volts (814), and the strength of the current measured in amperes (814), for a given speed of rotation be expressed by a curve, it is found that this curve has the form of a slanting straight line starting from the origin, and then begins to bend away, approaching a horizontal line. The point at which it begins to bend away is when the electromotive force is about two-thirds of its maximum, and this is called by Dr. Hopkinson the *critical current*: it has this physical meaning, that below this point any change in the speed of rotation, with a steady external resistance, or any change in the external resistance, with a constant speed of rotation, produces considerable changes in the current.

The principal application which has been made of the currents produced by dynamo machines is to the production of the electrical light (837). In this respect it may be said that the arrangements for producing the electricity are more perfect than those for producing the light; for while over 90 per cent. of the mechanical power used appears in the form of current, only about half of that which is transmitted to the machine appears in the electrical arc.

For electrodes of a definite material, kept at a definite distance apart, and under the ordinary atmospheric pressure, the difference of potential is approximately constant for a constant speed of rotation. The product of difference of potential into the current passing, is the work developed in the arc, and the ratio of this, to the mechanical power expended in driving the machine, is the *efficiency of the electrical arc.*

Comparing together the relative costs of producing a certain degree of illumination—*a*, by means of gas ; *b*, by the electrical arc with alternating currents ; *c*, by one with continuous currents, the machines for the production of the last two being worked by a gas engine—it was found that the ratio was as 116 : 62 : 15 ; when the machine was heated by coal instead of gas, the cost was as 116 : 50 : 10, it being assumed that four pounds of coal produced one horse-power per hour. The actual cost of lighting the British Museum with a light representing 18,800 candles was six shillings an hour, of which the carbons cost nearly one-half.

Dr. Hopkinson gave the following illustration of the luminous effect produced by converting energy into heat in a closed space. One hundred and twenty cubic feet of what is called 15-candle gas (509) produce a light of 360 standard candles for an hour. The heat produced in this combustion is equivalent to about 60 millions of foot-pounds (484). If this gas be burned in a gas-engine (476) about 8 million foot-pounds of work will be done outside the engine, or 4 horse-power for an hour (472). This power is sufficient to drive an A Gramme machine for an hour ; the amount of energy which is converted into current is 6,400,000 foot-pounds, of which about one-half, or 3,200,000, appear in the form of energy in the electric arc. Viewed horizontally this radiates a light of 2,000 candles, and two or three times as much when viewed from below. Hence about 3 million foot-pounds changed into heat in the electric arc will affect our eyes six times as powerfully as 60 millions changed into heat in a gas burner.

Both for arc and incandescent lamps the relative efficiency is greater the higher the illuminating power. Thus with a Swan lamp of 16 candles the work required is at the rate of a horse-power for 272 candles, or about $2\frac{1}{2}$ watts per candle, while with a 32-candle lamp the number of candles equivalent to a horse-power is 415.

Although the temperature of the electric arc is exceedingly high (838), yet from the small amount of radiating surface the heating effect is far less than that produced by other sources of equal illumination. Thus the late Sir C. Siemens found that an electric arc light of 4,000 candles radiated 142·5 thermal units in a minute, while to produce this light by gas would require 200 Argand burners, which would emit 15,000 units, or over a hundred times as much. So too it has been found that incandescent lamps produce less than five per cent. of the heat from other sources of equal intensity as regards their light.

Siemens made a series of experiments on the influence of the electrical light on vegetation. The light was produced by a dynamo-electrical machine of his construction, and was equal in illuminating power to 1,400 candles. Of a series of four sets of quickly growing plants in pots, one set was left in the dark, and two other sets were exposed to the action of the daylight and of the electric light separately ; while the fourth was exposed to the joint action of the two lights. The first set sowed withered and died ; those exposed to the electric light grew and flourished, but not so vigorously as those exposed to daylight alone ; those, however, which had been exposed to the conjoint action of both lights showed the most vigorous growth. Plants did not seem to require a period of repose, but made increased and vigorous progress if subjected at daytime to sunlight, and by night to the electric light.

The electric light was also found beneficial in promoting the formation of aromatic and saccharine substances on which the ripening of fruits depends.

Abney found that the luminosity and also the actinic action of the light produced by the electric arc increased more rapidly than in direct ratio to the velocity of rotation, and the horse-power required to produce it. This increase was slowest for red light, more rapid with blue, and most rapid of all with the actinic action. With a speed of 565 rotations, and an expenditure of 9 horse-power, the actinic action was equal to that of 11,000 candles. Cohn found that the electrical light is more favourable for the pure perception of colour than any other light of equal luminosity.

Electrical furnace.—It is probable that the temperature which can be produced by the oxyhydrogen flame is limited and has been already reached, and that we must look to the electrical arc for the production of higher temperatures than those at which carbonic acid and water are decomposed. Direct experiments by Siemens with the electric arc show not only that it produces a very high temperature within a contracted space, but also that it will conveniently and economically produce such larger effects as will render it useful for many purposes in the arts, like the fusion of platinum and steel. He constructed an arrangement by which the electric arc was formed within a crucible made of the most refractory materials; the one electrode passed through the bottom of the crucible and the other through the lid, and there was an arrangement by which the distance of the electrodes could be automatically regulated; another important point was to constitute the positive pole of the material to be fused, as it is at this pole that the heat is principally developed, the arrangement formed in short an *electrical furnace.* A dynamo-machine capable of producing a current of 36 amperes, and which produces a light equal to 6,000 candles, fused a kilogramme of steel within half an hour. Siemens calculated that the heat in his furnace represented one-third of the horse-power expended in working the machine; and as a good engine only utilises about one-sixth of the combustible value of the coal employed in working it, it follows that the electrical furnace utilises one-eighteenth of the energy residing in the fuel under the engine. The electrical furnace is theoretically more economical than the ordinary air furnaces. Not only is the furnace thus a source of intense heat, but in certain operations the reducing action of the electrodes plays an important part, as in Cowles' method for the direct production of aluminum bronze. A charge of 35 kgr. powdered *corundum*, an aluminous mineral, mixed with powdered charcoal and twice its weight of granulated copper, was placed between carbon electrodes in a suitable vessel. On passing a powerful current the alumina was reduced and united directly with the copper to form aluminum bronze. The current actually employed was one of 5,000 amperes with an E.M.F. of 50 volts, or with a power of 250,000 watts-second. The current was continued for an hour and a half, and produced about 82 kgr. of the alloy. Each kgr. of aluminum in the alloy represents a work of 44 horse-power for an hour. Minet obtained pure aluminum by electrolysing between carbon electrodes a mixture of cryolite and sodium chloride, kept in an iron trough at a temperature of 1000°.

920*a*. **Electrical transmission of power.**—When a magneto or dynamo machine is coupled up with a second one, on working the first the second is put in rotation, and in a direction opposed to that of the first. Two such machines coupled in this way are called the *generator* and the *motor*. This motor may be geared up with any machine, such as a saw wheel, a lathe, or a pump, which is thereby made to do its special work. On this depends the possibility of transmitting by electricity to great distances power from a common centre, and of thereby utilising natural sources of power, such as rivers, waterfalls, windmills, and the like.

The efficiency of any magneto machine, as we have seen, is the ratio of the energy w' developed in the machine to the mechanical power w, expended in producing it. Apart from friction, more than 90 per cent. of the power can be thus converted; if such a machine works on short circuit the whole of this energy would appear as heat; when external work is done, such as in producing the electric light, the energy is shared between the various parts of the circuit, and the amount of this energy in any part can be easily obtained if we know the fall of potential between the part in question and the current which is passing.

When a motor is connected with a generator at work, the former is set in motion, and in a direction opposed to that of the generator; it thereby develops an electromotive force expressed in volts of v, opposed to that of the generator V. The total work W of the generator in unit time is $\frac{VA}{746}$ horse-power. Part of this work appears in the heating of the conducting wires, and the rest in the form of the energy of the motor w, which is $\frac{vA}{746}$ h.p., where v is the difference of potentials at the two terminals of the machine. The ratio $\frac{w}{W} = \frac{v}{V}$; that is, the work of the motor is to that of the generator in the ratio of their electromotive forces—in other words, to the differences of potentials at the respective terminals. In practice the best condition of working is to arrange so that the generator has twice the electromotive force of the motor, the current being, of course, the same in each.

Suppose a source of energy available of 50,000 watts, for example, and that this is to be transmitted in the form of electrical energy to a certain distance, there to be utilised. Since a watt represents the product of two factors, a volt and an ampere, we may vary these factors, which make up the total, in any way we like. We may transmit it in the form of a strong current of low electromotive force, or of a weak current with very high electromotive force. Thus, for instance, we can transmit the above quantity as a current of 500 amperes under a pressure of 100 volts. But for this purpose the resistance of the conductor through which the current passes must be small, which necessitates that it have a large section and consist of very good conducting material—that is, of copper. The great weight and cost of such a conductor quite preclude its use for transmission to any considerable distance.

The energy could, however, also be transmitted in the form of a weak current, say of 10 amperes, under a pressure of 5,000 volts; the conductor required for this purpose might be very much thinner, and therefore far less costly.

By increasing the number of coils of a magneto-electrical machine, and the speed of rotation, currents may be produced of very high electromotive force. But in practice it is found that a certain speed of rotation cannot be exceeded, and, moreover, the difficulty of insulation in the machines themselves is greatly increased ; the electricity is apt to discharge in the body of the machine itself, and thus destroy the insulation. In practice it is not safe to go beyond 2,000 volts.

Here comes in the great utility of *transformers* (923*a*). At the place at which the energy is available this is converted into a strong electrical current of low potential, which is then passed into a suitable transformer and, on the principle $VA = V'A'$, is converted into a small current of high electromotive force ; this is then transmitted along a thin wire to the place where it is to be utilised, where it again enters a transformer, and is changed into a current of the desired practical dimensions.

The greatest progress hitherto made in this direction was at the Electrical Exhibition of 1891, at Frankfort-on-the-Main, in which mechanical power was transmitted from Lauffen-on-the-Neckar through a distance of 110 miles with a loss of 28 per cent. The water-power available at Lauffen worked a turbine producing 300 horse-power ; this generated an enormous current of 4,000 amperes and 55 volts, or 220,000 watts ; by means of copper rods over an inch in diameter this was passed into a transformer, where it was changed into a current of 27,000 volts and 8 amperes. This passed through the conductor, which consisted of three copper wires, each $\frac{3}{16}$ of an inch in diameter, and at Frankfort this again entered a transformer, and was converted into a current of 100 volt potential.

The chief practical difficulty in the transmission of power economically seems now to be one of insulation.

Magneto-electrical machines have been applied to propelling carriages along a railway. A narrow-gauge railway was laid down, and upon this a train of three or four carriages was laid, and on the first of these a medium-sized dynamo machine, so fixed and connected with the axle of one pair of wheels as to give motion to the same. The two rails, being laid upon wooden sleepers, were sufficiently insulated to serve for electrical conductors. Between the two rails a bar of iron was fixed on wooden supports, through which the current was conveyed to the train by brushes fixed to the driving carriage, while the return circuit was completed through the rails. At the station the centre bar and rails were electrically connected with the poles of a dynamo machine like that on the carriage, and which was worked from a fixed steam-engine on the ground. The magneto machine exerted 5 horse-power, and it travelled with a velocity of 15 to 20 miles an hour.

Another application is to what is called *telpherage*, by which is meant a means of propelling light carriages or buckets along a single metal rope or rod, supported on posts at some height above the ground. A working line has been already constructed and used with success, and this method of electrical haulage will probably be of great service in conveying minerals in mountainous countries, from the facility with which it can be constructed on uneven ground, and particularly in those cases in which water supply is available.

921. **Inductorium. Ruhmkorff's coil.** These are arrangements for producing induced currents, in which a current is induced by the action of an electric current, whose circuit is alternately opened and closed in rapid succession. These instruments, known as *inductoriums*, or *induction coils*, present considerable variety in their construction, but all consist essentially of a hollow cylinder in which is a bar of soft iron, or bundle of iron wires, with two helices coiled round it, one connected with the poles of a battery, the current of which is alternately opened and closed by a self-acting arrangement, and the other serving for the development of the induced current. By means of these apparatus, and with a current of three or four Grove's cells, physical, chemical, and physiological effects are produced equal and superior to those obtainable with electrical machines and even the most powerful Leyden batteries.

Of all the forms those constructed by Ruhmkorff are the most powerful. Fig. 931 is a representation of one, the coil of which is about 14 inches in length. The *primary* or *inducing* wire is of copper, and is about 2 mm. in

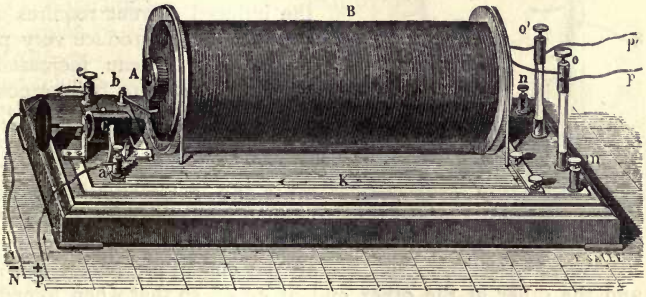

Fig. 931.

diameter, and 40 or 50 yards in length. It is coiled directly on a cylinder of cardboard, which forms the nucleus of the apparatus, and is enclosed in an insulating cylinder of glass or of caoutchouc. On these is coiled the *secondary* or *induced* wire, which is also of copper, and is about $\frac{1}{2}$ mm. in diameter. A great point in these apparatus is the insulation. The wires are not merely insulated by being in the first case covered with silk, but each individual coil is separated from the rest by a layer of melted shellac. The length of the secondary wire varies greatly; in the largest size hitherto made, that of the late Mr. Spottiswoode, it is as much as 280 miles, while the primary was 1,164 yards. With these great lengths the wire is thinner, about $\frac{1}{8}$ mm. The thinner and longer the wire the higher the potential of the induced electricity.

The following is the working of the apparatus:—The current arriving by the wire P at a binding screw, *a*, passes thence in the commutator C, to be afterwards described (fig. 934), thence by the binding screw *b* it enters the primary wire, where it acts inductively on the secondary wire; having traversed the primary wire, it emerges by the wire *s* (fig. 932). Following the direction of the arrows, it will be seen that the current ascends in the binding screw *i*, reaches an oscillating piece of iron, *o*, called the *hammer*, descends by the *anvil h*, and passes into a copper plate, K, which takes it

to the commutator C. It goes from there to the binding screw *c*, and finally to the negative pole of the battery by the wire N.

The current in the primary wire only acts inductively on the secondary wire (901), when it opens or closes, and hence must be constantly interrupted. This is effected by means of the oscillating hammer *o* (fig. 932). In the centre of the bobbin is a bundle of soft iron wires, forming together a cylinder a little longer than the bobbin, and thus projecting at the end as seen at A. When the current passes in the primary wire this hammer, *o*, is attracted ; but immediately, there being no contact between *o* and *h*, the current is broken, the magnetisation ceases, and the hammer falls ; the current again passing, the same series of phenomena recommences, so that the hammer oscillates with great rapidity.

922. **Condenser.**—In proportion as the current passes thus intermittently in the primary wire of the bobbin, an induced current, alternately direct and inverse, is produced at each interruption in the secondary wire. But as this is perfectly insulated, the induced current requires such a strength as to produce very powerful effects. Fizeau increased this strength still more by interposing a condenser in the primary circuit.

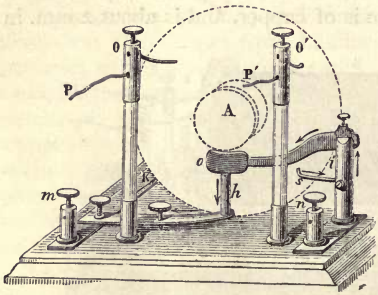

Fig. 932.

This condenser (fig. 933) consists of sheets of tinfoil placed over each other and insulated by larger sheets of stout paper, *v*, soaked in paraffine or resin. The sheets of tinfoil project at the end of the paper, one set at *s s' s''*, and the other at the other end, at *e e' e''*, so that when joined by a binding screw the odd numbers form one coating of a condenser, and the even numbers the other coating. In large condensers, the surface of each condenser is as much as 75 square yards. The whole being placed in a box at the base of the apparatus, one of the coatings, the positive, is connected with the binding screw *i*, which receives the current on emerging from the bobbin ; and the other, the negative, is connected with the binding screw *m*, which communicates by the plate K with the commutator C, and with the battery.

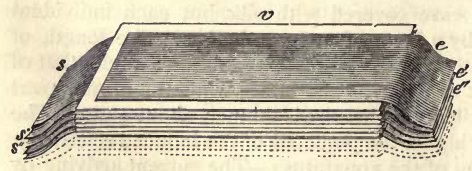

Fig. 933.

The capacity of the condenser should bear a relation to the self-induction of the primary circuit and the rapidity of the contact-breaker. To understand the effect of the condenser, it must be observed that at each break of the inducing current an extra current is produced in the same direction, which, continuing in a certain manner, prolongs its duration. It is this extra current which produces the spark that passes at each break between the hammer and the anvil ; when

the current is strong this spark rapidly alters the surface of the hammer and anvil, though they are of platinum. By interposing the condenser in the inducing circuit, the extra current, instead of producing so strong a spark, passes into the condenser—the positive electricity in the coating connected with *i*, and the negative in that connected with *m*. But the opposite electricities combining quickly by the thick wire of the primary coil, by the battery, and the circuit C K *m*, give rise to a current contrary to that of the battery, which instantaneously demagnetises the bundle of soft iron : the induced current is thus shorter and more intense. The binding screws *m* and *n* on the base of the apparatus are for receiving this extra current.

The *commutator* or *key* serves to break contact or send the current in either direction. The section in fig. 934 is entirely of brass, excepting the core, A, which is of ebonite : on the two sides are two brass plates, C C′. Against these press two elastic brass springs, joined to two binding screws, *a* and *c*, with which are also connected the electrodes of the battery. The current arriving at *a* rises in C ; thence by a screw, *y*, it reaches the binding screw *b* and the bobbin : then returning by the plate K, which is connected with the hammer, the current goes to C′ by the screw *x*, descends to *c*, and rejoins the battery by the wire N. If, by means of the milled head, the key is turned 180 degrees, it is easy to see that exactly the opposite takes place ; the current reaches the hammer by the plate K and emerges at *b*. If, lastly, it is only turned through 90 degrees, the elastic plates rest on the ebonite A instead of on the plate C C′, and the current is broken.

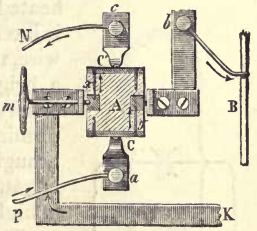

Fig. 934.

The two wires from the bobbin at *o* and *o*′ (fig. 931) are the two ends of the secondary wire. They are connected with the thicker wires P P′, so that the current can be sent in any desired direction. With large coils the hammer cannot be used, for the surfaces become so much heated as to melt. But Foucault invented a mercury contact-breaker which is free from this inconvenience, and which is an important improvement. Very powerful discharges were obtained by Spottiswoode from his coil by disconnecting the contact-breaker and sending into it the alternate currents of a powerful magneto-machine.

923. **Effects produced by Ruhmkorff's coil.**—The high potential of the electricity of induction coils has long been known, and many luminous and heating effects have been obtained by their means. But it is only since the improvements which Ruhmkorff introduced into his coil that it has been possible to utilise all the potential of induced currents, and to show that these currents possess powerful statical as well as dynamical properties.

Induced currents are produced in the coil at each opening and breaking of contact. But these currents are not equal either in duration or in potential. The direct current, or that on *opening*, is of shorter duration, but higher potential ; that of *closing* of longer duration, but lower potential. Hence if the two ends P and P′ of the fine wire (figs. 931 and 932) are connected, as there are two equal and contrary quantities of electricity in the

wire the two currents neutralise each other. If a galvanometer is placed in the circuit, only a very feeble deflection is produced in the direction of the direct current. This is not the case if the two ends P and P′ of the wire are separated. As the resistance of the air is then opposed to the passage of the currents, that which has highest potential—that is, the direct one or that on opening—passes in excess, and the more so the greater the distance of P and P′ up to a certain limit at which neither passes. There are then at P and P′ nothing but potentials which are alternately contrary.

A striking distance of 1 mm. (788) corresponds to an electromotive force of 5,490 volts, and the striking distance of 1 inch which is furnished by even a small machine represents a potential of over 70,000 volts. How enormous must then be the potential of Spottiswoode's larger machine !

The *physiological* effects of Ruhmkorff's coil are very powerful ; in fact, shocks are so violent that many experimenters have been suddenly prostrated by them. A rabbit may be killed with two of Bunsen's elements, and a somewhat larger number of couples would kill a man.

The *heating* effects are also easily observed ; an air thermometer is heated by the alternating currents ; if a very fine iron wire is interposed between the two ends P and P′ of the induced wire, this iron wire is immediately melted, and burns with a bright light. A curious phenomenon may here be observed, namely, that when each of the wires P and P′ terminates in a very fine iron wire, and these two are brought near each other, the wire corresponding to the negative pole alone melts, showing that its temperature is higher.

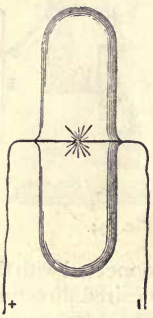

Fig. 935.

The *chemical* effects are very varied ; thus, according to the shape and distance of the platinum electrodes immersed in water, and to the degree of acidulation of the water, either luminous effects may be produced in water without decomposition, or the water may be decomposed and the mixed gases disengaged at the two poles, or the decomposition may take place, and the mixed gases separate either at a single pole or at both poles.

Gases may also be decomposed or combined by the continued action of the spark from the coil. If the current of a Ruhmkorff's coil be passed through an hermetically sealed tube containing air, as shown in fig. 935, nitrogen and oxygen combine to form nitrous acid.

The *luminous* effects of Ruhmkorff's coil are also very remarkable, and vary according as they take place in air, in vapour, or in very rarefied vapours. In air the coil produces a very bright loud spark, which, with the largest sized coil hitherto made, that of the late Mr. Spottiswoode, has a length of 42 inches. In vacuo the effects are also remarkable. The experiment is made by connecting the two wires of the coil P and P′ with the two rods of the electric egg (fig. 742) used for producing in vacuo the luminous effects of the electrical machine. Exhaustion having been produced up to 1 or 2 mm., a beautiful luminous trail is produced from one knob to the other, which is virtually constant, and has the same intensity as that obtained with a powerful electrical machine when the plate is rapidly turned. This experi-

ment is shown in figs. 939 and 940. Fig. 938 represents a remarkable deviation which light undergoes when the hand is presented to the egg.

The positive pole of the current shows the greatest brilliancy; its light is of a fiery red, while that of the negative pole is of a feeble violet colour; moreover, the latter extends along all the length of the negative rod, which is not the case with the positive pole.

The coil also produces mechanical effects so powerful that, with the largest apparatus, glass plates two inches thick have been perforated. This result, however, is not obtained by a single charge, but by several successive charges.

The experiment is arranged as shown in fig. 936. The two poles of the induced current correspond to the binding screws *a* and *b*; by means of a copper wire, the pole *a* is connected with the lower part of an apparatus for piercing glass like that already described (fig. 748); the other pole is attached to the other conductor by a wire, *d*. The latter is insulated in a large

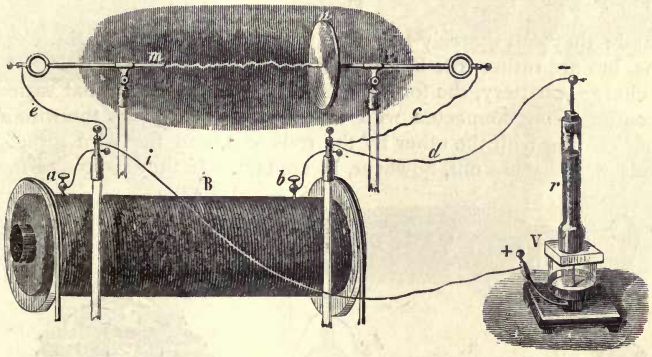

Fig. 936.

glass tube, *r*, filled with shellac, which is run in while in a state of fusion. Between the two conductors is the glass to be perforated, V. When this presents too great a resistance, there is danger lest the spark pass in the coil itself, perforating the insulating layers which separate the wires, and then the coil is destroyed. To avoid this, two wires, *e* and *c*, connect the poles of the coil with two metallic rods whose distance from each other can be regulated. If then the spark cannot penetrate through the glass, it strikes across, and the coil is not injured.

The coil can also be used to charge Leyden jars. With a large coil giving sparks of 6 to 8 inches, and using 6 Bunsen's elements with a large surface, Ruhmkorff charged large batteries of 6 jars each, having about 3 square yards of coated surface.

The experiment with a single Leyden jar (fig. 937) is made as follows:— The coatings of the latter are in connection with the poles of the coil by the wires *d* and *i*, and these same poles are also connected, by means of the wires *e* and *c*, with the two horizontal rods of a universal discharger (fig. 731). The jar is then being constantly charged by the wires *i* and *d*, sometimes in one direction and sometimes in another, and as constantly

discharged by the wires *e* and *c*; the discharges from *m* to *n* taking place as sparks two or three inches in length, very luminous, and producing a deafen-

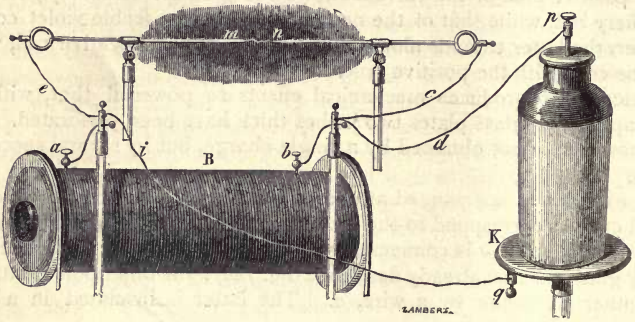

Fig. 937.

ing sound; they can scarcely be compared with the sparks of the electrical machine, but are rather true lightning flashes.

To charge a battery, the form of the experiment is somewhat varied, the outer coating being connected with one pole of the coil by the wire *d*, and the inner coating with the other by the rods *m n*, and the wire *c* (fig. 938). The rods *m* and *n* are not, however, in contact. If they were—as the two

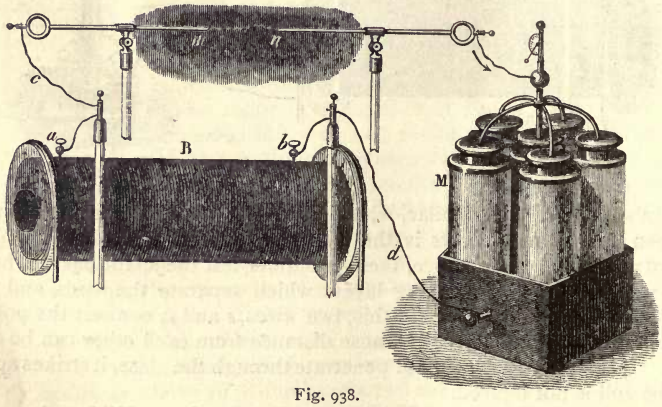

Fig. 938.

currents, the inverse and direct, pass equally—the battery would not be constantly charged and discharged; while from the distance between *m* and *n* the direct current, that of breaking, which has higher potential, passes alone, and it is this which charges the battery.

923*a*. **Transformers.**—Ruhmkorff's coil, as we have seen, is an arrangement by means of which we can transform electricity of low into electricity of high potential. There is no creation of electricity; the energy produced in the secondary circuit is produced at the cost of that in the primary. The apparatus acts in short as a *transformer* or *converter*, and it is reversible, for

if we connect the long thin wire with a source of electricity yielding alternating discharges at high potential, we get alternating discharges in the short thick wire of low potential but of much stronger current. The functions of the wires are reversed in this case; the thin long wire is the primary and the short thick wire the secondary. This modification of the principle of Ruhmkorff's coil is, as we have seen (920*a*), of great practical importance in the transmission of electrical energy.

One of the chief types of transformer, represented in fig. 939, consists of a closed iron ring, or preferably a bundle of soft iron wires or strips, so as to hinder the formation of Foucault currents (929). In either case the magnetic circuit is closed, the primary wire A B is coiled on this, and over it the secondary, *a b*, both carefully insulated; or the two sets are coiled separately in individual sections, as shown in the figure.

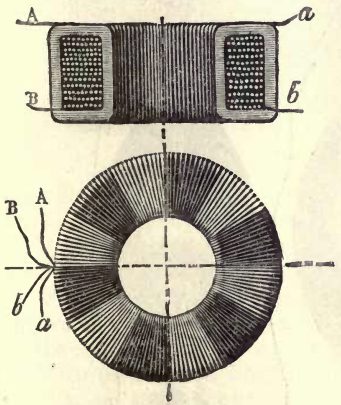

Fig. 939.

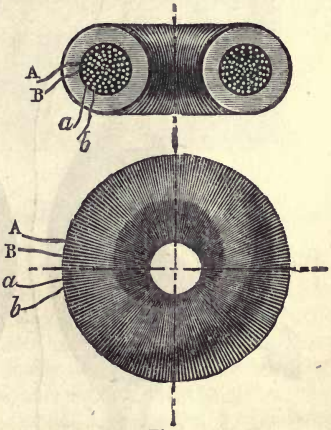

Fig. 940.

In another type, represented in fig. 940, the primary and secondary wires are wound together and form the core, while on this is coiled in a continuous circuit at right angles iron wire or strips, coated on the surface with oxide of iron which, being an insulator, tends to prevent the formation of Foucault currents.

It is of great importance that the insulation of transformers be good, more especially when, as in recent times, it is requisite to transform potentials of 30,000 to 40,000 volts, or even more. In such cases the whole transformer is placed in oil, which penetrates the pores and insulates admirably.

The chief requirement in any transformer is that it shall effect the transformation of electrical energy without great loss. In the ideal case the transformed electrical energy would be equal to the original, which, however, is impossible; for in the first case heat due to the Joule effect is produced both in the primary and the secondary wire, which is lost as far as transmission of energy is concerned. Then, further, some energy is consumed, in Foucault currents, and, moreover, the magnetisation and demagnetisation of the iron core or envelope represents a loss of energy, which is spoken of as the loss by hysterisis (829).

Owing to all these causes, the effect in the secondary wire is always less than that in the primary, and the ratio of the two is called the *efficiency*. In the most improved modern forms an efficiency of 96 per cent. is attained. The efficiency is greater when the transformer works under a full load. The magnetisation then oscillates within narrower limits, and the losses due to hysterisis are smaller.

Lane Fox showed that secondary batteries could be used as transformers for direct currents of high electromotive force.

924. **Stratification of the electric light.**—Quet observed, in studying the electric light which Ruhmkorff's coil gives in a vacuum, that if some of the vapour of turpentine, wood spirit, alcohol, or bisulphide of carbon, &c.,

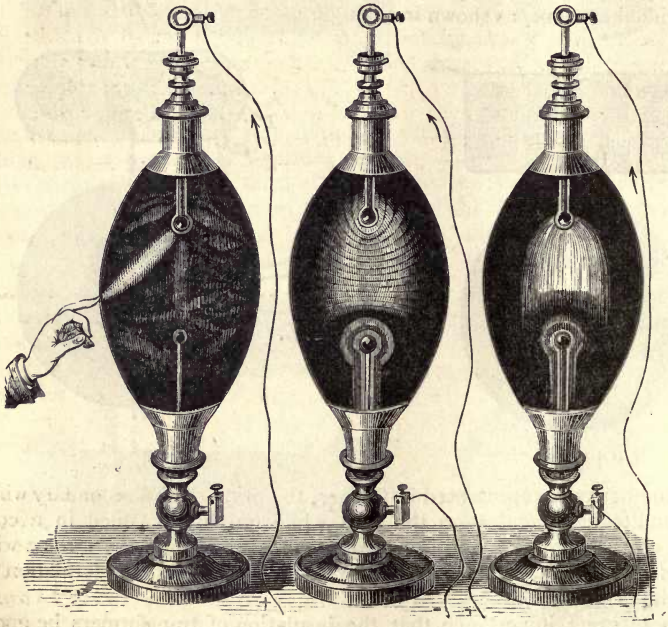

Fig. 941. Fig. 942. Fig. 943.

be introduced into the vessel before exhaustion, the aspect of the light is totally modified. It appears then like a series of alternately bright and dark zones, forming a pile of electric light between the two poles (fig. 942).

In this experiment it follows, from the discontinuity of the current of induction, that the light is not continuous, but consists of a series of discharges which are near each other in proportion as the hammer *o* (fig. 932) oscillates more rapidly. The zones appear to possess a rapid gyratory and undulatory motion. Quet considers this as an optical illusion: for if the hammer is slowly moved by the hand, the zones appear very distinct and fixed.

The light of the positive pole is most frequently red, and that of the negative pole violet. The tint varies, however, with the vapour or gas in the globe.

925. **Geissler's tubes.**—The brilliancy and beauty of the stratification of the electric light are most remarkable when the discharge of the Ruhmkorff coil takes place in glass tubes containing a highly rarefied vapour or gas. These phenomena, which were originally investigated by Gassiott, are produced by means of sealed glass tubes first constructed by Geissler, of Bonn, and generally known as *Geissler's tubes* (578). The tubes are filled with different gases or vapours, and are then exhausted, so that the pressure

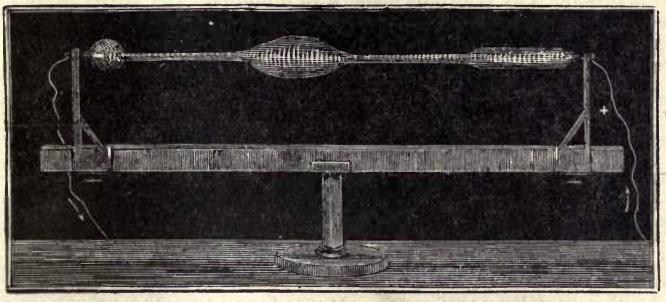

Fig. 944.

does not exceed half a millimetre. At the ends of the tubes two platinum or aluminum wires are soldered into the glass.

When the two platinum wires are connected with the ends of a Ruhmkorff coil magnificent lustrous striæ, separated by dark bands, are produced all through the tube. These striæ vary in shape, colour, and lustre with the degree of the vacuum, the nature of the gas or vapour, and the dimensions of the tube. The phenomenon has occasionally a still more brilliant aspect from the fluorescence which the electric discharge excites in the glass.

Fig. 944 shows the striæ in carbonic acid under a quarter of a millimetre pressure; the colour is greenish, and the striæ have not the same form as hydrogen. In nitrogen the light is orange-yellow.

Plücker found that the light in a Geissler's tube did not depend on the substance of the electrodes, but simply on the nature of the gas or vapour in the tube. He found that the lights furnished by hydrogen, nitrogen, carbonic oxide, &c., give different spectra when they are decomposed by a prism. The discharge of the coil which passes through a highly rarefied gas would not pass through a perfect vacuum, from which it follows that the presence of a ponderable substance is absolutely necessary for the passage of electricity.

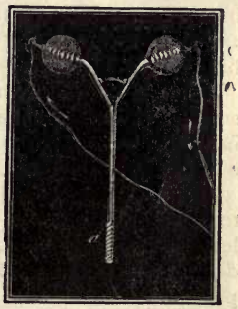

Fig. 945.

By the aid of a powerful magnet Plücker tried the action of magnetism on the electric discharge in a Geissler's tube, as Davy had done with the ordinary voltaic arc, and obtained many curious results, one of which may be mentioned. He found that where the discharge is perpendicular to the line of the poles, it is separated into two distinct

parts, which can be referred to the different action exerted by the electro-magnet on the two extra currents produced in the discharge.

The light of Geissler's tubes has been applied to medical purposes. A long capillary tube is soldered to two bulbs provided with platinum wires; this tube is bent in the middle, so that the two branches touch, and their extremities are twisted as shown at *a* (fig. 945). This tube contains a highly rarefied gas, like those previously described, and when the discharge passes a light is produced at *a*, bright enough to illuminate any cavity of the body into which the tube is introduced.

926. **De la Rue and Müller's experiments.**—These physicists have made a very extensive and elaborate series of experiments on the stratification of the electric light by means of the currents produced by their battery (812). They employed for some of these experiments as many as 14,400 cells, which is by far the most powerful battery ever put together. It is impossible to attempt here even a condensed account of these experiments; but the following, which are some of the results obtained, may be mentioned.

The discharge in a vacuum tube is essentially of the same nature as that which takes place in gases under the ordinary atmospheric pressure. A vacuum tube was interposed in the circuit of a battery of 2,400 cells, together with a very long resistance. It was found that the potentials at the two ends of the tube are virtually the same; now according to Ohm's law there should be a fall of potential along the entire circuit; it is accordingly concluded that the discharge is not a current in the ordinary sense of the term, but is disruptive, the electricity being carried by the molecules of the gas. At no degree of exhaustion is air a conductor.

All the strata start from the positive pole. For a definite pressure an aureole is formed at the positive pole; with a diminished pressure this detaches itself, is succeeded by others, and so on.

One of the most curious results is the definite and stationary character of the striæ for given conditions; they are remarkably permanent, and seem almost as if they could be manipulated; a single stratum may be seen falling down a tube like a feather in a vacuum, or like a drop of water. They are not produced in the same way as drops falling, but all and each of the little strata are so many Leyden jars.

The length of the arc found between two terminals varies with the square of the number of cells; thus while 1,000 cells give a spark of 0·0051 inch under ordinary atmospheric pressure, 11,000 cells give a spark of 0·62 inch.

With an increase of exhaustion the potential necessary to cause a current to pass diminishes to a certain pressure which represents an exhaustion of least resistance; from this it again increases, and the strata thicken and diminish in number until a point is reached at which no discharge takes place, however high be the potential.

A change in the current often produces an entire change in the colour of the stratification; thus in hydrogen the change is from blue to pink. If the discharge is irregular and the strata indistinct, an alteration in the strength of the current makes the strata distinct and steady. Even when the strata are apparently quite steady and permanent, a pulsation may be detected in the current by means of the telephone.

In the same tube, and with the same gas, a very great variety of phenomena can be produced by varying the pressure and the current. The peculiar luminosity and form of stratification in their various forms can be reproduced in the same tube or in others having similar dimensions.

The colour of the discharge in one and the same gas greatly depends on the degree of rarefaction. The least resistance to the discharge in hydrogen, and when its brilliancy is greatest, is at a pressure of 0·642 mm. or 845 M (M is a very convenient symbol for the millionth of an atmosphere). When the rarefaction has attained 0·002 mm. or 3 M, the discharge only just passes even with a potential of 11,330 volts ; while with an exhaustion of 0·000055 mm., the nearest approach to a perfect vacuum ever attained, not only does this fail to produce a discharge, but the 1-inch spark of an induction coil does not pass.

Air offers a greater resistance than hydrogen ; a spark which passes in hydrogen across a distance of 5·6 mm. will only strike across a distance of 3 mm. in air.

In air at a pressure of 62 mm., which corresponds to an atmospheric height of 12·4 miles, the electric discharge has the carmine tint so often seen in the display of the aurora borealis (1003) ; at a pressure of 1·5 mm., corresponding to a height of 30·96 miles, it is salmon-coloured ; and at a pressure of 0·8 mm., representing a height of 33·96 miles, it is of a pale white. Under a pressure of 0·379 mm. the discharge has the greatest brilliancy. This represents a height of 37·67 miles, and would be visible at a distance of 585 miles ; it is probably the upper limit of the height, though on the other hand it is possible that the discharge may sometimes take place at a height of a few thousand feet.

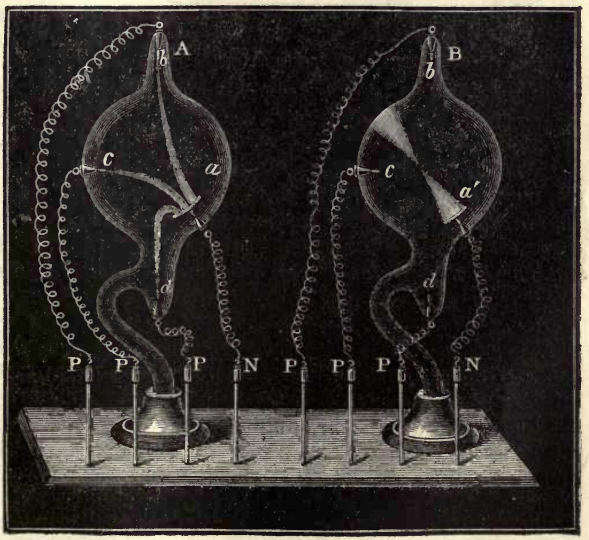

Fig. 946.

927. **Crookes's experiments.** —Dr. Crookes has made a remarkable series of experiments on the phenomena which occur when the electrical discharge takes place in tubes very highly exhausted, that is, beyond the point at which the best effects of the stratification are produced.

When the electrical discharge is passed through a Geissler's tube in which the exhaustion is as low as 2 mm., the negative pole is surrounded by a narrow layer, and then by a relatively dark bluish space, the rest of the tube being filled by layers of reddish-yellow light, separated by dark spaces ; as the rarefaction proceeds, the bluish light extends, and under certain circumstances fills the entire tube. Wherever the light strikes against the glass it excites the brightest fluorescence (882). But the most remarkable feature is that when the vacuum is almost complete the nature of the phenomenon changes. The light now proceeds from the electrode in straight lines, and does not follow any bends in the tubes. This rectilinear propagation is well illustrated by the following experiment of Crookes. In fig. 946, A, the negative pole of the induction coil, is connected with the electrode *a*, which is made of aluminum, and forms a slightly concave mirror. If the exhaustion is not more than 2 mm. pressure, and the positive pole is connected successively with the electrodes *b*, *c*, *d*, the discharge takes place in curved lines as shown in the figure. But when the rarefaction is exceedingly great, about a millionth of an atmosphere, the appearance is that presented in fig. 946, B. With whatever electrode the positive pole is connected, the rays proceeding in straight lines cross in the focus, and, striking against the opposite side, excite there the most brilliant fluorescence.

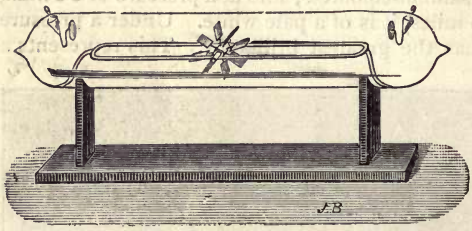

Fig. 947.

If a screen of mica of any shape be interposed in the path of the rays, it stops the light on its path, and a shadow is formed at the other end of its own shape, surrounded by a bright fluorescence.

The discharge can also produce mechanical effects. A Geissler's tube (fig. 947) is constructed with a pair of glass rails in it, on which rolls the axis of a light wheel, on the spokes of which are mica vanes. If now the discharge be directed against the top of the vanes, the wheel moves along towards the positive pole.

The experiment represented in fig. 948 shows the very great heat which the glow light can produce ; *a* is the negative electrode in the form of a concave mirror, *b* a strip of platinum foil. With a sufficiently powerful induction coil the platinum can be made white-hot or even melted.

Some of the most beautiful of these experiments are those made by directing the discharge on various precious stones. In these circumstances diamond emits a splendid green fluorescence, ruby a brilliant red, emerald a carmine, and so forth.

The electrical discharge does not pass through a vacuum, as is shown by the following experiment. A small tube containing caustic potash is fused to a Geissler's tube connected with a Sprengel pump. By continual exhaustion while the caustic potash is being heated, as complete a vacuum as possible is made and the tube sealed. The last minute trace of aqueous vapour is absorbed by the caustic potash as it cools. In this complete

vacuum the discharge, however strong, no longer passes; the vacuum acts as a complete non-conductor.

If, however, the caustic potash is gently heated, a trace of aqueous vapour is given off, and a green fluorescent light flashes along the tube; as the heating is continued and the vapour becomes denser we get the stratification, until ultimately the electricity passes along the tube in the form of a narrow purple line. If the tube is allowed again to cool, the phenomena reproduce themselves in the reverse order.

The phenomena here described are regarded by Crookes as due to an *ultra-gaseous* state, which he calls *radiant matter*. In gas under the ordinary pressure the average free path of a molecule of air is 0·0001 mm.; as the gas is more rarefied the length of the path increases, so that with the high degrees of exhaustion which Crookes employs in his later experiments—as much as the one twenty-millionth of an atmosphere—the length of the mean path is so much increased that its dimensions are comparable with those of the vessel, and along with this increase the number of intramolecular shocks diminishes in a corresponding ratio. It is to this condition, in which the molecules move forward with their own motion, and, striking against the sides, give rise to the fluorescence, that Crookes accounts for the effects produced.

Fig. 948.

The theoretical views to which Crookes has been led by his experiments have met with a considerable degree of criticism, and it must be added that none of the explanations of these singularly beautiful experiments have met with general adoption.

928. **Rotation of induced currents by magnets.**—De la Rive devised an experiment which shows in a most ingenious manner that magnets act on the light in Geissler's tubes in accordance with the laws with which they act on any other movable conductor.

This apparatus consists of a glass globe or electrical egg (fig. 948), provided at one end with two stopcocks, one of which can be screwed on the air-pump, and the other, which is a stopcock like that of Gay Lussac (383), serves to introduce a few drops of liquid into the globe. At the other end a tubulure is cemented, through which passes a soft iron rod about $\frac{1}{5}$ of an inch in diameter, the top of which is about the centre of the globe. Except at the two ends, this rod is entirely covered with a very thick insulating layer of shellac, then with a glass tube also coated with shellac, and finally with another glass tube uniformly coated with a layer of wax. The insulating layer must be at least $\frac{2}{5}$ of an inch thick. Inside the globe, the insulating layer is surrounded at x with a copper ring, connected with a binding screw, c, by means of a copper wire.

The vessel having been exhausted as completely as possible, a few drops of ether or of turpentine are introduced by means of a stopcock a; it is again exhausted, so that the vapour remaining is highly rarefied.

A thick disc of soft iron, *o*, provided with a binding screw, is then placed on one of the branches of a powerful electromagnet, and the end *m* of the rod *mn* is placed on this disc, while at the same time one of the ends of the secondary wire of Ruhmkorff's coil is connected with the binding screw, *c*, and the other with the knob, *o*. If then the coil is worked without setting in action the electromagnet, the electricity of the wire *s* passes to the top, *n*, of the soft iron rod, and that of the second wire to the ring *x*, and a more or less irregular luminous sheaf appears on the inside of the globe round the rod, as in the experiment of the electric egg.

But if a voltaic current passes into the electromagnet, the phenomenon is different; instead of starting from different points of the upper surface *n*, and the ring *x*, the light is condensed and emits a single arc, from *n* to *x*. Further—and this is the most remarkable part of the experiment—this arc turns slowly round the magnetised cylinder *mn*, sometimes in one direction, and sometimes in another, according to the direction of the induced current, or the direction of the magnetisation. As soon as the magnetisation ceases, the luminous phenomenon reverts to its original appearance.

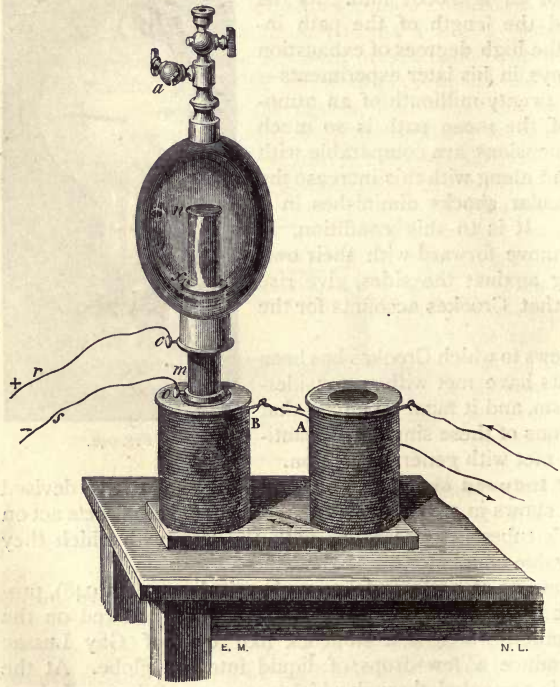

Fig. 949.

This experiment is remarkable as having been devised *à priori* by De la Rive to explain, by the influence of terrestrial magnetism, a kind of rotatory motion, from east to west, observed in the aurora borealis. The rotation of the luminous arc in the above experiment can evidently be referred to the rotation of currents by magnets (868).

Geissler has constructed a very useful form of the above experiment, in which the globe is exhausted once for all. Apart from the purpose for which it was originally devised, it is a very convenient arrangement for demonstrating the action of magnets on movable currents.

929. **Heat developed by the induction of powerful magnets on bodies in motion.**—We have already seen in Arago's experiments (914) that a rotating copper disc acts at a distance on a magnetic needle, communicating to it a rotatory motion. We shall presently see that a cube of copper, rotating with great velocity, is suddenly stopped by the influence of the poles of two strong magnets (938). It is clear that, in order to prevent the rotation of the needle or of the copper, a certain mechanical force must be expended in overcoming the resistance which arises from the inductive action of the magnet. Reasoning upon the theory of the transformation of mechanical work into heat (497), it has been attempted to ascertain what quantity of heat is developed by the action of induced currents under the influence of powerful magnets. Joule, with a view of determining the mechanical equivalent of heat, coiled a quantity of copper wire round a cylinder of soft iron, and having enclosed the whole in a glass tube full of water, he caused the system to rotate rapidly between the branches of an electromagnet. A thermometer placed in the liquid served to measure the quantity of heat produced by the induced currents in the soft iron and the wire round it.

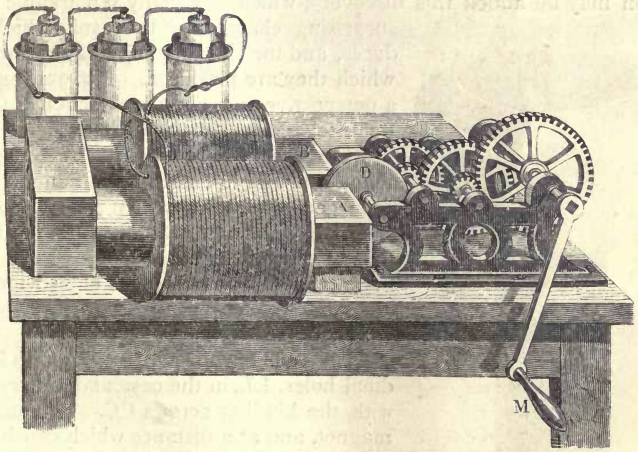

Fig. 950.

It was thus found that the heat developed was proportional to the square of the magnetism evoked, and was equivalent to the work used in the rotation.

Foucault made a remarkable experiment by means of the apparatus represented in fig. 950. It consists of a powerful electromagnet fixed horizontally on a table. Two pieces of soft iron, A and B, are in contact with the poles of the magnet, and, becoming magnetised by induction, they concentrate their magnetic inductive action on the two faces of a copper disc, D, 3 inches in diameter and a quarter of an inch thick; this disc partly projects between the pieces A and B, and can be rotated by means of a handle and a series of toothed wheels with a velocity of 150 to 200 turns in a second.

So long as the current does not pass through the wire of the electro-

magnet, very little resistance is experienced in turning the handle, and when once it has begun to rotate rapidly, and is left to itself, the rotation continues in virtue of the acquired velocity. But when the current is made to pass, the disc and other pieces stop almost instantaneously ; and if the handle is turned considerable resistance is felt. If, in spite of this, the rotation be continued, the force used is transformed into heat, and the disc becomes heated to a remarkable extent. In an experiment made by Foucault the temperature of the disc rose from 10° to 61°, the current being formed by three of Bunsen's elements ; with six the resistance was such that the rotation could not long be continued. The currents thus produced in solid conductors, and which are converted into heat, are often spoken of as *Foucault* or *eddy currents*.

Such currents are of constant occurrence in magneto-electrical machines, and weaken their force, first, by owing their existence to some part of the work expended ; secondly, they weaken the magnetism of the armatures by their direction ; and, lastly, they are converted into heat, which increases the internal resistance of the machine.

930. **The Telephone.**—To the number of instruments depending on induction may be added this discovery, which is equally remarkable for the surprising character of the results which it produces, and for the simplicity of the means by which they are produced. Fig. 951 represents a perspective, and fig. 952 a section of Graham Bell's telephone.

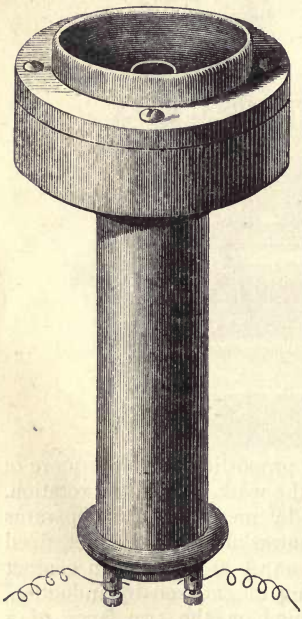

Fig. 951.

It consists essentially of a steel magnet, of about 4 inches in length by half an inch in diameter, enclosed in a wooden case. Round one end of this magnet is fitted a thin flat bobbin, BB, of fine insulated copper wire. For a magnet of this size a length of 250 metres of No. 38 wire, offering a resistance of 350 ohms, is well suited.

The ends of this coil pass through longitudinal holes, LL, in the case, and are connected with the binding screws CC. In front of the magnet, and at a distance which can be regulated by a screw, but which is something less than a millimetre, is the essential feature of the instrument, a diaphragm, D, of soft iron, not much thicker than a sheet of stout letter-paper. This diaphragm is screwed down by the mouthpiece E, which is similar to, though somewhat larger than, that of a stethoscope.

The instruments are connected by wires, for one of which the earth may be substituted, as in ordinary telegraphic communication (886). Each instrument can be used either as sender or receiver, though in actual practice it is more convenient for each operator to have two telephones, one of which is held to the ear, while the other is used for speaking into ; the latter being larger and more powerful than the receiver.

The action of the instrument depends on the fact that whenever the relative positions of a magnet and of a closed coil of wire are altered there is produced within the coil a current or currents of electricity. This may be illustrated by reference to fig. 892. When the magnet is suddenly brought into the coil, a current is produced in the coil in a particular direction. There is no current so long as the coil and the magnet are stationary. When, however, the magnet is suddenly withdrawn, a current is produced in the opposite direction. Similar effects are produced if, while the magnet is in the coil, its magnetism is by any means increased or diminished.

Now in the telephone the magnet and the coil, when once properly adjusted, remain fixed. But the magnet M magnetises by induction the soft iron membrane D in front of it—that is, converts it into a magnet. When, by the mouthpiece being spoken into, this iron membrane vibrates backwards and forwards, these vibrations give rise to an alteration in the magnetism of the permanent magnet, the effect of which is that currents are produced in alternate directions in the coil surrounding the pole. Moreover, the alteration in the relative positions of the magnetised diaphragm, thus magnetised by induction, and of the coil, give rise to currents in the same direction as the above. These alternating currents, being transmitted through the circuit to the distant coil, alternately attract, and cease to attract, the corresponding diaphragm. They thereby put this in vibration, and when the mouthpiece of this telephone is held to the ear, these vibrations are perceived as sound corresponding to that which is transmitted. Hence, whatever sound produces the vibration of the diaphragm of the sending instrument is repeated by that of the receiver.

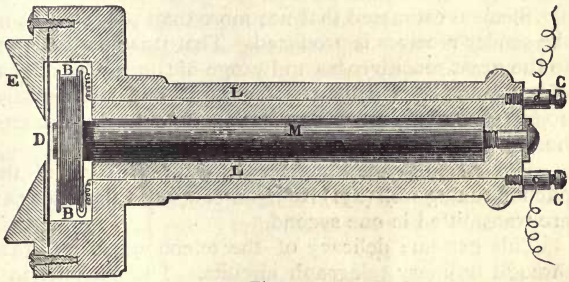

Fig. 952.

The reproduction of the sound in the receiving instrument is perfect as far as articulation is concerned, but it is considerably enfeebled, as might be expected. The sound has something of a metallic character, appearing as if heard through a long length of tubing, while it faithfully reproduces the characteristics of the person speaking. It does not result from a series of sharp and distinct makes and breaks, but in each of the momentary currents there is a continuous rise and fall, corresponding in every gradation and inflection to the motion of the air agitated by the speaker.

Various attempts have been made to improve the loudness of the sounds produced in the telephone, by varying the form of the various parts, and using more powerful magnets of horseshoe and circular forms; *Ader's* telephone, which is largely used in France, is constructed with a circular horseshoe magnet. Experiment shows that with telephones increased loudness is in general produced at the expense of distinctness.

The amplitude of the vibration of the disc is extremely small. According to Bosscha a unit current produced a displacement of 0·034 of a mm., and as currents of $\frac{1}{10000}$ of this are perceptible, it follows that the amount of displacement must be about the $\frac{1}{200}$ of the wave-length of yellow light (637).

The current in a telephone is estimated by De la Rue as not exceeding that which would be produced by one Daniell's cell in a circuit of copper wire 4 mm. in diameter of a length sufficient to go 290 times round the earth. This current would have to pass 19 years through a voltameter, to produce 1 cc. of detonating gas. This is about 1,000 million times less than the currents in ordinary use. Such currents are, however, sufficient to cause the contraction of a frog's leg (797). According to Pellat, the energy contained in one heat unit (water-gramme-degree) would maintain a continuous sound for 10,000 years.

Siemens estimated that not more than $\frac{1}{100000}$ of the mass of sound which the sender receives is produced. That it is possible to perceive this, is due to the great sensitiveness and range of the ear, which can endure the sound of a cannon at a distance of 5 yards, and still perceives it at a distance 10,000 times as great. This represents a ratio of intensities of one to one hundred millions.

From some experiments on the transmission of the sound of a high-pitched tuning-fork (251) Röntgen concludes that no less than 24,000 currents are transmitted in one second.

This extreme delicacy of the telephone is its drawback to speaking through ordinary telegraph circuits. The currents in adjacent wires, the vibration of the posts and of the insulators, or the passage of a cart over the streets, acts by induction on the telephone circuit, and overpowers its indications. When a telephone circuit was placed at a distance of 20 metres from a well-insulated line, through which signals were sent by means of a battery of a few elements, sounds were distinctly heard in the telephone. Speaking under such circumstances is like speaking in a storm. The powerful currents used for systems of electric lighting produce such a roar in an adjacent telephone circuit that it is impossible to speak through the telephone. The only effective way of diminishing the inductive action of adjacent systems is to have two insulated wires close to each other, forming a *loop circuit*. They are thus at the same distance from the inducing circuit, and as one of the wires is used for going and the other for returning, the similar influences must be in opposite directions, and therefore neutralise each other.

Iron wires present a special difficulty in telephoning through long distances. Telephone currents are alternating ones, and at each reversal an extra current is produced, which enfeebles the original one, and alters its character. This extra current is more pronounced the longer the circuit, and with iron it is 300 times as strong as with copper. Hence for long distances a loop circuit of copper or bronze wire is used, and with such circuits it is possible to telephone through very long distances. In America, cities at a distance of 750 miles apart are in telephonic communication ; the greatest distance in Europe is from London to Marseilles, *viâ* Paris.

If a telephone is inserted in the circuit of a Morse's instrument, the sound of the working is heard, and the messages can be read ; this is the

case also of the telephone in the branch circuit of a Morse. If one telephone is joined up with the primary, and another with the secondary wire of an induction coil, communication is almost as good as if the two apparatus were directly united.

Telephones have been constructed in which the thin iron plate is replaced by a thicker one, or by an unmagnetic one; or if the telephone is held close to the ear, the plate can be dispensed with altogether. In the latter two cases the sounds are only perceived when the spiral surrounding the magnet can vibrate with it.

A telephone may be constructed with a rod of soft iron instead of a magnet; when the rod is held in the line of dip, and the mouthpiece is sung into, the sounds are reproduced.

From its extreme sensitiveness, being perhaps the most delicate galvanoscope we possess, the telephone has become of great service in scientific research. It may be used instead of a galvanometer in a Wheatstone's bridge. If inserted in either of the circuits of an induction coil, the number of breaks can be determined from the height of the tone which is produced. When inserted in the current of a Holtz's machine, the disc of which is rotating with a uniform velocity, the height of the note varies with the resistance of the circuit, and with the capacity of the condensers. It can be shown also that the circumstances most favourable for the production of a most distinct stratification in a Geissler's tube correspond to a definite pitch in the telephone.

The telephone has been used to test hardness of hearing. If the magnetism of a telephone be excited by galvanic currents which are made intermittent by a vibrating tuning-fork, and if a telephone is inserted in a branch circuit (961), then by varying the strength of the principal current, by the insertion of resistances, the strength of the sounds in the telephone may be varied at will.

When a telephone is held to the ear during a thunderstorm, every lightning flash in the sky is simultaneously heard to be accompanied by a sharp crack.

Dolbear has constructed a telephone in which the electrostatic action of currents is used. It consists of two circular flat discs of metal rigidly fixed to each other in an insulated case of ebonite. One of the discs is in metallic connection with the line wire, in which is a battery and an induction coil; in this way, while one disc is electrified positively, the other is negatively electrified by induction, and if the current be varied by speaking through a transmitter in the circuit their varying effects are faithfully reproduced, and reappear as sound vibrations on the receiver.

931. **The Microphone.**—When the wires of an electrical circuit, in which is interposed a telephone, are broken, and rest loosely on each other, sounds produced near the point of contact are reproduced and magnified in the telephone. The *microphone*, invented by Prof. Hughes, depends on this fact; its arrangement may be greatly varied; one of the simplest and most convenient forms is that represented in fig. 953. A piece of thin wood is fitted vertically on a base of the same material; two small rods of gas carbon, C C, about $\frac{1}{8}$ of an inch thick, are fixed horizontally in the upright; by means of binding screws, they are in metallic connection with the wires of a circuit

in which is a small battery and a telephone ; and in each of them is a cavity. A third piece, D, of the same material, and about one inch long, is pointed at each end, one of which rests in the lower cavity, while the other pivots loosely in the upper one. When a watch is placed on the base B, its ticking is heard in the telephone with surprising loudness ; the walking of a fly on the base suggests the stamping of a horse ; the scratching of a quill, the rustling of silk, the beating of the pulse, are perceived in the telephone at a distance of a hundred miles from the source of sound ; while a drop of water falling on the base has a loud crashing sound. To obtain the best results with a particular instrument, the position of the carbon must be carefully adjusted by trial ; and indeed the form of the instrument itself must be variously modified for the special object in view : in some cases great sensitiveness is required : in others great range. In order to eliminate as far as possible the effect of accidental vibrations due to the supports, the base should rest on pieces of vulcanised tubing, or on wadding.

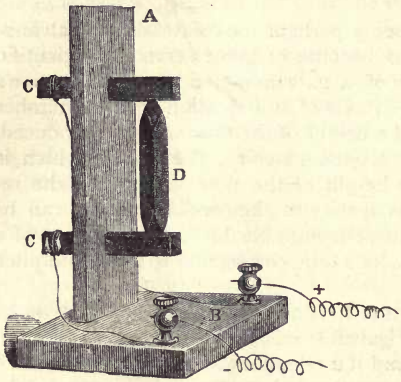

Fig. 953.

It is known that the compression of a semiconductor, such as carbon, diminishes its resistance, while a diminution in the compression increases the resistance. The action of the microphone is to be ascribed to this ; in consequence of the minute alterations in the pressure and in the degree of contact at the break, the electrical resistance in the circuit varies in accordance with the sound-waves, and consequently the strength of the current varies too. The result of this is, that what we may call undulating currents of electricity are produced, whose amplitude, height, and form are in exact correspondence with the sound-waves. The effect of the microphone is to draw supplies of energy from the battery, which then appear in the telephone.

932. **Hughes's induction balance.**—The principle of this apparatus may be thus stated :—Suppose we have two exactly equal primary induction coils, A and A′, and near them two secondary coils, B and B′, also exactly equal, and connected up with a galvanometer, so that the coils act upon it in opposite directions. If now the current of a battery be sent through the primary coils, joined in series, the inductive effects on each of the secondary coils will be the same, and, as their action on the galvanometer is opposed, no deflection of the needle will be produced. If, however, a piece of iron be introduced into the core of one of the secondary coils, the equality in the induction effects will be destroyed, and the needle of the galvanometer at once deflected.

This principle was first applied by Babbage, Herschel, and in a special apparatus by Dove ; but the microphone and the telephone have led the inventor of the former to the invention of an apparatus which has opened

out new possibilities, and has placed in the hands of the physicist an elegant and powerful engine of research, which in certain departments of investigation promises to be of great service.

The form of instrument as devised by Professor Hughes is represented in fig. 954, where the essential parts are drawn to scale, though the relative distances of the parts are not so; a and a' are the two primary coils, each of which consists of 100 metres of No. 32 silk-covered copper wire (0·009 in diameter) wound on a flat boxwood spool 10 inches in depth; b and b' are two secondary coils, all four coils being, in intention at least, exactly alike. The two primary coils are joined in series with a battery of three or four small Daniell's cells, in which circuit a microphone, m, is also inserted; the ticking of a small clock on the table acts as make and break.

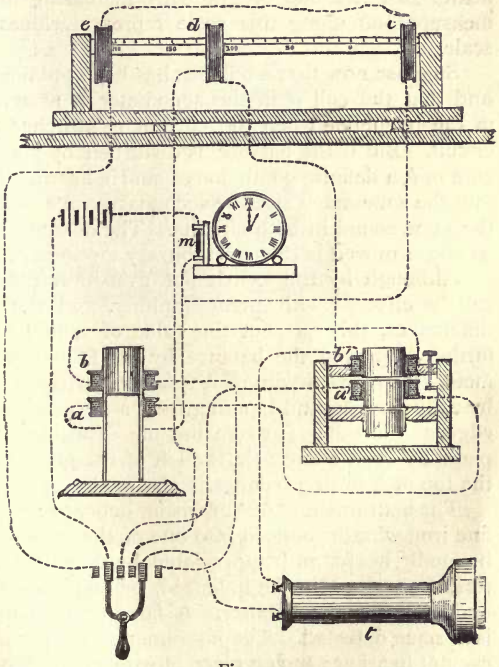

Fig. 954.

The secondary coils are joined up with a telephone in such a manner that their action upon it is opposed.

Now, whatever care be taken in winding the wire on the coils, it is not possible to get at the outset an exact balance. Hence, while one of the secondary coils, b, is at a fixed distance from a, the corresponding one, b', is not so; its distance from a' can be slightly modified by means of a micrometric screw, and thus, connection with the battery circuit having been made, a balance is obtained by slightly varying the adjustment, and the accomplishment of this is known by there being silence in the telephone. But if now any metal whatever be introduced in one of the secondary coils, a sound is at once heard.

This arrangement is so far a simple differential one, and furnishes as yet no means of measuring the forces brought into play, and for this purpose Hughes uses what is called a *sonometer* or *audiometer*. This consists of three similar coils, c, d, and e, placed vertically on a horizontal graduated rule along which d can be moved. By means of a *switching key* or *switch*, the primary coils c and e can be put in communication with the battery and microphone circuit quite independently of the balance, and it is so arranged that the ends of the coils c and e facing each other are of the same

polarity; the third coil, *d*, the secondary one, is connected with the telephone circuit.

If these primary coils *c* and *e* were quite equal, then, when connected up with the battery circuit, no sound would be heard in the telephone, when the secondary *d* is exactly midway between them. But as the coil is moved from this position either towards *c* or *e* a sound is heard, due to the preponderance of one or the other. In practice the coils are so arranged that a balance is obtained when the secondary circuit is near one of the coils, *c* for instance; this represents a zero of sound, and as the coil *d* is moved nearer to *e* a sound of gradually increasing intensity is heard; distances measured off along this scale represent values of sound on an arbitrary scale.

Suppose now that a balance has been obtained in the induction balance, and that the coil *d* in the sonometer is at zero; no sound is then heard in the telephone when the current is switched either in one or the other circuit. But if the balance is disturbed by placing a piece of metal in the core of *b*, a definite continuous sound is heard. The current is then switched into the sonometer, and the secondary coil *e* is moved until the ear perceives the same sound in both circuits. The distance then along which the coil *d* has been moved is thus an arbitrary measure of the effect produced.

Although by the switch the transition from one circuit to the other can be effected with great rapidity, and the ear can appreciate minute differences, this has not the value of a null method. Hughes has still further improved the balance by the following device, in which the sonometer is dispensed with:—A graduated strip of zinc about 200 mm. in length by 25 mm. wide, and tapering from a thickness of 4 mm. at one end to a fine edge at the other, is made use of. The metal to be tested is placed in a plane between *a* and *b* on the left of the plate, and the strip is moved along the top of *b'* until a balance is obtained.

The instrument is of surprising delicacy; a milligramme of copper or a fine iron wire introduced into one of the coils which has been balanced can be loudly heard, and appreciated by direct measurement. If two shillings fresh from the Mint be balanced, rubbing one of them or breathing on it at once disturbs the balance. A false coin balanced against a genuine one is at once detected. The instrument furnishes a means of testing the delicacy of hearing; such a piece of wire as the above, or a fine spiral of copper, furnishes a kind of test object for this purpose.

933. **Tasimeter.**—This instrument, invented by Edison, consists essentially of an arrangement by which a disc of carbon forming part of a voltaic circuit is exposed to varying pressure. It depends on the fact that the resistance of carbon varies very greatly with the pressure to which it is exposed. It consists of an iron base, on which are two rigid supports (fig. 955), one of which, *a*, is connected with the galvanometer, *g*, by means of a wire. An ebonite disc, *d*, is screwed into *a*, and in a circular cavity in this ebonite is a small carbon disc, not shown in the figure, in the outer surface of which is a strip of platinum in metallic connection with one pole of an element, *l*. The disc of carbon is closed in the cavity by a metal plug, *c*, in which is a cavity. There is a similar plug, *e*, with a corresponding cavity at the end of a screw, *b*, which works in the upright support; in

the two cavities is placed the strip of substance, *f*, with which the experiment is made.

A gentle pressure being applied by the screw, the needle is deflected through a few degrees, and its position, when it comes to rest, is noted. The slightest subsequent contraction or expansion is indicated by a deflection of the needle of the galvanometer.

The sensitiveness of the instrument is very great ; a thin strip of ebonite is expanded oy the heat of the hand held near it, so as to affect a not very delicate galvanometer. A strip of gelatine, inserted instead of the ebonite,

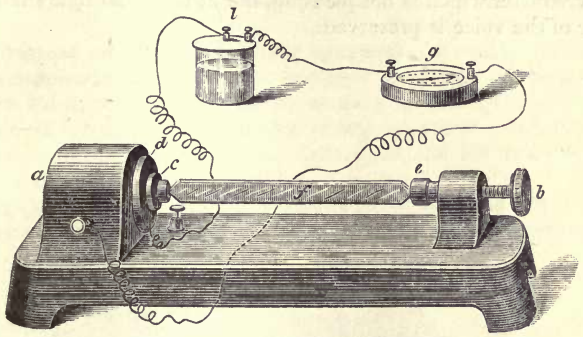

Fig. 955.

is expanded by the moisture of a damp strip of paper held two or three inches away.

The apparatus seems well adapted for the qualitative observation of minute changes in length ; it has been used, for instance, to show the very small elongation of an iron rod when it is magnetised (883). Great care is required in the preparation of the carbon disc ; the best kind seems to be made from lampblack prepared at a low temperature, and then powerfully compressed into a button.

934. **Edison's loud-speaking telephone.**—Although depending on a different principle, we may give a description here of this instrument.

An adjustable metal spring passes on the surface of a small cylinder, made of chalk, moistened with solutions of caustic potash and acetate of mercury ; both the spring and the cylinder form part of a circuit in which is a battery and a Reis's transmitter (885). The spring is connected in a suitable manner with a mica disc, which is the vibrating part of a mouthpiece like that of an ordinary telephone. The cylinder can be turned at a uniform rate, either by hand or by an automatic clockwork arrangement.

Now while the spring is pressing on the cylinder, if the latter be rotated in a direction away from the mouthpiece, in consequence of the friction between the spring and the surface of the cylinder, a certain pull will be exerted on the disc, which will tend to drag it outwards. If the direction of rotation were the opposite, the disc would be pushed inwards. Now the amount of pull or push will depend on the friction between the point and the surface. If a momentary current be passed, there will be a momentary decomposition at the surface of the cylinder, its friction will be altered in

consequence of this momentary decomposition, the effect of which is that the disc moves inwards, and a series of such intermissions of the current produces a corresponding series of pulsations of the disc which if sufficiently rapid produce a sound. The friction of the surfaces in contact is in fact modified by means of electrical decomposition, a lubricator is liberated in correspondence with the sound-waves, and thus the sound which they represent is reproduced. The reproduction is so loud as to be heard throughout a room, the sounding instrument being at a distance. Although ordinary speech and music can thus be transmitted, yet the sounds have a harsh metallic character which is not pleasing, but at the same time the individual character of the voice is preserved.

CHAPTER VII

OPTICAL EFFECTS OF POWERFUL MAGNETS. DIAMAGNETISM

935. **Optical effects of powerful magnets.**—Faraday discovered, in 1845, that a powerful electromagnet exercises an action on the propagation of light in substances exposed to its action; thus if a polarised ray traverses them in the direction of the line of the magnetic poles, the plane of polarisation is deviated either to the right or to the left according to the direction of the magnetisation.

Fig. 956 represents Faraday's apparatus, as constructed by Ruhmkorff. It consists of two very powerful electromagnets, M and N, fixed on two iron

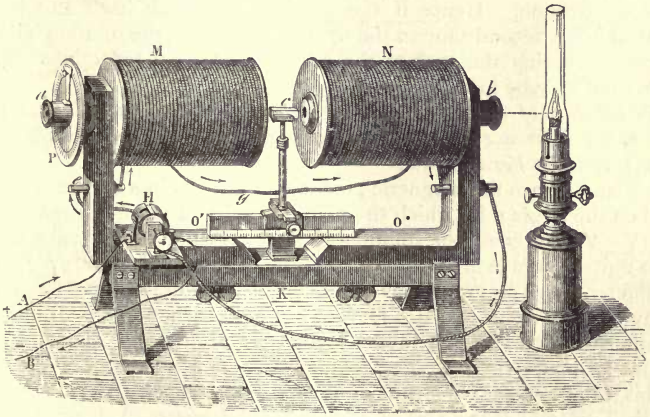

Fig. 956.

supports, O O′, which can be moved on a support, K. The current from a battery of 10 or 12 Bunsen's elements passes by the wire A to the commutator, H, the coil M, and then to the coil N, by the wire *g*, descends in the wire *i*, passes again to the commutator, and emerges at B. The two cylinders of soft iron, which are in the axis of the coils, are perforated by cylindrical holes, to allow the light to pass. At *b* and *a* there are two Nicol's prisms, *b* serving as polariser and *a* as analyser. By means of a limb this latter is turned round the centre of a graduated circle, P.

The two prisms being then placed so that their principal sections are perpendicular to each other, the prism *a* completely extinguishes the light transmitted through the prism *b*. If at *c*, on the axis of the two coils, a plate be placed with parallel faces, either of ordinary or flint glass, light supposed

to be monochromatic is still extinguished so long as the current does not pass; but when the connections are made, the light reappears, and in order to extinguish it the analyser must be turned through an angle which can be read off on the limb, and which measures the rotation. By reversing the direction of the current twice the rotation is observed. If the source of light is not monochromatic, and if the analyser be turned from left or right, according to the direction of the current, the light passes through the different tints of the spectrum, as is the case with plates of quartz cut perpendicularly to the axis (675). Becquerel showed that a large number of substances can also rotate the plane of polarisation under the influence of powerful magnets. The direction of the rotation for a given substance is independent of the direction in which the rays of light pass; and also of whether the propagation of the light is in the direction of the lines of force, or in the opposite direction. Hence if the ray is reflected on itself and traverses the substance a second time in the opposite direction, the rotation is doubled. By thus increasing the path of the ray by successive reflections (fig. 957), the rotation may be increased in the same proportion.

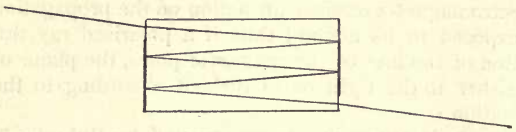

Fig. 957.

The rotation of the plane of polarisation between two points is proportional to the difference of magnetic potential which exists *between these points.* This is known as *Verdet's law.*

If V and V′ are the magnetic potentials at two points on the path of the ray, then the angle θ by which the plane of polarisation has been turned is $\theta = \omega\,(V - V')$; ω being the rotation which for the body in question would be due to unit difference of potential. This quantity is called *Verdet's constant.* For different rays it is *nearly* as the inverse square of the wave length. For the ray D and at 0° it is 0′·040 for bisulphide of carbon and 0′·013 for water. It diminishes with rise of temperature. It is positive for diamagnetic and negative for magnetic substances.

This leads to a simple means of determining a current in absolute measure by optical means; it is a sort of *optical galvanometer.* A long tube closed at each end by glass plates (fig. 958) is filled with some liquid which has a high value for *Verdet's constant*, and is surrounded by a coil consisting of n turns traversed by a current i. If the tube is so long that the influence of the coil at the ends may be neglected, the ray passes from a point where the potential is zero to another where it is also zero, and in so doing traverses the current n times, and the potential has each time varied $4\pi i$; accordingly the observed rotation θ is

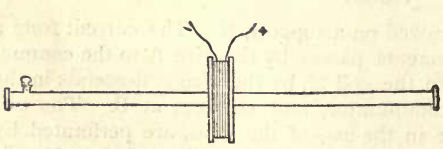

Fig. 958.

$$\theta = \omega 4\pi n i$$

from which i can be calculated.

By means of Faraday's apparatus it has been found that thin layers of iron, cobalt, and nickel, so fine as to be transparent, exert a powerful rotation of the plane of polarisation for transmitted light. The rotation for the central rays of the spectrum in iron is 32,000 times that of glass of the same thickness. In all the above substances the rotation is in the direction of the magnetising current.

936. **Photophone.**—Mr. Graham Bell, the inventor of the telephone has invented an apparatus by which articulate speech can be transmitted to a considerable distance by the simple agency of a ray of light.

The essential features of the apparatus are represented in fig. 959, in which *m* is the *transmitter*. This consists of a wooden box closed by a thin plate of microscope glass silvered in front, which acts as mirror; in the

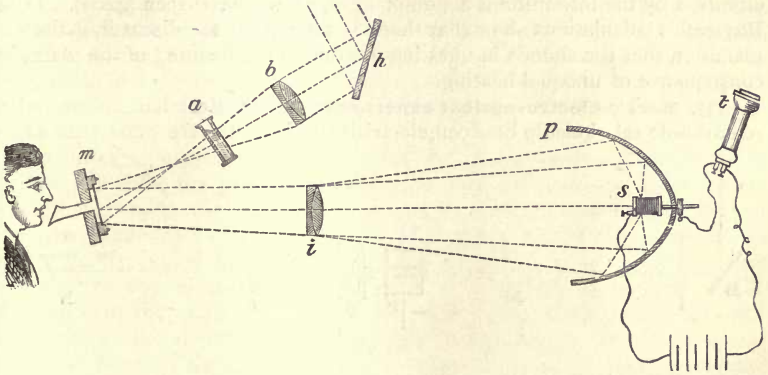

Fig. 959.

back of the box is an aperture provided with a flexible tube and mouthpiece. A powerful beam of solar or of the electrical light falls against a large mirror, *h*, and is reflected by it on a lens, *b*, by which the rays are concentrated on the mirror, *m*, of the transmitter. An alum cell, *a*, is sometimes interposed, to cut off the influence of the heating rays.

From the mirror *m* the reflected rays pass through a lens, *i*, by which they are rendered parallel, and fall on a parabolic mirror, *p*, at the distant station. Here they are concentrated on what may be called a *selenium rheostate*, *s* which is interposed in a circuit consisting of a few Leclanché cells and a telephone, *t*.

The action depends on the alterations in the resistance of selenium produced by the action of light. The construction of the rheostat is as follows:—A number of discs of thin sheet brass are taken, separated from each other by thin discs of mica of somewhat smaller diameter, and, the whole having been tightly screwed together, the interstitial spaces are filled with melted selenium. All the odd numbers of brass discs are in metallic connection with each other and with one pole of the circuit, and all the even ones are also in metallic connection with each other and with the other pole. In this way two conditions are realised—namely, that the surface of selenium exposed to the action of light is as large, and its resistance as small, as possible.

This being premised, when light falls on the plane mirror at rest, its rays are reflected parallel against the parabolic mirror by which they are concentrated on the cell, the cylindrical shape being well adapted for this. But if, by being spoken against, the transmitting mirror *m* is put in vibration, it bulges in and out—that is, becomes convex and concave—and the rays no longer fall parallel on the parabolic mirror; they diverge or converge—in other words, the whole of the light is no longer concentrated on the selenium cell: its intensity changes at every instant, and these variations in the action of the light produce corresponding variations in the resistance of the selenium, which again produce corresponding variations in the strength of the current, and these are revealed by the articulate sounds of the telephone.

Mr. Bell has found that a great number of substances are thrown into vibration by the intermittent action of light, as we have seen (446*a*). Lord Rayleigh's calculations show that there is no reason for discarding the explanation that the sounds in question are due to the bending of the plates in consequence of unequal heating.

937. **Kerr's electro-optical experiments.**—Dr. Kerr has discovered a remarkable relationship between electricity and light. He finds that when

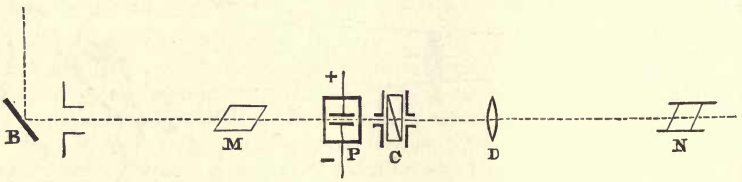

Fig. 960.

certain dielectrics are subjected to a state of electrical strain, they develop doubly refringent properties (639). The general arrangement of the experiments is as follows: a cell, P (fig. 960), is suitably constructed of stout glass plates, in which is placed the liquid under examination; its dimensions are 4 inches in length by 1 inch in width, and about $\frac{1}{8}$ of an inch in thickness. Two copper plates placed horizontally, and kept at a distance of about $\frac{1}{12}$ of an inch, can be connected with the poles of a Holtz machine (fig. 705), or, what is more convenient, with the opposite coatings of a Leyden jar, which in turn is worked by such a machine. B is the mirror of a heliostat, by which a beam of light may be sent in any direction. M and N are two Nicol's prisms (660); C is a compensator, while D is a condensing lens.

Of the two Nicol's prisms, M serves as polariser, and N as analyser (656); at the outset they are arranged so that their principal sections are at right angles to each other, and make an angle of 45° with the vertical. Thus the light polarised by the prism M is extinguished by the analyser N, so that the field between them is quite dark, and remains so even when the cell is filled with liquid; the cell is so arranged that the observer looks through the slit of dielectric which is between the conductors in the cell.

If now the plates are placed in opposite electrical conditions, the field at once becomes clear. Of all dielectrics hitherto examined, carbon bisulphide

is that which best exhibits the phenomenon. A fraction of a turn of a Holtz machine is at once sufficient to produce light in the field, which disappears immediately the plates are discharged. As the machine is worked and the potential rises, the light between the conductors gradually increases in brightness until a pure and brilliant white is obtained; with increase of potential a fine progression of chromatic effects is obtained; the luminous band between the conductors changes first from white to a straw colour, which deepens gradually to a rich yellow; it then passes through orange to a deep brown, next to a pure and dense red, through purple and violet to a rich and full blue, and then to green. All the colours are beautifully dense and pure, and as fine as anything seen in experiments with crystals in the polariscope. The phenomenon generally ceases at the green of the second order with a discharge of electric sparks. The action of bisulphide of carbon under electrical strain is similar to that of glass stretched in a direction parallel to the lines of force; it is an action of the same kind as that of a uniaxial bi-refringent crystal (640); in this respect carbon bisulphide occupies a place among dielectrics similar to that of Iceland spar among crystals.

In order to measure the effect produced, a compensator, C, is placed behind the cell; the plates are connected with a Thomson's electrometer in such a manner that the potential can be directly measured, and then compared simultaneously with the difference of the path of the extraordinary and ordinary ray in the dielectric. Kerr arrived thus at the law: 'the strength of the electro-optical action of a given dielectric, that is, the difference in the path of the ordinary and extraordinary rays, for unit thickness of the dielectric, varies directly as the square of the resultant electrical force.' Kerr also found that when a pencil of plane polarised light is reflected from the polished surface of either pole of an electromagnet of iron, it undergoes a rotation in a direction contrary to that of the magnetising current. This result is also obtained when it is reflected from the sides of the electromagnet when the magnet is excited.

938. **Diamagnetism.**—Coulomb observed, in 1802, that magnets act upon all bodies in a more or less marked degree: this action was at first attributed to the presence of ferruginous particles. Brugmann also found that certain bodies—for instance, bars of bismuth—when suspended between the poles of a powerful magnet, do not set *axially* between the poles, that is, in the line joining the poles, but *equatorially*, or at right angles to that line. In other words, while a magnetic substance such as iron sets *along* the lines of force of the magnetic field, a bar of bismuth sets *at right angles* to the field. This phenomenon was explained by the assumption that the bodies were transversely magnetic. Faraday made the important discovery in 1845 that *all* solids and liquids which he examined are either attracted or repelled by a powerful electromagnet. The bodies which are attracted are called magnetic or *paramagnetic*, or also *ferromagnetic*, substances, and those which are repelled or take a magnetisation opposite that of the lines of force are *diamagnetic* bodies. Among the metals, iron, nickel, cobalt, manganese, platinum, cerium, osmium, and palladium are magnetic; while bismuth, antimony, zinc, tin, mercury, lead, silver, copper, gold, and arsenic are diamagnetic, bismuth being the most so and arsenic the least. Diamagnetic

effects were first observed by Faraday in a particular kind of glass called *Faraday's heavy glass*; they can only be produced by means of very powerful magnets, and it is by means of Faraday's apparatus that they have been discovered and studied. In experimenting on the diamagnetic effects—solids, liquids, and gases—armatures of soft iron, S and Q (figs. 961–963), of different shapes, are screwed on the magnets.

i. *Diamagnetism of solids.* If a small cube of copper, suspended by a fine silk thread between the poles of the magnet (fig. 962), be in rapid rotation between the poles of an electromagnet, it stops the moment the current passes through the coils. If the movable piece have the form of a small rectangular bar it sets *equatorially*, or at right angles to the axis of the bobbins, if it is a diamagnetic substance, such as bismuth, antimony, or copper; but *axially*, or in the direction of the axis, if it is a magnetic substance, such as iron, nickel, or cobalt. Besides the substances enumerated above, the following are diamagnetic: rock crystal, alum, glass, phosphorus, iodine, sulphur, sugar, bread; and the following are magnetic: many kinds of paper and sealing-wax, fluorspar, graphite, charcoal, &c.

ii. *Diamagnetism of liquids.* To experiment with liquids, very thin glass tubes filled with the substance are suspended between the poles instead of the cube *m* in the figure 962. If the liquids are magnetic, such as solutions

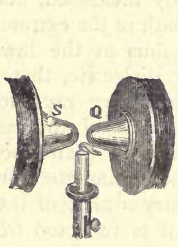

Fig. 961.

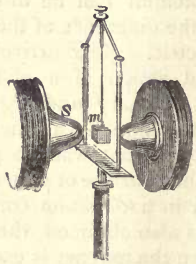

Fig. 962.

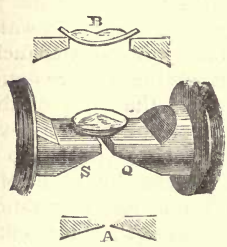

Fig. 963.

of iron or cobalt, the tubes set axially; if diamagnetic, like water, blood, milk, alcohol, ether, oil of turpentine, and most saline solutions, the tubes set equatorially. Very remarkable changes take place in the direction of magnetic and diamagnetic substances when they are suspended in liquids. A magnetic substance is indifferent in an equally strong magnetic liquid; it sets equatorially in a stronger magnetic substance, and axially in a substance which is less strongly magnetic; it sets axially in all diamagnetic liquids.

A diamagnetic substance surrounded by a magnetic or diamagnetic substance sets equatorially. According to its composition glass is sometimes magnetic and sometimes diamagnetic, and as glass tubes are used for containing the liquids in these investigations, its deportment must first be determined, and then taken into account in the experiment.

The action of powerful magnets on liquids may also be observed in the following experiment devised by Plücker. A solution of a magnetic liquid is placed on a watch-glass between the two poles, S and Q, of a powerful

electromagnet. When the current passes, the solution forms the enlargement represented in fig. 963 ; this continues as long as the current passes, and is produced to different extents with all magnetic liquids. The changes in the aspects of the liquids are, however, so small as to require careful scrutiny to detect their existence. A method of magnifying these changes so as to render them visible to larger audiences was devised by Prof. Barrett. A source of light is placed above the watch-glass containing a drop of the solution to be tried. Below the watch-glass, and between the legs of the magnet, is placed a mirror at an angle of 45°. By this means the beam of light passing through the watch-glass is reflected at right angles on to a screen, where an image of the drop is focussed by the lens. If now a drop of diamagnetic liquid, such as water, or, better, sulphuric acid, be placed on the watch-glass, as soon as the current passes, the flattened drop retreats from the two poles, and gathers itself up into a little heap, as at A (fig. 963). So doing, it forms a double convex lens, by which the light is brought to a short focus below the drop, an effect instantly seen on the screen. When the current is interrupted the drop falls, and the light returns to its former appearance. A magnetic liquid, such as a solution of perchloride of iron, has exactly the opposite effect. The drop attached to the two poles becomes flattened, and instead of a plano-convex shape, at which it rests, it becomes nearly concavo-convex, as at B. The light is dispersed, and the effect manifest on the screen. Instead of a mirror and lens, a sheet of white paper may be placed in an inclined position under the watch-glass, and the effects are somewhat varied, but equally well pronounced.

iii. *Diamagnetism of gases.* Bancalari observed that the flame of a candle placed between the two poles in Faraday's apparatus was strongly repelled (fig. 961). All flames present the same phenomenon to different extents, resinous flames or smoke being most powerfully affected.

The magnetic deportment of gases may be exhibited for lecture purposes by inflating soap bubbles with them between the poles of the electromagnet, and projecting on them either the lime or the electric light.

Faraday experimented on the magnetic or diamagnetic nature of gases. He allowed gas mixed with a small quantity of a visible gas or vapour, so as to render it perceptible, to ascend between the two poles of a magnet, and observed their deflections from the vertical line in the axial or equatorial direction ; in this way he found that oxygen was least, nitrogen more, and hydrogen most diamagnetic. With iodine vapour, produced by placing a little iodine on a hot plate between the two poles, the repulsion is strongly marked. Becquerel found that oxygen is the most strongly magnetic of all gases, and that a cubic yard of this gas condensed would act on a magnetic needle like 5·5 grains of iron. This magnetism of gases may be shown by suspending a glass globe to the pan of a balance, above the pole of a powerful magnet ; this globe being exhausted, it is exactly counterpoised, and having been filled with a given gas, the weight is ascertained which is required to detach them. With oxygen the attraction is appreciable, and is five times as much as air under the same pressure. Faraday found that oxygen, although magnetic under ordinary circumstances, became diamagnetic when the temperature was much raised, and that the magnetism or diamagnetism of a substance depends on the medium in which it is placed.

A substance, for instance, which is magnetic in vacuo may be diamagnetic in air.

In the crystallised bodies which do not belong to the regular system, the directions in which the magnetism or diamagnetism of a body is most easily excited are generally related to the crystallographic axis of the substance. The optic axis of the uniaxial crystals sets either axially or equatorially when a crystal is suspended between the poles of an electromagnet. Faraday assumed from this the existence of a *magneto-crystalline* force, but it appears probable from Knoblauch's researches that the action arises from an unequal density in different directions, inasmuch as unequal pressure in different directions produces the same result.

According to Plücker, for a given unit of magnetising force, the specific magnetisms developed in equal weights of the undermentioned substances are represented by the following numbers, those bodies with the minus signs prefixed being diamagnetic :—

Iron . . .	1,000,000	Nickel oxide . .	287
Cobalt . . .	1,009,000	Water	−25
Nickel . . .	465,800	Bismuth . . .	−23·6
Iron oxide . .	759	Phosphorus . . .	−13·1

iv. *Detonation produced by the rupture of a current under the influence of a powerful electromagnet.* The following experiment by Ruhmkorff is a remarkable effect of Faraday's apparatus. When the two ends of a stout wire in which the current of the electromagnet passes are placed between the two poles S and Q of fig. 961—that is to say, when the current is closed between S and Q—this closing takes place without a spark and without noise, or merely a feeble noise and a spark. But when the two ends are separated, and the current is hence broken, a violent noise is heard, almost as strong as the report of a pistol. This appears to be the extra current, the intensity of which is greatly increased by the influence of two poles.

The repulsion produced in a diamagnetic body under the influence of a powerful magnet is due to the fact that the magnet develops in the end nearest to it like polarity, and in the end furthest away unlike polarity; a phenomenon the exact opposite of that of iron.

The following experiment, which is due to Weber, is considered to prove that diamagnetism is a polar force. A coil was placed near the end of an electromagnet, its axis being in the prolongation of the axis of the magnet, and its ends being connected with a sensitive galvanometer. When a bar of bismuth was suddenly introduced and removed from the coil, induction currents were produced in the circuit, the direction of which, as shown by the galvanometer, was the exact opposite of that which iron would have produced under the same circumstances.

CHAPTER VIII

THERMO-ELECTRIC CURRENT

939. **Thermo-electricity.**—In 1821, Professor Seebeck, of Berlin, found that by heating one of the junctions of a metallic circuit, consisting of two metals soldered together, an electric current was produced. This phenomenon may be shown by means of the apparatus represented in fig. 964, which consists of a plate of copper, *mn*, the ends of which are bent and soldered to a plate of bismuth, *op*. Inside the circuit is a magnetic needle *a*, moving on a pivot. When the apparatus is placed in the magnetic meridian, and one of the solderings gently heated, as shown in the figure, the needle is deflected in a manner which indicates the passage of a current from *n* to *m*—that is, from the heated to the cool junction in the copper. If, instead of heating the junction *n*, it is cooled by ice, or by placing upon it cotton-wool moistened with ether, the other junction remaining at the ordinary temperature, a current is produced, but in the opposite direction—that is to say, from *m* to *n* ; in both cases the current is in general stronger in proportion as the *difference* in temperature of the solderings is greater.

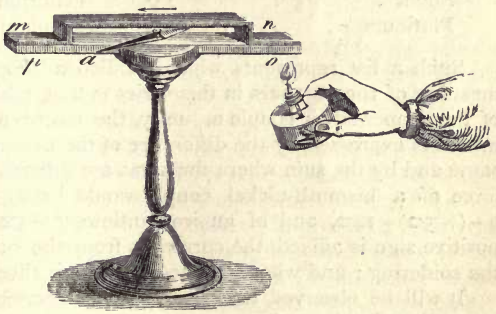

Fig. 964.

Seebeck gave the name *thermo-electric* to this current, and to the couple which produces it, to distinguish it from the *hydro-electric* or ordinary voltaic current and couple.

940. **Thermo-electric series.**—If small bars of two different metals are soldered together at one end while the free ends are connected with the wires of a galvanometer, and if now the point of junction of the two metals be heated, a current is produced, the direction of which is indicated by the deflection of the needle of the galvanometer. Moreover, the strength of the current, calculated from the deflection of the galvanometer, is proportional to the electromotive force of the *thermo-element*. By experimenting in this way with different metals, they may be formed in a list such that each metal gives rise to positive electricity when associated with one of the following, and negative electricity with one of those that precede :—that is, in heating the soldering, the positive current goes from the positive to the negative

metal *across* the soldering, just as if the soldering represented the liquid in a hydro-electrical element; hence out of the element—in the connecting wire and the galvanometer, for instance—the current goes from the negative to the positive metal.

Thus a couple, bismuth-antimony, heated at the junction would correspond to a couple, zinc-copper, immersed in sulphuric acid. The following is a list drawn up from Matthiessen's researches, which also gives comparative numerical values for the electromotive force :—

Bismuth	+25	Gas coke	−0·1
Cobalt	9	Zinc	0·2
Potassium	5·5	Cadmium	0·3
Nickel	5	Strontium	2·0
Sodium	3	Arsenic	3·8
Lead	1·03	Iron	5·2
Tin	1	Red phosphorus	9·6
Copper	1	Antimony	9·8
Silver	1·0	Tellurium	179·9
Platinum	+0·7	Selenium	−290·0

Such a list represents what is called a *thermo-electric series*, and the meaning of the numbers in this series is that, taking the electromotive force of the copper-silver couple as unity, the electromotive force of any pair of metals is expressed by the difference of the numbers where the signs are the same and by the sum where the signs are different. Thus the electromotive force of a bismuth-nickel couple would be $25-5=20$; of a cobalt-iron $9-(-5{\cdot}2)=14{\cdot}2$, and of an iron-antimony $-5{\cdot}2-9{\cdot}8=-4{\cdot}6$. Where the positive sign is affixed, the current is from the other metal to silver across the soldering; and where the negative, from silver to that metal.

It will be observed how great is the electromotive force of the highly crystalline metals. Alloys are not always intermediate to the metals of which they are composed, and, therefore, the position of the metals is greatly affected by slight admixtures. The thermo-electric behaviour of substances is greatly affected by hardness, direction of crystallisation, and so forth, and to this is no doubt due many of the discrepancies in the lists given by different observers.

Of all the bodies mentioned in the above series, bismuth and selenium produce the greatest electromotive force; but from the expense of this latter element, and on account of its low conducting power and the difficulty of making good joints, antimony is generally substituted. The antimony is the negative metal but the positive pole, and the bismuth the positive metal but the negative pole, and the current goes from bismuth to antimony across the junction.

If copper wires connected with the ends of a galvanometer are soldered together to the ends of an antimony rod, and if one of the junctions is heated to 50°, the other being maintained at 0°, a certain deflection is observed in the galvanometer. If, similarly, a compound bar, consisting of antimony and tin soldered together, be connected with the ends of the galvanometer, and if the junction copper-tin as well as the junction tin-antimony be heated to 50°, while the junction antimony-copper is kept at 0°, the deflection is the same

as in the previous case. Hence the electromotive force produced by heating the two junctions, copper-tin and tin-antimony, is equal to the electromotive force produced by heating the copper-antimony; and, generally, if a metal, *b*, is associated with a metal, *a*, which is *above* it in the list, and in like manner if *b* is associated with *c*, which is below it in the list, then the electromotive force produced by heating the combination *ac* is equal to the sum of the electromotive forces produced by heating *ab* and *bc* separately.

If the two junctions of a given couple be heated to the temperatures t and θ, and then to θ and t', respectively, the electromotive force produced by heating the junctions to the temperatures, t and t', is equal to the sum of the electromotive forces produced in the other two cases; that is, for small intervals the electromotive force is directly proportional to the temperature.

With greater ranges this no longer holds; as the temperature increases the differences of potential gradually diminish, and at a certain temperature of the hot junction no current is produced; this temperature is called the *neutral temperature.* In the case of a silver-iron couple this is when one junction is at 0°, the other is at 223°; in the case of copper-iron, it is when the hot junction is at 276°.

When the couple is heated beyond the neutral temperature, the phenomenon of *inversion* now takes place—that is, the direction of the current changes. Thus, with iron-copper, whereas below 276° copper is positive to iron, above that temperature iron is positive to copper.

There is another general case in which no current is produced by heating the two junctions, and that is whenever the arithmetical mean of the temperatures of the junction is equal to this neutral temperature. Thus, for silver and iron this temperature is 228·5°, and no current is produced when the temperature, t, of the one is 186, 145, and 118, the corresponding one of the other being 260, 302, and 328. If the mean temperature in one case is above and in another below, the current has different directions in the two cases; hence the electromotive force cannot always be increased by raising the temperature of one or lowering the temperature of another.

As compared with ordinary hydro-electric currents, the electromotive force of thermo currents is very small; thus the electromotive force of a bismuth-copper element with a difference of 100° C. in the temperatures of their junctions is, according to Neumann, $\frac{1}{250}$ that of a Daniell's element: the electromotive force of an iron-argentan couple with 10° to 15° difference of temperature at their junctions is $\frac{1}{6000}$ that of a Daniell's, according to Kohlrausch that of a copper-argentan couple by $\frac{1}{10000}$ of a Daniell for 100° C. The E.M.F. of a bismuth-antimony couple is 0·000057 volt for a degree Centigrade.

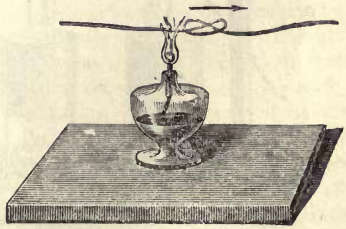

Fig. 965.

941. **Causes of thermo-electric currents.**—Thermo-electric currents are probably to be attributed to an electromotive force produced by the contact of heterogeneous substances, a force which varies with the tempera-

ture. When all the parts of a circuit are homogeneous, no current is produced on heating, because the heat is equally propagated in all directions. This is the case if the wires of the galvanometer are connected by a second copper wire. But if the uniformity of this is destroyed by coiling it in a spiral, or by knotting it (fig. 965), the needle indicates by its deflection a current going from the heated part to that in which the homogeneity has been destroyed by the twisting produced ; this is not seen with platinum wire. If the ends of the galvanometer wires be coiled in a spiral, and one end heated and touched with the other, the current goes from the heated to the cooled end.

When two plates of the same metal, but at different temperatures, are placed in a fused salt such as borax, which conducts electricity but exerts no chemical action, a current passes from the hotter metal through the fused salt to the colder one. Hot and cold water in contact produce a current which goes from the warm water to the cold.

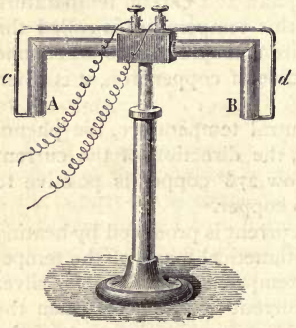

Fig. 966.

Svanberg found that the thermo-electromotive force is influenced by the crystallisation ; for instance, if the cleavage of bismuth is parallel to the face of contact, it is greater than if both are at right angles, and that the reverse is the case with antimony. Thermo-electric elements may be constructed of either two pieces of bismuth or two pieces of antimony, if in the one the principal cleavage is parallel to the place of contact, and in the other is at right angles. Hence the position of metals in thermo-electric series is influenced by their crystalline structure.

Many crystallised minerals have great electromotive force when heated with metals or with each other. Thus the combination copper pyrites—copper when heated in a spirit lamp has an electromotive force of 0·12, and copper pyrites—iron pyrites of 0·18 of a volt.

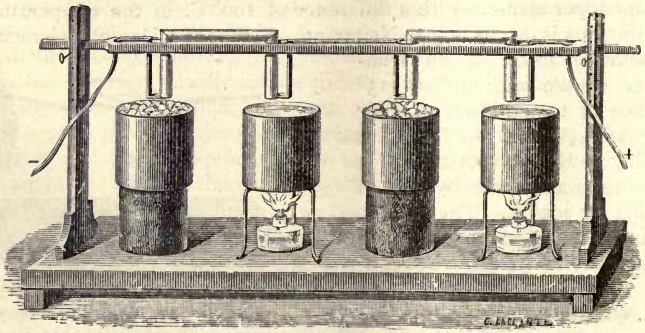
Fig. 967.

942. **Thermo-electric battery.**—From what has been said it will be understood that a thermo-electric couple consists of two metals soldered

together, the two ends of which can be joined by a conductor. Fig. 966 represents a bismuth-copper couple; fig. 967 represents a series of couples used by Pouillet. The former consists of a bar of bismuth bent twice at right angles, at the ends of which are soldered two copper strips, *c*, *d*, which terminate in two binding screws fixed on some insulating material.

When several of these couples are joined so that the second copper of the first is soldered to the bismuth of the second, then the second copper of this to the bismuth of the third, and so on, this arrangement constitutes a thermo-electric battery, which is worked by keeping the odd solderings, for instance, in ice, and the even ones in water, which is heated to 100°.

943. **Nobili's thermo-electric pile.**—Nobili devised a form of thermo-electric battery, or *pile*, as it is usually termed, in which there are a large number of elements in a very small space. For this purpose he joined the couples of bismuth and antimony in such a manner that, after having formed a series of five couples, as represented in fig. 969, the bismuth from *b* was soldered to the antimony of a second series arranged similarly; the last bismuth of this to the antimony of a third, and so on for four vertical series, containing together 20 couples, commencing by antimony, finishing by bismuth.

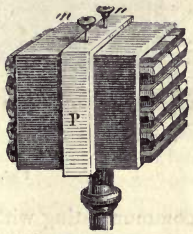

Fig. 968.

Fig. 969.

Thus arranged, the couples are insulated from one another by means of small paper bands covered with varnish, and are then enclosed in a copper frame, P (fig. 968), so that only the solderings appear at the two ends of the pile. Two small copper binding screws, *m* and *n*, insulated in an ivory ring, communicate in the interior, one with the first antimony, representing the positive pole, and the other with the last bismuth, representing the negative pole. These binding screws communicate with the extremities of a galvanometer wire when the thermo-electric current is to be observed.

944. **Becquerel's thermo-electric battery.**—Becquerel found that artificial sulphuret of copper heated from 200° to 300° is powerfully positive, and that a couple of this substance and copper has an electromotive force nearly ten times as great as that of the bismuth and copper couple in fig. 966. Native sulphuret, on the contrary, is powerfully negative. As the artificial sulphuret only melts at about 1,035°, it may be used at very high temperatures. The metal joined with it is German silver (90 of copper and 10 of nickel). Fig. 970 represents the arrangement of a battery of 64 couples arranged in two series of 32. Fig. 971 gives on a larger scale the view of a single couple, and fig. 972 that of 6 couples in two series of 3. The sulphuret is cut in the form of rectangular prisms, 10 centimetres in length, by 18 mm. in breadth, and 12 mm. thick. In front is a plate of German silver, *m*, intended to protect the sulphuret from roasting when it is placed in a gas flame. Below there is a plate of German silver MM, which is bent several times so as to be joined to the sulphuret of the next, and so on. The couples, thus arranged in two series of 32, are fixed to a wooden frame supported by two

brass columns, A, B, on which it can be more or less raised. Below the couples is a brass trough, through which water is constantly flowing, arriving by the tube *b* and emerging by the stopcock *r*. The plates of German silver are thus kept at a constant temperature. On each side of the trough are two long Argand burners, fed by gas from a caoutchouc tube, *a*. The frame being sufficiently lowered, the ends are kept at a temperature of 200° or 300°. For utilising the current, two binding screws are placed on the left

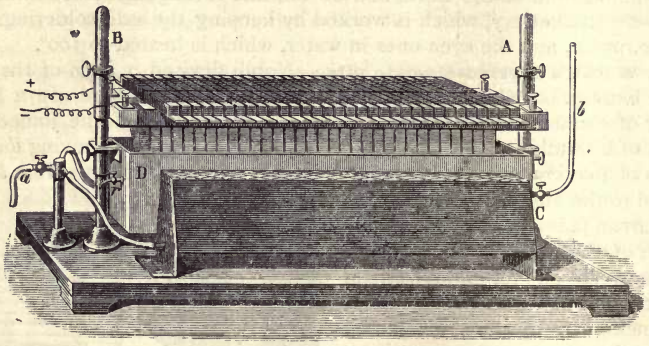

Fig. 970.

of the frame, one communicating with the first sulphuret—that is, the positive pole—and the other with the last German silver, or the negative pole. At the other end of the frame are two binding screws, which facilitate the arrangement of the couples in different ways.

945. **Clamond's thermo-electric battery.**—Of the attempts which have been made to apply thermo-electric currents to directly practical purposes,

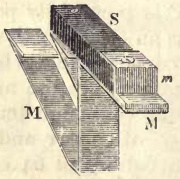

Fig. 971.

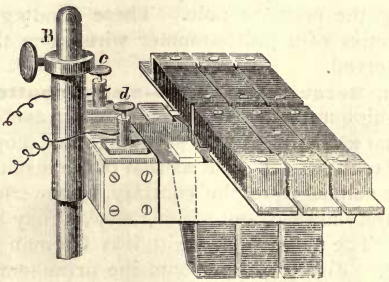

Fig. 972.

perhaps the most successful has been Clamond's, which has been used for telegraphic purposes and also for electroplating. Its characteristic features are the construction and arrangement of the elements, and the manner in which the heating is effected.

The negative element consists of an alloy of two parts of antimony and one of zinc, forming a flat spindle-shaped bar from 2 to 3 inches in length, by

$\frac{3}{8}$ inch in thickness (fig. 974). The positive metal is a thin strip or lug of tin-plate, stamped as represented at *a a'* in fig. 973; this is then bent in as shown at *c*, and being held in a mould, the alloy, which melts at 260° C., is poured in. The individual elements have then the appearance represented in fig. 974, and to connect them together the tin lugs are bent into shape, and joined in a circle of elements (fig. 975), being kept in their position by a paste of asbestos and soluble glass; flat rings of this composition are also made, and are placed between each series of rings piled over each other; the connection between the individual elements and between the sets of rings is made by soldering together the projecting ends of the tin lugs. Thin plates of mica are placed between the alloy and the tin-plate, excepting at the place of soldering. Looked at from the inside, the faces of the battery present the appearance of a perfect cylinder.

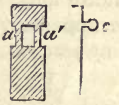

Fig. 973.

Fig. 974.

The heating is effected by means of coal gas, admitted through an earthenware tube, A B, fig. 976, perforated by numerous small holes; this is surrounded by a somewhat larger iron tube, C D, reaching nearly to the top of the cylinder, which is closed by a lid, E F. Air enters at the bottom of this tube, and the heated gases, passing up the tube, curl over the top, descend on the outside, and escape by a chimney, G H. This arrangement economises gas and prevents danger from overheating, as the gas-jets do not impinge directly on the element. The supply of gas is

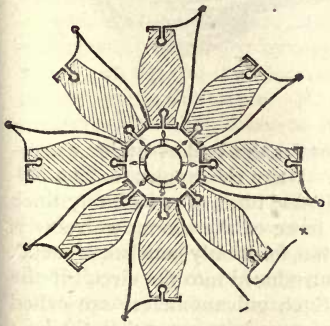
Fig. 975.

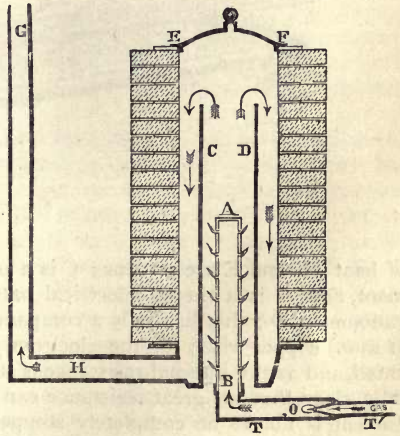

Fig. 976.

regulated by an automatic arrangement, so that the temperature is not higher than about 200°.

Although sometimes convenient, thermo-electric batteries are by no means an economical source of electricity. Thus a Clamond's battery of 120 elements has an E.M.F. of 8 volts, and a resistance of 3·2 ohms; its maximum

available work can be shown to be 5 watts per second; and the consumption of gas per hour is 180 litres. The heat of combustion of a litre of gas gives 5,200 gramme calories; the heat expended per second is, therefore, 260 calories, which would correspond to 1,084 watts. The yield is therefore only about $\frac{1}{200}$ of the heat supplied.

Taking the ordinary price of gas in large towns, the cost of producing a horse-power by a thermo-electric battery would be nearly three shillings an hour for gas alone.

946. **Melloni's thermomultiplier.**—We have already noticed (434) the use which Melloni made of Nobili's pile, in conjunction with the galvanometer, for measuring the most feeble alterations of temperature. The arrangement he used for his experiment is represented in fig. 977.

On a wooden base, provided with levelling screws, a graduated copper rule, about a metre long, is fixed edgeways. On this rule the various parts composing the apparatus are placed, and their distance can be fixed by means of binding screws. *a* is a support for a Locatelli's lamp, or other source

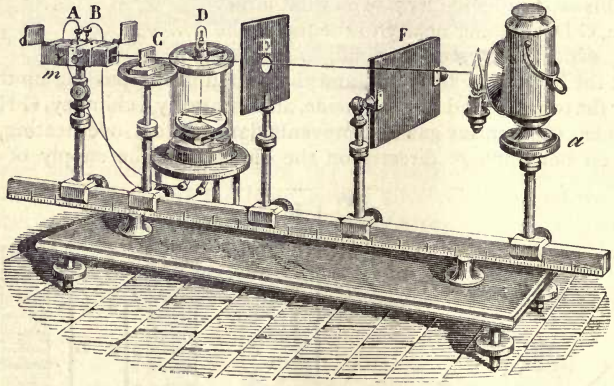

Fig. 977.

of heat; F and E are screens; C is a support for the bodies under experiment, and *m* is a thermo-electrical battery. Near the apparatus is a galvanometer, D; this has only a comparatively few turns of a tolerably thick (1 mm.) copper wire; for the electromotive force of the thermo-currents is small, and as the internal resistance is small too, for it only consists of metal, it is clear that no great resistance can be introduced into the circuit if the current is not to be completely stopped. Such galvanometers are called *thermomultipliers*. The delicacy of this apparatus is so great that the heat of the hand is enough, at a distance of a yard from the pile, to deflect the needle of the galvanometer.

In using it for measuring temperature, the relation of the deflection of the needle, and therefore of the strength of the current, to the difference of the temperatures of the two ends must be determined. That known, the temperatures of the ends not exposed to the source of heat also being known, the

observed deflection gives the temperature of the other, and therewith the intensity of the source of heat.

The most sensitive arrangement of this class is the *radiomicrometer* invented by Mr. Boys. It consists of a light thermojunction suspended by a thin quartz thread (89) between the poles of a strong horse-shoe magnet; it resembles in fact D'Arsonval's galvanometer (fig. 84). With the slightest difference in the temperature of the two ends of the bars of the thermo pair a current is produced in its circuit, and this being in a magnetic field is deflected like any current under the influence of a field. And as the force tending to deflect it, is the product of the current into the strength of the field, it follows that with a strong field only an extremely feeble current is necessary to produce a considerable deflection. By its means Mr. Boys can detect differences of less than one-millionth of a degree Centigrade. It will clearly respond to a quantity of heat not greater than that which would be received on a halfpenny by the flame of a candle at a distance of 1,530 feet.

947. **Properties and uses of thermo-electric currents.**—Thermo-electric currents are of extremely low potential, but of great constancy; for their opposite junctions can easily be kept at 0° and 100° C. by means of melting ice and boiling water. On this account, Ohm used them in the experimental establishment of his law. They can produce all the actions of the ordinary battery in kind, though in less degree. By means of a thermo-electrical pile consisting of 769 elements of iron and German silver, the ends of which differed in temperature by about 10° to 15°, Kohlrausch proved the presence of free positive and negative electricity at the two ends of the open pile respectively. He found that the potential of the free electricity was nearly proportional to the number of elements, and also that the electromotive force of a single element under the above circumstances was about $\frac{1}{6600}$ that of a single Daniell's element. On account of their low potential, thermo-electric piles produce only feeble chemical actions. Botto, however, with 120 platinum and iron wires, decomposed water.

948. **Thermo-electric diagram.**—Thermo-electric relations may be very conveniently illustrated by means of what is called the *thermo-electric diagram*. In fig. 978 the abscissæ represent the temperatures of the junctions on the centigrade scale. If, now, the thermo-electric deportment of any metal with another, taken as standard, be determined for any given temperature, the corresponding differences of potential are represented by an ordinate according to a definite scale. In the diagram the ordinates represent microvolts (964), and lead is taken as standard. A line which connects the ordinates thus determined is called a *thermo-electric line*; the lines are here represented as straight, though some, such as iron and nickel, present distinct sinuosities, and may thus cross the straight line belonging to another metal more than once, indicating therefore more than one neutral temperature.

It will be seen that, if we know the differences of potential of any two metals in respect of lead, the thermo-electrical lines give us the differences of potential of these two metals directly. If, for example, for the metals copper and iron the junctions are heated to 0° and 100° respectively, the mean temperature is 50°, and the difference of the two ordinates yy' gives the thermo-electric force of the combination for this mean temperature, the metal at the top, copper, being electropositive; the area $x0 - 15x_1$ represents

the total thermo-electric force in the circuit. If the temperatures of the two junctions were 300° and 500°, the mean temperature will now be 400°, and the difference, $y\,y'$, would represent the thermo-electric force, which in this case would be from iron to copper; that is, iron is now electropositive to copper.

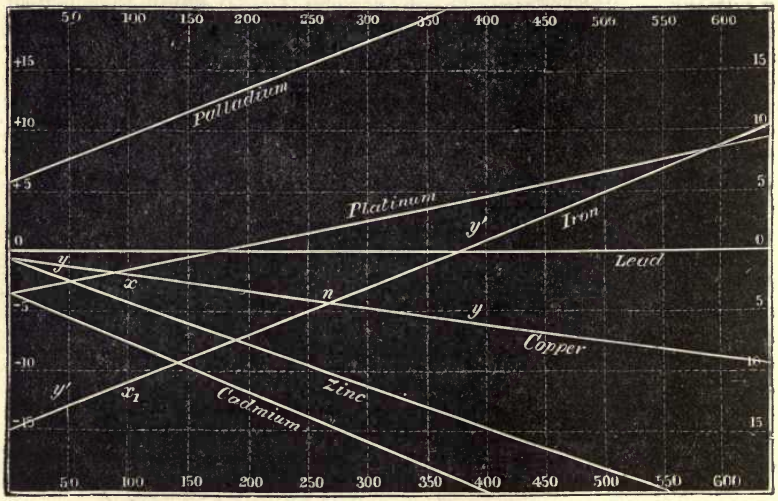

Fig. 978.

The point *n* where two lines cross one another and where, therefore, there is no electromotive force, represents the neutral temperature, or temperature of inversion (940); for copper-iron this is at 276°, for iron-cadmium is at 140°.

949. **Becquerel's electric pyrometer.**—This apparatus is an improved form of one originally devised by Pouillet. It consists (fig. 979) of two wires, one of platinum and the other of palladium, both two metres in length and a square millimetre in section. They are not soldered at the ends, but firmly tied for a distance of a centimetre with fine platinum wire. The palladium wire is enclosed in a thin porcelain tube; the platinum wire is on the outside, and the whole is enclosed in a larger porcelain tube, P. At the end of this is the junction, which is adjusted in the place the temperature of which is to be investigated. At the other end project the platinum and palladium wires *m* and *n*, which are soldered to two copper wires that lead the current to a *magnetometer*, G. These wires at the junction are placed in a glass tube immersed in ice, so that, being both at the same temperature, they give rise to no current.

The magnetometer, which was devised by Weber, is in effect a large galvanometer. It consists of a magnetised bar, *ab*, placed in the centre of a copper frame, which damps the oscillations (905) and rests on a stirrup, H, which in turn is suspended to a long and very fine platinum wire. On the stirrup is fixed a mirror, M, which moves with the magnet, and gives by reflection the image of divisions traced on a horizontal scale, E, at a

distance. These divisions are observed by a telescope. With this view, before the current passes the image of the zero of the scale is made to coincide with the micrometer wire of the telescope : then the slightest deflection of the mirror gives the image of another division, and therefore the angular deflection of the bar magnet. This angle is always small, and should not exceed 3 or 4 degrees : this is effected by placing, if necessary, a rheostat or a resistance coil in the circuit. The angular deflection being known, the intensity of the current and the temperature of the junction are deduced from pyrometric tables. These are constructed by interpolation when the strengths

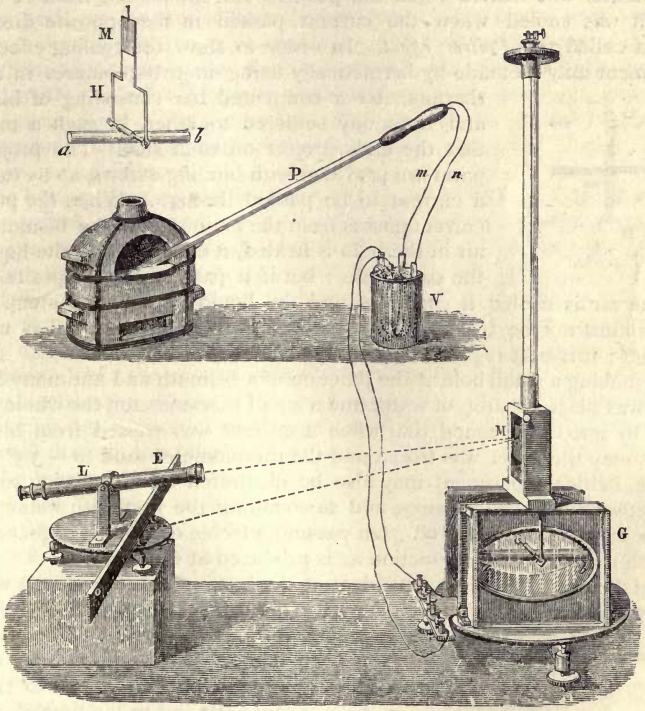

Fig. 979.

are known which correspond to two temperatures near those to be observed. The indications of the pyrometer extend to the fusing point of palladium.

Lechatellier has greatly improved this method by using a couple of platinum associated with an alloy of platinum with 10 per cent. of rhodium, both in the form of wires. These are connected with a Deprez and D'Arsonval's dead-beat galvanometer (821). The couple is placed in various metals and salts at their melting points, the temperatures of which are known, and from the observed deflections a scale is empirically graduated. In this way temperatures up to 1,200° C. may be determined which are accurate to within 10°.

950. **Peltier's experiment.**—When on a bar of bismuth, BB′, cut halfway through at its centre (fig. 980), is soldered a bar of antimony with a similar cut, and when the ends A and B are connected with a galvanometer, the needle of the galvanometer is deflected in one direction when the junction is heated, and in the other when it is cooled.

Peltier found by means of this apparatus, which is known as *Peltier's cross*, that when the end A′ was connected with one pole, and B′ with the other pole of a voltaic element, so that a current passed from A′ through the junction to B′, the needle was deflected in such a direction as to show that the junction was heated when the positive current passed from A′ to B′, while it was cooled when the current passed in the opposite direction. This is called the *Peltier effect*. In order to show the cooling effect, this experiment may be made by hermetically fixing in two tubulures in an air thermometer a compound bar consisting of bismuth and antimony soldered together, in such a manner that the ends project on each side. The projecting parts are provided with binding screws, so as to allow a current to be passed through. When the positive current passes from the antimony to the bismuth, the air in the bulb is heated, it expands, and the liquid in the stem sinks; but if it passes in the opposite direction the air is cooled, it contracts, and the liquid rises in the stem. The current must not be too strong; that of a single Bunsen's cell is usually sufficient; it is best regulated by a rheostat (949).

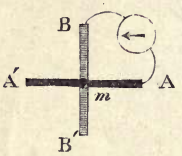

Fig. 980.

By making a small hole at the junction of a bismuth and antimony bar, in which was placed a drop of water and a small thermometer, the whole being cooled to zero, Lenz found that when a current was passed from bismuth to antimony the water was frozen and the thermometer sank to $-3{\cdot}5°$ C.

The Peltier experiment may also be illustrated by interposing an iron wire between two copper wires, and surrounding the first with water at 0°, and the second with ice at 0°. On passing a feeble current, it is found that as much ice melts at one junction as is produced at the other.

The Peltier effect is independent of the heating effect produced when a current traverses any conductor, and which may be called the *frictional heating* or *Joule effect*. The heat due to this cause is proportional to the square of the current, C, to the resistance, R, and to the time, t, and is independent of the direction of the current (830); while the Peltier effect is proportional to the strength of the current and to the time, and is reversible with its direction. This suggests a method of determining the effect in question. If this be called $\mathfrak{H}$, the heat due to it will be $\mathfrak{H}Ct$, and that due to the frictional heating will be C^2Rt. Hence if the current be passed so that in one case the Peltier effect coincides with the Joule effect, while in the other it is opposed to that effect,

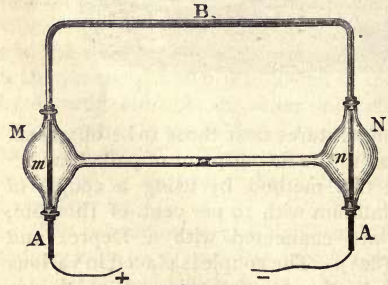

Fig. 981.

we shall have for the total heat H and H′ in the two cases: $H = C^2Rt + \mathfrak{H}Ct$, and $H' = C^2Rt - \mathfrak{H}Ct$, from which

$$\mathfrak{H} = \frac{H - H'}{2Ct}.$$

That the Peltier effect is independent of the Joule heating has been investigated by Edlund, by a method the principle of which is represented in fig. 981. M and N are two bulbs, and are connected by a narrow glass tube, in which is a drop of liquid serving as index. The rods of metal A and B are fixed airtight in the bulbs, and are soldered at *m* and *n*, while the free ends can be connected with a battery. If the pieces *m* and *n* inside the glass vessels offer the same resistance, and these vessels are of the same size, when the current passes the Joule effect is the same in each case, and consequently the index is equally pressed in opposite directions, and therefore does not move. But the Peltier effect is opposite in the two vessels, and produces a displacement of the index, from which the change of temperature can be deduced.

The Peltier effect, as will be seen, is greater as the term 2C*t*, or the strength of the current, is less, and hence it can only be shown with feeble currents.

These experiments form an interesting illustration of the principle, that whenever the effects of heat are reversed heat is produced; and whenever the effects ordinarily produced by heat are otherwise produced, cold is the result; for cooling takes place when the current is in the same direction as the thermo-current which would be produced by heating the junctions, and heating when the current is in the opposite direction.

950*a*. **Thomson effect.**—If we take two bars of the same metal A B and A′ B′, which are connected at the ends A A′, by a wire, while a current can be passed through the other, then the temperature of each part of the bar due to the Joule effect would be the same when the stationary condition is attained. If the two ends B B′ are kept at a constant temperature of 100° by being immersed in boiling water, while the others A A′, are placed in melting ice, and are thus at 0°, and if now a thermopile be placed with its two opposite faces in contact with symmetrical positions of the two bars, it will be found that when a current passes through the system at A A′, the galvanometer of the thermopile is deflected, showing that there is a difference of temperature at the two ends of the pile—that is, the corresponding parts of the bars are not at the same temperature. In the case of copper, silver, zinc, and antimony the point would be hotter on that bar along which the positive current passes from cold to hot; in the case of tin, aluminum, platinum, bismuth, and iron it is hotter when the negative current passes.

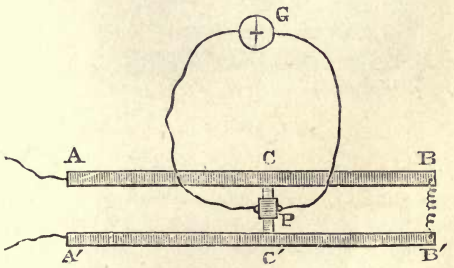

Fig. 982.

This phenomenon, which is known as the *Thomson effect* from its discoverer, Lord Kelvin, is most marked in antimony among positive metals, and in iron; it is a sort of *electrical convection of heat*; in copper the positive current carries electricity along with it more readily than iron; it has, in short, a greater *specific heat of electricity*.

Le Roux found that lead has no Thomson effect. He also showed that this effect is proportioned to the strength of the current.

CHAPTER IX

DETERMINATION OF ELECTRICAL CONSTANTS

951. **Rheostat.**—A *Rheostat* is an instrument by which the resistance of any given circuit can be increased or diminished without opening the circuit. The original form invented by Wheatstone consists of two parallel cylinders, one, A, of brass, the other, B, of wood (fig. 983). In the latter there is a spiral groove, which terminates at *a* in a brass ring, to which is fixed the end of a fine brass wire. This wire, which is about 40 yards long, is partially coiled on the groove; it passes to the cylinder A, and, after a great number of turns on this cylinder, is fixed at the extremity *e*. Two binding screws, *n* and *o*, connected with the battery, communicate by two steel plates; one with the cylinder A, the other with the ring *a*.

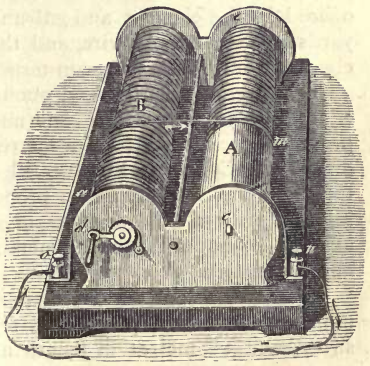

Fig. 983.

When a current enters at *o*, it simply traverses that portion of the wire rolled on the cylinder B, where the windings are insulated by the grooves; passing thence to the cylinder A, which is of metal, and in contact with the wire, the current passes directly to *m*, and thence to *n*. Hence, if the length of the current is to be increased, the handle *d* must be turned from right to left. If, on the contrary, it is to be diminished, the handle is to be fixed on the axis *c*, and turning then from left to right, the wire is coiled on the cylinder A. The length of the circuit is indicated in feet and inches, by two needles, at the end of the apparatus not seen in the figure, which are moved by the cylinders A and B.

952. **Determination of the resistance of a conductor. Reduced length.**—If in the circuit of a constant element a tangent galvanometer be interposed, a certain deflection of the needle will be produced. If, then, different lengths of copper wire of the same diameter be successively interposed, corresponding deflections will in each case be produced. Let us suppose that in a particular case the tangent of the angle of deflection (823) observed with the element and tangent galvanometer alone was 1·88, and that when 5, 40, 70, and 100 yards of copper wire were successively placed in the circuit, the tangents of the corresponding deflections were 0·849, 0·172,

0·105, and 0·074. Now, in this experiment, the total resistance consists of two components—the resistance offered by the element and the tangent galvanometer, and the resistance offered by the wire in each case. The former resistance may be supposed to be equal to the resistance of x yards of copper wire of the same diameter as that used, and then we have the following relations :—

Length of wire	*Tangent of angle of deflection*
x yards	1·88
$x+5$ „	0·849
$x+40$ „	0·172
$x+70$ „	0·105
$x+100$ „	0·074

If the intensities of the currents are inversely as the resistances—that is, as the lengths of the circuits—the proportion must prevail,

$$x : x+5 = 0{\cdot}849 : 1{\cdot}88 ;$$

from which $x = 4{\cdot}11$. Combining, in like manner, the other observations, we get a series of numbers, the mean of which is 4·08. That is, the resistance offered by the element and galvanometer is equal to the resistance of 4·08 yards of such copper wire, and this is said to be the *reduced length* of the element and galvanometer in terms of the copper wire.

It is of great scientific and practical importance to have a *unit* or *standard of comparison* of resistance, and numerous such have been proposed. Jacobi proposed the resistance of a metre of a special copper wire a millimetre in diameter. Copper is, however, ill adapted for this purpose, as it is difficult to obtain pure. Matthiessen proposed an alloy of gold and silver, containing two parts of gold and one of silver ; its conducting power is very little affected by impurities in the metals, by annealing, or by moderate changes of temperature.

Siemens' unit is a metre of pure mercury, having a section of a square millimetre. Its actual material reproduction for ordinary use is a German silver wire 3·8 metres in length and 0·9 mm. in diameter. It is 0·9534 of an ohm (963). A mile of No. 16 pure copper wire represents a resistance o 13·67 ohms.

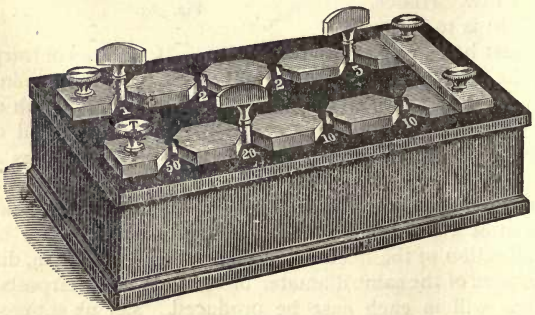

Fig. 984.

953. **Resistance coils.**—The actua material production of a standard resistance is ordinarily a given length of wire of a certain definite material, and is known as a *resistance coil.* An alloy of silver with about $\frac{1}{3}$ of platinum is best, as it is very permanent, and its resistance varies little with increase of temperature. Such resistance coils are

usually employed in what are called *resistance boxes* (fig. 984). Fig. 985 represents the way in which resistance coils are affixed inside the box. On the top of the box, which is of slate or ebonite, are a number of solid prismatic pieces of brass fixed a little distance apart ; at their ends are conical perforations in which fit brass plugs. Inside the box are fitted to these brass pieces the various lengths of wires which represent very accurately the resistances ; they are covered with insulated wire, and are wound double, so as to neutralise any extraneous inductive action. If the terminals of a circuit are connected with TT′, fig. 985, and all the plugs are inserted, the resistance box offers no appreciable resistance, for the current passes by the plugs and the massive metal ; but by taking out any of the plugs the current has to pass through the wire coil between the two brass pieces, and thus its resistance is introduced. In fig. 984 this represents the use of a resistance of 74 ohms.

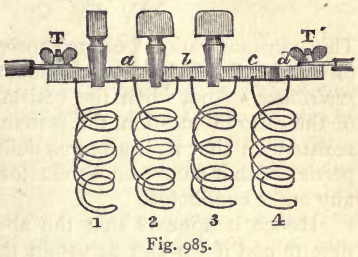

Fig. 985.

The coils are in multiples and submultiples of ohms, and are so arranged that their combination may be as greatly varied with as few resistances as possible. Thus a set of eleven coils of 0·1, 0·2, 0·2, 0·5, 2, 2, 5, 10, 10, 20, and 50 enables us to introduce any resistance from 0·1 to 100 into the circuit.

Resistance boxes have almost entirely superseded the rheostat and similar instruments. They are more accurate, and not nearly so likely to suffer from use.

954. **Absolute measure of electrical resistance.**—When the resistance of any conductor has been measured and expressed by reference to any of the standards of resistance mentioned in the preceding paragraph, the number denoting the result of the measurement still does not tell us what the resistance of the conductor in question really is ; it only tells us what multiple it is of the resistance of the particular conductor with which the comparison has been made. It gives us merely a *relative* and not an *absolute* measure. Just in the same way, if we are told that the pressure of the steam in a boiler is equal to (say) 8 *atmospheres* (157), this statement does not in itself enable us to form any estimate of what the actual pressure of the steam is ; it only tells us that, whatever the pressure of an atmosphere may be, that of the steam is 8 times as great. In order that we may be able to calculate what effects the pressure of the steam is capable of producing, we require to have it stated in *absolute* measure—that is, not how much greater or less it is than some other pressure—but what actual force is exerted by it on each unit of surface. So, for very many purposes, we require absolute measures of electrical resistance, instead of mere comparisons of the resistance of one conductor with that of another.

To see how it is possible to get an absolute measure of resistance, we must go back to the fundamental meaning expressed by the term. If, by any means whatever, a definite electromotive force or difference of potential is maintained between any two given cross-sections of a conductor, a constant electric current flows from one cross-section to the other, and, for the same

conductor, *the ratio of the electromotive force to the strength of the resulting current is constant.* That is, if E_1, E_2, E_3, . . . be various values successively given to the electromotive force, and C_1, C_2, C_3, . . . be the corresponding strengths of the current, then

$$\frac{E_1}{C_1} = \frac{E_2}{C_2} = \frac{E_3}{C_3} = \text{ . . . } = R \text{ (a constant).}$$

This constant ratio of electromotive force to strength of current is characteristic of the individual conductor employed, and is called its *electrical resistance.* And, when the resistance of a conductor is stated as the value of the ratio in question, the statement gives us the absolute measure of the resistance ; that is, it gives us definite information about the electrical properties of that particular conductor without implying a comparison of it with any other conductor.

Hence it appears that the absolute resistance of a given conductor is determined if we can ascertain the ratio of any electromotive force to the strength of the current which it is capable of producing in the conductor in question. It is not, however, needful to make an independent measurement of this ratio in the case of every conductor whose resistance we require to know ; it is sufficient to determine it once for all for some one conductor, and then, taking this conductor as a standard, to compare the resistance of other conductors with that of this one, by means of Wheatstone's Bridge (948), or any other convenient method.

The methods available for determining the ratio between electromotive force and resistance, required for an absolute measurement of resistance, depend on the electromagnetic phenomena presented by electric conductors and currents ; it will be sufficient here to indicate the general principles upon which such methods can be founded. From what has been said it will be seen that any method for this purpose involves a measurement of electromotive force and a measurement of the strength of a current. It will be convenient to treat these two parts of the process separately.

A. *Absolute measurement of electromotive force.*—When any electric conductor is moved in a magnetic field (707)—that is to say, in any region where there is magnetic force—an electromotive force is in general developed in the conductor during its motion. The magnitude of this electromotive force depends upon the strength of the magnetic field, on the length and form of the conductor, and on the velocity and direction of its motion. The simplest case is presented by a straight conductor, with its length perpendicular to the direction of the force in a uniform magnetic field, and moving at right angles to its length and to the direction of the force. If T be the strength of the field, l the length of the conductor, and v the velocity, the electromotive force E is

$$E = kTlv,$$

where k is a constant, depending on the unit adopted for the measurement of electromotive force. If we define the unit of electromotive force as that which is developed in a *conductor of unit length moving* (in the way specified above) *with unit velocity in a magnetic field of unit intensity*, the constant k becomes $= 1$, and the value of E is

$$E = Tlv.$$

If the length and the direction of motion of the conductor are not at right angles to the direction of magnetic force, we must project both on a plane perpendicular to the direction of the force ; thus, if the conductor is inclined at an angle α, and moves in a direction making an angle β, both being measured from the direction of magnetic force, the electromotive force becomes

$$E = Tl \sin \alpha . v \sin \beta.$$

If the conductor is bent in any way, so that α has different values for different parts, and if the direction or velocity of its motion varies from one part to another, we may conceive of it as divided into a great number of equal parts, each so small that no sensible variation of α, β, or v can occur within it, we may calculate the electromotive force due to each of these small parts taken separately by the last formula, and then, adding all the results together, we obtain the electromotive force developed in the whole conductor. A little consideration will show that the following statement is equivalent to that just given : namely, the electromotive force generated in a conductor moving in any manner in a magnetic field is proportional at each instant to the *rate of variation of the area swept over by its projection on a plane perpendicular to the direction of the magnetic force* ; and the average electromotive force acting in the conductor during any interval of time is proportional directly to the total area swept over by its projection during the interval, and inversely to the length of the interval.

In order to apply practically the principles that have been pointed out, it is most convenient to take advantage of the magnetic field due to the magnetism of the earth. Throughout any moderate space at a distance from magnets or masses of iron, the magnetic force due to the earth is uniform in intensity and direction. Suppose, then, a circular conducting ring, placed so that its plane is perpendicular to the direction of the earth's magnetic force—that is, to the direction of the dipping needle—to be turned through half a revolution about one of its diameters ; we may regard its projection on a plane perpendicular to the direction of the earth's force to be made up of the projections of the two semicircles into which it is divided by the axis of rotation. During the half-turn made by the ring, the projection of each semicircle sweeps through an area equal to that of the whole ring ; but one projection passes over this area in one direction, and the other in the opposite direction. Consequently, equal electromotive forces are generated in the two halves of the ring, in opposite directions as regarded from outside, but both in the same direction if considered as tending to produce a current round the ring : the total electromotive force is therefore the sum of the forces in the two halves, and if r be the radius of the ring, and therefore πr^2 its area, and n the number of revolutions per second, so that the time occupied by each half-revolution is $\frac{1}{2n}$, the average electromotive force acting in the ring as it rotates uniformly about a diameter is

$$2T . \pi r^2 \div \frac{1}{2n} = 4T\pi r^2 n,$$

where T stands for the whole intensity of the earth's magnetic force. If, instead of a single ring, we have a circular coil of wire of u convolutions.

and if the axis of rotation makes any angle a with the line of dip, the electromotive force due to the rotation of the coil is

$$E = 4T\pi r^2 nu \sin a.$$

Consequently, the rotation of a coil of wire under the circumstances named furnishes the means of obtaining an electromotive force, the absolute value of which is given by the intensity of the magnetic field, the dimensions and speed of the coil, and the position of its axes of rotation. If we can determine the strength of current which this electromotive force is capable of producing in a given conductor, the absolute resistance of the conductor is at once known.

B. *Absolute measurement of the strength of currents.*—The method of measuring the strength of electric currents is founded on the fact that a force is exerted between a conductor carrying a current and any magnetic pole in its neighbourhood. In general, both the distance and the direction, as seen from a given magnetic pole, vary from point to point of the conductor, so that it is generally impossible to give any simple statement of the law according to which a given current acts upon a magnetic pole in a given position. But, if we consider only a very small length of a current, neither the distance of its various points from a given magnetic pole, nor their directions, can vary to a sensible extent; and when these two conditions are constant, the law of the force between the current and the pole may be stated as follows: As to direction the force is perpendicular to a plane containing the current and the pole, and acts upon a north pole, towards the left hand of an observer looking at the pole from the line of the current, and so placed that the nominal direction of the current is from his feet to his head, or, upon a south pole, towards the right hand of an observer similarly placed; as to magnitude, the force is proportional directly to the length (l) and to the strength (C) of the current, to the strength of the magnetic pole (m), and to the sine of the angle (θ) made by the direction of the current with a straight line drawn from it to the pole, and inversely to the square of the distance (r') from the current to the pole. Hence, if the force be denoted by f, we have

$$f = k\frac{Cml}{r'^2}\sin\theta,$$

where k is a constant, depending on the units in which the numerical values of the various quantities are expressed. If we define the unit strength of current as the *strength of a current of which unit length placed at unit distance from a magnetic pole of unit strength, and making everywhere a right angle with a line drawn from it to the pole, exerts unit force on the pole*, k becomes unity, and we have

$$f = \frac{Cml}{r'^2}\sin\theta, \text{ or } C = \frac{fr'^2}{ml\sin\theta}.$$

The most convenient way of founding a practical measurement of the strength of a current upon these principles is to cause the current to go one or more times round a vertical circle of known radius placed in the plane of the magnetic meridian, with a very short magnet suspended at the centre. This is the arrangement of the tangent galvanometer already described (823). If H is the intensity of the horizontal component of the earth's mag-

netic force, the force which must be exerted upon each pole of a magnet whose poles are of the strength $+m$ and $-m$, in a direction perpendicular to the magnetic meridian, in order to deflect the magnet through an angle γ, is

$$f = \mathrm{H}m \tan \gamma.$$

Putting this value of f into the expression given above for the strength of a current, we have

$$\mathrm{C} = \frac{\mathrm{H}m \tan \gamma \, r'^2}{ml \sin \theta}$$

But in the case supposed, that of a tangent-galvanometer with the current going u' times round the circle, we have $l = u' 2\pi a$, if a is the radius of the circle; moreover, the distance r' of each part of the current from the magnet is constant and equal to the radius, or $r' = a$, and the angle θ is also constant, being everywhere a right angle, so that $\sin \theta = 1$; consequently we get for the strength of the current in absolute measure,

$$\mathrm{C} = \frac{\mathrm{H}mr'^2}{mu' 2\pi a} \tan \gamma = \frac{\mathrm{H}r'}{2\pi u'} \tan \gamma.$$

We have thus shown how both electromotive force and strength of current can be measured in absolute units, and if these two measurements be combined, the ratio of the numerical value of the electromotive force, acting in a conductor, to that of the strength of the resulting current, is the measure of the resistance of the conductor in question. Using the notation employed above, this leads to the following expression for the absolute measure of resistance.

$$\mathrm{R} = \frac{\mathrm{E}}{\mathrm{C}} = \frac{4\mathrm{T}\pi r^2 un \sin a \, . \, 2\pi u'}{\mathrm{H}\, r' \tan \gamma}.$$

Various practical methods of measurement founded upon this principle have been devised, and when any of them is employed the value of the resistance under investigation is obtained by putting in this formula the values of electromotive force and strength of current that result from the particular arrangement adopted.

It may be observed with regard to the above expression, that the factors π, u, u', $\sin a$ and $\tan \gamma$, are all of them simple numbers, that T and H are quantities of the same kind, so that their ratio is also a pure number. The only factors which involve reference to physical units are therefore r^2, r' and n, and the two former being both distances, the ratio $r^2 \div r'$ is the first power of a distance, while n, the number of revolutions per unit of time, is the reciprocal of the time occupied by a single revolution. Hence the expression for the absolute resistance of a conductor is in all cases reducible to

$$\frac{\text{a distance}}{\text{a time}} \times \text{a numerical factor};$$

that is to say, electrical resistance may be expressed in terms of the units of length (or distance) and time in the same manner as a velocity, and the natural unit of resistance, like the natural unit of velocity, would be represented by a unit of length per unit of time. Adopting the C.G.S. system, the absolute unit of resistance becomes *one centimetre per second*; such a resistance, however, is so small that resistances commonly occurring in practice would

have to be represented by inconveniently great multiples of it. As a practical standard of resistance, it is, therefore, more usual to employ the *ohm* (964), which is a resistance of one thousand million centimetres per second, or,

$$\frac{10^9 \text{ centimetres}}{1 \text{ second}}.$$

955. **Wheatstone's bridge.**—The various methods of determining the electrical conductivity of a body consist essentially in ascertaining the ratio between the resistance of a certain length of the conductor in question, having a given section, to that of a known length of a known section of some substance taken as standard. The most convenient method of ascertaining experimentally the ratio between the resistance of two conductors is by a method known as that of *Wheatstone's bridge*, the general principle of which may be thus stated :—

The conductors, which may be denoted by AB and BC, are connected end to end, as shown in fig. 986, and one end of each is also connected with a battery, say the end A of AB with the positive pole, and the end C of BC with the negative pole ; the ends that are in connection with the battery are likewise connected together by another conductor, AB′C. A current will thus pass from A to C by each of the two paths ABC and AB′C, and there

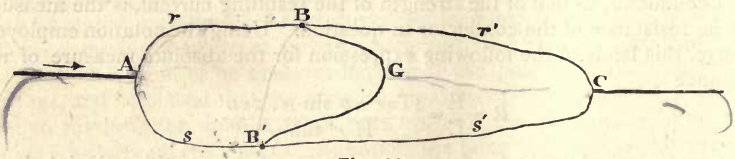

Fig. 986.

will be a gradual fall of potential in passing from A to C along either path, so that for every point in the conductors AB and BC there is a point in the wire AB′C which has the same potential. If one end of a galvanometer wire BGB′ be connected with the point of junction B, the point of AB′C which has the same potential as the point B can be found by applying the other end of the galvanometer wire to AB′C, and shifting the point of contact towards A or C until the galvanometer shows no deflection. Let B′ be the point so found ; the fact that when it is connected with B by the bridge BGB′ no current passes from one to the other proves that the potential at B′ is the same as the potential at B. From this it follows that if r and r' are the resistances of AB and BC respectively, and s and s' the resistances of AB′ and B′C,

$$r : r' = s : s'.$$

If the conductor AB′C is a wire of uniform material and diameter, the ratio of the resistances s and s' will be the ratio of the lengths of the corresponding portions of wire, and can therefore be at once really ascertained.

To prove this, let MN, NO, MN′ and N′O′ (fig. 987) be taken in the same straight line, proportional respectively to the several resistances r, r', s, s' ; and let MP be drawn at right angles to O′MO of a length

proportional to the difference of potential between the points A and C. Then if the straight lines PO and PO′ be drawn, the potential at N (the point of junction of the conductors whose resistances r and r' are to be compared—*i.e.* the point corresponding to B in the previous figure) will be given by the length of the line NQ, drawn from N at right angles to NO ; and the point

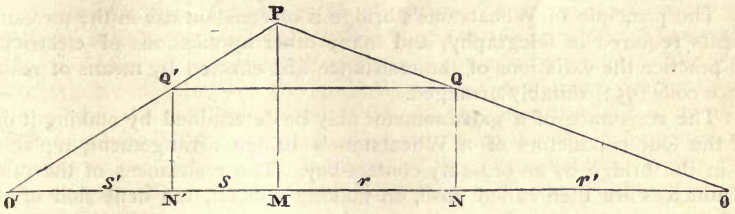

Fig. 987.

N′ (corresponding to B′ in the previous figure), where the potential is the same as at N, will be found by drawing QQ′ parallel to OO′, and letting fall from Q′ the perpendicular Q′N′ upon O′M. The geometry of the figure gives obviously,

$$\frac{r}{r+r'} = \frac{NQ}{MP} \text{ and } \frac{s}{s+s'} = \frac{N_1Q_1}{MP},$$

and therefore since $NQ = N_1Q_1$

$$\frac{r}{r'} = \frac{s}{s'}.$$

A convenient form of Wheatstone's bridge, and one well adapted for purposes of instruction, is that represented in fig. 988. It consists of a long mahogany board, on which is fixed a thick copper band, which practically offers no resistance. To the ends of this band is fixed a straight platinum wire, near which is a scale divided into 100 parts. At c and d are breaks in the copper band, provided with binding screws, in which are introduced the resistances to be compared, o and x. The wires, from an element which gives only a weak current, so as not to introduce heating effects, are

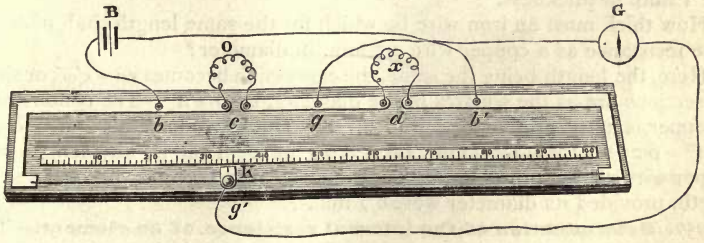

Fig. 988.

connected with the binding screws b and b'. Another wire connects the binding screw g and one end of a sensitive galvanometer, the other end of which is connected with a sliding spring contact-key g', which is so

constructed that when the knob is depressed a knife-edge makes contact with any part of the wire. The resistances to be compared having been introduced at c and d, the position on the platinum wire is found by trial, at which, when the key is depressed, the needle of the galvanometer is not deflected. When this is found, for instance, at 34, the resistance of 0 : the resistance of $x = 34 : 66$.

The principle of Wheatstone's bridge is of constant use in the measurements required in telegraphy, and many other applications of electricity. In practice the variations of the resistance are effected by means of resistance coils (953) suitably arranged.

The resistance of a galvanometer may be determined by making it one of the four conductors of a Wheatstone's bridge arrangement, replacing it in the bridge by an ordinary contact-key. The resistances of the other conductors are then varied until, on making contact, the deflection of the galvanometer is constant.

956. **Equivalent conductors.**—The resistance of a conductor depends, as we have seen (825), on its length, section, and conductivity. Two conductors, C and C′, whose length, conductivity, and section are respectively λ,λ', κ,κ', ω,ω', would offer the same resistance, and might be substituted for each other in any voltaic circuit, without altering its strength, provided that $\frac{\lambda}{\kappa\omega} = \frac{\lambda'}{\kappa'\omega'}$; and such conductors are said to be *equivalent* to each other. An example will best illustrate the application of this principle.

It is required to know what length of a cylindrical copper wire 4 mm. in diameter would be equivalent to 12 metres of copper wire 1 mm. in diameter.

Let $\lambda = 12$ the length of the copper wire 1 mm. in diameter, and λ' the length of the other wire; then since in this case the material is the same, the conductivity is also the same, and the equation becomes $\frac{\lambda}{\omega} = \frac{\lambda'}{\omega'}$. Now the sections of the wires are directly as the squares of the diameters, and hence we have $\frac{12}{1^2} = \frac{\lambda'}{4^2}$, or $\lambda' = 12 \times 16 = 192$. That is, 192 metres of copper wire 4 mm. in thickness would only offer the same resistance as 12 metres of copper wire 1 mm. in thickness.

How thick must an iron wire be which for the same length shall offer the same resistance as a copper wire 2·5 mm. in diameter?

Here, the length being the same, the expression becomes $\kappa\omega = \kappa'\omega'$, or since the sections are as the squares of the diameter, $\kappa d^2 = \kappa' d'^2$. The conductivity of copper is unity, and that of iron 0·138. Hence we have $2{\cdot}5^2 = d'^2 \times 0{\cdot}138$ or $d'^2 = 6{\cdot}25 \div 0{\cdot}138 = 45{\cdot}3$ mm. or $d' = 6{\cdot}7$ mm. That is, any length of a copper wire 2·5 mm. in diameter might be replaced by iron wire of the same length, provided its diameter were 6·7 mm.

957. **Determination of the internal resistance of an element.**—The following is the method of determining the internal resistance of an element. A circuit is formed consisting of one element, a rheostat, and a galvanometer, and the strength C is noted on the galvanometer. A second element is then joined with the first, so as to form one of double the size, and therefore half the resistance, and then by adding a length, l, of the rheostat wire, the

strength is brought to what it originally was. Then if E is the electromotive force, and R the resistance of the element, r the resistance of the galvanometer and the other parts of the circuit; the current strength C in the one case is $C = \frac{E}{R + r}$, and in the other $= \frac{E}{\frac{1}{2}R + r + l}$, and since the strength in both cases is the same, $R = 2l$.

Another method is that due to Sir H. Mance. The element whose internal resistance is to be determined is placed in one of the arms of a Wheatstone bridge as at fig. 988, a resistance box being placed in the other. The galvanometer is connected with the ends of the wire, and a simple contact-key is interposed in the ordinary position of the galvanometer, and by trial its position is found for the sliding contact such that when the key is depressed no alteration is produced in the deflection of the galvanometer. When this is found, the ordinary conditions of the bridge hold—that is, the cross products of the resistances are equal.

958. **Electrical conductivity.**—We may regard conductors in two aspects, and consider them as endowed with a greater or less facility for allowing electricity to traverse them, a property which is termed *conductivity*, or we may consider conductors interposed in a circuit as offering an obstacle to the passage of electricity—that is, a resistance which it must overcome. A good conductor offers a feeble resistance, and a bad conductor a great resistance. Conductivity and resistance are the inverse of each other.

The conductivity of metals has been investigated by many physicists by methods analogous in general to that described in the preceding paragraphs, and very different results have been obtained. This arises mainly from the various degrees of purity of the specimens investigated, but their molecular condition has also great influence. Matthiessen found the difference in conductivity between hard-drawn and annealed silver wire to amount to 8·5, for copper 2·2, and for gold 1·9 per cent. The following are results of a series of careful experiments by Matthiessen on the electrical conductivity of metals at 0° C. compared with silver as a standard.

Silver	100·0	Platinum	18·0
Copper	99·9	Iron	16·8
Gold	80·0	Tin	13·1
Sodium	37·4	Lead	8·3
Aluminium	34·0	German silver	7·7
Zinc	29·0	Antimony	4·6
Cadmium	23·7	Mercury	1·6
Brass	22·0	Bismuth	1·2
Potassium	20·8	Graphite	0·07

Silver and copper have the smallest resistance for a given *volume*, while aluminium has the smallest for a given *weight*.

The conductivity of metals is *diminished* by an increase in temperature. The law of this diminution is expressed by the formula

$$\kappa_t = \kappa_0 (1 - at + bt^2);$$

where κ and κ_0 are the conductivities at t and 0° respectively, and a and b are constants, which are probably the same for all pure metals. For ten

metals investigated by Matthiessen he found that the conductivity is expressed by the formula

$$\kappa_t = \kappa_0 (1 - 0{\cdot}00376471 + 0{\cdot}0000834t^2).$$

It seems that this value is about 0·00368 for each degree C. This coefficient agrees in a surprising manner with the coefficient of expansion of gases, which is $\frac{1}{273}$.

Liquids are far worse conductors than metals. The conductivity of a solution of one part of chloride of sodium in 100 parts of water is $\frac{1}{30000000}$ that of copper.

Liquids only conduct by being electrolysed; such liquids as perfectly pure ether, turpentine, benzole, and even water do not conduct; they are not electrolysed, and have no free ions (845). In general, acids have the highest, and solutions of alkalies, alkaline earths, and neutral salts the lowest conductivity. The conductivity of a 27 per cent. solution of sal ammoniac is the greatest of all known salts, and is about half that of the best conductors among the acids.

The most important researches on the conductivity are those of Kohlrausch. Owing to the effects of polarisation of the electrodes, which give rise to unknown electromotive forces, it is difficult to get very accurate results with the ordinary methods, and Kohlrausch introduced the use of alternating currents, employing for this purpose those of a small induction coil. His apparatus consisted essentially of a Wheatstone bridge arrangement, in one arm of which was introduced the liquid resistance to be determined. For the rarer liquids he used such vessels as are represented in fig. 989, which consists of two small glass vessels joined by a glass tube 3 to 9 mm. in the clear. In these are placed cup-shaped electrodes of platinised platinum, in the centre of which are small apertures to allow gas to escape. In place of the galvanometer an electro-dynamometer was used, and more recently the telephone was found of great service. The resistances were varied in the other branches until equality was obtained, which was recognised by there being no sound in the telephone.

Fig. 989.

The conductivity of a salt depends on the concentration; for dilute solutions it increases with the strength, though not in direct proportion. For each solution there is a certain strength at which the conductivity is greatest; this strength, in many cases, is short of that at which the liquid is saturated. For copper sulphate this is 18 per cent., and for sodium chloride 26 per cent.

The conductivity of liquids increases with the temperature. The coefficient of expansion for 1° C. varies within wide limits in the case of strong solutions of various liquids—thus, from 0·0087 in the case of hydropotassic sulphate to 0·072 in the case of sodium hydrate. With dilute solutions, however, the limits are much narrower—from 0·0211 to 0·233, or about six times as much as the coefficient of expansion of gases (333).

Of the elements which influence the resistance of a solution, the temperature, the percentage strength, and the specific conductivity, the latter is susceptible of the most accurate measurement, and, if it were necessary, a determination of the other two might be based on it. Since the conductivity

of a liquid depends on the number of free ions, a determination of the conductivity gives also a determination of the number of free ions. This becomes of great importance, from the fact that the number of free ions is also a measure of the facility with which a body enters into chemical action. Bodies which are chemically inert, such as benzole, the neutral alcohols, and ether, do not conduct electricity; they have no free ions. The measurement of electrical conductivity furnishes thus a measure of chemical activity.

If two liquids be mixed, each of which conducts badly, for instance, alcohol and water each in a state of perfect purity, the conductivity of the mixture is greater than that of either of the constituents. Here we may suppose that each of the liquids acts as a diluent to the other, and thus favours the production of ions.

The following is a list of the conductivity of a few liquids as compared with that of pure silver :—

Pure silver	100,000,000,000
Nitrate of copper, saturated solution	8990
Sulphate of copper ditto	5420
Chloride of sodium ditto	31520
Sulphate of zinc ditto	5770
Sulphuric acid, 1·10 sp. gr.	99070
„ „ 1·24 sp. gr.	132750
„ „ 1·40 sp. gr.	90750
Nitric acid, commercial	88680
Distilled water	7

The last number was that found by Kohlrausch for distilled water, which had been specially purified. Accordingly, a disc of water a millimetre in thickness offers the same resistance as a column of silver of the same diameter, but of a length equal to that of the moon's orbit. The least trace of impurity in water markedly raises its conductivity : thus standing in the air for 5 hours doubles it; the addition of a millionth part of sulphuric acid—that is, a drop in about 17 gallons—increases the conductivity tenfold. Accordingly we may say in effect that perfectly pure water is not a conductor, and therefore is not appreciably decomposed.

Liquids and fused conductors increase in conductivity by an increase of temperature (845). This increase is expressed by the formula

$$\kappa_t = \kappa_o (1 + at),$$

and the values of a are considerable. Thus for a saturated solution of sulphate of copper it is 0·0286.

The influence of *light* upon electrical conductivity in the case of selenium has been already alluded to (930), and is directly proved by the following experiment. A thin strip of this metalloid, about 38 mm. in length by 13 in breadth, was provided at the ends with conducting wires and placed in a box with a draw-lid. The selenium, having been carefully balanced in a Wheatstone's bridge, was exposed to diffused light by withdrawing the lid, when the resistance at once fell in the ratio of 11 to 9. On exposure to the various spectral colours, after having been in the dark it was found to be most affected by the red; but the maximum action was just outside the red, where

the resistance fell in the ratio of 3 to 2. Momentary exposure to the light of a gas lamp, or even to that of a candle, caused a diminution of resistance. Exposure to full sunlight diminished the resistance to one half.

The effect produced on exposure to light is immediate, while recurrence to the normal state takes place more slowly. A vessel of hot water placed near the strip produced no effect, and hence the phenomenon cannot be due to heat, but there appear to be certain rays which have the power of producing a molecular change in the selenium by which its conductivity is increased.

If the two electrodes of a Ruhmkorff's coil are connected with a Geissler's tube, suitably exhausted, so that a discharge just does not pass when the apparatus is in the dark, it is at once formed when the path is exposed to the ultra violet rays of light.

When a large and small induction coil are inserted in the same circuit so that they spark simultaneously, it is found that by interposing a screen between the two the smaller spark is shorter than when it is exposed to the light of the other. The action diminishes as the distance between the two sets of sparks increases. By varying the nature of the screen and other experiments, it has clearly been established that this alteration is not due to any electrical effect, but is to be ascribed solely to a special action of the ultra violet rays.

959. **Determination of electromotive force.—Wheatstone's method.** In the circuit of the element whose electromotive force is to be determined a tangent galvanometer and a rheostat are inserted, the latter being so arranged that the strength, C, of the current is a definite amount ; for example, the galvanometer indicates 45°. By increasing the amount of the rheostat wire by the length, l, a diminished strength, c (for instance, 40°), is obtained.

A second standard element is then substituted for that under trial, and by arranging the rheostat, the strength of the current is first made equal to C, and then, by addition of l, length of the rheostat, is made $= c$.

Then if E and E_1 are the two electromotive forces, R and R_1 their resistances when they have the intensity I, and l and l_1 the lengths added, we have

Trial Element	Standard Element
$C = \frac{E}{R}$	$C = \frac{E_1}{R_1}$
$c = \frac{E}{R + l}$	$c = \frac{E_1}{R_1 + l_1}$,

from which we have

$$E = E_1 \frac{l}{l_1}.$$

Hence the electromotive forces of the elements compared are directly as the lengths of the wire interposed.

Another method is that of Wiedemann. The two elements are connected in the same circuit with a tangent galvanometer, or other apparatus for measuring strength, first, in such a manner that their currents go in the same direction, and secondly, that they are opposed. Then if the electromotive forces are E and E′, their resistances are R and R′, the other resistances in the circuits r, while C_s is the intensity when the elements are

in the same direction, and C_d the intensity when they go in opposite directions, then

$$C_s = \frac{E + E'}{R + R' + r} \text{ and } C = \frac{E - E'}{R + R' + r'},$$

whence

$$E' = \frac{E\,(C - C_d)}{C_s + C_d}.$$

The difference of potentials or E.M.F. between any two points of a circuit conveying a current, such as that of a magneto machine, may be determined by charging a condenser from the terminals at the points in question, and discharging it through a galvanometer with a high resistance, and then repeating the operation with a standard cell, such as that of Latimer Clarke, the E.M.F. of which is 1·434 volt (964). If d is the deflection of the galvanometer when the standard cell is used, and D the deflection after the discharge of the current, and if a shunt be used so that only $\frac{1}{n}$ of the current passes through the galvanometer, then E.M.F. $= \frac{nD}{d} \times 1{\cdot}434$.

959*a*. **Measurement of capacity.**—In order to compare the capacities of two condensers, the two armatures are severally connected with the two poles of a battery, and are then discharged through a ballistic galvanometer; the amount of charge, and therefore the capacities, are proportioned to the angles of throw of the needle.

If we have a condenser of known capacity this method may be used to measure the E.M.F. of a battery, or rather to compare the E.M.F. of two couples. Two capacities, C and C′, may also be compared, by an arrangement (fig. 990) resembling that of Wheatstone's bridge, by connecting the

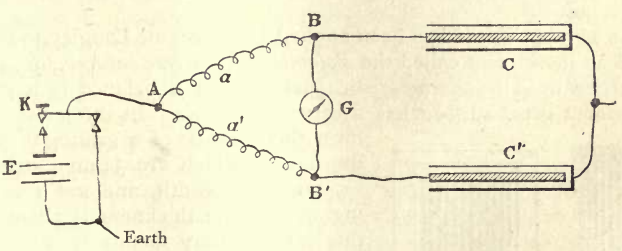

Fig. 990.

inner coatings at c and d respectively (fig. 990), their outer coatings being put to earth. The two resistances a and a' are adjusted, so that by raising or lowering the key K, which puts the battery E in connection with A, no current is shown in the galvanometer.

The condition of equilibrium is that the two points of the bridge B and B′ are at the same potential—that is to say, at a given time the charges Q and Q, are proportional to the capacities C and C,; as these charges are proportional to the currents which produce them, and as these latter again are inversely as the resistances, we have the proportion

$$C : C_, = a' : a.$$

960. **Siemens' electrical resistance thermometer.**—Supposing in a Wheatstone's bridge arrangement, after the ratio $r : r_1 = s : s_1$ has been established, the temperature of one of the coils, r, for instance, be increased, the above ratio will no longer prevail, for the resistance of r will have been altered by the temperature (958), and the ratio of s and s_1 must be altered so as to produce equivalence. On this idea Siemens based a mode of observing the temperature of places which are difficult of direct access. He placed a coil of known resistance in the particular locality whose temperature was to be observed ; it was connected by means of long good conducting wires with the place of observation, where it formed part of a Wheatstone's bridge arrangement. The resistance of the coil was known in terms of the rheostat, and by preliminary trials it had been ascertained how much additional wire must be introduced to balance a given increase in the temperature of the resistance coil. This being known, and the apparatus adjusted at the ordinary temperature, when the temperature of the resistance coils varied, this variation in either direction was at once known by observing the quantity which must be brought in or out of the rheostat to produce equivalence.

This apparatus has been of essential service in watching the temperature of large coils of telegraph wire, which, stowed away in the hold of vessels, are very liable to become heated. It might also be used for the continuous and convenient observation of underground and submarine temperatures. If a coil of platinum wire were substituted for the copper, the apparatus could be used for watching the temperature of the interior of a furnace. It has been found that the magnetism of ships (715) excited so perturbing an influence on the needle of the galvanometer as to make its indications untrustworthy. Hence for use in such cases Siemens replaced the galvanometer, as an indicator, by a voltameter specially constructed for the purpose.

The same principle has been applied by Professor Langley to the invention of an instrument called the *Bolometer*, or *actinic balance*, for measuring radiant heat. The characteristic part of such a balance is a resistance which for a great surface has a very small mass. In the latest improved form this consists of a grating of platinum, the bars of which are 32 mm. in length by 1·5 mm. in breadth, and are 1 mm. apart (fig. 991). The thickness is about 1 μ, and this extraordinary tenuity is attained by a process based on that by which Wollaston obtained fine wire. Such a grating is fixed in a slate frame (fig. 992) ; two of them being placed in the two arms of a Wheatstone bridge, one of them is exposed to the action of radiant heat. The sensitiveness is far greater than that of the most sensitive thermopile, and makes it possible to measure a difference of temperature of the $\frac{1}{10000}$ of a degree between the two resistances. It has been used by the inventor to measure the distribution of heat in the solar spectrum. By its means he has been able to map the dark heat of the spectrum, and to extend it far beyond the limits which were previously known. It has also been used in the investigation of electrical vibrations (965).

Fig. 991.

Fig. 992.

961. **Divided or branch currents.**—In fig. 993 the current from Bunsen's element traverses the wire *rqpnm*. Let us take the case in which any two points of this circuit, *n* and *q*, are joined by a second wire, *nxq*. The current will then divide at the point *q* into two others, one of which goes in the direction *qpnm*, while another takes the direction *qxnm*. The two points *q* and *n* from which the second conductor starts, and at which it ends, are called the *points of derivation*; the wire *qpm* and the wire *qxn* are *derived wires* or shunts. The currents which traverse these wires are called the *derived* or *partial currents*; the current which traverses the circuit *rqpnm* before it branches is the *primitive current*; and the name *principal current* is given to the whole of the current which traverses the circuit when the shunt has been added. The principal current is stronger than the primitive one, because the interposition of the wire *qxn* lessens the total resistance of the circuit.

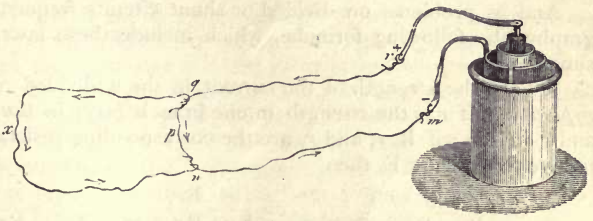

Fig. 993.

If the two wires *qpn* and *qxn* were of the same length and the same section, their action would be the same as if they were juxtaposed, and they might be replaced by a single wire of the same length but of twice the section, and therefore with half the resistance. Hence the current would divide into two equal parts along the two conductors.

When the two wires are of the same length but of different sections, the current would divide unequally, and the quantity which traversed each wire would be proportional to its section, just as when a river divides into two branches, the quantity of water which passes in each branch is proportional to its dimensions. Hence the resistance of the two conductors joined would be the same as that of a single wire of the same length, the section of which would be the sum of the two sections.

If the two conductors *qpn* and *qxn* are different, both in kind, length, and section, they could always be replaced by two wires of the same kind and length, with such sections that their resistances would be equal to the two conductors; in short, they might be replaced by equivalent conductors. These two wires would produce in the circuit the same effect as a single wire, which had this common length, and whose section would be the sum of the sections thus calculated. The current divides at the junction into two parts proportional to these sections, or inversely as the resistances of the two wires.

Suppose, for instance, *qpn* is an iron wire 5 metres in length and 3 square mm. in section, and *qxn* a copper wire. The first might be replaced by a copper wire a metre in length, whose section would be $\frac{3}{5} \times \frac{1}{7}$ (taking the conductivity of copper at 7 times that of iron) or $\frac{3}{35}$ square mm. The second wire might be replaced by a copper wire a metre in length with a section of $\frac{3}{5}$ square mm. These two wires would present the same resistance as a copper wire a metre in length, and with a section of $\frac{3}{35} + \frac{3}{5} = \frac{24}{35}$ square millimetres.

The principal current would divide along the wires into two portions, which would be as $\frac{3}{35} : \frac{2}{3}$, or as 27 : 70.

The most important laws of divided circuits are as follows :—

i. *The sum of the strengths in the divided parts of a circuit is equal to the strength of the principal current.*

ii. *The strengths of the currents in the divided parts of a circuit are inversely as their resistances; or, what is the same, the division of a current into partial currents which lie between two points is directly as the respective conductivities of these branches.*

And as problems on divided or shunt circuits frequently occur in telegraphy, the following formulæ, which include these laws, are given for a simple case.

If C be the strength of the current in the undivided part of the circuit *rqpnm*, and if *c* is the strength in one branch (say) in the above figure *qpn* and *c'* in *qxn*; if R, *r*, and r_1 are the corresponding resistances, the electromotive force being E, then

$$C = \frac{E\,(r + r_1)}{Rr + Rr_1 + rr_1} \qquad c = \frac{Er_1}{Rr + Rr_1 + rr_1} \qquad c' = \frac{Er}{Rr + Rr_1 + rr_1}.$$

The resistance R_1 of the whole circuit is

$$R_1 = R + \frac{rr_1}{r + r'},$$

and therefore the total resistance of the branch currents *qpn* and *qxn* is

$$\frac{rr_1}{r + r_1}.$$

961*a*. **Use of shunts.**—The principle of divided or branch circuits has an important application in *shunts*, by which any given proportion of even a powerful current may be transmitted through a delicate galvanometer, and thus their range is greatly extended.

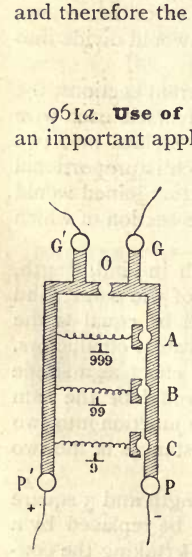

Fig. 994.

They consist of a set of resistances usually $\frac{1}{9}$, $\frac{1}{99}$, and $\frac{1}{999}$, of that of the galvanometer, arranged as represented in fig. 994. G and G' are in connection with the terminals of the galvanometer, and P, P' with those of the battery. The gaps, O, A, B, C can be closed by plugs, and thus the corresponding resistances introduced. When they are all open, the entire current would pass through the galvanometer. By plugging O the current is short-circuited, and none of it passes through the galvanometer.

If *g* is the resistance of the galvanometer, *s* that of the shunt, C the total current, and *c* that which passes through the galvanometer and produces the deflection, we may deduce from the laws of branch circuits

$$gc = s\,(C - c) \text{ or } C = \frac{g + s}{s}c.$$

The expression $\frac{g + s}{s} = m$ is called the *multiplying power* of the shunt; it is the value by which the observed current must be multiplied to obtain

the principal current. In the above cases the multiplying powers are 10, 100, and 1,000 respectively.

962. **Electrical measuring instruments.**—The numerous and important technical applications of electricity have given rise to the invention of numerous instruments for the simple and direct measurement of powerful electrical currents. The *amperometer*, or briefly *ammeter*, for instance, gives at once the strength of a current in amperes.

As a type of this instrument we may take a recent form of that invented by Professors Ayrton and Perry; it depends on the principle that when a portion of an iron core is partly within and partly without a magnetising coil, it is drawn inwards when a current is passed through the coil. The essential feature of the apparatus is a coil of insulated wire, in the axis of which is a spiral attached at one end to an index moving over a graduated scale. At the other end of the spiral is a brass cap to which is attached a thin cylinder of fine sheet iron, which is in fact the core; it encircles the spiral and projects outside the coil. The spiral itself is formed of a ribbon of thin phosphorus bronze coiled so as to form a very narrow cylinder. This construction gives it the property that, unlike ordinary spirals, when its length increases the free end rotates through a considerable distance. Accordingly, when the current passes through the coil, the iron tube is drawn within the spiral to an extent varying with the strength of the current; this thereby elongates the spiral to which it is attached, and the index attached to the latter moves over the scale, finally taking up a position which depends on the strength of the current. Such instruments are graduated empirically and within any desired range by observing the deflection caused by passing through them currents of known strength.

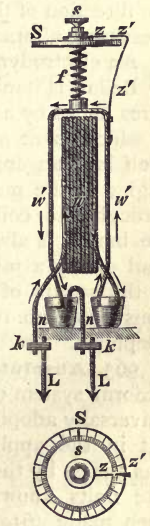

Fig. 995.

The *voltmeter*, which is not to be confounded with the voltameter (846), measures the difference of potential between any two points of a circuit. It consists essentially of a coil such as the above, but with a great length of long fine wire, offering, therefore, a great resistance. This can be inserted as a shunt without appreciably altering the resistance of the circuit. It is empirically graduated like the ammeter.

Cardew's voltmeter depends on the heating effect produced when a current traverses a wire and consists essentially of a long fine platinum wire, stretched by a spring or a weight to which is attached a multiplying motion and an index. This wire, being introduced between the points of the circuit to be measured, becomes heated to an extent proportional to the square of the difference of potentials, and the motion of the index is a measure of this heating.

The principle of the *electrodynamometer* is that of measuring the repulsion between parallel currents moving in opposite directions, one of them being fixed and the other movable. Fig. 995 represents the main features of a form devised by Siemens for measuring the strength of the powerful currents used in electric lighting; w is a coil of stout copper wire, and w'

a single wire ; *nn* are mercury cups, and *kk* binding screws, by which connection is made with the main circuit LL.

The wire *w'* is surrounded by a stout spiral spring, which is connected at one end with this wire, and at the other with a screw, *s* ; this is provided with an index, *z*, which moves over a graduated scale, S. An index, *z'z'*, is also fixed to the wire *w'*. At the outset both indexes point to zero ; when the current passes it will be seen from the direction of the arrows that it traverses the fixed and movable coils in opposite directions, and the point *z'* is displaced along the scale. By turning the screw *s* it is brought back to zero, in doing which the index *z* is moved through an angle which is a measure of the torsion of the spiral spring *f*, and this angle is proportional to the square of the strength of the current by which the movable coil is deflected.

This electrodynamometer has by no means the sensitiveness which can be readily obtained with galvanometers ; but it has the advantage that its indications are independent of the strength of the external field, and when the two coils are traversed by the same current they are also independent of the direction of the current ; and can accordingly be used with advantage in measuring alternating currents.

An electrodynamometer devised by Giltay on a principle first introduced by Bellati is remarkable for its great sensitiveness. A bundle of fine iron wires hangs by a bifilar suspension inside the coil of a multiplier, the plane of which is at an angle of 45° with the magnetic meridian. The bundle itself is at an angle of 45° with the plane of the coils, and is thus at right angles to the magnetic meridian. When alternating currents are passed through the coil they magnetise the wires with alternate poles, so that the bundle is always deflected in the same direction. The deflections are read off by a mirror and scale, and when small, are directly proportional to the square of the current. The apparatus is so sensitive that deflections of 18 cm. on the scale are produced by the currents of an ordinary telephone.

963. **Absolute electrical units.**—The great importance of having a uniform system of measurements of physical magnitudes which should be universally adopted is at once obvious, and this has been more especially felt in the applications of electricity. The first step in this direction was taken by the British Association, which adopted the system of absolute units known as the C.G.S. system, of which mention has already been made (61*a*, 709), and which this account is intended to supplement.

The essence of an absolute system of physical measurements is that the various units may be directly expressed in mechanical units (61*a*). A system of absolute electrical units may be based on either the electrostatic, the electromagnetic, or again on the electrodynamic actions. There is no theoretical reason why one should be preferred to another of these, but in practice only the two former are used. Of these the electrostatic system is perhaps the simpler, but that based on electromagnetism is most convenient, and best lends itself to the practical determination of the most important standards, such as those of electromotive force and resistance.

Electrostatic Units

We shall distinguish these units by small letters placed in brackets.

Quantity of Electricity. q. Coulomb's law given for the repulsive force between two equal quantities q, of electricity at the distance l, $f = \frac{q^2}{l^2}$ (734), from which $q = l\sqrt{f}$. Hence we have for the dimensions of unit quantity of electricity $[q] = lf^{\frac{1}{2}} = L^{\frac{3}{2}}M^{\frac{1}{2}}T^{-1}$.

Potential. v. The potential of a quantity of electricity at the distance l is the quotient of the quantity by the distance. Hence $[v] = \frac{q}{l} = L^{\frac{1}{2}}M^{\frac{1}{2}}T^{-1}$.

Capacity. c. The capacity of a conductor is the quotient of the quantity of electricity with which it is charged, by the potential which this quantity produces in it; $[c] = \frac{q}{v}$ from which $[c] = L$. Hence the capacity of a conductor is expressed by a length. Unit capacity is thus that of a body which is raised by unit quantity to unit potential. An insulated conducting sphere which has a radius of one centimetre has unit capacity.

Current. i. The strength of a current is the quantity of electricity which passes in a given time; $[i] = \frac{q}{t} = L^{\frac{3}{2}}M^{\frac{1}{2}}T^{-2}$. Accordingly unit current is that which conveys unit quantity of electricity in a second.

Resistance. r. From Ohm's law (825), the resistance of a conductor is the quotient of difference of potentials at the two ends of a wire by the strength of a current. Hence $[r] = \frac{v}{i} = L^{-1}T$, which shows that the dimensions of resistance are the inverse of a velocity.

Electromagnetic Units

Quantity of magnetism. From Coulomb's law $f = \frac{M^2}{l^2}$, from which $[M] = L^{\frac{3}{2}}M^{\frac{1}{2}}T^{-1}$—that is, the same as that of quantity of electricity on the electrostatic system. Unit magnetic pole is that which repels an equal pole at a distance of a centimetre with a force of a dyne.

Magnetic Field. H. Unit magnetic field is that field in which unit quantity magnetism is acted on by unit force. Hence $F = HM$, from which $[H] = L^{-1}M^{\frac{1}{2}}T^{-1}$.

Current. I. The unit of electrical current in the electromagnetic system is that which, traversing unit length of an arc of a circle of unit radius, exerts unit force on unit pole, or unit magnetism at its centre. Its dimensions are $[I] = L^{\frac{1}{2}}M^{\frac{1}{2}}T^{-1}$.

Quantity of electricity. Q. The quantity of electricity conveyed by a conductor is the product of the current by the time that it lasts. Hence unit quantity is that which passes in a second in a conductor in which unit current is flowing, $[Q] = IT = L^{\frac{1}{2}}M^{\frac{1}{2}}$.

Resistance. R. The resistance of a conductor may be defined by Joule's law, $W = I^2RT$. Hence $[R] = \frac{L}{T}$—that is, the resistance of a conductor is expressed by a velocity.

Electromotive force. Difference of potentials [E]. From Ohm's law, $E = IR = L^{\frac{1}{2}}M^{\frac{1}{2}}T^{-2}$.

964. **Practical units.**—The values of the absolute units in the C.G.S. system are not convenient for measuring the magnitudes which ordinarily occur. Thus the absolute unit of resistance is that represented by the twenty-thousandth part of a millimetre of pure copper wire a millimetre in diameter. It has therefore been necessary to choose units better suited for practical uses, and at the International Congress of Electricians at Paris in 1881 an International Commission was formed for the purpose of deciding on such units and determining their value. In 1892 a Committee of the Board of Trade agreed to recommend the following, which are in the main those originally introduced by the British Association.

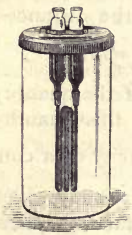

Fig. 996.

The practical unit of resistance is equal to 10^9 absolute electromagnetic C.G.S. units of resistance, and is called the *Ohm*. It has been decided to represent it by a column of pure mercury of uniform section 106·3 cm. in length at 0° C., and with a mass of 14·521 grammes. Copies of this standard may be made either in mercury (fig. 996), or in wire (fig. 995), and each copy has the value marked upon it, which is correct for a certain temperature. A wire of pure copper, a millimetre in diameter and 46·25 metres in length, has a resistance of one ohm. Siemens' unit (952) has a resistance of 0·94339 ohm. The copper conducting wire of an ordinary submarine cable has a resistance of about 11 ohms per mile.

In order to express multiples and submultiples the prefixes *mega* and *micro* are used, which are respectively a million times as great or as small. Thus a *megohm* is 10^6 ohms—that is, 10^{15} absolute units of resistance. In like manner a *microhm* is 10^{-6} ohm—that is, $10^3 = 1000$ such units.

The *Volt* is the practical unit of electromotive force or of difference of potentials, and is equal to 10^8 absolute units. From the difficulty of getting an element which is perfectly constant, more especially when it is closed, the standard of E.M.F. is best derived from measurements of resistance and of strength of current, which are both convenient and very accurate. Compared with the electrostatic unit of potential the volt is very small, being only $\frac{1}{300}$ of such a unit. The Board of Trade Committee takes the Clark cell standard of electromotive force as equal to 1·434 volt; so that

$$\text{Volt} = \frac{\text{electromotive force of Clark cell at } 15° \text{ C.}}{1{\cdot}434}.$$

The *Ampere* is the unit of current, and is the current produced by the electromotive force of a volt in a circuit having a resistance of an ohm. It is therefore equal to 10^{-1} C.G.S. units. It is equal to the current that can deposit 0·001118 gramme of silver per second from neutral solution of silver nitrate. A *millampere* is the thousandth of an ampere.

The resistance of a Daniell's element with an external cylinder of zinc, 8 inches high and $3\frac{1}{2}$ in diameter, surrounding the porous pot, is about 1·3 ohm, and taking its E.M.F. at 1·08 volt, its current when on short circuit is about 0·8 ampere. In like manner a medium-sized Bunsen has a resistance of about 0·1 ohm, and as its E.M.F. is 1·8 volt, the current on short circuit is 18 amperes. A Brush machine the current of which ignited 16 lamps had an

E.M.F. of 839 volts; its internal resistance was 10·55, and the external, including the lamps, was 73 ohms. Accordingly the current was 10·04 amperes. A Holtz machine has in electromagnetic measure the E.M.F. of 90,000 volts; its internal resistance, when it makes two turns in a second, is calculated at 27×10^8 ohms; thus its current is $\frac{1}{300000}$ of an ampere, or $\frac{1}{30}$ of a millampere. Such a current is too weak for telegraph work; the currents used with the ordinary Morse receivers have a strength of 14 to 16 millamperes.

The *Coulomb* is the unit of quantity of electricity, and is that quantity which traverses the section of a conductor in a second, when a current of an ampere is passing through it. A coulomb of electricity in traversing an electrolyte decomposes a weight of the body expressed by 0·00001038 times its chemical equivalent.

The *Farad* is the unit of capacity, and is such that in a condenser of that capacity the quantity of a coulomb produces a difference of potential of a volt. It is 10^{-9} C.G.S. units. The farad is far too large a unit for practical use; thus the capacity of the globe is only 0·000636 of a farad, that of the Sun does not amount to a farad. Accordingly the technical unit of capacity is the millionth part of this, and is called the *microfarad*. This is 10^{-15} units. A Leyden jar with a total coated surface of a square metre, and the glass of which is 1 mm. thick, has a capacity of $\frac{1}{55}$ of a microfarad. The capacity of an ordinary submarine cable may be taken at about $\frac{1}{3}$ of a microfarad per *knot* or nautical mile of 1852 metres. A sphere nine kilometres in radius has a capacity of a microfarad.

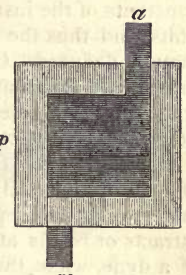

Fig. 997.

The practical standards consist of circular or square sheets of tinfoil with projecting tongues, *a* and *a'* (fig. 997), fastened on thin sheets of mica. Between each such coated sheet is placed an uncoated one of mica, the two sets of tongues being severally connected with each other, and thus the coatings represent the coated surfaces of a condenser. The whole is enclosed in a box; a condenser having a capacity of a microfarad will represent a coated surface of over 6 square yards.

Watt.—The energy, W, of an electrical current in unit time may be variously expressed; thus $W = C^2R = \frac{E^2}{R} = CE$. This latter expression is the most convenient for practical purposes; if the factors which express the watt are given in practical units, it represents the work done by unit current (ampere) when impelled by an E.M.F. of a volt. It is thus a *voltampere*, and on the proposal of the late Sir W. Siemens has been called a *watt*. If the factors are given in absolute units, V A is equal to 10^7 ergs. It may also be defined as the work done by the quantity of electricity of a coulomb falling through a difference of potentials equal to a volt, and in this form the definition is closely analogous to that of a kilogramme metre.

The watt is $\frac{1}{746}$ of an English horse-power, or one horse-power = 746 watts. The French *cheval vapeur* of 75 kilogramme-metres or 542·4 foot-pounds per second is equal to 736 watts.

965. **Relation of the electrostatic to the electromagnetic unit.**—If we compare the dimensions of the units of quantity and of the other electrical magnitudes in the electrostatic with those of the corresponding dimensions as expressed in the electromagnetic system, we find that the ratios are independent of the unit of mass, and that $\frac{L}{T}$, that is, the expression of a velocity, always enters into the ratio between them. Now the ratio of the two sets of units may be determined experimentally. Suppose that a condenser is charged with electricity. Knowing its dimensions, the quantity, q, of the charge may be determined in electrostatic measure, by measuring, for instance, the repulsion which a given proportion of the total charge produces in a torsion balance of known dimensions. The same condenser, being charged to the same extent, may be discharged through a galvanometer, and by measuring the deflection produced, and knowing the constants of the instrument, the quantity may be obtained in electromagnetic units, and thus the ratio of the quantity expressed in the two sets of units may be deduced. Or, again, the E.M.F. of a Daniell's cell may be measured first by the aid of an absolute electrometer (781), which will give in electrostatic units of potential about 0·0036. On the other hand, the potential determined in electromagnetic measure has the value $1·088 \times 10^8$. Hence it would thus be found that in round numbers the electromagnetic unit of quantity is equal to 3×10^{10} electrostatic units of quantity. This can be understood if we consider that the latter is the quantity of electricity which attracts or repels another equal quantity at a distance of 1 cm. with a force of a dyne, while the former is the quantity which traverses the wire in a second when the current has unit intensity. Similarly, by making determinations of the ratio in all cases in which the same magnitude may be determined in electrostatic as well as in electromagnetic measure, it is found that the agreement in the numbers found is very close, and as the mean of the best results is $2·9857 \times 10^{10}$. As the ratio between the units is always of the dimensions of a velocity, and holds under the condition that the centimetre is the unit of length, and the second is the unit of time, this velocity is 298,570 kilometres, or 185,530 miles in a second. Now this number agrees very closely with that which has been experimentally found for the velocity of light—185,420 miles (507).

Faraday, discarding the idea of action at a distance, considered that electrical forces are transmitted through an elastic medium, and that this was the luminiferous ether (637). Maxwell, starting from these ideas, was led to the development of his *electromagnetic theory of light*; this theory requires that an electromagnetic wave motion must be transmitted with a velocity represented by the ratio of the electrostatic to the electromagnetic unit of quantity of electricity; this, as we have seen, is equal to the velocity of light. Now, if luminous and electromagnetic waves are transmitted in one and the same medium and with the same velocity, it is natural to suppose that they are identical in kind. The theory also requires the relation between the refractive index of a body and the dielectric constant which we have already found to exist (748).

These theoretical previsions of what is known as the Faraday-Maxwell theory have quite recently received a striking confirmation in a most remark-

able and beautiful series of experiments by Professor Hertz, of which we can only give an outline of some of the principal results.

In order to demonstrate that light is essentially an electromagnetic phenomenon, it would be necessary to produce, with a vibratory motion of a purely electromagnetic origin, the same class of phenomena as can be produced with ordinary light, such, more especially, as interference and refraction. The difficulty is the great length of the waves with which we have to deal; for from the laws of wave motion (253), if the frequency of the electrical oscillations were even as great as ten thousand in a second, that would represent a wave-length of 30 kilometres, and for a wave-length of 3 metres the duration should not be greater than the hundred-millionth of a second. Now in the discharge of a Leyden jar, or the still more rapid one which takes place between the ends of the secondary wire of a Ruhmkorff's coil, the duration of the oscillation is comprised within the ten-thousandth and the hundred-thousandth of a second.

Hertz devised an apparatus which he calls a *vibrator* or discharger for obtaining continuous but very rapid electrical oscillations, true rays of electrical force. Two spheres or plates of metal, AA′ (fig. 998), are provided with straight metal rods with small knobs at the end, the distance, C, of which can be adjusted. The rods are in connection with the poles of a small Ruhmkorff's coil B, which charges the two spheres to different potentials and a spark passes at C. This spark, by heating the air, forms, as it were, a path for the subsequent oscillations, and the vibrator now discharges itself independently, as if it were detached from the coil, forming between each discharge of the Ruhmkorff a series of oscillations of extreme rapidity. Theory shows that if the resistance of the circuit is so small as to be neglected, the period, t, of such an oscillation is proportional to the square root of the capacity, and the coefficient of self-induction, $t = 2\pi\sqrt{LC}$; and accordingly by suitably choosing the dimensions of the apparatus it has been possible to obtain electrical oscillations with a frequency of 5×10^9 in a second, representing a wave-length of 60 cm.

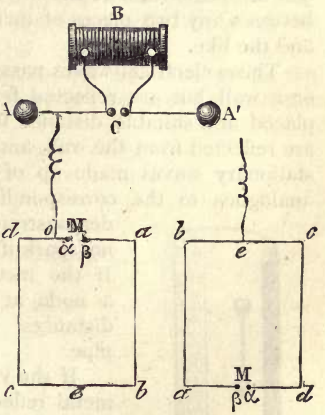

Fig. 998.

If a wire frame be connected with one of the spheres, as shown on the right of the figure, the potentials are equal at α and β, and no spark passes between them. But if the connection is made dissymmetrically, as shown in the left—that is to say, is nearer one knob than the other—there is a difference of potential, and very minute sparks pass continuously.

If we have a vibrating tuning-fork producing sound waves therefore, and we approach to it a body tuned in unison with the fork, the body in question begins to vibrate also; such bodies as we have seen are called *resonators* (255). In order to investigate the distribution of electrical waves

in the region about a vibrator, Hertz used what he calls an *electrical resonator*. This consists (fig. 999) of a wire ring, one end terminating in a point and the other in a knob, which by a micrometric arrangement, not shown in the figure, may be kept at any desired distance. The dimensions of the frame are adjusted—tuned as it were—so that its oscillations synchronise with those of the vibrator. If now the resonator is placed with its axis parallel to the axis of the vibrator—that is, to the line joining α and β—positions are found in which a flow of minute sparks passes between the ends of the resonator; their quantity and strength diminish as the distance from the vibrator increases, but are perceptible at even 50 or 60 feet. These waves are transverse to the direction of propagation, as appears from the fact that when in a given position the resonator is giving sparks, it ceases to do so when turned at right angles. When the vibrator works well the whole room is pervaded by electrical waves, and by varying the position and distance of the resonator in reference to the vibrator, it is possible to plot out the exact form of the wave motion in the field. Sparks can be taken between any two pieces of metal; by presenting a penknife to a gaspipe and the like.

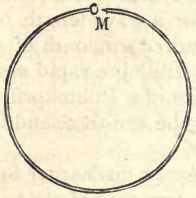

Fig. 999.

These electrical waves pass through ordinary conductors, such as a door or a wall, but are reflected from a conducting surface. If the vibrator is placed at a suitable distance in front of a large sheet of metal, the waves are reflected from the wall, and interfering with incident rays give rise to stationary waves made up of nodes and loops at regular intervals, quite analogous to the corresponding acoustical phenomenon. This may be demonstrated by means of the resonator, which gives no spark if placed at a node, but does so if in a loop. If the metal is a perfect conductor, there is formed a node at the reflecting surface and others at equal distances. This is analogous to the case of a stopped pipe.

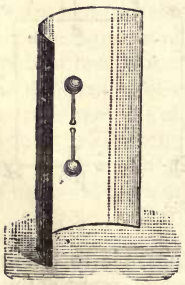
Fig. 1000.

If the vibrator is placed in front of a tall cylindrical metal reflector with a parabolic section (fig. 1000), the effects produced are more pronounced, and can be perceived at a greater distance than before. A mass of electrical rays parallel to the focal line is formed, and the experiment of the conjugate mirrors (420) may be repeated. An insulating screen placed between the two mirrors does not stop the action, but a conducting screen does. It forms an *electrical shadow*.

The electrical rays undergo a refraction on passing from one medium to another. This Hertz demonstrated by means of a huge prism of pitch, 5 feet in height, with a refracting angle of 30°, and with a face of over a square yard. When the rays, rendered parallel by the mirror, fell on this they were deflected towards the base, and by means of the resonator the refractive index was found to be 1·69; the optical refractive index is between 1·5 and 1·6.

By allowing electrical rays to fall on a plane reflecting surface, part are absorbed and part reflected, and it is readily shown that the angle of reflection, as with light and heat, is equal to the angle of incidence (511).

If the electrical rays concentrated by a mirror fall on a grating formed of parallel copper wires, it is found that when the grating is in the direction of the rays—that is, when the wires are parallel to the focal line of the mirror —they are transmitted, but are stopped when the wires are turned at right angles to the direction. This is a phenomenon of polarisation ; the grating acts in regard to the rays like a tourmaline in respect of plane polarised light (666). In another experiment Hertz reproduced the phenomena of diffraction (646). He also showed that electrical waves can exert a mechanical action by causing them to strike against a small tube of gold paper very delicately suspended.

Hertz's experiments have been reproduced by many observers, and with other resonators and modifications in the way of experimenting. Dragoumis found that Geissler's tubes were well fitted for this purpose. Lecher's method of investigation is convenient. The vibrator is formed of two metal plates, *ab* (fig. 1001), connected with the terminals of a Ruhmkorff's coil as in fig. 998, opposite which are two similar ones, *a'b'* ; from these pass long wires, *st s't'*, parallel from *s* to *t*, which are tightly stretched by means of strings.

If a Geissler's tube *g* with or without electrodes be placed across the wires it becomes luminous. If now a metal wire, *xx'*, be placed across, the

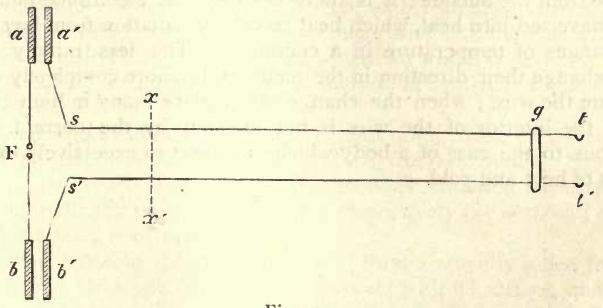

Fig. 1001.

luminosity ceases, but by moving the cross wires backwards or forwards positions are found in which it again appears. These positions represent the nodes. According to Lecher, this is a phenomenon of electrical resonance ; a principal vibration is found from *a'sxx's'b'*, which by induction produces secondary vibrations in the rest of the wire.

If a portion of each of the wires is cut off, the resonance is disturbed, the tube is dark, and to restore the luminosity the bridge *xx'* must be moved nearer to F' ; the amount of displacement is half the length of the pieces thus cut off.

If a sheet of tinfoil is attached to each end, this increases the capacity ; the period of the vibration is increased, the wave-length greater, and to keep the tubes luminous the cross wires must be moved nearer the ends *tt'*. This

leads to a method of determining capacities and dielectric constants by means of very rapid vibrations. The apparatus furnishes also a ready means of determining the electrical wave-length. In certain experiments with a period of vibration of the hundred-thousandth of a second, the distance of two consecutive nodes, or half a wave-length, was found to be 1·4 m., so that from the formula (253) we get for the velocity of electricity 280,000 kilometres—that is, virtually the same as that of light (507).

The velocity is the same whatever be the nature of the wires used, from which it follows that the transmission is effected by the surrounding medium and not by the wire itself.

All these experiments show that the analogy is complete between the waves of light and of electricity, and we are led to the conclusion that electrical, thermal, and luminous phenomena have one and the same origin, which is a vibratory motion of the ether, and that they only differ in the length of the waves. The wave-length of the longest visible rays is about 0·00005 cm., that of the longest dark thermal ray hitherto observed is 0·003 cm., while the shortest electrical wave is 50 cm., or a million times that of light.

These experiments lead to a fundamental change in our views as to the way in which the electrical current is transmitted. It has hitherto been considered that when the current is closed the wire itself is the agency by which it is transmitted. We must for the future consider that the surrounding medium, the ether, transmits the electrical energy, and that this energy enters the wire from the outside; it is there destroyed as electromagnetic energy, but is converted into heat, which heat travels by radiation from layer to layer like changes of temperature in a conductor. The less rapidly electrical forces change their direction in the medium the more completely does heat penetrate the wire; when the change takes place many million times in a second the interior of the wire is not affected by the current. This is analogous to the case of a body which is subject to excessively rapid alternations of heat and cold.

CHAPTER X

ANIMAL ELECTRICITY

966. **Muscular currents.**—The existence of electrical currents in living muscle was first indicated by Galvani, but his researches fell into oblivion after the discovery of the voltaic pile, which was supposed to explain all the phenomena. Since then, Nobili, Matteucci, and others, especially, in late years, Du Bois Reymond, have shown that electric currents do exist in living muscles and nerves, and have investigated their laws.

For investigating these currents it is necessary to have a delicate galvanometer, and also electrodes which will not become polarised or give a current of their own, and which will not in any way alter the muscle when placed in contact with it; the electrodes which satisfy these conditions best are those of Du Bois Reymond, as modified by Donders. Each consists of a glass tube, one end of which is narrowed and stopped by a plug of paste made by moistening china-clay with a half per cent. solution of common salt; the tube is then partially filled with a saturated solution of sulphate of zinc; and into this dips the end of a piece of thoroughly amalgamated zinc wire, the other end of which is connected by a copper wire with the galvanometer; the moistened china-clay is a conducting medium which is perfectly neutral to the muscle, and amalgamated zinc in solution of sulphate of zinc does not become polarised.

967. **Currents of muscle at rest.**—In describing these experiments the surface of the muscle is called the *natural longitudinal section*; the tendon the *natural transverse section*; and the sections obtained by cutting the muscle longitudinally or transversely are respectively the *artificial longitudinal* and *artificial transverse sections.*

If a living irritable muscle be removed from a recently killed frog, and the clay of one electrode be placed in contact with its surface, and of the other with its tendon, the galvanometer will indicate a current from the former to the latter; showing, therefore, that the surface of the muscle is positive with respect to the tendon. By varying the position of the electrodes, and making various artificial sections, it is found—

1. That any longitudinal section is positive to any transverse section.

2. That any point of a longitudinal section nearer the middle of the muscle is positive to any other point of the same section farther from the centre.

3. In any artificial transverse section any point nearer the periphery is positive to one nearer the centre.

4. The current obtained between two points in a longitudinal or in a transverse section is always much more feeble than that obtained between two different sections.

5. No current is obtained if two points of the same section equidistant from its centre be taken.

6. To obtain these currents it is not necessary to employ a whole muscle, or a considerable part of one, but the smallest fragment that can be experimented with is sufficient.

7. If a muscle be cut straight across, the most powerful current is that from the centre of the natural longitudinal section to the centre of the artificial transverse; but if the muscle be cut across obliquely, as in fig. 1002, the most positive point is moved from *c* towards *b*, and the most negative from *d* towards *a* ('*currents of inclination*').

Fig. 1002.

To explain the existence and relations of these muscular currents, it may be supposed that each muscle is made up of regularly disposed electromotor elements, which may be regarded as cylinders whose axes are parallel to that of the muscle, and whose sides are charged with positive and their ends with negative electricity; and, further, that all are suspended and enveloped in a conducting medium. In such a case (fig. 1002) it is clear that throughout most of the muscle the positive electricities of the opposed surfaces would neutralise one another, as would also the negative charges of the ends of the cylinders; so that, so long as the muscle was intact, only the charges at its sides and ends would be left to manifest themselves by the production of electromotive phenomena; the whole muscle being enveloped in a conducting stratum, a current would constantly be passing from the longitudinal to the transverse section, and, a part of this being led off by the wire circuit, would manifest itself in the galvanometer.

This theory also explains the currents between two different points on the same section; the positive charge at *b*, for instance (fig. 1003), would have more resistance to overcome in getting to the transverse section than that at *d*, therefore it has a higher tension; and if *b* and *d* are connected by the electrodes, *b* will be found positive to *d*, and a current will pass from the former to the latter. What are called *currents of inclination* are also explicable on the above hypothesis, for the oblique section can be represented as a number of elements arranged as in fig. 1004, so that both the longitudinal surfaces and the ends of the cylinders are laid bare, and it can thus be regarded as a sort of oblique pile whose positive pole is towards *b* and its negative at *a*, and whose current adds itself algebraically to the ordinary current and displaces its poles as above mentioned.

Fig. 1003.

A perfectly fresh muscle, very carefully removed, with the least possible contact with foreign matters, sometimes gives almost no current between its different natural sections, and the current always becomes more marked after

the muscle has been exposed a short time ; nevertheless, the phenomena are vital, for the currents disappear completely with the life of the muscle, sometimes becoming first irregular or even reversed in direction.

968. **Rheoscopic frog. Contraction without metals.**—The existence of the muscular currents can be manifested without a galvanometer, by using another muscle as a galvanoscope. Thus, if the nerve of one living muscle of a frog be dropped suddenly on another living muscle, so as to come in contact with its longitudinal and transverse sections, a contraction of the first muscle will occur, due to the stimulation of its nerve by the passage through it of the electric current derived from the surface of the second.

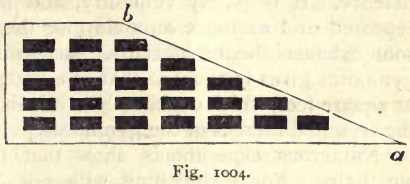

Fig. 1004.

969. **Currents in active muscle.**—When a muscle is made to contract there occurs a sudden diminution of its natural electric current, as indicated by the galvanometer. This is so instantaneous that, in the case of a single muscular contraction, it does not overcome the inertia of the needle of the galvanometer ; but if the contractions be made to succeed one another very rapidly—that is, if the muscle be *tetanised* (827)—then the needle swings steadily back towards zero from the position in which the current of the resting muscle had kept it, often gaining such momentum in the swing as to pass beyond the zero point, but soon reverting to some point between zero and its original position.

The negative variation in the case of a simple muscular contraction can, however, be made manifest by using another muscle as a rheoscope ; if the nerve of this second muscle be laid over the first muscle in such a position that the muscular current passes through it, and the first muscle be then made to contract, the sudden alteration in its strength of the current stimulates the nerve laid on it (827), and so causes a contraction of the muscle to which the latter belongs.

The same phenomenon can be demonstrated in the muscles of warm-blooded animals ; but with less ease, on account of the difficulty of keeping them alive after they are laid bare or removed from the body. Experiments made by placing electrodes outside the skin, or passing them through it, are inexact and unsatisfactory.

970. **Electric currents in nerve.**—The same electromotor indications can be obtained from nerves as from muscles—at least, as far as their smaller size will permit ; the currents are more feeble than the muscular ones, but can be demonstrated by the galvanometer in a similar way. Negative variation has been proved to occur in active nerve as in active muscle. The effect of a constant current passed through one part of a nerve on the amount of the normal nerve-current, measured at another part, has already been described (Chapter III., ELECTROTONUS).

971. **Electrical fish.**—Electrical fish are those fish which have the remarkable property of giving, when touched, shocks like those of the Leyden jar. Of these fish there are several species, the best known of which are the torpedo, the gymnotus, and the silurus. The torpedo, which is very common

in the Mediterranean, has been carefully studied by Becquerel and Breschet in France, and by Matteucci in Italy. The gymnotus was investigated by Humboldt and Bonpland in South America, and in England by Faraday, who had the opportunity of examining live specimens.

The shock which they give serves both as a means of offence and of defence. It is purely voluntary, and becomes gradually weaker as it is repeated and as these animals lose their vitality, for the electrical action soon exhausts them materially. According to Faraday, the shock which the gymnotus gives is equal to that of a battery of 15 jars exposing a coating of 25 square feet, which explains how it is that horses frequently give way under the repeated attacks of the gymnotus.

Numerous experiments show that these shocks are due to ordinary electricity. For if, touching with one hand the back of the animal, the belly is touched with the other, or with a metal rod, a violent shock is felt in the wrists and arms ; while no shock is felt if the animal is touched with an insulating body. Further, when the back is connected with one end of a galvanometer wire and the belly with the other, at each discharge the needle is deflected, but immediately returns to zero, which shows that there is an instantaneous current ; and, moreover, the direction of the needle also shows that the current goes from the back to the belly of the fish. Lastly, if the current of a torpedo be passed through a helix in the centre of which is a small steel bar, the latter is magnetised by the passage of a discharge.

By means of the galvanometer, Matteucci established the following facts :—

1. When a torpedo is lively, it can give a shock in any part of its body, but as its vitality diminishes, the parts at which it can give a shock are nearer the organ which is the seat of the development of electricity. 2. Any point of the back is always positive as compared with the corresponding point of the belly. 3. Of any two points at different distances from the electrical organ, the nearest always plays the part of a positive pole, and the farthest that of a negative pole. With the belly the reverse is the case.

The organ where the electricity is produced in the torpedo is double, and formed of two parts symmetrically situated on two sides of the head and attached to the skull-bone by the internal face. Each part consists of nearly parallel lamellæ of connective tissue inclosing small chambers, in which lie the so-called *electrical plates*, each of which has a final nerve ramification distributed on one of its faces. The face, on which the nerve ends, is turned the same way in all the plates, and when the discharge takes place is always negative to the other.

Matteucci investigated the influence of the brain on the discharge. For this purpose he laid bare the brain of a living torpedo, and found that the first three lobes could be irritated without the discharge being produced, and that when they were removed the animal still possessed the faculty of giving a shock. The fourth lobe, on the contrary, could not be irritated without an immediate production of the discharge ; but if it was removed, all disengagement of electricity disappeared, even if the other lobes remained untouched. Hence it would appear that the primary source of the electricity elaborated is the fourth lobe, whence it is transmitted by means of the nerves

to the two organs described above, which act as multipliers. In the silurus the head appears also to be the seat of the electricity ; but in the gymnotus it is found in the tail.

972. **Application of electricity to medicine.**—The first applications of electricity to medicine date from the discovery of the Leyden jar. Nollet and Boze appear to have been the first who thought of the application, and soon the spark and electrical friction became a universal panacea ; but it must be admitted that the results of subsequent trials did not come up to the hopes of the early experimentalists.

After the discovery of dynamic electricity Galvani proposed its application to medicine ; since which time many physicists and physiologists have been engaged upon this subject, and yet there is still much uncertainty as to the real effects of electricity, the cases in which it is to be applied, and the best mode of applying it. Practical men prefer the use of currents to that of statical electricity, and, except in a few cases, discontinuous to continuous currents. There is, finally, a choice between the currents of the battery and induction currents ; further, the effects of the latter differ according as induction currents of the first or second order are used. In fact, since induction currents, although very intense, have a very feeble chemical action, it follows that when they traverse the organs they do not produce the chemical effects of the current of the battery, and hence do not tend to produce the same disorganisation. Further, in electrifying the muscles of the face, induction currents are to be preferred, for these currents only act feebly on the retina, while the currents of the battery act energetically on this organ, and may affect it dangerously. There is a difference in the action of induced currents of different orders ; for while the primary induced current causes lively muscular actions, but has little action on the cutaneous sensibility, the secondary induced current, on the contrary, increases the cutaneous sensibility to such a point that its use ought to be proscribed to persons whose skin is very irritable.

Hence electrical currents should not be applied in therapeutics without a thorough knowledge of their various properties. They ought to be used with great prudence, for their continued action may produce serious accidents. Matteucci says : ' In commencing, a feeble current must always be used. This precaution now seems to me the more important as I did not think it so before seeing a paralytic person seized with almost tetanic convulsions under the action of a current formed of a single element. Take care not to continue the application too long, especially if the current is energetic. Rather apply a frequently-interrupted current than a continuous one, especially if it be strong ; but after twenty or thirty shocks, at most, let the patient take a few moments' rest.'

Of late years, however, feeble continuous currents have come more into use. They are frequently of great service when applied skilfully, so as to throw the nerves of the diseased part into a state of cathelectrotonus or anelectrotonus (827), according to the object which is sought to be attained in any given case.

ELEMENTARY OUTLINES

OF

METEOROLOGY AND CLIMATOLOGY

METEOROLOGY

973. **Meteorology.**—The phenomena which are produced in the atmosphere are called *meteors* ; and *meteorology* is that part of physics which is concerned with the study of these phenomena.

A distinction is made between *aerial* meteors, such as winds, hurricanes, and whirlwinds ; *aqueous* meteors, comprising fogs, clouds, rain, dew, snow, and hail ; and *luminous* meteors, as lightning, the rainbow, and the aurora borealis.

974. **Meteorograph.**—The importance of being able to make continuous observations of various meteorological phenomena has led to the construction of various forms of automatic arrangements for this purpose, of which that of Osler in England may be specially mentioned. One of the most comprehensive and complete is Secchi's *meteorograph*, of which we will give here a description.

It consists of a base of masonry about 2 feet high (fig. 1005) ; on this are fixed four columns, about 2½ yards high, which support a table on which is a clockwork regulating the whole of the movements. The phenomena are registered on two sheets which move downwards on two opposite sides, their motion being regulated by the clockwork. One of them occupies ten days in so doing, and on it are registered the direction and velocity of the wind, the temperature of the air, the height of the barometer, and the occurrence of rain ; on the second, which only takes two days, the barometric height and the occurrence of rain are repeated, but on a much larger scale ; this gives, moreover, the moisture of the air.

Direction of the wind.—The four principal directions of the wind are registered by means of four pencils fixed at the top of thin brass rods, *a*, *b*, *c*, *d* (fig. 1005), which are provided at the bottom ends with soft iron keepers attracted by two electromagnets, E E′, for west and north, and by two other electromagnets lower down for south and east. These four electromagnets, as well as all the others on the apparatus, are worked by a single sand battery (886) of twenty-four elements. The passage of the current in one or

the other of these electromagnets is regulated by means of a vane (fig. 1006) consisting of two plates at an angle of thirty degrees with each other, by

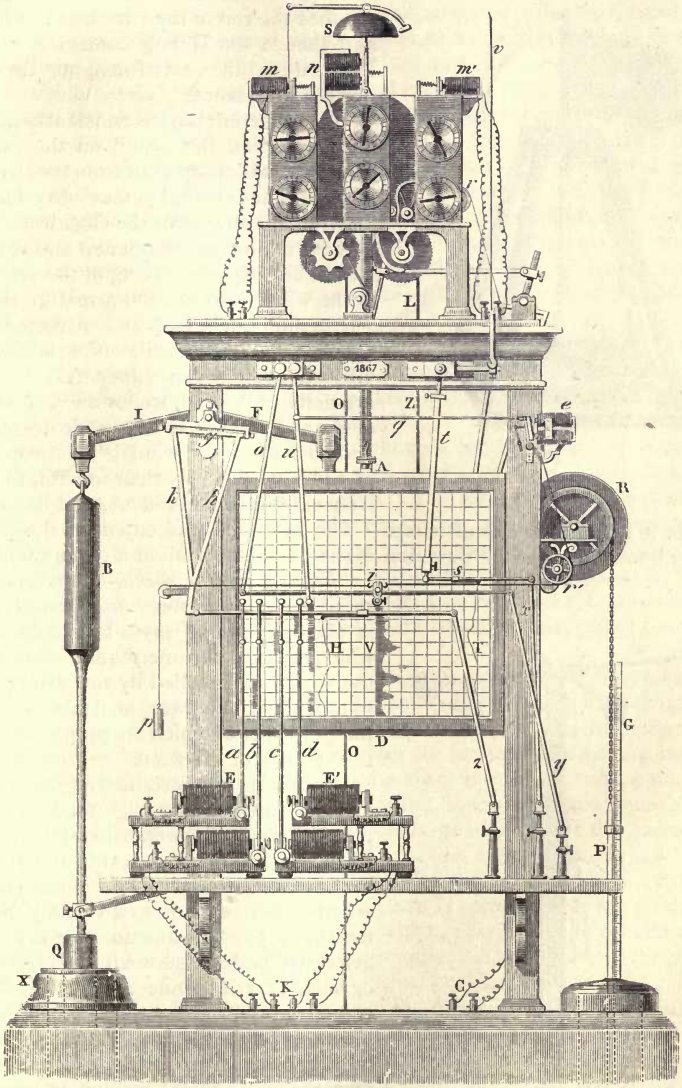

Fig. 1005.

which greater steadiness is obtained than with a single plate. In the rod of the vane is a small brass plate, *o*; this part is in the centre of four metal

sectors, insulated from each other, and each provided with a binding-screw, by which connection is established with the binding-screw K, and the electromagnets E E′. The battery current reaches the rod of the vane by the wire *a*, and thence the sliding contact *o*, which leads it to the electromagnet for the north, for instance.

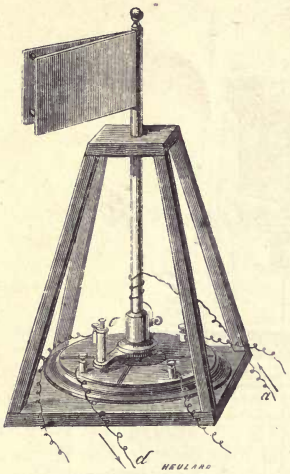

Fig. 1006.

If the current passed constantly in this electromagnet, the pencil on the rod *d* would be stationary ; but from the electromagnet E′ the current passes into a second electromagnet, *n*, over the clockwork, and is thereby alternately opened and closed, as will be seen in speaking of the velocity of the wind. Hence the armature of the rod *d*, alternately free and attracted, oscillates ; and its pencil, which is always pressed against the paper AD by the elasticity of the rod, traces on it a series of parallel dashes as the paper descends, and so long as the wind is in the north. If the wind changes then to west, for instance, the rod *a* oscillates, and its pencil traces a different series of marks being known, we get the direction of the prevalent wind at a given moment. The rate of displacement of the paper

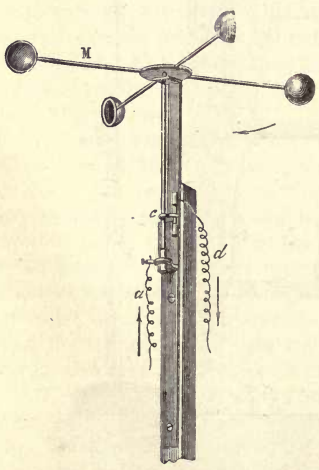

Fig. 1007.

Velocity of the wind.—This is indicated by a Robinson's *anemometer*, and is registered in two ways : by two counters which mark in decametres and kilometres the distance travelled by the wind ; and by a pencil which traces on a table a curve, the ordinates of which are proportional to the velocity of the wind.

Robinson, who originally devised this form of anemometer (fig. 1007), proved that its velocity is proportional to that of the wind ; in this apparatus the length of the arms is so calculated that each revolution corresponds to a velocity of ten metres (975). The anemometer is placed at a considerable distance from the meteorograph, and is connected with it by a copper wire, *d*, which passes to the electromagnet, *n*, of the counter. On its rod there is, moreover, an excentric, which at each turn touches a metal contact in connection with the wire *d*. The battery current reaches the anemometer by a wire *a*, the current is closed once at each rotation, and passes to the electromagnet *n*, which moves the needle of

the dial through one division. There are fifty such divisions, which represent as many turns of the vane, and therefore so many multiples of ten metres. The lower dial marks the kilometres.

The curve of velocities is traced on the sheet by a pencil, *i*, fixed to a horizontal rod. This is joined at its two ends to two guide-rods, *o* and *y*, which keep it parallel. The pencil and the rod are moved laterally by a chain which passes over two pulleys, *r'* and *r*, and is then coiled over a pulley placed on the shaft of the counter, but connected with it merely by a ratchet-wheel; and moved thus by the counter and the chain, the pencil traces every hour on the sheet a line the length of which is proportioned to the velocity of the wind. From hour to hour an excentric moved by clockwork detaches, from the shaft of the counter, the pulley on which is coiled the chain, and this pulley becoming out of gear, a weight, *p*, connected with the pencil *i*, restores this to its starting-point. All the lines, V, traced successively by the pencil, start from the same straight line as ordinates, and their ends give the curve of velocities.

The counters on the right and left are worked by electromagnets, *m m'*, and are intended to denote the velocity of special winds; for instance, those of the north and south, by connecting their electromagnets with the north and south sectors of the vane (fig. 1006).

Temperature of the air.—This is indicated by the expansion and contraction of a copper wire of 16 metres in length stretched backwards and forwards on a fir post 8 metres in length. The whole being placed on the outside—on the roof, for instance—the expansion and contraction are transmitted by a system of levers to a wire, *v*, which passes to the meteorograph, where it is jointed to a bent lever, *t*. This is jointed to a horizontal rod, *s*, which supports a pencil, and at the other end is jointed to a guide-rod, *x*. Thus the pencil, sharing the oscillations of the whole system, traces the curve of the temperatures.

Pressure of the atmosphere.—This is registered by the oscillations of a barometer, B, suspended at one end of a bent scale-beam, I F, playing on a knife-edge (fig. 1009). The arm F supports a counterpoise; to the arm I is suspended the barometer B, which is wider at the top than at the bottom. A wooden flange or floater, Q, fixed to the lower part of the tube, plunges in a bath of mercury, so that the buoyancy of the liquid counterbalances part of the weight of the barometer. Owing to the large diameter of the barometric chamber, a very slight variation of level in this chamber makes the tube oscillate, and with it the scale-beam I F. To the axis of this is a triangle, *ghk*, jointed to a horizontal rod, which in turn is connected with a guide-rod, *z*. In the middle of this rod is a pencil which, sharing in the oscillations of the triangle *ghk*, traces the curve H of pressure. A bent lever at the bottom of the barometer tube keeps this in a vertical position.

Rainfall.—This is registered between the direction of the winds and the curve H by a pencil at the end of a rod, *u*, which is worked by an electromagnet, *e*. On the roof is a funnel which collects the rain, and a long tube leads the water to a small water-balance, with the cups placed near the meteorograph (fig. 1008). To the axis of the scale-beam one pole of the battery is connected; the left cup being full, tips up, and a contact, *a*, closes the current, which passes then to one of the binding-screws, C, and

hence to the electromagnet, *e*. Then the right cup, being in turn full, tips in the opposite direction, and the contact *b* now transmits the current to the electromagnet. Thus, at each oscillation this latter attracts its armature, and with it the rod *a*, which makes a mark by means of a pencil at the end. If the rain is abundant, the oscillations of the beam are rapid, and the marks, being very close together, give a deep shade; if, on the contrary, the oscillations are slow, the marks are at a greater distance, and give a light shade. When the rain ceases the oscillations cease also, and the pencil makes no mark.

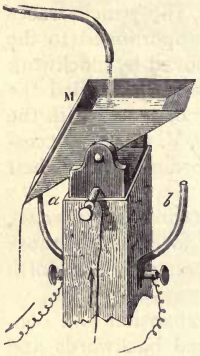

Fig. 1008.

To complete this description of the first face of the meteorograph: S is the alarum-bell of the clockwork; OO a cord supporting a weight which moves the works of the hour-hand; LZ is a second cord that supports the weight which works the alarum; the wheel U, placed below the clockwork, winds up the sheet AD when it is at the bottom of its course.

The second sheet (fig. 1009) gives the barometric height and the rainfall like the first, but on a larger scale, since the motion of the sheet is five times as rapid. Its principal function is that of registering the moisture of the air. This is effected by means of the *psychrometer* (fig. 1010). T and T′ are two thermometers fixed on two plates. The muslin which covers the second is kept continually moist by water dropping on it. In each of the bulbs are fused two platinum wires; the stems of the thermometers are open at the top, and in them are two platinum wires, *m* and *n*, suspended to a metal frame movable on four pulleys supported by a fixed piece, B. The frame A, in contact with the current of the battery, is suspended to a steel wire, L, which passes over a pulley to the meteorograph (fig. 1008). Here is a long triangular lever, W, which supports a small wheel, to which is fixed the wire L. The lever W, which turns about an axis, *f*, is moved by a rod, *a*, by means of an excentric, which the clock works every quarter of an hour. At each oscillation the lever W transmits its motion to a small chariot, on which is an electromagnet, *x*, and at the same time to the steel wire L, which supports the frame A (fig. 1010). The chariot, moved towards the left by the rotation of the excentric, lets the frame sink. The moment the first platinum wire reaches the mercurial column of the dry-bulb thermometer, which is the highest, the current is closed, and passes into the electromagnet of the chariot. An armature at once causes a pencil to mark a point on the sheet which is the beginning of a line representing the path of the dry-bulb thermometer. As the frame continues to descend, the second platinum wire touches the mercury of the wet bulb, and closes a current in a relay, M, which opens the circuit of the electromagnet, *x*. The pencil is then detached; then, returning upon itself, the chariot reproduces the closing and opening of the circuit in the opposite direction, the pencil making another mark, which is the end of the line. There are thus formed two series of dots arranged in two curves, one of which represents the path of the dry, and the other the path of the wet, bulb. The horizontal distance of the two points of these curves is proportional to the difference

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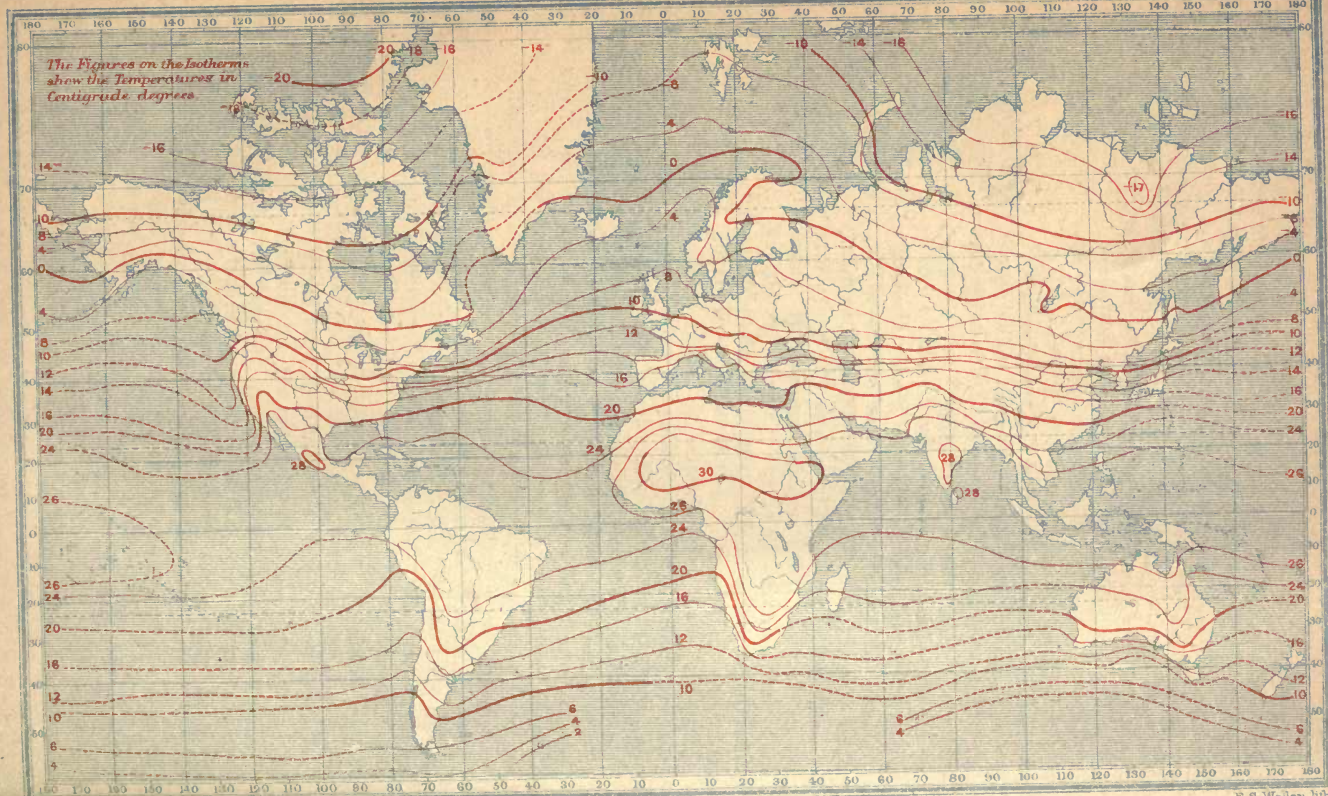

$t-t_1$ of the temperatures indicated at the same moment by the thermometers (fig. 1010).

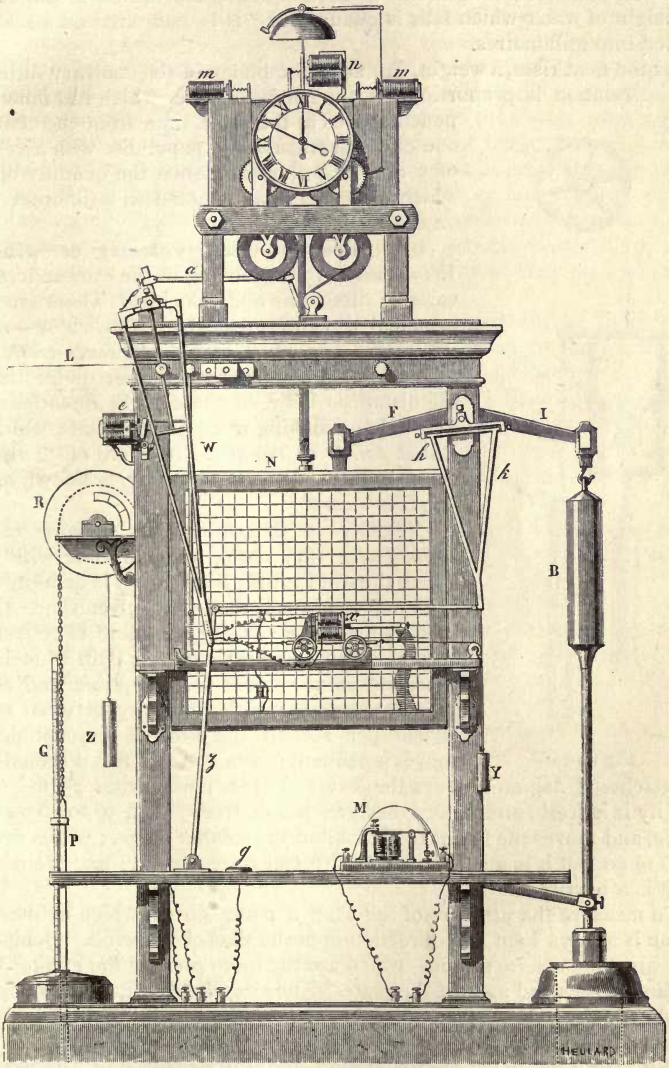

Fig. 1009.

Quantity of rain.—The quantity of rain which falls in a given time is registered on a disc of paper on a pulley, R. On the groove of this is coiled a chain to which is suspended a brass tube, P. This is fixed at the

bottom to a float, which plunges in a reservoir placed in the base of the meteorograph. On passing out of the water-balance (fig. 1005) the water passes into this reservoir, and as its section is one-fourth that of the funnel, the height of water which falls is quadrupled; it is measured on a scale, G, divided into millimetres.

As the float rises, a weight, Z, moves the pulley in the contrary direction, and its rotation is proportional to the height of water which has fallen. A pencil moves at the same time from the centre to one circumference of the paper disc with a velocity of 5 mm. in 24 hours: hence the quantity of rain which falls every day is noted on a different place on the paper disc.

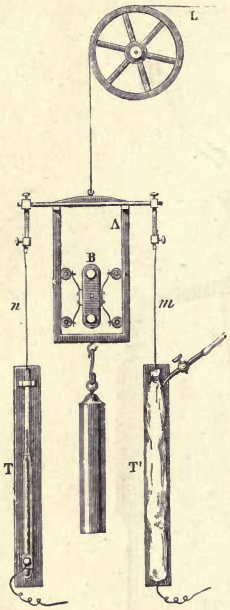

Fig. 1010.

975. **Direction and velocity of winds.**—*Winds* are currents moving in the atmosphere with variable directions and velocities. There are eight principal directions in which they blow—*north*, *north-east*, *east*, *south-east*, *south*, *south-west*, *west*, and *north-west*. Mariners further divide each of the distances between these eight directions into four others, making in all 32 directions, which are called *points* or *rhumbs*. A figure of 32 rhumbs on a circle, in the form of a star, is known as the *mariner's card*.

Velocity is determined by means of the *anemometer* (fig. 1006), a small vane with fans, which the wind turns; the velocity is deduced from the number of turns made in a given time. In our climate the mean velocity is from 18 to 20 feet in a second. With a velocity of less than 18 inches in a second no movement is perceptible, and smoke ascends straight; with a velocity between 1½ and 2 feet per second the wind is perceptible and moves a pennant; from 13 to 22 feet it is moderate, it stretches a flag and moves the leaves of trees; with from 23 to 36 feet velocity it is fresh, and moves the branches of trees; with 36 to 56 feet it is strong, and moves the larger branches and the smaller stems; with a velocity of 56 to 90 feet it is a storm, and entire trees are moved; and from 90 to 120 it is a hurricane.

To measure the pressure of the wind a plate is used, which by means of a vane is always kept in a direction opposite that of the wind. Behind the plate are one or more springs, which are the more pressed the greater is the pressure of the wind against the plate. Knowing the distance through which the plate is pressed, we can calculate the pressure which the wind exerts on the plate in question.

With some degree of approximation, and for low velocities, the pressure may be taken as proportional to the square of the velocity. Thus, if the pressure on the square foot is 0·005 pound, with a velocity of 1·5 foot in a second, it is 0·02 pound with a velocity of 3 feet, and 0·123 with a velocity of 7·33 feet.

976. **Causes of winds.**—Winds are produced by a disturbance of the equilibrium in some part of the atmosphere; a disturbance always resulting from a difference in temperature between adjacent countries. Thus, if the temperature of a certain extent of ground becomes higher, the air in contact with it becomes heated, it expands and rises towards the higher regions of the atmosphere; whence it flows, producing winds which blow from hot to cold countries. But at the same time the equilibrium is destroyed at the surface of the earth, for the barometric pressure on the colder adjacent parts is greater than on that which has been heated, and hence a current will be produced with a velocity dependent on the difference between these pressures; thus two distinct winds will be produced—an upper one setting *outwards* from the heated region, and a lower one setting *inwards* towards it.

977. **Regular, periodical, and variable winds.**—According to the more or less constant directions in which winds blow, they may be classed as regular, periodical, and variable winds.

i. *Regular winds* are those which blow all the year through in a virtually constant direction. These winds, which are also known as the *trade winds*, are uninterruptedly observed far from the land in equatorial regions, blowing from the north-east to the south-west in the Northern Hemisphere, and from the south-east to the north-west in the Southern Hemisphere. They prevail on the two sides of the equator as far as 30° of latitude, and they blow in the same direction as the apparent motion of the sun—that is, from east to west.

The air above the equator being gradually heated, rises as the sun passes round from east to west, and its place is supplied by the colder air from the north or south. The direction of the wind, however, is modified by this fact, that the velocity which this colder air has derived from the rotation of the earth—namely, the velocity of the surface of the earth at the point from which it started—is less than the velocity of the surface of the earth at the point at which it has now arrived: hence the currents acquire, in reference to the equator, the constant direction which constitutes the trade winds.

ii. *Periodical winds* are those which blow regularly in the same direction at the same seasons and at the same hours of the day: the monsoon, simoom, and the land and sea breeze are examples of this class. The name *monsoon* is given to winds which blow for six months in one direction and for six months in another. They are principally observed in the Red Sea and in the Arabian Gulf, in the Bay of Bengal and in the Chinese Sea. These winds blow towards the continents in summer, and in a contrary direction in winter. The *simoom* is a hot wind that blows over the deserts of Asia and Africa, and which is characterised by its high temperature and by the sands which it raises in the atmosphere and carries with it. During the prevalence of this wind the air is darkened, the skin feels dry, the respiration is accelerated, and a burning thirst is experienced.

This wind is known under the name of *sirocco* in Italy and Algiers, where it blows from the great desert of Sahara. In Egypt, where it prevails from the end of April to June, it is called *kamsin*. The natives of Africa, in order to protect themselves from the effects of the too rapid perspiration occasioned by this wind, cover themselves with fatty substances.

A wind characteristic of Switzerland and known as the *Föhn* originates as

follows : a mass of air coming from the south-east being impelled over a mountain ridge becomes rarefied as it ascends ; the temperature falls, and it deposits its moisture on the other side as rain or snow. Being driven still forward into the valleys, the superincumbent pressure being greater, the air is condensed and its temperature rises, and having parted with its moisture it appears as a wind which is at once hot and dry. One observation gave the temperature at 31·4 C., while it only contained 20 per cent. of moisture.

The *land* and *sea breeze* is a wind which blows on the sea-coast, during the day from the sea towards the land, and during the night from the land to the sea. For during the day the land becomes more heated than the sea, in consequence of its lower specific heat and greater conductivity, and hence, as the superincumbent air becomes more heated than that upon the sea, it ascends and is replaced by a current of colder and denser air flowing from the sea towards the land. During the night the land cools more rapidly than the sea, and hence the same phenomenon is produced, but in a contrary direction. The sea breeze commences after sunrise, increases up to three o'clock in the afternoon, decreases towards evening, and is changed into a land breeze after sunset. These winds are only perceived at a slight distance from the shores. They are regular in the tropics, but less so in our climates ; and traces of them are seen as far as the coasts of Greenland. The proximity of mountains, and also of forests, likewise gives rise to periodical daily breezes.

iii. *Variable winds* are those which blow sometimes in one direction and sometimes in another, alternately, without being subject to any law. In mean latitudes the direction of the winds is very variable ; towards the poles this irregularity increases, and under the arctic zone the winds frequently blow from several points of the horizon at once. On the other hand, in approaching the torrid zone they become more regular. The south-west wind prevails in England, in the north of France, and in Germany ; in the south of France the direction inclines towards the north, and in Spain and Italy the north wind predominates.

978. **Law of the rotation of winds.**—Notwithstanding the great irregularity which characterises the direction of the winds in our latitude, it has been ascertained that the wind has a preponderating tendency to veer round according to the sun's motion—that is, to pass from north, through north-east, east-south-east to south, and so on round in the same direction from west to north ; that it often makes a complete circuit in that direction, or more than one in succession, occupying many days in doing so, but that it rarely veers, and very rarely or never makes a complete circuit in the opposite direction. This course of the winds is most regularly observed in winter. According to Leverrier, the displacement of the north-east by the south-west wind arises from the occurrence of a whirlwind formed upon the Gulf Stream. For a station in south latitude a contrary law of rotation prevails.

This law, though more or less suspected for a long time, was first formally enunciated and explained by Dove, and is known as *Dove's law of rotation of winds.*

979. **Weather charts.**—A considerable advance has been made in weather forecasts by the frequent and systematic publication of *weather charts* ; that is to say, maps in which the barometric pressure, the temperature, the force of the wind, &c., are expressed for considerable areas in an

exact and comprehensive manner. A careful study of such maps renders possible a forecast of the weather for a day or more in advance. We can here do little more than explain the meaning of the principal terms in use.

If lines are drawn through those places on the earth's surface where the corrected barometric height at a given time is the same, such lines are called *isobarometric lines*, or, more briefly, *isobaric lines*, or *isobars*. Between any two points on the same isobar there is no difference of pressure. Isobars are usually drawn either for a difference of 5 mm. or of $\frac{1}{10}$ of an inch.

If we take a horizontal line between two isobars, and at that point at which the pressure is greatest draw a perpendicular line on any suitable scale, which shall represent the *difference* in pressure between the two places, the line drawn from the top of this perpendicular to the lower isobar will form an angle with the horizontal, and the steepness of this angle is a measure of the fall in pressure between the two stations, and is called the *barometric gradient*. Gradients are usually expressed in England and America in hundredths of an inch of mercury for one degree of sixty nautical miles, and on the Continent in millimetres for the same distance. The closer are the isobars the steeper is the gradient, and the more powerful the wind; and though no exact numerical relationship can be proved to exist between the steepness of the gradient and the force of the wind, it may be taken that a gradient of about 6 represents a strong breeze; and a gradient of 10, or a difference in pressure of $\frac{1}{10}$ of an inch for 60 miles, is a stiff gale.

The direction of the wind is from the place of higher pressure to that of lower, and in this respect the law of Buys Ballot may be mentioned, which has been found to hold in all cases of the Northern Hemisphere, where local configuration does not come into play. *If we stand with our back to the wind, the line of lower pressure is on the left hand.* For places in the Southern Hemisphere exactly the opposite law holds.

If within any area the pressure is lower, the wind blows round that area, the place of lowest pressure being on the left. The direction of the wind is, in short, opposite that of the hands of a watch. Such a circulation is called *cyclonic*; it is that which is characteristic of the West Indian hurricanes, which are known as *cyclones*. Conversely, the wind blows round an area of higher pressure in the same direction as the hands of a watch; and this circulation is called *anti-cyclonic*.

Cyclonic systems are by far the most frequent, and are characterised by steep gradients; the air in them tends to move in towards the centre, and thence to the upper regions of the atmosphere. They bring with them over the greater part of the region which they cover much moisture, an abundance of cloud, and heavy rain. An anti-cyclonic system has the opposite characteristics: the gradients are slight, the wind light, and moves with the hands of a watch. The air is dry, so that there is but little cloud, and no rain. Cyclonic systems, from the dampness of the air, produce warm weather in winter, and cold, wet weather in summer. Anti-cyclonic systems bring our hardest frosts in winter and greatest heat in summer, as there is but little moisture in the air to temper the extremes of climate. Both systems travel over the earth's surface—the cyclones rapidly, but the anti-cyclones more slowly.

980. **Fogs and Mists.**—When aqueous vapour rising from a vessel of boiling water diffuses in the colder air, it is condensed; a sort of cloud is formed which consists of a number of small hollow vesicles of water, which remain suspended in the air. These are usually spoken of as vapour, yet they are not so—at any rate, not in the physical sense of the word, for in reality they are partially condensed vapour.

When this condensation of aqueous vapour is not occasioned by contact with cold, solid bodies, but takes place throughout large spaces of the atmosphere, it constitutes *fogs* or *mists*, which, in fact, are essentially the same, the appearance seen over a vessel of hot water.

A chief cause of fogs consists in the moist soil being at a higher temperature than the air. The vapours which then ascend condense and become visible. In all cases, however, the air must have reached its point of saturation before condensation takes place. Fogs may also be produced when a current of hot and moist air passes over a river at a lower temperature than its own; for then, the air being cooled, as soon as it is saturated, the excess of vapour present is condensed. The distinction between mists and fogs is one of degree rather than of kind. A fog is a very thick mist.

By observations based on diffraction phenomena (650), the diameter of fog vesicles has been found to vary from 0·0154 to 0·0521 mm.; the longer the continuance of fine weather, the smaller are the vesicles; before rains they increase rapidly.

Dines, by direct microscopic measurement, found that the diameter of fog particles varied with the same fog from 0·015 to 0·127 mm.; the larger occur in dense fogs, in lighter fogs they sink to 0·0033. Kämtz found from 0·014 to 0·035 mm.

When water is coated with a layer of coal-tar, it is prevented from evaporating. Frankland ascribes the *dry fog* met with in London to the large quantities of coal-tar and paraffin vapour which are sent into the atmosphere, and which, condensing on the vesicles of fog, prevent their evaporation.

Aitkin has shown that aqueous vapour never condenses unless some liquid or solid is present on which it is deposited. Particles of dust in the air are the nuclei for clouds and fogs. This he showed by passing steam into filtered air; it remained quite clear, while a turbidity was produced under the same circumstances in unfiltered air. The density of the cloud was found to depend on the number of particles of dust in the air. A most abundant source of dust is the combustion of coal. The sulphur in the coal in burning also forms sulphurous acid, which, though a gas, is found to act as a nucleus.

981. **Clouds.**—*Clouds* are masses of vapour condensed into little drops or vesicles of extreme minuteness, like fogs. There is no difference of kind between fogs and clouds. Fogs are clouds resting on the ground. To a person enveloped in it, a cloud on a mountain appears like a fog. They always result from the condensation of vapour which rises from the earth. The horizontal base of a cloud denotes the layer of air in which the ascending current of air has attained the dew-point. According to their appearance, clouds were divided by Howard into four principal kinds: the *nimbus*, the *stratus*, the *cumulus*, and the *cirrus*. These four kinds are represented in

fig. 1011, and are designated respectively by one, two, three, and four birds on the wing.

The *cirrus* consists of small whitish clouds, which have a fibrous or wispy appearance, and occupy the highest regions of the atmosphere. The name of *mares' tails*, by which they are generally known, well describes their appearance. From the low temperature of the spaces which they occupy, it is more than probable that cirrus clouds consist of frozen particles ; and hence it is that halos, coronæ, and other optical appearances, produced by refraction and reflection from ice-crystals, appear almost always in these clouds and their derivatives. Their appearance often precedes a change of weather.

The *cumulus* are rounded spherical forms which look like mountains piled one on the other. They are more frequent in summer than in winter,

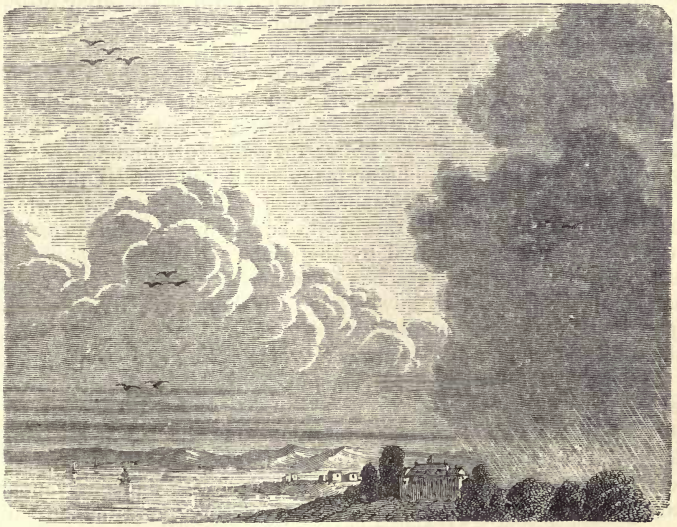

Fig. 1011.

and after being formed in the morning, they generally disappear towards evening. If, on the contrary, they become more numerous, and especially if surmounted by cirrus clouds, rain or storms may be expected.

Stratus clouds consist of very large and continuous horizontal sheets, which form chiefly at sunset and disappear at sunrise. They are frequent in autumn and unusual in spring-time, and are lower than the preceding.

The *nimbus*, or rain clouds, which are sometimes classed as one of the fundamental varieties, are properly a combination of the three preceding kinds. They affect no particular form, and are solely distinguished by a uniform grey tint and by fringed edges. They are indicated on the right of the figure by the presence of one bird.

The fundamental forms pass into one another in the most varied manner.

Howard has classed these transitional forms as *cirro-cumulus*, *cirro-stratus*, and *cumulo-stratus*, and it is often very difficult to tell, from the appearance of a cloud, which type it most resembles. The *cirro-cumulus* is most characteristically known as a 'mackerel sky'; it consists of small roundish masses, disposed with more or less irregularity. It is frequent in summer, and attendant on warm and dry weather. *Cirro-stratus* appears to result from the subsidence of the fibres of cirrus to a horizontal position, which at the same time approach laterally. The form and relative position when seen in the distance frequently give the idea of shoals of fish. The tendency of *cumulo-stratus* is to spread, settle down into the *nimbus*, and finally fall as rain.

The height of clouds varies greatly; in the mean it is from 1,300 to 1,500 yards in winter, and from 3,300 to 4,300 yards in summer. But they often exist at greater heights; Gay-Lussac, in his balloon ascent, at a height of 7,630 yards, observed cirrus clouds above him, which appeared to be at a considerable height. In Ethiopia, D'Abbadie observed storm-clouds whose height was only 230 yards above the ground.

In order to explain the suspension of clouds in the atmosphere, Halley first proposed the hypothesis of vesicular vapours. He supposed that clouds are formed of an infinity of extremely minute vesicles, hollow, like soap-bubbles filled with air, which are hotter than the surrounding air; so that these vesicles float in the air like so many small balloons. Others assume that clouds and fogs consist of extremely minute droplets of water, which are retained in the atmosphere by the ascensional force of currents of hot air, just as light powders are raised by the wind. Ordinarily, clouds do not appear to descend, but this absence of downward motion is only apparent, In fact, clouds do usually fall slowly, but then the lower part is continually dissipated on coming in contact with the lower and more heated layers; at the same time the upper part is always increasing from the condensation of new vapours, so that from these two actions clouds appear to retain the same height.

982. **Formation of clouds.**—Many causes may concur in the formation of clouds. The usual cause of the formation of a cloud is the ascent, into higher regions of the atmosphere, of air laden with aqueous vapour; it thereby expands, being under diminished pressure; and in consequence of this expansion it is cooled, and this cooling produces a condensation of vapour. Hence it is that high mountains, stopping the currents of air and forcing them to rise, are an abundant source of rain. If the air is quite dry, its temperature would be one degree lower for every 300 metres. The case is different with moist air; for when the air has ascended so high that its temperature has fallen to the dew-point, aqueous vapour is condensed, and in consequence of this heat is liberated; when the dew-point is thus attained, and the air is saturated, the cooling due to the ascent and expansion of air is counteracted by this liberation of latent heat, so that the diminution of temperature with the height is considerably slower in the case of moist than of dry air. About one-half of the entire quantity of moisture in the air is contained in the first six or seven thousand feet upon the ground.

The following calculation will give us the quantity of water separated in a given case: Suppose air at a temperature of 20° to be saturated with

aqueous vapour at that temperature; the pressure of the vapour will be 17·4 mm., and the weight contained in one cubic metre of air 17·1 grammes.

If the air has risen to a height of 3,500 metres, it has come under a pressure which is only $\frac{2}{3}$ of what it was: its temperature is 4°, and its volume about $1\frac{1}{2}$ times what it originally was. As it remains saturated the pressure will be 6·1 mm., and the quantity of vapour will be 6·4 grammes in a cubic metre—that is to say, $6{\cdot}4 \times 1\frac{1}{2} = 9{\cdot}6$ grammes in the whole mass of what was originally a cubic metre. The pressure of aqueous vapour has sunk during the ascent from 17·4 mm. to 6·1 mm., and its weight 17·1 grammes to 9·6 grammes; that is, a weight of 7·5 grammes has been deposited for that mass of air which at the sea-level occupied a space of one cubic metre. These 7·5 grammes are in the form of the small droplets which constitute fogs or clouds.

If the mass of air had risen to a height of 8,500 metres, where the pressure is only one-third that on the sea-level, the temperature is −28°, and the space it occupies three times as great as at first. The pressure of aqueous vapour is 0·5 mm., and its weight 0·6 gramme in a cubic metre. Hence there is now only 1·8 gramme left of the entire quantity of aqueous vapour originally present, and the remaining 15·3 grammes would be separated as water or ice. A similar calculation will show that at a height of 4,200 metres, where the temperature is zero and the pressure $\frac{3}{5}$, the quantity of water present in the original cubic metre is only 0·82 gramme, the rest being deposited.

Thus, a mass of air which, at the sea-level, occupies a space of a cubic metre, and is saturated with aqueous vapour at 20°, and then contains 17·1 grammes, will only contain 9·6 grammes at a height of 3,500 metres, 8·2 grammes at 4,200 metres, and 1·8 gramme at 8,500 metres. Hence, while a mass of air rises from the sea-level to a height of 4,200 feet, 8·9 grammes of aqueous vapour are separated as cloud-vesicles; at 8,500 metres, or about double the height, 6·4 grammes are separated in the form of ice.

A hot moist current of air mixing with a colder current undergoes a cooling, which brings about a condensation of the vapour. Thus, the hot and moist winds of the south and south-west, mixing with the colder air of our latitudes, give rain. The winds of the north and north-east tend also, in mixing with our atmosphere, to condense the vapours; but as these winds, owing to their low temperature, are very dry, the mixture rarely attains saturation, and generally gives no rain.

The formation of clouds in this way is thus explained by Hutton. The pressure of aqueous vapour, and therewith the quantity present in a given space when saturated, diminishes according to a geometric progression, while the temperature falls in arithmetical progression, and therefore the elasticity of the vapour present at any time is reduced by a fall of temperature more rapidly than in direct proportion to the fall. Hence, if a current of warm air, saturated with aqueous vapour, meets a current of cold air also saturated, the air acquires the mean temperature of the two, but can only retain a portion of the vapour in the invisible condition, and a cloud or mist is formed. Thus, suppose a cubic metre of air at 10° C. mixes with a cubic metre of air at 20° C., and that they are respectively saturated with aqueous vapour. By formula (401) it is easily calculated that the weight of water

contained in the cubic metre of air at 10° C. is 9·397 grammes, and in that at 20° C. is 17·153 grammes, or 26·559 grammes in all. When mixed they produce two cubic metres of air at 15° C.; but as the weight of water required to saturate this is only $2 \times 12{\cdot}8 = 25{\cdot}6$ grammes, the excess, 0·95 gramme, will be deposited in the form of mist or clouds.

983. **Rain.**—When the individual vapour-vesicles become larger and heavier by the condensation of aqueous vapour, and when, finally, individual vesicles unite, they form regular drops, which fall as *rain*.

The quantity of rain which falls annually in any given place, or the annual rainfall, is measured by means of a *rain-gauge*, or *pluviometer*. Ordinarily it consists of a cylindrical vessel M (figs. 1012 and 1013), closed at the top by a funnel-shaped lid, in which there is a very small hole, through which the rain falls. At the bottom of the vessel is a glass tube, A, in which the water rises to the same height as inside the rain-gauge, and is measured by a scale on the side, as shown in the figures.

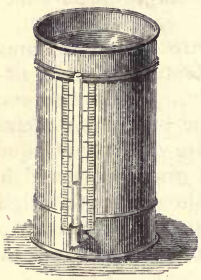

Fig. 1012.

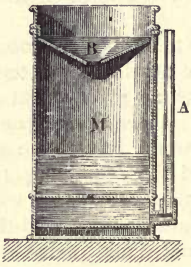

Fig. 1013.

The apparatus being placed in an exposed situation, if at the end of a month the height of water in the tube is two inches, for example, it shows that the water has attained this height in the vessel, and, consequently, that a layer of two inches in depth expresses the quantity of rain which this extent of surface has received.

It has been noticed that the quantity of rain indicated by the rain-gauge is greater the nearer this instrument is to the ground. This has been ascribed to the fact that the raindrops, which are generally colder than the layers of air which they traverse, condense the vapour in these layers, and therefore constantly increase in volume. Hence more rain falls on the surface of the ground than at a certain height. But it has been objected that the excess of the quantity of rain which falls, over that at a certain height, is six or seven times that which could arise from condensation, even during the whole course of the raindrops from the clouds to the earth. The difference must therefore be ascribed to purely local causes, and it is now assumed that the difference arises from eddies produced in the air about the rain-gauge, which are more perceptible the higher it is above the ground; as these eddies disperse the drops which would otherwise fall into the instrument, they diminish the quantity of water which it receives.

In any case it is clear that, if rain-drops traverse moist air, they will, from their temperature, condense aqueous vapour and increase in volume. If, on the contrary, they traverse dry air, the drops tend to vaporise, and less rain falls than at a certain height; it might even happen that the rain did not reach the earth.

From measurements of the coronæ (981), Delezenne determined the diameter of the globules in the case of rain-clouds just about to fall, and in

the case of the cloud from a low-pressure steam-engine (471). The former was found to vary from 0·0565 to 0·0226 mm., and the latter from 0·0051 to 0·0042 mm. With the former, 5,500 droplets would be needed to make a drop of water a millimetre in diameter, and with the latter 50,000.

According to the same author, there would be about 15 mgr. of globules in a cubic metre of a cloud which produced a rainfall of 10 mm. of water in an hour. With this number the mean distances of the vesicles with the above magnitudes are respectively 1·845, 0·706, 0·167, and 0·148 mm.

Many local circumstances may affect the quantity of rain which falls in different countries ; but, other things being equal, most rain falls in hot climates, for there the vaporisation is most abundant. The rainfall decreases, in fact, from the equator to the poles. At London it is 23·5 inches ; at Bordeaux it is 25·8 ; at Madeira it is 27·7 ; at Havannah it is 91·2 ; and at St. Domingo it is 107·6. The quantity varies with the season : in Paris, in winter, it is 4·2 inches ; in spring, 6·9 ; in summer, 6·3 ; and in autumn, 4·8 inches. The heaviest annual rainfall at any place on the globe is on the Khasi Hills, in Bengal, where it is 600 inches ; of which 500 inches fall in seven months. On July 1, 1851, a rainfall of 25½ inches on one day was observed at Cherrapoonjee. At Kurrachee, in the north-west of India, the rainfall is only 7 inches.

The rainfall varies with the height of a station above the sea-level at the rate of 3 or 4 per cent. for each 100 feet of altitude above the sea.

The driest recorded place in England is Lincoln, where the mean rainfall is 20 inches ; and the wettest is Stye, at the head of Borrowdale, in Cumberland, where it amounts to 165 inches. The greatest average amount of rainfall in any one day, taking the means of all stations, is 1½ inch ; though individual stations far exceed this amount, sometimes reaching 4 inches.

An inch of rain on a square yard of surface expresses a fall of 46·74 pounds, or 4·67 gallons. On an acre it corresponds to 22,622 gallons, or 100·9935 tons. 100 *tons per inch per acre* is a ready way of remembering this.

984. **Waterspouts.**—On hot summer days, and when the weather is otherwise calm, we often notice sand and dust carried forward in a column with a whirling motion. As storms come on, larger whirlwinds of this kind are formed, which carry with them leaves, straw, and even small branches. When they are of larger dimensions they form real whirlwinds. They are probably due to the contest of two winds blowing in the upper regions of the atmosphere. When they pass over land they form large conical-shaped masses of dust, which make them visible at a distance ; when they pass over rivers or the sea they present a curious phenomenon : the water is disturbed, and rises in the form of a cone, while the clouds are depressed in the form of an inverted cone ; the two cones then unite and form a continuous column from the sea to the clouds (fig. 1014). Even, however, on the high seas the water of these waterspouts is never salt, proving that they are formed of condensed vapour, and not of sea-water raised by aspiration.

985. **Influence of aqueous vapour on climate.**—Tyndall applied the property possessed by aqueous vapour of powerfully absorbing and radiating heat to the explanation of some obscure points in meteorology. He esta-

blished the fact that in a tube 4 feet long the atmospheric vapour on a day of average dryness absorbs 10 per cent. of obscure heat. With the earth warmed by the sun as a source, at the very least 10 per cent. of its heat is intercepted within 10 feet of the surface. The absorption and radiation of aqueous vapour is more than 16,000 times that possessed by dry air.

The *radiative* power of aqueous vapour may be the main cause of the torrent-like rains that occur in the tropics, and also of the formation of cumulus clouds in our own latitudes. The same property probably causes the descent of very fine rain, called *sérein*, which has more the characteristics of falling dew, as it appears a short time after sunset, when the sky is clear ; its production has therefore been attributed to the cold resulting from the radiation of the air. It is not the air, however, but the aqueous vapour in

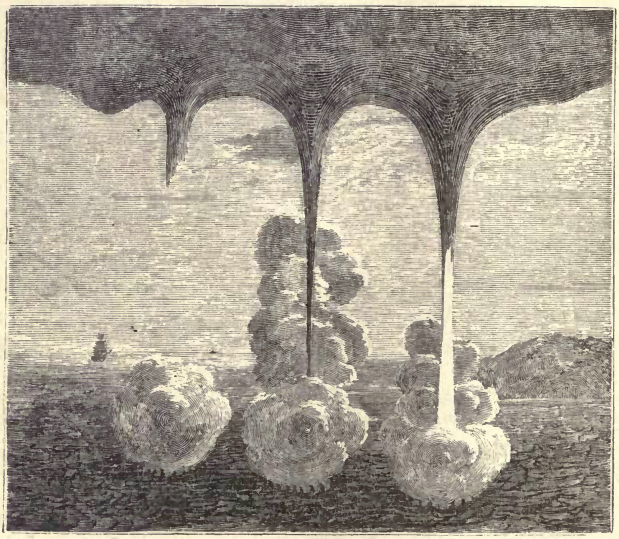

Fig. 1014.

the air, which by its own radiation chills itself, so that it condenses into *serein.*

The *absorbent* power of aqueous vapour is of even greater importance. Whenever the air is dry, terrestrial radiation at night is so rapid as to cause intense cold. Thus, in the central parts of Asia, Africa, and Australia, the daily range of the thermometer is enormous ; in the interior of the last-named continent a difference in temperature of no less than 40° C. has been recorded within 24 hours. In India, and even in the Sahara, ice has been formed at night, owing to the copious radiation. But the heat which aqueous vapour absorbs most largely is of the kind emitted from sources of low temperature ; it is to a large extent transparent to the heat emitted from the sun, whilst it is almost opaque to the heat radiated from the earth. Consequently, the solar rays penetrate our atmosphere with a loss, as estimated

by Pouillet, of only 25 per cent., when directed vertically downwards, but after warming the earth they cannot retraverse the atmosphere. Through thus preventing the escape of terrestrial heat, the aqueous vapour in the air moderates the extreme chilling which is due to the unchecked radiation from the earth, and raises the temperature of that region over which it is spread. In Tyndall's words, 'aqueous vapour is a blanket more necessary to the vegetable life of England than clothing is to man. Remove for a single summer night the aqueous vapour from the air which overspreads this country, and every plant capable of being destroyed by a freezing temperature would perish. The warmth of our fields and gardens would pour itself unrequited into space, and the sun would rise upon an island held fast in the iron grip of frost.'

986. **Tyndall's researches.**—Tyndall found that by the action of the sun and the electric light on vapours under a great degree of attenuation they are decomposed. He used a glass tube with glass ends, which could be exhausted and then filled with air charged with the vapours of volatile liquids, by allowing the air to bubble through small Wolff bottles containing them. By mixing the air charged with vapour with different proportions of pure air, and by varying the degrees of exhaustion, it was possible to have a vapour under any degree of attenuation. The tube could also be filled with the vapour of a liquid alone. The tube having been filled with air charged with vapour of nitrite of amyle, a somewhat convergent beam from the electric lamp was passed into the tube. For a moment the tube appeared optically empty, but suddenly a shower of liquid spherules was precipitated on the path of the beam, forming a luminous white cloud. The nature of the substance thus precipitated was not specially investigated. This effect was not due to any chemical action between the vapour and the air, for when either dry oxygen or dry hydrogen was used instead of air, or when the vapour was admitted alone, the effect was substantially the same. Nor was it due to any heating effect, for the beam had been previously sifted by passing through a solution of alum, and through the thick glass of the lens. The unsifted beam produced the same effect ; the obscure calorific rays did not seem to affect the result. The sun's light also effects the decomposition of nitrite of amyle vapour ; and this decomposition was found to be mainly due to the more refrangible rays. When the electric light, before entering the experimental tube, was made to pass through a layer of liquid nitrite of amyle an eighth of an inch in thickness, the luminous effect was not appreciably diminished, but the chemical action was almost entirely stopped. Thus, that special constituent of the luminous radiation which effects the decomposition of the vapour is absorbed by the liquid. The decomposition of liquid nitrite of amyle by light, if it take place at all, is far less rapid and distinct than that of the vapour. The circumstance that the absorption is the same whether the nitrite is in the liquid or in the vaporous state, is considered by Tyndall as a proof that the absorption is not the act of the molecule as a whole, but that it is atomic ; that is, that it is to the atoms that the peculiar rate of vibration is transferred which brings about the decomposition of the body. By varying the nature of the vapour the shape of a cloud could be greatly varied, and in many cases presented the most fantastic and beautiful forms.

It was also found that a vapour which when alone resists the action of light may, by being associated with another gas or vapour, exhibit a vigorous action. Thus, when the tube was filled with atmospheric air, mixed with nitrite of butyle vapour, the electric light produced very little effect; but with half an atmosphere of this mixture, and half an atmosphere of air which had passed through hydrochloric acid, the action of the light was almost instantaneous. In another case, mixed air and nitrite of butyle vapour were passed into the tube so that the mixture was under a pressure 2·5 mm. Air passed through aqueous hydrochloric acid was introduced until the pressure was 3 inches. The condensed beam passed through at first without change, but afterwards a superb blue cloud was formed.

In cases where the vapours are under a sufficient degree of attenuation, whatever otherwise be their nature, the visible action commences with the formation of a *blue cloud.* The term cloud, however, must not be understood in its ordinary sense; the blue cloud is invisible in ordinary daylight, and to be seen must be surrounded by darkness, *it alone* being illuminated by a powerful beam of light. The blue cloud differs in many important particulars from the finest ordinary clouds, and may be considered to occupy an intermediate position between these clouds and true cloudless vapour.

By graduating the quantity of vapour, the precipitation may be obtained of any required degree of fineness; forming either particles distinguishable by the naked eye, or particles beyond the reach of the highest microscopic power. The case is similar to that of carbonic acid gas, which, diffused in the atmosphere, resists the decomposing action of solar light, but is decomposed when in contact with the chlorophyll in the leaves of plants.

When the blue cloud produced in these experiments was examined by any polarising arrangement, the light emitted laterally from the beam—that is, in a direction at right angles to its axis—was found to be perfectly polarised. This phenomenon was observed in its greatest perfection the more perfect the blue of the cloud. It is produced by any particles, provided they are sufficiently fine. This is quite analogous to the light of the blue sky. When this is examined by a Nicol's prism, or any other analyser, it is found that the light emitted at right angles to the path of the sun's rays is polarised.

The phenomena of the firmamental blue, and the polarisation of the sky-light, thus find definite explanations in these experiments. We need only assume the existence, in the higher regions of the atmosphere, of excessively fine particles of water; for particles of any kind produce this effect. It is easy to conceive the existence of such particles in the higher regions, even on a hot summer's day. For the vapour must there be in a state of extreme attenuation; and inasmuch as the oxygen and nitrogen of the atmosphere behave like a vacuum to radiant heat, the extremely attenuated particles of aqueous vapour are practically in contact with the absolute cold of space.

'Suppose the atmosphere surrounded by an envelope impervious to light, but with an aperture on the sunward side, through which a parallel beam of solar light could enter and traverse the atmosphere. Surrounded on all sides by air not directly illuminated, the track of such a beam would resemble that of the parallel beam of the electric light through an incipient cloud. The sunbeam would be blue, and it would discharge light laterally in the

same condition as that discharged by the incipient cloud. The azure revealed by such a beam would be to all intents and purposes a blue cloud.'

987. **Dew. Hoarfrost.**—*Dew* is aqueous vapour which has condensed on bodies during the night in the form of minute globules. It is occasioned by the chilling which bodies near the surface of the earth experience in consequence of nocturnal radiation. Their temperature having then sunk several degrees below that of the air, it frequently happens, especially in hot seasons, that this temperature is below that at which the atmosphere is saturated. The layer of air which is immediately in contact with the chilled bodies, and which has virtually the same temperature, then deposits a portion of the vapour which it contains (396); just as when a bottle of cold water is brought into a warm room it becomes covered with moisture, owing to the condensation of aqueous vapour upon it.

According to this theory, which was first propounded by Dr. Wells, all causes which promote the cooling of bodies increase the quantity of dew. These causes are the emissive power of bodies, the state of the sky, and the agitation of the air. Bodies which have a great radiating power more readily become cool, and therefore ought to condense more vapour. In fact there is generally no deposit of dew on metals, whose radiating power is very small, especially when they are polished; while the ground, sand, glass and plants, which have a great radiating power, become abundantly covered with dew.

The state of the sky also exercises a great influence on the formation of dew. If the sky is cloudless, the planetary spaces send to the earth an inappreciable quantity of heat, while the earth radiates very considerably, and therefore, becoming very much chilled, there is an abundant deposit of dew. But if there are clouds, as their temperature is far higher than that of the planetary spaces, they radiate in turn towards the earth, and as bodies on the surface of the earth only experience a feeble chilling, no deposit of dew takes place.

Wind also influences the quantity of vapour deposited. If it is feeble, it increases it, inasmuch as it renews the air; if it is strong, it diminishes it, as it heats the body by contact, and thus does not allow the air time to become cooled. Finally, the deposit of dew is more abundant according as the air is moister, for then it is nearer its point of saturation.

Hoarfrost and *rime* are dew which has been deposited on bodies cooled below zero, and has become frozen. The flocculent form which the small crystals present, of which rime is formed, shows that the vapour solidifies directly without passing through the liquid state. Hoarfrost, like dew, is formed on bodies which radiate most, such as the stalks and leaves of vegetables, and is chiefly deposited on the parts turned towards the sky.

We must distinguish between the dew formed in consequence of lowering of temperature by radiation, and the deposit formed by warm moist air passing over a cold wall; in mild weather this deposit forms a liquid, and in severe weather a snow or icy coating. Unlike dew, a deposit of this kind is most abundantly found on good conductors, for they are the coldest.

988. **Snow. Sleet.**—*Snow* is water solidified in stellate crystals, variously modified, and floating in the atmosphere. These crystals arise from the congelation of the minute vesicles which constitute the clouds, when the

temperature of the latter is below zero. They are more regular when formed in a calm atmosphere. Their form may be investigated by collecting them on a black surface and viewing them through a strong lens. The regularity, and at the same time variety, of their forms are truly beautiful. Fig. 1015 shows some of these forms as seen through a microscope. Very roughly, a fall of one foot of snow may be taken as equal to an inch of rain.

It snows most in countries near the poles, or which are high above the sea-level. By the limit of perpetual snow—or, briefly *snow-line*—is meant that height above the sea-level at which the snow does not melt, even in the hottest summers. It is lower nearer the poles than the equator: it does not depend solely on the latitude, but is influenced by many local circumstances.

Sleet is also solidified water, and consists of small icy needles pressed together in a confused manner. Its formation is ascribed to the sudden congelation of the minute globules of the clouds in an agitated atmosphere.

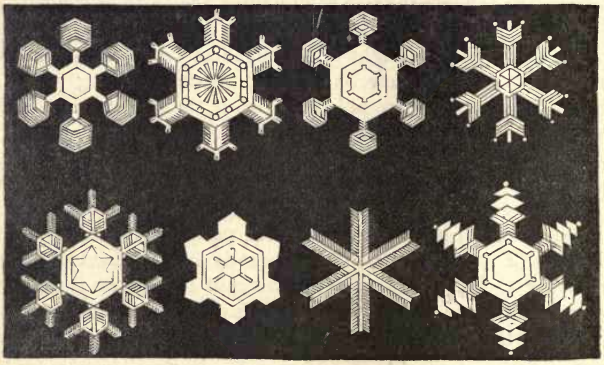

Fig. 1015.

When the ground is cooled below zero after severe frost and a thaw sets in, the moist air passing over the ground deposits its moisture, which is converted into a continuous sheet of ice; this is known as *glazed frost* (the French *verglas*); it may also occur when raindrops which have been cooled below zero in the higher regions of the air, and are accordingly in a state of superfusion (345), fall on the ground, which may even be above the freezing-point.

989. **Hail.**—*Hail* is a mass of compact globules of ice of different sizes which fall in the atmosphere. In our climate hail falls principally during spring and summer, and at the hottest times of the day; it rarely falls at night. The fall of hail is always preceded by a peculiar noise.

Hail is generally the precursor of storms; it rarely accompanies them, and follows them more rarely still. Hail falls from the size of small peas to that of an egg or an orange, with a core of compressed snow which is surrounded by concentric layers of ice. While snowstorms may last for days, hailstorms do not last for more than a quarter of an hour. The formation of hailstones has never been altogether satisfactorily accounted for; nor, more especially, their great size.

990. **Ice. Regelation.**—Ice is an aggregate of snow-crystals, such as are shown in fig. 1015. The transparency of ice is due to the close contact of these crystals, which causes the individual particles to blend into an unbroken mass, and renders the substance *optically*, as well as mechanically, continuous. When large masses of ice slowly melt away, a crystalline form is sometimes seen by the gradual disintegration into rude hexagonal prisms; a similar structure is frequently met with, but in greater perfection, in the ice-caves or glaciers of cold regions.

An experiment of Tyndall shows the beautiful structure of ice. When a piece of ice is cut parallel to its planes of freezing, and the radiation from any source of light is permitted to pass through it, the disintegration of the substance proceeds in a remarkable way. By observing the plate of ice through a lens, numerous small crystals will be seen studding the interior of the block; as the heat continues these crystals expand, and finally assume the shape of six-rayed stars of exquisite beauty.

This is a kind of negative crystallisation, the crystals produced being composed of water; they owe their formation to the molecular disturbance caused by the absorption of heat from the source. Nothing is easier than to reproduce this phenomenon, if care be taken in cutting the ice. The planes of freezing can be found by noting the direction of the bubbles in ice, which are either sparsely arranged in striæ at right-angles to the surface, or thickly collected in beds parallel to the surface of the water. A warm and smooth metal plate should be used to level and reduce the ice to a slab not exceeding half an inch in thickness.

A still more important property of ice remains to be noticed. Faraday discovered that when two pieces of melting ice are pressed together they freeze into one at their points of contact. This curious phenomenon is now known under the name of *Regelation*. The cause of it has been the subject of much controversy, but the simplest explanation seems to be that given by its discoverer. The particles on the exterior of a block of ice are held by cohesion on one side only: when the temperature is at o° C., these exterior particles being partly free, are the first to pass into the liquid state, and a film of water covers the solid. But the particles in the interior of the block are bounded on all sides by the solid ice, the force of cohesion is here a maximum, and hence the interior ice has no tendency to pass into a liquid, even when the whole mass is at o°. If the block be now split in halves, a liquid film instantly covers the fractured surfaces, for the force of cohesion on the fractured surfaces has been lessened by the act. By placing the halves together, so that their original position shall be regained, the liquid films on the two fractured surfaces again become bounded by ice on both sides. The film being excessively thin, the force of cohesion is able to act across it; the consequence of this is, the liquid particles pass back into the solid state, and the block is reunited by *regelation*. Not only do ice and ice thus freeze together, but regelation also takes place between moist ice and any non-conducting solid body, as flannel or sawdust; a similar explanation to that just given has been applied here, substituting another solid for the ice on one side. It must be remarked, however, that many eminent philosophers dissent from the explanation here given.

Whatever may be the true cause of regelation, there can be no doubt that

this interesting observation of Faraday's explains many natural phenomena. For example, the formation of a snowball depends on the regelation of the snow-granules composing it; and as regelation cannot take place at temperatures below o° C., for then both snow and ice are dry, it is only possible to make a coherent snowball when the snow is melting.

The snow-bridges, also, which span wide chasms in the Alps and elsewhere, and over which men can walk in safety, owe their existence to the regelation of gradually accumulating particles of snow.

Bottomley has made a very instructive experiment which illustrates regelation. A block of ice is suspended on two supports, and a fine piano wire with heavy weights at each end is laid across it. After some time the wire has slowly cut its way through, but the cut surfaces have reunited, and, excepting a few bubbles, show no trace of the operation; the wire is below zero, as is proved by placing it in cold water, upon which some ice forms round it.

991. **Glaciers.**—Tyndall has applied this regelating property of ice to an explanation of the formation and motion of glaciers, of which the following is a brief description: In elevated regions, the *snow-line* (988) marks the boundary of eternal snow, for above this the heat of summer is unable to melt the winter's snow. By the heat of the sun and the consequent percolation of water melted from the surface, the lower portions of the snow-field are raised to o° C.; at the same time this part is closely pressed together by the weight of the snow above; regelation therefore sets in, converting the loose snow into a coherent mass.

By increasing pressure the intermingled air which renders snow opaque becomes ejected and the snow becomes transparent; ice then results. Its own gravity, and the pressure from behind, urge downwards the glacier which has thus been formed. In its descent from the mountain the glacier behaves in all respects like a river, passing through narrow gorges with comparative velocity, and then spreading out and moving slowly as its bed widens. Further, just as the central portions of a river move faster than the sides, so Forbes ascertained that the centre of a glacier moves quicker than its margin, and from the same reason (the difference in the friction encountered) the surface moves more rapidly than the bottom. To explain these facts Forbes assumed ice to be a viscous body capable of flexure, and flowing like lava; but as ice has not the properties of a viscous substance, the now generally accepted explanation of glacier motion is that supplied by the theory of regelation. According to this theory, the brittle ice of the glacier is crushed and broken in its passage through narrow channels, such as that of Trélaporte on Mont Blanc; and then, as it emerges from the gorge which confined it, becomes reunited by virtue of regelation; in this instance forming the well-known Mer de Glace. By numerous experiments Tyndall has established that regelation is adequate to furnish this explanation, and has artificially imitated, on a small scale, the moulding of glaciers by the crushing and subsequent regelation of ice.

We see an example of this formation of ice from pressure from the glazed appearance of the tracks in snow in roads over which heavy carts have passed.

992. **Atmospheric electricity. Franklin's experiment.**—The most frequent luminous phenomena, and the most remarkable for their effects,

are those produced by the free electricity in the atmosphere. The first physicists who observed the electric spark compared it to the gleam of lightning, and its crackling to the sound of thunder. But Franklin, by the aid of powerful electrical batteries, first established a complete parallel between lightning and electricity ; and he indicated, in a memoir published in 1749, the experiments necessary to attract electricity from the clouds by means of pointed rods. The experiment was tried by Dalibard, in France ; and Franklin, pending the erection of a pointed rod on a spire in Philadelphia, had the happy idea of flying a kite, provided with a metal point, which could reach the higher regions of the atmosphere. In June, 1752, during stormy weather, he flew the kite in a field near Philadelphia. The kite was flown with ordinary packthread, at the end of which Franklin attached a key, and to the key a silk cord, in order to insulate the apparatus : he then fixed the silk cord to a tree, and having presented his hand to the key, at first he obtained no spark. He was beginning to despair of success, when, rain having fallen, the cord became a good conductor, and a spark passed. Franklin, in his letters, describes his emotion on witnessing the success of the experiment as being so great that he could not refrain from tears.

Franklin imagined that the kite drew from the cloud its electricity ; it is, in fact, a simple case of induction, and depends on the inductive action which the thunder-cloud exerts upon the kite and the cord.

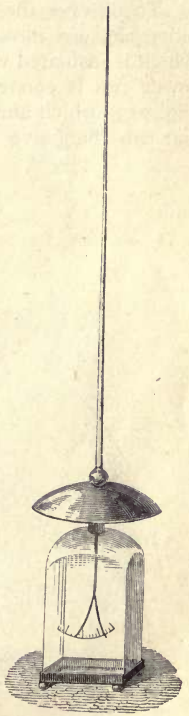

Fig. 1016.

993. **Apparatus to investigate the electricity of the atmosphere.**—To observe the electricity in fine weather, when the quantity is generally small, an apparatus may be used as devised by Saussure for this kind of investigation. It is an electroscope similar to that already described (751), but the rod to which the gold leaves are fixed is surmounted by a conductor 2 feet in length, and terminates either in a knob or a point (fig. 1016). To protect the apparatus against rain, it is covered with a metal shield 4 inches in diameter. The glass case is square instead of being round, and a divided scale on its inside face indicates the divergence of the gold leaves. This electrometer only gives signs of atmospheric electricity as long as it is raised in the atmosphere so that it is in layers of air of higher electrical potential than its own.

To ascertain the electricity of the atmosphere Saussure also used a copper ball, which he projected vertically with his hand. This ball was fixed to one end of a metal wire, the other end of which was attached to a ring, which could glide along the conductor of the electrometer. From the divergence of the gold leaves, the electrical condition of the air at the height which the ball attained could be determined. Becquerel, in experiments made on the St. Bernard, improved Saussure's apparatus by

substituting for the knob an arrow, which was projected into the atmosphere by means of a bow. A gilt silk thread, 88 yards long, was fixed with one end to the arrow, while the other end was attached to the stem of an electroscope. Peltier used a gold-leaf electroscope, at the top of which was a somewhat large copper globe. Provided with this instrument, the observer places himself in a prominent position ; it is then quite sufficient to raise the electroscope even a foot or so to obtain signs of electricity.

To observe the electricity of clouds, where the potential is very considerable, use is made of a long bar terminating in a point. This bar, which is insulated with care, is fixed to the summit of a building, and its lower end is connected with an electrometer, or even with electric chimes (fig. 713), which announce the presence of thunder-clouds. As, however, the bar can then give dangerous shocks, a metal ball must be placed near it,

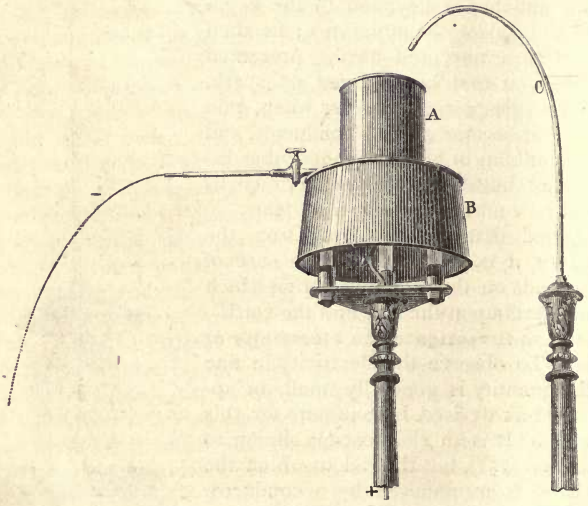

Fig. 1017.

which is well connected with the ground, and which is nearer the bar than the observer himself ; so that if a discharge should ensue, it will strike the ball, and not the observer. Richmann, of St. Petersburg, was killed in an experiment of this kind, by a discharge which struck him on the forehead.

Sometimes also captive balloons or kites have been used, provided with a point, and connected by means of a gilt cord with an electrometer.

A good collector of atmospheric electricity consists of a fishing-rod with an insulated handle which projects from an upper window. At the top is a bit of lighted tinder held in a metallic forceps, the smoke of which, being an excellent conductor, conveys the electricity of the air down a wire attached to the rod. A sponge moistened with alcohol, and set on fire, is also an excellent conductor.

A convenient instrument for investigating atmospheric electricity has

been introduced by Lord Kelvin, one form of which, used in the Meteorological Observatory of Montsouris, is represented in fig. 1017. It consists of a large metal vessel, A, resting on three insulating glass legs fixed to the top of a tall column of cast iron. A sheet-metal mantle, B, protects the supports from the rain. The apparatus is arranged in the open, and can be filled with water from a pipe, C. The water issues through a long, lateral jet in A, in a stream so fine that the volume of the water is not appreciably altered. An insulated wire, *i*, passing through the column, connects the vessel A with an electrometer placed indoors. This plan of collecting the atmo-

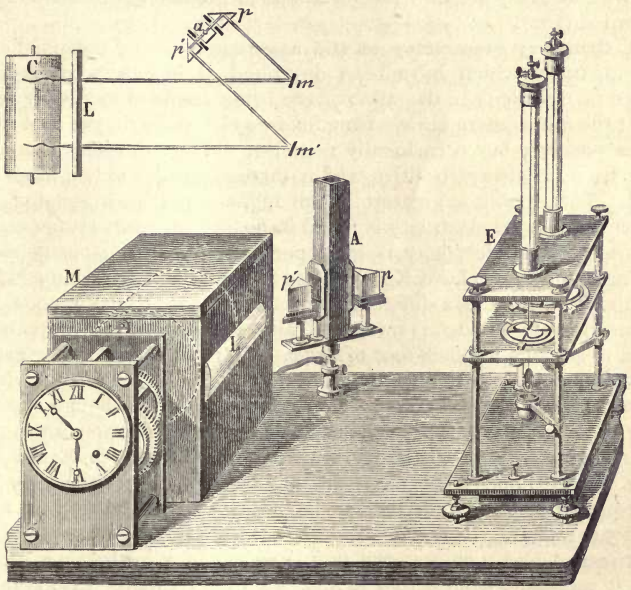

Fig. 1018.

spheric electricity is adopted in balloons, where a flame, for instance, is out of the question.

The manner in which the electricity of the atmosphere is registered is seen from fig. 1018, which represents the form in use at the above observatory. In a light-tight box is a band of sensitised photographic paper, stretched on the surface of a cylinder and moved by clockwork.

In one side of the box is a long cylindrical glass lens, in front of which, at E, are two quadrant electrometers (780). Both of these are connected with the same collector of electricity, placed outside, and their sectors are charged by the same source of electricity, but one of them is ten times as sensitive as the other. Near one side of the box is a gas-burner with an opaque chimney, A, in two opposite sides of which are longitudinal slits, through which the light passes to two total-reflection prisms (545), *pp'*, which are

arranged so as to send two pencils of light on the mirrors, *m m'*, of the electrometer. This is shown on a larger scale on the left of the figure : the two pencils fall upon the lens, L, which concentrates in a point the slices of light issuing from the chimney and reflected from the mirror. These follow the motion of the mirror, and thus impress on the sensitive paper the curves which measure the electrical potential of the air. There is also an arrangement by which an electromagnet puts the electrometers to earth for a few minutes at every hour, and thus discharges them. The mirrors revert then to their original position and commence a new trace.

If we replace the electrometer with its mirror attached by a magnetometer, we can easily see how the variations in the magnetic declination may be recorded (702).

994. **Ordinary electricity of the atmosphere.**—By means of the different apparatus which have been described, it has been found that the presence of electricity in the atmosphere is not confined to stormy weather, but that the atmosphere always contains free electricity, in the vast majority of cases positive, but occasionally negative. When the sky is unclouded the electricity is always positive, and it increases with the height above the ground. The amount is greatest in the highest and most isolated places. No trace of positive electricity is found in houses, streets, and under trees : in towns, positive electricity is most perceptible in large open spaces, on quays, or on bridges. Lord Kelvin found in the Isle of Arran, at a height of 9 feet above the ground, a difference of potential equal to 200 to 400 Daniell's elements, or from 216 to 432 volts. This represents a rise of potential of from 24 to 48 volts for each foot of ascent. This is subject to great variation ; with winds from the north and north-east the potential was often 6 to 10 times as much as the higher of these amounts. The change of potential is most rapid in cold dry weather, when the quantity of moisture in the air is at its lowest. Thus, at a temperature of $-8°$ to $-12°$ C., Exner found a change of 600 Daniells per metre in the direction of the vertical. With a vapour-pressure of 2·3 mm. the change was 325, with 6·8 it was 116, and with 12·5 it was 68.

Between 5 and 7.30 A.M. the positive electricity in the air is at a minimum ; it increases from 7 to 9.30 A.M., according to the season, and then attains its first maximum. It then decreases rapidly until from 2.30 to 4.30 P.M., and again increases till it reaches its second maximum, from 6.30 to 9.30 P.M. ; the remainder of the night the electricity decreases until sunrise. Thus the greatest amount of electricity is observed when the barometric pressure is highest. These increasing and decreasing periods, which are observed all the year, are more perceptible when the sky is clearer and the weather more settled. The positive electricity of fine weather is much stronger in winter than in summer. It may, in short, be said that electricity of the air follows the opposite course to that of temperature and moisture.

When the sky is clouded, the electricity is sometimes positive and sometimes negative. According to Palmieri, the occurrence of negative electricity is a certain indication that within a distance of 40 miles it either rains, snows, or hails. It often happens that the electricity changes its sign several times in the course of the day, owing to the passage of an electrified

cloud. During storms, and when it rains or snows, the atmosphere may be positively electrified one day, and negatively the next, and the number of the two sets of days are virtually equal.

During a thunderstorm the changes in potential and sign of electricity are so rapid that the photographic method of registration fails.

From a long series of observations on the electricity of the atmosphere made in the early morning, Dellman found that the electricity increased with the density of the fog, but in a far more rapid ratio.

The electricity of the ground has been found by Peltier to be always negative, and this is the cardinal fact in reference to atmospheric electricity; it is so, however, to different extents, according to the hygrometric state and temperature of the air. The density is, moreover, exceedingly small, being calculated at 0·00036 dyne per square centimetre, from which it follows that the electrical pressure (737) is 0·00000082 dyne per square centimetre, or less than the millionth of a milligramme in weight. Even if the pressure were ten times as great, it would be insufficient to raise even the lightest bodies.

995. **Causes of atmospheric electricity.**—Although many hypotheses have been propounded to explain the origin of atmospheric electricity, it must be confessed that our knowledge is in an unsatisfactory state.

Volta first showed that the evaporation of water produced electricity. Pouillet subsequently showed that no electricity is produced by the evaporation of distilled water; but that if an alkali or a salt is dissolved, even in small quantity, the vapour is positively and the solution is negatively electrified. The reverse is the case if the water contains acid. Hence it has been assumed, that as the waters which exist on the surface of the earth and on the sea always contain salt dissolved, the vapours disengaged ought to be positively and the earth negatively electrified. The development of electricity by evaporation may be observed by heating strongly a platinum dish, adding to it a small quantity of liquid, and placing it on the upper plate of the condensing electroscope (fig. 734), taking care to connect the lower plate with the ground. When the water of the capsule is evaporated, the connection with the ground is broken, and the upper plate raised. The gold leaves then diverge if the water contains salts, but remain quiescent if the water is pure.

Reasoning from such experiments, Pouillet ascribed the development of electricity by evaporation to the separation of particles of water from the substances dissolved; but Reich and Riess showed that the electricity disengaged during evaporation could be attributed to the friction which the particles of water carried away in the current of vapour exert against the sides of the vessel, just as in Armstrong's electrical machine (758). By a recent series of experiments, Gaugain has arrived at the same result.

Sohncke recalls an experiment of Faraday which he has repeated—that the friction of minute vesicles of water against dry ice is an abundant source of electricity; he ascribes atmospheric electricity to this origin, showing that in the upper regions both particles of water and of ice may coexist. The ice particles become positively electrified, while those of water are negative. When these fall in rain, they carry with them their negative electricity. A similar theory has been propounded by Luvini.

996. **Electricity of clouds.**—Clouds are in general electrified usually positively, but sometimes negatively, and only differ in their higher or lower potential. The formation of positive clouds is by some ascribed to the vapour disengaged from the ground and condensed in the higher regions. Negative clouds are supposed to result from fogs, which, by their contact with the ground, become charged with negative electricity, which they retain on rising into the atmosphere; or that, separated from the ground by layers of moist air, they have been negatively electrified by induction from the positive clouds, which have repelled into the ground positive electricity.

Whatever be the origin of atmospheric electricity, there can be no doubt that the invisible aqueous vapour is the carrier of it, and it is easy to explain the high potential of clouds from the condensation of this vapour. For suppose 1,000 vapour-particles, each possessing the same charge of electricity, coalesce to form a single droplet, the diameter of such a droplet will be ten times that of the individual particles—that is, its capacity is ten times as great, since the capacity is equal to the radius (739); but the quantity of electricity will be 1,000 times as great as on the small one, and therefore the potential will be 100 times as great. Now the number of vapour-particles which go to form a single droplet is rather to be counted by billions; hence, however small be the finite value which we assign to the potential of the electricity of the vapour-particles, that of the drops will be infinitely greater, and sufficient to account for the high potential of clouds. Thunder-clouds are sometimes as low as 700 to 1,000 feet; but their usual height appears to be 3,000 to 6,000 feet.

997. **Lightning.**—This, as is well known, is the dazzling light emitted by the electric spark when it shoots from clouds charged with electricity. In the lower regions of the atmosphere the light is white, but in the higher regions, where the air is more rarefied, it takes a violet tint; as does the spark of the electrical machine in a rarefied medium (788).

The flashes of lightning are often more than a mile, and sometimes extend to four or five miles, in length; they generally pass through the atmosphere in a zigzag direction—a phenomenon ascribed to the resistance offered by the air condensed by the passage of a strong discharge. The spark then diverges from a right line, and takes the direction of least resistance. In a vacuum, electricity passes in a straight line.

De la Rue and Müller have calculated that the potential required to produce a flash a mile in length would be that of 3,516,480 of their cells (812).

We cannot, however, regard the length of a lightning flash as the direct striking distance between two conductors. Owing to the number of droplets met on its path, the discharge is rather to be compared with that of the luminous tubes and panes (789). The experiments of Mascart on the relation between the striking distance (788) and the potential required to produce it, show that the striking distance increases far more rapidly than the potential. Thus, while the potential required for a striking distance of 1 cm. is represented by 8·3, for 4 cm. it is 15·9, for 8 cm. 20·5, and for 15 cm. 23·3. From this it is possible that a lightning discharge is produced by a

difference of potentials between two clouds which is not out of proportion with those obtained by our electrical machines.

Several kinds of lightning flashes may be distinguished—1, the *zigzag* flashes, which move with extreme velocity in the form of a line of fire with sharp outlines, and which closely resemble the spark of an electrical machine. The recent investigation of the shape of lightning discharges by means of extra rapid photographic dry plates (610) has shown that the path of a discharge is not so sharply zigzag as is usually represented, but has more the shape of the course of a river as shown on a map, and with frequent branchings ; 2, the *sheet* flashes, which, instead of being linear, like the preceding, fill the entire horizon without having any distinct shape. This kind, which is most frequent, appears to be produced in the cloud itself, and to illuminate the mass. According to Kundt, the number of sheet discharges are to the zigzag discharged as 11 : 6; and from spectrum observations it would appear that the former are brush discharges between clouds, while the latter are true electrical discharges between the clouds and the earth. Another kind, called *heat lightning*, is ascribed to distant lightning flashes which are below the horizon, but illuminate the higher strata of clouds, so that their brightness is visible at great distances ; they produce no sound, probably in consequence of the fact of their being so far off that the rolling of thunder cannot reach the ear of the observer. There is, further, the very unusual phenomenon of *globe lightning*, or the flashes which appear in the form of globes of fire 18 inches in diameter. These, which are sometimes visible for as much as ten seconds, descend from the clouds to the earth with such slowness that the eye can follow them. They often rebound on reaching the ground ; at other times they burst and explode with a noise like that of the report of many cannon. No adequate explanation has been given of these, though Planté with a large battery of his cells has imitated the phenomena.

The duration of the light of the first three kinds does not amount to the millionth of a second, as was determined by Wheatstone by means of his rotating wheel, which was turned so rapidly that the spokes were invisible ; on illuminating it by the lightning flash, its duration was so short that whatever the velocity of rotation of the wheel, it appeared quite stationary ; that is, its displacement is not perceptible during the time the lightning exists. The light produced by a lightning flash must be comparable to the sun in brightness, though it does not appear to us brighter than ordinary moonlight. But considering its excessively brief duration, and that the full effect of any light on the eye is only produced when its duration is at least the tenth of a second, it follows that a landscape continuously illuminated by the lightning flash would appear 100,000 times as bright as it actually appears to us during the flash.

Here also may be mentioned the phenomenon known as *St. Elmo's fire*, which occurs in a highly electrical state of the atmosphere when the clouds are low. It is a sort of brush discharge (787), appearing like small flames issuing from prominent point-objects such as masts, tops of trees, lightning-conductors ; it has also been observed on the points of helmets or lances, alpenstocks ; it is of course most easily seen in the dark, and is accompanied by a slight rustling noise. On the sea it is not uncommon in thunderstorms on mastheads and yard-arms.

998. **Thunder.**—*Thunder* is the violent report which succeeds lightning in stormy weather. The occurrence of lightning and thunder is practically simultaneous, but an interval of several seconds is generally observed between the perception of these two phenomena, which arises from the fact that sound only travels at the rate of about 1,100 feet in a second (232), while the passage of light is almost instantaneous. Hence an observer will only hear the noise of thunder five or six seconds, for instance, after the lightning, according as the distance of the thunder-cloud is five or six times 1,100 feet. The noise of thunder arises in some such manner as the crack of a whip or the report of a gun. The lightning discharge, whether by heating the air or by a purely mechanical action, such as is illustrated with Kinnersley's thermometer (fig. 749), is expanded with explosive violence, which is only possible by a compression of the surrounding air. This compressed air rushes in to fill the partial vacuum, forming itself, in turn, a partial vacuum, and thus, giving rise to alternate condensation and rarefaction, constitutes the wave-motion producing the sound. The depth of the note represents a great wave-length, and shows that the disturbance must have a great length. Near the place where the lightning strikes the sound is sharp and of short duration. At a greater distance a series of reports are heard in rapid succession. At a still greater distance the noise, feeble at first, changes into a prolonged rolling sound of varying intensity. If the lightning is at a greater distance than 14 or 15 miles it is no longer heard, for sound is more imperfectly propagated through air than through solid bodies: hence there are lightning discharges without thunder; these occur at times when the sky is cloudless.

The rolling of thunder, the alternate rise and fall of the sound, occurs ordinarily with sheet lightning, less so with forked lightning, when the sound is short and crackling.

Various causes contribute to produce the rolling; one cause is the reflection from the ground, from clouds, and even from layers of air of unequal density. Lightning, too, is not a single discharge, but a series of discharges, each of which gives rise to a particular sound. But as these partial discharges proceed from points at different distances, and from zones of unequal density, it follows not only that they reach the ear of the observer successively, but that they bring sounds of unequal density, which occasion the duration and inequality of the rolling. The phenomenon has finally been ascribed to the zigzags of lightning themselves, assuming that the air at each salient angle is at its greatest compression, which would produce the unequal intensity of the sound. The distance between the nearest point of a lightning flash is obtained in kilometres if we divide the time in seconds which elapses between the lightning flash and the beginning of the thunder by 3.

999. **Effects of lightning.**—The lightning discharge is the electric discharge which strikes between a thunder-cloud and the ground. The latter, by the induction of the electricity from the cloud, becomes charged with contrary electricity; and when the tendency of the two electricities to combine exceeds the resistance of the air, the spark passes, which is often expressed by saying that 'a thunderbolt has fallen.' Lightning in general strikes from above, but *ascending lightning* is also sometimes observed;

probably this is the case when the clouds being negatively the earth is positively electrified ; for experiments show that at the ordinary pressure positive electricity passes through the atmosphere more easily than negative electricity.

The discharge usually falls first on the nearest and best conducting objects, and, in fact, trees, elevated buildings, metals, are particularly struck by the discharge. Hence it is imprudent to stand under trees during a thunderstorm.

According to Hellman, the frequency with which trees are struck is : fir, 5, beech 7, oak 18 ; in like manner of soils the ratio is : chalk 1, clay 7, sand 9, and loam 22.

The effects of lightning are very varied, and of the same kind as those of batteries (783), but of far greater power. The lightning discharge kills men and animals, ignites combustibles, melts metals, breaks bad conductors in pieces. When it penetrates the ground it melts the silicious substances on its path, and thus produces in the direction of the discharge those remarkable vitrified tubes called *fulgurites*, some of which are as much as 12 yards in length ; in most cases there are found to be accumulations of water below such fulgurites. When it strikes bars of iron it magnetises them, and often inverts the poles of compass needles.

The action of lightning on trees is very singular. When struck by it they are sometimes stripped of their bark, either wholly or partially, or the wood is often split into thin laths, or into a mass of fibres. Franklin ascribed this to the sudden evaporation of the water.

After the passage of lightning a highly peculiar odour is frequently produced, like that perceived in a room in which an electrical machine is being worked. This is due to the formation of *ozone*, a peculiar allotropic modification of oxygen (793). An electrified cloud forms with the earth below a condenser, the intervening mass of air being the dielectric. This mass of air is therefore in a state of strain, like the dielectric in a charged Leyden jar, and it is to this state of strain which precedes the actual discharge, rather than to the discharge itself, that is due the production of ozone.

Heated air conducts better than cold air, probably only owing to its lesser density. Hence it is that large numbers of animals are often killed by a single discharge, as they crowd together in a storm, and a column of warm air rises from the group.

1000. **Return shock.**—This is a violent and sometimes fatal shock which men and animals experience, even when at a great distance from the place where the lightning discharge passes. It is caused by the inductive action which the thunder-cloud exerts on bodies placed within the sphere of its activity. These bodies are then, like the ground, charged with the opposite electricity to that of the cloud ; but when the latter is discharged by the recombination of its electricity with that of the ground, the induction ceases, and the bodies reverting rapidly from the electrical state to the neutral state, the concussion in question is produced—the *return* or *back shock*. A gradual decomposition and reunion of the electricity produces no visible effects ; yet it is alleged that such disturbances of the electrical equilibrium are perceived by nervous persons.

The return shock is always less violent than the direct one ; there is no instance of its having produced any inflammation, yet plenty of cases in which it has killed both men and animals ; in such cases no broken limbs, wounds, or burns are observed.

The return shock may be imitated by placing a gold-leaf electroscope connected by a wire with the ground near an electrical machine ; when the machine is worked, at each spark taken from the prime conductor the gold leaves of the electroscope suddenly diverge.

It is stated that persons struck by lightning often lose their lives only by a temporary injury to the nerves which control the act of respiration ; so that under favourable circumstances such persons might probably be saved by producing artificial respiration.

1001. **Lightning-conductor.**—This was invented by Franklin in 1755.

There are two principal parts in a lightning-conductor, the rod and the conductor. The *rod* (fig. 1019) is a pointed bar of iron, preferably galvanised, P, fixed vertically to a tube or rod of iron, which, by means of a collar *a a*, and tube *g*, is fitted on the roof of the edifice to be protected ; it is from 6 to 10 feet in height, and its basal section is about 2 or 3 inches in diameter. The *conductor* is best formed of a wire rope, C, such as those which are used for rigging or for telegraph wires, attached to the rod by a metal collar, *b*. The section of the metallic conductor ought to be about half a square inch, and the individual wires 0·04 to 0·06 inch in diameter ; they ought to be twisted in strands, like an ordinary cord. The conductor is usually led into a well, a pond, or other continuous mass of water, and to connect it better with the ground it should terminate in a plate called an earth plate, or if a strand of wires the separate wires should be spread out. This plate should be of the same metal as the conductor, so as to avoid the possibility of local galvanic action (816), by which one or the other metal would be eaten away and the continuity destroyed. If there is no well near, a hole is dug in the soil to the depth of 6 or 7 yards, or to where the earth is permanently damp ; where the ground is naturally dry it is advantageous to direct the rainfall from the roof towards where the plate is placed, and the ends of the conductor having been introduced, the hole is filled with powdered coke, which conducts very well. A good earth contact is obtained when it is possible to connect the wire conductors with large iron gas or water pipes.

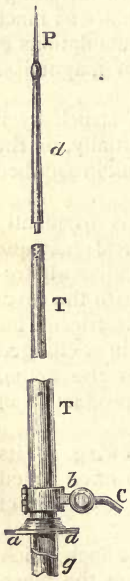

Fig. 1019.

The action of a lightning-conductor is regarded as an illustration of the action of induction and of the property of points (742) ; when a storm cloud positively electrified, for instance, forms in the atmosphere, it acts inductively on the earth, repels the positive and attracts the negative electricity, which accumulates on bodies placed on the surface of the soil, the more abundantly as these bodies are at a greater height. The density is then greatest on the highest bodies, which are therefore most exposed to the electric discharge ; but if these bodies are provided with metal points, like the rods of conductors, the negative electricity, withdrawn from the soil by the influence of the cloud,

flows into the atmosphere, and neutralises the positive electricity of the cloud. Hence the action of the lightning-conductor is twofold ; not only does it tend to prevent the accumulation of electricity on the surface of the earth, but it also tends to restore the clouds to their natural state, both which concur in preventing lightning discharges. This mode of action of lightning-conductors is often overlooked ; it is stated in reference to Pietermaritzburg that until lightning-conductors became common in that town it was constantly visited by thunderstorms at certain seasons. They come as frequently as ever, but cease to give flashes on reaching the town ; they do so, however, when they have passed over it. The quantity of electricity is, however, sometimes so abundant that the lightning-conductor is inadequate to discharge the electricity accumulated, and the lightning strikes ; but the conductor receives the discharge, in consequence of its greater conductivity, and the edifice is preserved.

A conductor, to be efficient, ought to satisfy the following conditions :— (i.) The rod ought to be so large as not to be melted if the discharge passes. (ii.) It ought to terminate in a point, or in several points, to give readier issue to the electricity disengaged by induction from the ground. (iii.) Copper was formerly preferred to iron owing to its greater conductivity. But the lightning discharge is strictly analogous to the discharge of a Leyden jar, which according to circumstances may form either a continuous discharge like a steady current, or a series of oscillations (783). In the latter case the discharge is restricted to the surface owing to the effect of impedance (910), which may have a greater influence than the ohmic resistance, so that the advantage of the greater conducting power of copper disappears. The conductor must be continuous from the point to the ground, and the connection between the rod and the ground must be as intimate as possible ; this is the most important of all, and the one point most frequently neglected in the older arrangements. A lightning-conductor with bad earth contact is not only useless but dangerous. In regard to this, it may be said that the best earth for contact is water. The continuity of the conductor may be tested by means of a voltaic cell and a portable form of galvanometer. (iv.) If the building which is provided with a lightning-conductor contains metallic surfaces of any extent, such as zinc roofs, metal gutters, or ironwork, these ought to be connected with the conductor, or, still better, have each a separate earth connection. If the last two conditions are not fulfilled, there is a great danger of lateral discharges—that is to say, that the discharge takes place between the conductor and the edifice, and then it increases the danger.

Colladon concludes, from the observation of a series of lightning discharges, that a tall tree, such as a poplar, whose roots are in moist ground, may act as a good lightning-conductor, if on the other side of the house there does not happen to be a well or pool, towards which the electricity can spring through the house.

The requirements above laid down are based on the older views of the protection of buildings. Another principle is based on the screening action of a closed conducting surface, or even of a wire gauze. This we have seen (735) protects a body from external electrical action, which probably holds also for violent and sudden electrical discharges. If a building could be surrounded by a wire cage which itself had good earth contact, this would

be an efficient protection. Accordingly, an alternative plan aims at providing all the ridges, eaves and corners, and chimneys of a building with abundance of galvanised iron wire, preferably barbed, and with wire netting, all in metallic connection with each other and with the earth.

1002. **Rainbow.**—The *rainbow* is a luminous phenomenon which appears in the clouds opposite the sun when they are resolved into rain. It consists of seven concentric arcs, presenting successively the colours of the solar spectrum. Sometimes only a single bow is perceived, but there are usually two: a lower one, the colours of which are very bright; and an external or *secondary* one, which is paler, and in which the order of the colours is reversed. In the interior rainbow the red is the highest colour; in the other rainbow the violet is. It is seldom that three bows are seen; theoretically a greater number may exist, but their colours become so faint that they cannot be perceived.

The phenomenon of the rainbow is produced by decomposition of the white light of the sun when it passes into the drops, and by its reflection from their inside face. In fact, the same phenomenon is witnessed in dew-drops and in jets of water—in short, wherever sunlight passes into drops of water under a certain angle.

The appearance and the extent of the rainbow depend on the position of the observer, and on the height of the sun above the horizon; hence only some of the rays refracted by the raindrops, and reflected in their concavity to the eye of the spectator, are adapted to produce the phenomenon. Those which do so are called *effective rays.*

To explain this let *n* (fig. 1020) be a drop of water, into which a solar ray S *a* penetrates. At a point of incidence, *a*, part of the light is reflected from the surface of the liquid; another, entering it, is decomposed and traverses the drop in the direction *a b*. Arrived at *b*, part of the light emerges from the raindrop, the other part is reflected from the concave surface, and tends to emerge at *g*. At this point the light is again partially reflected; the remainder emerges in a direction *g*O, which forms with the incident ray, S *a*, an angle called the *angle of deviation.* It is such rays as *g*O, proceeding from the side next the observer, which produce on the retina the sensation of colours, provided the light is sufficiently intense.

It can be shown mathematically that in the case of a series of rays which impinge on the same drop, and only undergo one reflection in the interior, the angle of deviation increases from the ray S″*n*, for which it is zero, up to a certain limit, beyond which it decreases, and that near this limit rays passing parallel into a drop of rain also emerge parallel. From this parallelism a beam of light is produced sufficiently intense to impress the retina; these are the rays which emerge parallel and are efficient.

As the different colours which compose white light are unequally refrangible, the maximum angle of deviation is not the same for all. For red rays the angle of deviation corresponding to the active rays is 42° 2′, and for violet rays it is 40° 17′. Hence, for all drops placed so that rays proceeding from the sun to the drop make, with those proceeding from the drop to the eye, an angle of 42° 2′, this organ will receive the sensation of red light; this will be the case with all drops situated on the circumference of the base of a cone, the summit of which is the spectator's eye; the axis of

this cone is parallel to the sun's rays, and the angle formed by the two opposed generating lines is 84° 4′. This explains the formation of the red band in the rainbow ; the angle of the cone in the case of the violet band is 80° 34′.

The cones corresponding to each band have a common axis called the *visual axis*. As this right line is parallel to the rays of the sun, it follows that when this axis is on the horizon, the visual axis is itself horizontal, and the rainbow appears as a semicircle. If the sun rises, the visual axis sinks, and with it the rainbow. Lastly, when the sun is at a height of 42° 2′, the arc disappears entirely below the horizon. Hence the phenomenon of the rainbow never takes place except in the morning and evening.

What has been said refers to the interior arc. The secondary bow is formed by rays which have undergone two reflections, as shown by the ray S′*i d f e* O, in the drop *p*. The angle S′IO formed by the emergent and incident rays is called the angle of deviation. The angle is no longer susceptible of a maximum, but of a minimum deviation, which varies for each kind of

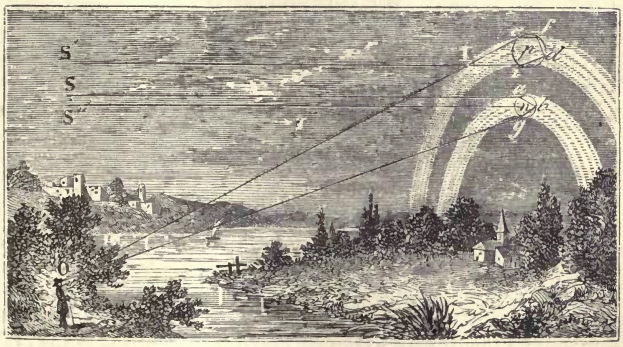

Fig. 102c.

rays, and to which also efficient rays correspond. It is calculated that the minimum angle from violet rays is 54° 7′, and for red rays only 50° 57′ ; hence it is that the red bow is here on the inside, and the violet arc on the outside. There is a loss of light for every internal reflection in the drop of rain, and therefore the colours of the secondary bow are always feebler than those of the internal one. The secondary bow ceases to be visible when the sun is 54° above the horizon.

The moon sometimes produces rainbows like the sun, but they are very pale.

1003. **Aurora borealis.**—The *aurora borealis*, or northern light, or more properly *polar aurora*, is a remarkable luminous phenomenon which is frequently seen in the atmosphere at the two terrestrial poles. The following is a description of an aurora borealis observed at Bossekop, in Lapland, lat. 70°, in the winter of 1838–39 :—

In the evening, between 4 and 8 o'clock, the upper part of the fog which usually prevails to the north of Bossekop became coloured. This light

became more regular, and formed an indistinct arc of a pale yellow, with its concave side turned towards the earth, while its summit was in the magnetic meridian.

Blackish rays soon separated the luminous parts of the arc. Luminous rays formed, becoming alternately rapidly and slowly longer and shorter, their lustre suddenly increasing and diminishing. The bottom of these rays always showed the brightest light, and formed a more or less regular arc. The length of the rays was very variable, but they always converged towards the same point of the horizon, which was in the prolongation of the north end of the dipping-needle ; sometimes the rays were prolonged as far as their point of meeting, and thus appeared like a fragment of an immense cupola.

The arc continued to rise in an undulatory motion towards the zenith. Sometimes one of its feet or even both left the horizon ; the folds became more distinct and more numerous ; the arc was now nothing more than a long band of rays convoluted in very graceful shapes, forming what is called the boreal crown. The lustre of the rays varied suddenly in intensity, and attained that of stars of the first magnitude ; the rays darted with rapidity, the curves formed and re-formed like the folds of a serpent, or like a flag moved by the wind (fig. 1021), the base was red, the middle green, while the remainder retained its bright yellow colour. Lastly, the lustre diminished, the colours disappeared ; everything became feebler or suddenly went out.

Plate III. represents a very beautiful aurora observed by Lemström on the north coast of Norway. The radial divergence of the aurora and the convergence towards a corona is due to an effect of perspective. The work of this author (*L'Aurore Boréale*, Gauthier Villars, Paris) is a storehouse of information on this subject.

A French scientific commission to the North observed 150 auroræ boreales in 200 days ; it appears that at the poles, nights without an aurora borealis are quite exceptional, so that it may be assumed that they take place every night, though with varying intensity. They are visible at a considerable distance from the poles, and over an immense area. Sometimes the same aurora borealis has been seen at the same time at places so widely apart as Moscow, Warsaw, Rome, and Cadiz. It seems difficult to assign a very high limit for the occurrence of the aurora ; this is probably lower than has generally been stated. Lemström holds that from 22 to 44 miles is a close approximation to the truth ; and it may be regarded as certain that even in more southern latitudes the aurora is often seen much lower—at a height of two or three miles, for instance. In polar countries certain forms of aurora, more especially those of weak flames, are seen to proceed from the ground on the tops of certain mountains. They are most frequent at the equinoxes, and least so at the solstices. The number differs in different years, attaining a maximum every 11 years at the same time as the sun-spots, and like these a minimum which is about five or six years from the maximum. The years 1844, 1855, 1866, and 1877 were poor in the appearance of the aurora.

There is, moreover, a period of about 60 years ; for the years 1728, 1780, and 1842 have been remarkable for the prevalence of the aurora. The last

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Aurora Borealis, North Coast of Norway, Oct. 18, 1868

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two periods are also remarkable for the occurrence of disturbances in the earth's magnetism.

Numerous hypotheses have been devised to account for the auroræ boreales. As they share the rotation of the earth, they must have an atmospheric origin. Their direction is not due north and south, but is always parallel to that of the dipping-needle, pointing to the magnetic pole; this, together with their action on the magnetic needle (702), seems to prove that they ought to be attributed to electric currents in the higher regions of the atmosphere. In high latitudes the aurora borealis acts powerfully on the wires of the electric telegraph; the alarums are for a long time violently rung, and telegraphic messages frequently interrupted by the spontaneous abnormal working of the apparatus (894). In the lower discharges a crack-

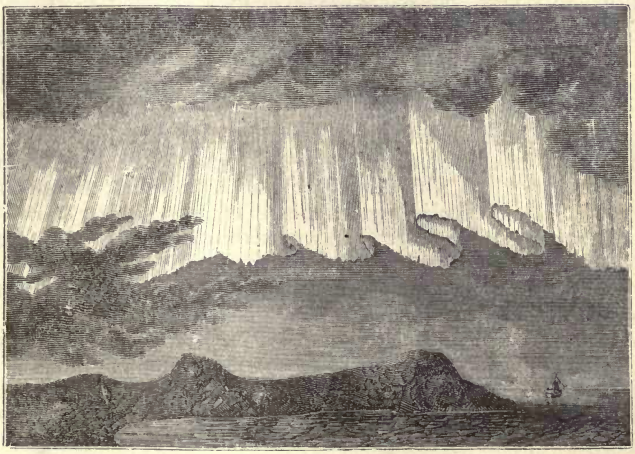


Fig. 1021.

ling sound has been heard, and during balloon ascents a strong smell of ozone has been perceived when the balloon was among the luminous rays.

The spectrum of the aurora borealis has been found to consist of several lines in the green, and of an indistinct line in the blue; to which must be added a red line due to the red protuberances; these lines are the same as those of nitrogen, greatly rarefied and at a low temperature; one special line between the green and the yellow, and called the *yellow* line, is so characteristic of the aurora that it is visible even when the eye can discern no other trace of this light; this line has not been produced in laboratory experiments.

De la Rive held that auroræ boreales were due to electric discharges which take place in polar regions between the positive electricity of the atmosphere and the negative electricity of the earth. The positively electrified aqueous vapours are supposed to be carried by the equatorial current in the higher regions of the atmosphere to the poles, where the neutralisation takes place. These discharges produce luminous appearances of the

same kind as are observed in Geissler's tubes; and De la Rive showed by means of an apparatus specially devised for the purpose (fig. 949) that the forms of the luminous phenomena are in accordance with this theory.

By direct experiments Lemström has been able to imitate and reproduce a peculiar form of aurora observed in winter as a flame-like appearance on the tops of two mountains 800 and 1,100 metres in height, and to show that it is of electrical origin. He erected on the summit of a hill a system of pointed rods extending over a surface of nearly 4,000 square feet; each rod was carefully insulated from the earth by means of a Mascart's insulator (fig. 688), but was connected with the rest, and an insulated wire led down from this system, into the valley where it was connected with one terminal of a galvanometer, the other being put to earth. The existence of a positive current from the air to the earth was observed, and at the same time yellowish-white columns of light, reaching to a height of 120 metres, were observed to issue from the points. Observed with the spectroscope it gave the characteristic lines between D and E.

Making similar experiments on even a larger scale in Lapland on a detached peak, he observed that the characteristic luminous phenomena were produced there, while the neighbouring peaks remained dark.

The recent investigations of Exner relative to the fall of atmospheric electrical potential lend a further support to the view that the aurora is due to electricity. In the polar regions the fall of potential is 13 times greater in summer, and 18 times greater in winter than at the equator. Hence an electrical phenomenon, which depends on the magnitude of this fall of potential, must be more intense in winter and in high latitudes than in summer and in the torrid zones.

The occurrence of irregular currents of electricity which manifest themselves by abnormal disturbances of telegraphic communications is not infrequent: such currents have received the name of *earth currents* (894). Sabine held that these magnetic disturbances are due to a peculiar action of the sun, and probably independent of its radiant heat and light. It has also been ascertained that the aurora borealis as well as earth currents invariably accompanies these magnetic disturbances. According to the late Balfour Stewart, auroræ and earth currents are to be regarded as secondary currents due to small but rapid changes in the earth's magnetism: he likened the body of the earth to the magnetic core of a Ruhmkorff's machine (905); the lower strata of the atmosphere forming the insulator, while the upper and rarer, and therefore electrically conducting, strata may be considered as the secondary coil.

On this analogy the sun may perhaps be likened to the primary current which performs the part of producing changes in the magnetic state of the core. Now in Ruhmkorff's machine the energy of the secondary current is derived from that of the primary current. Thus, if the analogy be correct, the energy of the aurora borealis may in like manner come from the sun; but until we know more of the connection between the sun and terrestrial magnetism, these ideas are to be accepted with some reserve.

CLIMATOLOGY

1004. **Mean temperature.**—The *mean daily temperature*, or simply *temperature*, is that obtained by adding together 24 hourly observations, and dividing by 24. A very close approximation to the mean temperature is obtained by taking the mean of the highest and lowest temperatures of the day and of the night, which are determined by means of the maximum and minimum thermometers. These ought to be protected from the sun's rays, to be raised above the ground, and far from all objects which might influence them by their radiation.

The temperature of a month is the mean of the temperature of 30 days, and the temperature of the year is the mean of those of 12 months. Finally, the temperature of a place is the mean of its annual temperature for a great series of years. The mean temperature of London is 8·28° C., or 46·9° F. The temperatures in all cases are those of the air, and not those of the ground.

1005. **Causes which modify the temperature of the air.**—The principal causes which modify the temperature of the air are the latitude of a place, its height, the direction of the winds, and proximity of seas.

Influence of the latitude.—The influence of the latitude arises from the greater or less obliquity of the solar rays, for as the quantity of heat absorbed is greater the more perpendicular are the rays (414), the heat absorbed decreases from the equator to the poles, for the rays are then more oblique. This loss is, however, in summer, in the temperate and arctic zones, partially compensated by the length of the days. Under the equator, where the length of the days is constant, the temperature is almost invariable ; in the latitude of London, and in more northerly countries, where the days are very unequal, the temperature varies greatly ; but in summer it sometimes rises almost as high as under the equator. The lowering of the temperature produced by the latitude is small ; thus, in a latitude 115 miles north of France, the temperature is only 1° C. lower.

Influence of height.—The height of a place has a much more considerable influence on the temperature than its latitude. In the temperate zone a diminution of 1° C. corresponds in the mean to an ascent of 180 yards.

The cooling on ascending in the atmosphere has been observed in balloon ascents, and a proof of it has been seen in the perpetual snows which cover the highest mountains. It is due in part to the greater rarefaction of the air, which necessarily diminishes its absorbing power ; besides which the air is at a greater distance from the ground, which heats it by contact ; and finally, dry air is very diathermanous.

The law of the diminution of temperature corresponding to greater heights in the atmosphere has not been made out, in consequence of the numerous disturbing causes which modify it, such as the prevalent winds, the hygrometric state, the time of day, the season of the year, &c. The difference between the temperatures of two places at unequal heights is not proportional to the difference of level, but for moderate heights an approximation to the law may be made. As the mean of a series of very careful observations made during balloon ascents, a diminution of 1° C. corresponded to an increase in height of 232 yards.

It will thus be seen that at a certain height above the ground, there must be a surface or layer where the temperature is uniformly zero. The height of this isothermal surface (1007) will vary materially with the time of the year, being lower in the cold months : it varies also with the time of day, rising rapidly about mid-day. In summer this height may be taken at from 3,400 to 3,700 metres above the sea-level.

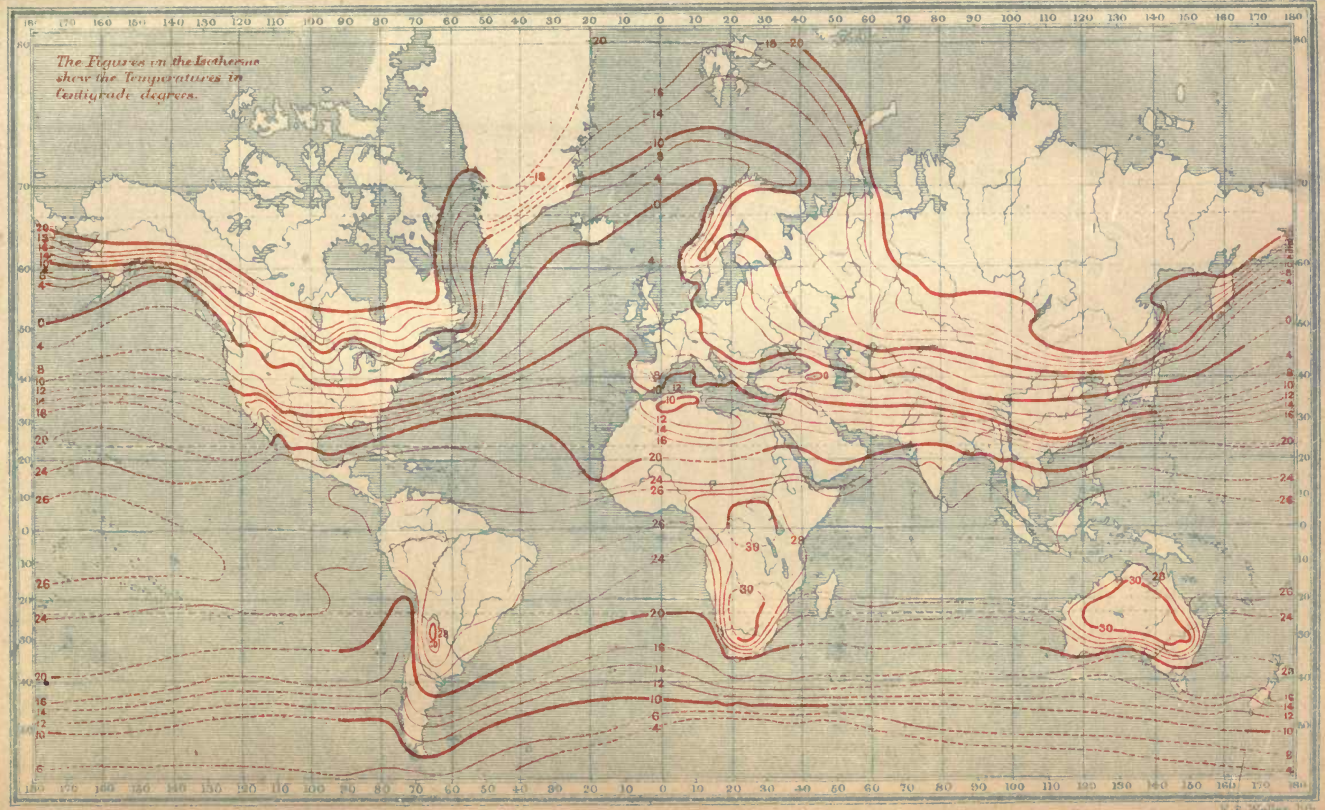
Direction of winds.—As winds share the temperature of the countries which they have traversed, their direction exercises great influence on the air in any place. In Paris, the hottest winds are the south ; then come the south-east, the south-west, the west, the east, the north-west, north, and lastly, the north-east, which is the coldest. The character of the wind changes with the seasons ; the east wind, which is cold in winter, is warm in summer.

Proximity of the sea.—The neighbourhood of the sea tends to raise the temperature of the air, and to render it uniform. The average temperature of the sea in equatorial and polar countries is always higher than that of the atmosphere. With reference to the uniformity of the temperature, it has been found that in temperate regions—that is, from 25° to 50° of latitude—the difference between the highest and lowest temperature of a day does not exceed, on the sea, 2° to 3° ; while upon the continent this amounts to from 12° to 15°. In islands the uniformity of temperature is very perceptible, even during the greatest heats. In continents, on the contrary, the winters for the same latitudes become colder, and the difference between the temperature of summer and winter becomes greater.

1006. **Gulf Stream.**—A similar influence to that of the winds is exerted by currents of warm water. To one of these, the Gulf Stream, the mildness of the climate in the north-west of Europe is mainly due. This great body of water, taking its origin in equatorial regions, flows through the Gulf of Mexico, whence it derives its name ; passing by the southern shores of North America, it makes its way in a north-westerly direction across the Atlantic, and finally washes the coast of Ireland and the north-west of Europe generally. Its temperature in the Gulf is about 28° C. ; and it is usually a little more than 5° C. higher than the rest of the ocean on which it floats, owing to its lower specific gravity. To its influence is due the milder climate of West Europe as compared with that of the opposite coast of America ; thus the river Hudson, in the latitude of Rome, is frozen over three months in the year. It also causes the polar regions to be separated from the coasts of Europe by a girdle of open sea ; and thus the harbour of Hammerfest is open the year round. Besides its influence in thus moderating climate, the Gulf Stream is an important help to navigators.

1007. **Isothermal lines.**—When on a map all the points whose temperature is known to be the same are joined, curves are obtained which Humboldt first noticed, and which he called *isothermal lines*. If the temperature of a place only varied with the obliquity of the sun's rays—that is, with the latitude—isothermal lines would all be parallel to the equator ; but as the temperature is influenced by many local causes, especially by the height, the isothermal lines are always more or less curved. On the sea, however, they are almost parallel. Maps 4, 5, and 6 represent these lines for the Year, for January and for July.

ISOTHERMS FOR JANUARY.



J. & C. Walker lith.

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A distinction is made between *isothermal lines*, *isothiral lines*, and *isochimenal lines*, where the *mean general*, the *mean summer*, and the *mean winter* temperatures are respectively constant. An *isothermal zone* is the space comprised between two isothermal lines. Kupffer also distinguishes *isogeothermic lines* where the mean temperature of the soil is constant.

1008. **Climate.**—By the climate of a place is understood the whole of the meteorological conditions to which a place is subjected ; its mean annual temperature, summer and winter temperatures, and the extremes within which these are comprised. Some writers distinguish seven classes of climates, according to their mean annual temperature : a *hot climate* from 30° to 25° C. ; a *warm climate* from 25° to 20° C. ; a *mild climate* from 20° to 15° C. ; a *temperate climate* from 15° to 10° C. ; a *cold climate* from 10° to 5° C. ; a *very cold climate* from 5° to zero C. ; and an *arctic climate* where the temperature is below zero.

Those climates, again, are classed as *constant climates* where the difference between the mean and summer and winter temperature does not exceed 6° to 8° ; *variable climates*, where the difference amounts to from 16° to 20° ; and *extreme climates*, where the difference is greater than 30°. The climates of Paris and London are variable ; those of Pekin and New York are extreme. Island climates are generally little variable, as the temperature of the sea is constant ; and hence the distinction between land and sea climates. Marine climates are characterised by the fact that the difference between the temperature of summer and winter is always less than in the case of continental climates. But the temperature is by no mean the only character which influences climates ; there are, in addition, the moisture of the air, the quantity and frequency of the rains, the number of storms, the direction and intensity of the winds, and the nature of the soil.

1009. **Distribution of temperature on the surface of the globe.**—The temperature of the air on the surface of the globe decreases from the equator to the poles ; but it is subject to perturbing causes so numerous and so purely local, that its decrease cannot be expressed by any law. It has hitherto not been possible to do more than obtain by numerous observations the mean temperature of each place, or the maximum and minimum temperatures. The following table gives a general idea of the distribution of heat in the Northern Hemisphere :—

Mean temperature at different latitudes

Abyssinia	31·0° C.	Cairo	22·4° C.
Calcutta	28·5	Constantine	17·2
Jamaica	26·1	Naples	16·7
Senegal	24·6	Mexico	16·6
Rio de Janeiro	23·1	Marseilles	14·1
Constantinople	13·7	London	8·3
Pekin	12·7	Stockholm	5·6
Paris	10·8	Moscow	3·6
Brussels	10·2	St. Petersburg	3·5
Strasburg	9·8	St. Gothard	−1·0
Geneva	9·7	Greenland	−7·7
Boston	9·3	Melville Island	−18·7

These are mean yearly temperatures. The highest temperature which has been observed on the surface of the globe is 47·4° at Esne, in Egypt, and the lowest is −75° in the Arctic Expedition of 1876; which gives a difference of 122° between the extreme temperatures observed on the surface of the globe.

The highest temperature observed at Paris was 38·4° on July 8, 1793, and the lowest −23·5° on December 26, 1798. The highest observed at Greenwich was 35° C. in 1808, and the lowest −20° C. in 1838.

No arctic voyagers have succeeded in reaching the poles, in consequence of these seas being completely frozen, and hence the temperature there is not known. In our hemisphere the existence of a single *glacial pole*—that is, a place where there was the maximum cold—has been long assumed. But the bendings which the isothermal lines present in the Northern Hemisphere have shown that in this hemisphere there are two cold poles—one in Asia, to the north of Gulf Tamour; and the other in America, north of Barrow's Straits, about 15° from the earth's north pole. The mean temperature of the first of these poles has been estimated at −17°, and that of the second at −19°. With respect to the austral hemispheres, the observations are not sufficiently numerous to tell whether there are one or two poles of greatest cold, or to determine their position.

1010. **Temperature of lakes, seas, and springs.**—In the tropics the temperature of the sea is generally the same as that of the air; in polar regions the sea is always warmer than the atmosphere.

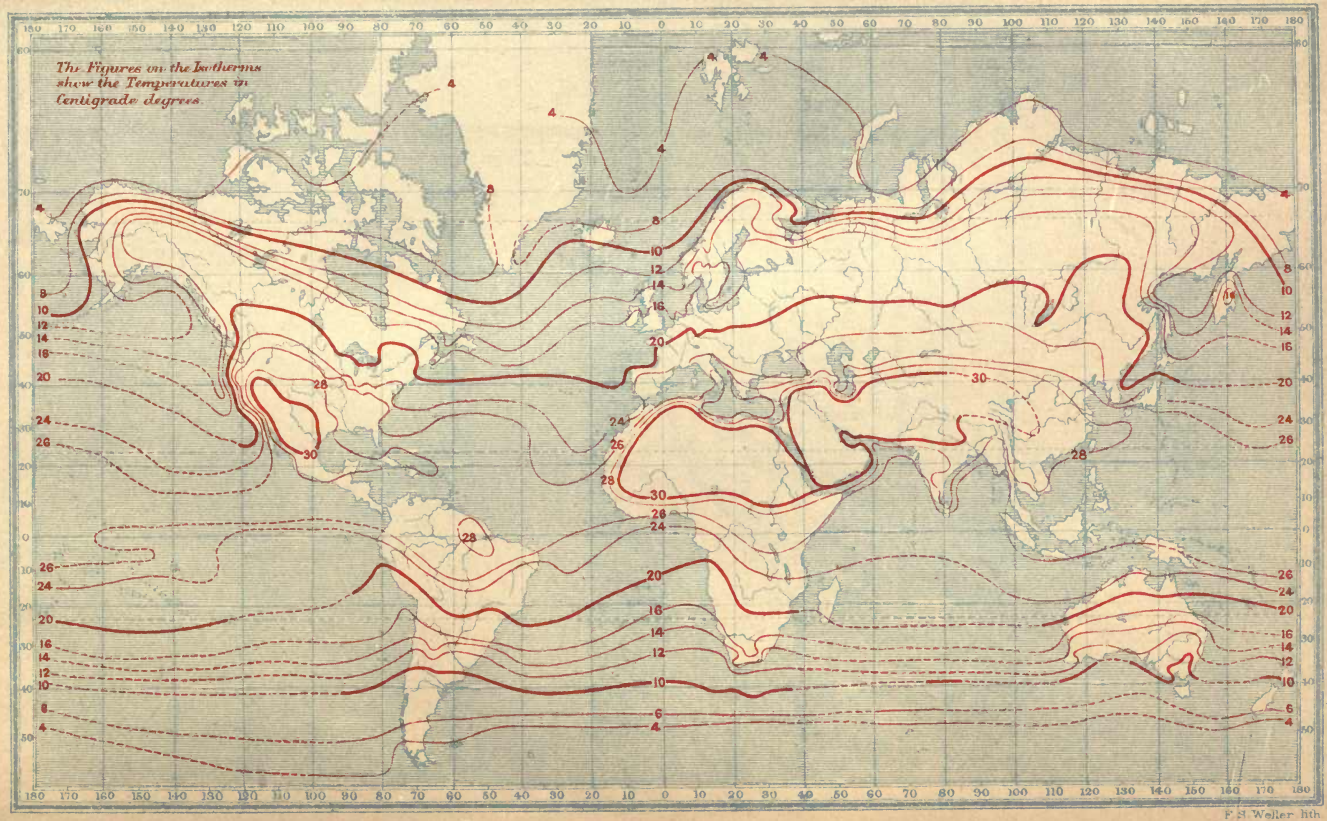
The temperature of the sea under the torrid zone is always about 26° to 27° at the surface: it diminishes as the depth increases, and in temperate as well as in tropical regions the temperature of the sea at great depths is between 2·5° and 3·5°. The temperature of the lower layers is caused by submarine currents which carry the cold water of the polar seas towards the equator.

The variations in the temperature of lakes are more considerable; their surface, which becomes frozen in winter, may become heated to 20° or 25° in summer. The temperature of the bottom, on the contrary, is virtually 4°, which is that of the maximum density of water.

Springs, which arise from rain water which has penetrated into the crust of the globe to a greater or less depth, necessarily tend to assume the temperature of the terrestrial layers which they traverse. Hence, when they reach the surface their temperature depends on the depth which they have attained. If this depth is that of the layer of invariable temperature, the springs have a temperature of 10° or 11° in this country, for this is the temperature of this layer, or about the mean annual temperature. If the springs are not very copious, their temperature is raised in summer and cooled in winter by that of the layers which they traverse in passing from the invariable layer to the surface. But if they come from below the layer of invariable temperature their temperature may considerably exceed the mean temperature of the place, and they are then called *thermal springs*. The following list gives the temperature of some of them:—

ISOTHERMS FOR JULY.

No. 5.



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Wildbad	37·5° C.
Vichy	40
Bath	46
Ems	46
Baden-Baden	67·5
Chaudes-Aigues	88
Trincheras	97
Great Geyser, in Iceland, at a depth of 66 feet	124

From their high temperature they have the property of dissolving many mineral substances which they traverse in their passage, and hence form *mineral waters*. The temperature of mineral waters is not modified in general by the abundance of rain or of dryness; but it is by earthquakes, after which they have sometimes been found to rise and at others to sink.

1011. **Distribution of land and water.**—The distribution of water on the surface of the earth exercises great influence on climate. The area covered by water is considerably greater than that of the dry land; and the distribution is unequal in the two hemispheres. The entire surface of the globe occupies about 200 millions of square miles, nearly three-fourths of which are covered by water; that is, the extent of the water is nearly three times as great as that of the land. The surface of the sea in the Southern Hemisphere is to that in the Northern in about the ratio of 13 to 9.

The depth of the open sea is very variable; the lead generally reaches the bottom at about 300 to 450 yards; in the ocean it is often 1,300 yards, and instances are known in which a bottom has not been reached at a depth of 4,500. It has been computed that the total mass of the water does not exceed that of a liquid layer surrounding the earth with a depth of about 1,100 yards.

PROBLEMS AND EXAMPLES IN PHYSICS

I. EQUILIBRIUM

1. A body being placed successively in the two pans of a balance, requires 180 grammes to hold it in equilibrium in one pan, and 181 grammes in the other; required the weight of the body to a milligramme.

From the formula $x = \sqrt{pp}$, we have

$$x = \sqrt{180 \times 181} = 180^{gr}, 499.$$

2. What resistance does a nut offer when placed in a pair of nutcrackers at a distance of $\frac{3}{4}$ of an inch from the joint, if a pressure of 5 pounds applied at a distance of 4 inches from the joint is just sufficient to crack it? *Ans.* $26\frac{2}{3}$ pounds.

3. What force is required to raise a cask weighing 6 cwt. into a cart 0·8 metre high along a ladder 2·75 metres in length? *Ans.* $19\frac{5}{2}$ pounds.

4. If a horse can move 30 cwt. along a level road, what can it move along a road the inclination of which is 1 in 80, the coefficient of friction on each road being $\frac{1}{10}$?

Ans. $26\frac{2}{3}$ cwt.

5. The piston of a force-pump has a diameter of 8 centimetres, and the arms of the lever by which it is worked are respectively 12 and 96 centimetres in length; what force must be exerted at the longer arm if a pressure of 12·36 pounds on a square centimetre is to be applied? *Ans.* 77·69 pounds.

II. GRAVITATION

6. A stone is thrown from a balloon with a velocity of 50 metres in a second. How soon will the velocity amount to 99 metres in a second, and through what distance will the stone have fallen?

To find the time requisite for the body to have acquired the velocity of 99 metres in a second, we have

$$v = V + gt;$$

in which V is the initial velocity, g the acceleration of gravity, which, with sufficient approximation, is equal to 9·8 metres in a second, and t the time. Substituting these values, we have

$$t = \frac{99 - 50}{9{\cdot}8} = \frac{49}{9{\cdot}8} = 5 \text{ seconds.}$$

For the space traversed we have

$$s = Vt + \tfrac{1}{2}gt^2 = 50 \times 5 + 4{\cdot}9 \times 25 = 372{\cdot}5 \text{ metres.}$$

7. A projectile was thrown vertically upwards to a height of $510^{m}{\cdot}22$. Disregarding the resistance of the air, what was the initial velocity of the body?

The velocity is the same as that which the body would have acquired on falling from a height of 510·22 metres.

From the formula $v = \sqrt{2gs}$ we get

$$v = \sqrt{2 \times 9{\cdot}8 \times 510{\cdot}22} = \sqrt{10000} = 100 \text{ metres.}$$

8. A stone is thrown vertically upwards with an initial velocity of 100 metres. After what time would it return to its original position?

The time of rising and falling is the same, but the time of falling s $\frac{v}{g}$ (from the formula $v=gt$) or $\frac{100}{9{\cdot}8}=10{\cdot}2$, which is half the time required ; therefore $t=20{\cdot}4$ sec.

9. A stone is thrown vertically upwards with an initial velocity of 100 metres ; after x seconds a second stone is thrown with the same velocity. The second stone is rising 8·7 seconds before it meets the first. What interval separated the throws?

The rising stone will have the velocity $v = V - gt$, whence $v = 100 - 9{\cdot}8 \times 8{\cdot}7$. On the other hand, the falling stone, at the moment the stones meet, will have the velocity given by the equation $v = gt'$, in which t' is the time during which the stone falls before it meets the second one. This time is equal to 8·7 seconds $+ x - \frac{100}{9{\cdot}8}$. Hence its velocity is

$$v = 9{\cdot}8 \left(8{\cdot}7 + x - \frac{100}{9{\cdot}8}\right).$$

Equating the two values of v and reducing, we obtain $x = 3$ seconds.

10. A body moving with a uniformly accelerated motion traverses a space of 1000 metres in 10 seconds. What would be the space traversed during the eighteenth second if the motion continued in the same manner?

The formula $s = \frac{1}{2} gt^2$ gives for the accelerating force $g = 20$ metres per second.

The space traversed during the eighteenth second will be equal to the difference of the space traversed in 18 seconds and that traversed at the end of the seventeenth.

$$x = \frac{20 \times 18^2}{2} - \frac{20 \times 17^2}{2} = 350 \text{ metres.}$$

11. A cannon-ball has been shot vertically upwards with a velocity of 250 metres in a second. After what interval of time would its velocity have been reduced to 54 metres under the retarding influence of gravity, and what space would have been traversed by the ball at the end of this time?

If t be the time, then at the end of each second the initial velocity would be diminished by $9^m{\cdot}8$. Hence we shall have

$$54 = 250 - t \times 9{\cdot}8, \text{ whence } t = 20 \text{ seconds ;}$$

and for the space traversed

$$= 250 \times 20 - \frac{9{\cdot}8 \times 20^2}{2} = 3040 \text{ metres.}$$

12. Required the time in which a body would fall through a height of 2000 metres, neglecting the resistance of the air.

From $s = \frac{1}{2} gt^2$ and substituting the values, we have

$$2000 = \frac{9{\cdot}8}{2} t^2, \text{ whence } t = 20{\cdot}2 \text{ seconds.}$$

13. A body falls in air from a height of 4000 metres. Required the time of its fall and its velocity when it strikes the ground.

From the formula $s = \frac{1}{2} gt^2$ we have for the time $t = \sqrt{\frac{2s}{g}}$; and, on the other hand, from the formula for velocity $v = gt$ we have $t = \frac{v}{g} = \frac{200}{9{\cdot}8} = 20{\cdot}4$.

Hence $\frac{v}{g} = \sqrt{\frac{2s}{g}}$, from which $v = \sqrt{2sg}$, and substituting the values for s and g, $v = 280$ metres.

14. A stone is thrown into a pit 150 metres deep and reaches the bottom in 4 seconds. With what velocity was it thrown, and what velocity had it acquired on reaching the ground? *Ans.* The stone was thrown with a velocity of 17·9, and on reaching the ground had acquired the velocity 57·1.

15. A stone is thrown downwards from a height of 150 metres with a velocity of 10 metres per second. How long will it require to fall?

The distance through which the stone falls is equal to the sum of the distances

through which it would fall in virtue of its initial impulse and of that which it would traverse under the influence of gravity alone; that is, $150 = 10t + \frac{9{\cdot}8\,t^2}{2}$.

Taking the positive value only we get $t = 4{\cdot}61$ seconds.

16. How far will a heavy body fall in vacuo during the time in which its velocity increases from 40·25 feet per second to 88·55 feet per second?

Ans. Taking the value of g at 32·2 feet, the body falls through 96·6 feet.

17. Required the time of oscillation of a single pendulum whose length is 0·9938, and in a place where the intensity of gravity is 9·81.

From the general formula $t = \pi\sqrt{\frac{l}{g}}$, in which t expresses the time of one oscillation, l the length of the pendulum, and g the intensity of gravity, we have

$$t = 3{\cdot}1416\sqrt{\frac{0{\cdot}99384}{9{\cdot}81}} = 1 \text{ second.}$$

18. What is the intensity of gravity in a place in which the length of the seconds pendulum is $0^m{\cdot}991$?

In this case $t = \pi\sqrt{\frac{l}{g'}}$; and also $t = \pi\sqrt{\frac{l'}{g'}}$; and therefore $\frac{l'}{g'} = \frac{l}{g}$, from which $g' = \frac{gl'}{l}$. Substituting in this latter equation the values of g', l and l', we have $g' = 9^m{\cdot}782$.

19. In a place at which the length of the seconds pendulum is 0·99384, t is required to know the length of a pendulum which makes one oscillation in 5 seconds.

In the present case, as g remains the same in the general formula, and t varies, the length l must vary also. We shall have, then,

$$1 : 5 = \pi\sqrt{\frac{l}{g}} : \pi\sqrt{\frac{l'}{g'}}$$

from which, reducing and introducing the values, we have

$$l' = 5^2 \times 0{\cdot}99384 = 24{\cdot}846.$$

20. A pendulum, the length of which is $1^m{\cdot}95$, makes 61,682 oscillations in a day. Required the length of the seconds pendulum. *Ans.* 0·99385 metre.

21. A pendulum clock loses 5 seconds in a day. By how much must it be shortened to keep correct time?

Let s = the number of seconds in one day, and s' the number indicated by the clock, then $s : s' = n : n' = t' : t = \sqrt{l} : \sqrt{l}$ $\therefore$ 86400 : 86395 = 1 : $\sqrt{x}$ $\therefore x = {\cdot}9998843$.

Hence $1 - x = 0{\cdot}00011157$ *Ans.*

22. What is the normal acceleration of a body which traverses a circle of 4·2 metres diameter with a rectangular velocity of 3 metres? *Ans.* 4·286 metres.

23. An iron ball falls from a height of 68 cm. on a horizontal iron plate, and rebounds to a height of 27 cm. Required the coefficient of elasticity of the iron.

If an imperfectly elastic ball with the velocity v strikes against a plate, it rebounds with the velocity $v_1 = -kv$, from which, disregarding the sign, $k = \frac{v_1}{v}$. Now we have the velocity $v_1 = \sqrt{2gh_1}$ and $v = \sqrt{2gh}$, from which $k = \frac{\sqrt{h_1}}{\sqrt{h}}$. Substituting the corresponding values, we get $k = 0{\cdot}63$.

24. Two inelastic bodies, weighing respectively 100 and 200 pounds, strike against each other with velocities of 50 and 20 feet; what is their common velocity, after the impact? *Ans.* 30, or 3·3, according as they move in the same or in opposite directions before impact.

III. ON LIQUIDS AND GASES

25. The force with which a hydraulic press is worked is 20 pounds ; the arm of the lever on which this force acts is 5 times as long as that of the resistance ; lastly, the area of the large piston is 70 times that of the smaller one. Required the pressure transmitted to the large piston.

If F be the power, and p the pressure transmitted to the smaller piston, we have from the principle of the lever $p \times 1 = F \times 5$. Moreover, from the principle of the equality of pressure

$$F \times 1 = p \times 70 = 5 \times 20 \times 70 = 7000 \text{ pounds.}$$

26. The force with which a hydraulic press is worked being 30 kilos and the arm of the lever by which this force is applied being 10 times as long as that of the resistance, and the diameter of the small piston being two centimetres ; find the diameter of the large piston, in order that a pressure of 2000 kilos may be produced.

Ans. 5·164 centimetres.

27. One of the limbs of a U-shaped glass tube contains mercury, to a height of $0^{m}{\cdot}575$; the other contains a different liquid to a height of $0^{m}{\cdot}42$; the two columns being in equilibrium, required the density of the second liquid with reference to mercury and to water.

If d is the density of the liquid as compared with mercury, and d_1 the density compared with water, then $1 \times 0{\cdot}175 = 0{\cdot}42 \times d$; and $13{\cdot}6 \times 0{\cdot}175 = 0{\cdot}42 \times d_1$; whence $d = 0{\cdot}416$ and $d_1 = 5{\cdot}66$.

28. What force would be necessary to support a cubic decimetre of platinum in mercury at zero? Density of mercury 13·6 and that of platinum 21·5.

From the formula $P = VD$ the weight of a cubic decimetre of platinum is $1 \times 21{\cdot}5 = 21^{k}{\cdot}5$ and that of a cubic decimetre of mercury is $1 \times 13{\cdot}6 = 13^{k}{\cdot}6$. From the principle of Archimedes, the immersed platinum loses part of its weight equal to that of the mercury which it displaces. Its weight in the liquid is therefore $21{\cdot}5 - 13{\cdot}6 = 7{\cdot}9$, and this represents the force required.

29. Given a body A which weighs 7·55 grammes in air, 5·17 gr. in water, and 6·35 gr. in another liquid, B ; required from these data the density of the body A and that of the liquid B.

The weight of the body A loses in water $7{\cdot}55 - 5{\cdot}17 = 2{\cdot}38$ grammes ; this represents the weight of the displaced water. In the liquid B it loses $7{\cdot}55 - 6{\cdot}35 = 1{\cdot}2$ gr. ; this is the weight of the same volume of the body B, as that of A and of the displaced water. The specific gravity of A is therefore

$$\frac{755}{238} = 3{\cdot}172, \text{ and that of } B \ \frac{120}{238} = 0{\cdot}504.$$

30. A cube of lead, the side of which is 4 cm., is to be supported in water by being suspended to a sphere of cork. What must be the diameter of the latter, the specific gravity of cork being 0·24, and that of lead 11·35?

The volume of the lead is 64 cubic centimetres ; its weight in air is therefore $64 \times 11{\cdot}35$, and its weight in water $64 \times 11{\cdot}35 - 64 = 662{\cdot}4$ gr.

If r be the radius of the sphere in centimetres, its volume in cubic centimetres will be $\frac{4\pi r^3}{3}$, and its weight in grammes is $\frac{4\pi r^3 \times 0{\cdot}24}{3}$. Now, as the weight of the displaced water is obviously $\frac{4}{3}\pi r^3$ in grammes, there will be an upward buoyancy represented by $\frac{4\pi r^3}{3} - \frac{4\pi r^3 \times 0{\cdot}24}{3} = \frac{4\pi r^3 \times 0{\cdot}76}{3}$ which must be equal to the weight of the lead ; that is, $\frac{4\pi r^3 \times 0{\cdot}76}{3} = 662{\cdot}5$, from which $r = 5^{cm}{\cdot}925$ and the diameter $= 11{\cdot}85$.

31. A cylindrical steel magnet 15 cm. in length and 1·2 mm. in diameter is loaded at one end with a cylinder of platinum of the same diameter and of such a length that when the solid thus formed is in mercury, the free end of the steel projects 10 mm. above the surface. Required the length of this platinum, specific gravity of steel being 7·8 and of platinum 21·5.

The weight of the steel in grammes will be $15\pi r^2 \times 7{\cdot}8$ and of the platinum $x\ r^2 \times 21{\cdot}5$.

These are together equal to the weight of the displaced mercury, which is

$$\pi r^2 (14 + x)\ 13{\cdot}6, \text{ from which } x = 9{\cdot}29 \text{ cm.}$$

32. A cylindrical silver wire $0^m{\cdot}0015$ in diameter weighs 3·2875 grammes; it is to be covered with a layer of gold $0^m{\cdot}0002$ in thickness. Required the weight of the gold, the specific gravity of silver being 10·47 and that of gold 19·26.

If r is the radius of the silver wire and R its radius when covered with gold, then $r = 0^c{\cdot}075$ and $R = 0^c{\cdot}095$. The volume of the silver wire will be $\pi r^2 l$ and its weight $\pi r^2 l\ 10{\cdot}47$, from which $l = 17^c{\cdot}768$.

The volume of the layer of gold is

$$\pi (R^2 - r^2)\ 17{\cdot}768,$$

and its weight

$$\pi (0{\cdot}095^2 - 0{\cdot}075^2) \times 17{\cdot}768 \times 19{\cdot}26 = 3{\cdot}656 \text{ nearly.}$$

33. A kilogramme of copper is to be drawn into wire having a diameter of 0·16 centimetre. What length will it yield? Specific gravity of copper 8·88.

The wire produced represents a cylinder l cm. in length, the weight of which is $\pi r^2 l\ 8{\cdot}88$, and this is equal to 1000 grammes. Hence $l = 56^m{\cdot}0085$.

34. The specific gravity of cast copper being 8·79, and that of copper wire being 8·88, what change of volume does a kilogramme of cast copper undergo in being drawn into wire?

Ans. $\frac{100}{86617}$.

35. Determine the volumes of two liquids, the densities of which are respectively 1·3 and 0·7, and which produce a mixture of three volumes having the density 0·9.

If x and y be the volumes, then from $P = VD$, $1{\cdot}3x + 0{\cdot}7y = 3 \times 0{\cdot}9$ and $x + y = 3$, from which $x = 1$ and $y = 2$.

36. The specific gravity of zinc being 7 and that of copper 9, what weight of each metal must be taken to form 50 grammes of an alloy having the specific gravity 8·2, it being assumed that the volume of the alloy is exactly the sum of the alloyed metals?

Let x = the weight of the zinc, and y that of the copper, then $x + y = 50$, and from the formula $P = VD$, which gives $V = \frac{P}{D}$, the volumes of the two metals and of the alloy are respectively $\frac{x}{7} + \frac{y}{9} = \frac{50}{8{\cdot}2}$. From these two equations we get $x = 17{\cdot}07$ and $y = 32{\cdot}93$.

37. A platinum sphere 3 cm. in diameter is suspended to the beam of a very accurate balance, and is completely immersed in mercury. It is exactly counterbalanced by a copper cylinder of the same diameter completely immersed in water. Required the height of the cylinder. Specific gravity of mercury 13·6, of copper 8·8, and of platinum 21·5.

Ans. 2·025 centimetres.

38. To balance an ingot of platinum 27 grammes of brass are placed in the other pan of the balance. What weight would have been necessary if the weighing had been effected in vacuo? The density of platinum is 21·5, that of brass 8·3, and air under a pressure of 760 mm. and at the temperature 0° has $\frac{1}{770}$ the density of water.

The weight of brass in air is not 27 grammes, but this weight minus the weight of a volume of air equal to its own.

Since $P = VD$ $\therefore V = \frac{P}{D}$ and the weight of the air is $\frac{P}{D \times 770} = \frac{27}{8{\cdot}3 \times 770}$.

By similar considerations, if x is the weight of platinum in vacuo, its weight in air

will be x minus the weight of air displaced, that is $x - \frac{x}{21{\cdot}5 \times 770}$, and this weight is equal to that of the true weight of the brass; and we have

$$x - \frac{x}{21{\cdot}5 \times 770} = 27 - \frac{27}{8{\cdot}3 \times 770}; \text{ from which } x = 26{\cdot}996.$$

39. A body loses in carbonic acid 1·15 gr. of its weight. What would be its loss of weight in air and in hydrogen respectively?

Since a litre of air at 0° and 760 mm. weighs 1·293 gramme, the same volume of carbonic acid weighs 1·293 × 1·524 = 1·97 gramme. We shall, therefore, obtain the volume of carbonic acid corresponding to 1·15 gr. by dividing this number by 1·97, which gives 0·5837 litre. This being then the volume of the body, it displaces that volume of air, and therefore its loss of weight in air is 0·5837 × 1·293 = 0·7547 gramme, and in hydrogen 0·5837 × 1·293 × 0·069 = 0·052076.

40. Calculate the ascensional force of a spherical balloon of oiled silk which, when empty, weighs 62·5 kilos, and which is filled with impure hydrogen, the density of which is $\frac{1}{13}$ that of air. The oiled silk weighs 0·250 kilo the square metre.

The surface of the balloon is $\frac{62{\cdot}5}{0{\cdot}25} = 250$ square metres. This surface being that of a sphere, is equal to $4\pi R^2$, whence $4\pi R^2 = 250$ and $R = 4{\cdot}459$; therefore $V = \frac{4\pi R^3}{3}$ = 371·52 cubic metres.

The weight of air displaced is 371·52 × 1·293 kilo = 480·375 kilos; the weight of the hydrogen is 36·88 kilos, and therefore the ascensional force is

$$480{\cdot}375 - (36{\cdot}88 + 62{\cdot}5) = 380{\cdot}995.$$

41. A balloon 4 metres in diameter is made of the same material and filled with the same hydrogen as above. How much hydrogen is required to fill it, and what weight can it support?

The volume is $\frac{4}{3}\pi R^3 = 33{\cdot}51$ cubic metres, and the surface $4\pi R^2 = 50{\cdot}265$ square metres. The weight of the air displaced is 33·51 × 1·293 = 43·328 kilos, and that of the hydrogen is from the above data 3·333 kilos, while the weight of the material is 12·566 kilos. Hence the weight which the balloon can support is

$$43{\cdot}328 - (12{\cdot}566 + 3{\cdot}333) = 27{\cdot}429 \text{ kil.}$$

42. Under the receiver of an air-pump is placed a balance, to which are suspended two cubes; one of these is 3 centimetres in the side, and weighs 26·324 gr.; and the other is 5 centimetres in the side, and weighs 26·2597 grammes. When a partial vacuum is made these cubes just balance each other. What is the pressure? *Ans.* 0^{m}·374.

43. A soap-bubble 8 centimetres in diameter was filled with a mixture of one volume of hydrogen gas and 15 volumes air. The bubble just floated in the air; required the thickness of the film.

The weight of the volume of air displaced is $\frac{4}{3}\pi r^3 \times 0{\cdot}001293$ gramme, and that of the mixture of gases $\frac{4}{3}\pi r^3 \times 0{\cdot}001293 \times \frac{15 + 0{\cdot}0693}{16}$; and the difference of these will equal the weight of the soap-bubble.

This weight is that of a spherical shell, which, since its thickness t is very small, is with sufficient accuracy $4\pi r^2 t s$ in grammes, where s is the specific gravity = 1·1. Hence

$$\frac{4}{3}\pi r^3 \left(0{\cdot}001293 - 0{\cdot}001293 \times \frac{15{\cdot}0693}{16}\right) = 4\pi r^2 t \, 1{\cdot}1.$$

Dividing each side by $\frac{4}{3}\pi r^2$, and putting $r = 4$, we get

$$4 \times 0{\cdot}001293 \left(1 - \frac{15{\cdot}0693}{16}\right) = 3{\cdot}3t;$$

or

$$\cdot 001293 \times \frac{\cdot 9307}{4} = 3\cdot 3\, t :$$

whence $t = \cdot 00009116629$ cm.

44. In a vessel whose capacity is 3 litres, there are introduced 2 litres of hydrogen under the pressure of 5 atmospheres; 3 litres of nitrogen under the pressure of half an atmosphere, and 4 litres of carbonic acid under the pressure of 4 atmospheres. What is the final pressure of the gas, the temperature being supposed constant during the experiment?

The pressure of the hydrogen, from Dalton's law, will be $\frac{2 \times 5}{3}$, that of the nitrogen will remain unchanged, and that of the carbonic acid will be $\frac{4 \times 4}{3}$. Hence the total pressure will be

$$\frac{10}{3} + \frac{1}{2} + \frac{16}{3} = 9\tfrac{1}{6} \text{ atmospheres.}$$

45. A vessel containing 10 litres of water is first exposed in contact with oxygen under a pressure of 78 cm. until the water is completely saturated. It is then placed in a confined space containing 100 litres of carbonic acid under a pressure of 72 cm. Required the volumes of the two gases when equilibrium is established. The coefficient of absorption of oxygen is 0·042, and that of carbonic acid unity.

The volume of oxygen dissolved is 0·42. Being placed in carbonic acid it will act as if it alone occupied the space of the carbonic acid, and its pressure will be

$$78 \times \frac{0\cdot 42}{100\cdot 42} = 0\cdot 326 \text{ cm.}$$

Similarly the 10 litres of water will dissolve 10 litres of carbonic acid gas, the total volume of which will be 110, of which 100 are in the gaseous state and 10 are dissolved. Its pressure is therefore $72 \times \frac{100}{110} = 65\cdot 454$ cm.

Hence the total pressure when equilibrium is established is

$$0\cdot 326 + 65\cdot 454 = 65\cdot 78 \text{ cm.};$$

and the volume of the oxygen dissolved reduced to the pressure 65·78 is

$$0^{\text{litre}}\cdot 42 \times \frac{0\cdot 326}{65\cdot 78} = 0^{\text{litre}}\cdot 00208,$$ and that of the carbonic acid $10 \times \frac{65\cdot 454}{65\cdot 78} = 9\cdot 95$.

46. In a barometer which is immersed in a deep bath the mercury stands 743 mm. above the level of the bath. The tube is lowered until the barometric space, which contains air, is reduced to one-third, and the mercury is then at a height of 701 mm. Required the atmospheric pressure at the time of observation. *Ans.* $= 764^{\text{mm}}$.

47. What is the pressure on the piston of a steam boiler of 8 decimetres diameter if the pressure in the boiler is 3 atmospheres? *Ans.* 10385·85 kilos.

48. What is the pressure of the atmosphere at that height at which an ascent of 21 metres corresponds to a diminution of 1^{mm} in the barometric height? *Ans.* $378\cdot 9^{\text{mm}}$.

49. What would be the height of the atmosphere if its density were everywhere uniform? *Ans.* 7954·1 metres, or nearly 5 miles.

50. How high must we ascend at the sea-level to produce a depression of 1 mm. in the height of the barometer?

Ans. Taking mercury as 10,500 times as heavy as air, the height will be 10·5 metres.

51. Mercury is poured into a barometer tube so that it contains 15 cc. of air under the ordinary atmospheric pressure. The tube is then inverted in a mercury bath and the air then occupies a space of 25 cc.; the mercury occupying a height of 302 mm. What is the pressure of the atmosphere?

Let x be the amount of this pressure, the air in the upper part of the tube will have a pressure represented by $\frac{15x}{25}$, and this, together with the height of the mercurial column 302, will be the pressure exerted in the interior of the tube on the level of the

mercury in the bath, which is equal to the atmospheric pressure ; that is $\frac{15x}{25} + 302 = x$, from which $x = 755$ mm.

52. What effort is necessary to support a cylindrical bell-jar full of mercury immersed in mercury ; its internal diameter being 6 centimetres, its height *ob* above the surface of the mercury (fig. 1) 18 centimetres, and the pressure of the atmosphere 0·77 centimetre?

The bell-jar supports on the outside a pressure equal to that of a column of mercury the section of whose base is *cd*, and the height that of the barometer. This pressure is equal to

$$\pi R^2 \times 0{\cdot}77 \times 13{\cdot}6.$$

The pressure on the inside is that of the atmosphere less the weight of a column of mercury whose base is *cd* and height *ob*. This is equal to $\pi R^2 \times (0{\cdot}77 - 0{\cdot}18) \times 13{\cdot}6$; and the effort necessary is the difference of these two pressures. Making $R = 3$ cm., this is found to be 69·216 kilogrammes.

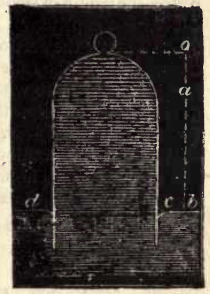

Fig. 1.

53. A barometer is placed within a tube which is afterwards hermetically closed. At the moment of closing, the temperature is 15° and the pressure 750 mm. The external space is then heated to 30°. What will be the height of the barometer?

The effect of the increase of temperature would be to raise the mercury in the tube in the ratio $1 + \frac{30}{5550}$ to $1 + \frac{15}{5550}$, and the height h would therefore be

$$= \frac{75\left(1 + \frac{30}{5550}\right)}{1 + \frac{15}{5550}}$$

and since in the closed space the elastic force of the air increases in the ratio $1 + 30\alpha : 1 + 15\alpha$, we shall have finally $h = 301{\cdot}74$ mm.

54. The heights of two barometers A and B have been observed at − 10° and + 15°, respectively, to be $A = 737$ and $B = 763$. Required their corrected heights at 0°. *Ans.* $A = 738{\cdot}33$. $B = 760{\cdot}94$.

55. A voltaic current gives in an hour 840 cubic centimetres of detonating gas under a pressure of 760 and at the temperature 12°·5 ; a second voltaic current gives in the same time 960 cubic centimetres under a pressure of 755 and at the temperature 15°·5. Compare the quantities of gas given by the two currents. *Ans.* 1 : 1·129.

56. The volume of air in the pressure gauge of an apparatus for compressing gases is equal to 152 parts. By the working of the machine this is reduced to 7 parts, and the mercury is raised through 0·48 metre. What is the pressure of the gas?

Here $AB = 152$, $AC = 37$ parts, and $BC = 0^{m}{\cdot}48$. The pressure of air therefore in AC is, from Boyle's law,

$$\frac{152}{37} = 4^{atm}{\cdot}108 = 3^{m}{\cdot}122.$$

The pressure in the receiver is therefore

$$3{\cdot}122 + 0{\cdot}48 = 3^{m}{\cdot}602,$$

which is equal to 4·74 atmospheres.

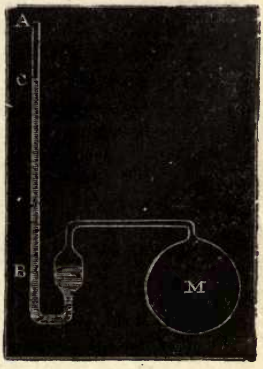

Fig. 2.

57. An airtight bladder holding two litres of air at the standard pressure and temperature is immersed in sea-water to a depth of 100 metres, where the temperature is 4°. Required the volume of the gas.

The specific gravity of sea-water being 1·026, the depth of 100 metres will represent a column of pure water 102·6 metres in height. As the pressure of an atmosphere is equal to a pressure of 10·33 metres of pure water, the pressure of this column

$$= \frac{102{\cdot}68}{10{\cdot}33} = 9{\cdot}94 \text{ atm.}$$

Hence, adding the atmospheric pressure, the bladder is now under a pressure of 10·94 atmospheres, and its volume being inversely as the pressure will be $\frac{2}{10{\cdot}94} = 0{\cdot}183$ litre, if the temperature be unaltered. But the temperature is increased by 4°, and therefore the volume is increased in the ratio 277 to 273, and becomes

$$0{\cdot}183 \times \frac{277}{273} = 0{\cdot}18568 \text{ litre.}$$

58. To what height will water be raised in the tube of a pump by the first stroke of the piston, the length of stroke of which is 0·5m., the height of the tube 6 metres, and its section $\frac{1}{10}$ that of the piston? At starting the air in the tube is under a pressure of 10 metres.

If we take the section of the tube as unity, that of the body of the pump is 10; and the volumes of the tube and of the body of the pump are in the ratio of 6 to 5. Then if x is the height to which the water is raised in the pipe, the volumes of air in the pump before and after the working of the pump are 6 at the pressure 10, and $5 + 6 - x$ at the pressure $10 - x$.

Forming an equation from these terms, and solving, we have two values, $x' = 18^{m}\,26$ and $x'' = 2{\cdot}74$. The first of these must be rejected as being physically impossible; and the true height is $x = 2{\cdot}75$ metres.

59. A receiver with a capacity of 10 litres contains air under the pressure 76 cm. It is closed by a valve, the section of which is 32 square centimetres, and is weighted with 25 kilogrammes. The temperature of the air is 30°; its density at 0° and 76 cm. pressure is $\frac{1}{773}$ that of water. The coefficient of the expansion of gases is 0·00366. Required the weight of air which must be admitted to raise the valve.

The air already present need not be taken into account as it is under the pressure of the atmosphere. Let x be the pressure in centimetres of mercury of that which is admitted, $\frac{x \times 13{\cdot}6}{1000}$ will represent in kilogrammes its pressure on a square centimetre; and therefore the internal pressure on the valve, and which is equal to the external pressure of 25 kilogrammes, is $\frac{x \times 13{\cdot}6 \times 32}{1000} = 25\,\text{k}$. From which $x = 57{\cdot}44$.

For the weight we shall have

$$P = \frac{10 \times 0{\cdot}001293}{1 + 0{\cdot}00366 \times 30} \times \frac{57{\cdot}44}{76{\cdot}00} = 8{\cdot}8055 \text{ grammes.}$$

60. A bell-jar contains 3·17 litres of air; a pressure gauge connected with it marks zero when in contact with the air (fig. 3). The jar is closed and the machine worked; the mercury rises to 65 cm. A second barometer stands at 76 cm. during the experiment. Required the weight of air withdrawn from the bell-jar and the weight of that which remains.

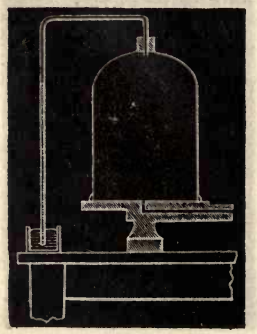

Fig. 3.

At 0° and 76 cm. the weight of air in the bell-jar is

$$1{\cdot}293 \times 3{\cdot}17 = 4{\cdot}09881.$$

At 0° and under the pressure $76 - 65$ the weight of the residual air is

$$\frac{4{\cdot}09881 \times 11}{76} = 0{\cdot}5932,$$

and therefore the weight of that which is withdrawn is

$$4{\cdot}0988 - 0{\cdot}5932 = 3{\cdot}5056 \text{ gr.}$$

61. The capacity of the receiver of an air-pump

is 7·53; it is full of air under the ordinary atmospheric pressure and at 0°. Required the weight of air when the pressure is reduced to 0·21; the weight withdrawn by the piston; and the weight which would be left at 15°.

The weight of 7·53 litres of air under the ordinary conditions is 9·736 grammes.

Under a pressure of 0·21 it will be 2·69 grammes, and at the temperature 15° it will be $\frac{2{\cdot}69}{1 + 0{\cdot}00366 \times 15} = 0{\cdot}255$ gramme.

62. In a theoretically perfect air-pump, how great is the rarefaction after 10 strokes, if the volumes of the barrel and the receiver are respectively 2 and 3?

Ans. = 4·59^{mm}; or about $\frac{1}{166}$ of an atmosphere.

63. What must be the capacity of the barrel of an air-pump if the air in a receiver of 4 litres is to be reduced to $\frac{1}{3}$ the density in two strokes? *Ans.* 2·9.

64. The reservoir of an air-gun, the capacity of which is 40 cubic inches, contains air whose density is 8 times that of the mean atmospheric pressure. A shot is fired when the atmospheric pressure is 741 mm. and the gas which escapes occupies a volume of 80 cubic inches. What is the elastic force of the residual air? *Ans.* 6·05 atmospheres.

65. Suppose that at the limit of the atmosphere the pressure of the attenuated air is the $\frac{1}{1000}$ of a millimetre of mercury and the temperature − 135°, and that in a place at the sea-level, in latitude 45°, the pressure of the atmosphere is 760^{mm} and its temperature 15° C. Determine from these data the height of the atmosphere.

From the formula $18400 \left\{ 1 + 0{\cdot}002\{T + t\} \right\} \log \frac{H}{H_1}$, we get for the height in metres 82237, which is equal to 51·1 miles.

66. If water is continually flowing through an aperture of 3 square inches with a velocity of 10 feet, how many cubic feet will flow out in an hour? *Ans.* 750 cubic feet.

67. With what velocity does water issue from an aperture of 3 square inches, if 37·5 cubic feet flow out every minute? *Ans.* 30 feet.

68. What is the ratio of the pressure in the above two cases? *Ans.* 1 : 9.

69. What is the theoretical velocity of water from an aperture which is 9 feet below the surface of water? *Ans.* 24 feet.

70. In a cylinder, water stands 2 feet above the aperture and is loaded by a piston which presses with a force of 6 pounds on the square inch. Required the velocity of the effluent water. *Ans.* 32 feet.

71. How deep must the aperture of the longer leg of a syphon, which has a section of 4 square centimetres, be below the surface of the water in order that 25 litres may flow out in a minute? *Ans.* 5·535 cm.

72. Through a *circular* aperture having an area of 0·196 square cm. in the bottom of a reservoir of water which was kept at a constant level, 55 cm. above the bottom, it was found that 98·5 grammes of water flowed in 22 seconds. Required the coefficient of efflux.

Since the velocity of efflux through an aperture in the bottom of a vessel is given by the formula $v = \sqrt{2gh}$, it will readily be seen that the weight in grammes of water which flows in a given time, t, will be given by the formula $w = a\,\alpha\,t\sqrt{2gh}$, where a is the area in square centimetres, α the coefficient of efflux, t the time in seconds, and h the height in centimetres. Hence in this case $\alpha = 0{\cdot}699$.

73. Similarly through a *square* aperture, the area of which was almost exactly the same as the above, and at the same depth, 104·4 grammes flowed out in 21·6 seconds. In this case $\alpha = 0{\cdot}73$.

IV. ON SOUND

74. A stone is dropped into a well, and 4 seconds afterwards the report of its striking the water is heard. Required the depth, knowing that the temperature of the air in the pit was 10°·74.

From the formula $v = 333\sqrt{1 + at}$ we get for the velocity of sound at the temperature in question 339·05 metres.

Let t be the time which the stone occupies in falling; then $\frac{1}{2}gt^2 = x$ will represent the depth of the well; on the other hand, the time occupied by the report will be $4 - t$, and the distance will be $(4 - t)v = x$ (i); thus $(4 - t)v = \frac{1}{2}gt^2$ (ii), from which, substituting the values,

$$(4 - t)\,339{\cdot}5 = 4{\cdot}9\,t^2$$

$t = 3{\cdot}793$ seconds, and substituting this value in either of the equations (i) or (ii), we have the depth $= 72{\cdot}6$ metres nearly.

75. A bullet is fired from a rifle with a velocity of 414 metres, and is heard to strike a target 4 seconds afterwards. Required the distance of the target from the marksman, the temperature being assumed to be zero.

$$\frac{x}{414} + \frac{x}{333} = 4;\ x = 738{\cdot}2.$$

76. At what distance is an observer from an echo which repeats a sound after 3 seconds, the temperature of the air being 10°?

In these 3 seconds the sound traverses a distance of $3 \times 339 = 1017$ metres; this distance is twice that between the observer and the reflecting surface; hence the distance is

$$\frac{1017}{2} = 508{\cdot}5 \text{ metres.}$$

77. Between a flash of lightning and the moment at which the corresponding thunder is first heard, the interval is the same as that between two beats of the pulse. Knowing that the pulse makes 80 beats in a minute, and assuming the temperature of the air to be 15° C., what is the distance of the discharge? *Ans.* 454·1 metres.

78. A stone is thrown into a well with a velocity of 12 metres, and is heard to strike the water 4 seconds afterwards. Required the depth of the well.

Ans. About 110 metres.

79. What is the velocity of sound in coal gas at 0°, the density being 0·5?

Ans. 470·9 metres.

80. What must be the temperature of air in order that sound may travel in it with the same velocity as in hydrogen at 0°? *Ans.* About 3680° C.

81. What must be the temperature of air in order that the velocity of sound may be the same as in carbonic acid at 0°? *Ans.* $-105°{\cdot}5$ C.

82. Kendall, in a North Pole Expedition, found the velocity of sound at $-40°$ was 314 m. How closely does this agree with that calculated from the value we have assumed for 0°? *Ans.* 6·64 metres too much.

83. The report of a cannon is heard 15 seconds after the flash is seen. Required the distance of the cannon, the temperature of the air being 22°.

From the formula for the velocity of sound we have

$$15 \times 333\sqrt{1 + 0{\cdot}003665 \times 22} = 5190 \text{ metres.}$$

84. If a bell is struck immediately at the level of the sea, and its sound, reflected from the bottom, is heard 3 seconds after, what is the depth of the sea?

Ans. 7140 feet.

85. A person stands 150 feet on one side of the line of fire of a rifle range 450 feet in length and at right angles to a point 150 feet in front of the target. What is the velocity of the bullet if the person hears it strike the target $\frac{1}{9}$ of a second later than the report of the gun? The temperature is assumed to be 16°·5. *Ans.* 2038 feet.

86. An echo repeats five syllables, each of which requires a quarter of a second to pronounce, and half a second elapses between the time the last syllable is heard and the first syllable is repeated. What is the distance of the echo, the temperature of the air being 10° C.? *Ans.* 297·47 metres.

87. The note given by a silver wire a millimetre in diameter and a metre in length being the middle C, what is the tension of the wire? Density of silver 10·47. *Ans.* 22·67 kilogrammes.

88. The density of iron being 7·8 and that of copper 8·8, what must be the thickness of wires of these materials, of the same length and equally stretched, so that they may give the same note?

From the formula for the transverse vibration of strings we have for the number of vibrations $n = \frac{1}{rl}\sqrt{\frac{P}{\pi d}}$. As in the present case, the tensions, the length of the strings, and the number of vibrations are the same, we have

$$\frac{1}{rl}\sqrt{\frac{P}{\pi d}} = \frac{1}{r_1 l}\sqrt{\frac{P}{\pi d_1}}, \text{ from which } \frac{1}{r}\sqrt{\frac{1}{d}} = \frac{1}{r_1}\sqrt{\frac{1}{d_1}};$$

$$\text{whence } \frac{r^2}{r_1^2} = \frac{d'}{d} = \frac{8\cdot8}{7\cdot8}; \text{ hence } \frac{r}{r_1} = \sqrt{\frac{8\cdot8}{7\cdot8}} = 1\cdot062.$$

89. A wire stretched by a weight of 13 kilos sounds a certain note. What must be the stretching weight to produce the major third?

The major third having $\frac{5}{4}$ the number of vibrations of the fundamental note, and as, all other things being the same, the numbers of vibrations are directly as the square roots of the stretching weight, we shall have $x = 20\cdot312$ kilos.

90. The diameters of two wires of the same length and material are 0·0015 and 0·0038 m.; and their stretching weights 400 and 1600 grammes respectively. Required the ratio of the numbers of their vibrations. *Ans.* $n : n_1 = 1\cdot266 : 1$.

91. A brass wire 1 metre in length stretched by a weight of 2 kilogrammes, and a silver wire of the same diameter, but 3·165 metres in length, give the same number of vibrations. What is the stretching weight in the latter case?

Since the number of vibrations is equal, we shall have

$$\frac{1}{rl}\sqrt{\frac{P}{\pi d}} = \frac{1}{rl_1}\sqrt{\frac{x}{\pi d_1}};$$

from which, replacing the numbers, we get $x = 25$ kilos.

92. A brass and a silver wire of the same diameter are stretched by the weights of 2 and 25 kilogrammes respectively, and produce the same note. What are their lengths, knowing that the density of brass is 8·39, and of silver 10·47?

Ans. The length of the silver wire is 3·16 times that of the brass.

93. A copper wire 1·25 mm. in diameter and a platinum one of 0·75 mm. are stretched by equal weights. What is the ratio of their lengths, if, when the copper wire gives the note C, the platinum gives F on the diatonic scale?

Ans. The length of the copper is to the length of the platinum = 1·264 : 1.

94. An organ pipe gives the note C at a temperature 0°; at what temperature will it yield the major third of this note? *Ans.* 153° C.

95. A brass wire a metre in length, and stretched by a weight of a kilogramme, yields the same note as a silver wire of the same diameter but 2·5 metres in length and stretched by a weight of 7·5 kilogrammes. Required the specific gravity of the silver. *Ans.* 10·068.

96. How many beats are produced in a second by two notes, whose rates of vibration are respectively 340 and 354? *Ans.* 14.

V. ON HEAT

97. Two mercurial thermometers are constructed of the same glass; the internal diameter of one of the bulbs is $7^{mm}\cdot5$ and of its tube $2\cdot5$; the bulb of the other is $6\cdot2$ in diameter and its tube $1\cdot5$. What is the ratio of the length of a degree of the first thermometer to a degree of the second?

Let A and B be the two thermometers, D and D' the diameters of the bulbs, and d and d' the diameters of the tubes. Let us imagine a third thermometer C with the same bulb as B and the same tube as A, and let l, l', and l'' denote the length of a degree in each of the thermometers respectively. Since the stems of A and C have the equal diameters, the lengths l and l'' are directly as the volumes of the tubes, or what is the same, as the cubes of their diameters; and as B and C have the same bulk, the lengths l' and l'' are inversely proportionate to the sections of the stems, or what amounts to the same, to the squares of their diameters. We have then

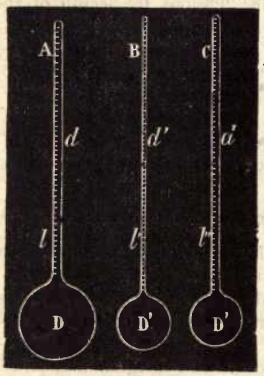

Fig. 4.

$$\frac{l}{l''} = \frac{D^3}{D'^3} \text{ and } \frac{l'}{l'} = \frac{d'^2}{d^2};$$

introducing the values and solving, we have

$$\frac{l}{l''} = 0\cdot638.$$

98. At what temperature is the number on the Centigrade and Fahrenheit thermometers the same?

Ans. $-40°$.

99. The same question for the Fahrenheit and Réaumur scales. *Ans.* $-25\cdot6$.

100. A capillary tube is divided into 180 parts of equal capacity, 25 of which weigh $1\cdot2$ gramme. What must be the radius of a spherical bulb to be blown to it so that 180 divisions correspond to 150 degrees Centigrade?

Since 25 divisions of the tube contain $1\cdot2$ gramme, 180 divisions contain $\frac{1\cdot2 \times 180}{25} = 8\cdot64$. And since these 180 divisions are to represent 150 degrees, the weight of mercury corresponding to a single degree is $\frac{8\cdot64}{150}$. But as the expansion corresponding to one degree is only the apparent expansion of mercury in glass, the weight $\frac{8\cdot64}{150}$ is $\frac{1}{6480}$ of the mercury in the reservoir, which is $\frac{4}{3}\pi R^3$. From this $R = 1\cdot8755$ centimetre.

101. By how much is the circumference of an iron wheel, whose diameter is 6 feet, increased when its temperature is raised 400 degrees? Coefficient of expansion of iron $= 0\cdot0000122$. *Ans.* By $0\cdot092$ foot.

102. What must be the length of a wire of this metal which for a temperature of $1°$ expands by one foot? *Ans.* 81967 feet.

103. A pendulum consists of a platinum rod, on a flattening at the end of which rests a spherical zinc bob. The length of the platinum is l at $0°$. What must be the diameter of the bob, so that its centre is always at the same distance from the point of suspension whatever be the temperature? Coefficient of expansion of platinum $0\cdot0000088$ and of zinc $0\cdot0000294$.

Ans. The diameter of the bob must be $\frac{3}{5}$ of the length of the platinum.

104. Two walls, which when perpendicular are 30 feet apart, have bulged outwards to the extent of $2\cdot4$ inches. They are to be made perpendicular by the contrac-

tion of an iron bar. By how much must its temperature be raised above that of the air, which is taken at 0°? *Ans.* 546·4.

105. An iron wire 4 sq. mm. in cross section is stretched $\frac{1}{81200}$ of its length by a weight of 1 kilogramme. What weight must be applied to a bar 9 sq. mm. in cross section, when it is heated from 0° to 20°, in order to prevent it from expanding? *Ans.* 44·5 kilos.

106. At the temperature zero a solid is immersed 0·975 of its total volume in alcohol. At the temperature 25° the solid is wholly immersed. The coefficient of expansion of the solid being 0·000026, required the coefficient of expansion of the alcohol. *Ans.* 0·001052.

107. Into a glass globe, the capacity of which at 0° is 250 cc., are introduced 25 cc. of air measured at 0° and 76 cm. The flask being closed and heated to 100°, required the internal pressure. Coefficient of cubical expansion of glass $\frac{1}{38700}$.

At 100° the capacity of the flask is $250\left(1 + \frac{100}{38700}\right)$; again at 100° the volume of the free air under the pressure 76 is 25 (1 + 100 × 0·00366). But its real volume is $250 \times \frac{388}{387}$ under a pressure x. Hence

$$76 : x = 250 \times \frac{388}{387} : 25 \times 1{\cdot}366,$$ from which $x = 10{\cdot}3548$ cm.

108. The specific gravity of mercury at 0° being 13·6, required the volume of 3 kilogrammes at 85°. Coefficient of expansion $\frac{1}{5550}$.

The volume at 0° will be $\frac{3}{13{\cdot}6}$ and at 85° $\frac{3}{13{\cdot}6} \times \left(1 + \frac{85}{5550}\right) = 0{\cdot}2239$ litre.

109. A hollow copper sphere 20 cm. in diameter is filled with air at 0° under a pressure of 1½ atmosphere; what is the total pressure on the interior surface when the enclosed air is heated to a temperature of 600°? *Ans.* 6226·5 kilogrammes.

110. Between the limits of pressure 700 to 780 mm. the boiling-point of water varies 0°·0375 C. for each mm. of pressure. Between what limits of temperature does the boiling point vary, when the height of the barometer is between 735 and 755 mm.? *Ans.* Between 99°·0625 and 99°·8125.

111. Liquid phosphorus cooled down to 30°, is made to solidify at this temperature. Required to know if the solidification will be complete, and if not, what weight will remain melted? The melting point of phosphorus is 44·2; its latent heat of fusion 5·4, and its specific heat 0·2.

Let x be the weight of phosphorus which solidifies; in so doing it will give out a quantity of heat = 5·4 x; this is expended in raising the whole weight of the phosphorus from 30 to 44·2. Hence we have $5{\cdot}4x = 1 \times (44{\cdot}2 - 30)\ 0{\cdot}2$, from which $x = \frac{2{\cdot}84}{5{\cdot}4} = 0{\cdot}526$, so that 0·474 of phosphorus will remain liquid.

112. A pound of ice at 0° is placed in two pounds of water at 0°; required the weight of steam at 100° which will melt the ice and raise the temperature of the mixture to 30°. The latent heat of the liquefaction of ice is 79·2, and that of the vaporisation of water 536. *Ans.* ·279 pound.

113. 65·5 grammes of ice at − 20° having been placed in x grammes of oil of turpentine at 13·3°, the final temperature is found to be 3·1°. The specific heat of turpentine is 0·4, and it is contained in a vessel weighing 25 grammes, whose specific heat is 0·1. The specific heat of ice is 0·5. Required the value of x.

Ans. x = 1475 grammes.

114. In what proportion must water at a temperature of 30° and linseed oil (sp. heat = 0·5) at a temperature of 50° be mixed so that there are 20 kilogrammes of the mixture at 40°? *Ans.* Water = 6·66 kilos and linseed oil = 13·34.

115. By how much will mercury at 0° be raised by an equal volume of water at 100°? *Ans.* 68°·9 C.

116. The specific heat of gold being 0·03244, what weight of it at 45° will raise a kilogramme of water from 12°·3 to 15°·7?

Let x be the weight sought; then x kilogrammes of gold in sinking from 45° to 15°·7 will give out a quantity of heat represented by x (45° − 15°·7) 0·0324, and this is equal to the heat gained by the water, that is to 1 (15·7 − 12·3) = 3·4, that is $x = 3{\cdot}58$.

117. The specific heat of sulphide of copper is 0·1212, and that of sulphide of silver 0·0746. 5 kilos of a mixture of these two bodies at 40°, when immersed in 6 kilos of water at 7·669°, raises its temperature to 10°. How much of each sulphuret did the mixture contain?

The weight of the copper sulphuret = 2, and that of the silver sulphuret 3.

118. Into a mass of water at 0°, 100 grammes of ice at − 12° are introduced; a weight of 7·2 grammes of water at 0° freezes about the lump immersed, while its temperature rises to zero. Required the specific heat of ice. Latent heat of water 79·2. *Ans.* 0·4752.

119. Four pounds of copper filings at 130° are placed in 20 pounds of water at 20°, the temperature of which is thereby raised 2 degrees. What is the specific heat, c, of copper? *Ans.* c = 0·0926.

120. Two pieces of metal weighing 300 and 350 grammes, heated to a temperature x, have been immersed, the former in 3351·6 grammes of water at 10°, and the latter in 1935·4 grammes at the same temperature. The temperature in the first case rises to 20°, and in the second to 30°. Required the original temperature and the specific heat of the metal. *Ans.* x the temperature = 1000°; c the specific heat = 0·114.

121. In what proportions must a kilogramme of water at 50° be divided in order that the heat which one portion gives out in cooling to ice at zero may be sufficient to change the other into steam at 100°? *Ans.* x = 0·8203.

122. Three mixtures are formed by mixing two and two together, equal quantities of ice, salt, and water at 0°. Which of these mixtures will have the highest and which the lowest temperature? *Ans.* The mixture of ice and salt will produce the lowest temperature, while that of ice and water will produce no lowering of temperature.

123. In 25·45 kilogrammes of water at 12°·5 are placed 6·17 kilos of a body at a temperature of 80°; the mixture acquires the temperature 14°·1. Required the specific heat of the body.

If c is the specific heat required, then $mc\ (t' - \theta)$ represents the heat lost by the body in cooling from 80° to 14°·1; and that absorbed by the water in rising from 12°·5 to 14°·1 is $m'\ (\theta - t)$. These two values are equal. Substituting the numbers, we have c = 0·10014.

124. Equal lengths of the same thin wire traversed by the same electrical current are placed respectively in 1 kilogramme of water and in 3 kilogrammes of mercury. The water is raised 10° in temperature; by how much will the mercury be raised?

Ans. 100°·04.

125. How many cubic feet of air under constant pressure are heated through 1° C. by one thermal unit? *Ans.* 55·3 cubic feet.

126. Given two pieces of metal, one x weighing 2 kilos heated to 80°, and the other y weighing 3 kilos, and at the temperature 50°. To determine their specific heats they are immersed in a kilogramme of water at 10°, which is thereby raised to 26°·3.

The experiment is repeated, the two metals being at the temperature 100° and 40° respectively, and, as before, they are placed in a kilogramme of water at 10°, which this time is raised to 28°·4. Required the specific heats of the two metals.

Ans. x = 0·115; y = 0·0555.

127. For high temperatures the specific heat of iron is 0·1053 + 0·000017 t. What is the temperature of a red-hot iron ball weighing a kilogramme, which, plunged in 16

kilogrammes of water, raises its temperature from 12° to 24°? What was the temperature of the iron?

$$(0{\cdot}1053 + 0{\cdot}000017\,t)\,(t - 24) = 16\,(24 - 12),$$

or
$$\cdot000017\,t^2 + \cdot1048892\,t - 2{\cdot}5272 = 192;$$

transposing and dividing by the coefficient of t^2, we get

$$t^2 + 6170\,t = 11442776,$$

$$t^2 + 6170\,t + (3085)^2 = 20960001;$$

hence
$$t + 3085 = 4578{\cdot}3 \text{ nearly}; \quad \therefore\ t = 1493{\cdot}3.$$

128. A kilogramme of the vapour of alcohol at 80° passes through a copper worm placed in 10·8 kilogrammes of water at 12°, the temperature of which is thereby raised to 36°. The copper worm and copper vessel in which the water is contained weigh together 3 kilogrammes. Required the latent heat of alcohol vapour. *Ans.* 238·77.

129. Determine the temperature of combustion of charcoal in burning to form carbonic acid.

We know from chemistry that one part by weight of carbon in burning unites with $2\frac{2}{3}$ parts by weight of oxygen to form $3\frac{2}{3}$ parts by weight of carbonic acid. Again the number of thermal units produced by the combustion of a pound of charcoal is 8080; the whole of this heat is contained in the $3\frac{2}{3}$ parts of carbonic acid produced, and if its specific heat were the same as that of water, its temperature would be $\frac{8080}{3\frac{2}{3}} = 2204°$ C.; but since the specific heat of carbonic acid is 0·2163 that of an equal weight of water, the temperature will be $\frac{2204}{0{\cdot}2163} = 10189°$ C.

130. A glass globe measuring 60 cubic centimetres is found to weigh 19·515 grammes when filled with air under a pressure of 752·3^{mm} and at a temperature of 10° C. Some ether is introduced and vaporised at a temperature of 60°, whereupon the flask is sealed while quite full of vapour, the pressure being 753·4^{mm}. Its weight is now found to be 19·6025 grammes. Required the density of the ether vapour compared with that of hydrogen. *Ans.* 37.

131. Calculate the density of alcohol vapour as compared with air by Gay-Lussac's method from the following data:—

Weight of alcohol 0·1047 grm.; vol. of vapour at 110° C. = 82·55 c.c.; height of mercury above the level in the bath, 98 mm.; barometric height, 752·3 mm.; temperature of the room, 15° C. *Ans.* 1·6.

132. In a determination of the vapour density by Gay-Lussac's method, 0·1163 gramme of substance was employed. The volume observed was 50·79 cc, the height of the mercury above the level of that in the bath was 80·0^{mm}, the height of the oil column reduced to millimetres of mercury 16·9; the temperature 215° C., and the height of the barometer at the time of observation 755·5^{mm}. Required the specific gravity of the vapour as compared with that of hydrogen. *Ans.* 50·1.

133. Through a U-tube containing pumice saturated with sulphuric acid a cubic metre of air at 15° is passed, and the tube is found to weigh 3·95 grammes more. Required the hygrometric state of the air.

The pressure of aqueous vapour at 15° is 12·699; hence the weight of a cubic metre of aqueous vapour saturated at 15° is $\frac{1293 \times 12{\cdot}699 \times 5}{\left(1 + \frac{15}{273}\right) 760 \times 8} = 12{\cdot}79$ grammes,

and the hygrometric state is $\frac{3{\cdot}95}{12{\cdot}79} = 0{\cdot}309$.

134. The quantity of water given out by the lungs and skin may be taken at 30 ounces in 24 hours. How many cubic inches of air already half saturated at 10° will be fully saturated by the moisture exhaled from the above two sources by one man? Tension of aqueous vapour at 10° in inches = 0·361. Pressure of the atmosphere = 30 inches. *Ans.* 6121 cubic feet.

135. A mass of air extending over an area of 60,000 square metres to a height of 300 metres has the dew point at 15°, its temperature being 20°. How much rain will fall if the temperature sinks to 10°?

The weight of vapour condensed from one cubic metre under these circumstances will be 3·1435 grammes, and therefore from 18,000,000 cubic metres it will be 56,583 kilogrammes, which is equal to a rainfall 0·0943 mm. in depth.

136. When 3 cubic metres of air at 10° and 5 cubic metres at 18°, each saturated with aqueous vapour at those temperatures, are mixed together, is any water precipitated? And if so, how much?

The weight of water contained in the two masses under the given conditions are respectively 28·18 and 76·59 grammes; the weight required to saturate the mixture at the temperature of 15° is 102·39 grammes, and therefore 2·38 grammes will be precipitated.

137. The temperature of the air at sunset being 10°, what must be the lowest hygrometric state, in order that dew may be deposited, it being assumed that in consequence of nocturnal radiation the temperature of the ground is 7° below that of the air?

Ans. The hygrometric state must be at least 0·62 of total saturation.

138. It is stated as a practical rule that when the tension of aqueous vapour present in the atmosphere, as indicated by the dew point, is equal to x mm. of mercury, the weight of water present in a cubic metre of that air is x grammes. What is the error in this statement for a pressure of 10 mm. and the temperature 15° C.?

Ans. 0·172 gr.

139. A raindrop falls to the ground from a height of a mile; by how much would its temperature be raised, assuming that it imparts no heat to the air or to the ground? *Ans.* 3°·8 C.

140. A lead bullet falls through a height of 10 metres; by what amount will its temperature have been raised when it reaches the ground, if all the heat is expended in raising the temperature of the bullet? *Ans.* 0·7515° C.

141. From what height must a lead bullet fall in order that its temperature may be raised n degrees?—and what velocity will it have acquired? It is assumed that all the heat is expended in raising the temperature of the bullet; the specific heat of lead is taken at 0·0314, and Joule's equivalent in metres at 424.

Ans. 13·31 × n metres; $v = 28{\cdot}8\sqrt{n}$.

142. How much heat is disengaged if a bullet weighing 50 grammes and having a velocity of 50 metres strikes a target?

Ans. Sufficient to raise one gramme of water through 15° C.

143. How much heat is produced in the room of a manufactory in which 1·2 horse-power of the motor is consumed each second in overcoming the resistance of friction?

Ans. A quantity sufficient to raise 102561 pounds of water one degree Centigrade.

144. What is the ratio between the quantities of heat which are respectively produced, when a bullet weighing 50 grammes and having a velocity of 500 metres, and a cannon-ball weighing 40 kilogrammes with a velocity of 400 metres, strike a target? *Ans.* 1 : 512.

145. The specific heat of lead is 0·031, and its latent heat 5·37. What is the mechanical equivalent of the heat necessary to raise 5 pounds of lead from a temperature of 270° C. to its melting-point 335° C., and then to melt it?

Ans. 51326 foot-pounds.

146. Assuming that the temperature at which heat leaves a perfect engine is 16° C., at what temperature must it be taken in in order to obtain a theoretical useful effect of $\frac{1}{3}$?

Ans. 160·5° C.

147. Assuming that in a perfect engine heat is taken in at a temperature of 144°, and is given out at a temperature of 36°: what is the greatest theoretical useful effect?

Ans. 0·259.

VI. ON LIGHT

148. How many candles are required to produce at a distance of 2·5 metres, the same illuminating effect as one candle at a distance of 0·45 m.? *Ans.* 31.

149. Two sources of light whose intensities are as 1 : 2 are two metres apart. At what position is a space between them equally illuminated?
Ans. 0·828 metre from the less intense light.

150. A candle sends its rays vertically against a plane surface. When the candle is removed to thrice the distance and the surface makes an angle of 60° with the original position, what is the ratio of the illuminations in the two cases? *Ans.* $1 : \frac{1}{18}$.

151. An observer, whose eye is 6 feet above the ground, stands at a distance of 18 feet from the near edge of a still pond, and sees there the image of the top of a tree, the base of which is at a distance of 100 yards from the place at which the image is formed. Required the height of the tree. *Ans.* 100 feet.

152. What is the height of a tower which casts a shadow 56·4 m. in length when a vertical rod 0·95 m. in height produces a shadow 1·38 m. in length? *Ans.* 38·8.

153. A minute hole is made in the shutter of a dark room, and at a distance of 2·5 metres a screen is held. What is the size of the image of a tree which is 15·3 metres high and is at a distance of 40 metres? *Ans.* 0·95625 metre.

154. What is the length of the shadow of a tree 50 feet high when the sun is 30° above the horizon? What when it is 45°, and 60°? *Ans.* 86·6; 50, and 28·867 feet.

155. Under what visual angle does a line of 30 feet appear at a distance of 18 feet?
Ans. 79°·36.

156. The apparent diameter of the moon amounts to 31′ 3″. What is its real diameter if its distance from the earth is taken at 239000 geographical miles?
Ans. 2158 geographical miles.

157. For an ordinary eye an object is visible with a moderate illumination and pure air under a visual angle of 40 seconds. At what distance, therefore, can a black circle (6 inches in diameter) be seen on a white ground? *Ans.* 2578 feet.

158. At what distance from a circle with a diameter of *one* foot is the visual angle a second? *Ans.* 206265 feet.

159. At what distance would a circular disc 1 inch in diameter, of the same brightness as the sun's surface, illuminate a given object to the same extent as a vertical sun in the tropics, the light absorbed by the air being neglected?
Ans. Taking the sun's angular diameter at 30′, $x = 38$ inches.

160. What is the minimum deviation for a glass prism ($n = 1·53$), whose refracting angle is 60°? *Ans.* 39° 50′.

161. What is the minimum deviation for a prism of the same substance when the refracting angle is 45°? *Ans.* 63° 38′.

162. The refracting angle of a prism of silicate of lead has been found by measurement to be 21°·12′, and the minimum deviation to be 24°·46′. Required the refractive index of the substance. *Ans.* 2·122.

163. Construct the path of a ray which falls on an equiangular crown-glass prism at an angle of 30°; and find its deviation. *Ans.* 70°·45.

164. What are the angles of refraction upon a ray which passes from air into glass at an angle of 40°; from air into water at an angle of 65°; and from air into diamond at an angle of 80°? *Ans.* 25°·20; 44°·5; 23°·12.

165. The focal distance of a concave mirror is 8 metres. What is the distance of the image from the mirror when the object is at a distance of 12, 5, and 7 metres respectively? *Ans.* 24 ; − 13·3, and − 56.

166. An object at a distance of 10 feet produces a distinct image at a distance of 3 feet. What is the focal distance of the mirror? *Ans.* 2·3077 feet.

167. Required the focal distance of a crown-glass meniscus, the radius of curvature of the concave face being 45 mm., and that of the convex face 30 mm. ; the index of refraction being 1·5. *Ans.* f = 180 mm.

168. What is the principal focal distance of a double-convex lens of diamond, the radius of curvature of each of whose faces is 4 mm., and the refractive index of diamond 2·487? *Ans.* 1·34 mm.

169. A watch-glass with ground edges, the curvature of which was 4·5 cm., was filled with water, and a glass plate slid over it. The focus of the plano-convex lens thus formed was found to be 13·5 cm. Required the refractive index of the water. *Ans.* n = 1·33.

170. What is the focal distance of a double-convex lens when the distances of the image and object are respectively 5 and 36 centimetres? *Ans.* 4·4 centimetres.

171. The radii of curvature of a double-convex lens of crown glass are six and eight inches. What is the focal distance? *Ans.* 6·85 inches.

172. The focal distance of a double-convex lens is 4 inches ; the radius of curvature of one of its faces is 3 inches. What is that of the second? *Ans.* 6 inches.

173. The radius of curvature of a plano-convex lens is 12 inches. Required its focal distance. *Ans.* 24 inches.

174. If the focal distance of a double-convex lens is 1 centimetre, at what distance must a luminous object be placed so that its image is formed at 2 centimetres distance from the lens? *Ans.* 2 centimetres.

175. A candle at a distance of 120 centimetres from a lens forms an image on the other side of the lens at a distance of 200 cm. Required the nature of the lens and its focal distance. *Ans.* It is a convex lens, and its focal distance is 75 cm.

176. A plano-convex lens was found to produce at a distance of 62 cm. a sharp image of an infinitely distant object. In front of the same lens, at a distance of 84 cm., a millimetre scale was placed, and a sharp image was formed at a distance of 250 cm. It was thus found that 10 millimetres in the object corresponded to 29 in the image. From these observations determine the focal distance of the lens. *Ans.* The mean of the results is 62·4.

177. The image of a distant tree was sharply formed at a distance of 31 cm. from the centre of a concave mirror.

In another case the image of an object 18 mm. in length at a distance of 405 mm. from the mirror was formed at 1350 mm. from the mirror and had a length of 61 mm. In another experiment the distances of object and image and the size of the image were respectively 2200, 355, and 3 mm.

Deduce from these several data the focal distance of the mirror. *Ans.* 31·2 ; 30·5.

178. What must be the radii of curvature of the faces of a lens of best form made of glass (n=1·5) if its focal distance is to be 6 inches? *Ans.* 3·5 inches and 21 inches.

179. A diffraction grating, with lines 0·05 mm. apart, is held in front of a Bunsen's burner in which common salt is volatilised, and when viewed through a telescope it is found that the angular distances of the first, second, fourth, and sixth bright bands from the central one are respectively 0° 41′, 1° 21′, 2° 42′, and 4° 3′. Required the wave-length of sodium light.

The formula $\lambda = \frac{d \sin \phi}{n}$, where λ is the wave-length, ϕ the angular distance of any bright line of order n from the central one, gives as the mean of the 4 observations : *Ans.* 0·00059088 mm.

VII. MAGNETISM AND FRICTIONAL ELECTRICITY

180. A compass needle at the magnetic equator makes 15 oscillations in a minute; how many will it make in a place where the horizontal force of the earth's magnetism is $\frac{16}{25}$ as great? *Ans.* 12.

181. A compass needle makes 9 oscillations a minute under the influence of the earth's magnetism alone; how many will it make when re-magnetised so as to be half as strong again as before? *Ans.* 11.

182. A small magnetic needle makes 100 oscillations in 7 min. 42 secs. under the influence of the earth's force only; when the south pole of a long bar magnet A is placed 10 inches north of it, it makes 100 oscillations in 4 min. 3 secs.; and with the south pole of another magnet B in the same place, it makes 100 oscillations in 4 min. 48 secs. What are the relative strengths of the magnets A and B?

Ans. A = 1·404 B.

183. On a table where the earth's magnetism is counteracted, the north pole of a compass needle makes 20 oscillations in a minute under the attraction of a south pole 4 inches distant; how many will it make when the south pole is 3 inches distant?

Ans. 26·6.

184. If the oscillating magnet be re-magnetised so as to be twice as strong as before, how many oscillations in a minute will it make in the two positions respectively?

Ans. 28·28 and 50·27.

185. At one end of a light glass thread, carefully balanced so as to oscillate in a vertical plane, is a pith ball. Over this and in contact with it is a fixed pith ball of the same dimensions. Both balls being charged with the same electricity, it is found that to keep them 1·4 inch apart, a weight of ·9 mgr. must be placed at the free end of the glass thread. What weight must be placed there to keep the balls 1·05 inch apart?

Ans. 1·6 mgr.

186. A small insulated sphere A charged with the quantity of + electricity 2 is at a distance of 25 mm. from a second similar sphere B charged with the quantity 5; the latter is momentarily touched with an unelectrified sphere B, of the same size, and the distance altered to 20 mm. What is the ratio of the repulsive forces in the two cases? *Ans.* 32 : 25.

187. Two insulated spheres A and B, whose diameters are respectively as 7 : 10, have equal quantities of electricity imparted to them. In what ratio are their electrical densities? *Ans.* 100 : 49.

188. Two such spheres whose diameters are as 3 : 5 contain respectively the quantities of electricity 7 and 10. In what ratio are their densities? *Ans.* 35 : 18.

189. Three insulated conducting spheres, A, B, and C, whose radii are respectively 1, 2, and 3, are charged with electricity, so that their respective potentials are as 3 : 2 : 1, and are then connected by wires, whose capacity may be neglected. What is the total quantity and potential of the system? *Ans.* Q = 10; V = 1·66.

190. Supposing each of the spheres discharged separately, what would be the total work they would produce, as compared with that produced by the discharge of the whole system? *Ans.* 30 : 25.

VIII. VOLTAIC ELECTRICITY

191. A galvanometer offering no appreciable resistance is connected by short thick wires with the poles of a cell, and deflects 20°. By how much will it be deflected if two exactly similar cells are connected with the first side by side? *Ans.* 47°·30′.

192. By how much if the three cells are connected in series? *Ans.* 20°.

193. Two cells each of 1 ohm resistance are connected in series by a wire the resistance of which is also 1 ohm. If each of these when connected singly by short thick wires to a galvanometer of no appreciable resistance deflects it 25°, how much will the combination deflect it, the connections being made by short thick wires?

Ans. 17°·16′.

A Siemens unit is equal to the resistance of a column of pure mercury a metre in length and a square mm. in cross section. It is equal to 0·9536 of an ohm or BA unit; or a BA unit equals 1·0485 Siemens unit, or equals a column of mercury 1·0485 metre in length and a square mm. in cross section.

194. A single thermo-electric couple deflects a galvanometer of 100 ohms resistance 0° 30′; how much will a series of 30 such couples deflect it, the connections being made by short thick wires? *Ans.* 14°·40′.

195. Suppose a sine galvanometer had been used in the last question, and the first reading had been 0°·30′, what would the second be? *Ans.* 15°·10′.

196. The internal resistance of a cell is half an ohm; when a tangent galvanometer of 1 ohm resistance is connected with it by short thick wires it is deflected 15°; by how much will it be deflected if for one of the thick wires a thin wire of $1\frac{1}{2}$ ohm resistance is substituted? *Ans.* 7°·37′.

197. What will be the deflection if each of the wires is replaced by a thin wire of $1\frac{1}{2}$ ohm resistance? *Ans.* 5° 6′.

198. A cell of one-third of an ohm resistance deflects a tangent galvanometer of unknown resistance 45°, the connection being made by two short thick wires. If a wire of 3 ohms resistance be substituted for one of the short wires the deflection is 30°. What is the resistance of the galvanometer? *Ans.* 3·75 ohms.

199. What would be the deflection if for the cell in the last question three exactly similar cells in series were substituted (*a*) when the galvanometer alone is in circuit; (*b*) when both the galvanometer and the thin wire are in circuit?

Ans. a 68°·48. *b* = 57°·41′.

200. A galvanometer offering no sensible resistance is deflected 50° by a cell connected with it by short thick wires. If a resistance of 3 ohms be put in the circuit, the deflection is 20°. Find the internal resistance of the cell. *Ans.* 1·32.

201. Suppose the results in the last question were produced by two exactly similar cells in series, find the internal resistance of each. *Ans.* 0·659.

202. Suppose they were produced by two exactly similar cells placed side by side, find the internal resistance of each. *Ans.* 2·639.

203. If the resistance of 130 yards of a particular copper wire $\frac{1}{16}$ of an inch in diameter is an ohm, express in that unit the resistance of 8242 yards of copper wire $\frac{1}{12}$ of an inch in diameter. *Ans.* 35·66.

204. One form of fuse for firing mines by voltaic electricity consists of a platinum wire $\frac{3}{8}$ of an inch long, of which a yard weighs 2 grains. Required its resistance in terms of a Siemens unit. Specific gravity of platinum 22, and its conducting power 11·25 that of mercury. *Ans.* 0·131.

205. Express in ohms the resistance of one mile of copper wire $\frac{1}{4}$ of an inch in diameter of the same quality as that referred to in 203. *Ans.* 0·8461.

206. The whole resistance of a copper wire going round the earth (24800 miles) is 221650 ohms. Find its diameter in inches. *Ans.* 0·0738.

207. What length of platinum wire 0·05 of an inch in diameter must be taken to get a resistance equal to 1 ohm, the specific resistance of platinum being taken at 5·55 that of copper? *Ans.* 14·9 metres.

208. 660 yards of iron wire 0·0625 of an inch in diameter have the same electrical resistance as a mile of copper wire 0·0416 of an inch in diameter. Find the specific resistance of iron, that of copper being unity. *Ans.* 6·02.

209. Ten exactly similar cells in series produce a deflection of 45° in a tangent galvanometer, the external resistance of the circuit being 10 ohms. If arranged so that there is a series of 5 cells, of two abreast, a deflection of 33°·42′ is produced; find the internal resistance of the cell. *Ans.* ½ ohm.

210. On the bobbins of the new Post Office pattern of a single needle instrument are coiled 225 yards of No. 35 copper wire 0·0087 inch in diameter, the resistance of which is about 92 ohms. Required the conducting power of the wire in terms of mercury. *Ans.* 55·6.

211. Ten exactly similar cells each of $\frac{2}{3}$ of an ohm resistance give, when arranged in 2 series of five each, a deflection of 23°·57′; but when arranged in 5 series of 2 each a deflection of 33°·42′. Required the external resistance of the circuit including that of the galvanometer. *Ans.* $\frac{5}{36}$.

212. A cell in a certain circuit deflects a tangent galvanometer 18° 26′; two such cells abreast in the same circuit deflect it 23° 57′; two such cells in series in the same circuit diminished by 1 ohm deflect it 29°·2′. Find the internal resistance of one cell and that of the circuit. *Ans.* $R = r = 1·66$.

213. What is the best arrangement of 6 cells, each of $\frac{2}{3}$ of an ohm resistance, against an external resistance of 2 ohms?

Ans. Indifferent whether in 6 cells of 1 each or in 3 cells of 2 each.

214. What is the best arrangement of 20 cells, each of 0·8 ohm resistance, against an external resistance of 4 ohms? *Ans.* 10 cells of 2 each.

215. In a circuit containing a galvanometer and a voltameter, the current which deflects the galvanometer 45° produces 10·32 cubic centimetres of mixed gas in a minute. The electrodes are put farther apart, and the deflection is now 20°; find how much gas is now produced per minute. *Ans.* 3·757 cc.

216. 100 inches of copper wire weighing 100 grains has a resistance of 0·1516 ohm. Required the resistance of 50 inches weighing 200 grains. *Ans.* 0·0379.

217. A knot of nearly pure copper wire weighing one pound has a resistance of 1200 ohms at 15°·5 C.; what is the resistance at the same temperature of a knot of the same quality of wire weighing 125 pounds? *Ans.* 9·6 ohms.

218. Find the length in yards of a wire of the same diameter and quality as the knot pound in 217, having a resistance of 2 ohms. *Ans.* 3·38 yards.

219. Find the length in yards of a wire of the same quality and total resistance as the knot pound in 217, but of three times the diameter. *Ans.* 18261 yards.

220. The specific gravity of platinum is 2½ times that of copper; its resistance $5\frac{5}{9}$ as great. What length of platinum wire weighing 100 grains has the same resistance as 100 inches of copper wire also weighing 100 grains? *Ans.* 27.

221. A cell with a resistance of an ohm is connected by very short thick wires with the binding screws of a tangent galvanometer, the resistance of which is half an ohm, and the deflection is 45°; if the screws of the galvanometer be also connected at the same time by a wire of 1 ohm resistance, find the deflection. *Ans.* 36° 52′.

222. The resistance of a galvanometer is half an ohm, and the deflection when

the current of a cell is passed through it is 30°. When a wire of 2 ohms resistance is introduced into the circuit the deflection is 15° ; find the internal resistance of the cell.

Ans. 1·23.

223. When the current of a cell, the resistance of which is $\frac{2}{3}$ of an ohm, is passed through a galvanometer connected with it by very short thick wires, the deflection is 45° ; when the binding screws are also connected by a shunt having a resistance of 1 the deflection is 33°·42′. Find the resistance of the galvanometer. *Ans.* 2.

224. A cell whose internal resistance is 2 ohms has its copper pole connected with the binding screw A of a galvanometer formed of a thick band of copper. From the other screw B a wire of 20 ohms resistance passes to the zinc pole, and the deflection read off is 7°·8′. Find the deflection when B is at the same time connected with the zinc pole by a second wire of 30 ohms resistance. *Ans.* 11° 8′.

225. What would be the deflection in 224 if the second wire instead of passing from B to the zinc pole passed directly from the zinc pole to the copper pole?

Ans. 6°·47′.

226. A Leclanché cell deflects a galvanometer 30° when 200 ohms resistance are introduced into the circuit, 15° when 570 ohms are introduced; a standard Daniell cell deflects it 30° when 100 ohms are in circuit, and 15° when 250 additional ohms are introduced. Required the electromotive force of the Leclanché in terms of that of the Daniell. *Ans.* 1·48.

227. A Bunsen and a Daniell cell are placed in the same circuit in the first case so that the carbon of the first is united to the zinc of the Daniell ; and in the second case so that their currents oppose each other. The currents are respectively 30°·2′, and in the second 10°·6′. Required the electromotive force of the Bunsen in terms of the Daniell. *Ans.* 1·89.

228. A telegraph line constructed of copper wire, a kilometre of which weighs 30·5 kilogrammes, is to be replaced by iron wire a kilometre of which weighs 135·6 kilogrammes. In what ratio does the resistance alter? *Ans.* The resistance of the iron wire will be 1·18 times that of the copper wire for which it is substituted.

229. A telegraph line which has previously consisted of copper wire weighing 30·5 kilogrammes to the kilometre is to be replaced by an iron wire of the same length which shall offer the same resistance. What must be the section of the latter, and what its weight per kilometre?

Ans. The section of the copper wire is 3·4357 sq. mm., that of the iron by which it is replaced is 20·6 sq. mm., and its weight per kilometre is 160·4 kilogrammes.

230. When the poles of a voltaic cell are connected by a conductor of resistance 1, a current of strength 1·32 is produced ; and when they are connected by a conductor of resistance 5 the strength of the current is 0·33. Find from these data the internal resistance and the electromotive force of the cell. *Ans.* $R=\frac{1}{3}$ $E=1·76$.

231. A silver wire is joined end to end to an iron wire of the same length, but of double the diameter, and six times the specific resistance ; the other ends are joined to the battery, the current of which is transmitted for five minutes, during which time a total quantity of 45 units of heat is generated in the two wires. How is it shared between them? *Ans.* $Ag : Fe = 18 : 27$.

232. A window casement of iron faces the south, and the hinges which support it are on the east. What electrical phenomena are observed (*a*) when the window is opened, and (*b*) when it is closed?

233. Two points 135° apart in a uniform circular conducting ring are connected with the opposite poles of a voltaic battery. Compare the strength of the current in the two portions of the ring.

234. A mile of cable with a resistance of 3·59 ohms was put in water, with the end *B* insulated ; its core having been pricked with a needle the resistance tested from the end *A* was found to be 2·81 ohms. *A* being insulated, a test from *B* showed the resistance to be 2·76. Required the distance from *B* to the injured spot.

Ans. 867 yards.

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